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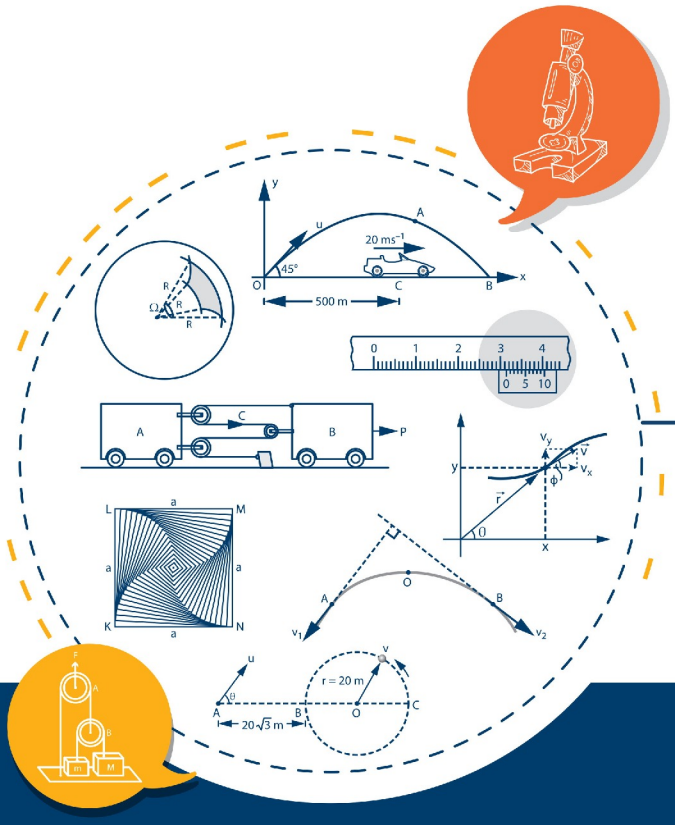

JEE ADVANCED PHYSICS

Mechanics – I

Rahul Sardana

Pearson

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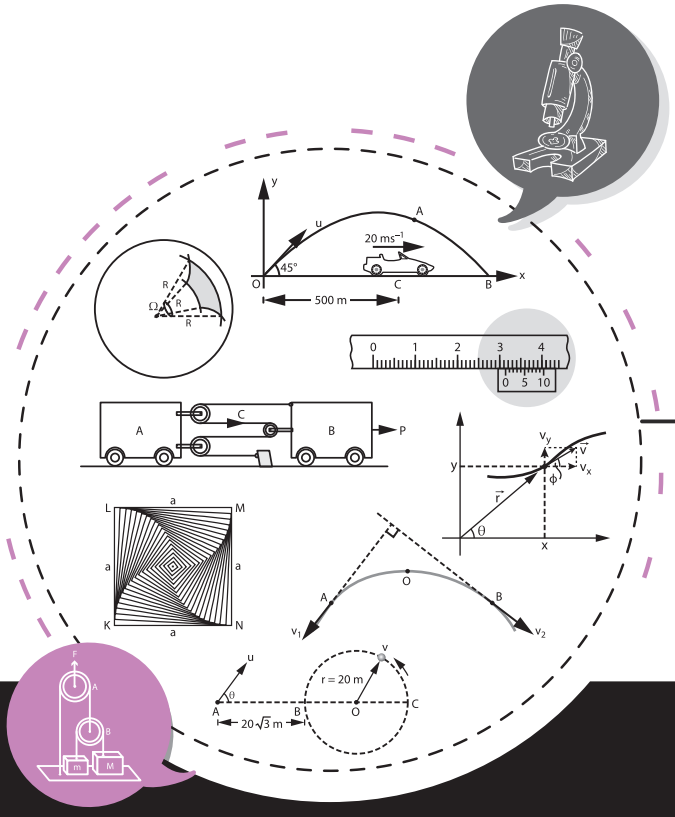

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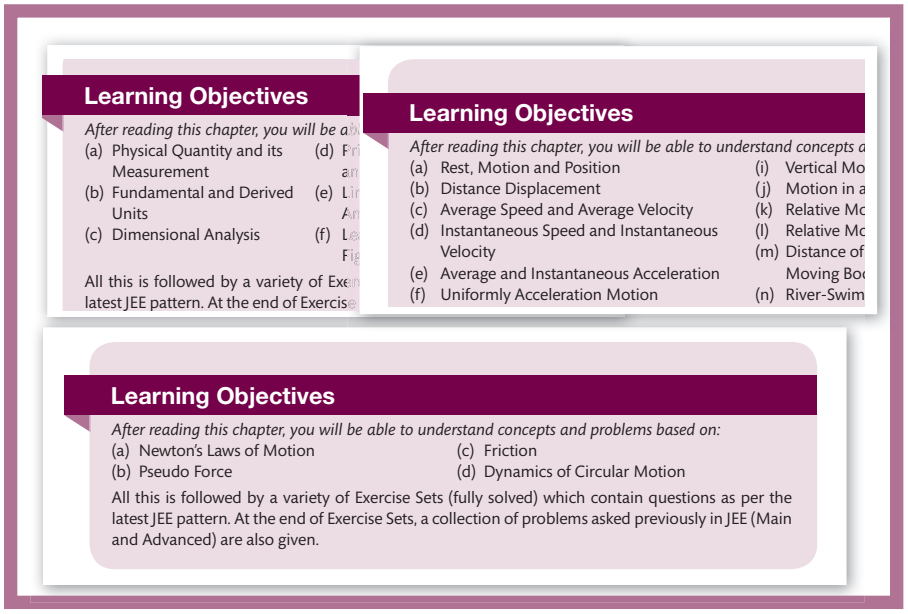
CHAPTER INSIGHT

Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.



Learning Objectives

After reading this chapter, you will be a

(a) Physical Quantity and its Measurement
(b) Fundamental and Derived Units
(c) Dimensional Analysis
(d) Fr
(e) Li
(f) Le

All this is followed by a variety of Exe
latest JEE pattern. At the end of Exercis

Learning Objectives

After reading this chapter, you will be able to understand concepts a

(a) Rest, Motion and Position
(b) Distance Displacement
(c) Average Speed and Average Velocity
(d) Instantaneous Speed and Instantaneous Velocity
(e) Average and Instantaneous Acceleration
(f) Uniformly Acceleration Motion
(i) Vertical Mo
(j) Motion in a
(k) Relative Mo
(l) Relative Mo
(m) Distance of Moving Bo
(n) River-Swim

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Newton's Laws of Motion
(b) Pseudo Force
(c) Friction
(d) Dynamics of Circular Motion

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

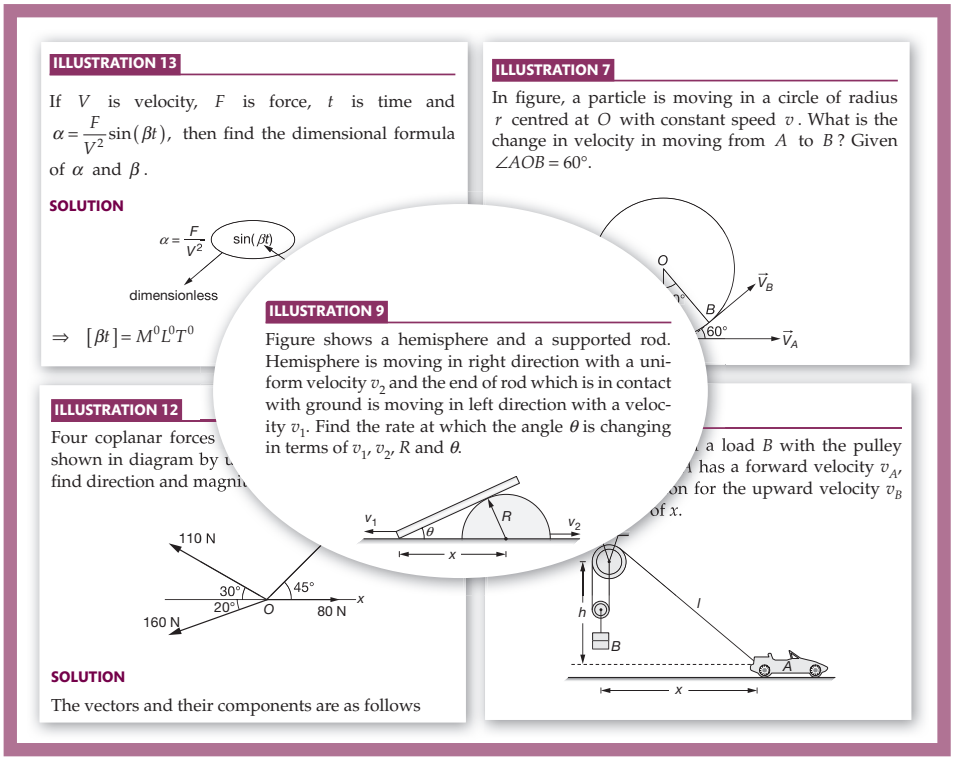


ILLUSTRATION 13

If V is velocity, F is force, t is time and $\alpha = \frac{F}{V^2}\sin(\beta t)$, then find the dimensional formula of α and β.

SOLUTION

$\alpha = \frac{F}{V^2}$, $\sin(\beta t)$ — dimensionless

$\Rightarrow \quad [\beta t] = M^0L^0T^0$

ILLUSTRATION 7

In figure, a particle is moving in a circle of radius r centred at O with constant speed v. What is the change in velocity in moving from A to B? Given $\angle AOB = 60°$.

ILLUSTRATION 9

Figure shows a hemisphere and a supported rod. Hemisphere is moving in right direction with a uniform velocity v_2 and the end of rod which is in contact with ground is moving in left direction with a velocity v_1. Find the rate at which the angle θ is changing in terms of v_1, v_2, R and θ.

ILLUSTRATION 12

Four coplanar forces
shown in diagram by
find direction and magni

SOLUTION

The vectors and their components are as follows

a load B with the pulley
has a forward velocity v_A,
on for the upward velocity v_B
of x.

Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF → THEN → ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

Test Your Concepts-I

Based on Principle of Homogeneity and Verification

(Solutions on page H.5)

1. If a composite physical quantity in terms of moment of inertia I, force F, velocity v, work W and length L is defined as, $Q=\left(\frac{IFv^2}{WL^3}\right)$. Find the dimensions of Q.
2. Can two physical quantities have same dimensions? Explain with example.
5. Check the correctness of the equation: $y=a\sin(\omega t+\phi)$, where y = displacement, $V=\frac{\pi}{8}\frac{Pr^4}{\eta l}$ Check the dimensional consistency of this equation.

Test Your Concepts-I

Based on Addition, S

1. Can three vectors not in one plane give a ze resultant? Can four vectors do?
2. The x and y components of vector $\vec{A}$ are 4 m ar 6 m respectively. The x and y components of vect $\vec{A}+\vec{B}$ are 10 m and 9 m respectively. Calculate f the vector $\vec{B}$ the following:
   (a) its x and y components,
   (b) its length and
   (c) the angle its makes with x-axis.

Test Your Concepts-II

Based on Ho

1. A body is thrown horizontally from the top of tower and strikes the ground after three seconds a an angle of 45° with the horizontal. Find the heigh of the tower and the speed with which the bod was projected. Take $g=9.8\ ms^{-2}$.
2. With what minimum horizontal velocity u ca a boy throw a ball at A and have it just clear th obstruction at B?

Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

Problem Solving Technique(s)

(a) Δy is always positive i.e. $\Delta y>0$.
(b) Δy has units same as that of y.
(c) A quantity in terms of absolute error is expresse as
$y=(y_t\pm\Delta y)$ units
(d) If least count is not given a… measurement … given, then error in the m… … + … in last digit.

Problem Solving Technique(s)

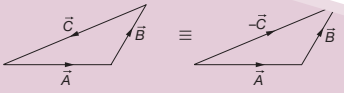
So, we observe that
(a) when an interval is divided into n equal time parts, then the "average speed is the simple average of the speeds in the respective intervals".
(b) when an interval is divided into n equal length parts, then the "reciprocal of the average speed is equal to the average of the reciprocals of the speeds in respective intervals".
(c) **Time Average Speed:** When particle moves with different uniform speed v_1, v_2, v_3 ... etc. in different …tervals t_1, t_2, t_3, ... etc. respectively, its aver… …he total time of journey is given

Problem Solving Technique(s)

(a) For motion with uniform angular acceleration, we can use the following equations
$\omega=\omega_0+\alpha t$
$\theta=\omega_0 t+\frac{1}{2}\alpha t^2$

Problem Solving Tecl…

(a) If the vectors form a clo… the sides in the same order then …



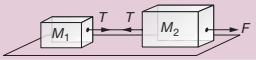
Addition of Vectors
(b) To find the sum of any number of vectors we must represent the vectors by the directed line

…nique(s)

…ted with strings, we can have …o analyse constraint equations.
(a) Branch… …ysis.
(b) Law of Conservation of Length of strings connecting the pulleys.
(c) If one end of a string passing over a moving pulley is fixed, then the acceleration of the other

Conceptual Note(s)

(a) Tension in any branch of string must be shown as a pair.
(The pair is shown with two arrows facing each other).
When two blocks (M_1 and M_2 say) connected by a string are pulled, then tension on M_1 acts towards M_2 and that on M_2 acts towards M_1 as shown in the figure.



(b) If a
strin
hor

Conceptual Note(s)

(a) In the arrangement shown, for translational equilibrium to exist, we have

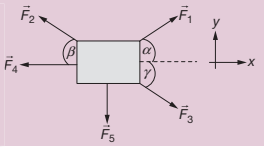
$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

$\sum F_x = 0$ gives

$$F_1\cos\alpha + F_3\cos\gamma - F_4 - F_2\cos\beta = 0 \quad ...(1)$$

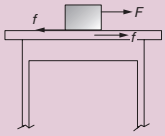
$\sum F_y = 0$ gives

$$F_1\sin\alpha + F_2\sin\beta - F_5 - F_3\sin\gamma = 0 \quad ...(2)$$



Conceptual Note(s)

Note that in figure the block, of course exerts an equal and opposite frictional force on the table (as shown in figure), tending to drag it in the direction of the horizontal force we exert. This frictional force is due to the bonding of the molecules of the block and the table at the places where the surfaces are in very close contact. If we focus on the table only, we see that friction tends to move the table in the same direction in which force F tends to move the block. This observation leads us to two very important conclusions.



Conceptual Notes

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

Chapter End Solved Problems

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.

SOLVED PROBLEMS

PROBLEM 1

P.A.M. Dirac, a great Physicist of the 20th century found that from the following basic constants a number having dimensional formula same as that of time can be constructed i.e.,

(a) the charge on the electron e
(b) permittivity of free space ε_o
(c) mass of the electron m_e
(d) mass of the proton m_P
(e) speed of light c
(f) Universal Gravitational constant G.

$\Rightarrow \quad q = -2$

So, $p = 4$, $q = -2$, $s = -1$, $r = -3$

Hence, we get

$$t = e^4\varepsilon_0^{-2}m_e^{-2}m_p^{-1}c^{-3}G^{-1}$$

PROBLEM 1

The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces?

SOLUTION

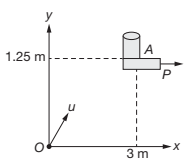
Let F_1 and F_2 be the two forces where $F_1 < F_2$

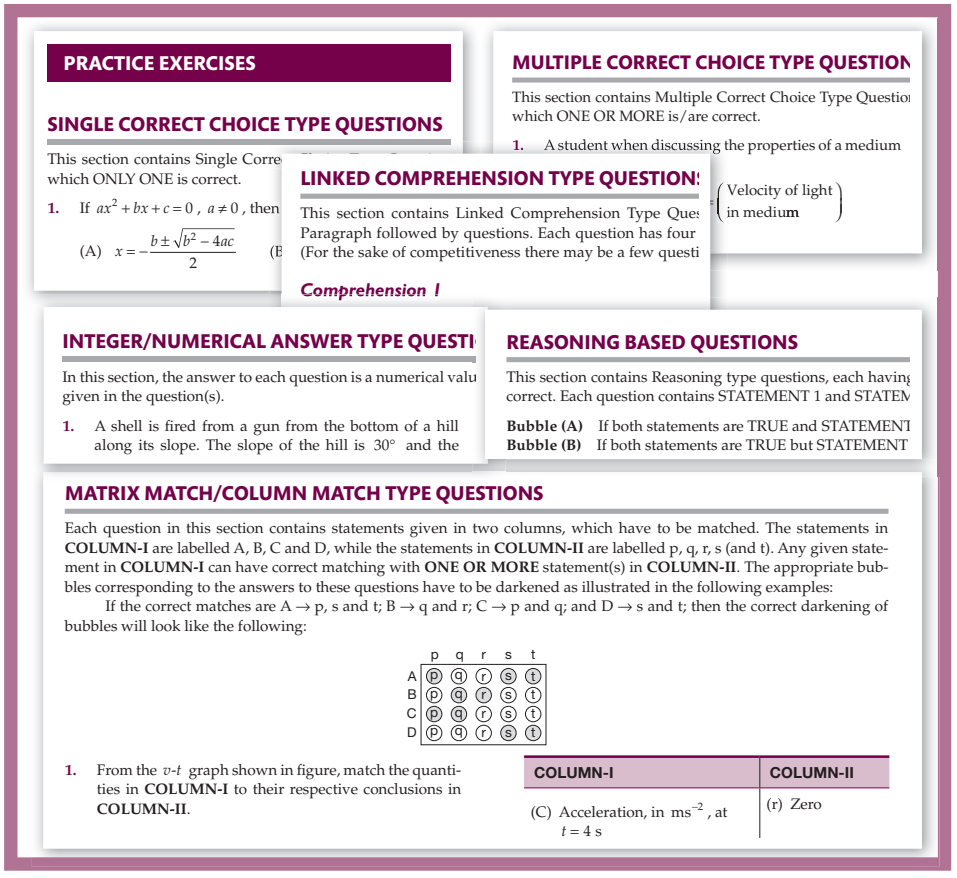
Here, $F_1 + F_2 = 18$ ...(1)

The statement to the question can be diagrammatically drawn as shown in Figure.

PROBLEM 2

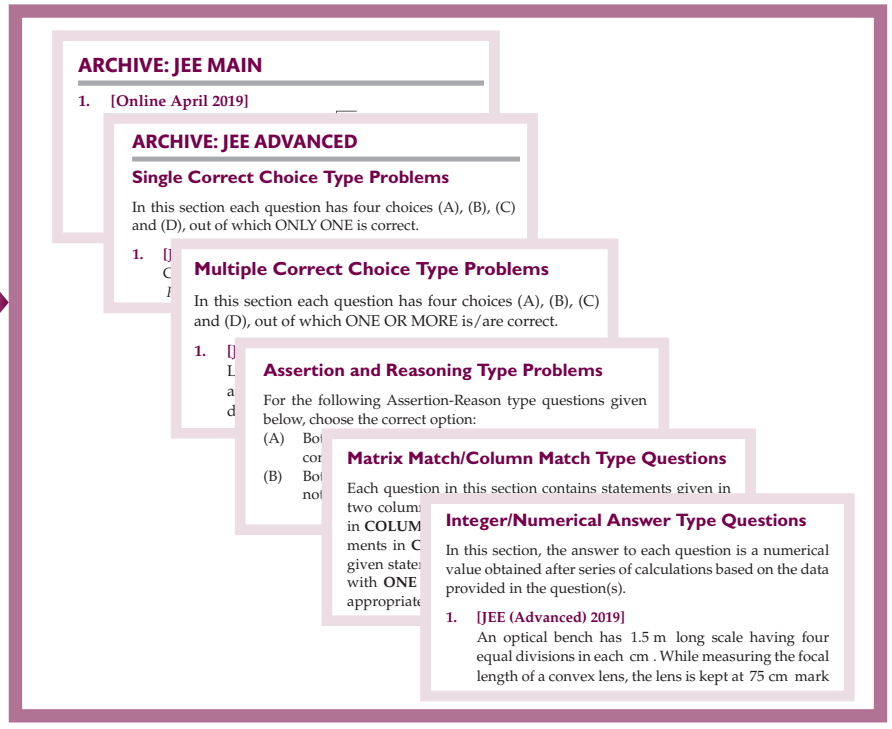
An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$, the plank starts moving along the $+x$ direction with an acceleration 1.5 ms^{-2}. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the x-y plane. Find u and the time after which the stone hits the object. Take $g = 10$ ms^{-2}.




PRACTICE EXERCISES
SINGLE CORRECT CHOICE TYPE QUESTIONS
This section contains Single Corre
which ONLY ONE is correct.
1. If $ax^2+bx+c=0$, $a\neq 0$, then
(A) $x=-\frac{b\pm\sqrt{b^2-4ac}}{2}$
MULTIPLE CORRECT CHOICE TYPE QUESTION
This section contains Multiple Correct Choice Type Questio
which ONE OR MORE is/are correct.
1. A student when discussing the properties of a medium
(Velocity of light in medium)
LINKED COMPREHENSION TYPE QUESTION
This section contains Linked Comprehension Type Que
Paragraph followed by questions. Each question has four
(For the sake of competitiveness there may be a few questi
Comprehension I
INTEGER/NUMERICAL ANSWER TYPE QUESTI
In this section, the answer to each question is a numerical valu
given in the question(s).
1. A shell is fired from a gun from the bottom of a hill
along its slope. The slope of the hill is 30° and the
REASONING BASED QUESTIONS
This section contains Reasoning type questions, each having
correct. Each question contains STATEMENT 1 and STATEM
Bubble (A) If both statements are TRUE and STATEMENT
Bubble (B) If both statements are TRUE but STATEMENT
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS
Each question in this section contains statements given in two columns, which have to be matched. The statements in COLUMN-I are labelled A, B, C and D, while the statements in COLUMN-II are labelled p, q, r, s (and t). Any given statement in COLUMN-I can have correct matching with ONE OR MORE statement(s) in COLUMN-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:
p q r s t
A (p) (q) (r) (s) (t)
B (p) (q) (r) (s) (t)
C (p) (q) (r) (s) (t)
D (p) (q) (r) (s) (t)
1. From the v-t graph shown in figure, match the quantities in COLUMN-I to their respective conclusions in COLUMN-II.
COLUMN-I
(C) Acceleration, in ms^{-2}, at t = 4 s
COLUMN-II
(r) Zero

Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice


ARCHIVE: JEE MAIN
1. [Online April 2019]
ARCHIVE: JEE ADVANCED
Single Correct Choice Type Problems
In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.
Multiple Correct Choice Type Problems
In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.
Assertion and Reasoning Type Problems
For the following Assertion-Reason type questions given below, choose the correct option:
Matrix Match/Column Match Type Questions
Each question in this section contains statements given in
Integer/Numerical Answer Type Questions
In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).
1. [JEE (Advanced) 2019]
An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark

Archive JEE Main and Advaned

From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.

CHAPTER 6: NEWTON'S LAWS OF MOTION

Test Your Concepts-I (Based on Impulse Momentum)

1. Since $\Delta\vec{p} = \vec{F}\Delta t$

Now, $\Delta p_y = m(v_y - u_y) = mv[\cos(30°) - \cos(30°)]$

$\Rightarrow \quad \Delta p_y = 0$

Similarly, $\Delta p_x = m(v_x - u_x)$

$\Rightarrow \quad \Delta p_x = mv[-\sin(30°) - \sin(30°)]$

(c) From the graph we observe that the peak force exerted on the ball is 16000 N = 16 kN.

4. $F = \dfrac{\Delta p_{\text{water}}}{\Delta t} = \dfrac{m(v-u)}{\Delta t}$

$\Rightarrow \quad F = (0.6)(25-0) \qquad \left\{\because \dfrac{m}{\Delta t} = 0.6\right\}$

$\Rightarrow \quad F = 15 \text{ N}$

Now due to Newton's Third Law, the water exerts a force of equal magnitude back on the hose, hence the gardener must apply a 15 N force in the direction of

Single Correct Choice Type Questions

1. Maximum possible error

$$\left|\frac{\Delta H}{H}\times 100\right|_{\max} = 2\frac{\Delta \text{I}}{\text{I}}\times 100 + \frac{\Delta R}{R}\times 100 + \frac{\Delta t}{t}\times 100$$

Hence, the correct answer is (D).

2. The MKS unit of $\eta = \dfrac{\text{kg}}{\text{ms}}$ is $kgm^{-1}s^{-1}$

The CGS of $\eta = \dfrac{\text{g}}{\text{cms}}$ is $gcm^{-1}s^{-1}$

$$\frac{\eta_{\text{MKS}}}{\eta_{\text{CGS}}} = \frac{\text{kg}}{\text{ms}} \times \frac{\text{cms}}{\text{g}}$$

$$\frac{\eta_{\text{MKS}}}{\eta_{\text{CGS}}} = \frac{10^3 \text{ g} \times \text{cm} \times \text{s}}{100 \text{ cm} \times \text{s} \times \text{g}} = 10$$

Hence, the correct answer is (A).

3. $i = K\tan\theta$ $\{\because H = \text{I}^2Rt\}$

Integer/Numerical Answer Type Problems

1. Linear impulse, $J = mv_0$

$\Rightarrow \quad v_0 = \dfrac{J}{m} = 2.5 \text{ ms}^{-1}$

$\Rightarrow \quad v = v_0 e^{-t/\tau}$

$\Rightarrow \quad \dfrac{dx}{dt} = v_0 e^{-t/\tau}$

$\Rightarrow \quad \displaystyle\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau}\, dt$

Multiple Correct Choice Type Questions

1. (A) is correct when the bicycle is being pedalled because during pedalling, the force is acting in the backward direction on the rear wheel and thus frictional force acts in the forward direction.

(D) is correct when bicycle is not pedalled.

Hence, (A) and (D) are correct.

2. All accelerated frames are Non-inertial frames. Since earth rotates about its own axis and revolves around the sun, so it is a Non-Inertial frame (STRICTLY SPEAKING), whereas for a good number of cases we assume the earth to be an Inertial frame of reference as the value of acceleration is extremely small.

Hence, (B) and (D) are correct.

3. Let F be the force exerted on the mirror by the photon. Since a photon will be reflected from the mirror with the same value of momentum so, change in momentum $\Delta p = p - (-p) = 2p = \dfrac{2E}{\quad}$.

Hints and Explanations

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions. I hope this book will nourish you with the concepts involved so that you get a great rank at JEE.

PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

Rahul Sardana

ABOUT THE AUTHOR

Rahul Sardana, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

CHAPTER

1 Mathematical Physics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Quadratic Equations
(b) Logarithmic Functions
(c) Linear Equations
(d) Determinants
(e) Arithmetic Progression (AP)
(f) Geometric Progression (GP)
(g) Coordinate Geometry
(h) Trigonometry
(i) Factorial
(j) Series Expansions
(k) Functions
(l) Limit of a Function
(m) Differentiation
(n) Integration

All this is followed by an Exercise Set which contains questions for your practice. Please be advised that you must first thoroughly study this chapter for your command on the Mathematical tools used in Physics hereafter.

GENERAL MATHEMATICS: A REVIEW

This part in mathematics is introduced here to give you a fundamental review of operations and methods. To have an extra understanding in Physics you should be totally familiar with basic algebraic techniques, analytic geometry, and trigonometry. Differential and integral calculus are discussed in detail and are intended for those students who have difficulties in applying calculus concepts to physical situations.

Table 1.1 Mathematical symbols used in the text and their meaning

Symbol	Meaning
$=$	is equal to
$\neq$	is not equal to
$\propto$	is proportional to
$>$	is greater than
$<$	is less than
$\gg (\ll)$	much greater (less) than
$\approx$	is approximately equal to
$\sim$	positive difference between two numbers
Δx	the change in x
$\sum_{i=1}^{N} x_i$	the sum of all quantities x_i from $i=1$ to $i=N$
$\lvert x \rvert$	the magnitude of x (always a positive quantity)
$\Delta x \to 0$	Δx approaches zero
$\frac{dx}{dt}$	the derivative of x with respect to t
$\frac{\partial x}{\partial t}$	the partial derivative of x with respect to t
$\int$	Integral

(*Continued*)

ALGEBRA

When algebraic operations are performed, the laws of arithmetic apply. Symbols such as x, y and z are usually used to represent quantities that are not specified, and symbols such as a, b and c are used to represent numbers.
You should be familiar with the following operations:

Fractions

(i) $a\left(\frac{b}{c}\right)=\frac{ab}{c}$ **(ii)** $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=\frac{ac}{bd}$

(iii) $\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{ad}{bc}$ **(iv)** $\frac{a}{b}\pm\frac{c}{d}=\frac{ad\pm cb}{bd}$

Factoring and Combinations

(i) $ax+bx=x(a+b)$

(ii) $x^2-y^2=(x+y)(x-y)$

(iii) $x^2-2x-15=(x+3)(x-5)$ {Quadratic Equation}

Roots of a Quadratic Equation

If $ax^2+bx+c=0\,(a\neq 0)$, has two roots given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

If α and β are two roots of equation, then

$$\alpha+\beta=-\frac{b}{a} \text{ and } \alpha\beta=\frac{c}{a}$$

ILLUSTRATION 1

Solve the equation $10x^2-27x+5=0$.

SOLUTION

By comparing the given equation with standard equation $a=10$, $b=-27$, and $c=5$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow\quad x=\frac{-(-27)\pm\sqrt{(-27)^2-4\times10\times5}}{2\times10}=\frac{27\pm23}{20}$$

$$\Rightarrow\quad x_1=\frac{27+23}{20}=\frac{5}{2} \text{ and } x_2=\frac{27-23}{20}=\frac{1}{5}$$

Roots of the equation are $\frac{5}{2}$ and $\frac{1}{5}$.

MULTIPLYING POWERS OF A GIVEN QUANTITY

(i) $x^n x^m=x^{n+m}$ **(ii)** $\frac{x^n}{x^m}=x^{n-m}$

(iii) $(x^n)^m=x^{nm}$

Logarithmic Functions

(i) ln = logarithm to base e, also called as natural log.

(ii) log = logarithm to base 10

(iii) $\ln(e)=1$

(iv) $\ln(e^x)=x$

(v) $\ln(xy)=\ln x+\ln y$

(vi) $\ln\left(\frac{x}{y}\right)=\ln x-\ln y$

(vii) $\ln\left(\frac{1}{x}\right)=-\ln x$

(viii) $\ln\left(x^n\right)=n(\ln x)$

(ix) $\ln(a)=2.3026\ \log(a)$

(x) $\ln a=\log_e a=2.303\log_{10} a$

(xi) $\log_a x=\frac{\log_e x}{\log_e a}$

(xii) If $\log_e x=\alpha$, then $x=e^{\alpha}$

Simultaneous Linear Equations

In order to solve two simultaneous equations involving two unknowns, x and y, we solve one of the equations for x in terms of y and substitute this expression into the other equation.

ILLUSTRATION 2

Solve the following given equations

$$5x+y=-8 \qquad \ldots(1)$$

$$2x-2y=4 \qquad \ldots(2)$$

SOLUTION

From (2), $x = y + 2$, substituting this in (1) gives

$$5(y+2)+y=-8$$

$$\Rightarrow \quad 6y=-18$$

$$\Rightarrow \quad y=-3$$

$$\Rightarrow \quad x=y+2=-1$$

Alternate Solution:
Multiplying (1) by 2 and add the result to (2)

$$\begin{array}{r} 10x+2y=-16 \\ 2x-2y=4 \\ \hline 12x=-12 \end{array}$$

$$\Rightarrow \quad x=-1$$

$$\Rightarrow \quad y=x-2=-3$$

Determinants

2nd order (four elements) i.e., a 2×2 determinant is evaluated as

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

3rd order (nine elements) i.e., a 3×3 determinant is evaluated as

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

POWERS OF TEN

You should be familiar with the usage of powers of ten. It is a compact form of writing very large or very small numbers. For example, instead of 10000, we write 10^4, where the exponent represents the number of zeros; that is $10^4 = 10\times10\times10\times10 = 10000$. Likewise, a small number like 0.0001 can be expressed as 10^{-4}, where the negative exponent indicates that we are dealing with a number less than one. Some other examples of the use of powers of ten are

$1000 = 10^3$ $\qquad$ $0.003 = 3\times10^{-3}$

$85\,000 = 8.5\times10^4$ $\qquad$ $0.00085 = 8.5\times10^{-4}$

$3\,200\,000 = 3.2\times10^6$ $\qquad$ $0.00002 = 2\times10^{-5}$

If numbers written as powers of ten are multiplied, we simply add the exponents, maintaining their signs. For example,

$$(3\times10^3)\times(5\times10^4) = 15\times10^7 = 1.5\times10^8$$

$$(2\times10^5)\times(4\times10^{-2}) = 8\times10^3$$

$$(5.6\times10^4)\times(4.3\times10^8) = 24\times10^{12}$$

When numbers written as powers of ten are divided, we can bring the power of ten from the denominator to the numerator by changing its sign. For example,

$$\frac{8\times10^5}{2\times10^2} = 4\times10^5\times10^{-2} = 4\times10^3$$

$$\frac{12\times10^{-4}}{4\times10^{-9}} = 3\times10^{-4}\times10^9 = 3\times10^5$$

In general,

$$10^n10^m = 10^{n+m}$$

$$\frac{10^n}{10^m} = 10^{n-m}$$

$$(10^n)^m = 10^{nm}$$

ARITHMETIC PROGRESSION (AP)

It is a sequence of numbers which are arranged in increasing order and having a constant difference between them.

EXAMPLE: 1, 3, 5, 7, 9, 11, 13, ... or 2, 4, 6, 8, 10, 12, ...
In general arithmetic progression can be written as $a_0, a_1, a_2, a_3, a_4, a_5 \ldots$

(a) n^{th} term of an arithmetic progression is given by

$$a_n = a_0 + (n-1)d$$

where a_0 is the first term, n is the number of terms, d is the common difference given by

$$d = (a_1 - a_0) = (a_2 - a_1) = (a_3 - a_2)$$

(b) Sum of arithmetic progression

$$S_n = \frac{n}{2}(2a_0 + (n-1)d) = \frac{n}{2}(a_0 + a_n)$$

ILLUSTRATION 3

Find the sum of series $7+10+13+16+19+22+25$.

SOLUTION

Since $n=7$, $a_0=7$ and $a_n = a_7 = 25$, so

$$S_n = \frac{n}{2}(a_0 + a_n) = \frac{7}{2}(7+25) = 112$$

GEOMETRIC PROGRESSION (GP)

It is a sequence of numbers in which every successive term is obtained by multiplying the previous term by a constant quantity. This constant quantity is called the common ratio (r).

EXAMPLE: 4, 8, 16, 32, 64, 128 ... where $a = 4$ and $r = 2$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \cdots$ where $a = 1$ and $r = \frac{1}{2}$

In general geometric progression can be written as a, ar, ar^2, ar^3, ar^4, … where a is the first term and r is the common ratio

(a) Sum of n terms of G.P. is

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{if} \quad r < 1$$

$$S_n = \frac{a(r^n-1)}{r-1} \quad \text{if} \quad r > 1$$

(b) Sum of infinite terms of G.P.

$$S_\infty = \frac{a}{1-r} \quad \text{if} \quad r < 1$$

ILLUSTRATION 4

Find the sum of series $Q = 2q + \frac{q}{3} + \frac{q}{9} + \frac{q}{27} + ...$

SOLUTION

Above equation can be re-written as

$$Q = q + \left[q + \frac{q}{3} + \frac{q}{9} + \frac{q}{27} + ... \right]$$

By using the formula of sum of infinite terms of G.P.

$$Q = q + \left[\frac{q}{1-\frac{1}{3}} \right] = q + \frac{3}{2}q = \frac{5}{2}q$$

COORDINATE GEOMETRY

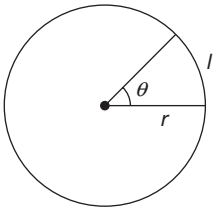
(a) The distance d between two points whose coordinates are (x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(b) Radian: The arc length l is proportional to the radius r for a fixed value of θ (in radians)

$$l = r\theta$$

$$\Rightarrow \quad \theta = \frac{l}{r}$$



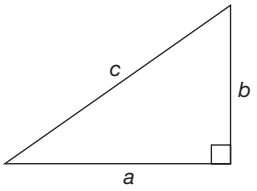
(c) Circumference of a circle is $C = 2\pi r$

(d) Area of a circle is $A = \pi r^2$

(e) Sector area $= \frac{1}{2} r^2 \theta$, where θ is in radian

(f) The pythagorean theorem, which relates the three sides of a right triangle

$$c^2 = a^2 + b^2$$



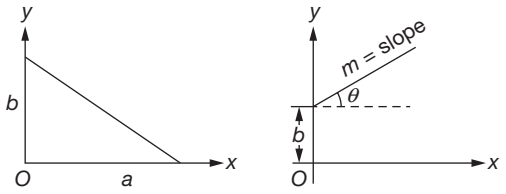
(g) Area of a triangle is $A = \frac{1}{2}(\text{Base})(\text{Altitude})$

(h) Surface area of a sphere is $A = 4\pi r^2$

(i) Volume of a sphere is $V = \frac{4}{3}\pi r^3$. If a spherical shell (hollow sphere) of radius x and thickness dx is cut out from the centre then the surface area and volume of the solid part is given by $4\pi r^2$ and $4\pi x^2 dx$ respectively

(j) Volume of a cylinder is $V = \pi r^2 l$

(k) Equation of a straight line is $y = mx + b$ ($b = y$ intercept, $m =$ slope $= \tan\theta$). The equation of a straight line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$.

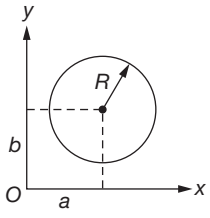


(l) Equation of a circle of radius R centred at the origin is $x^2 + y^2 = R^2$. Equation of a circle of radius R centred at (a, b) is the general form of equation of a circle is

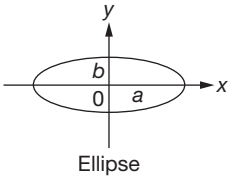
$$x^2 + y^2 + 2gx + 2fy + c = 0$$

This circle has centre at $(-g, -f)$ with radius $r = \sqrt{g^2 + f^2 - c}$.

$$(x-a)^2 + (y-b)^2 = R^2$$

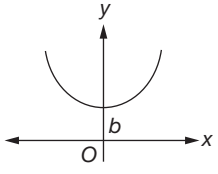


(m) Equation of an ellipse with the origin at its center is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a = semi-major axis, b = semi-minor axis).



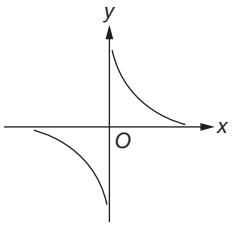
Ellipse

(n) Equation of a parabola whose vertex is at $y = b$ is $y = ax^2 + b$.



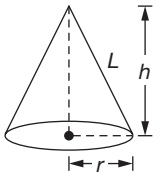
Parabola

(o) Equation of a rectangular hyperbola is xy = constant.



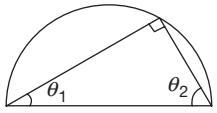
Rectangular hyperbola

(p) For a right circular cone of height h, radius r, the volume is $V = \frac{1}{3}\pi r^2 h$, Lateral surface area $= \pi r L$, where $L = \sqrt{r^2 + h^2}$.



Right circular cone

(q) Every triangle inscribed within a semicircle is a right triangle.



$\theta_1 + \theta_2 = \pi/2$

Trigonometry

(a) The angle θ is measured in radian.

$$180^\circ = \pi \text{ radian}$$

$$\Rightarrow \quad 1^\circ = \frac{\pi}{180} \text{ radian} \approx 0.02 \text{ radian}$$

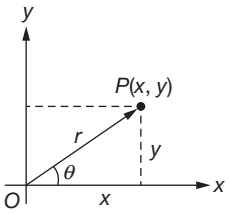
$$\text{Similarly 1 radian} = \frac{180}{\pi} \text{ degree} \approx 57^\circ$$

Degree to Radian: Multiply by $\frac{\pi}{180}$

Radian to Degree: Multiply by $\frac{180}{\pi}$

(b) Basic Trigonometric Quantities

The sine, cosine and tangent functions in trigonometry are defined in terms of the ratio of the sides of a right triangle:



(i) $\sin\theta = \dfrac{\text{Side opposite } \theta}{\text{Hypotenuse}}$

$\Rightarrow \sin\theta = \dfrac{a}{c} = \dfrac{\text{Perpendicular } (P)}{\text{Hypotenuse } (H)}$

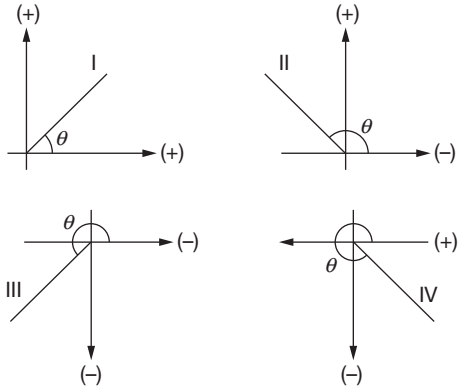
(ii) $\cos\theta = \dfrac{\text{Side adjacent } \theta}{\text{Hypotenuse}}$

$\Rightarrow \cos\theta = \dfrac{b}{c} = \dfrac{\text{Base } (B)}{\text{Hypotenuse } (H)}$

(iii) $\tan\theta = \dfrac{\text{Side opposite } \theta}{\text{Side adjacent } \theta}$

$\Rightarrow \sin\theta = \dfrac{a}{b} = \dfrac{\text{Perpendicular } (P)}{\text{Base } (B)}$

(c) Signs in the four quadrants.



	sinθ	cosθ	tanθ	cosecθ	secθ	cotθ
Quad I (All)	⊕	⊕	⊕	⊕	⊕	⊕
Quad II (sine)	⊕	⊖	⊖	⊕	⊖	⊖
Quad III (tan)	⊖	⊖	⊕	⊖	⊖	⊕
Quad IV (cos)	⊖	⊕	⊖	⊖	⊕	⊖

(d) From the above basic units and using the Pythagorean theorem, it follows that

$$\sin^2\theta + \cos^2\theta = 1;\quad \sec^2\theta = 1 + \tan^2\theta;$$

$$\text{cosec}^2\theta = 1 + \cot^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

(e) The cosecant, secant, and cotangent functions are defined by

(i) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ (ii) $\sec\theta = \dfrac{1}{\cos\theta}$

(iii) $\cot\theta = \dfrac{1}{\tan\theta}$

(f) The relations at the right follows directly from the right triangle above:

In First Quadrant

$\sin\theta = \cos(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$

$\cot\theta = \tan(90° - \theta)$

In Second Quadrant:

$\sin(90° + \theta) = \cos\theta$	$\sin(180° - \theta) = \sin\theta$
$\cos(90° + \theta) = -\sin\theta$	$\cos(180° - \theta) = -\cos\theta$
$\tan(90° + \theta) = -\cot\theta$	$\tan(180° - \theta) = -\tan\theta$

In Third Quadrant:

$\sin(180° + \theta) = -\sin\theta$	$\sin(270° - \theta) = -\cos\theta$
$\cos(180° + \theta) = -\cos\theta$	$\cos(270° - \theta) = -\sin\theta$
$\tan(180° + \theta) = \tan\theta$	$\tan(270° - \theta) = \cot\theta$

In Fourth Quadrant:

$\sin(270° + \theta) = -\cos\theta$	$\sin(360° - \theta) = -\sin\theta$
$\cos(270° + \theta) = \sin\theta$	$\cos(360° - \theta) = \cos\theta$
$\tan(270° + \theta) = -\cot\theta$	$\tan(360° - \theta) = -\tan\theta$

(g) Some properties of trigonometric functions:

(i) $\sin(-\theta) = -\sin\theta$

(ii) $\cos(-\theta) = \cos\theta$

(iii) $\tan(-\theta) = -\tan\theta$

(h) Values for certain angles

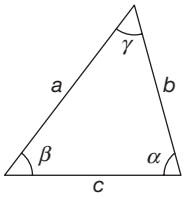
θ (in degree)	θ (in radian)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not Defined
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
180°	π	0	-1	0

Also take a note that $\sin(37°) = \cos(53°) = \frac{3}{5}$

Sides and Angles of Triangle

(i) Sum of all the angles of Triangle is 180°.

$\Rightarrow \quad \alpha + \beta + \gamma = 180°$



(j) Law of cosine

(i) $a^2 = b^2 + c^2 - 2bc\cos\alpha$

(ii) $b^2 = a^2 + c^2 - 2ac\cos\beta$

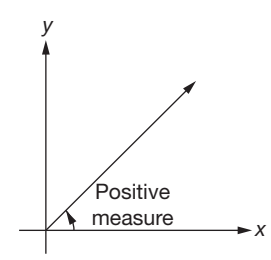
(iii) $c^2 = a^2 + b^2 - 2ab\cos\gamma$

(k) Law of sines (Lami's Theorem), for any triangle

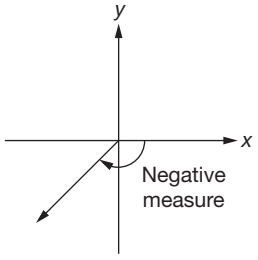
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

MEASUREMENT OF POSITIVE AND NEGATIVE ANGLES

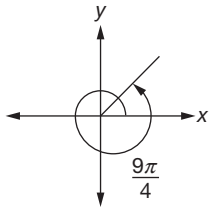
Angles measured counterclockwise (CCW) from the positive x-axis are assigned positive measures whereas angles measured clockwise (CW) are assigned negative measures.

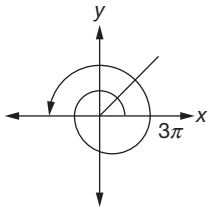


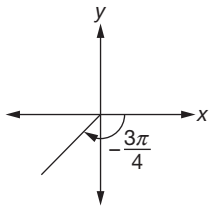
(Counter Clockwise Sense)

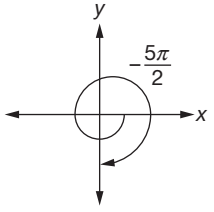


(Clockwise Sense)









Some Trigonometric Identities

(a) $\sin^2\theta + \cos^2\theta = 1$

(b) $\text{cosec}^2\theta = 1 + \cot^2\theta$

(c) $\sec^2\theta = 1 + \tan^2\theta$

(d) $\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 - \cos\theta)$

(e) $\sin(2\theta) = 2\sin\theta\cos\theta$

(f) $\cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 + \cos\theta)$

(g) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

(h) $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

(i) $\tan(2\theta)=\dfrac{2\tan\theta}{1-\tan^2\theta}$

(j) $\tan\left(\dfrac{\theta}{2}\right)=\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

(k) $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$

(l) $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

(m) $\sin C+\sin D=2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

(n) $\cos A+\cos B=2\cos\left[\dfrac{1}{2}(A+B)\right]\cos\left[\dfrac{1}{2}(A-B)\right]$

(o) $\sin(3\theta)=3\sin\theta-4\sin^3\theta$

(p) $\cos(3\theta)=4\cos^3\theta-3\cos\theta$

(q) $\tan(3\theta)=\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

(r) $\cos(2\theta)=\cos^2\theta-\sin^2\theta=\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(s) $\sin^2 A-\sin^2 B=\sin(A+B)\sin(A-B)$

(t) $\cos^2 A-\sin^2 B=\cos(A+B)\cos(A-B)$

(u) $\cos C+\cos D=2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

(v) $\sin C-\sin D=2\sin\left(\dfrac{C-D}{2}\right)\cos\left(\dfrac{C+D}{2}\right)$

(w) $\cos C-\cos D=-2\sin\left(\dfrac{C-D}{2}\right)\cos\left(\dfrac{C+D}{2}\right)$

ILLUSTRATION 5

Convert the following angles to radian.

(a) 45°	**(b)** 60°	**(c)** 120°
(d) 135°	**(e)** 210°	**(f)** 225°
(g) 270°	**(h)** 300°	**(i)** 330°

SOLUTION

(a) $45^\circ=45\times(1^\circ)=45\times\left(\dfrac{\pi}{180}\right)\text{ radian}=\dfrac{\pi}{4}\text{ radian}$

(b) $60^\circ=60\times(1^\circ)=60\times\left(\dfrac{\pi}{180}\right)\text{ radian}=\dfrac{\pi}{3}\text{ radian}$

(c) $120^\circ=120\times(1^\circ)=120\times\left(\dfrac{\pi}{180}\right)\text{ radian}$

$\Rightarrow\ 120^\circ=\dfrac{2\pi}{3}\text{ radian}$

(d) $135^\circ=135\times(1^\circ)=135\times\left(\dfrac{\pi}{180}\right)\text{ radian}$

$\Rightarrow\ 135^\circ=\dfrac{3\pi}{4}\text{ radian}$

(e) $210^\circ=210\times(1^\circ)=210\times\left(\dfrac{\pi}{180}\right)\text{ radian}$

$\Rightarrow\ 210^\circ=\dfrac{7\pi}{6}\text{ radian}$

(f) $225^\circ=225\times(1^\circ)=225\times\left(\dfrac{\pi}{180}\right)\text{ radian}$

$\Rightarrow\ 225^\circ=\dfrac{9\pi}{4}\text{ radian}$

(g) $270^\circ=270\times(1^\circ)=270\times\left(\dfrac{\pi}{180}\right)\text{ radian}$

$\Rightarrow\ 270^\circ=\dfrac{3\pi}{2}\text{ radian}$

(h) $300^\circ=300\times(1^\circ)=300\times\left(\dfrac{\pi}{180}\right)\text{ radian}$

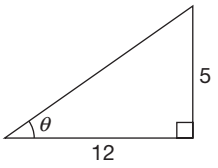
$\Rightarrow\ 300^\circ=\dfrac{5\pi}{3}\text{ radian}$

(i) $330^\circ=330\times(1^\circ)=330\times\left(\dfrac{\pi}{180}\right)\text{ radian}$

$\Rightarrow\ 330^\circ=\dfrac{11\pi}{6}\text{ radian}$

ILLUSTRATION 6

Find the six trigonometric ratios from the given figure.



SOLUTION

By Pythagoras Theorem, we have

$H^2=P^2+B^2$

$\Rightarrow\ H^2=5^2+12^2=169$

$\Rightarrow\ H=13$

$\Rightarrow\ \sin\theta=\dfrac{P}{H}=\dfrac{5}{13},\ \cos\theta=\dfrac{B}{H}=\dfrac{12}{13},\ \tan\theta=\dfrac{P}{B}=\dfrac{5}{12}$

$\Rightarrow\ \text{cosec}\,\theta=\dfrac{13}{5},\ \sec\theta=\dfrac{13}{12},\ \cot\theta=\dfrac{12}{5}$

ILLUSTRATION 7

Find the values of

(a) $\cos(120°)$ **(b)** $\sin(-1485°)$ **(c)** $\sin(300°)$
(d) $\cos(-60°)$ **(e)** $\tan(210°)$

SOLUTION

(a) $\cos(120°)=\cos(90°+30°)=-\sin(30°)=-\frac{1}{2}$

(b) $\sin(-1485°)=-\sin(3\times360°+45°)$

$\Rightarrow \sin(-1485°)=-\sin(45°)=-\frac{1}{\sqrt{2}}$

(c) $\sin(300°)=\sin(360°-60°)=-\sin(60°)=\frac{-\sqrt{3}}{2}$

(d) $\cos(-60°)=\cos(60°)=\frac{1}{2}$

(e) $\tan(210°)=\tan(180°+30°)=\tan(30°)=\frac{1}{\sqrt{3}}$

ILLUSTRATION 8

If $A=60°$, then find the value of $\sin(2A)$.

SOLUTION

Since $\sin(2A)=2\sin A\cos A$

So for $A=60°$, we have

$$\sin(2A)=2\sin A\cos A=2\sin(60°)\cos(60°)$$

$$\Rightarrow \sin(2A)=2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$$

FACTORIAL

Factorial is always defined for a positive integral number, say n. Then

$$\underline{|n}=n!=n(n-1)(n-2)(n-3).....\times3\times2\times1$$

(read as n factorial)

Also, we observe that $\underline{|n}=n\underline{|n-1}=n(n-1)!$

Further more, we have $\underline{|1}=1$ and $\underline{|0}=1$.

SERIES EXPANSIONS

(a) $(a+b)^n=a^n+\frac{n}{1!}a^{n-1}b+\frac{n(n-1)}{2!}a^{n-2}b^2+.....$

{Binomial Expansion}

(b) $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+.....$

{Binomial Series}

(c) $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$ {Exponential Series}

(d) $\ln(1\pm x)=\pm x-\frac{1}{2}x^2\pm\frac{1}{3}x^3-\cdots$

{Logarithmic Series}

(e) $\sin x=x-\frac{x^3}{3!}+\frac{x^3}{5!}-\cdots$

(f) $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots$ {x in radian}

(g) $\tan x=x+\frac{x^3}{3}+\frac{2x^3}{15}+\cdots|x|<\frac{\pi}{2}$

{Trigonometric Expansion}

(h) For $x\ll1$, the following approximations can be used:

(i) $(1+x)^n\approx1+nx$ {Binomial Approximation}

(ii) $\ln(1\pm x)\approx\pm x$ {Logarithmic Approximation}

(iii) $e^x\approx1+x$ {Exponential Approximation}

(iv) $\sin x\approx x$ {Trigonometric Approximation}

(v) $\cos x\approx1$ {Trigonometric Approximation}

(vi) $\tan x\approx x$ {Trigonometric Approximation}

Conceptual Note(s)

In Binomial Series, if $|x|\ll1$, then only the first two terms are significant. It is so because the values of second and the higher order terms become very small and hence can be neglected. So the following expressions can be re-written as

$$(1+x)^n=1+nx$$

$$\Rightarrow (1+x)^{-n}=1-nx$$

$$\Rightarrow (1-x)^n=1-nx$$

$$\Rightarrow (1-x)^{-n}=1+nx$$

ILLUSTRATION 9

Evaluate $(1001)^{1/3}$ upto six places of decimal.

SOLUTION

$$(1001)^{1/3} = (1000+1)^{1/3} = 10(1+0.001)^{1/3}$$

By comparing the given equation with standard equation

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+.....$$

$$\Rightarrow \quad x = 0.001 \text{ and } n = \frac{1}{3}$$

$$\Rightarrow \quad 10(1+0.001)^{1/3} = 10\left[1+\frac{1}{3}(0.001)+\begin{pmatrix}\text{Neglected}\\ \text{Terms}\end{pmatrix}\right]$$

$$\Rightarrow \quad (1001)^{1/3} = 10\left[1+0.00033-\frac{1}{9}(0.000001)+.....\right]$$

$$\Rightarrow \quad (1001)^{1/3} = 10(1.0003301) = 10.003301 \text{ (Approx.)}$$

ILLUSTRATION 10

The value of acceleration due to gravity (g_h) at a height h above the surface of earth is given by $g_h = \frac{gR^2}{(R+h)^2}$. Find the approximate value of g_h when $h \ll R$.

SOLUTION

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad g_h = g\left(\frac{1}{1+\frac{h}{R}}\right)^2$$

$$\Rightarrow \quad g_h = \left(1+\frac{h}{R}\right)^{-2}$$

$$\Rightarrow \quad g_h = g\left[1+(-2)\frac{h}{R}+\frac{(-2)(-3)}{2!}\left(\frac{h}{R}\right)^2+.....\right]$$

$$\Rightarrow \quad g_h = g\left(1-\frac{2h}{R}\right)$$

FUNCTION: AN INTRODUCTION

A key idea in mathematical analysis and in Physics is the idea of dependence of one quantity on the other. A quantity depends on another if the variation of one of them is accompanied by a variation of other. We must have seen Mathematicians speaking about an **independent variable** and the **dependent variable**. Similarly in Physics, it is better to think in terms of **cause** and **effect** or **interdependent quantities**.

We are aware of the fact that the area of a circle depends upon its radius. Mathematically speaking, the area of a circle is a function of its radius.

Similarly in Physics we observe that the volume of a given mass of a gas at a fixed temperature is a function of the pressure of the gas. That is we can say that the **cause** of the change in temperature will produce an **effect** that produces the change in pressure of the gas.

REPRESENTATION OF A FUNCTION

A function is denoted by symbols like of $f(x)$, $F(x)$, $\phi(x)$... . and is read as function of x. Thus if y is a function of x, we may write $y = f(x)$.

Conceptual Note(s)

It must be clearly understood that *f*(*x*) does not mean *f* into *x*, but is only a symbolic way of representing some function of *x*.

The dependence of one quantity on another can be quantitatively expressed in three different ways:

(a) Tabular Presentation
(b) Graphical Presentation
(c) Mathematical Presentation/Equations

Tabular Presentation

Let us consider the distance covered by an automobile, moving at constant speed, as a function of time. Data for a particular example of such motion may be presented numerically, as in the following **Table.** The exact mathematical relationship between the time and the distance in this example is not immediately obvious while examining the table. This is one of the

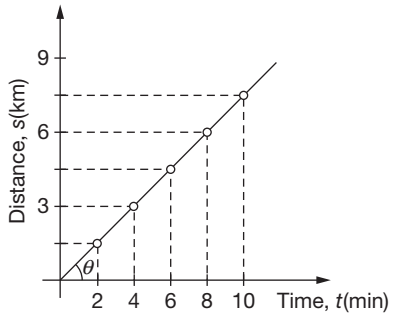
disadvantages of tabular presentation. Although the numerical values can be precisely specified, they do not at once convey the clear picture of the dependence of variables on one another. This can be better visualised by drawing the graph for the varying quantities.

Table 1.2 Time and Distance for a moving Automobile

Elapsed Time (min)	Distance (km)
0	0
2	1.5
4	3.0
6	4.5
8	6.0
10	7.5

Graphical Presentation

Let us plot the same data on a graph as shown in the figure.



Variation of distance with time

Here we plot the Time (an independent variable) horizontally and the Distance (a dependent variable) vertically. Each pair of numbers in the Table 10.2 gives a single point on the graph. It is immediately seen that the points when joined give us a straight line.

Mathematical Presentation/Equations

The equation that fits the above tabular and graphical data is

$$s = 0.75t$$

Where s represents the distance in kilometre and t represents the time in minutes.

This equation can also be expressed as

$$s = v_0 t$$

where v_0 is a constant whose value in this example is

$$v_0 = 0.75 \text{ kmmin}^{-1}$$

The equation provides the most concise expression of a functional relationship.

SLOPE OF A LINE

The **slope** of a line in a graph is defined as the **tangent of the angle** (measured in anticlockwise direction) that the line makes with the positive direction of the horizontal axis. This angle is designated by θ. From the distance vs time plot shown in "Graphical Presentation", we have

$$\tan\theta = \frac{s}{t} = v_0$$

is the slope of the straight line that has been plotted for the variation of the distance with time.

Here we observe that the quantity $\tan\theta$ is a dimensional quantity. We always measure slope as a vertical increment divided by a horizontal increment on a graph, each increment being measured in the appropriate unit for the quantity in the problem.

ILLUSTRATION 11

If $y = f(x) = x^2 - 3x + 5$, find $f(0)$ and $f(1)$.

SOLUTION

To find the value of a function at a particular value of the independent variable, let us put the given value in the expression of the function and simplify. The result followings.

Here, $f(x) = x^2 - 3x + 5$

$$f(0) = (0)^2 - 3(0) + 5 = 5$$

$$f(1) = (1)^2 - 3(1) + 5 = 3$$

CONCEPT OF LIMIT OF FUNCTIONS: MEANING OF THE SYMBOL $x \to a$

When the independent variable is gradually taken to a definite value, say a, the dependent variable, i.e., the function will lead to another definite value, say l. This value is defined as the limiting value of the

function as the independent variable approaches the given value a.

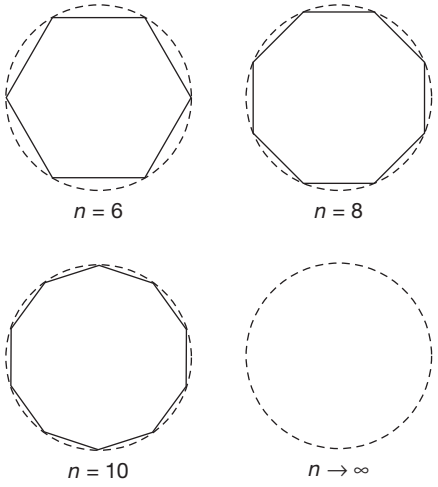
The arrow in the above symbol stands for gradual approach of x to a and the symbol is read as x tending to a. If $y = f(x)$ approaches a value l, as x approaches a, we say that the limiting value of $f(x)$ is l when x approaches a and this is symbolically written as

$$\lim_{x \to a} f(x) = l$$

This symbol is read as the limit of the function is l when x tends to a.

EXAMPLES

(a) Let us inscribe a polygon of n sides in a circle of radius a. The area A of the polygon will depend on the number of sides n. Hence $A = f(n)$.



As we increase the number of sides, the sides will be shorter and shorter in size, the area of the polygon will increase and ultimately when n is made infinitely large, the area of the polygon will become equal to the area of circle. Thus we may say that the limiting value of the area of a polygon of n sides inscribed in a circle is the area of the circle itself, as n tends to infinity. (Just think that the circle is a polygon having infinite number of sides)

Mathematically, $\lim_{n \to \infty} \text{Area} = \pi a^2$

(b) Let us consider the function $y = f(x) = x^2 - 1$. Let us find $\lim_{x \to 2} f(x)$.

When x approaches 2 from left, we observe $f(x)$ approaches 3 from the left.

x	$y = f(x) = x^2 - 1$
1.9	2.6100
1.99	2.96010
1.999	2.996001
1.9999	2.99960

When x approaches 2 from right, we observe $f(x)$ approaches 3 from the right.

x	$y = f(x) = x^2 - 1$
2.2	3.84000
2.1	3.41000
2.01	3.04010
2.001	3.004
2.0001	3.00040

From the above two tables, it is clear that as x tends to 2, $y = f(x) = x^2 - 1$ approaches or tends to 3

$\Rightarrow \lim_{x \to 2} x^2 - 1 = 3$

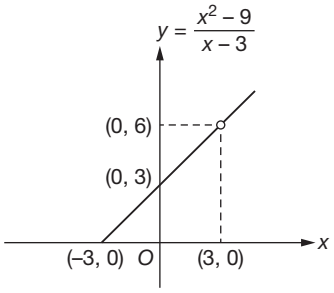
The process of finding the limiting value of a function in the above way is actually the fundamental way of finding the limiting value of a function called the **first principle**. But this is considered to be a lengthy process. Every time it is not possible to find the limiting value by this process and that too for all kinds of functions.

Simply by putting $x = 2$ in the above expression will give us the limiting value. Hence the shortcut method of finding the limiting value of a function is to make direct substitution of the limiting the value of the independent variable in the expression of the function. But this technique may not work for all kinds of functions. Actually we have learnt the concept of limit in an easy manner, but this concept is useful whenever we have to find the value of the functions at a point where they do not have indeterminate values. Consider another example to have a clear concept.

(c) Let us take the function

$$y = f(x) = \frac{x^2 - 9}{x - 3}$$

Here the student generally concludes that the above expression could also have been easily written as $y = f(x) = x + 3$. But this is only true when $x \neq 3$. Actually we shall not be able to find the value of this function at $x = 3$, because at $x = 3$ the function has a value $\frac{0}{0}$ which is an indeterminate value. For this let us again consider the values of x approaching from the left as well as the right towards 3.



When x approaches 3 from left, we observe $f(x)$ approaches 6 from the left.

x	$y = f(x) = \frac{x^2 - 9}{x - 3}$
2.9	5.9
2.99	5.99
2.999	2.9999
5.999	5.9999

When x approaches 3 from right, we observe $f(x)$ approaches 6 from the right.

x	$y = f(x) = \frac{x^2 - 9}{x - 3}$
3.1	6.1
3.01	6.01
3.001	3.0001
6.001	6.0001

From the above two tables, it is clear that as x tends to 3, $y = f(x) = \frac{x^2 - 9}{x - 3}$ approaches (or tends to) 6

$$\Rightarrow \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$$

Conceptual Note(s)

(a) Please note here that the function $y = \frac{x^2 - 9}{x - 3}$ is discontinuous at $x = 3$. A function is said to be continuous when you can draw it in one go with your pen without its tip losing contact with the paper. However, if we consider the function $y = x + 3$, we observe that it is continuous.

(b) Remember that you must not cancel the common factor from the numerator and the denominator till you are sure that the common factor is non zero.

So, the expression $\frac{x^3 y^4}{xy^2} = x^2 y^2$ only if $x \neq 0$ and $y \neq 0$.

(c) A very important formula of limit extensively used in Physics is

$$\lim_{\phi \to 0} \frac{\sin\phi}{\phi} = 1$$

DERIVATIVE OF A FUNCTION

Meaning of Δx

For a finite but small increment in x, we use symbol Δx. Please note that this is also not to be read as Δ multiplied by x. It stands for a small but finite increment in x and is treated as a single quantity. This should be read as 'delta x'.

Meaning of dx

Whenever a variable is changed by an infinitesimal (extremely small) amount, then that change is called the differential of the variable. This is denoted by dx and read as 'dee x'. Again please do not misinterpret this as d into x. dx is merely a representation standing for a very, very small increment in x. This

should be treated as a single quantity just as $\sin\theta$ which again is not the product of sine and θ.

Slope has a simple physical meaning. It is the rate of change of the quantity being plotted vertically with respect to the quantity being plotted horizontally or we can say that it is the rate of change of dependent variable with respect to an independent variable. Mathematically, **derivative of the function** gives the instantaneous slope of that function.

Slope and Derivative of a Function

Slope has a simple physical meaning. It is the rate of change of the quantity being plotted vertically with respect to the quantity being plotted horizontally or we can say that it is the rate of change of dependent variable with respect to an independent variable. Mathematically,

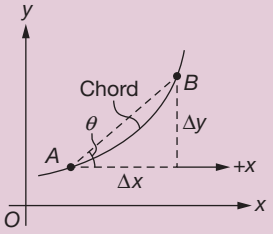
(a) $\frac{dy}{dx}$ (derivative of the function y w.r.t. x) gives the instantaneous slope of the function at a point. $\frac{dy}{dx}$ is also called as the Rate Measurer.

(b) $\frac{\Delta y}{\Delta x}$ gives the average slope of a curve y between two points.

Conceptual Note(s)

Please note that:

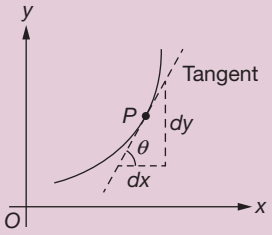
(a) Average slope is always in an interval i.e. between two points and it is the slope of the chord that joins the two points.



So, average slope of the curve between the points A and B is

$$\frac{\Delta y}{\Delta x} = \tan\theta = \tan\left(\begin{array}{c}\text{of angle which chord}\\ \text{joining points } A \text{ and } B\\ \text{makes with} + x \text{ direction}\end{array}\right)$$

(b) Instantaneous slope is at a point or an instant and it is the slope of the tangent drawn to the curve at that point.



So, instantaneous slope of the curve at the point P is

$$\frac{dy}{dx} = \tan\theta = \tan\left(\begin{array}{c}\text{of angle which tangent to}\\ \text{point } P \text{ makes with}\\ +x \text{ direction}\end{array}\right)$$

DEFINITION OF DIFFERENTIAL COEFFICIENT

Consider a function $y = f(x)$. Let the value of x changes to $x + \Delta x$. Correspondingly the value of y changes from y to $y + \Delta y$. The limiting value of the ratio $\frac{\Delta y}{\Delta x}$ when Δx tends to zero is called the differential coefficient of y with respect to x. It is denoted by the symbol $\frac{dy}{dx}$.

Thus, $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

The process of finding the differential coefficient of a function is called differentiation or the derivative of the function and this signifies the instantaneous slope of that function.

MATHEMATICAL DEFINITION

Consider a function $y = f(x)$. Let the value of x changes to $x + \Delta x$. Correspondingly the value of y changes from y to $y + \Delta y$. The limiting value of the ratio $\frac{\Delta y}{\Delta x}$ when Δx tends to zero is called the differential coefficient of y with respect to x. It is denoted by the symbol $\frac{dy}{dx}$.

Thus, $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

So, $\frac{dy}{dx} = \frac{d}{dx}[f(x)] = f'(x)$

If $y = f(x)$

then, $y + \Delta y = f(x + \Delta x)$

$\Rightarrow \quad y + \Delta y - y = f(x + \Delta x) - f(x)$

$\Rightarrow \quad \Delta y = f(x + \Delta x) - f(x)$

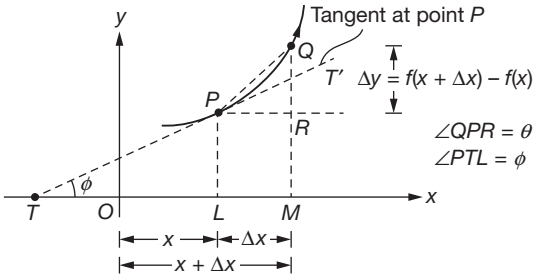
$\Rightarrow \quad \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$\Rightarrow \quad \frac{dy}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

The derivative of $f(x)$ at $x = a$ is denoted by $f'(a)$. In other words the slope of the function $y = f(x)$ at $x = a$ is given by $f'(a)$.

GEOMETRICAL INTERPRETATION OF DERIVATIVE

Let us consider the graph of $y = f(x)$ as shown in figure.



Let P and Q be the two points on it. Then

$PR = LM = \Delta x$

$QR = \Delta y$

$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

The average slope of the curve in the internal PQ.

As $Q \to P$ along the curve, $\Delta x \to 0$, $\theta \to \phi$ and PQ becomes tangent TPT' at P.

$\therefore \quad \tan\phi = \lim_{\theta \to \phi} \tan\theta = \frac{d}{dx}(x^n) = \frac{dy}{dx}$ at P

$\tan\phi \left(\text{i.e., } \frac{dy}{dx} \text{ at } P \right)$ is the slope of tangent at P.

RULES OF DIFFERENTIATION

The process of finding the derivative of a function is called differentiating the function.

Differentiation obeys several simple rules that are worth **Committing To Memory (CTM)**.

RULE 1:

Derivative of a constant is zero.

$$\frac{d}{dx}(\text{constant}) = 0$$

RULE 2:

The derivative of a constant times a function is the constant times the derivative of the function.

$$\frac{d}{dx}(au) = a\frac{du}{dx}$$

RULE 3:

The derivative of the sum of the functions is the sum of their derivatives.

$$\frac{d}{dx}(u \pm v \pm w \pm \ldots.) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \ldots..$$

RULE 4: PRODUCT RULE

Derivative of product of two functions is given as

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(① \times ②) = ① \frac{d}{dx} ② + ② \frac{d}{dx} ①$$

RULE 5: QUOTIENT RULE

Derivative of a quotient is given as

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}\left(\frac{\text{Num}^{\text{r}}}{\text{Den}^{r}}\right)=\frac{(\text{Den}^{\text{r}})\frac{d}{dx}(\text{Num}^{\text{r}})-(\text{Num}^{\text{r}})\frac{d}{dx}(\text{Den}^{\text{r}})}{(\text{Den}^{r})^2}$$

RULE 6: CHAIN RULE

Suppose f is a function of u, which in turn is a function of x. The derivative $\frac{df}{dx}$ can be written as the product of two derivatives.

$$\frac{df}{dx}=\frac{df}{du}\times\frac{du}{dx}$$

IMPORTANT DIFFERENTIAL FORMULAE

$\frac{d}{dx}(\text{constant})=\text{Zero}$	$\frac{d}{dx}[\cos(ax+b)]=-a\sin(ax+b)$
$\frac{d}{dx}(u\pm v\pm w)=\frac{du}{dx}\pm\frac{dv}{dx}\pm\frac{dw}{dx}$	$\frac{d}{dx}[\tan(ax+b)]=a\sec^2(ax+b)$
$\frac{d}{dx}(x^n)=nx^{n-1}$	$\frac{d}{dx}[\sec(ax+b)]=a\sec(ax+b)\tan(ax+b)$
$\frac{d}{dx}(kx^n)=k(nx^{n-1})$	$\frac{d}{dx}[\cot(ax+b)]=-a\,\text{cosec}^2(ax+b)$
$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[\text{cosec}(ax+b)]=-a\,\text{cosec}(ax+b)\cot(ax+b)$
$\frac{d}{dx}(uvw)=uv\frac{dw}{dx}+uw\frac{dv}{dx}+vw\frac{du}{dx}$	$\frac{d}{dx}(\log_e x)=\frac{1}{x}$
$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\log_e(ax+b)]=\frac{a}{ax+b}$
$\frac{d}{dx}\left(\frac{\text{Num}^{\text{r}}}{\text{Den}^{r}}\right)=\frac{(\text{Den}^{\text{r}})\frac{d}{dx}(\text{Num}^{\text{r}})-(\text{Num}^{\text{r}})\frac{d}{dx}(\text{Den}^{\text{r}})}{(\text{Den}^{r})^2}$	$\frac{d}{dx}(e^x)=e^x$
If y is a function of u and u is a function of x, then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$	$\frac{d}{dx}(a^x)=a^x\log_e a$
If y is a function of u, u a function of v, v a function of w and w a function of x, then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dv}\times\frac{dv}{dw}\times\frac{dw}{dx}$	$\frac{d}{dx}(e^{kx})=ke^{kx}$
$\frac{d}{dx}(\sin x)=\cos x$	$\frac{d}{dx}(a^{kx})=ka^{kx}\log_e a$
$\frac{d}{dx}(\cos x)=-\sin x$	$\frac{d}{dx}[\log_e(\sec x+\tan x)]=\sec x$

$\frac{d}{dx}(\tan x)=\sec^2 x$	$\frac{d}{dx}\left[\log_e(\cot x+\operatorname{cosec} x)\right]=-\operatorname{cosec} x$
$\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x$	$\frac{d}{dx}(x\log_e x-x)=\log_e x$
$\frac{d}{dx}(\sec x)=\sec x\tan x$	$\frac{d}{dx}\left[\log_e(\cos x)\right]=-\tan x$
$\frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x\cot x$	$\frac{d}{dx}\left[\log_e(\sin x)\right]=\cot x$
$\frac{d}{dx}[\sin(ax+b)]=a\cos(ax+b)$	$\frac{d}{dx}(ax^2+bx+c)^n=(2ax+b)\left[n(ax^2+bx+c)^{n-1}\right]$

ILLUSTRATION 12

Find the derivative of $y=3x^2$.

SOLUTION

$$\frac{dy}{dx}=3\frac{d}{dx}(x^2)=3(2x)=6x$$

$$\left\{\because \frac{d}{dx}(x^n)=nx^{n-1}\right\}$$

ILLUSTRATION 13

Find the derivative of $y=x^3+3x^2$.

SOLUTION

Since the derivative of the sum is the sum of the derivatives, so

$$\frac{dy}{dx}=\frac{d}{dx}(x^3+3x^2)$$

$$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}(x^3)+\frac{d}{dx}(3x^2) \qquad \text{(using rule 3)}$$

$$\Rightarrow \frac{dy}{dx}=3x^2+6x$$

ILLUSTRATION 14

Find the derivative of $y=\sin(x^2)$.

SOLUTION

Let us assume $u=x^2$, then $y=\sin u$.

Then $\frac{dy}{du}=\cos u$ and $\frac{du}{dx}=2x$

Since, $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$ ($\because$ of chain rule)

$$\Rightarrow \frac{dy}{dx}=(\cos u)(2x)=2x\cos u$$

$$\Rightarrow \frac{dy}{dx}=2x\cos(x^2)$$

ILLUSTRATION 15

If $y=\sin x+\cos x$, then find $\frac{dy}{dx}$.

SOLUTION

Since $y=\sin x+\cos x$

$$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}(\sin x+\cos x)$$

Using RULE 3, we get

$$\frac{dy}{dx}=\frac{d}{dx}(\sin x)+\frac{d}{dx}(\cos x)$$

$$\Rightarrow \frac{dy}{dx}=\cos x-\sin x$$

ILLUSTRATION 16

Find the derivative of $y=x\sin x$.

SOLUTION

$$\frac{dy}{dx}=x\frac{d}{dx}(\sin x)+(\sin x)\left(\frac{dx}{dx}\right)=x\cos x+\sin x$$

(using product rule)

ILLUSTRATION 17

Differentiate the following w.r.t. x.

(a) $\sin x-\cos x$

(b) $\sin x+e^x$

SOLUTION

(a) $\frac{d}{dx}(\sin x-\cos x)=\frac{d}{dx}(\sin x)-\frac{d}{dx}(\cos x)$

$\Rightarrow \frac{d}{dx}(\sin x-\cos x)=\cos x+\sin x$

(b) $\frac{d}{dx}(\sin x+e^x)=\frac{d}{dx}(\sin x)+\frac{d}{dx}(e^x)=\cos x+e^x$

ILLUSTRATION 18

Find the derivative of $y=\frac{x^2+1}{x}$.

SOLUTION

$$\frac{dy}{dx}=\frac{x\frac{d}{dx}\left(x^2+1\right)-\left[\frac{d}{dx}(x)\right]\left(x^2+1\right)}{x^2}$$

$\Rightarrow \frac{dy}{dx}=\frac{2x^2-\left(x^2+1\right)}{x^2}=\frac{x^2-1}{x^2}$ (using quotient rule)

ILLUSTRATION 19

If $y=\sqrt{x}$, then find $\frac{dy}{dx}$.

SOLUTION

Given $y=\sqrt{x}=x^{\frac{1}{2}}$

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^{\frac{1}{2}}\right)=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2\sqrt{x}}\quad\left\{\because \frac{d}{dx}(x^n)=nx^{n-1}\right\}$$

ILLUSTRATION 20

Differentiate the following w.r.t. x.

(a) x^3 **(b)** $\sqrt{x}$ **(c)** ax^2+bx+c

(d) $2x^3-e^x$ **(e)** $6\log e^x-\sqrt{x}-7$

SOLUTION

(a) $\frac{d}{dx}(x^3)=3x^2$

(b) $\frac{d}{dx}(x)^{1/2}=\frac{1}{2}(x)^{\frac{1}{2}-1}=\frac{1}{2}(x)^{-1/2}=\frac{1}{2\sqrt{x}}$

(c) $\frac{d}{dx}(ax^2+bx+c)=a\frac{d}{dx}(x^2)+b\frac{d}{dx}(x)+\frac{d}{dx}(c)$

$\Rightarrow \frac{d}{dx}(ax^2+bx+c)=2ax+b$

(d) $\frac{d}{dx}(2x^3-e^x)=2\frac{d}{dx}(x^3)-\frac{d}{dx}(e^x)=6x^2-e^x$

(e) Let $y=6\log e^x-\sqrt{x}-7$

$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}\left(6\log_e x-\sqrt{x}-7\right)$

$\Rightarrow \frac{dy}{dx}=6\frac{d}{dx}(\log_e x)-\frac{d}{dx}(x^{1/2})-\frac{d}{dx}(7)$

$\Rightarrow \frac{dy}{dx}=\frac{6}{x}-\frac{1}{2\sqrt{x}}$

ILLUSTRATION 21

Differentiate the following w.r.t. t.

(a) $\sin(t^2)$ **(b)** $e^{\sin t}$

(c) $\sin(\omega t+\theta)$

SOLUTION

(a) $\frac{d}{dt}(\sin t^2)=\cos t^2\frac{d}{dt}(t^2)=2t\cos t^2$

(b) $\frac{d}{dt}(e^{\sin t})=e^{\sin t}\frac{d}{dt}(\sin t)=e^{\sin t}.\cos t$

(c) $\frac{d}{dt}[\sin(\omega t+\theta)]=\cos(\omega t+\theta)\frac{d}{dt}(\omega t+\theta)$

$\Rightarrow \frac{d}{dt}[\sin(\omega t+\theta)]=\omega\cos(\omega t+\theta)$

ILLUSTRATION 22

Differentiate $\frac{x^2+e^x}{\log x+20}$ w.r.t. x.

SOLUTION

Let $y=\frac{x^2+e^x}{\log x+20}$

Then $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+e^x}{\log x+20}\right)$

$$\frac{dy}{dx}=\frac{(\log x+20)\frac{d}{dx}\left(x^2+e^x\right)-\left(x^2+e^x\right)\frac{d}{dx}(\log x+20)}{(\log x+20)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(\log x+20)\left(2x+e^x\right)-\left(x^2+e^x\right)\left(\frac{1}{x}+0\right)}{(\log x+20)^2}$$

ILLUSTRATION 23

If $y = a\sin(\omega t)$, where a and ω are constants, find $\frac{dy}{dt}$.

SOLUTION

Given $y = a\sin\omega t$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{d}{dt}(a\sin\omega t) = a\frac{d}{dt}(\sin\omega t)$$

$$\Rightarrow \quad \frac{dy}{dt} = a\cos(\omega t)\frac{d}{dt}(\omega t) \qquad (\because \text{ of chain rule})$$

$$\Rightarrow \quad \frac{dy}{dt} = a\omega\cos(\omega t)$$

ILLUSTRATION 24

Find the slope of the function

$$y = \sin x + \frac{1}{x^2} - 3\log_e x \text{ at } x = \frac{\pi}{2}.$$

SOLUTION

We have $y = \sin x + \frac{1}{x^2} - 3\log_e x$

Slope of the function is the derivative of the function. So,

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) + \frac{d}{dx}\left(\frac{1}{x^2}\right) - 3\frac{d}{dx}(\log_e x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \cos x - \frac{2}{x^3} - \frac{3}{x}$$

Slope at $x = \frac{\pi}{2}$ is given by

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - \frac{2}{(\pi/3)^3} - \frac{3}{(\pi/2)}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0 - \frac{16}{\pi^3} - \frac{6}{\pi} = -\frac{16}{\pi^3} - \frac{6}{\pi}$$

ILLUSTRATION 25

If $y = 3e^x - 5x^3 + 3$, then find $\frac{dy}{dx}$.

SOLUTION

Given $y = 3e^x - 5x^3 + 3$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}\left(3e^x\right) - \frac{d}{dx}\left(5x^3\right) + \frac{d}{dx}(3) \qquad (\because \text{ of rule 3})$$

$$\Rightarrow \quad \frac{dy}{dx} = 3e^x - 5\left(3x^2\right) + 0 \;=\; 3e^x - 15x^2.$$

ILLUSTRATION 26

If $y = x^2\sin x$, find $\frac{dy}{dx}$.

SOLUTION

Given $y = x^2\sin x$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}\left(x^2\sin x\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = x^2 \times \frac{d}{dx}(\sin x) + \sin x \times \frac{d}{dx}\left(x^2\right)$$

(using product rule)

$$\Rightarrow \quad \frac{dy}{dx} = x^2\cos x + 2x\sin x.$$

ILLUSTRATION 27

Find the slope of the tangent to the curve $y = 3x^2 - 5$ at the point $(2, 7)$.

SOLUTION

We have

$$y = 3x^2 - 5$$

$$\Rightarrow \quad \frac{dy}{dx} = 6x$$

At point $(2,\ 7)$, we have $\frac{dy}{dx} = \tan\theta = 6(2) = 12$

$$\Rightarrow \quad \tan\theta = 12$$

ILLUSTRATION 28

Find the inclination with the x-axis of the tangent to the curve $y^2 = 4x$ at $(1,\ 2)$.

SOLUTION

Given $y^2 = 4x$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 4 \qquad \text{(taking derivative of both sides)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2}{y}$$

At the point $(1, 2)$

$$\frac{dy}{dx}=\frac{2}{2}=1$$

$$\Rightarrow \quad \tan\theta=1$$

$$\Rightarrow \quad \theta=45^\circ$$

ILLUSTRATION 29

Find $\frac{dy}{dx}$ for $y=\frac{2}{\sqrt{x}}+x\sqrt{x}$.

SOLUTION

$$\frac{dy}{dx}=\frac{d}{dx}\left(2x^{-\frac{1}{2}}+x^{\frac{3}{2}}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=2\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}+\frac{3}{2}x^{\frac{3}{2}-1}$$

$$\Rightarrow \quad \frac{dy}{dx}=-\frac{1}{x^{\frac{3}{2}}}+\frac{3}{2\sqrt{x}}$$

ILLUSTRATION 30

If we have $2y-y^2-x+x^3+9=0$, then calculate $\frac{dy}{dx}$ at $x=2$.

SOLUTION

Let us firstly calculate value of y, when x equals 2. For this, we have

$$2y-y^2-2+8+9=0$$

$$\Rightarrow \quad 2y-y^2+15=0$$

$$\Rightarrow \quad y^2-2y-15=0$$

$$\Rightarrow \quad y^2-5y+3y-15=0$$

$$\Rightarrow \quad (y-5)(y+3)=0$$

$$\Rightarrow \quad y=5,\ y=-3$$

So, we are to calculate $\frac{dy}{dx}$ at $(2,\ 5)$ and $(2,\ -3)$.

Now, $2y-y^2-x+x^3+9=0$

$$\Rightarrow \quad 2\frac{dy}{dx}-2y\frac{dy}{dx}-1+3x^2=0$$

$$\Rightarrow \quad 2\frac{dy}{dx}-2y\frac{dy}{dx}=1-3x^2$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1-2x^2}{2-2y}$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{x=2,\ y=-5}=\frac{1-2(2)^2}{2-2(5)}=\frac{1-8}{2-10}=\frac{7}{8}$$

and $\left.\frac{dy}{dx}\right|_{x=2,\ y=-3}=\frac{1-2(-3)^2}{1-2(-3)}=\frac{1-18}{1+6}=-\frac{17}{7}$

APPLICATIONS OF DERIVATIVE

With the help of differentiation, we can

(a) check whether the function is increasing or decreasing.
(b) find the maximum and minimum value(s) of a function.
(c) calculate the rate of change of quantity (say y) w.r.t. another quantity (say x).

INCREASING AND DECREASING FUNCTION

A function $f(x)$ is said to be increasing if $f(x)$ increases as x increases, and decreasing if $f(x)$ decreases as x increases.
In other words, if $x_1<x_2$

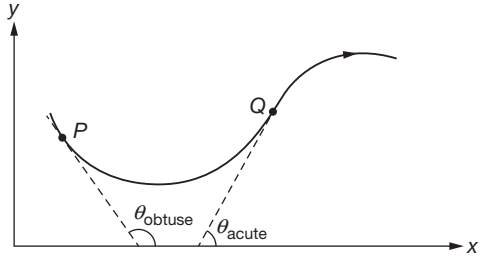
$$\Rightarrow \quad f(x_1)<f(x_2) \text{ then } f(x) \text{ is increasing if } x_1<x_2$$

$$\Rightarrow \quad f(x_1)>f(x_2) \text{ then } f(x) \text{ is decreasing}$$

As shown in the figure, when $f(x)$ is increasing, the tangent to the curve at any point, say P, makes an acute angle with positive x-axis. The slope of the tangent is positive.

Thus, $\tan\theta=\frac{dy}{dx}>0$

As shown in the figure, when $f(x)$ is decreasing, the tangent to the curve at any point, say P, makes an obtuse angle with positive x-axis. The slope of the tangent is negative.



Thus, $\tan\theta=\frac{dy}{dx}<0$

MAXIMUM AND MINIMUM VALUES OF A FUNCTION

Let us suppose a quantity y depends upon another quantity x in a manner shown in the figure. It becomes maximum at x_1 and minimum at x_2. At those points, the tangent to the curve is parallel to the x-axis and hence its slope is $\tan\theta = 0$.

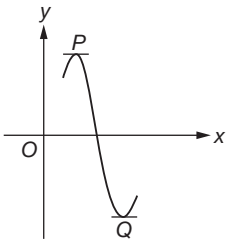
But the slope of curve $= \frac{dy}{dx}$

So, for the maximum or minimum value of y, we have $\frac{dy}{dx} = 0$

Just before the maximum point, the slope is positive. At the maximum point, it is zero And just after the maximum point, it is negative. Thus, $\frac{dy}{dx}$ decreases at the maximum point, i.e., the rate of change of $\frac{dy}{dx}$ is negative at the maximum point,

So $\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0$

$\Rightarrow \frac{d^2y}{dx^2} < 0$



The maximum and minimum of a function

Hence, condition for the maximum value of y is

$\frac{dy}{dx} = 0$ and

$\frac{d^2y}{dx^2} < 0$

Similarly, at a minimum point, the slope changes from negative to positive. The slope increases at such a point. Hence, $\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$

Hence, condition for the minimum value of y is

$\frac{dy}{dx} = 0$ and

$\frac{d^2y}{dx^2} > 0$

As shown in the figure, at the point of maximum and minimum of a function the slope of the tangent at the point is zero.

Thus, $\tan\theta = \frac{dy}{dx} = 0$

Conceptual Note(s)

(a) $f(x)$ is maximum at a point $x = a$, if

(i) $f'(a) = 0$ and

(ii) $f'(x)$ changes in sign from positive to negative when x passes through the point $x = a$. In other words, the second derivative of the function at $x = a$ is negative

i.e., $f''(a) < 0$.

(b) $f(x)$ is minimum at a point $x = a$, if

(i) $f'(a) = 0$ and

(ii) $f'(x)$ changes in sign from negative to positive when x passes through the point $x = a$. In other words, the second derivative of the function at $x = a$ is positive.

i.e., $f''(a) > 0$.

ILLUSTRATION 31

Consider a function $y = \sin x + \cos x$. Find the maximum value of the function.

SOLUTION

$y = \sin x + \cos x$

$\Rightarrow \frac{dy}{dx} = \cos x - \sin x$

For a function to be a MAXIMUM

$\Rightarrow \frac{dy}{dx} = 0$

$\Rightarrow \cos x - \sin x = 0$

$\Rightarrow \tan x = 1$

$\Rightarrow \alpha = \frac{\pi}{4}$

Verification

$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\text{at } x=\frac{\pi}{4}} = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = -\sqrt{2}$$

Since, at $x = \frac{\pi}{4}$, $\frac{d^2y}{dx^2} < 0$

So y is MAXIMUM at $x = \frac{\pi}{4}$

Maximum Value

$$y = \sin x + \cos x$$

$$\Rightarrow y|_{\text{at } x=\frac{\pi}{4}} = \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = \sqrt{2}$$

ILLUSTRATION 32

The height reached in time t by a particle thrown upward with a speed u is given by $h = ut - \frac{1}{2}gt^2$. Find the time taken in reaching the maximum height.

SOLUTION

For maximum height $\frac{dh}{dt} = 0$

$$\frac{d}{dt}\left[ut - \frac{1}{2}gt^2\right] = u - \frac{2gt}{2} = 0$$

$$\Rightarrow t = \frac{u}{g}$$

ILLUSTRATION 33

The distance travelled by a body as a function of time is given by $x = t^3 - 3t^2 + 6$, where x is m and t is in S. Find the maximum and minimum displacement of the body from the origin. Also find the time at which it occurs.

SOLUTION

Differentiating x with respect to t, we get

$$\frac{dx}{dt} = 3t^2 - 6t$$

For maximum and minimum displacements $\frac{dx}{dt} = 0$

$$\therefore \quad 3t^2 - 6t = 0$$

$$\Rightarrow \quad 3t(t-2) = 0$$

$$\Rightarrow \quad t = 0 \text{ and } t = 2 \text{ s}$$

Once again differentiating w.r.t. time t, we get

$$\frac{d^2x}{dt^2} = 6t - 6$$

At $t = 0$, $\frac{d^2x}{dt^2} = -6 < 0$ (maximum)

At $t = 2$ s, $\frac{d^2x}{dt^2} = 6 > 0$ (minimum)

The maximum displacement occurs at $t = 0$ and is equal to

$$(x)_{\max} = (0)^3 - 3(0)^2 + 6 = 6 \text{ m}$$

The minimum displacement occurs at $t = 2$ s and is equal to

$$(x)_{\min} = (2)^3 - 3(2)^2 + 6 = 2 \text{ m}$$

$\frac{dy}{dx}$ AS RATE MEASURE

The rate of change of a quantity (y) with respect to another quantity (x) is defined as the ratio of the change of y to change is x, however small the change in x may be

If Δy be the change in y corresponding to a change Δx is x, then according to the definition, the rate of change of y with respect to x is the limiting value of the ratio $\frac{\Delta y}{\Delta x}$ when Δx tends to zero.

So, rate of change of y with respect to x is

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Conceptual Note(s)

When we simply say rate of change y, we mean change of y with respect to time. So, the rate of change of y is $\frac{dy}{dt}$.

ILLUSTRATION 34

The area of a blot of ink is growing such that after t second, its area is given by $A = (3t^2 + 7)$ cm^2. Calculate the rate of increase of area at $t = 5$ seconds.

SOLUTION

Given $A = 3t^2 + 7$

$$\therefore \quad \frac{dA}{dt} = 6t$$

$$\Rightarrow \quad \left(\frac{dA}{dt}\right)_{t=5} = 6 \times 5 = 30 \text{ cms}^{-2}$$

ILLUSTRATION 35

A metal ring is being heated so that at any instant of time t in second, its area is given by $A = 3t^2 + \frac{t}{3} + 2 \text{ m}^2$. What will be the rate of increase of area at $t = 10$ sec.

SOLUTION

Rate of increase of area

$$\frac{dA}{dt} = \frac{d}{dt}\left(3t^2 + \frac{t}{3} + 2\right) = 6t + \frac{1}{3}$$

$$\left(\frac{dA}{dt}\right)_{t=10\,\text{sec}} = 6 \times 10 + \frac{1}{3} = \frac{181}{3} \text{ m}^2\text{s}^{-1}$$

ILLUSTRATION 36

Find the rate of change in area of a square of side 4 cm when its side is increasing at the rate of 0.01 cm per second.

SOLUTION

Let A be the area of the square; a be the length of the side

We have

$$A = a^2$$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{d}{dt}\left(a^2\right) = \frac{d}{da}\left(a^2\right) \times \frac{da}{dt}$$

$$\Rightarrow \quad \frac{dA}{dt} = 2a \times \frac{da}{dt}$$

$$\Rightarrow \quad \frac{dA}{dt} = 2 \times 4 \times .01 = 0.8 \text{ cms}^{-2}$$

ILLUSTRATION 37

The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cms^{-1}. Determine the rate of increase in its volume when the radius is 1 cm.

SOLUTION

Volume of the spherical bubble $V = \frac{4}{3}\pi R^3$

Differentiating both sides w.r.t. time

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi R^3\right)$$

$$\Rightarrow \quad \frac{dV}{dt} = \left(\frac{4}{3}\pi\right)\left(3R^2 \frac{dR}{dt}\right)$$

$$\Rightarrow \quad \frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$$

at $R = 1$ cm and when $\frac{dR}{dt} = \frac{1}{2}$ cms^{-1}, we have

$$\frac{dV}{dt} = 4\pi \times (1)^2 \times \frac{1}{2} = 2\pi \text{ cm}^3\text{s}^{-1}$$

INTEGRATION: AN INTRODUCTION

The word "integral" simply means "the whole", and the process of adding or summing up a large number of infinitesimal elements of a quantity is called integration.

If x be supposed to be made up of a large number of infinitesimal elements each equal to dx, it is obvious that if we add up all these dx together, we shall get the total x. Mathematically we put it as $\int dx = x$ (read it as integral of dx equals x). **Integration is the inverse operation of differentiation.**

Integration of $f(x)$ simply means finding the function $I(x)$ whose derivative is equal to $f(x)$.

Mathematically, $f(x) = \frac{dI}{dx}$

$$\Rightarrow \quad I(x) = \int f(x)\, dx$$

In the above expression, $f(x)$ is called the integrand. The symbol, $\int$ is the symbol of integration and dx indicates the variable of integration.

The function $I(x)$ is also known sometimes as the anti derivative of $f(x)$.

"The result of an indefinite integral, when differentiated, will give you the function that has been integrated."

Table 1.3 Some Indefinite Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1}$ (provided $n \neq -1$)	$\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax-1)$
$\int \frac{dx}{x} = \int x^{-1} dx = \ln x$	$\int \frac{dx}{a+be^{cx}} = \frac{x}{a} - \frac{1}{ac}\ln(a+be^{cx})$
$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)$
$\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax)$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	$\int \tan(ax)dx = -\frac{1}{a}\ln(\cos(ax)) = \frac{1}{a}(\sec(ax))$
$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$ $(a^2-x^2>0)$	$\int \cot(ax)dx = \frac{1}{a}\ln(\sin(ax))$
$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$ $(x^2-a^2>0)$	$\int \sec(ax)dx = \frac{1}{a}\ln(\sec(ax)+\tan(ax)) = \frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right]$
$\int \frac{xdx}{a^2 \pm x^2} = \pm\frac{1}{2}\ln(a^2 \pm x^2)$	$\int \text{cosec}(ax)dx = \frac{1}{a}\ln(\text{cosec}(ax)-\cot(ax)) = \frac{1}{a}\left[\ln\left(\tan\frac{ax}{2}\right)\right]$
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)(a^2-x^2>0)$	$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$	$\int \cos^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x+\sqrt{x^2-a^2}\right)$	$\int \frac{dx}{\sin^2(ax)} = \int \text{cosec}^2(ax)dx = -\frac{1}{a}\cot(ax)$
$\int \frac{xdx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$	$\int \frac{dx}{\cos^2(ax)} = \int \sec^2(ax)dx = \frac{1}{a}\tan(ax)$
$\int \sqrt{a^2-x^2}dx = -\frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right)$	$\int \tan^2(ax)dx = \frac{1}{a}(\tan(ax)) - x$
$\int x\sqrt{a^2-x^2}dx = -\frac{1}{3}\left(a^2-x^2\right)^{\frac{3}{2}}$	$\int \cot^2(ax)dx = -\frac{1}{a}(\cot(ax)) - x$
$\int \sqrt{x^2 \pm a^2}dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln(x+\sqrt{x^2 \pm a^2})\right]$	$\int \sin^{-1}(ax)dx = x(\sin^{-1}(ax)) + \frac{\sqrt{1-a^2x^2}}{a}$
$\int x\sqrt{x^2} \pm a^2 dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2}$	$\int \cos^{-1}(ax)dx = x(\cos^{-1}(ax)) - \frac{\sqrt{1-a^2x^2}}{a}$
$\int e^{ax}dx = \frac{1}{a}e^{ax}$	$\int \tan^{-1}(ax)dx = x(\tan^{-1}(ax)) - \frac{1}{2a}\ln(1+a^2x^2)$
$\int e^{-ax}dx = -\frac{1}{a}e^{-ax}$	$\int \cot^{-1}(ax)dx = x(\cot^{-1}(ax)) + \frac{1}{2a}\ln(1+a^2x^2)$

(an arbitrary constant should be added to each of these integrals)

After each integral one must add a constant. The reason for adding a constant is given as follows:
The differential of x^{n+1} is $(n+1)x^n dx$. The differential of $\left(x^{n+1}+c\right)$ is $(n+1)x^n dx$ because the differential co-efficient of a constant is zero. Hence in general one has to add a constant after performing an integration. This constant is called the **constant of integration**.

RULES FOR INTEGRATION

(a) $\int c\,dx = c\int dx$, where c is a constant.

(b) $\int (u \pm v)dx = \int u\,dx \pm \int v\,dx$, where u and v are the function of x.

(c) $\int u\,dv = uv - \int v\,du$

$$\left\{\because \int u\,dv + \int v\,du = \int d(uv) = uv\right\}$$

ILLUSTRATION 38

Evaluate $\int dx$.

SOLUTION

Let $I = \int dx = \int (1)dx = \int x^0 dx = \dfrac{x^0+1}{0+1}$

$\Rightarrow \quad I = x + c$, where c is a constant of integration.

ILLUSTRATION 39

Evaluate $\int x^2 dx$

SOLUTION

$I = \int x^2 dx = \dfrac{x^{2+1}}{2+1} = \dfrac{x^3}{3} + c$, where c is a constant of integration.

ILLUSTRATION 40

Evaluate $\int \dfrac{1}{x^2} dx$

SOLUTION

$$I = \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-2}+1}{-2+1} = -\frac{1}{x} + c$$

ILLUSTRATION 41

Find $\int \sin^2 x\,dx$.

SOLUTION

Let $I = \int \sin^2 x\,dx$

Since, $\sin^2 x = \dfrac{1-\cos(2x)}{2}$

$$\Rightarrow \quad I = \int\left(\frac{1-\cos 2x}{2}\right)dx = \frac{1}{2}\int (1-\cos 2x)\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}\left(\int dx - \int \cos(2x)\,dx\right)$$

$$\Rightarrow \quad I = \frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right) + c$$

where c is a constant of integration.

ILLUSTRATION 42

Integrate the following w.r.t. x.

(a) $x^{1/2}$ **(b)** $\cot^2 x$ **(c)** $\dfrac{1}{1-\sin x}$

SOLUTION

(a) $\int x^{\frac{1}{2}} dx = \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \dfrac{2}{3}\left(x^{\frac{3}{2}}\right)$

(b) $I = \int \cot^2 x\,dx$

$$\Rightarrow \quad I = \int (\operatorname{cosec}^2 x - 1)dx$$

$$\Rightarrow \quad I = \int \operatorname{cosec}^2 x\,dx - \int dx$$

$$\Rightarrow \quad I = -\cot x - x$$

(c) $I = \int \dfrac{1}{1-\sin x} dx$

$$\Rightarrow \quad I = \int\left(\frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}\right)dx$$

$$\Rightarrow \quad I = \int \frac{1+\sin x}{1-\sin^2 x} dx$$

$$\Rightarrow \quad I = \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} dx$$

$$\Rightarrow \quad I = \int (\sec^2 x + \tan x \sec x)dx = \tan x + \sec x$$

ILLUSTRATION 43

Evaluate $\int \cos^2 x\, dx$

SOLUTION

Let $I = \int \cos^2 x\, dx$

$$\Rightarrow \quad I = \int \left(\frac{1+\cos(2x)}{2} \right) dx \quad \left\{ \because \cos^2 x = \frac{1+\cos(2x)}{2} \right\}$$

$$\Rightarrow \quad I = \frac{1}{2} \int \left[dx + \int \cos 2x \right] dx$$

$$\Rightarrow \quad I = \frac{1}{2} \left[\int dx + \int \cos(2x)\, dx \right] = \frac{1}{2} \left(x + \frac{\sin(2x)}{2} \right) + c$$

ILLUSTRATION 44

Evaluate $\int e^{5x}\, dx$.

SOLUTION

Let $I = \int e^{5x} dx = \frac{e^{5x}}{5} + c$, where c is a constant of integration.

DEFINITE INTEGRALS

When an integral is defined between two limits, it is called a **definite integral**. The lower value of the limit is called the **lower limit** and the higher value of the limit is called **the upper limit**.

For example, if the integral of the function $f(x)$ is to be determined between the limits, $x = a$ and $x = b$, we represent it symbolically as

$$\int_a^b f(x)\, dx$$

where b is the upper limit and a is the lower limit. We read it as "integral from a to b of the function $f(x)$ with respect to x".

If $\int f(x)\, dx = \phi(x) + c$, then

$$\int_a^b f(x)\, dx = \left(\phi(x) + c \right) \Big|_a^b$$

$$\Rightarrow \quad \int_a^b f(x)\, dx = \left(\phi(b) + c \right) - \left(\phi(a) + c \right) = \phi(b) - \phi(a)$$

The constant of integration has disappeared during the process of integrating a function within limits.

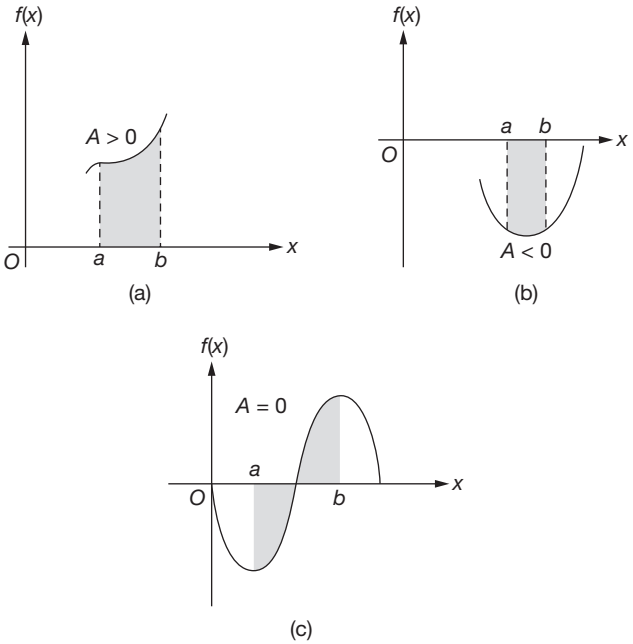
GEOMETRICAL INTERPRETATION OF DEFINITE INTEGRATION

As we have learnt, the graphical interpretation of differentiation is finding the slope of a curve. Integration also has a simple graphical meaning. It is related to finding the area under a curve.

If a function $f(x)$ is expressed graphically in the form $f(x)$ vs x, the **area under the curve between the limits** a **and** b means the area bounded by the curve of $f(x)$, the x-axis and two lines $x = a$ and $x = b$.

The area under the graph of a positive function is defined to be positive. The area under (actually, above) the graph of a negative function is defined to be negative. As shown in the figure (c), positive and negative area add algebraically and may even cancel. **The total area between definite limits of** x **is called a definite integral.** The notation for the definite integral is

$$\text{Area} = A = \int_a^b f(x)\, dx$$



Definition of the definite integral as area under the curve. Area above the x-axis is defined to be positive, area below the x-axis is negative.

Conceptual Note(s)

A definite integral between fixed limits is a fixed quantity, not a function. It has a specific numerical value and generally has a unit, which need not be a unit of area. Just as the idea of slope is generalized from its purely geometric meaning and acquires a unit determined by the quotient of the vertically plotted quantity and the horizontally plotted quantity, the idea of areas is also generalized and acquires a **unit determined by the product of the vertically and horizontally plotted quantities**.

ILLUSTRATION 45

Evaluate $\int_2^3 x^2 dx$

SOLUTION

Let $I = \int_2^3 x^2 dx = \left(\frac{x^3}{3}\right)\Big|_2^3 = \left[\frac{3^3}{3} - \frac{2^3}{2}\right]$

$\Rightarrow \quad I = \frac{27}{3} - \frac{8}{2} = \frac{54-24}{6} = \frac{30}{6} = 5$

ILLUSTRATION 46

Evaluate $\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta.$

SOLUTION

Let $I = \int_0^{\frac{\pi}{2}} \sin\theta d\theta = (-\cos\theta)\Big|_0^{\frac{\pi}{2}}$

$\Rightarrow \quad I = -\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] = -[0-1] = 1$

ILLUSTRATION 47

Find $\int_a^b \frac{dr}{r}$.

SOLUTION

Let $I = \int_a^b \frac{dr}{r}$

$\Rightarrow \quad I = \int_a^b \frac{dr}{r} = \left(\log_e r\right)\Big|_a^b$

$\Rightarrow \quad I = \log_e b - \log_e a = \log_e\left(\frac{b}{a}\right)$

ILLUSTRATION 48

The momentum p of a particle changes with time t according to the relation $\frac{dp}{dt} = (10 + 2t)$. If the momentum is zero at $t = 0$, what will be the momentum at $t = 10$ s?

SOLUTION

Given

$\frac{dp}{dt} = (10 + 2t)$

$\Rightarrow \quad dp = (10 + 2t)dt$

$\Rightarrow \quad \int_0^p dp = \int_0^{10} (10 + 2t)dt$

$\Rightarrow \quad p = \left(10t + t^2\right)\Big|_0^{10}$

$\Rightarrow \quad p = \left[10(10) + (10)^2\right] - \left[10(0) + (0)^2\right]$

$\Rightarrow \quad p = 200 \text{ kgms}^{-1}$

ILLUSTRATION 49

Evaluate the integral $\int_0^t A\sin(\omega t)dt$, where A and ω are constants.

SOLUTION

Let $I = \int_0^t A\sin(\omega t)\,dt$

$$\Rightarrow \quad I = A\left[-\frac{\cos(\omega t)}{\omega}\Big|_0^t\right]$$

$$\Rightarrow \quad I = -\frac{A}{\omega}\left[\cos(\omega t) - \cos 0^\circ\right]$$

$$\Rightarrow \quad I = \frac{A}{\omega}\left[1 - \cos(\omega t)\right]$$

ILLUSTRATION 50

The velocity v and displacement x of a particle executing simple harmonic motion are related as

$$v\frac{dv}{dx} = -\omega^2 x$$

At $x = 0$, $v = v_0$. Find the velocity v when the displacement becomes x.

SOLUTION

The given equation is

$$v\frac{dv}{dx} = -\omega^2 x$$

$$\Rightarrow \quad v\,dv = -\omega^2 x\,dx$$

$$\Rightarrow \quad \int_{v_0}^{v} v\,dv = -\omega^2 \int_0^x x\,dx$$

$$\Rightarrow \quad \left|\frac{v^2}{2}\right|_{v_0}^{v} = -\omega^2 \left|\frac{x^2}{2}\right|_0^x$$

$$\Rightarrow \quad v^2 - v_0^2 = -\omega^2 x^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 - \omega^2 x^2}$$

PRACTICE EXERCISE

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. If $ax^2+bx+c=0$, $a\neq 0$, then

(A) $x=-\dfrac{b\pm\sqrt{b^2-4ac}}{2}$ (B) $x=-\dfrac{b\pm\sqrt{b^2-4ac}}{4}$

(C) $x=-\dfrac{b\pm\sqrt{b^2-4ac}}{2a}$ (D) $x=-\dfrac{b\pm\sqrt{b^2-4ac}}{4a}$

2. $2^x3^y=72$, then

(A) $x=2$, $y=3$ (B) $x=3$, $y=2$

(C) $x=-2$, $y=-3$ (D) $x=-3$, $y=-2$

3. $\log_e x+\log_e y=$

(A) $\log_e(x+y)$ (B) $\log_e(xy)$

(C) $\log_e\left(\dfrac{x}{y}\right)$ (D) $\log_e\left(\dfrac{y}{x}\right)$

4. If $\log_e x+\log_e y=2\log_e z$, then

(A) $2z=x+y$ (B) $z=2x+2y$

(C) $z=xy$ (D) $z=\sqrt{xy}$

5. $\log_e x=k\log_{10}(x^2)$, then k equals

(A) 2.303 (B) 4.606

(C) 1.151 (D) 3.303

6. If $\log_e(x^4)=k\log_{10}x$, then k equals

(A) 2.303 (B) 4.606

(C) 9.212 (D) 13.818

7. For $x\ll 1$, the value of $(1+x)^n$ is

(A) $1-nx$ (B) $\dfrac{1}{2}(1-nx)$

(C) $1+nx$ (D) $\dfrac{1}{2}(1+nx)$

8. $\log_a x$ equals

(A) $\log_e\left(\dfrac{x}{a}\right)$ (B) $\dfrac{\log_e x}{\log_a e}$

(C) $\dfrac{\log_e x}{\log_e a}$ (D) $\dfrac{\log_x e}{\log_a e}$

9. $\log_2(8)=$

(A) 1 (B) 3

(C) 4 (D) 6

10. $\log_{10}100=$

(A) 1 (B) 2

(C) 3 (D) 4

11. $\log_e(mn)=$

(A) $\log_e m\times\log_e n$ (B) $\log_e m-\log_e n$

(C) $\log_e\left(\dfrac{m}{n}\right)$ (D) $\log_e m-\log_e\left(\dfrac{1}{n}\right)$

12. If $3x^2+8x+5=0$, then

(A) $x=1$ (B) $x=\dfrac{5}{3}$

(C) $x=-1$ (D) $x=-\dfrac{5}{3}$

13. $\log_b a\times\log_a b=$

(A) 0 (B) $\log_a(ab)$

(C) 1 (D) $\log_b(ab)$

14. In $\log_a x$, the value of a must be

(A) between 0 and 1
(B) positive, but not 1
(C) positive but not zero
(D) in some other interval

15. The ratio of area of circle of radius r and surface area of sphere of radius r, is

(A) $\dfrac{1}{4}$ (B) 4

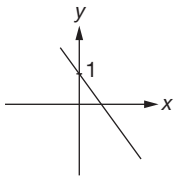
(C) $\dfrac{3}{4r}$ (D) $\dfrac{1}{4r}$

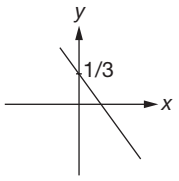
16. An equation of straight line $ay=bx+c$ is given, where a, b and c are constants. The slope of the given straight line is

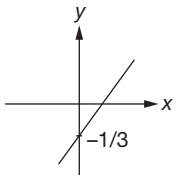
(A) $-\dfrac{a}{b}$ (B) $\dfrac{b}{a}$

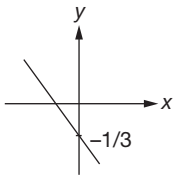
(C) b (D) c

17. Correct graph of $3x + 4y + 1 = 0$ is

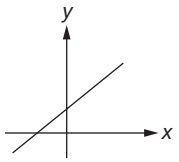
(A) 

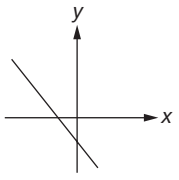
(B) 

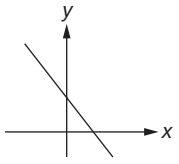
(C) 

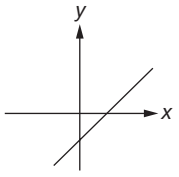
(D) 

18. Graph of $y = 2x - 3$ is

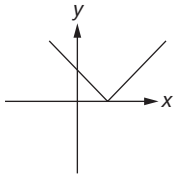
(A) 

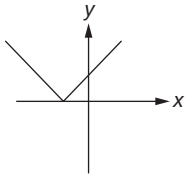
(B) 

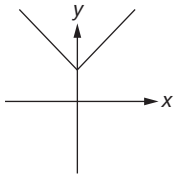
(C) 

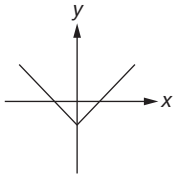
(D) 

19. Correct graph of $y = |3x + 4|$ is

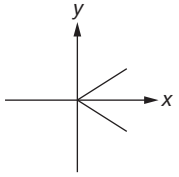
(A) 

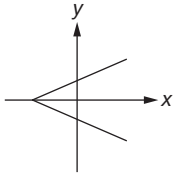
(B) 

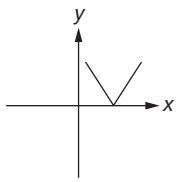
(C) 

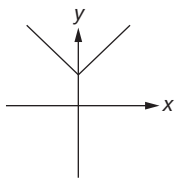
(D) 

20. Correct graph of $|y| = x + 1$ is

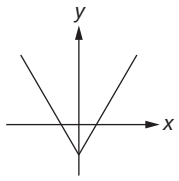
(A) 

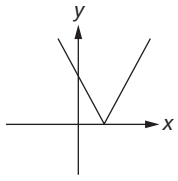
(B) 

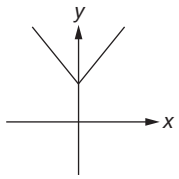
(C) 

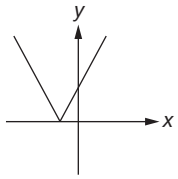
(D) 

21. Graph of $y = |x| + 2$ is

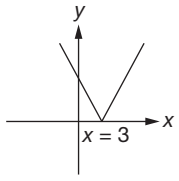
(A) 

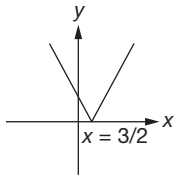
(B) 

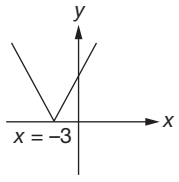
(C) 

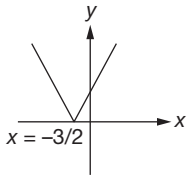
(D) 

22. Graph of $y = |2x - 3|$ is

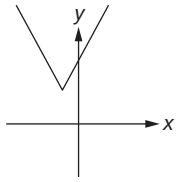
(A) 

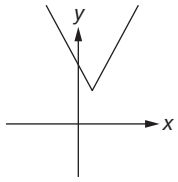
(B) 

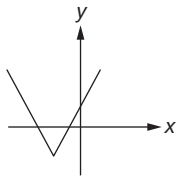
(C) 

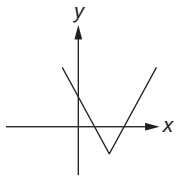
(D) 

23. Graph of $y = |2x + 1| + 1$ is

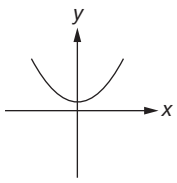
(A) 

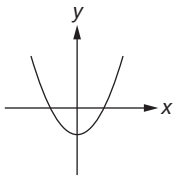
(B) 

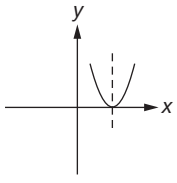
(C) 

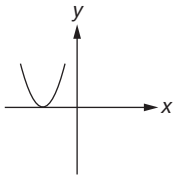
(D) 

24. Correct graph of $y-1=x^2$ is

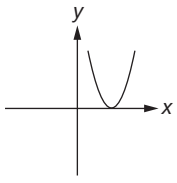
(A) 

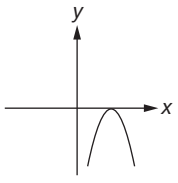
(B) 

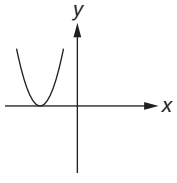
(C) 

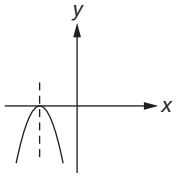
(D) 

25. Correct graph of $y=-(x+2)^2$ is

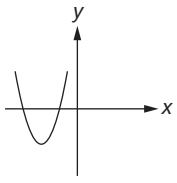
(A) 

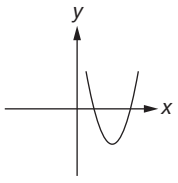
(B) 

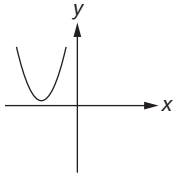
(C) 

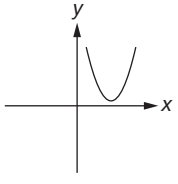
(D) 

26. Correct graph of $y=2x^2+3x+1$ is

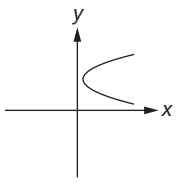
(A) 

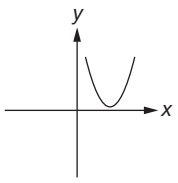
(B) 

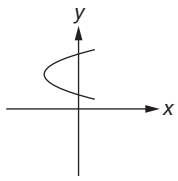
(C) 

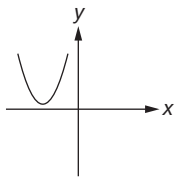
(D) 

27. Graph of $y=2(x-1)^2+2$ is

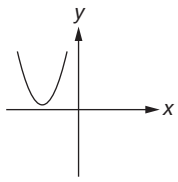
(A) 

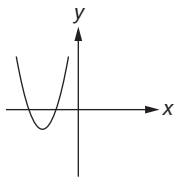
(B) 

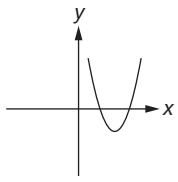
(C) 

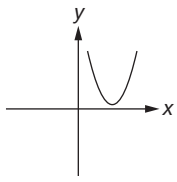
(D) 

28. Graph of $y=3x^2-4x+1$ is

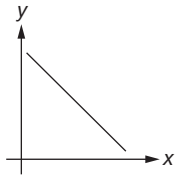
(A) 

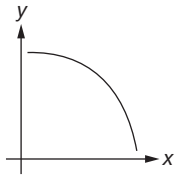
(B) 

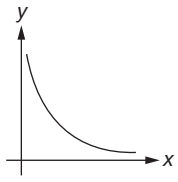
(C) 

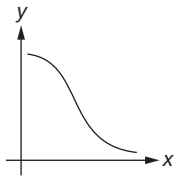
(D) 

29. Graph of $x^2y=2$ is best represented by

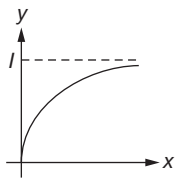
(A) 

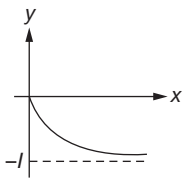
(B) 

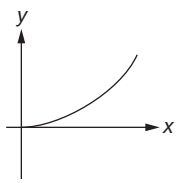
(C) 

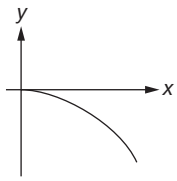
(D) 

30. Graph of $y=1-e^{-x}$ is best represented by (for $x>0$)

(A) 

(B) 

(C) 

(D) 

31. 1 radian is equal to

(A) $\frac{\pi}{180}$degree (B) $\frac{180}{\pi}$degree

(C) $\frac{90}{\pi}$degree (D) $\frac{18}{\pi}$degree

32. $\tan\left(\frac{\pi}{6}\right)=$

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $-\sqrt{3}$ (D) $-\frac{1}{\sqrt{3}}$

33. $\tan\left(\frac{3\pi}{4}\right)=$

(A) 1 (B) –1

(C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

34. $\tan\left(\frac{3\pi}{2}+\theta\right)=$

(A) $\cot\theta$ (B) $-\tan\theta$

(C) $-\cot\theta$ (D) $\tan\theta$

35. $\cos\left(\frac{3\pi}{2}\right)=$

(A) 1 (B) –1

(C) $\frac{1}{\sqrt{2}}$ (D) zero

36. $\sin(2\theta)=$

(A) $2\sin\theta$ (B) $2\sin\theta\cos\theta$

(C) $\frac{1}{2}\sin\theta\cos\theta$ (D) $\sin\theta\cos\theta$

37. $(1-\cos\theta)$ equals

(A) $2\cos^2\left(\frac{\theta}{2}\right)$ (B) $2\cos^2\theta$

(C) $2\sin^2\theta$ (D) $2\sin^2\left(\frac{\theta}{2}\right)$

38. $(1+\cos\theta)$ equals

(A) $2\cos^2\left(\frac{\theta}{2}\right)$ (B) $2\cos^2\theta$

(C) $2\sin^2\theta$ (D) $2\sin^2\left(\frac{\theta}{2}\right)$

39. $\tan(2\theta)$ equals

(A) $\frac{2\tan\theta}{1-\tan^2\theta}$ (B) $2\tan\theta$

(C) $\frac{\tan\theta}{1-\tan^2\theta}$ (D) $\frac{2\tan\theta}{1+\tan^2\theta}$

40. $\cos^2\theta-\sin^2\theta$ equals

(A) $\cos(2\theta)$ (B) $\sin(2\theta)$

(C) $\tan(2\theta)$ (D) $\cot(2\theta)$

41. $\sin(2\theta)$ equals

(A) $\frac{2\tan\theta}{1+\tan^2\theta}$ (B) $2\sin\theta$

(C) $\frac{\tan\theta}{1+\tan^2\theta}$ (D) $\frac{2\tan\theta}{1-\tan^2\theta}$

42. The value of $\sin(15°)$ is

(A) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}+1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (D) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

43. The value of $\sin(75°)$ is

(A) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}+1}{4}$ (D) $\frac{\sqrt{3}-1}{4}$

44. The value of $\cos(75°)$ is

(A) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}+1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (D) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

45. $(\cos A-\cos B)=$

(A) $2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

(B) $2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

(C) $2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)$

(D) $2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

46. $\sin^2(37°)+\sin^2(53°)=$

(A) 0 (B) 1

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{4}{5}$

47. $\sin\theta$ can never have a value

(A) 1 (B) –1

(C) $\frac{1}{4}$ (D) 2

48. The value of $\sin^2\theta$ always lies between

(A) –1 and 1 (B) –1 and zero

(C) zero and 1 (D) zero and 2

49. $\sin(100\pi)$ is equal to

(A) 1 (B) 100

(C) zero (D) $\frac{1}{2}$

50. $\cos(180-\theta)$ is equal to

(A) $-\cos\theta$ (B) $\cos\theta$

(C) $\sin\theta$ (D) $-\sin\theta$

51. $\lim\limits_{x\to3}\frac{x^2-9}{x-3}=$

(A) 3 (B) 4

(C) 5 (D) 6

52. $\lim\limits_{x\to-5}\frac{x^2-25}{x+5}=$

(A) 10 (B) –10

(C) 0 (D) 5

53. $\frac{d}{dx}\left(x^{\frac{5}{2}}\right)=$

(A) $\frac{5}{2}x^{\frac{5}{2}}$ (B) $\frac{5}{2}x^{\frac{3}{2}}$

(C) $\frac{5}{2}x^{\frac{7}{2}}$ (D) $\frac{2}{7}x^{\frac{7}{2}}$

54. $\frac{d}{dx}(u+v-w)=$

(A) $\left(\frac{du}{dx}\pm\frac{dv}{dx}\right)\frac{dw}{dx}$ (B) $-\frac{du}{dx}+\frac{dv}{dx}-\frac{dw}{dx}$

(C) $\frac{du}{dx}-\frac{dw}{dx}+\frac{dv}{dx}$ (D) $\frac{du}{dx}-\frac{dv}{dx}+\frac{dw}{dx}$

55. $\frac{d}{dx}(uvwz)=$

(A) $uvw\frac{dz}{dx}+uvz\frac{dw}{dx}-uwz\frac{dv}{dx}-vwz\frac{du}{dx}$

(B) $-uvw\frac{dz}{dx}-uvz\frac{dw}{dx}-uwz\frac{dv}{dx}-vwz\frac{du}{dx}$

(C) $uwz\frac{dv}{dx}+vwz\frac{du}{dx}+uvz\frac{dw}{dx}+uvw\frac{dz}{dx}$

(D) $-uvw\frac{dz}{dx}-uvz\frac{dw}{dx}+uwz\frac{dv}{dx}+vwz\frac{du}{dx}$

56. $\frac{d}{dx}\left(\frac{u}{v}\right)=$

(A) $\frac{1}{v}\left(\frac{du}{dx}-\frac{u}{v}\frac{dv}{dx}\right)$ (B) $\frac{1}{u}\left(\frac{v}{u}\frac{du}{dx}-\frac{dv}{dx}\right)$

(C) $\frac{1}{u}\left(\frac{u}{v}\frac{du}{dx}-\frac{dv}{dx}\right)$ (D) $\frac{1}{v}\left(\frac{u}{v}\frac{dv}{dx}-\frac{du}{dx}\right)$

57. $\frac{d}{dx}(u^n)=$

(A) nu^{n-1} (B) $nu^{n-1}\frac{du}{dx}$

(C) zero (D) None of these

58. $\frac{d}{dx}(\sin u)=$

(A) $\cos u$ (B) $-\cos u$

(C) $(\cos u)\frac{du}{dx}$ (D) $-(\cos u)\frac{du}{dx}$

59. $\frac{d}{dx}(\tan x)=$

(A) $\sec x\tan x$ (B) $1+\tan^2 x$

(C) $\sec x$ (D) None of these

60. $\frac{d}{dx}(\sec x)=$

(A) $-\sec x\tan x$ (B) $\sec^2 x\cot x$

(C) $\frac{\sin x}{\cos^2 x}$ (D) $\frac{\cos x}{\sin^2 x}$

61. $\frac{d}{dx}(\operatorname{cosec} x)=$

(A) $-\operatorname{cosec} x\tan x$ (B) $-\frac{\sin x}{\cos^2 x}$

(C) $-\frac{\cos x}{\sin^2 x}$ (D) $\frac{\cos x}{\sin^2 x}$

62. $\frac{d}{dx}(x^5 + x^7 + x^9) =$

(A) $\frac{x^6}{6} + \frac{x^8}{8} + \frac{x^{10}}{10}$ (B) $5x^4 + 7x^6 + 9x^8$

(C) $\frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9}$ (D) $5x^3 + 7x^5 + 9x^7$

63. $\frac{d}{dx}(\log_e x) =$

(A) $\frac{1}{e}$ (B) $\frac{1}{\log_e x}$

(C) $\frac{1}{x}$ (D) $x\log_e x - x$

64. $\frac{d}{dx}(e^{2x}) =$

(A) e^{2x} (B) e^x

(C) $2e^{2x}$ (D) $2e^x$

65. $\frac{d}{dx}(x\log_e x - x) =$

(A) zero (B) 1

(C) $\log_e x$ (D) $x\log_e x$

66. The tangent to the curve $y^2 = 4x$ at $(1, 2)$ is inclined to the x-axis at an angle of

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

67. The maximum value of the function $y = \sin x + \cos x$ is

(A) 1 (B) 2

(C) $\sqrt{2}$ (D) $-\sqrt{2}$

68. $\frac{d}{dx}[\log_e(\sin x)] =$

(A) $\cot x$ (B) $-\cot x$

(C) $\tan x$ (D) $-\tan x$

69. $\frac{d}{dx}[\log_e(\cos x)] =$

(A) $\cot x$ (B) $-\cot x$

(C) $\tan x$ (D) $-\tan x$

70. $\frac{d}{dx}[\log_e(\sec x + \tan x)] =$

(A) $\sec x$ (B) $-\sec x$

(C) $\operatorname{cosec} x$ (D) $-\operatorname{cosec} x$

71. $\frac{d}{dx}[\log_e(\operatorname{cosec} x + \cot x)] =$

(A) $-\sec x$ (B) $\sec x$

(C) $-\operatorname{cosec} x$ (D) $\operatorname{cosec} x$

72. $\frac{d}{dx}(x^2 \sin x) =$

(A) $x^2 \cos x$ (B) $x^2 \sin x + x^2 \cos x$

(C) $x^2 \cos x + 2x \cos x$ (D) $x^2 \cos x + 2x \sin x$

73. The tangent to the curve $y = 3x^2 - 5$ at the point $(2, 7)$ makes an angle θ with the positive x-axis. Then

(A) $\tan\theta = 7$ (B) $\tan\theta = 10$

(C) $\tan\theta = 11$ (D) $\tan\theta = 12$

74. $\frac{d}{dx}(\log_e x + \tan x) =$

(A) $\frac{1}{x} + \sec x \tan x$ (B) $x + \sec x \tan x$

(C) $\frac{1}{x} + \sec^2 x$ (D) $\frac{1}{x} - \sec^2 x$

75. $\frac{d}{dx}(e^x \sin x) =$

(A) $e^x \cos x$ (B) $e^x(\cos x + \sin x)$

(C) $-e^x \cos x$ (D) $e^x(\sin x - \cos x)$

76. $\frac{d}{dx}(x\log_e x) =$

(A) 1 (B) $1 + \log_e x$

(C) $1 - \log_e x$ (D) $\log_e x$

77. $\frac{d}{dx}(e^x + 6^x) =$

(A) $e^{x+6}\log_e 6$ (B) $6^{x+e}\log_6 e$

(C) $e^x \log_e 6$ (D) $e^x + 6^x \log_e 6$

78. $\frac{d}{dx}\left(\frac{1}{x} + \tan x + x^2 + \log_e x\right) =$

(A) $-\frac{1}{x^2} + \sec^2 x + 2x + \frac{1}{x}$

(B) $\frac{1}{x^2} - \sec^2 x - 2x + \frac{1}{x}$

(C) $-\frac{1}{x^2} - \sec^2 x + 2x - \frac{1}{x}$

(D) $-\frac{1}{x^2} + \sec x \tan x + 2x + \frac{1}{x}$

79. $\frac{d}{dx}\left(\frac{\tan x+\cot x}{x}\right)=$

(A) $\frac{(\sec^2 x-\operatorname{cosec}^2 x)}{x}$

(B) $\left(\frac{\sec^2 x-\operatorname{cosec}^2 x}{x}\right)-\left(\frac{\tan x+\cot x}{x^2}\right)$

(C) $-\left(\frac{\tan x+\cot x}{x^2}\right)$

(D) None of these

80. $\frac{d}{dx}(e^x\tan x)=$

(A) $e^x\sec x\tan x$ (B) $e^x\sec^2 x$

(C) $e^x(\tan x+\sec^2 x)$ (D) $e^x(\tan x-\sec^2 x)$

81. $\frac{d}{dx}\left(2x^5-4x^2-\frac{1}{\sqrt{x}}\right)=$

(A) $2x^5-8x-\frac{1}{2\sqrt{x}}$ (B) $10x^4-8x-\sqrt{x}$

(C) $10x^4-8x+\frac{1}{2x\sqrt{x}}$ (D) $10x^4-8x-\frac{1}{2\sqrt{x}}$

82. $\frac{d}{dx}\left(\frac{3x^2+1}{2x-1}\right)=$

(A) $\frac{6x}{2x-1}$

(B) $\frac{(2x-1)(6x+1)-(3x^2+1)2x}{(2x-1)^2}$

(C) $\frac{12x^3-6x^2-8x}{4x^2-4x+1}$

(D) None of these

83. $\frac{d}{dx}(e^x\log_e x)=$

(A) $e^x\left(\log_e x+\frac{1}{x}\right)$ (B) $\frac{e^x}{x}$

(C) $e^x\log_e x+1$ (D) None of these

84. $y=\sin x+x^4$

(A) $\frac{dy}{dx}=-\cos x+4x^3$ (B) $\frac{dy}{dx}=\sin x+4x^3$

(C) $\frac{d^2y}{dx^2}=-\sin x+12x^2$ (D) $\frac{d^2y}{dx^2}=-\cos x+6x^2$

85. $y=x\log_e x$

(A) $\frac{dy}{dx}=1$ (B) $\frac{dy}{dx}=\log_e x$

(C) $\frac{d^2y}{dx^2}=\frac{1}{x}$ (D) $\frac{d^2y}{dx^2}=x$

86. $y=e^x\sin x$

(A) $\frac{dy}{dx}=e^x(\cos x-\sin x)$ (B) $\frac{dy}{dx}=e^x\cos x$

(C) $\frac{d^2y}{dx^2}=2e^x\sin x$ (D) $\frac{d^2y}{dx^2}=2e^x\cos x$

87. $\frac{d}{dx}\sqrt{\tan x}$

(A) $2\sec^2 x(\tan x)^{-1/2}$ (B) $\frac{1}{2}\sec^2 x(\tan x)^{-1/2}$

(C) $\frac{1}{2}(\tan x)^{-1/2}$ (D) $2(\tan x)^{-1/2}$

88. $\frac{d}{dx}\sin(\log x)$

(A) $\cos(\log x)$ (B) $\log(\cos x)$

(C) $x\cos(\log x)$ (D) $\frac{\cos(\log x)}{x}$

89. $\frac{d}{dx}\left(\sqrt{2x^2+1}\right)$

(A) $2x(2x^2+1)^{1/2}$ (B) $2x(2x^2+1)^{-1/2}$

(C) $(2x^2+1)^{1/2}$ (D) $(2x^2+1)^{-1/2}$

90. $\frac{d}{dx}\left(e^{\sqrt{2x}}\right)$

(A) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (B) $\sqrt{2x}e^{\sqrt{2x}}$

(C) $e^{\sqrt{2x}}$ (D) $e^{(2x)^{-1/2}}$

91. $\frac{d}{dx}(x^4-2\sin x+3\cos x)$

(A) $4x^3-2\cos x+3\sin x$ (B) $3x^2+2\cos x+3\sin x$

(C) $4x^3+2\cos x-3\sin x$ (D) $4x^3-2\cos x-3\sin x$

92. $\frac{d}{dx}(x^2\sin x\log x)$

(A) $2x\sin x\log x+x^2\cos x\log x+x\sin x$

(B) $x^2\sin x\log x+2x\cos x\log x+x\sin x$

(C) $2x\sin x\log x+x^2\cos x\log x+\sin x$

(D) None of these

93. $\frac{d}{dx}\frac{(x^2+1)}{x+1}$

(A) $\frac{x^2+2x-1}{(x+1)^2}$ (B) $\frac{x^2-2x+1}{(x+1)^2}$

(C) $\frac{x^2+2x-1}{x+1}$ (D) $\frac{x^2+2x+1}{(x+1)^2}$

94. $xy=c^2$, then $\frac{dy}{dx}$

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$

(C) $-\frac{x}{y}$ (D) $-\frac{y}{x}$

95. If $x=at^2$ and $y=2at$, then $\frac{dy}{dx}$

(A) t (B) $\frac{1}{t}$

(C) 1 (D) None of these

96. If $y=\sin^3 x-3\sec^2 x$, then $\frac{dy}{dx}$ at $x=\frac{\pi}{3}$ is

(A) $\frac{9-96\sqrt{3}}{4}$ (B) $\frac{9-86\sqrt{3}}{4}$

(C) $\frac{9-76\sqrt{3}}{2}$ (D) None of these

97. If $y=\sin(2x^2)$, then $\frac{dy}{dx}$ is

(A) $4x\cos(2x^2)$ (B) $2\cos(2x^2)$

(C) $4\cos(2x^2)$ (D) $-4\cos(2x^2)$

98. The minimum value of $y=2x^2-x+1$ is

(A) $-\frac{3}{8}$ (B) $-\frac{5}{8}$

(C) $-\frac{7}{8}$ (D) $-\frac{9}{8}$

99. If $y=\sin^2 x-2\tan^2 x$, then $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$ is

(A) –11 (B) –7

(C) –13 (D) –15

100. If $y=x^3+2x+1$, then $\frac{dy}{dx}$ at $x=1$ is

(A) 6 (B) 7

(C) 8 (D) 5

101. $y=\frac{1+x}{e^x}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x}{e^x}$ (B) $-\frac{x}{e^x}$

(C) $\frac{(x+1)}{e^x}$ (D) None of these

102. $\frac{d}{dx}\left(\frac{x+1}{(x+2)^2}\right)$

(A) $\frac{x}{(x+2)^3}$ (B) $\frac{-x}{(x+2)^3}$

(C) $\frac{1}{(x+2)^3}$ (D) $\frac{-1}{(x+2)^3}$

103. If $y=x^3+2x+1$ then $\frac{dy}{dx}$ at $x=1$ is

(A) 6 (B) 7

(C) 8 (D) 5

104. $y=\frac{1+x}{e^x}$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{x}{e^x}$ (B) $-\frac{x}{e^x}$

(C) $\frac{(x+1)}{e^x}$ (D) None of these

105. $\int \log_e x =$

(A) $\frac{1}{x}$ (B) $\frac{1}{e}$

(C) $x\log_e x-x$ (D) $x\log_e x$

106. $\int x^n dx$ for $n=-1$ is

(A) Not defined (B) $\frac{x^{n+1}}{n+2}$

(C) $\log_e x$ (D) $2\log_e x$

107. $\int(x^5+x^7+x^9)dx =$

(A) $5x^4+7x^6+9x^8$ (B) $\frac{x^5}{5}+\frac{x^7}{7}+\frac{x^9}{9}$

(C) $x^5\left(x+\frac{x^3}{3}+\frac{x^5}{5}\right)$ (D) $\frac{x^6}{6}+\frac{x^8}{8}+\frac{x^{10}}{10}$

108. $\int_a^b 2\frac{dx}{x} =$

(A) $\log_e b - \log_e a$ (B) $2\log_e(b-a)$

(C) $\log_e\left(\frac{b^2}{a^2}\right)$ (D) $2\log_e\left(\frac{a}{b}\right)$

109. $\int_0^x \frac{dx}{2+3x} =$

(A) $\frac{1}{3}\log_e(2+3x)$ (B) $\frac{1}{3}\log(3+2x)$

(C) $\frac{1}{3}\log_e\left(\frac{2+3x}{2}\right)$ (D) $\frac{1}{3}\log_e\left(\frac{2}{2+3x}\right)$

110. $\int 5x^4 dx =$

(A) $4x^3$ (B) $6x^2$

(C) $20x^3$ (D) x^5

111. $\int \sin(2x)dx =$

(A) $-\cos(2x)$ (B) $-\frac{1}{2}\cos(2x)$

(C) $-\cos x$ (D) None of these

112. $\int\left(x+\frac{1}{x}\right)^2 dx =$

(A) $x^3+\frac{1}{x}+2x$ (B) $\frac{x^3}{3}-\frac{1}{x}+2x$

(C) $\frac{x^3}{3}+\frac{1}{x^2}-2x$ (D) $\frac{x^3}{3}-\frac{1}{x^2}+2x$

113. $\int \frac{\text{cosec}^2 x\,dx}{1+\cot x} =$

(A) $-\log|1+\cot x|+C$ (B) $\log|1+\cot x|+C$

(C) $\log|1+\tan x|+C$ (D) $-\log|1+\tan x|+C$

114. Value of $\int_0^1 (3x^2-4x+1)dx$ is

(A) 0 (B) 1

(C) 2 (D) 3

115. The value of $\int_0^{\pi/2} \sin(2\theta)d\theta$ is

(A) 0 (B) 1

(C) –1 (D) 2

116. Value of $\int_3^5 \frac{1}{2x+3}dx$ is

(A) $\ln\left(\frac{13}{9}\right)$ (B) $\frac{1}{2}\ln\left(\frac{13}{9}\right)$

(C) $\frac{1}{2}\ln\left(\frac{15}{9}\right)$ (D) None of these

117. Value of $\int_0^1 \frac{1}{(3-2x)^2}dx$ is

(A) $-\frac{1}{9}$ (B) $-\frac{2}{9}$

(C) $-\frac{4}{9}$ (D) None of these

118. $\int(\sqrt{1+\cos x})dx =$

(A) $\frac{(\sqrt{1+\cos x})^{3/2}}{\frac{3}{2}}+C$ (B) $2\sqrt{2}\cos\frac{x}{2}+C$

(C) $2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$ (D) None of these

119. $\int_0^2 2t\,dt$ is equal to

(A) 0 (B) 4

(C) 2 (D) $\frac{1}{2}$

120. $\int_{\pi/6}^{\pi/2} \sin x\,dx$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) 0

121. $\int \frac{dt}{(6t-1)}$ is equal to

(A) $\frac{1}{6}\log_e(6t-1)+C$ (B) $\log_e(6t-1)+C$

(C) $-\frac{1}{6}\log_e(6t-1)+C$ (D) None of these

122. $\int (4\cos t + t^2)dt$ is equal to

(A) $-4\sin t + \frac{t^3}{3}+C$ (B) $-4\sin t + t^2 + C$

(C) $4\sin t + \frac{t^3}{3}+C$ (D) $4\sin t + 2t^3 + C$

ANSWER KEY–PRACTICE EXERCISE

Single Correct Choice Type Questions

1. C	**2.** B	**3.** B	**4.** D	**5.** C	**6.** C	**7.** C	**8.** C	**9.** B	**10.** B
11. D	**12.** C, D	**13.** C	**14.** B	**15.** A	**16.** B	**17.** D	**18.** D	**19.** B	**20.** B
21. C	**22.** B	**23.** A	**24.** A	**25.** D	**26.** A	**27.** B	**28.** C	**29.** C	**30.** A
31. B	**32.** B	**33.** B	**34.** C	**35.** D	**36.** B	**37.** D	**38.** A	**39.** A	**40.** A
41. A	**42.** C	**43.** B	**44.** C	**45.** B, C	**46.** B	**47.** D	**48.** C	**49.** C	**50.** A
51. D	**52.** B	**53.** B	**54.** C	**55.** C	**56.** A	**57.** B	**58.** C	**59.** B	**60.** C
61. C	**62.** B	**63.** C	**64.** C	**65.** C	**66.** D	**67.** C	**68.** A	**69.** D	**70.** A
71. C	**72.** D	**73.** D	**74.** C	**75.** B	**76.** B	**77.** D	**78.** A	**79.** B	**80.** C
81. C	**82.** C	**83.** A	**84.** C	**85.** C	**86.** D	**87.** B	**88.** D	**89.** B	**90.** A
91. D	**92.** A	**93.** A	**94.** D	**95.** B	**96.** A	**97.** A	**98.** C	**99.** D	**100.** D
101. B	**102.** B	**103.** D	**104.** B	**105.** C	**106.** C	**107.** D	**108.** C	**109.** C	**110.** D
111. B	**112.** B	**113.** A	**114.** A	**115.** B	**116.** B	**117.** B	**118.** C	**119.** B	**120.** C
121. A	**122.** C								

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CHAPTER 2

Measurements and General Physics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Physical Quantity and its Measurement
(b) Fundamental and Derived Units
(c) Dimensional Analysis
(d) Principle of Homogeneity and its uses
(e) Limitations of Dimensional Analysis
(f) Least Count, Significant Figures and Rounding off
(g) Errors and their Propagation
(h) Measurements done using Vernier Calliper (VC)
(i) Measurements done using Screw Gauge (SG).

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

SCIENTIFIC PROCESS

The history of science reveals that it has evolved through a series of steps. Let us have a small discussion of the steps involved.

STEP-1: Observation
STEP-2: Proposing/propounding a theory based on those observations.
STEP-3: Verification of the theory as applied to those observations.
STEP-4: Modification in the theory, if at all necessary.

OBSERVATION

Observations are basically of two types

Subjective Observation

An observation that varies from person to person is called **Subjective Observation**. Physics never deals with subjective observations like beauty, emotion, personality etc.

Objective Observation

An observation that remains identical for all the observers (persons) is called an **Objective Observation**.

EXAMPLE:
Four observers viewing the same painting may feel differently about the "**Beauty"** of the painting, where as they shall report identical length, breadth or area of the painting, when asked. So, Beauty is not an **Objective Observation** as it cannot be assigned a numerical value along with some appropriate unit (that could have measured it).
Physics always deals with **Objective Observation**.

PHYSICAL QUANTITY

The objective quantities to which a numerical value can be attached along with some unit (specified to measure it) are called Physical Quantities. Else, they may also be defined as the quantities through which

(through the help of which) we can describe the Laws of Physics appropriately. A physical quantity can be completely specified if it has

(a) only magnitude (some constants or some ratios) e.g., specific gravity, dielectric constant, strain, refractive index etc.
(b) magnitude along with a unit (scalars) e.g., mass, length, time, energy, current etc.
(c) magnitude along with a unit in a specified direction (vectors) such that laws of vector algebra are obeyed by this quantity e.g., displacement, force, momentum, torque, angular momentum, electric field, magnetic field etc.
(d) magnitude and phase (phasors) e.g., superposition of mechanical wave, AC voltages and currents, SHM etc.
(e) magnitude of varying values in different directions (having no specified directions) (are called Tensors) e.g., Moment of Inertia (cannot be defined without specifying the axis of rotation).

In few anisotropic media the physical quantity(ies) like density, refractive index, dielectric constant, stress, strain, electric conductivity all become Tensors.

MEASUREMENT OF A PHYSICAL QUANTITY

The process of measurement is basically a comparison process. To measure a physical quantity we need to have the following two things.

(a) Firstly, a unit u (of same nature) to measure that physical quantity.
(b) Secondly, the number of times (n) this selected unit is contained in the required physical quantity.

e.g., if we are asked to measure the length and breadth of a room with the help of a scale that is half a metre in length (half metre scale) and we observe that this unit (half metre scale) is contained 10 times in the length of the room and 8 times in its width, then

$$\text{Length of the room} = 10\left(\frac{1}{2}\text{ m}\right) = 5\text{ m}$$

$$\text{Breadth of the room} = 8\left(\frac{1}{2}\text{ m}\right) = 4\text{ m}$$

So, measurement of a Physical Quantity = nu
where, u is the unit selected (of same nature) to measure the physical quantity.
n is the number of times this selected unit is contained in it.
For a particular measurement, irrespective of the system in which the unit is selected, measurement of a physical quantity is a constant.

$$\Rightarrow \quad nu = \text{constant}$$

$$\Rightarrow \quad n_1u_1 = n_2u_2 = n_3u_3 = ...$$

where,
u_1, u_2, u_3, ... are the units selected to measure a physical quantity in system 1, 2, 3, …….. respectively.
n_1, n_2, n_3, ... are the numerical values that the measured physical quantity contains corresponding to the respective systems.
Also, we observe that

$$n \propto \frac{1}{u}$$

i.e., the bigger the unit selected to measure the physical quantity, the smaller the numerical values vice-versa.

FUNDAMENTAL AND DERIVED UNITS

The exact specification of the measurement of a physical quantity requires

(a) the standard or unit in which the quantity is measured and
(b) the numerical value representing the number of times the quantity contains that unit.

The physical quantities which do not depend upon other quantities are called **fundamental quantities.** In M.K.S. system the fundamental quantities are mass, length and time, while in more general Standard International (S.I.) system the fundamental quantities are **mass, length, time, temperature, illuminating power (or luminous intensity), current and amount of substance.** The units of fundamental quantities are called **fundamental units**. The units of physical quantities which may be derived from fundamental units are called **derived units**.

Conceptual Note(s)

Measurement of a Physical Quantity means a comparison of it with some reference standard, also called as unit, which does not change (under any circumstances). So, these standards must be

(a) invariable
(b) easily accessible
(c) precise and
(d) universally agreed.

SYSTEM OF UNITS

Following principal systems of units are used commonly.

C.G.S. System

In this system the unit of length is centimetre (cm), that of mass is gram (g) and that of time is second (s).

F.P.S. System

In this system the unit of length is foot, weight is pound (lb) and time is second.

Conceptual Note(s)

(a) In fps system the unit pound is the unit of weight and not of mass.
(b) 1 pound = 0.4536 kgwt
i.e. 1 pound is equivalent to the weight of a body having a mass of 0.4536 kg.
(c) The unit of mass in fps system is slug.

M.K.S. System

In this system the units of length is metre (m), mass is kilogram (kg) and time is second(s).

S.I. System

In this system there are seven fundamental quantities whose units and symbols are as follows:

Fundamental Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Luminous Intensity	candela	cd
Electric Current	ampere	A
Amount of Substance	mole	mol

(*Continued*)

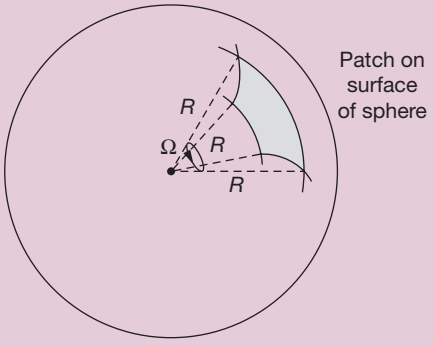
In S.I. system there are two supplementary units.

(a) Radian (rad): Unit of plane angle.
(b) Steradian (sr): Unit of solid angle.

Conceptual Note(s)

To understand the concept of solid angle, let us consider a football having black and white patches on it. Now consider any one patch, say black. Then on the boundary of this patch lie infinite number of points. If you join all these points with the centre of the football, then you observe all these points to be at equal distance from it and this distance happens to be the **radius of the football**. Now, if you join all these points to the centre of the football then the angle enclosed by this patch at the centre of the football is called the Solid Angle, just like the angle at the end of an empty ice-cream cone. Now this solid angle, denoted by Ω, has a value given by

$$\Omega = \frac{\text{Area of patch on the surface of sphere}}{(\text{Radius of sphere})^2}$$



Prefixes for Power of Ten

Prefix	Abbreviation	Power of Ten
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}
peta	P	10^{15}
exa	E	10^{18}

(Continued)

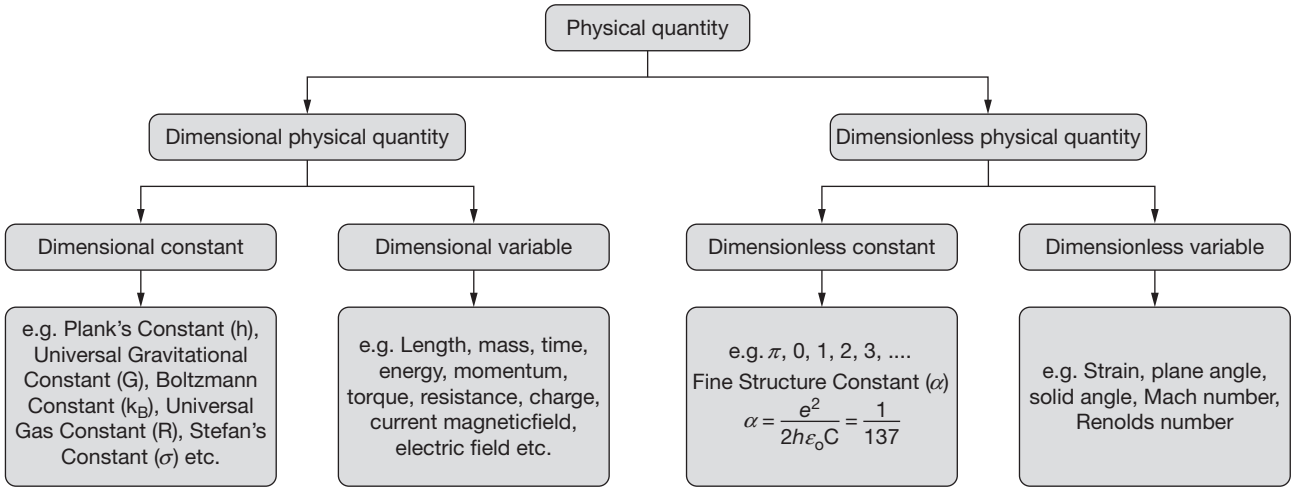
EXAMPLES:

Derived units

1 micro second = 1 μs = 10^{-6} s

1 nano second = 1 ns = 10^{-9} s

1 kilo-metre = 1 km = 10^{-3} m


Physical quantity
Dimensional physical quantity
Dimensionless physical quantity
Dimensional constant
Dimensional variable
Dimensionless constant
Dimensionless variable
e.g. Plank's Constant (h), Universal Gravitational Constant (G), Boltzmann Constant (k_B), Universal Gas Constant (R), Stefan's Constant (σ) etc.
e.g. Length, mass, time, energy, momentum, torque, resistance, charge, current magneticfield, electric field etc.
e.g. π, 0, 1, 2, 3, ...
Fine Structure Constant (α)
$\alpha = \frac{e^2}{2h\varepsilon_0 C} = \frac{1}{137}$
e.g. Strain, plane angle, solid angle, Mach number, Renolds number

Some Important Commonly Used Units

(a) micrometer $(\mu m) = 1\mu m = 10^{-6}$ m

(b) Angstrom $1\text{Å} = 10^{-10}$ m

(c) Astronomical unit

This is the mean distance of earth from sun

$$1\,AU = 1{\cdot}496\times10^{11}\,m \approx 1{\cdot}5\times10^{11}\,m$$

(d) Light year

It is the distance traversed by light in vacuum in 1 year,

$$1\text{light year} = 365\times24\times60\times60\times3\times10^{8} = 9{\cdot}45\times10^{15}\,m$$

(e) parsec

It is the distance at which an arc of length one Astronomical unit subtends an angle of 1 second i.e. 1″.

Since $l = r\theta$

$$\Rightarrow \quad r = \frac{l}{\theta}$$

$$\text{where } l = 1\,AU \text{ and } \theta = 1'' = \left(\frac{1}{3600}\right)\left(\frac{\pi}{180}\right)\text{rad}$$

$$\Rightarrow \quad r = 1 \text{ parsec} = \frac{1.496\times10^{11}\,m}{\frac{1}{3600}\times\frac{\pi}{180}(\text{rad})}$$

$\Rightarrow$ $r = 1$ parsec $= 3{\cdot}07 \times 10^{16}$ m

$\Rightarrow$ 1 parsec $= 3{\cdot}26$ light years

(f) X-ray Unit (XU) $= 1$ XU $= 10^{-13}$ m

(g) 1 Bar $= 10^5$ Nm^{-2} $= 10^5$ pascal

(h) 1 atmosphere (atm) $= 1.013 \times 10^5$ Pa

(i) 1 torr $=$ 1 mm of Hg $= 133.3$ Pa

(j) 1 barn $= 10^{-28}$ m^2

(k) 1 horse power $= 746$ watt

(l) 1 ft $= 0{\cdot}3048$ m

(m) 1 pound $= 453.6$ g $= 0{\cdot}4536$ kg

(n) 1 slug $= 14.57$ kg

(o) 1 poiseuille (Pl) $= 10$ poise

(p) 1 metricton $= 10$ quintal $= 1000$ kg

(q) 1 Chandra Shekhar Limit (CSL) $= 1{\cdot}4\ M_s$

where M_s = mass of sun

(r) 1 Shake $= 10^{-8}$ s

DIMENSIONS

The powers to which the fundamental units of mass, length and time are raised so as to get the required physical quantity.

EXAMPLE:
To get the physical quantity "force", mass has to be raised to the power 1, length to the power 1 and time to the power –2. So, dimensions of force are 1 in mass, 1 in length and –2 in time.

DIMENSIONAL FORMULA

$M^a L^b T^c$ is the dimensional formula of a physical quantity which has dimensions a, b and c in mass, length and time respectively. So, dimensional formula of force is MLT^{-2}. Whenever a physical quantity is written in square brackets, it just means that dimensional formula of the physical quantity has to be taken.

DIMENSIONAL EQUATION

Whenever a physical quantity is equated to its dimensional formula, we get a dimensional equation. So, dimensional equation for force is

$$[F] = MLT^{-2}$$

In general, any physical quantity X, having dimensional formula $M^a L^b T^c$, has a dimensional equation

$$[X] = M^a L^b T^c$$

DIMENSIONS OF SOME PHYSICAL QUANTITIES

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
1.	Area (A)	Length × Breadth	m^2	$M^0L^2T^0$
2.	Volume (V)	Length × Breadth × Height	m^3	$M^0L^3T^0$

(*Continued*)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
3.	Mass density (ρ)	$\frac{\text{Mass}}{\text{Volume}}$	kgm^{-3}	$ML^{-3}T^{0}$
4.	Surface Mass Density (σ)	$\frac{\text{Mass}}{\text{Area}}$	kgm^{-2}	$ML^{-2}T^{0}$
5.	Linear Mass Density (λ)	$\frac{\text{Mass}}{\text{Length}}$	kgm^{-1}	$ML^{-1}T^{0}$
6.	Frequency (ν)	$\frac{1}{\text{Time period}}$	Hz	$M^{0}L^{0}T^{-1}$
7.	Velocity, Speed (v)	$\frac{\text{Displacement}}{\text{Time}}, \frac{\text{Distance}}{\text{Time}}$	ms^{-1}	$M^{0}LT^{-1}$
8.	Acceleration (a)	$\frac{\text{Velocity}}{\text{Time}}$	ms^{-2}	$M^{0}LT^{-2}$
9.	Force (F)	Mass × Acceleration	newton (N)	MLT^{-2}
10.	Impulse (I)	Force × Time	Ns	MLT^{-1}
11.	Work, Energy (W, E)	Force × Distance	joule (J)	$ML^{2}T^{-2}$
12.	Power (P)	$\frac{\text{Work}}{\text{Time}}$	watt (W)	$ML^{2}T^{-3}$
13.	Momentum (p)	Mass × Velocity	kgms^{-1}	MLT^{-1}
14.	Kinetic energy (K or K.E.)	$\left(\frac{1}{2}\right) \times \text{Mass} \times (\text{Velocity})^{2}$	J	$ML^{2}T^{-2}$
15.	Potential energy (U)	Mass × Acceleration due to gravity × Height	J	$ML^{2}T^{-2}$
16.	Spring Constant (k)	$\frac{\text{Force}}{\text{Extension}}$	Nm^{-1}	$ML^{0}T^{-2}$
17.	Elastic Potential Energy (U)	$\frac{1}{2}(\text{Spring constant})(\text{Extension})^{2}$	J	$ML^{2}T^{-2}$
18.	Angle, Angular displacement (θ)	$\frac{\text{Arc}}{\text{Radius}}$	radian	$M^{0}L^{0}T^{0}$

(*Continued*)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
19.	Trigonometric ratio ($\sin\theta$, $\cos\theta$, $\tan\theta$, etc.)	$\frac{\text{Length}}{\text{Length}}$	No Units	$M^0L^0T^0$
20.	Angular velocity (ω)	$\frac{\text{Angle}}{\text{Time}}$	rads^{-1}	$M^0L^0T^{-1}$
21.	Angular acceleration (α)	$\frac{\text{Angular velocity}}{\text{Time}}$	rads^{-2}	$M^0L^0T^{-2}$
22.	Radius of gyration (k)	Distance	m	M^0LT^0
23.	Moment of inertia (I)	$\text{Mass} \times (\text{Radius of gyration})^2$	kgm^2	ML^2T^0
24.	Angular momentum (L)	Moment of inertia $\times$ Angular velocity	$\text{kgm}^2\text{s}^{-1}$	ML^2T^{-1}
25.	Moment of force, moment of couple (τ)	Force $\times$ Distance	Nm	ML^2T^{-2}
26.	Torque (τ)	$\frac{\text{Angular momentum}}{\text{Time}}$ or Force $\times$ Distance	Nm	ML^2T^{-2}
27.	Angular frequency (ω)	$2\pi \times$ Frequency	rads^{-1}	$M^0L^0T^{-1}$
28.	Rotational kinetic energy (RKE)	$\frac{1}{2} \times$ Moment of inertia $\times (\text{Angular velocity})^2$	J	ML^2T^{-2}
29.	Angular impulse (J)	Torque $\times$ Time	Js	ML^2T^{-2}
30.	Centripetal acceleration (a_c)	$\frac{(\text{Velocity})^2}{\text{Radius}}$	ms^{-2}	M^0LT^{-2}
31.	Pressure (P)	$\frac{\text{Force}}{\text{Area}}$	Nm^{-2}	$ML^{-1}T^{-2}$
32.	Stress	$\frac{\text{Restoring force}}{\text{Area}}$	Nm^{-2}	$ML^{-1}T^{-2}$
33.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	No units	$M^0L^0T^0$
34.	Modulus of elasticity (E)	$\frac{\text{Stress}}{\text{Strain}}$	Nm^{-2}	$ML^{-1}T^{-2}$

(*Continued*)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
35.	Surface tension (T or σ)	$\frac{\text{Force}}{\text{Length}}$	Nm^{-1}	ML^0T^{-2}
36.	Surface energy	$\frac{\text{Energy}}{\text{Area}}$	Jm^{-2} $(=\text{Nm}^{-1})$	ML^0T^{-2}
37.	Speed gradient	$\frac{\text{Speed}}{\text{Distance}}$	s^{-1}	$M^0L^0T^{-1}$
38.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	Nm^{-3}	$ML^{-2}T^{-2}$
39.	Pressure energy	$\text{Pressure}\times\text{Volume}$	$\text{Nm}(=\text{J})$	ML^2T^{-2}
40.	Fluid flow rate (V)	$\left(\frac{\pi}{8}\right)\frac{(\text{Pressure})\times(\text{Radius})^4}{(\text{Viscosity coefficient})\times(\text{Length})}$	m^3s^{-1}	$M^0L^3T^{-1}$
41.	Bulk modulus (B) or $(\text{Compressibility})^{-1}$	$\frac{\text{Volume}\times(\text{Change in pressure})}{(\text{Change in volume})}$	Nm^{-2}	$ML^{-1}T^{-2}$
42.	Coefficient of viscosity (η)	$\frac{\text{Force}}{\text{Area}\times\text{Speed gradient}}$	$\text{kgm}^{-1}\text{s}^{-1}$	$ML^{-1}T^{-1}$
43.	Wavelength (λ)	Distance	m	M^0LT^0
44.	Hubble constant (H)	$\frac{\text{Recession speed}}{\text{Distance}}$	s^{-1}	$M^0L^0T^{-1}$
45.	Solid Angle (Ω)	$\frac{\text{Area of patch on surface of sphere}}{(\text{Radius of sphere})^2}$	steradian	$M^0L^0T^0$
46.	Intensity of wave (I)	$\frac{\left(\frac{\text{Energy}}{\text{Time}}\right)}{\text{Area}}$	Wm^{-2}	ML^0T^{-3}
47.	Radiation pressure (P)	$\frac{\text{Intensity of wave}}{\text{Speed of light}}$	Wm^{-3}s	$ML^{-1}T^{-2}$
48.	Energy density (u)	$\frac{\text{Energy}}{\text{Volume}}$	Jm^{-3}	$ML^{-1}T^{-2}$

(*Continued*)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
49.	Critical velocity (v_c)	$\frac{\text{Renold's number} \times \text{Coefficient of viscocity}}{\text{Mass density} \times \text{Diameter}}$	ms^{-1}	M^0LT^{-1}
50.	Escape velocity (v_e)	$\sqrt{2 \times \text{Acceleration due to gravity} \times \text{Earth's radius}}$	ms^{-1}	M^0LT^{-1}
51.	Heat energy, internal energy (Q, U)	Work (= Force × Distance)	J	ML^2T^{-2}
52.	Efficiency (η)	$\frac{\text{Output work or energy}}{\text{Input work or energy}}$		$M^0L^0T^0$
53.	Gravitational constant (G)	$\frac{\text{Force} \times (\text{Distance})^2}{\text{Mass} \times \text{Mass}}$	Nm^2kg^{-2}	$M^{-1}L^3T^{-2}$
54.	Planck constant (h)	$\frac{\text{Energy}}{\text{Frequency}}$	Js	ML^2T^{-1}
55.	Heat capacity (C), entropy (S)	$\frac{\text{Heat energy}}{\text{Temperature}}$	JK^{-1}	$ML^2T^{-2}K^{-1}$
56.	Specific heat capacity (c)	$\frac{\text{Heat Energy}}{\text{Mass} \times \text{Temperature}}$	$Jkg^{-1}K^{-1}$	$M^0L^2T^{-2}K^{-1}$
57.	Latent heat (L)	$\frac{\text{Heat energy}}{\text{Mass}}$	Jkg^{-1}	$M^0L^2T^{-2}$
58.	Thermal expansion coefficient or Thermal expansivity $(\alpha, \beta \text{ or } \gamma)$	$\frac{\text{Change in dimension}}{\text{Original dimension} \times \text{temperature}}$	K^{-1}	$M^0L^0K^{-1}$
59.	Thermal conductivity (κ)	$\frac{\text{Heat energy} \times \text{Thickness}}{\text{Area} \times \text{Temperature} \times \text{Time}}$	$Js^{-1}m^{-1}K^{-1}$ $(= Wm^{-1}K^{-1})$	$MLT^{-3}K^{-1}$
60.	Stefan's constant (σ)	$\frac{\left(\frac{\text{Energy}}{\text{Area} \times \text{Time}}\right)}{(\text{Temperature})^4}$	$Wm^{-2}K^{-4}$	$ML^0T^{-3}K^{-4}$
61.	Wien constant (b)	Wavelength × Temperature	Km	M^0LT^0K
62.	Boltzmann constant (k_B)	$\frac{\text{Energy}}{\text{Temperature}}$	JK^{-1}	$ML^2T^{-2}K^{-1}$

(*Continued*)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
63.	Universal gas constant (R)	$\frac{\text{Pressure} \times \text{Volume}}{\text{Mole} \times \text{Temperature}}$	$JK^{-1}mol^{-1}$	$ML^2T^{-2}K^{-1}mol^{-1}$
64.	Charge (Q)	Current × Time	C (coulomb)	M^0L^0TA
65.	Current density (J)	$\frac{\text{Current}}{\text{Area}}$	Am^{-2}	$M^0L^{-2}T^0A$
66.	Voltage, electric potential, electromotive force $(V$ or $E)$	$\frac{\text{Work}}{\text{Charge}}$	V (volt)	$ML^2T^{-3}A^{-1}$
67.	Resistance (R)	$\frac{\text{Potential difference}}{\text{Current}}$	ohm (Ω)	$ML^2T^{-3}A^{-2}$
68.	Capacitance (C)	$\frac{\text{Charge}}{\text{Potential difference}}$	farad (F)	$M^{-1}L^{-2}T^4A^2$
69.	Electrical resistivity (ρ) or $\left(\text{Electrical conductivity}\right)^{-1}$	$\frac{\text{Resistance} \times \text{Area}}{\text{Length}}$	Ωm	$ML^3T^{-3}A^{-2}$
70.	Electric field $(\vec{E})$	$\frac{\text{Electrical force}}{\text{Charge}}$	NC^{-1} $(\text{or } Vm^{-1})$	$MLT^{-3}A^{-1}$
71.	Electric flux (ϕ_E)	Electric field × Area	$NC^{-1}m^2$ (or Vm)	$ML^3T^{-3}A^{-1}$
72.	Electric dipole moment $(\vec{p})$	$\frac{\text{Torque}}{\text{Electric field}}$	Cm (Coulomb metre)	M^0LTA
73.	Electric field strength or electric intensity $(\vec{E})$	$\frac{\text{Potential difference}}{\text{Distance}}$	Vm^{-1}	$MLT^{-3}A^{-1}$
74.	Magnetic field, magnetic flux density, magnetic induction $(\vec{B})$	$\frac{\text{Force}}{\text{Current} \times \text{Length}}$	tesla (T)	$ML^0T^{-2}A^{-1}$
75.	Magnetic flux (ϕ_B)	Magnetic field × Area	weber $(= Tm^2)$	$ML^2T^{-2}A^{-1}$

(*Continued*)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
76.	Inductance (L)	$\frac{\text{Magnetic flux}}{\text{Current}}$	henry (H)	$ML^2T^{-2}A^{-2}$
77.	Magnetic dipole moment $(\vec{m})$	$\frac{\text{Torque}}{\text{Magnetic field}}$ or Current × Area	Am^2	$M^0L^2T^0A$
78.	Magnetic field strength, magnetic intensity or magnetic moment density $(\vec{B}$ or $\vec{H})$	$\frac{\text{Magnetic moment}}{\text{Volume}}$	Am^{-1}	$M^0L^{-1}T^0A$
79.	Permittivity constant (of free space) (ε_0)	$\frac{\text{Charge} \times \text{Charge}}{4\pi \times \text{Electric force} \times (\text{Distance})^2}$	$\text{C}^2\text{N}^{-1}\text{m}^{-2}$	$M^{-1}L^{-3}T^4A^2$
80.	Permeability constant (of free space) (μ_0)	$\frac{2\pi \times \text{Force} \times \text{Distance}}{\text{Current} \times \text{Current length}}$	NA^{-2}	$MLT^{-2}A^{-2}$
81.	Refractive index (μ)	$\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$		$M^0L^0T^0$
82.	Faraday constant (F)	Avogadro constant × Elementary charge	Cmol^{-1}	$M^0L^0T\ \text{Amol}^{-1}$
83.	Wave number $(\bar{\lambda})$	$\frac{2\pi}{\text{Wavelength}}$	m^{-1}	$M^0L^{-1}T^0$
84.	Radiant flux, Radiant power	$\frac{\text{Energy emitted}}{\text{Time}}$	W	ML^2T^{-3}
85.	Luminosity of radiant flux or radiant intensity	$\frac{\text{Radiant power of radiant flux of source}}{\text{Solid angle}}$	Wsr^{-1}	ML^2T^{-3}
86.	Luminous power or luminous flux of source	$\frac{\text{Luminous energy emitted}}{\text{Time}}$	W	ML^2T^{-3}
87.	Luminous intensity or illuminating power of source	$\frac{\text{Luminous flux}}{\text{Solid angle}}$	Wsr^{-1}	ML^2T^{-3}
88.	Intensity of illumination or luminance	$\frac{\text{Luminous intensity}}{(\text{Distance})^2}$	$\text{Wm}^{-2}\text{sr}^{-1}$	ML^0T^{-3}

(*Continued*)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
89.	Relative luminosity	$\dfrac{\left(\begin{array}{c}\text{Luminous flux of a source}\\ \text{of given wavelength}\end{array}\right)}{\left(\begin{array}{c}\text{Luminous flux of peak sensitivity wavelength}\\ (555\text{ nm})\text{source of same power}\end{array}\right)}$		$M^0L^0T^0$
90.	Luminous efficiency	$\dfrac{\text{Total luminous flux}}{\text{Total radiant flux}}$		$M^0L^0T^0$
91.	Illuminance or illumination	$\dfrac{\text{Luminous flux incident}}{\text{Area}}$	Wm^{-2}	ML^0T^{-3}
92.	Mass defect (Δm)	(Sum of masses of nucleons) – (Mass of the nucleus)	kg	ML^0T^0
93.	Binding energy of nucleus (BE)	Mass defect $\times$ (Speed of light in vacuum)^2	J	ML^2T^{-2}
94.	Decay constant (λ)	$\dfrac{0.693}{\text{Half life}}$	s^{-1}	$M^0L^0T^{-1}$
95.	Resonant frequency	$(\text{Inductance} \times \text{Capacitance})^{-\frac{1}{2}}$	Hz	$M^0L^0A^0T^{-1}$
96.	Quality factor of Q-factor of coil (Q)	$\dfrac{\text{Resonant frequency} \times \text{Inductance}}{\text{Resistance}}$		$M^0L^0T^0$
97.	Power of lens (P)	$(\text{Focal length})^{-1}$	dioptre (D)	$M^0L^{-1}T^0$
98.	Magnification (m)	$\dfrac{\text{Image distance}}{\text{Object distance}}$		$M^0L^0T^0$
99.	Capacitive reactance (X_C)	$(\text{Angular frequency} \times \text{Capacitance})^{-1}$	ohm (Ω)	$ML^2T^{-3}A^{-2}$
100.	Inductive reactance (X_L)	(Angular frequency $\times$ Inductance)	ohm (Ω)	$ML^2T^{-3}A^{-2}$

QUANTITIES HAVING SAME DIMENSIONS

Physical Quantities	Dimensional Formula
Frequency, angular frequency, angular velocity, velocity gradient and decay constant	$[M^0L^0T^{-1}]$
Work, internal energy, potential energy, kinetic energy, torque, moment of force.	$[M^1L^2T^{-2}]$

(*Continued*)

Physical Quantities	Dimensional Formula
Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density.	$[M^1L^{-1}T^{-2}]$
Momentum, impulse.	$[M^1L^1T^{-1}]$
Acceleration due to gravity, gravitational field intensity.	$[M^0L^1T^{-2}]$
Thrust, force, weight, energy gradient.	$[M^1L^1T^{-2}]$
Angular momentum and Planck's constant	$[M^1L^2T^{-1}]$
Surface tension, Surface energy (energy per unit area).	$[M^1L^0T^{-2}]$
Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.	$[M^0L^0T^0]$
Latent heat and gravitational potential.	$[M^0L^2T^{-2}]$
Thermal capacity, gas constant, Boltzmann constant and entropy.	$[ML^2T^{-2}Q^{-1}]$
$\sqrt{\frac{l}{g}}, \sqrt{\frac{m}{k}}, \sqrt{\frac{R}{g}}$, where l = length, g = acceleration due to gravity, m = mass, k = spring constant, R = Radius of earth.	$[M^0L^0T^1]$
$\frac{L}{R}, \sqrt{LC}, RC$ where L = inductance, R = resistance, C = capacitance.	$[M^0L^0T^1]$
$I^2Rt, \frac{V^2}{R}t, VIt, qV, LI^2, \frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance	$[ML^2T^{-2}]$

SYMBOLS

Following table gives the international symbols for SI units. In addition to the symbols of fundamental units, the symbols of derived units have also been included in this table.

Unit	Symbol	Unit	Symbol
metre	m	joule	J
kilogram	kg	watt	W
second	s	coulomb	C

(*Continued*)

Unit	Symbol	Unit	Symbol
ampere	A	weber	Wb
kelvin	K	ohm	ohm Ω
candela	cd	volt	V
radian	rad	farad	F
steradian	sr	henry	H
newton	N	siemen	S
hertz	Hz	tesla	T

Following points must be noted regarding the symbolic representation of various units.

(a) Small letters are used as symbols of units. However, if the symbol is derived from a proper name, then capital letter is used. As an example, the symbol of newton is "N" and **not** "n". It may be pointed out here only that if a unit is derived from the name of a person, only the symbol will be represented by capital letter. The unit itself will start with small letter. Thus, the unit of force will be written as "newton" and **not** as "Newton".

(b) The symbols of units are regarded as algebraic symbols. They are not followed by full stop, dots, dashes etc. Thus, SI unit of impulse will be represented by Ns and **not** N-s or N s.

(c) Some space is always left between the number and the symbol of the unit. Thus, it will be **incorrect** to write 2.4 kg. The **correct** representation is 2.4 kg.

(d) Even if the unit is in the plural form, "s" is not mentioned at the end of the unit. The same is true for its symbolic representation. Thus, it will be **incorrect** to write "metres". This will be written as "metre". In the same way, the symbol will be m and **not** ms.

PRINCIPLE OF HOMOGENEITY AND USES OF DIMENSIONAL ANALYSIS

According to this principle, the dimensions of all the terms of the two sides of an equation must be the same. If

$$X = A \pm (BC)^2 \pm \sqrt{\frac{DE}{F}}$$

then according to Principle of Homogeneity

$$[X] = [A] = \left[(BC)^2\right] = \left[\sqrt{\frac{DE}{F}}\right] = M^a L^b T^c$$

Taking help from the Principle of Homogeneity and knowing the dimensional formulae of various physical quantities, dimensional analysis can be employed to

(a) check the dimensional correctness of a physical relation.

(b) convert units from one system to another.

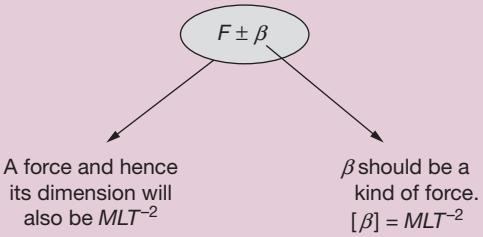
(c) find dependency of a physical quantity on other physical quantities.

Conceptual Note(s)

(a) Suppose in any formula, $(L \pm \alpha)$ term is coming (where L is length). As length can be added only with a length, so α should also be a kind of length.

$\Rightarrow \quad [\alpha] = M^0 L^1 T^0 = L$

(b) Similarly consider a term $(F \pm \beta)$ where F is force. A force can be added/subtracted with a force only and give rise to force. So β should be a kind of force and its result $(F \pm \beta)$ should also be a kind of force.



So, only like Physical Quantities (scalars with scalars and vectors with vectors) can be added or subtracted. However there is no restriction on multiplication and division of Physical Quantities.

ILLUSTRATION 1

Check the dimensional correctness of $Fs = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

SOLUTION

Given $\quad Fs = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad \ldots(1)$

According to Principle of Homogeneity

$$[Fs] = \left[\frac{1}{2}mv^2\right] = \left[\frac{1}{2}mv_0^2\right]$$

$\Rightarrow \quad (MLT^{-2})L = M(L^2T^{-2}) = M(L^2T^{-2})$

$\Rightarrow \quad ML^2T^{-2} = ML^2T^2 = ML^2T^{-2}$

ILLUSTRATION 2

A student finds that pressure can be expressed as $P = \frac{4FV^2}{\pi t^2 x}$, where F is force, V is velocity, t is time and x is distance. However his teacher is doubtful

about the expression. Express the way his teacher confirms the correctness of expression.

SOLUTION

Dimension of LHS $= [P] = M^1L^{-1}T^{-2}$

Dimension of RHS is $\left[\frac{4FV^2}{\pi t^2 x}\right] = \frac{[4][F][v^2]}{[\pi][t^2][x]}$

$$\Rightarrow \left[\frac{4FV^2}{\pi t^2 x}\right] = \frac{(M^1L^1T^{-2})(L^2T^{-2})}{(T^2)(L)} = M^1L^2T^{-6}$$

Dimension of LHS and RHS are not same. So the relation cannot be correct.

ILLUSTRATION 3

For n moles of gas, Vander Waal's equation is $\left(P+\frac{a}{V^2}\right)(V-b) = nRT$. Find the dimensions of a and b, where P is gas pressure. V is volume of gas and T is temperature of gas.

SOLUTION

$$\underbrace{\left(P+\frac{a}{V^2}\right)}_{\text{pressure}} \times \underbrace{(V-b)}_{\text{volume}} = nRT$$

$$\Rightarrow [P] = \left[\frac{a}{V^2}\right] \text{ and } [b] = [V] = L^3$$

$$\Rightarrow \frac{[a]}{[V^2]} = M^1L^{-1}T^{-2}$$

$$\Rightarrow \frac{[a]}{(L^3)^2} = M^{-1}L^{-1}T^{-2}$$

$$\Rightarrow [a] = M^1L^5T^{-2} \text{ and } [b] = L^3$$

ILLUSTRATION 4

If, $\frac{\alpha}{t^2} = Fv + \frac{\beta}{x^2}$, then find dimension formula for α and β, ω here t is time, F is force, V is velocity, x is distance.

SOLUTION

Since $[Fv] = M^1L^2T^{-3}$

So, $\left[\frac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$$\Rightarrow \frac{[\beta]}{[x^2]} = M^1L^2T^{-3}$$

$$\Rightarrow [\beta] = M^1L^4T^{-3}$$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$

$$\Rightarrow \frac{[\alpha]}{[t^2]} = M^1L^2T^{-3}$$

$$\Rightarrow [\alpha] = M^1L^2T^{-1}$$

Test Your Concepts-I

Based on Principle of Homogeneity and Verification

(Solutions on page H.5)

1. If a composite physical quantity in terms of moment of inertia I, force F, velocity v, work W and length L is defined as, $Q = \left(\frac{IFv^2}{WL^3}\right)$. Find the dimensions of Q.
2. Can two physical quantities have same dimensions? Explain with example.
3. Find dimensions of universal gas constant R, universal gravitational constant G.
4. The rate of flow (V) of a liquid flowing through a pipe of radius r and a pressure gradient $\left(\frac{P}{\ell}\right)$ is given by Poiseuille's equation:

$$V = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

Check the dimensional consistency of this equation.

5. Check the correctness of the equation: $y = a\sin(\omega t + \phi)$, where y = displacement, a = amplitude, ω = angular frequency and ϕ is an angle.
6. If E, M, J and G respectively denote energy, mass, angular momentum and gravitational constant, calculate the dimensions of $\frac{EJ^2}{M^5G^2}$.

7. In the formula $x = 3yz^2$, x and z are the dimensions of capacitance and magnetic induction respectively. Find the dimensions of y in MKSQ system.

8. State whether the following statement is true or false. Give very brief reason in support of your answer.

The quantity $\frac{e^2}{2\varepsilon_0 hc}$ is dimensionless. Here e, h and c are electronic charge, Planck's constant and velocity of light respectively and ε_0 is the permittivity constant of free space.

9. When white light travels through glass, the refractive index of glass (μ = velocity of light in air/velocity of light in glass) is found to vary with wavelength as $\mu = A + \frac{B}{\lambda^2}$. Using the particle of homogeneity of dimensions, find the SI unit in which the constants A and B must be expressed.

10. A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v as $\tan\theta = v$ and checks that the relation has a correct limit: as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess at the correct relation.

CONVERSION OF UNITS FROM ONE SYSTEM TO ANOTHER

The measure of a physical quantity is given by

$$nu = \text{constant}$$

If a physical quantity X has dimensional formula $M^a L^b T^c$ and if (derived) units of that physical quantity in two systems are $M_1^a L_1^b T_1^c$ and $M_2^a L_2^b T_2^c$, respectively and n_1 and n_2 be the numerical values in the two systems respectively, then

$$n_1[u_1] = n_2[u_2]$$

$$\Rightarrow \quad n_1\left(M_1^a L_1^b T_1^c\right) = n_2\left(M_2^a L_2^b T_2^c\right)$$

$$\Rightarrow \quad n_2 = n_1\left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

a, b and c are the respective dimensions in mass, length and time of the physical quantity to be converted.

M_1, L_1 and T_1 are fundamental units of mass, length and time in the **first (known)** system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the **second (unknown)** system.

Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

ILLUSTRATION 5

If 1 hp is 746 watt, comment on the statement "1 hp is 550 ft lb/s".

SOLUTION

hp is unit of power, so dimensional formula will be ML^2T^{-3}. Hence $a = 1$, $b = 2$ and $c = -3$.

So 1 hp = 746 watt = 746 kg m^2s^{-3}

Now $M_1 = 1$ kg $\quad M_2 = 1$ lb

$L_1 = 1$ m $\quad L_2 = 1$ ft

$T_1 = 1$ sec $\quad T_2 = 1$ sec

$n_1 = 746 \quad n_2 = ?$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

where $a = 1$, $b = 2$, $c = -3$

$$n_2 = 746\left(\frac{1\text{ kg}}{0.4536\text{ kg}}\right)^1 \left(\frac{1\text{ m}}{0.3048\text{ m}}\right)^2 \left(\frac{1\text{ sec}}{1\text{ sec}}\right)^{-3}$$

$$\Rightarrow \quad n_2 = 746 \times 23.73$$

{as 1 lb = 0.4536 kg, 1 ft = 0.3048 m}

$$\Rightarrow \quad n_2 = 17702.55$$

As $n_1u_1 = n_2u_2$

1 hp = 746 watt = 17702.55 lbft^{2}s^{-3}

Since $(1\text{ lb})\times\left(1\,\frac{\text{ft}}{\text{s}^2}\right)=1$ poundal

$$17702.55\text{ lbft}^2\text{s}^{-3}=17702.55\ \frac{\text{foot-poundal}}{\text{sec}}$$

Since 1 lb = 32.2 poundal

$$\Rightarrow\quad 1\text{ hp}=\frac{17702.55}{32.2}\ \frac{\text{ftlb}}{\text{s}}$$

$$\Rightarrow\quad 1\text{ hp}\approx 550\text{ ftlbs}^{-1}$$

Conceptual Note(s)

The poundal is a non-SI unit of force and is a part of FPS system of units. It is equal to $1\ \frac{(\text{lb})(\text{ft})}{\text{s}^2}$.

ILLUSTRATION 6

If velocity of light in air $\left(=3\times10^8\ \text{ms}^{-1}\right)$, acceleration due to gravity $\left(=9.8\ \text{ms}^{-2}\right)$ and density of mercury at 0°C $\left(13600\ \text{kgm}^{-3}\right)$ be chosen as fundamental units, find the unit of mass, length and time.

SOLUTION

Given

$$LT^{-1}=3\times10^8\ \text{ms}^{-1}\qquad\ldots(1)$$

$$LT^{-2}=9.8\ \text{ms}^{-2}\qquad\ldots(2)$$

$$ML^{-3}=13600\ \text{kgm}^{-3}\qquad\ldots(3)$$

Dividing (1) by (2)

$$\frac{LT^{-1}}{LT^{-2}}=\frac{3\times10^8}{9.8}$$

$$\Rightarrow\quad T=3.061\times10^7$$

Substituting for T in equation (1)

$$L\left(3.061\times10^7\right)^{-1}=3\times10^8$$

$$\Rightarrow\quad L=3\times3.061\times10^{15}$$

$$\Rightarrow\quad L=9.918\times10^{15}\text{ m}$$

Substituting for L in equation (3)

$$M\left(9.183\times10^{15}\right)^{-3}=13600$$

$$\Rightarrow\quad M=\frac{13600}{\left(9.183\times10^{15}\right)^{-3}}$$

$$\Rightarrow\quad M=13600\times\left(9.183\right)^3\times10^{45}$$

$$\Rightarrow\quad M=1.29\times10^{51}\text{ kg}$$

So, $M=1.29\times10^{51}\text{kg}$,

$L=9.183\times10^{15}\text{m}$,

$T=3.061\times10^7\text{ s}$

ILLUSTRATION 7

In two systems of relations between velocity, acceleration and force are respectively given by

$$v_2=\frac{\alpha^2}{\beta}v_1,\ a_2=\alpha\beta a_1\ \text{and}\ F_2=\frac{F_1}{\alpha\beta}.$$

If α and β are constants then find the relations between mass, length and time in two systems.

SOLUTION

$$v_2=v_1\frac{\alpha^2}{\beta}$$

$$\Rightarrow\quad\left(L_2T_2^{-1}\right)=\left(L_1T_1^{-1}\right)\frac{\alpha^2}{\beta}\qquad\ldots(1)$$

$$a_2=a_1\alpha\beta$$

$$\Rightarrow\quad\left(L_2T_2^{-2}\right)=\left(L_1T_1^{-2}\right)\alpha\beta\qquad\ldots(2)$$

Since $F_2=\frac{F_1}{\alpha\beta}$

$$\Rightarrow\quad\left(M_2L_2T_2^{-2}\right)=\left(M_1L_1T_1^{-2}\right)\frac{1}{\alpha\beta}\qquad\ldots(3)$$

Dividing equation (3) by equation (2) we get

$$M_2=\frac{M_1}{(\alpha\beta)\alpha\beta}=\frac{M_1}{\alpha^2\beta^2}$$

Squaring equation (1) and dividing by equation (2) we get

$$L_2=L_1\frac{\alpha^3}{\beta^3}$$

Dividing equation (1) by equation (2) we get

$$T_2=T_1\frac{\alpha}{\beta^2}$$

Test Your Concepts-II

Based on Principle of Homogeneity: Conversion

(Solutions on page H.5)

1. Calorie is the unit of heat energy and its value is 4.18 J. Suppose we use a new system of units in which the unit of mass is α kg, unit of length is β m and that of time is γ s. Find the value of calorie in terms of the new system of units.
2. Express the power of 100 W bulb in CGS unit with proper prefix.
3. The *CGS* unit of viscosity is poise (*P*). Find how many poise are there in 1 MKS unit of viscosity called poiseuille (*PI*)?
4. If the units of force, energy and velocity are 20 N, 200 J and 5 ms^{-1}, find the units of length, mass and time.
5. Density of a material in the cgs system is 8 gcc^{-1}. In a system of units in which the unit of mass is 20 g and that of length is 5 cm, what is the density of the material in this new system of units.
6. Given that 1 pound = 1 lb = 0.4536 kgwt, 1 foot = 0.3048 m, then by dimensional analysis find the value of 1 horse power. Given that 1 hp = 550 foot lbs^{-1}.
7. Calculate the dimensions of linear momentum and surface tension in terms of velocity *v*, density ρ and frequency v as fundamental quantities.
8. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and earth in terms of the new unit if light take 8 min and 20 s to cover this distance.
9. If the unit of force is 1 kN, unit of length 1 km and the unit of time is 100 s, what will be the unit of mass.
10. It is estimated that per minute each cm^2 of earth receives about 2 calorie of heat energy from the sun. This constant is called solar constant *S*. Express solar constant in SI units.

TO DERIVE THE NEW RELATIONS

If a physical quantity X depends on other physical quantities P, Q and R (say), then we may write

$$X \propto P^a Q^b R^c \qquad \ldots(1)$$

respectively.

Then writing dimensional formula for X, P, Q and R and equating the dimensions on either sides give the values of a, b and c. The substitution of these values in (1) gives the new dimensional relation.

ILLUSTRATION 8

The time period (T) of a simple pendulum depends upon the length of the thread (l), mass of bob (m), acceleration due to gravity (g) and the angle of swing (θ). Find the relation of t with other physical quantities.

SOLUTION

It is found experimentally that T depends upon length of thread (l), mass of bob (m), acceleration due to gravity g, and angle of swing (θ).

So $T = f(l, m, g, \theta)$

If the function is product of power functions of l, m, g, θ

$$T = kl^a m^b g^c \theta^d$$

where k is dimensionless constant

$$T = (L)^a (M)^b (LT^{-2})^c \qquad \{\theta \text{ is dimensionless}\}$$

$$[T] = [M^b L^{a+c} T^{-2c}]$$

$$\Rightarrow \quad T = M^b L^{a+c} T^{-2c}$$

Equating the exponents of similar quantities, we get

$$b = 0;\ a + c = 0;\ -2c = 1$$

Solving for a, b, c we get

$$a = \frac{1}{2},\ b = 0,\ c = -\frac{1}{2}$$

$$\Rightarrow \quad T = Kl^{1/2} g^{-1/2}$$

on experimental grounds

$$K = 2\pi$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g}}$$

From, this illustration, it is clearly that dimensional analysis does not provide any information about dependence of a physical quantity on dimensionless physical quantities (like θ angle of swing).

ILLUSTRATION 9

Given that the time period t of oscillation of a gas bubble from an explosion under water depends upon the static pressure p, the density of water d and the total energy of explosion E. Find a dimensional relation for t.

SOLUTION

$$t \propto p^a d^b E^c$$

$$\Rightarrow \quad t = kp^a d^b E^C$$

where k is a dimensionless constant

$$\Rightarrow \quad T = \left(ML^{-1}T^{-2}\right)^a \left(ML^{-3}\right)^b \left(ML^2T^{-2}\right)^c$$

$$\Rightarrow \quad T = M^{a+b+c}L^{-a-3b+2c}T^{-2a-2c}$$

Using Principle of Homogeneity we get

$$a+b+c=0 \qquad \ldots(1)$$

$$-a-3b+2c=0 \qquad \ldots(2)$$

$$-2a-2c=1 \qquad \ldots(3)$$

From (3), we get

$$a+c=-\frac{1}{2}$$

Substituting in (1),

$$-\frac{1}{2}+b=0$$

$$\Rightarrow \quad b=\frac{1}{2}$$

Substituting in (2), we get

$$-a-\frac{3}{2}+2c=0$$

$$\Rightarrow \quad -a+2c=\frac{3}{2} \qquad \ldots(4)$$

Adding (3) and (4), we get

$$-3a=\frac{5}{2}$$

$$\Rightarrow \quad a=-\frac{5}{6}$$

Substituting in (3), to get

$$-2\left(-\frac{5}{6}\right)-2c=1$$

$$\Rightarrow \quad \frac{5}{3}-2c=1$$

$$\Rightarrow \quad 2c=\frac{5}{3}-1$$

$$\Rightarrow \quad 2c=\frac{2}{3}$$

$$\Rightarrow \quad c=\frac{1}{3}$$

Hence, we get

$$a=-\frac{5}{6},\ b=\frac{1}{2},\ c=\frac{1}{3}$$

$$\Rightarrow \quad T=kp^{-\frac{5}{6}}d^{\frac{1}{2}}E^{\frac{1}{3}}$$

ILLUSTRATION 10

The planets move round the sun in nearly circular orbits. Assuming that the period of rotation depends upon the radius of the orbit, the mass of the Sun and the Universal Gravitational constant. Show that the planet obeys the Kepler's Third Law of Planetary Motion.

SOLUTION

To show that the planet obeys Kepler's Third Law of planetary motion, we have to prove that

$$T^2 \propto r^3$$

Now, according to the problem, we have

$$T \propto r^a M_S^b G^c$$

$$\Rightarrow \quad T = kr^a M_S^b G^c$$

where k is a dimensionless constant.

$$\Rightarrow \quad T = L^a M^b \left(M^{-1}L^3T^{-2}\right)^c$$

$$\Rightarrow \quad T = M^{b-c}L^{a+3c}T^{-2c}$$

Using the Principle of Homogeneity, we get

$$b-c=0 \qquad \ldots(1)$$

$$a+3c=0 \qquad \ldots(2)$$

$$-2c=1 \qquad \ldots(3)$$

$$\Rightarrow \quad c = -\frac{1}{2},\ b = -\frac{1}{2},\ a = \frac{3}{2}$$

So, we get

$$T = kr^{\frac{3}{2}} M_S^{-\frac{1}{2}} G^{-\frac{1}{2}}$$

$$\Rightarrow \quad T^2 = \left(\frac{k^2}{M_S G}\right) r^3$$

Since the product $M_S G$ is constant, so

$$T^2 \propto r^3$$

ILLUSTRATION 11

The height to which the liquid rises in the capillary tube of radius r depends upon in addition to r on the surface tension S of the liquid, the density d of the liquid and the acceleration due to gravity g. Is it possible to obtain dimensionally a relation for h without the experimental information that h is inversely proportional to r? What is the relation?

SOLUTION

No, it would not be possible to obtain dimensionally a relation for h without the additional experimental information that $h \propto \frac{1}{r}$. This is because without the information $h \propto \frac{1}{r}$, we will get four variables to be calculated from three equations which is just impossible. So, we shall be writing

$$h \propto r^{-1} S^a d^b g^c$$

$$\Rightarrow \quad h = kr^{-1} S^a d^b g^c$$

where k is a dimensionless constant

$$\Rightarrow \quad L = L^{-1} \left(MT^{-2}\right)^a \left(ML^{-3}\right)^b \left(LT^{-2}\right)^c$$

$$\Rightarrow \quad L = M^{a+b} L^{-1-3b+c} T^{-2a-2c}$$

Using Principle of Homogeneity, we get

$$a + b = 0 \qquad \ldots(1)$$

$$-1 - 3b + c = 1 \qquad \ldots(2)$$

$$-2a - 2c = 0 \qquad \ldots(3)$$

So, $2\times(1)+(3)$ gives

$$2b - 2c = 0 \qquad \ldots(4)$$

Again $2\times(2)+(4)$ gives

$$b = -1$$

$$\Rightarrow \quad a = 1 \qquad \{\because \text{ of } (1)\}$$

$$\Rightarrow \quad c = -1$$

So, $a = 1$, $b = -1$ and $c = -1$

$$\Rightarrow \quad h = kr^{-1} S d^{-1} g^{-1}$$

$$\Rightarrow \quad h = k\left(\frac{S}{rgd}\right)$$

ILLUSTRATION 12

If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then find the dimensional formula of mass in this new system.

SOLUTION

Let $m \propto c^x G^y h^z$ or $m = K c^x G^y h^z$

By substituting the dimension of each quantity in both sides

$$[M^1 L^0 T^0] = K[LT^{-1}]^x [M^{-1} L^3 T^{-2}]^y [ML^2 T^{-1}]^z$$

$$[M^1 L^0 T^0] = [M^{-y+z} L^{x+3y+2z} T^{-x-2y-z}]$$

By equating the power of M, L and T in both sides:

$$-y + z = 1,\ x + 3y + 2z = 0,\ -x - 2y - z = 0$$

By solving above three equations, we get

$$x = \frac{1}{2},\ y = -\frac{1}{2} \text{ and } z = \frac{1}{2}$$

So, $m \propto c^{1/2} G^{-1/2} h^{1/2}$

Test Your Concepts-III

Based on Principle of Homogeneity: Dependence

(Solutions on page H.6)

1. Given that the time period of oscillation of a small drop of liquid under the influence of surface tension depends upon the density d, radius r and the surface tension S. Find the expression for the time period.
2. Given that the time period t of oscillation of a gas bubble from an explosion under water depends upon the static pressure p, the density of water d and the total energy of explosion E. Find a dimensional relation for t.
3. The time period of vibration of a stretched string depends upon the mass m of the string, tension f in the string and the length of the string. Find an expression for the time period of oscillation of the string.
4. A small steel ball of radius r is allowed to fall under the gravity through a column of liquid of coefficient of viscosity η. After some time the ball attains a constant velocity called the terminal velocity v_T. This terminal velocity depends upon the weight W, the coefficient of viscosity η and the radius of the ball r. Find an expression for the terminal velocity.
5. A liquid is flowing steadily through a capillary tube. The rate of flow of the volume of the liquid depends upon the coefficient of viscosity η, radius of the tube r and the pressure gradient along the tube p. Find an expression for the rate of flow of the volume of the liquid through the tube.
6. The critical angular velocity ω_c of a cylinder inside another cylinder containing a liquid at which its turbulence occurs depends on viscosity η, density ρ and the distance d between the walls of the cylinder. Find the expression for ω_c.
7. The height to which the liquid rises in the capillary tube of radius r depends upon in addition to r on the surface tension S of the liquid, the density d of the liquid and the acceleration due to gravity g. Is it possible to obtain dimensionally a relation for h without the experimental information that h is inversely proportional to r? What is the relation?
8. The planets move round the sun in nearly circular orbits. Assuming that the period of rotation depends upon the radius of the orbit, the mass of the Sun and the Universal Gravitational constant. Show that the planet obeys the Kepler's Third Law of Planetary Motion.
9. If density ρ, acceleration due to gravity g and frequency ν are the basic quantities, find the dimensions of force.
10. If velocity, force and time are taken to be fundamental quantities find dimensional formula for
    (a) Mass, and
    (b) Energy.

LIMITATIONS OF DIMENSIONAL ANALYSIS

The following are the limitations of dimensional analysis

(a) One has to make a guess about the dependence of the Physical Quantity on other Physical Quantities, which may or may not work.

(b) This method fails to find the dependency on functions other than power functions. As an example we cannot find the relation $s = ut + \frac{1}{2}at^2$, using the method of dimensional analysis. However dimensional analysis does help us to check the physical correctness of this relation.

(c) This method would also fail to find the dependency of a Physical Quantity on the two Physical Quantities with identical dimensional formulae.

(d) It gives no information about dimensionless constants which are to be calculated either by experiments or by actual derivation.

(e) It gives no information about the dependence of a physical quantity on trigonometrical, exponential and logarithmic functions as all are dimensionless i.e. it cannot find dependence on dimensionless physical quantities.

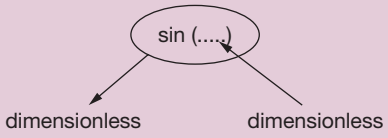
(f) If a physical quantity of mechanics depends on more than three physical quantities all having dimensional formulae, then dimensional analysis cannot be used to derive their relationship.

(g) Dimensional correctness does not establish numerical correctness but reverse is true.

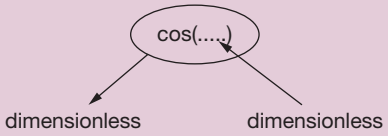
Conceptual Note(s)

Consider the terms, $\sin\theta$, $\cos\theta$, $\tan\theta$ (and their reciprocals) where θ is dimensionless and $\sin\theta = \left(\frac{\text{Perpendicular}}{\text{hypotenuse}}\right)$ is also dimensionless. Similarly $\cos\theta$ and $\tan\theta$ are also dimensionless. Whatever comes in $\sin(.....)$ is dimensionless and entire $[\sin(.....)]$ is also dimensionless. So,

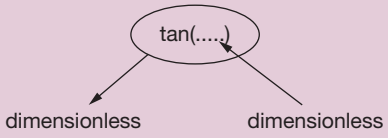
(a)



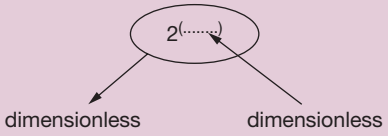
(b)



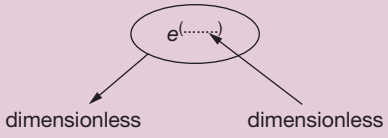
(c)



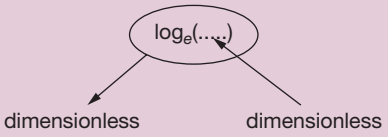
(d)



(e)



(f)



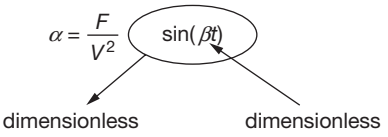
So, to conclude we have

1. $\sin x, \cos x, \tan x, \text{cosec}\, x, \sec x, \cot x, \log x, e^x, a^x$ all are dimensionless.
   i.e., $[\sin x] = [\cos x] = [\tan x] = [\text{cosec}\, x] = [\sec x] = [\log x] = [e^x] = [a^x] = M^0L^0T^0$
2. The argument of all the functions i.e. x is also dimensionless. Hence
   $[x] = M^0L^0T^0$

ILLUSTRATION 13

If V is velocity, F is force, t is time and $\alpha = \frac{F}{V^2}\sin(\beta t)$, then find the dimensional formula of α and β.

SOLUTION



$\Rightarrow \quad [\beta t] = M^0L^0T^0$

$\Rightarrow \quad [\beta] = T^{-1}$

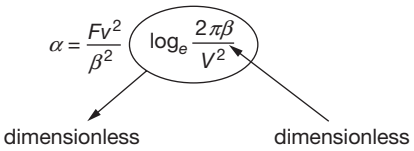
Similarly $[\alpha] = \frac{[F]}{[V^2]}$

$\Rightarrow \quad \alpha = \frac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$

ILLUSTRATION 14

If, $\alpha = \frac{FV^2}{\beta^2}\log_e\left(\frac{2\pi\beta}{V^2}\right)$ where F is force, V is velocity, then find the dimensional formula of α and β.

SOLUTION



$$\Rightarrow \quad [\alpha] = \frac{[F][V^2]}{[\beta^2]} \qquad \ldots(1)$$

Also, $\frac{[2\pi][\beta]}{[V^2]} = 1$

$$\Rightarrow \quad \frac{[1][\beta]}{L^2T^{-2}} = 1$$

$$\Rightarrow \quad [\beta] = L^2T^{-2}$$

Substituting β in (1), we get

$$[\alpha] = \frac{(M^1L^1T^{-2})(L^2T^{-2})}{(L^2T^{-2})^2}$$

$$\Rightarrow \quad [\alpha] = M^1L^{-1}T^0$$

LEAST COUNT

The minimum measurement that can be actually taken by an instrument is called the **least count**.

The least count of a metre scale graduated in millimetre mark is 1 mm.

The least count of a watch having second's hand is 1 second.

The least count of a stop watch is $\frac{1}{100}$th of a second i.e., $\frac{1}{100}$ s.

Conceptual Note(s)

(a) Least count (LC) of vernier calliper (VC) is given by

$$\text{LC} = \left\{\begin{matrix}\text{Value of one}\\ \text{part on main}\\ \text{scale}\end{matrix}\right\} - \left\{\begin{matrix}\text{Value of one}\\ \text{part on vernier}\\ \text{scale}\end{matrix}\right\}$$

$\Rightarrow$ LC = 1 MSD – 1 VSD

where

MSD = Main Scale Division

VSD = Vernier Scale Division

(b) Least count (LC) of vernier calliper (VC) is also given by

$$\text{LC} = \frac{\text{Value of 1 part on main scale } (s)}{\text{Number of parts on vernier scale } (n)}$$

(c) Least count (LC) of screw gauge (SG) is given by

$$\text{LC} = \frac{\text{Pitch}(p)}{\text{Number of parts on circular scale}(n)}$$

These have been discussed in detail at the end of the chapter.

(d)

Measurement from Device	Least Count
2.890 m	0.001 m
0.005 kg	0.001 kg
15.01 cm	0.01 cm
327.92 mm	0.01 mm
1111.111 m	0.001 m

SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity are the number of digits which are known reliably plus the one additional digit which is uncertain.

Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

RULE-1

All non-zero digits are significant.

EXAMPLE:

42.3 has three significant figures.
243.4 has four significant figures.
24.123 has five significant figures.

RULE-2

All zeros between two non-zero digits are significant.

EXAMPLE:

5.03 has three significant figures.
5.004 has four significant figures.
140.004 has six significant figures.

RULE-3

Leading zeros or the zeros placed to the left of the number are never significant

EXAMPLE:

0.543 has three significant figures.
0.045 has two significant figures.
0.006 has one significant figures.

RULE-4

Zeros that occur at the end of a number (i.e., on the right-hand side) without an expressed decimal point are ambiguous (i.e., we have no information on whether they are significant or not) and hence they are not considered to be significant.

EXAMPLE:

575000 has three significant figures.
3000 has one significant figure.

Conceptual Note(s)

(a) The measurements like 190 km may have 2 or 3 significant figures and 50800 calorie may have 3, 4 or 5 significant figures.

(b) The potential ambiguity here can be avoided by making use of Scientific Notation. Depending on whether 3, 4 or 5 significant figures are correct, we can write 50800 calorie as:

5.08×10^4 calorie has 3 significant figures.
5.080×10^4 calorie has 4 significant figures.
5.0800×10^4 calorie has 5 significant figures.

RULE-5

Trailing zeros or the zeros placed to the right of the number are significant.

EXAMPLE:

4.330 has four significant figures.
433.00 has five significant figures.
343.000 has six significant figures.

RULE-6

In exponential notation (or the results expressed in powers of 10), the numerical portion gives the number of significant figures i.e., the powers of 10 are not to be counted as significant figures.

EXAMPLE:

1.32×10^{-2} has three significant figures.
1.32×10^4 has three significant figures.

RULE-7

The number of significant figures does not depend upon the system of units used to measure the quantity.

EXAMPLE:

164 mm, 16.4 cm, 0.164 m, 0.000164 km, 164×10^{-6} km all have three significant figures.

RULE-8

There are certain measurements that are exact, like "If the number of students sitting in a class is say 125". Then the number of students sitting in the class is 125.00000000000000000 … and so this type of measurement is infinitely accurate and possesses infinite significant figures.

EXAMPLE:

Number of apples in a pack is 12,
Number of spheres in a box is 25
Both have infinite significant figures.

ROUNDING OFF

While rounding off measurements, we use the following rules by convention:

(a) *If the digit to be dropped is less than 5, then the preceding digit is left unchanged.*

EXAMPLE:

7.82 rounded off to one decimal place (RO1DP) is 7.8
3.94 rounded off to one decimal place (RO1DP) is 3.9
47.833 rounded off to one decimal place (RO1DP) is 47.8
47.862 rounded off to two decimal place (RO2DP) is 47.86

(b) *If the digit to be dropped is more than 5, then the preceding digit is raised by one.*

EXAMPLE:

6.87 rounded off to one decimal place (RO1DP) is 6.9
12.78 rounded off to one decimal place (RO1DP) is 12.8

47.862 rounded off to one decimal place (RO1DP) is 47.9
6.758 rounded off to two decimal place (RO2DP) is 6.76

(c) *If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.*

EXAMPLE:
16.351 rounded off to one decimal place (RO1DP) is 16.4
6.758 rounded off to one decimal place (RO1DP) is 6.8

(d) *If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.*

EXAMPLE:
3.250 rounded off to one decimal place (RO1DP) is 3.2
12.6850 rounded off to two decimal place (RO2DP) is 12.68
12.6850 rounded off to one decimal place (RO1DP) is 12.7

(e) *If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.*

EXAMPLE:
3.750 rounded off to one decimal place (RO1DP) is 3.8
16.150 rounded off to one decimal place (RO1DP) is 16.2

PRECISION AND ACCURACY OF A MEASUREMENT

Precision

Precision of measurement is the uncertainity in number. The precision of measurement is determined by the least count of measuring instrument. The smaller is the least count, larger is the precision of measurement.

Accuracy

The accuracy of a measurement is also determined by the number of significant figures. Larger the number of significant figures, more accurate is the measurement.

EXAMPLE:
If you find a length to be between values 3.375×10^7 m and 3.3765×10^7 m. The standard statement of your result is:

$$\text{Length} = (3.3760 \pm 0.0005)\times10^7 \text{ m}$$

The length 3.376×10^7 has an accuracy of four significant figures and precision of 0.0005×10^7 m.

SIGNIFICANT FIGURES IN CALCULATIONS: FEW EXAMPLES

In most of the experiments, the observations of various measurements are to be combined mathematically, i.e., added, subtracted, multiplied or divided as to achieve the final result. Since, all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.

1. In addition or subtraction of the numbers having different precisions, the result should be reported to the same number of decimal places as **are present in the number having the least number of decimal places**. In other words, the final result cannot be more precise than the least precise measurement. The rule is illustrated by the following examples:

(i)
$$\begin{array}{l} 33.3 \quad \leftarrow (\text{has only one decimal place}) \\ +3.11 \\ +0.313 \\ \hline 36.723 \leftarrow \left(\begin{array}{l}\text{answer should be reported}\\ \text{to one decimal place}\end{array}\right) \end{array}$$

Answer = 36.7

(ii)
$$\begin{array}{l} 3.1421 \\ +0.241 \\ +0.09 \quad \leftarrow (\text{has 2 decimal places}) \\ \hline 3.4731 \leftarrow \left(\begin{array}{l}\text{answer should be reported}\\ \text{to 2 decimal places}\end{array}\right) \end{array}$$

Answer = 3.47

(iii)
$$\begin{array}{l} 62.831 \quad \leftarrow (\text{has 3 decimal places}) \\ -24.5492 \\ \hline 38.2818 \leftarrow \left(\begin{array}{l}\text{answer should be reported}\\ \text{to 3 decimal places after}\\ \text{rounding off}\end{array}\right) \end{array}$$

Answer = 38.2818

2. In multiplication and division of numbers having different precisions, the final result should be reported to the same number of significant figures as that of the original number with **minimum number of significant figures**. In other words the result in multiplication or division **cannot be more accurate** than the **least accurate** measurement.

(i) 142.06
$\times 0.23 \quad \leftarrow (\text{two significant figures})$
32.6738

Answer $= 33$

(ii) 51.028
$\times 1.31 \quad \leftarrow (\text{three significant figures})$
66.84668

Answer $= 66.8$

(iii) $\frac{0.90}{4.26} = 0.2112676$

Answer $= 0.21$

ORDER OF MAGNITUDE: REVISITED

For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

(a) Speed of light in vacuum $= 3 \times 10^8\ \text{ms}^{-1} \approx 10^8\ \text{ms}^{-1}$ (ignoring $3 < 5$)

(b) Mass of electron $= 9.1 \times 10^{-31}$ kg $\approx 10^{-30}$ kg (as $9.1 > 5$)

Conceptual Note(s)

(a) Change in units of measurement of a quantity does not affect the number of significant figures.
(b) Significant figures are quoted for a measurement not for a pure number.
(c) Greater is number of significant figures in a measurement, smaller is the percentage error.

ORDER OF MAGNITUDE

In Physics, measurement of all things from atom to universe is done, so we have to deal with very small and very large magnitudes. Hence, we often talk about order of magnitude. In scientific notation the numbers are expressed as, Number $= M \times 10^x$. Where M is a number lies between 1 and 10 and x is integer. Order of magnitude of quantity is the power of 10 required to represent the quantity. Order of magnitude is expressed in terms of powers of 10 and is taken to be 10^0 if $M \le \sqrt{10}$ and 10^1 if $M > \sqrt{10}$ $(\text{where } \sqrt{10} = 3.16)$.

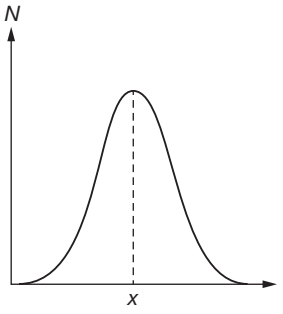
EXAMPLE:

Speed of light is $3 \times 10^8\ \text{ms}^{-1}$, its order of magnitude is and mass of electron is 9.1×10^{-31} kg, its order of magnitude is $10 \times 10^{-31} \approx 10^{-30}$ kg.

ERRORS IN A REPEATED MEASUREMENT

If we take a measurement experimentally, it necessarily involves errors, due to two factors.

(a) Human errors, which may be due to reaction time or carelessness.
(b) Experimental errors, which are due to least count of measuring instruments. For given measuring instruments, the human errors may be reduced by repeating experiment for a large number of times. If a graph is plotted between number of observations and the observed quantity x, the graph is shown in figure. Such a curve is called **Gaussian distribution** or **normal distribution curve**.



MEAN VALUE

If $x_1, x_2, x_3 ... x_n$ are n measured values of a physical quantity, then the mean value is given by

$$x_{av} = \langle x \rangle = \bar{x} = \frac{x_1 + x_2 + x_3 + ... x_n}{N} = \frac{\sum_{i=1}^{N} x_i}{N}$$

STANDARD DEVIATION (σ)

The spread of the experimental data is measured by the quantity called **Standard Deviation, defined as**

$$\sigma = \sqrt{\frac{1}{N-1}\sum_i (x_i - x_{av})^2}$$

STANDARD ERROR IN THE MEAN

The standard error (also called probable error of the mean) α, for a given set of readings (data) is

$$\alpha = \frac{\sigma}{\sqrt{N}}$$

The larger is the number of readings, the smaller is the error.

EXAMPLE:

$10^{\circ} \times 10^8 = 10^8$ ms^{-1}

A student takes 100 readings and standard error is e. If he takes 400 readings, then the error will be $\frac{e}{\sqrt{4}} = \frac{e}{2}$. So, on taking 400 readings the standard error will be halved.

ABSOLUTE ERRORS

The positive difference between arithmetic mean value and the measured value in the ith observation is called as the absolute error of that observation. The arithmetic mean value is also called as true value. Absolute error is

$$|(\text{mean value or true value}) - (i\text{th measured value})|$$

$$\Rightarrow \quad \Delta a_i = |a_{av} - a_i|$$

where Δa_i is absolute error in the ith observation. Then clearly

$$\Delta a_1 = |a_{av} - a_1|$$
$$\Delta a_2 = |a_{av} - a_2|$$
.....
.....
$$\Delta a_n = |a_{av} - a_n|$$

The arithmetic mean of all the absolute errors is called as mean absolute error and is given by

$$(\Delta a)_{av} = \langle \Delta a \rangle = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|}{n}$$

$$(\Delta a)_{av} = \langle \Delta a \rangle = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

And if we take the single measurement then the result of measurement will be $\langle a \rangle \pm \langle \Delta a \rangle = a_{av} + (\Delta a)_{av}$

RELATIVE AND PERCENTAGE ERROR

The relative error is defined as the ratio of the mean absolute error to the mean value or the true value. Mathematically,

$$\text{Relative error} = \frac{(\Delta a)_{av}}{a} = \frac{\langle \Delta a \rangle}{a}$$

$$\Rightarrow \quad \text{Percentage relative error} = \frac{\langle \Delta a \rangle}{a} \times 100\%$$

ILLUSTRATION 15

The length of a rod as measured in an experiment is recorded as 2.50 m, 2.54 m, 2.49 m, 2.58 m, 2.49 m, 2.57 m respectively. Find the mean/true length, absolute error in each case, mean absolute error and the percentage error.

SOLUTION

Mean length or true length

$$a_{av} = \langle a \rangle = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6}$$

$$\langle a \rangle = \frac{2 \cdot 50 + 2.54 + 2.49 + 2.58 + 2.49 + 2.57}{6}$$

$$\langle a \rangle = \frac{15.17}{6} = 2.528$$

$$\Rightarrow \quad \langle a \rangle \cong 2.53 \text{ m}$$

$$\Delta a_1 = |a_{av} - a_1| = |2.53 - 2.50| = 0.03$$

$$\Delta a_2 = |a_{av} - a_2| = |2.53 - 2.54| = 0.01$$

$$\Delta a_3 = |a_{av} - a_3| = |2.53 - 2.49| = 0.04$$

$$\Delta a_4 = |a_{av} - a_4| = |2.53 - 2.58| = 0.05$$

$$\Delta a_5 = |a_{av} - a_5| = |2.53 - 2.49| = 0.04$$

$$\Delta a_6 = |a_{av} - a_6| = |2.53 - 2.57| = 0.04$$

Δa_1, Δa_2, Δa_3, Δa_4, Δa_5, Δa_6 are the absolute errors in each case.

Mean absolute error i.e., $(\Delta a)_{av} = \langle \Delta a \rangle$ is

$$(\Delta a)_{av} = \langle \Delta a \rangle = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \Delta a_4 + \Delta a_5 + \Delta a_6}{6}$$

$$\Rightarrow \quad (\Delta a)_{av} = \langle \Delta a \rangle = \frac{0.03 + 0.01 + 0.04 + 0.05 + 0.04 + 0.04}{6}$$

$$\Rightarrow \quad (\Delta a)_{av} = \langle \Delta a \rangle = \frac{0.21}{6} = 0.035$$

So, mean length $= (2.53 \pm 0.035)$ m

$$\Rightarrow \quad \text{Percentage error} = \frac{\langle \Delta a \rangle}{a} \times 100\%$$

$$\Rightarrow \quad \%\text{ age error} = \frac{0.035}{2.53} \times 100\%$$

$$\Rightarrow \quad \%\text{ age error} = 1.38\%$$

COMBINATION OR PROPAGATION OF ERRORS

The formula used in an experiment may involve addition, subtraction, multiplication or division etc. of different quantities measured in the experiment. There may be some error in each of the reading. But all the errors will not affect the final result to the same extent. Different errors will affect the final result differently. So the final result will depend upon the way these errors are combined through mathematical operations. We shall calculate the maximum possible error in all the cases.

When the Result Involves the Sum of Two Observed Quantities

We suppose that the result X is given as the sum of two observed quantities A and B, i.e.

$$X = A + B$$

Let ΔA and ΔB the absolute errors in A and B. Then the values of A and B should be recorded as $A \pm \Delta A$ and $B \pm \Delta B$. If ΔX be the absolute error in the final result X, then

$$X \pm \Delta X = (A \pm \Delta A) + (B \pm \Delta B)$$

$$\Rightarrow \quad X \pm \Delta X = (A + B) \pm (\Delta A + \Delta B)$$

$$\Rightarrow \quad \pm \Delta X = \pm (\Delta A + \Delta B)$$

So, maximum possible error in X is

$$\Delta X = \Delta A + \Delta B$$

Thus when two quantities are added, the absolute error in the final result is the sum of the absolute errors of the quantities.

When the Result Involves the Difference of Two Observed Values

Let the result X be given as the different of two observed quantities A and B i.e.

$$X = A - B$$

Let ΔA and ΔB be absolute errors in A and B. If ΔX is the absolute error in the final result, then

$$X \pm \Delta X = (A \pm \Delta A) - (B \pm \Delta B)$$

$$\Rightarrow \quad X \pm \Delta X = (A + B) \pm (\Delta A \pm \Delta B)$$

$$\Rightarrow \quad \pm \Delta X = \pm (\Delta A \pm \Delta B)$$

But the error in X will be maximum if

$$\Delta X = \Delta A + \Delta B$$

Thus when the two quantities are subtracted, the absolute error (simply we may call it as error) in the final result is again the sum of absolute errors of the two quantities.

When the Result Involves the Product of Two Observed Quantities

We suppose that $X = AB$, then

$$X \pm \Delta X = (A \pm \Delta A)(B \pm \Delta B)$$

$$\Rightarrow \quad X \pm \Delta X = AB \pm A\Delta B \pm \Delta AB \pm \Delta A\, \Delta B$$

Dividing both sides of the above equation with X, we get

$$\frac{X \pm \Delta X}{X} = \frac{AB \pm A\Delta B \pm \Delta AB \pm \Delta A \Delta B}{AB}$$

$$\Rightarrow \quad 1 \pm \frac{\Delta X}{X} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A \Delta B}{AB}$$

Since ΔA and ΔB are small, so the product $\Delta A \Delta B$ can be neglected. So we get

$$\Rightarrow \quad \pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

To have the maximum relative error, we get

$$\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Alternative Method: The above result can be derived as follows:

$$X = AB$$

$$\Rightarrow \quad \log X = \log A + \log B$$

Differentiating on both the sides

$$\frac{dX}{X} = \frac{dA}{A} + \frac{dB}{B}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \text{ and}$$

$$\Rightarrow \quad \frac{\Delta X}{X}\% = \frac{\Delta A}{A} \times 100 + \frac{\Delta B}{B} \times 100$$

Thus, the final relative error, when the quantity is the product of two observed quantities is the sum of the relative errors of the two quantities.

When the Result Involves the Quotient of Two Observed Quantities

Suppose that $X = \frac{A}{B}$

Then $X \pm \Delta X = \frac{A \pm \Delta A}{B \pm \Delta B}$

$$\Rightarrow \quad X \pm \Delta X = (A \pm \Delta A)(B \pm \Delta B)^{-1}$$

$$\Rightarrow \quad X\left(1 \pm \frac{\Delta X}{X}\right) = A\left(1 \pm \frac{\Delta A}{A}\right) B^{-1} \left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

$$\Rightarrow \quad X\left(1 \pm \frac{\Delta X}{X}\right) = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

$$\Rightarrow \quad \left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A \Delta B}{AB}\right)$$

Neglecting $\frac{\Delta A \Delta B}{AB}$ $\{\because \Delta A \text{ and } \Delta B \text{ are very small}\}$

$$\Rightarrow \quad \left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}\right)$$

$$\Rightarrow \quad \pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

So, whether the quantities are being multiplied or divided, then

$$\Rightarrow \quad \begin{pmatrix}\text{Percentage}\\ \text{Error in}\\ \text{value of X}\end{pmatrix} = \begin{pmatrix}\text{Percentage}\\ \text{Error in}\\ \text{value of } A\end{pmatrix} + \begin{pmatrix}\text{Percentage}\\ \text{Error in}\\ \text{value of } B\end{pmatrix}$$

If we are to find the absolute error ΔX, then we have

$$\Delta X = X\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \quad X = AB$$

Now, if, then $\Delta X = B\Delta A + A\Delta B$

and if $X = \frac{A}{B}$, then $\Delta X = \frac{A}{B}\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

In Case of Power Functions

CASE-1

Suppose $X = kA^n$, where k is a constant

Then $X \pm \Delta X = k(A \pm \Delta A)^n$

$$\Rightarrow \quad \cancel{X}\left(1 \pm \frac{\Delta X}{X}\right) = \cancel{kA^n}\left(1 \pm \frac{\Delta A}{A}\right)^n$$

$$\Rightarrow \quad 1 \pm \frac{\Delta X}{X} = \left(1 \pm \frac{\Delta A}{A}\right)^n$$

Since $\frac{\Delta A}{A} \ll 1$, so from Binomial Theorem, we get

$$1 \pm \frac{\Delta X}{X} = 1 \pm n\left(\frac{\Delta A}{A}\right)$$

$$\Rightarrow \quad \pm \frac{\Delta X}{X} = \pm n\left(\frac{\Delta A}{A}\right)$$

$$\Rightarrow \quad \frac{\Delta X}{X} = n\left(\frac{\Delta A}{A}\right)$$

$\{\because$ Error in a measurement is always extremum during measurement$\}$

CASE-2

If $X = k\left(\frac{A^{\ell} B^{m}}{C^{n}}\right)$

$\{l > 0,\ m > 0,\ n > 0$ and k is a constant$\}$

If ΔA, ΔB and ΔC be the respective absolute errors in calculating the values of A, B and C, then let the corresponding error in calculating X (depending on A, B and C) be ΔX. Then

$$X \pm \Delta X = \frac{(A \pm \Delta A)^{l}(B \pm \Delta B)^{m}}{(C \pm \Delta C)^{n}}$$

$$\Rightarrow \quad X\left(1 \pm \frac{\Delta X}{X}\right) = \frac{A^{l}\left(1 \pm \frac{\Delta A}{A}\right)^{l} B^{m}\left(1 \pm \frac{\Delta B}{B}\right)^{m}}{C^{n}\left(1 \pm \frac{\Delta C}{C}\right)^{n}}$$

$$\Rightarrow \quad \cancel{X}\left(1 \pm \frac{\Delta X}{X}\right) = \frac{\cancel{A^{l}B^{m}}}{\cancel{C^{n}}} \frac{\left(1 \pm \frac{\Delta A}{A}\right)^{l}\left(1 \pm \frac{\Delta B}{B}\right)^{m}}{\left(1 \pm \frac{\Delta C}{C}\right)^{n}}$$

$$\Rightarrow \quad \left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A}\right)^{l}\left(1 \pm \frac{\Delta B}{B}\right)^{m}\left(1 \pm \frac{\Delta C}{C}\right)^{-n} \quad \ldots(1)$$

Since $\frac{\Delta A}{A} \ll 1$, $\frac{\Delta B}{B} \ll 1$ and $\frac{\Delta C}{C} \ll 1$, so from Binomial Theorem, we have

$$\left(1 \pm \frac{\Delta A}{A}\right)^{l} \cong 1 \pm l\frac{\Delta A}{A}$$

$$\left(1 \pm \frac{\Delta B}{B}\right)^{m} \cong 1 \pm m\frac{\Delta B}{B} \text{ and } \left(1 \pm \frac{\Delta C}{C}\right)^{-n} \cong 1 \mp n\frac{\Delta C}{C}$$

So, (1) becomes

$$1 \pm \frac{\Delta X}{X} = \left(1 \pm l\frac{\Delta A}{A}\right)\left(1 \pm m\frac{\Delta B}{B}\right)\left(1 \mp n\frac{\Delta C}{C}\right)$$

$$\Rightarrow \quad \pm\frac{\Delta X}{X} = \pm l\frac{\Delta A}{A} \pm m\frac{\Delta B}{B} \mp n\frac{\Delta C}{C} + \begin{pmatrix}\text{Neglected}\\ \text{Terms}\end{pmatrix}$$

$$\Rightarrow \quad \pm\frac{\Delta X}{X} = \pm l\frac{\Delta A}{A} \pm m\frac{\Delta B}{B} \mp n\frac{\Delta C}{C}$$

Since, we know that during propagation errors are always taken to be the extremum, so we have

$$+\frac{\Delta X}{X} = +l\frac{\Delta A}{A} + m\frac{\Delta B}{B} + n\frac{\Delta C}{C}$$

OR

$$-\frac{\Delta X}{X} = -l\frac{\Delta A}{A} - m\frac{\Delta B}{B} - n\frac{\Delta C}{C}$$

From both these relations, we get

$$\frac{\Delta X}{X} = l\frac{\Delta A}{A} + m\frac{\Delta B}{B} + n\frac{\Delta C}{C}$$

$$\Rightarrow \quad \begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } y\end{pmatrix} = l\begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } A\end{pmatrix} + m\begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } B\end{pmatrix} + n\begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } C\end{pmatrix}$$

Conceptual Note(s)

And if $X = kA^{\pm N}$, where k is a constant and N is any real number.

Then $\frac{\Delta X}{X} = N\frac{\Delta A}{A}$

$$\Rightarrow \quad \frac{\Delta X}{X}\% = N\left(\frac{\Delta A}{A}\right) \times 100\%$$

ILLUSTRATION 16

If $V = (50 \pm 2)V$ and $I = (5 \pm 0.1)A$, then find the percentage error in measuring the resistance. Also find the resistance with limits of error.

SOLUTION

(a) Given $V = (50 \pm 2)V$ and $I = (5 \pm 0.1)V$

We know that $R = \frac{V}{I}$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{\Delta V}{V} \pm \frac{\Delta I}{I} = \frac{2}{50} + \frac{0.1}{5}$$

$$\Rightarrow \quad \frac{\Delta R}{R}(\text{in } \%) = \frac{2}{50} \times 100 + \frac{0.1}{5} \times 100 = 6\ \%$$

(b) $R = \frac{V}{I} = \frac{50}{5} = 10$ ohm

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{2}{50} + \frac{0.1}{5}$$

$$\Rightarrow \quad \Delta R = \frac{2}{50} \times 10 + \frac{0.1}{5} \times 10 = 0.6$$

$$\Rightarrow \quad R \pm \Delta R = (10 \pm 0.8) \text{ ohm}$$

Problem Solving Technique(s)

(a) Δy is always positive i.e. $\Delta y > 0$.

(b) Δy has units same as that of y.

(c) A quantity in terms of absolute error is expressed as

$y = (y_t \pm \Delta y)$ units

(d) If least count is not given and a measurement is given, then error in the measurement will be ± 1 in last digit.

EXAMPLE

If $L = 5.216$ metre,

then $\Delta L = \pm\ 0.001$ metre

Also, if $M = 2.50$ Kg,

then $\Delta M = \pm\ 0.01$ kg

(e) The error in a measurement is equal to the least count of the instrument.

(f) For evaluating error in a formula

(i) the powers are changed into multiplication.

(ii) the multiplication and divisions are changed into addition.

(g) In case of addition and subtraction, it is advisable to calculate absolute error first and then relative error.

(h) In case of multiplication, division and power functions, it is advisable to calculate relative error first and then absolute error.

(i) Relative Error or Percentage relative error is dimensionless, hence a measurement can be expressed in terms of percentage relative error as

$$y = y_t \text{ units} \pm \left(\frac{\Delta y}{y_t} \times 100 \right)\%$$

(j) Errors never propagate in case of constants.

EXAMPLE

If $V = \frac{4}{3}\pi r^3$, then

$$\frac{\Delta V}{V} = \frac{3\Delta r}{r} \qquad \left\{ \text{Only when } \frac{\Delta r}{r} << 1 \right\}$$

(k) In general if $y = kx^n$, then

$$\frac{\Delta y}{y} = n\left(\frac{\Delta x}{x} \right) \qquad \left\{ \text{Only when } \frac{\Delta x}{x} << 1 \right\}$$

irrespective of value of k.

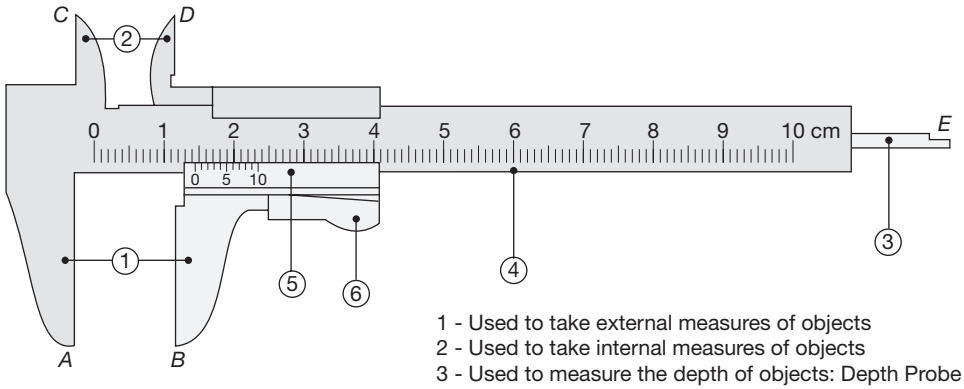
VERNIER CALLIPER

Vernier Calliper is a device used to measure the internal or external diameter as well as the depth of a vessel. The maximum precision up to which a scale can measure is 1 mm. For measuring lengths that are less than 1 mm, vernier calliper is used. Vernier calliper is also called as Slide Calliper.

A Vernier calliper consists of two scales (See Figure)

(a) **A Main Scale:** It is calibrated in cm on one side and in inches on the other side. The value of one main scale division (MSD) is 1 mm or 0.1 cm.

(b) **A Vernier Scale:** It is the movable scale that can slide on the main scale. Generally 10 vernier scale divisions (VSD) is equal to 9 main scale divisions (MSD). Therefore the value of one vernier scale division (VSD) is equal to $\frac{9}{10}$ mm.



1 - Used to take external measures of objects
2 - Used to take internal measures of objects
3 - Used to measure the depth of objects: Depth Probe
4 - Main scale (cm)
5 - Vernier (cm)
6 - Used to block movable part: Retainer

The upper two jaws C and D are used to find the internal diameter of a pipe, test tube and that of hollow cylinder. The lower two jaws A and B are used to measure the length and diameter of a cylinder or the diameter of a sphere by placing the object between the lower jaws. A thin metallic strip E projects at one end of the calliper which is used to measure the depth of the vessel. The smallest length up to which the vernier calliper can measure is called **Least Count (LC)** or **Vernier Constant**.

Least count of vernier calliper is defined as the ratio of the value of one MSD to the number of divisions on the vernier scale called Vernier Scale Division (VSD)

$$\text{Least count} = \frac{S}{N} = \frac{\text{Value of 1 MSD}}{\left(\begin{array}{c}\text{Number of divisions on}\\ \text{the vernier scale}\end{array}\right)}$$

Least count of vernier callipers is also defined as the difference between the value of one MSD and the value of one VSD.

$\Rightarrow$ Least count = 1 MSD – 1 VSD

It is observed that N VSD (divisions of vernier scale) coincide with $(N-1)$ MSD (divisions of the main scale)

$$N \text{ VSD} = (N-1) \text{ MSD}$$

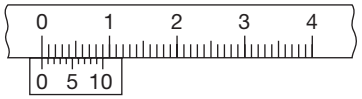
$$\Rightarrow \quad 1 \text{ VSD} = \frac{(N-1)}{N} \text{ MSD}$$

Least count = 1 MSD – 1 VSD

$$\text{Least Count} = 1 \text{ MSD} - \frac{(N-1)}{N} \text{ MSD} = \frac{1}{N} \text{ MSD}$$

CONCEPT OF ZERO ERROR

When the lower two jaws touch each other, the zero of the vernier scale coincides with the zero of the main scale. When this is the case then there is no zero error. However, if the zero of the vernier scale does not coincide with zero of the main scale (when the jaws are in contact), then error exists and necessary correction has to be applied.



No zero error

Positive Zero Error

The zero of the vernier scale lies to the right of zero of the main scale. When the two jaws are in contact, the distance between them is zero but the reading is positive. In other words, the instrument measures more than the actual distance and therefore zero error is positive. Figure A represents the case of positive zero error.

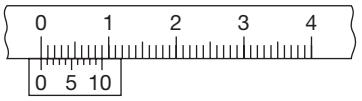


Figure A Positive zero error

Negative Zero Error

The zero of vernier scale lies to the left of zero of the main scale. When the two jaws are in contact, the distance between them is zero but now the reading is negative. In other words, the instrument measures less than the actual distance and therefore zero error is negative. Figure B represents the case of negative zero error.

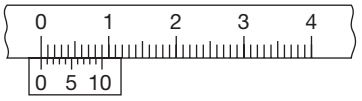


Figure B Negative zero error

CALCULATING ZERO ERROR

To calculate the zero error the two jaws are kept in contact and the number of vernier scale divisions coinciding with some main scale division is noted. For the arrangement shown in Figure A, the fourth vernier division coincides with some division of main scale.

$\Rightarrow$ Zero error $= +4 \times (\text{LC}) = +4 \times 0.01 = +0.04$ cm

$\Rightarrow$ Zero correction $= -0.04$ cm

For the arrangement shown in Figure B, fifth vernier division coincides with some main scale division. To calculate zero error, the number of division to be considered will be 5.

$\Rightarrow$ Zero error $= -5 \times (\text{LC}) = -5 \times 0.01 = -0.05$ cm

$\Rightarrow$ Zero correction $= +0.05$ cm

So, the correct length is

$$\text{Correct Length} = (\text{Observed Length}) \pm (\text{Zero Error})$$

STEPS TO BE FOLLOWED WHILE TAKING READINGS WITH VERNIER CALLIPERS

(a) Find the Least Count (LC).
(b) For taking the Main Scale Reading (MSR), the division next before the zero of the vernier should be considered.
(c) The division of the vernier scale which coincides with any main scale division should be noted. This is called Vernier Coincidence (VC).
(d) The Correct Reading (CR) is given by

$$CR = MSR + (LC) \times (VC)$$

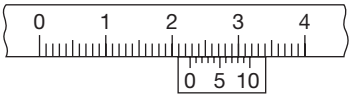
(e) Prefer using a lens to note Vernier Coincidence (VC).
(f) Vernier principle is also used for measurement of fraction of angles as in sextant.

Precautionary Measures

(a) As a precaution, the jaws (lower) of the callipers should not be pressed too hard on the object placed between them so as to get the actual reading.
(b) Also at any position, the diameter should be measured in two directions at right angles to each other.
(c) Same units should be used while calculating the result.
(d) Vernier coincidence must be noted without any parallax error by repeating the observations five times at different positions of the object. Slide calliper is called vernier callipers since it was first designed by French mathematician named Vernier.

HOW TO MEASURE

STEP-1: Measure the least count/vernier constant.
STEP-2: Place the object between the jaws and tighten them



STEP-3: Suppose that during this observation, zero of the vernier scale (having vernier constant 0.01 cm) lies between 2.2 cm and 2.3 cm mark. Therefore, in this observation, the length of the object under measurement is 2.2 cm plus some fraction (say y).
STEP-4: The value of the fraction is found by locating the vernier division which coincides with a main scale division. In present case it is 7. Therefore, fraction to be added is given by

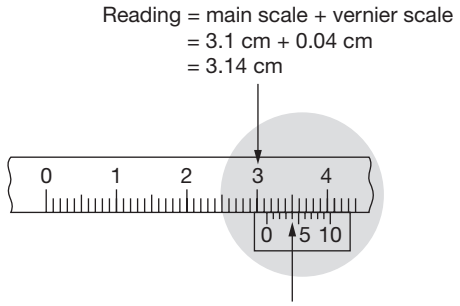
$$\text{Fraction} = 7 \times \text{V.C.} = 7 \times 0.01 \text{ cm} = 0.07 \text{ cm}$$

Hence, the reading of the vernier callipers is

$2.2 + 0.07$ i.e., 2.27 cm.

STEP-5: So, the correct reading is given by

$$CR = MSR + (LC) \times (VC)$$



The fourth vernier mark coincides with a marking on the main scale. This gives a reading of 0.4 mm or 0.04 cm to be added to main scale reading.

ILLUSTRATION 17

Least count of a vernier calliper is 0.01 cm. When the two jaws of the instrument touch each other the 5th division of the vernier scale coincide with a main scale division and the zero of the vernier scale lies to the left of the zero of the main scale. Further more while measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.4 cm and 2.5 cm and the 6th vernier division coincides with a main scale division. Calculate the diameter of the sphere.

SOLUTION

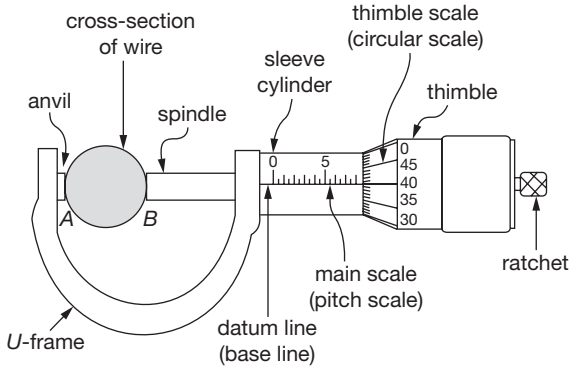
The instrument has a negative error,

$$e = (-5 \times 0.01) \text{ cm} = -0.05 \text{ cm}$$

Measured reading $= (2.4 + 6 \times 0.01) = 2.46$ cm
True reading = Measured reading $-e$
$\Rightarrow$ True Reading $= 2.46 - (-0.05)$
$\Rightarrow$ True Reading $= 2.51$ cm
Therefore, diameter of the sphere is 2.51 cm

SCREW GAUGE

A Screw Gauge is a device used to determine thickness (or diameter) of thin sheet or a wire. With its help one can measure the diameter of very thin wires or similar objects. Its measures accurately up to 0.001 cm and hence is also commonly called **micrometer screw gauge**.



CONSTRUCTION

The screw gauge consists of the following parts

U-Frame

It is a steel frame having a U shape. On one end of U shaped frame a screw is fixed permanently. This fixed screw is commonly called **Stud** or **Anvil** and forms the fixed jaw of the screw gauge. On the other end of U shaped frame is fixed a **Nut** through which slides a screw. The end A of the screw forms the movable jaw of screw gauge.

Nut and Screw

The nut (at B) is threaded from inside and the screw is threaded from outside. The screw can move in and out of nut by circular motion.

Thimble or Circular Cylinder

The screw is connected to a hollow circular cylinder(s) also called **Thimble**, which rotates along with the nut on turning.

Sleeve Cylinder

To the nut is attached a hollow cylinder, commonly called **Sleeve Cylinder**. The spindle of the screw passes through this sleeve cylinder.

Base Line

A reference line or Base Line or Datum Line is graduated in mm and is drawn on the sleeve cylinder parallel to the axis of nut. It is commonly called **Main Scale** or **Sleeve Scale** or **Pitch Scale.**

Circular Scale or Thimble Scale or Head Scale

The hollow cylinder that moves over the sleeve cylinder is tapered from one end. On the tapered end graduations are made, which are either 50 or 100 in number. The scale marked on sleeve is called **circular scale** or **thimble scale** or **head scale**.

Ratchet

The ratchet is attached to screw with the help of a spring. When the flattened end B of the screw comes in contact with the stud A, the ratchet becomes free and makes a rattling noise. Thus, end B of the screw will not be further pushed towards the stud A.

PITCH OF SCREW

The pitch of screw is defined as, the distance between two consecutive threads of screw when measured along the axis of screw

OR

Pitch of screw can also be defined as the forward distance travelled by the tip of screw (end B) when head of screw completes one rotation.

PRINCIPLE OF SCREW GAUGE

It works on fundamental screw principle that rotational motion can be converted into translational motion.

DETERMINATION OF PITCH OF SCREW

To determine the pitch of the screw gauge, the screw is given five complete rotations. The distance moved by the thimble on the main scale is then recorded. The pitch is calculated by the formula

$$\text{Pitch} = \left(\frac{\text{Distance moved by thimble on Main Scale (MS)}}{\text{Number of rotations of thimble}} \right)$$

ILLUSTRATION 18

If 5 mm is the distance moved by the thimble on the main scale for 5 rotations then calculate the pitch.

SOLUTION

$$\text{Pitch} = \frac{5 \text{ mm}}{5} = 1 \text{ mm}$$

LEAST COUNT OF SCREW

For a screw gauge, the least count is the smallest distance moved by its tip when the screw turns through division marked on it.

DETERMINATION OF LEAST COUNT

First of all we shall determine the pitch and also count the number of divisions on circular scale. Then least count is determined by the formula

$$\text{Least Count} = \frac{\text{Pitch}}{\left(\begin{array}{c}\text{Number of divisions on}\\ \text{the Circular Scale (CS)}\end{array}\right)}$$

For a screw gauge having a pitch of 1 mm and having 100 divisions marked on its thimble, the least count is

$$\text{Least Count} = \frac{1 \text{ mm}}{100} = 0.01 \text{ mm} = 0.001 \text{ cm}$$

BACKLASH ERROR

This type of error is associated with all instruments based on the Screw Principle. In the screw gauge, the screw moves in a nut. If the screw is not fitting properly in the grooves (threads), then there is irregularity in the movement of the screw, when the head is rotated. Error due to this is called **backlash error**. In order to avoid this error, during an observation the screw is always moved in one direction.

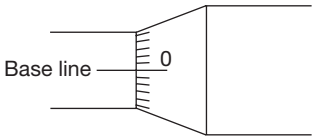
DETERMINATION OF ZERO ERROR

On bringing the screw end B in contact with the stud A, if the zero of the main scale (MS) does not coincide with zero of circular scale (CS), the micrometer or the screw gauge is said to have zero error.

No Zero Error

Before using the screw gauge, we must check for a zero error. Close the screw gauge so that the spindle touches the anvil.

If there is no zero error then the reading will be 0.00 mm, as shown. In this case, the zero of the circular scale coincides with Base Line (or Reference Line).

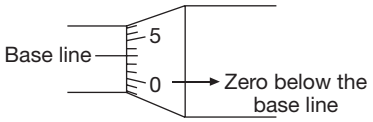


Positive Zero Error with Negative Correction

The zero of circular scale may be left behind the Base Line or below the Base Line (or Reference Line) as shown in figure.

In such a case when the studs are in contact (zero distance), the screw gauge is giving a positive reading. Such a screw gauge measures more and therefore it has got Positive Zero Error or Negative Zero Correction.

From figure we note that the zero of circular scale has been left behind by 3 circular divisions.



So, zero error is

$$\left(\begin{array}{c}\text{Zero}\\ \text{Error}\end{array}\right) = +\left(\begin{array}{c}\text{Number of divisions}\\ \text{left behind}\end{array}\right) \times \left(\begin{array}{c}\text{Least}\\ \text{Count}\end{array}\right)$$

$\Rightarrow$ Zero Error $= +3 \times 0.001$ cm $= +0.003$ cm

$\Rightarrow$ Zero Correction $= -0.003$ cm

So, to conclude we have

$$\left(\begin{array}{c}\text{Correct}\\ \text{Length}\end{array}\right) = \left(\begin{array}{c}\text{Observed}\\ \text{Length}\end{array}\right) + \left(\begin{array}{c}\text{Zero}\\ \text{Correction}\end{array}\right)$$

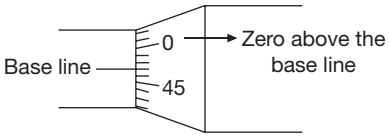
OR

$$\text{Correct Length} = \text{Observed Length} - \text{Zero Error}$$

Negative Zero Error with Positive Correction

When the zero line, marked on circular scale lies above the Base Line of the main scale, then there is a negative error and the correction is positive. The zero of circular scale may cross the reference line as shown in figure. In such a case, if we move the screw back so as to make the zero of circular scale to coincide with

reference line, a gap will be left between the studs. Thus such a screw gauge gives the zero reading even when there is gap between studs i.e., it measures less. Therefore, the screw gauge possesses negative zero error or positive zero correction.



To determine the zero correction, note the number of divisions through which the zero mark of circular scale has crossed the reference line. From figure, we note that the number of such divisions that cross the zero mark is 3.

So, Zero error is

$$\begin{pmatrix}\text{Zero}\\ \text{Error}\end{pmatrix} = -\begin{pmatrix}\text{Number of}\\ \text{divisions crossed}\end{pmatrix} \times \begin{pmatrix}\text{Least}\\ \text{Count}\end{pmatrix}$$

$\Rightarrow$ Zero Error $= -3 \times 0.001$ cm $= -0.003$ cm

$\Rightarrow$ Zero Correction $= +0.003$ cm

Here also, we have

$$\begin{pmatrix}\text{Correct}\\ \text{Length}\end{pmatrix} = \begin{pmatrix}\text{Observed}\\ \text{Length}\end{pmatrix} + \begin{pmatrix}\text{Zero}\\ \text{Correction}\end{pmatrix}$$

OR

Correct Length = Observed Length – Zero Error

READING A SCREW GAUGE

Place the object say, a piece of wire (whose diameter is to be found) between the two studs of the screw gauge. Turn the screw with the help of ratchet arrangement, till the ratchet becomes free with the sound of a click. It ensures the gentle placing of the wire between the studs.

Let the complete main scale reading (MSR) be read as a and the nth circular scale division (CSD) coinciding with the reference line.

Then, diameter of the wire,

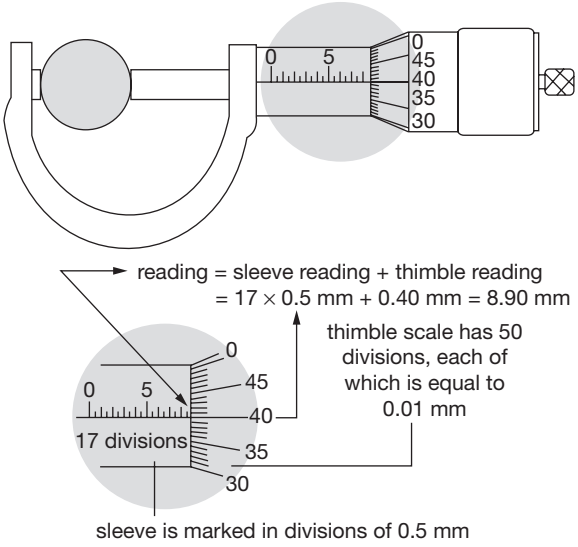
$$d = a \times \text{pitch} + n \times \text{least count}$$

Here $a = 17$, pitch $= 0.5$ mm, $n = 40$,

least count $= 0.01$ mm

Also, we can say that,

Reading = Sleeve Reading + Thimble Reading


0
5
0
45
40
35
30
reading = sleeve reading + thimble reading
= 17 × 0.5 mm + 0.40 mm = 8.90 mm
thimble scale has 50 divisions, each of which is equal to 0.01 mm
17 divisions
sleeve is marked in divisions of 0.5 mm

Conceptual Note(s)

So, in general the correct measurement taken by Screw Gauge is

$$\text{Correct Reading} = \text{MSR} + (\text{CSR} \times \text{LC}) + \text{ZC}$$

OR

$$\text{Correct Reading} = \text{MSR} + (\text{CSR} \times \text{LC}) - \text{ZE}$$

where, **MSR** is Main Scale Reading

CSR is Circular Scale Reading, **LC** is Least Count

ZC is Zero Correction and **ZE** is Zero Error

ILLUSTRATION 19

The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and B are just touching each without putting anything between the studs 32nd division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Find the thickness of the glass plate. Zero of linear scale is not hidden from circular scale when A and B touches each other.

SOLUTION

Least count

$$\text{L.C.} = \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$

$$\text{LC} = \frac{1}{100}\ \text{mm} = 0.01\ \text{mm}$$

As zero is not hidden from circular scale when A and B touches each other. Hence, the screw gauge has positive error.

$$e = +n(\text{L.C.}) = 32 \times 0.01 = 0.32\ \text{mm}$$

Linear scale reading $= 4 \times (1\ \text{mm}) = 4\ \text{mm}$

Circular scale reading $= 16 \times (0.01\ \text{mm}) = 0.16\ \text{mm}$

$\Rightarrow$ Measured reading $= (4 + 0.16)\ \text{mm} = 4.16\ \text{mm}$

$\Rightarrow$ Absolute reading = Measured reading $-e$

$\Rightarrow$ Absolute reading $= (4.16 - 0.32)\ \text{mm} = 3.84\ \text{mm}$

Therefore, thickness of the glass plate is 3.84 mm

ILLUSTRATION 20

The pitch of a screw gauge is 1 mm and there are 100 divisions on its circular scale. When nothing is put in between its jaws, the zero of the circular scale lies 6 divisions below the reference line. When a wire is placed between the jaws, 2 linear scale divisions are clearly visible while 62 divisions on circular scale coincide with the reference line. Determine the diameter of the wire.

SOLUTION

$$\text{L.C.} = \frac{p}{N} = \frac{1\ \text{mm}}{100} = 0.01\ \text{mm}$$

The instrument has a positive zero error.

$$e = +n(\text{L.C.}) = +(6 \times 0.01) = +0.06\ \text{mm}$$

Linear scale reading $= 2 \times (1\ \text{mm}) = 2\ \text{mm}$

Circular scale reading $= 62 \times (0.01\ \text{mm}) = 0.62\ \text{mm}$

So, measured reading $= 2 + 0.62 = 2.62\ \text{mm}$

i.e., True reading $= 2.62 - 0.06 = 2.56\ \text{mm}$

Test Your Concepts-IV

Based on Errors, Significant Figures, Vernier Calliper and Screw Gauge

(Solutions on page H.9)

1. In a circuit the potential difference across a resistance is $V = (8 \pm 0.5)$ volt and the current in the circuit is $I = (2 \pm 0.2)$ ampere. Calculate the resistance with absolute and the percentage error.
2. In an experiment for determining the density d of a rectangular block of metal the length, breadth and height are 5.12 cm, 2.56 cm, 0.37 cm and the mass is 39.3 g. Calculate the maximum permissible absolute and relative error in the determination of density.
3. In a vernier callipers N divisions of vernier coincide with $(N-1)$ divisions of main scale in which length of 1 division is 1 mm. What is the least count of the instrument in cm?
4. The length and the radius of a cylinder measured with slide callipers are found to be 4.54 cm and 1.75 cm respectively. Calculate the volume of the cylinder.
5. In the table shown, Column A gives few measured values and Column B has the number of significant figures to be correctly against the measured respective value. Complete the entries of Column B.

Column A	Column B
17236	_____
510 m	_____
270	_____
4.20	_____
7042.6	_____
0.017	_____
6.1×10^{14}	_____

6. Use the rules studied for rounding off to give the rounded off values to three significant figures.

Measured Value	Rounded off value to Three Significant Figures
7.364	_____
8.3251	_____
9.445	_____
15.75	_____
7.367	_____
9.4450	_____
15.7500	_____

7. Calculate :

(a) $6.789 + 3.45 + 1.2 =$ _____

(b) $12.63 - 10.2 =$ _____

(c) $36.72 \times 1.2 =$ _____

(d) $\dfrac{1100\ \text{ms}^{-1}}{10.2\ \text{ms}^{-1}} =$ _____

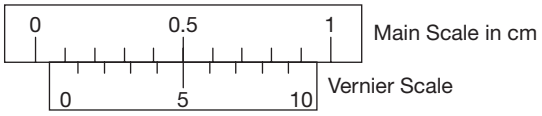
8. The smallest division on main scale of a vernier callipers is 1 mm and 10 vernier divisions coincide with 9 scale divisions. While measuring the length of a line, the zero mark of the vernier scale lies between 10.2 cm and 10.3 cm and the third division of vernier scale coincide with a main scale division.

(a) Determine the least count of the callipers

(b) Find the length of the line.

9. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are six divisions on the linear scale and forty divisions on circular scale coincides with the reference line. Find the diameter of the sphere.

10. In the diagram shown in figure, 9 MSD coincide with 10 VSD. Find the magnitude and nature of zero error.



SOLVED PROBLEMS

PROBLEM 1

P.A.M. Dirac, a great Physicist of the 20th century found that from the following basic constants a number having dimensional formula same as that of time can be constructed i.e.,

(a) the charge on the electron e
(b) permittivity of free space ε_o
(c) mass of the electron m_e
(d) mass of the proton m_P
(e) speed of light c
(f) Universal Gravitational constant G.

Obtain the relation for this Dirac's number. You are given that the desired number is proportional to m_p^{-1} and m_e^{-2}. What is the significance of this number? Assume constant of proportionality to be 1.

SOLUTION

According to the statement of the problem, we have

$$t \propto e^p \varepsilon_0^q m_e^{-2} m_p^{-1} c^r G^s$$

Taking constant of proportionality as 1 (given in question) and dimensions on both sides, we get

$$T = (AT)^p \left(M^{-1}L^{-3}A^2T^4\right)^q (M)^{-2} (M)^{-1} \times \left(LT^{-1}\right)^r \left(M^{-1}L^3T^{-2}\right)^s$$

$$\Rightarrow \quad T = M^{-q-3-s} L^{-3q+r+3s} T^{p+4q-r-2s} A^{p+2q}$$

Applying Principle of Homogeneity, we get

$$-q-3-s=0 \qquad \ldots(1)$$

$$-3q+r+3s=0 \qquad \ldots(2)$$

$$p+4q-r-2s=1 \qquad \ldots(3)$$

$$p+2q=0 \qquad \ldots(4)$$

Adding (2) and (3) for eliminating r, we get

$$q+p+s=1 \qquad \ldots(5)$$

From (5) $p=-2q$, put in (5) to get

$$q-2q+s=1$$

$$\Rightarrow \quad -q+s=1 \qquad \ldots(6)$$

Adding (1) and (6), we get

$$-2q=4$$

$$\Rightarrow \quad q=-2$$

So, $p=4$, $q=-2$, $s=-1$, $r=-3$

Hence, we get

$$t = e^4 \varepsilon_0^{-2} m_e^{-2} m_p^{-1} c^{-3} G^{-1}$$

$$\Rightarrow \quad t = \frac{e^4}{\varepsilon_0^2 m_e^2 m_p G c^3} \qquad \ldots(7)$$

Also, you can verify the dimensional correctness of equation (7) by substituting the dimensional formulae of the respective quantities on the right.

Now substituting the values of all constants in equation (7), we get the value of this time t as

$$t = 3.36 \times 10^{18} \text{ s} \cong 10^{11} \text{ years}$$

Dirac estimated this time to be the age of universe.

PROBLEM 2

The mean life of the neutral elementary particle pion is 2×10^{-7} ns. The age of the universe is about 4×10^9 year. Find a time that is midway between these two times on the logarithmic scale.

SOLUTION

Let t be the time half way between these two on the logarithmic scale. Then

$$\log_e t = \frac{\log_e t_1 + \log_e t_2}{2}$$

$$\Rightarrow \quad \log_e t = \frac{1}{2}\log_e (t_1 t_2) = \log_e \sqrt{t_1 t_2}$$

Taking antilog both sides, we get

$$t = \sqrt{t_1 t_2}$$

$$t_1 = 2\times10^{-7} \text{ ns} = 2\times10^{-16} \text{ s}$$

$$t_2 = 4\times10^9 \text{ year} = 4\times10^9 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$\Rightarrow \quad t_2 = 1.26\times10^{17} \text{ s}$$

So, $t_1 t_2 \cong 25$

$$\Rightarrow \quad \sqrt{t_1 t_2} = 5 \text{ s}$$

PROBLEM 3

Derive by the method of dimensions, an expression for the volume of a liquid flowing out per second through a narrow pipe. Assume that the rate of flow of liquid depends on

(a) the coefficient of viscosity η of the liquid

(b) the radius r of the pipe and

(c) the pressure gradient $\left(\frac{p}{l}\right)$ along the pipe.

Take $k = \frac{\pi}{8}$

SOLUTION

Let volume flowing out per second through the pipe be given by

$$V = k\eta^a r^b \left(\frac{p}{l}\right)^c \qquad \ldots(1)$$

where $k = a$ dimensionless constant. Dimensions of the various quantities are

$$[V] = \frac{\text{volume}}{\text{time}} = \frac{L^3}{T} = L^3T^{-1}$$

$$[\eta] = ML^{-1}T^{-1},\ [r] = L$$

$$\left[\frac{p}{l}\right] = \frac{\text{pressure}}{\text{length}} = \frac{\text{force}}{\text{area} \times \text{length}}$$

$$\Rightarrow \quad \left[\frac{p}{l}\right] = \frac{MLT^{-2}}{L^2.L} = ML^{-2}T^{-2}$$

Substituting these dimensions in equation (1), we get

$$\left[L^3T^{-1}\right] = \left[ML^{-1}T^{-1}\right]^a [L]^b \left[ML^{-2}T^{-2}\right]^c$$

$$\Rightarrow \quad M^0L^3T^{-1} = M^{a+c}L^{-a+b-2c}T^{-a-2c}$$

Equating the powers of M, L and T, we get

$$a + c = 0,\ -a + b - 2c = 3,\ -a - 2c = -1$$

On solving, $a = -1$, $b = 4$, $c = 1$.

$$\Rightarrow \quad V = k\eta^{-1}r^4\left(\frac{p}{l}\right)^1$$

$$\Rightarrow \quad V = \frac{\pi r^4 p}{8\eta l} \qquad \text{[Poiseuille's equation]}$$

PROBLEM 4

Finding dimensions of resistance R and inductance L, speculate what physical quantities $\left(\frac{L}{R}\right)$ and $\left(\frac{1}{2}\right)Li^2$ represents ?

SOLUTION

As $e = L\frac{di}{dt}$ i.e., $L = e\frac{dt}{di}$

So $\quad [L] = \left[\frac{W}{q}\right]\left[\frac{t}{i}\right] = \frac{ML^2T^{-2}}{AT} \times \frac{T}{A}$

i.e., $[L] = ML^2T^{-2}A^{-2}$

and as $V = IR$, i.e., $R = \frac{V}{I}$

So $\quad [R] = \left[\frac{W}{qA}\right] = \frac{ML^2T^{-2}}{ATA}$

$$\Rightarrow \quad [R] = ML^2T^{-3}A^{-2}$$

So $\quad \left[\frac{L}{R}\right] = \frac{ML^2T^{-2}A^{-2}}{ML^2T^{-3}A^{-2}} = \frac{1}{T^{-1}} = T$

and $\left[\frac{1}{2}Li^2\right] = \left(ML^2T^{-2}A^{-2}\right)\left(A^2\right) = ML^2T^{-2}$

Now as $\left(\frac{L}{R}\right)$ has dimensions of time and hence is called time constant of $L-R$ circuit and $\left(\frac{1}{2}\right)Li^2$ has dimensions of work or energy, so it represents magnetic energy stored in a coil.

PROBLEM 5

The Renold's number n_R for a liquid through a pipe depends upon the density of the liquid d, the coefficient of viscosity η, the speed of the liquid v and the radius of the tube r. Obtain by dimensional analysis an expression for n_R. Given $n_R \propto r$.

SOLUTION

As per the statement of the question, we get

$$n_R \propto r \qquad \ldots(1)$$

$$n_R \propto d^a \quad \ldots(2)$$

$$n_R \propto \eta^b \quad \ldots(3)$$

$$n_R \propto v^c \quad \ldots(4)$$

Combining (1), (2), (3) and (4), we get

$$n_R \propto rd^a\eta^b v^c$$

$$\Rightarrow \quad n_R = krd^a\eta^b v^c \quad \ldots(5)$$

where k is a dimensionless constant
Taking dimensions on both sides of (5), we get

$$M^0L^0T^0 = L\left(ML^{-3}\right)^a\left(ML^{-1}T^{-1}\right)^b\left(LT^{-1}\right)^c$$

$$\Rightarrow \quad M^0L^0T^0 = M^{a+b}L^{-3a-b+c+1}T^{-b-c}$$

So, from Principle of Homogeneity, we get

$$a+b=0,$$

$$-3a-b+c+1=0,$$

$$-b-c=0$$

$$\Rightarrow \quad a=-b \text{ and } c=-b$$

So, we get

$$-3(-b)-b-b+1=0$$

$$\Rightarrow \quad 3b-2b+1=0$$

$$\Rightarrow \quad b=-1$$

Hence $a=1$, $b=-1$ and $c=1$.
So, we get the final relation as

$$n_R = \frac{kvrd}{\eta}$$

PROBLEM 6

Two resistances are expressed as $R_1=(4\pm0.5)\ \Omega$ and $R_2=(12\pm0.5)\ \Omega$. Calculate the net resistance when both are connected in series and in parallel with absolute and %age error.

SOLUTION

$$R_S = R_1+R_2 = 16\ \Omega$$

$$R_P = \frac{R_1R_2}{R_1+R_2} = \frac{R_1R_2}{R_S} = 3\ \Omega$$

$$\Delta R_S = \Delta R_1 + \Delta R_2 = 1\ \Omega$$

$$\Rightarrow \quad \frac{\Delta R_S}{R_S}\times100 = \frac{1}{16}\times100\%$$

$$\Rightarrow \quad \frac{\Delta R_S}{R_S}\times100 = 6.25\%$$

$$\Rightarrow \quad R_S = 16\ \Omega \pm 6.25\%$$

Similarly

$$\frac{1}{R_P} = \frac{1}{R_1}+\frac{1}{R_2}$$

$$\Rightarrow \quad R_P^{-1} = R_1^{-1}+R_2^{-1}$$

$$\Rightarrow \quad \Delta\left(R_P^{-1}\right) = \Delta\left(R_1^{-1}\right)+\Delta\left(R_2^{-1}\right)$$

$$\Rightarrow \quad (-1)R_P^{-2}\Delta R_P = (-1)R_1^{-2}\Delta R_1 + (-1)R_2^{-2}\Delta R_2$$

$$\Rightarrow \quad \frac{\Delta R_P}{R_P^2} = \frac{\Delta R_1}{R_1^2}+\frac{\Delta R_2}{R_2^2} \text{ where}$$

$$R_P = \frac{R_1R_2}{R_1+R_2} = 3\ \Omega$$

$$\Rightarrow \quad \frac{\Delta R_P}{R_P} = \left(\frac{0.5}{16}+\frac{0.5}{144}\right)(3)$$

$$\Rightarrow \quad \frac{\Delta R_P}{R_P} = \frac{15}{144}$$

$$\Rightarrow \quad \%RE = 10.42\%$$

PROBLEM 7

In an experiment the following readings were recorded $L=2.890$ m, $M=3.00$ kg, $l=0.87$ cm and $D=0.041$ cm. Taking $g=9.8\ \text{ms}^{-2}$ and using the formula for Young's modulus $Y=\frac{MgL}{\pi r^2 l}$, calculate the maximum permissible error in the value of Y.

SOLUTION

$$Y = \frac{MgL}{\pi r^2 l} = \left(\frac{4g}{\pi}\right)\frac{ML}{D^2 l} \qquad \left\{\because\ r=\frac{D}{2}\right\}$$

$$\Rightarrow \quad \frac{\Delta Y}{Y} = \frac{\Delta M}{M}+\frac{\Delta L}{L}+2\frac{\Delta D}{D}+\frac{\Delta l}{l}$$

Since maximum permissible error can be equal to the least count of the device taking the measurement, so we have

$$\Delta M = 0.01 \text{ kg},\ \Delta L = 0.001 \text{ m},$$

$$\Delta D = 0.001 \text{ cm and } \Delta l = 0.01 \text{ cm}$$

$$\Rightarrow \quad \%\frac{\Delta Y}{Y}=\left[\frac{0.01}{3}+\frac{0.001}{2.89}+2\left(\frac{0.001}{0.041}\right)+\left(\frac{0.01}{0.87}\right)\right]\times 100$$

$$\Rightarrow \quad \%\frac{\Delta Y}{Y}=\frac{1}{3}+\frac{0.1}{2.890}+\frac{2(0.1)}{0.041}+\frac{1}{0.87}$$

$$\Rightarrow \quad \%\frac{\Delta Y}{Y}=\frac{1}{3}+\frac{10}{289}+\frac{200}{41}+\frac{100}{87}$$

$$\Rightarrow \quad \%\frac{\Delta Y}{Y}=0.34+0.035+4.9+1.2$$

$$\Rightarrow \quad \%\frac{\Delta Y}{Y}=6.5\%$$

PROBLEM 8

N divisions on the main scale of a vernier callipers coincide with $N+1$ divisions on the vernier scale. If each division on the main scale is of a units, determine the least count of the instrument.

SOLUTION

$(N+1)$ divisions on the vernier scale $=N$ divisions on main scale

1 division on vernier scale $=\frac{N}{N+1}$ divisions on main scale

Each division on the main scale is of a units.

So, 1 division on vernier scale $=\left(\frac{N}{N+1}\right)\ a$ unit $=a'$ (say)

Least count $=1$ MSD -1 VSD

$$\Rightarrow \quad \text{Least count } = a-a'=a-\left(\frac{N}{N+1}\right)a=\frac{a}{N+1}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. The amount of heat produced in an electric circuit depends upon the current (I), resistance (R) and time (t). If the errors created in the measurements of the above quantities are 2%, 1% and 1% respectively then the maximum possible error will be
(A) 1% (B) 2%
(C) 3% (D) 6%

2. The ratio of MKS and CGS units of the coefficient of viscosity is
(A) 10 (B) 10^2
(C) 10^{-1} (D) 10^{-2}

3. The unit of the reduction factor of tangent galvanometer is
(A) second (B) ampere
(C) radian (D) tesla

4. Which of the following pairs have the same dimensions?
(A) Pressure, stress and force
(B) Work, power and power factor
(C) Impulse and momentum
(D) Force, impulse and momentum

5. The unit of coefficient of viscosity in MKS system is
(A) $mkg^{-1}s^{-1}$ (B) $kgm^{-1}s^{-1}$
(C) $m^{-1}s^{-1}$ (D) mkgs

6. The unit of Vander Waal's gas constant a in MKS system will be
(A) kgm^5s^{-1} (B) kgm^4s^{-2}
(C) kgm^5s^{-2} (D) kgm^5s

7. The dimensions of $\frac{1}{\sqrt{\mu\varepsilon}}$ are equivalent to the dimensions of
(A) velocity (B) acceleration
(C) time (D) energy

8. How many significant figures are there in the numbers 108.023 and 0.19?
(A) 6 and 2 (B) 2 and 6
(C) 3 and 5 (D) 5 and 3

9. The number of particles crossing the unit area normal to x-axis per second is represented by the following formula $n = D\frac{n_2 - n_1}{x_2 - x_1}$, where D is the coefficient of diffusion and n_1 and n_2 are the number of molecules per unit volume at x_1 and x_2, respectively, then the dimensions of D are
(A) $M^0L^2T^{-1}$ (B) $M^0L^2T^{-2}$
(C) M^0LT^{-2} (D) M^0LT^{-1}

10. The dimensions of $\frac{L}{R}$ and RC are equivalent to the dimensions of
(A) time (B) frequency
(C) wavelength (D) energy

11. In the formula $V = d^a E^b$, if V, E and d are the velocity of longitudinal waves, bulk modulus of elasticity and density of the gaseous medium respectively, then the values of a and b are respectively.
(A) $-\frac{1}{2}$ and $\frac{1}{2}$ (B) $\frac{1}{2}$ and $-\frac{1}{2}$
(C) $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$ and $\frac{1}{2}$

12. The dimensional formula for $\frac{1}{2}CV^2$ or $\frac{1}{2}\frac{q^2}{C}$ or $\frac{1}{2}qV$ is
(A) MLT^{-2} (B) ML^2T^{-2}
(C) ML^2T^2 (D) $ML^{-2}T^{-3}$

13. If force (F), length (L) and time (T) are presumed to be the fundamental units, then the dimensional formula of mass will be
(A) $FL^{-1}T^2$ (B) $FL^{-1}T^{-2}$
(C) $FL^{-1}T^{-1}$ (D) FL^2T^2

14. If L, C and R denote the inductance, capacitance and resistance respectively, the dimensional formula for C^2LR is
(A) $ML^{-2}T^{-1}$ (B) $M^0L^0T^3$
(C) $M^{-1}L^{-2}T^6A^2$ (D) $M^0L^0T^2$

15. The position of a particle at time t is given by the relation $x_t = \left(\frac{v_0}{\alpha}\right)\left(1 - c^{-\alpha t}\right)$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively

(A) $M^0L^1T^{-1}$ and T^{-1} (B) $M^0L^1T^0$ and T^{-1}

(C) $M^0L^1T^{-11}$ (D) $M^0L^1T^{-1}$ and T

16. The potential energy of a particle varies with distance x from a fixed origin as $U = \frac{A\sqrt{x}}{x^2 + B}$, where A and B are dimensional constants then dimensional formula for AB is

(A) $ML^{\frac{7}{2}}T^{-2}$ (B) $ML^{\frac{11}{2}}T^{-2}$

(C) $M^2L^{\frac{9}{2}}T^{-2}$ (D) $ML^{\frac{13}{2}}T^{-3}$

17. Using dimensional analysis you can check on some results. In the integral $\int \frac{dx}{(2ax - x^2)^{1/2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$ the value of n is

(A) 1 (B) –1

(C) 0 (D) $\frac{1}{2}$

18. The equation of the stationary wave is $y = 2a\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$. Which of the following statements is wrong?

(A) The unit of ct is same as that of λ

(B) The unit of x is same as that of λ

(C) The unit of $\frac{2\pi c}{\lambda}$ is same as that of $\frac{2\pi x}{\lambda t}$

(D) The unit of $\frac{c}{\lambda}$ is same as that of $\frac{x}{\lambda}$

19. A physical quantity is measured and its value is found to be nu where n is numerical value and u is unit. Then which of the following relations is true

(A) $n \propto u^2$ (B) $n \propto u$

(C) $n \propto \sqrt{u}$ (D) $n \propto \frac{1}{u}$

20. In C.G.S. system the magnitude of the force is 100 dyne. In another system where the fundamental physical quantities are kilogram, metre and minute, the magnitude of the force is

(A) 0.036 (B) 0.36

(C) 3.6 (D) 36

21. The temperature of a body on Kelvin scale is found to be X K. When it is measured by a Fahrenheit thermometer, it is found to be X °F. Then X is

(A) 301.25 (B) 574.25

(C) 313 (D) 40

22. Which relation is incorrect?

(A) 1 calorie = 4.18 joule (B) $1\text{Å} = 10^{-10}$ m

(C) $1\text{ MeV} = 1.6 \times 10^{-13}$ J (D) 1 newton = 10^{-6} dyne

23. To determine the Young's modulus of a wire, the formula is $Y = \frac{F}{A}\frac{L}{\Delta L}$, where L is the length, A is the area of cross-section of the wire, ΔL is the change in length of the wire when stretched with a force F. The conversion factor to change it from CGS to MKS system is

(A) 1 (B) 10

(C) 0.1 (D) 0.01

24. Conversion of 1 MW power on a new system having basic units of mass, length and time as 10 kg, 1 dm and 1 minute respectively is

(A) 2.16×10^{12} unit (B) 1.26×10^{12} unit

(C) 2.16×10^{10} unit (D) 2×10^{14} unit

25. In two systems, the relations among velocity, acceleration and force are respectively $v_2 = \frac{\alpha^2}{\beta}v_1$, $a_2 = \alpha\beta a_1$ and $F_2 = \frac{F_1}{\alpha\beta}$. If α and β are constants then mass, length and time in two systems are related to each as

(A) $M_2 = \frac{\alpha}{\beta}M_1$, $L_2 = \frac{\alpha^2}{\beta^2}L_1$, $T_2 = \frac{\alpha^3 T_1}{\beta}$

(B) $M_2 = \frac{1}{\alpha^2\beta^2}M_1$, $L_2 = \frac{\alpha^3}{\beta^3}L_1$, $T_2 = T_1\frac{\alpha}{\beta^2}$

(C) $M_2 = \frac{\alpha^3}{\beta^3}M_1$, $L_2 = \frac{\alpha^2}{\beta^2}L_1$, $T_2 = \frac{\alpha}{\beta}T_1$

(D) $M_2 = \frac{\alpha^2}{\beta^2}M_1$, $L_2 = \frac{\alpha}{\beta^2}L_1$, $T_2 = \frac{\alpha^3}{\beta^3}T_1$

26. If the present units of length, time and mass (m, s, kg) are changed to 100 m, 100 s, and $\frac{1}{10}$ kg, then

(A) the new unit of velocity is increased 10 times

(B) the new unit of force is decreased $\frac{1}{1000}$ times

(C) the new unit of energy is increased 10 times

(D) the new unit of pressure is increased 1000 times

27. Suppose we employ a system in which the unit of mass equals 100 kg, the unit of length equals 1 km and the unit of time 100 s and call the unit of energy eluoj (joule written in reverse order), then

(A) $1\ \text{eluoj} = 10^4$ joule (B) $1\ \text{eluoj} = 10^{-3}$ joule

(C) $1\ \text{eluoj} = 10^{-4}$ joule (D) $1\ \text{joule} = 10^3$ eluoj

28. If $1\ \text{gcms}^{-1} = x\ \text{Ns}$, then number x is equivalent to

(A) 1×10^{-1} (B) 3×10^{-2}

(C) 6×10^{-4} (D) 1×10^{-5}

29. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn block A executes small oscillations. The time period of which is given by

(A) $2\pi\sqrt{\frac{M\eta}{L}}$ (B) $2\pi\sqrt{\frac{L}{M\eta}}$

(C) $2\pi\sqrt{\frac{ML}{\eta}}$ (D) $2\pi\sqrt{\frac{M}{\eta L}}$

30. If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then the dimensions of mass in new system is

(A) $c^{\frac{1}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$ (B) $c^{\frac{1}{2}}G^{-\frac{1}{2}}h^{\frac{1}{2}}$

(C) $c^{\frac{1}{2}}G^{\frac{1}{2}}h^{-\frac{1}{2}}$ (D) $c^{-\frac{1}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$

31. If P represents radiation pressure, C represents speed of light and Q represents radiation energy striking a unit area per second, the non-zero integers x, y and z such that $P^xQ^yC^z$ is dimensionless, are

(A) $x=1,\ y=1,\ z=-1$ (B) $x=1,\ y=-1,\ z=1$

(C) $x=-1,\ y=1,\ z=1$ (D) $x=1,\ y=1,\ z=1$

32. The volume V of water passing through a point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation $V \propto A^{\alpha}u^{\beta}t^{\gamma}$, which one of the following will be true

(A) $\alpha=\beta=\gamma$ (B) $\alpha\neq\beta=\gamma$

(C) $\alpha=\beta\neq\gamma$ (D) $\alpha\neq\beta\neq\gamma$

33. Given that the amplitude A of scattered light is

(i) directly proportional to the amplitude (A_0) of incident light

(ii) directly proportional to the volume (V) of the scattered particle

(iii) inversely proportional to the distance (r) from the scattered particle

(iv) depends upon the wavelength (λ) of the scattered light then

(A) $A \propto \frac{1}{\lambda}$ (B) $A \propto \frac{1}{\lambda^2}$

(C) $A \propto \frac{1}{\lambda^3}$ (D) $A \propto \frac{1}{\lambda^4}$

34. Each side a cube is measured to be 7.203 m. The volume of the cube up to appropriate significant figures is

(A) 373.714 (B) 373.71

(C) 373.7 (D) 373

35. Candela is the unit of

(A) acoustic intensity (B) electric intensity

(C) luminous intensity (D) magnetic intensity

36. The time taken by sunlight to penetrate a window pane is of the order of

(A) 10^{-5} s (B) 10^{-7} s

(C) 10^{-11} s (D) 10^{-19} s

37. Which of the following is the unit of latent heat?

(A) J (B) J mol^{-1}

(C) J kg^{-1} (D) $\text{J kg}^{-1}\text{mol}^{-1}$

38. The energy E radiated per unit area per second by a black body at temperature T is given by $E=\sigma T^4$, where σ is the Stefan's constant. The dimensions of σ are

(A) MT^2K^{-2} (B) $MT^{-3}K^{-4}$

(C) MT^3K^{-4} (D) $ML^4T^{-3}K^{-4}$

39. If length (L), mass (M) and force (F) are taken as fundamental quantities, then the dimensions of time will be

(A) $M^{\frac{1}{2}}L^{\frac{1}{2}}F^{-\frac{1}{2}}$ (B) $M^{-\frac{1}{2}}L^{\frac{1}{2}}F^{\frac{1}{2}}$

(C) $M^2L^2T^{-2}$ (D) $MLF^{\frac{1}{2}}$

40. The surface tension of a liquid is 70 dynecm^{-1}. It may be expressed in MKS as

(A) $70\ \text{Nm}^{-1}$ (B) $7\times10^{-2}\ \text{Nm}^{-1}$

(C) $7\times10^{2}\ \text{Nm}^{-1}$ (D) $7\times10^{3}\ \text{Nm}^{-1}$

41. Temperature can be expressed as a derived quantity in terms of any of the following

(A) length and mass

(B) mass and time

(C) length, mass and time

(D) in terms of none of these

42. The density of a material is $8\ \text{gcc}^{-1}$. In a system in which the unit of length is 5 cm and unit of mass is 20 g, the density of material will be
(A) 40 (B) 50
(C) 24 (D) 80

43. erg m^{-1} can be the unit of the measure of
(A) force (B) momentum
(C) power (D) acceleration

44. Which one of the following has the dimensions of pressure?
(A) $\frac{M}{L^2T^2}$ (B) $\frac{M}{LT}$
(C) $\frac{ML}{T^2}$ (D) $\frac{M}{LT^2}$

45. In a particular system, the units of length, mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equivalent to
(A) 0.1 N (B) 1 N
(C) 10 N (D) 100 N

46. The dimensions of resistivity in terms of M, L, T and Q are
(A) $ML^3T^{-1}Q^{-2}$ (B) $ML^3T^{-2}Q^{-1}$
(C) $ML^2T^{-1}Q^{-1}$ (D) $MLTQ^{-1}$

47. The SI unit of pressure is
(A) dyne cm^{-2} (B) atmosphere
(C) pascal (D) cm of Hg

48. A certain physical quantity is calculated from the formula $\frac{\pi}{3}(a^2-b^2)h$, where h, a and b are all lengths. The quantity being calculated is
(A) Velocity (B) Length
(C) Area (D) Volume

49. Joulesecond is a unit of
(A) energy (B) momentum
(C) power (D) angular momentum

50. If $x = at + bt^2 + c$, where x is in metre and t in hour (hr), then the unit of a, b and c are
(A) m, mhr^{-1}, mhr^{-2} (B) mhr^{-1}, mhr^{-2}, m
(C) m^2hr^{-1}, mhr^{-1}, m (D) mhr^{-2}, mhr^{-1}, m

51. The dimensions of $\frac{1}{\sqrt{\epsilon_0\,\mu_0}}$ are those of
(A) velocity (B) time
(C) capacitance (D) distance

52. Taking into account the significant figures, the product of 109.832 and 0.6107 should be written as
(A) 67.0744 (B) 67.1
(C) 67.07 (D) 67.074402

53. The dimensions of torque are
(A) ML^2T^{-2} (B) MLT^2
(C) MLT^{-1} (D) MT^{-2}

54. Which of the following quantities has the dimensional formula ML^2T^{-3}?
(A) Bulk modulus
(B) Coefficient of viscosity
(C) Energy
(D) Power

55. The dimensions of the ratio of angular to linear momentum is
(A) $M^0L^1T^0$ (B) $M^1L^1T^1$
(C) $M^1L^2T^{-1}$ (D) $M^{-1}L^{-1}T^{-1}$

56. The random error in the arithmetic mean of 100 observations is x. The random error in the arithmetic mean of 400 observations would be
(A) $4x$ (B) $\frac{x}{4}$
(C) $2x$ (D) $\frac{x}{2}$

57. The velocity v of a particle is given in terms of time t by the equation $v = at + \frac{b}{t+c}$. The dimensions of a, b, c are respectively
(A) L^2, T, LT^2 (B) LT^2, LT, L
(C) LT^{-2}, L, T (D) L, LT, T^2

58. The dimensions of mc^2 are
(A) MLT^{-1} (B) ML^2T^{-1}
(C) ML^2T^{-2} (D) ML^2T^2

59. Which of the following is not a unit of energy?
(A) Ws (B) kgms^{-1}
(C) Nm (D) joule

60. 1 micron (μ) is
(A) 10^{-9} m (B) 10^{-12} m
(C) 10^{-6} m (D) 10^{-15} m

61. The unit of Stefan's constant σ is
(A) $\frac{\text{watt}^4}{\text{mK}^4}$ (B) $\frac{\text{calorie}}{\text{m}^2\text{K}^4}$
(C) $\frac{\text{watt}}{\text{m}^2\text{K}^4}$ (D) $\frac{\text{joule}}{\text{m}^2\text{K}^4}$

62. Newtonsecond is a unit of
(A) force (B) impulse
(C) momentum (D) energy

63. If ΔX is absolute error in the measurement of X and ΔY is absolute error in the measurement of Y, then maximum absolute error in the measurement of difference of X and Y is

(A) $\pm(\Delta X - \Delta Y)$ (B) $(\Delta X + \Delta Y)$

(C) $\pm\dfrac{\Delta X}{\Delta Y}$ (D) $\pm\Delta X\Delta Y$

64. The dimensional formula of universal gravitational constant is

(A) MLT^{-2} (B) ML^3T^{-2}

(C) $M^{-1}L^3T^{-2}$ (D) MLT^2

65. If the units of M and L are doubled then the unit of kinetic energy will become

(A) 8 time (B) 16 time

(C) 4 time (D) 2 time

66. Dimensional analysis gives

(A) no information about dimensionless constants

(B) information about dimensionless constants

(C) information about dimensionless constants if quantity does not depend upon more than three variables

(D) information about dimensionless constants if quantity depends upon single variable

67. If force (F), area (A) and density (D) are taken as fundamental quantities, the dimensional formula for Young's modulus will be

(A) $FA^{-2}D^2$ (B) $FA^{-1}D^0$

(C) $F^{-1}A^{-1}D^{-1}$ (D) $FA^{-1}D$

68. The dimensions of the coefficient of viscosity are $[ML^{-1}T^{-1}]$. To convert the CGS unit poise (P) to the MKS unit poiseuille (PI), the poise has to be multiplied by

(A) 10^{-1} (B) 10

(C) 10^9 (D) 10^7

69. The dimensions of "light-year" are

(A) LT^{-1} (B) T

(C) ML^2T^{-2} (D) L

70. The velocity of a particle is given by $v = at^2 + bt + c$. If v is measured in ms^{-1} and t is measured in s, the unit of

(A) a is ms^{-1}

(B) b is ms^{-1}

(C) c is ms^{-1}

(D) a and b are same but that of c is different

71. The pair having the same dimension is

(A) angular momentum, work

(B) work, torque

(C) potential energy, linear momentum

(D) kinetic energy, velocity

72. Given that $y = a\cos\left(\dfrac{t}{p} - qx\right)$, where t represents time in second and x distance in metre. Which of the following statements is true?

(A) The unit of x is same as that of q.

(B) The unit of x is same as that of p.

(C) The unit of t is same as that of q.

(D) The unit of t is same as that of p.

73. Electronvolt (eV) is a unit of

(A) charge (B) potential

(C) energy (D) coulomb repulsion

74. A system has basic dimensions as density $[D]$, velocity $[V]$ and area $[A]$. The dimensional representation of force in this system is

(A) $[AV^2D]$ (B) $[A^2VD]$

(C) $[AVD^2]$ (D) $[A^0VD]$

75. A quantity X is given by $\varepsilon_0 L\dfrac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

(A) resistance (B) charge

(C) voltage (D) current

76. The dimensions of time constant $\dfrac{L}{R}$ during growth and decay of current in an inductive circuit are same as those of

(A) constant (B) resistance

(C) current (D) time

77. If the speed of light (c) acceleration due to gravity (g) and pressure (p) are taken as the fundamental quantities, then the dimensions of length will be

(A) $c^2g^{-1}p^0$ (B) cg^0p^{-1}

(C) $c^{-1}gp^0$ (D) $cg^{-1}p^0$

78. If $x = \dfrac{a}{b}$ and Δa and Δb are the errors in the measurement of a and b, respectively, then the maximum percentage error in the value of x will be

(A) $\left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)\times 100$ (B) $\left(\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)\times 100$

(C) $\left(\frac{\Delta a}{a-b}+\frac{\Delta b}{a-b}\right)\times 100$ (D) $\left(\frac{\Delta a}{a-b}-\frac{\Delta b}{a-b}\right)\times 100$

79. If the orbital velocity of a planet is given by $v = G^a M^b R^c$, then

(A) $a=\frac{1}{3}$, $b=\frac{1}{3}$, $c=-\frac{1}{3}$

(B) $a=\frac{1}{2}$, $b=\frac{1}{2}$, $c=-\frac{1}{2}$

(C) $a=\frac{1}{2}$, $b=-\frac{1}{2}$, $c=\frac{1}{2}$

(D) $a=\frac{1}{2}$, $b=-\frac{1}{2}$, $c=-\frac{1}{2}$

80. In an experiment to determine acceleration due to gravity by simple pendulum, a student commits 1% positive error in the measurement of length and 3% negative error in the measurement of time period. The percentage error in the value of g will be

(A) 7% (B) 10%
(C) 4% (D) 3%

81. The best method to reduce random errors is

(A) to change the instrument used for measurement
(B) to take help of experienced observer
(C) to repeat the experiment many times and to take the average results
(D) None of these

82. The least count of an instrument is 0.01 cm. Taking all precautions the most possible error in the measurement can be

(A) 0.005 cm (B) 0.001 cm
(C) 0.01 cm (D) 0.02 cm

83. The fundamental frequency of vibration of a stretched string is given by $\nu=\frac{1}{2l}\sqrt{\frac{T}{m}}$, where the symbols have their usual meanings. The dimensions of m are

(A) $ML^{-1}T^0$ (B) ML^0T^0
(C) $ML^{-1}T$ (D) $M^{-1}LT^0$

84. Which of the following pairs has the same dimensions?

(A) Pressure and density
(B) Impulse and momentum
(C) Stress and strain
(D) Momentum and inertia

85. The dimensions of PV are equivalent to those of

(A) work (B) force
(C) pressure (D) volume

86. If ρ is the mean density of the earth, r its radius, g the acceleration due to gravity and G the gravitational constant, on the basis of dimensional consistency, the correct expression is

(A) $\rho=\frac{3G}{4\pi rg}$ (B) $\rho=\frac{rG}{12\pi g}$

(C) $\rho=\frac{3g}{4\pi rG}$ (D) $\rho=\frac{3r}{4\pi gG}$

87. The dimension of $\left(\frac{1}{2}\right)\varepsilon_0 E^2$ (ε_0 : permittivity of free space; E : electric field) is

(A) MLT^{-1} (B) $ML^{-1}T^{-2}$
(C) MLT^{-2} (D) ML^2T^{-1}

88. Of the following quantities, which one has dimensions different from the remaining three?

(A) Energy per unit volume
(B) Force per unit area
(C) Product of voltage and charge per unit volume
(D) Angular momentum per unit mass

89. The time dependence of a physical quantity P is given by $P = P_0\exp(-\alpha t^2)$ where α is a constant and t is time. The constant α

(A) is dimensionless
(B) has dimensions of T^{-2}
(C) has dimensions as that of P
(D) has dimensions equal to the dimensions of PT^{-2}

90. The dimensional formula of magnetic moment is

(A) $M^{-2}A$ (B) M^2A
(C) L^2A (D) $L^{-2}A$

91. If energy E, velocity V and time T are taken as the fundamental units, the dimensional formula for surface tension is

(A) $EV^{-2}T^{-2}$ (B) $E^{-2}VT^{-2}$
(C) $E^{-2}V^{-2}T$ (D) $E^{-2}V^{-2}T^{-2}$

92. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in $MKSQ$ system?

(A) $M^{-3}L^{-1}T^3Q^4$ (B) $M^{-3}L^{-2}T^4Q^4$
(C) $M^{-2}L^{-2}T^4Q^4$ (D) $M^{-3}L^{-2}T^4Q^1$

93. E, m, L, G denote energy, mass, angular momentum and gravitational constant respectively. The dimensions of $\frac{EL^2}{m^5G^2}$ will be that of

(A) angle (B) length
(C) mass (D) time

94. Out of the following expressions for the couple per unit twist, time period of a compound pendulum, frequency of a stretched wire and time period of a simple pendulum respectively, the dimensionally inconsistent one is

(A) $C = \dfrac{\eta\pi r^4}{2l}$ (B) $T = 2\pi\sqrt{\dfrac{k^2}{g} + \dfrac{l}{g}}$

(C) $f = \dfrac{1}{l}\sqrt{\dfrac{T}{\rho}}$ (D) $T = 2\pi\sqrt{\dfrac{l}{g}}$

where η is coefficient of rigidity, r is the radius of the wire, K is the radius of gyration, and ρ is mass per unit length and l is the length corresponding to the particular case.

95. In the radioactive decay law $N = N_0 e^{-\lambda t}$, the dimensions of λ are

(A) $M^0 L^0 T^0$ (B) $M^0 L^0 T^{-1}$

(C) $M^0 L^0 T$ (D) $ML^0 T^{-1}$

96. Planck's constant has the dimensions of

(A) force (B) energy

(C) linear momentum (D) angular momentum

97. The SI unit of mobility of charges (μ) is

(A) Cskg^{-1} (B) Ckgs^{-1}

(C) Cskg (D) $\text{Cs}^{-1}\text{kg}^{-1}$

98. Turpentine oil is flowing through a tube of length l and radius r. The pressure difference between the two ends of the tube is p; the viscosity of the oil is given by $\eta = \dfrac{p(r^2 - x^2)}{4vl}$ where v is the velocity of oil at a distance x from the axis of the tube. From this relation, the dimensions of viscosity η are

(A) $M^0L^0T^0$ (B) MLT^{-1}

(C) ML^2T^{-2} (D) $ML^{-1}T^{-1}$

99. The SI unit of $\dfrac{1}{2\pi\sqrt{LC}}$ is equivalent to that of

(A) time period (B) frequency

(C) wave length (D) wave number

100. Given that $\tan\theta = \dfrac{v^2}{rg}$ gives the angle of banking of the cyclist going round the curve. Here v is the speed of cyclist, r is the radius of the curve and g is acceleration due to gravity. Which of the following statements about the relation is true?

(A) It is both dimensionally as well as numerically correct.

(B) It is neither dimensionally correct nor numerically correct.

(C) It is dimensionally correct but not numerically.

(D) It is numerically correct but not dimensionally.

101. If V denotes the potential difference across the plates of a capacitor of capacitance C, the dimensions of CV^2 are

(A) MLT^{-2}

(B) M^2LT^{-1}

(C) ML^2T^{-2}

(D) not expressible in terms of M, L, T

102. Unit of amplification factor is

(A) ohm (B) mho

(C) AV^{-1} (D) None of these

103. A physical quantity is represented by the relation $Y = M^aL^bT^{-c}$. If the percentage errors in the measurement of M, L and T are respectively $\alpha\%$, $\beta\%$ and $\gamma\%$, respectively then the total error will be

(A) $(\alpha a + \beta b - \gamma c)\%$ (B) $(\alpha a - \beta b - \gamma c)\%$

(C) $(\alpha a + \beta b + \gamma c)\%$ (D) $(\alpha a - \beta b + \gamma c)\%$

104. Given that T stands for time period and l stands for the length of simple pendulum. If g is the acceleration due to gravity, then which of the following statements about the relation $T^2 = \left(\dfrac{l}{g}\right)$ is correct?

(A) It is correct both dimensionally as well as numerically

(B) It is neither dimensionally correct nor numerically

(C) It is dimensionally correct but not numerically

(D) If is numerically correct but not dimensionally

105. Select the incorrect statement(s).

(A) If the zero of vernier scale does not coincide with the zero of the main scale, then the vernier calliper is said to be having zero error

(B) Zero correction has a magnitude equal to zero error but sign is opposite to that of zero error

(C) Zero error is positive when the zero of vernier scale lies to the left of the zero of the main scale

(D) Options (B) and (C) are incorrect.

106. 1 cm on the main scale of a vernier calliper is divided into 10 equal parts. If 10 divisions of vernier coincide with 8 small divisions of main scale, then the least count of the calliper is

(A) 0.01 cm (B) 0.02 cm

(C) 0.05 cm (D) 0.005 cm

107. The vernier constant of a travelling microscope is 0.001 cm. If 49 main scale divisions coincide with 50 main scale divisions, then the value of 1 main scale division is
(A) 0.1 mm (B) 0.5 mm
(C) 0.4 mm (D) 1 mm

108. Which of the following has the largest least count?
(A) Spherometer (B) Vernier callipers
(C) Screw gauge (D) Metre scale

109. 1 cm of main scale of a vernier caliper is divided into 10 divisions. The least count of the callipers is 0.005 cm, then the vernier scale must have
(A) 10 divisions (B) 20 divisions
(C) 25 divisions (D) 50 divisions

110. Least count of a vernier calliper is $\frac{1}{10N}$ cm. The value of one division on the main scale is 1 mm. Then the number of divisions of the main scale that coincide with N divisions of vernier scale is
(A) $10N$ (B) $\frac{N}{10}$
(C) $(N-1)$ (D) $N-10$

111. Each division on the main scale is 1 mm. Which of the following vernier scales give vernier constant equal to 0.01 mm?
(A) 9 mm divided into 10 divisions
(B) 90 mm divided into 100 divisions
(C) 99 mm divided into 100 divisions
(D) 9 mm divided into 100 divisions

112. The least count of vernier callipers is 0.01 cm. Then the error in the measurement is
(A) > 0.01 cm (B) ≥ 0.01 cm
(C) < 0.01 cm (D) ≤ 0.01 cm

113. A person performs an experiment with vernier callipers and takes 100 readings. He repeats the same experiment but now takes 400 readings. Then the probable error is
(A) the same
(B) halved
(C) doubled
(D) reduced by one fourth

114. Vernier constant is the
(A) value of one MSD divided by total number of divisions on the main scale
(B) value of one VSD divided by total number of divisions on the vernier scale
(C) total number of divisions on the main scale divided by total number of divisions on the vernier scale
(D) difference between value of one main scale division and one vernier scale division

115. Which of the following uses angular vernier?
(A) Metre scale (B) Vernier callipers
(C) Spherometer (D) Both (A) and (B)

116. Screw gauge can measure the diameter of thin wires or similar objects with accuracy upto
(A) 1 cm (B) 0.1 cm
(C) 0.01 cm (D) 0.001 cm

117. Pitch of screw of a screw gauge is the
(A) $\frac{\text{distance moved by thimble on main scale}}{\text{number of rotation of thimble}}$
(B) $\frac{\text{pitch}}{\text{number of circular scale divisions}}$
(C) $\frac{\text{number of rotation of thimble}}{\text{number of circular scale divisions}}$
(D) None of the above

118. Least count of screw gauge is defined as
(A) $\frac{\text{distance moved by thimble on main scale}}{\text{number of rotation of thimble}}$
(B) $\frac{\text{pitch of the screw}}{\text{number of circular scale divisions \& head scale}}$
(C) $\frac{\text{number of rotation of thimble}}{\text{number of circular scale divisions}}$
(D) None of the above

119. Screw gauge can be used to determine
(A) Thickness of glass piece
(B) Diameter of a wire
(C) To measure thickness of glass piece and to measure diameter of wire
(D) None of the above

120. Screw gauge contains following scales
(A) main scale, vernier scale
(B) main scale, ordinary scale
(C) main scale, head scale
(D) only main scale

121. Screw gauge is said to have negative error
(A) when head scale zeroth division coincides with base line of main scale
(B) when head scale zeroth division is above with base line of main scale

(C) when head scale zeroth division is below with base line of main scale
(D) None of the above

122. For negative error correction is
(A) positive (B) negative
(C) no correction (D) None of the above

123. Screw gauge is said to have positive error
(A) when head scale zeroth division coincides with base line of main scale
(B) when head scale zeroth division is above with base line of main scale
(C) when head scale zeroth division is below with base line of main scale
(D) None of the above

124. For positive error, the correction is
(A) positive (B) negative
(C) nil (D) None of the above

125. The diameter D of a wire is measured using screw gauge of zero error. Then

(A) $D = \begin{pmatrix}\text{main}\\ \text{scale}\\ \text{reading}\end{pmatrix} + \begin{pmatrix}\text{circular}\\ \text{scale}\\ \text{reading}\end{pmatrix} \times \begin{pmatrix}\text{least}\\ \text{count}\end{pmatrix}$

(B) $D = \begin{pmatrix}\text{circular}\\ \text{scale}\\ \text{reading}\end{pmatrix} + \begin{pmatrix}\text{main}\\ \text{scale}\\ \text{reading}\end{pmatrix} \times \begin{pmatrix}\text{least}\\ \text{count}\end{pmatrix}$

(C) $D = \begin{pmatrix}\text{main}\\ \text{scale}\\ \text{reading}\end{pmatrix} + \begin{pmatrix}\text{vernier}\\ \text{scale}\\ \text{reading}\end{pmatrix} \times \begin{pmatrix}\text{least}\\ \text{count}\end{pmatrix}$

(D) None of the above

126. The pitch of a screw gauge is 0.5 mm. Its head scale contains 50 divisions. The least count of the screw gauge is
(A) 0.01 mm (B) 0.001 mm
(C) 0.02 mm (D) 0.002 mm

127. Which of the following is correct?
(A) When the zero of the circular scale advances beyond the reference line, then the zero correction is negative
(B) When the zero of the circular scale has advanced beyond the reference line, then the zero correction is positive and when the zero of the circular scale is left behind the reference line, then the zero correction is negative
(C) When the zero of the circular scale is left behind the reference line, then the zero correction is positive
(D) Both (A) and (C)

128. The vernier of a circular scale has been divided into 30 divisions which coincide with 29 divisions of the main scale, having each division of $\frac{1}{2}^\circ$. The least count of the device is
(A) 10′ (B) 30′
(C) 0.1′ (D) 1′

129. The velocity v of a particle at time t is given by $v = \frac{a}{t} + \frac{bt}{t^2 + c}$. The dimensions of a, b, c are respectively
(A) LT^{-2}, L, T (B) L, L, T^2
(C) L, LT, T^{-2} (D) L, L, LT^2

130. What is the unit of k in the relation $U = \frac{ky}{y^2 + a^2}$, where U represents the potential energy, y represents the displacement and a represents the maximum displacement i.e., amplitude ?
(A) ms^{-1} (B) ms
(C) Jm (D) Js^{-1}

131. For a body moving along x-axis, the distance travelled by body from a reference point is given as function of time t as $x = at^2 + b$, where a and b are constants, then the dimension of $\sqrt{ab}$ is same as
(A) speed (B) distance travelled
(C) acceleration (D) None of these

132. If a quantity x is defined by the equation $x = 3CB^2$ where C is capacitance in farad and B represents magnetic field in tesla. The dimensions of x are
(A) ML^{-2} (B) $ML^{-2}T^{-2}A$
(C) $ML^{-2}T^{-2}A^2$ (D) $L^{-1}A^{-1}$

133. There are two different quantities A and B having different dimensions. Then which of the following operation is dimensionally correct?
(A) $A + B$ (B) $A - B$
(C) $\frac{A}{B}$ (D) $e^{A/B}$

134. The formula $S = ut - \frac{1}{4}at^2$ where S is the distance travelled, u is the initial velocity, a is the acceleration and t is the time is
(A) dimensionally correct only
(B) dimensionally incorrect only
(C) dimensionally and numerically correct
(D) dimensionally and numerically wrong

135. The dimensional formula of a physical quantity x is $[M^{-1}L^3T^{-2}]$. The error in measuring the quantities M, L and T are 2%, 3% and 4%. The maximum

percentage of error that occurs in measuring, the quantity x is

(A) 9 (B) 10
(C) 14 (D) 19

136. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = K|x|^3$ where K is a positive constant. If the amplitude of oscillation is a then its time period T is

(A) Proportional to $\frac{1}{\sqrt{a}}$ (B) Independent of a
(C) Proportional to $\sqrt{a}$ (D) Proportional to $a^{3/2}$

137. In a particular system, the unit of length, mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equivalent to

(A) $\frac{1}{10}$ N (B) 1 N
(C) 10 N (D) 100 N

138. Force acting on a particle is given by $F = \frac{A - x}{Bt}$, where x is in metre and t is in seconds. The dimensions of B is

(A) MLT^{-2} (B) $M^{-1}T^{-3}$
(C) $M^{-1}T$ (D) MT^{-1}

139. The viscosity η of a gas depends on the long-range attractive part of the intermolecular force, which varies with molecular separation r according to $F = \mu r^{-n}$ where n is a number and μ is a constant. If η is a function of mass m of the molecules, their mean speed v and the constant μ, then which of following is correct

(A) $\eta \propto m^{n+1}v^{n+3}\mu^{n-2}$ (B) $\eta \propto m^{\frac{n+1}{n-1}}v^{\frac{n+3}{n-1}}\mu^{\frac{-2}{n-1}}$
(C) $\eta \propto m^{n}v^{-n}\mu^{-2}$ (D) $\eta \propto mv\mu^{-n}$

140. A science student takes 100 observations in an experiment. Second time he takes 500 observations in the same experiment, by doing so the possible error becomes

(A) $\frac{1}{5}$ times (B) 5 times
(C) unchanged (D) None of these

141. The dimensions of $\frac{h}{e}$ (h = Planck's constant and e = electronic charge) are same as that of

(A) magnetic flux (B) electric flux
(C) electric field (D) magnetic field

142. $\frac{E^2}{\mu_0}$ has the dimensions (E = electric field, μ_0 = permeability of free space)

(A) $[M^2L^3T^{-2}A^2]$ (B) $[MLT^{-4}]$
(C) $[ML^3T^{-2}]$ (D) $[M^{-1}L^2TA^{-2}]$

143. The dimensions of σb^4 (σ = Stefan's constant and b = Wein's constant) are

(A) $[M^0L^0T^0]$ (B) $[ML^4T^{-3}]$
(C) $[ML^{-2}T]$ (D) $[ML^6T^{-3}]$

144. A quantity X is measured and expressed as $X = Nu$, where N numerical value and u is unit. Which of the following is independent of system of unit chosen?

(A) N (B) $N \times u$
(C) $\frac{N}{u}$ (D) None of these

145. Which of the following combination of three quantities P, Q, R having different dimension cannot be meaningful?

(A) $PQ - R$ (B) $PQ - 1$
(C) $\frac{(P-Q)}{R}$ (D) $\frac{PR - Q^2}{QR}$

146. A and B are two physical quantities having different dimensions. Then which of the following operation is dimensionally correct?

(A) $A + B$ (B) $\log\frac{A}{B}$
(C) $\frac{A}{B}$ (D) $e^{A/B}$

147. For $10^{(at+3)}$, the dimension of a is

(A) $M^0L^0T^0$ (B) $M^0L^0T^1$
(C) $M^0L^0T^{-1}$ (D) None of these

148. Choose the wrong statement

(A) The dimensions of $\frac{\omega L}{R}$ are same as that of strain
(B) The dimensions of $\frac{1}{\sqrt{LC}}$ are same as that of angular velocity
(C) The dimension of LCR are same as that of time
(D) None of these

149. After rounding off the number 4621 to 2 signification digits the value becomes

(A) 4600 (B) 4500
(C) 4700 (D) 4720

150. Which of the following relation cannot be derived using dimensional analysis (Neglect value of constant)

(A) $v=\sqrt{\frac{F}{\mu}}$ (B) $T=2\pi\sqrt{\frac{l}{g}}$

(C) $F=6\pi\eta r v_d$ (D) $F=\frac{Gm_1m_2}{r^2}$

151. Equation of plane progressive transverse wave in a dissipative medium has general form

$$y=Ae^{-\alpha x}\sin\beta(t-Bx)$$

where α, A, B, C are constant, x and y are displacement and t is time. Dimensions of α, β and B respectively are

(A) $M^0L^{-1}T^0$, $M^0L^{-1}T^{-1}$, M^0LT^{-1}

(B) $M^0L^1T^0$, $M^0L^0T^{-1}$, $M^0L^{-1}T^1$

(C) $M^0L^{-1}T^1$, $M^0L^{-1}T$, $M^0L^0T^{-1}$

(D) $M^0L^{-1}T^0$, $M^0L^0T^{-1}$, $M^0L^{-1}T^1$

152. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and unit of length is 10 m. In this system, one unit of power will correspond to

(A) 16 watt (B) $\frac{1}{16}$ watt

(C) 25 watt (D) None of these

153. In book, the answer for a particular question is expressed as $b=\frac{ma}{k}\left[\sqrt{1+\frac{2kl}{ma}}\right]$, where m represents mass, a represents acceleration, l represents length. Then the unit of b should be

(A) ms^{-1} (B) ms^{-2}

(C) meter (D) /sec

154. A student performs an experiment to determine the Young's modulus of a wire exactly 2 cm long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ±0.05 mm at a load of exactly 1 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ±0.01 mm. Take $g=9.8\ ms^{-2}$ (exact). The Young's modulus obtained from the reading is

(A) $(2\pm0.3)\times10^{11}\ Nm^{-2}$

(B) $(2\pm0.2)\times10^{11}\ Nm^{-2}$

(C) $(2\pm0.1)\times10^{11}\ Nm^{-2}$

(D) $(2\pm0.05)\times10^{11}\ Nm^{-2}$

155. The time dependence of physical quantity P is given by $P=P_0e^{-\alpha t^2+\beta t+\gamma}$, where α, β, γ are constants and their dimensions are given by (where t is time)

(A) $M^0L^0T^{-2}$, $M^0L^0T^{-1}$, $M^0L^0T^0$

(B) M^0L^{-1}, T^{-2}, $M^0L^0T^{-1}$, M^0L^0T

(C) $M^0L^0T^{-1}$, MLT^{-2}, $M^0L^0T^{-1}$

(D) M, L, T, MLT^0, $M^0L^0T^0$

156. The time period of a body under S.H.M. is represented by $T=P^\alpha D^\beta S^\gamma$, where P is pressure, D is density and S is surface tension, then values of α, β and γ are

(Given that surface tension $S=\frac{F}{l}$)

(A) $-\frac{3}{2}, \frac{1}{2}, 1$ (B) $1, 2, \frac{1}{3}$

(C) $-1, -2, 3$ (D) $\frac{1}{2}, \frac{-3}{2}, \frac{-1}{2}$

157. The relative error in calculating the value of g from the relation $T=2\pi\sqrt{\frac{l}{g}}$, if the relative errors in calculating T and l are $\pm x$ and $\pm y$ respectively is

(A) $x+y$ (B) $2x-y$

(C) $2x+y$ (D) $x-2y$

158. The dimensions of $\frac{e^2}{4\pi\varepsilon_0 hc}$, where e, ε_0, h and c are electronic charge, electric permittivity, Planck's constant and velocity of light in vacuum respectively is

(A) $[M^0L^0T^0]$ (B) $[M^1L^0T^0]$

(C) $[M^0L^1T^0]$ (D) $[M^0L^0T^1]$

159. If E be the energy, G is the gravitational constant, I be the impulse and M be the mass, then dimensions of $\frac{GIM^2}{E^2}$ are same as that of

(A) time (B) mass

(C) length (D) force

160. The equivalent focal length of a combination of two thin lenses (having focal length f_1 and f_2) can be given by $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$. If errors in measurement of f_1 and f_2 are ±0.5 cm and ±0.3 cm and given that $f_1=f_2$. Then errors in equivalent focal length is

(A) ±0.1 (B) ±0.8

(C) ±0.2 (D) ±0.4

161. If y represents distance and x represents time, dimensions of $\frac{d^2y}{dx^2}$ are

(A) LT^{-1} (B) L^2T^2

(C) L^2T^{-1} (D) LT^{-2}

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A student when discussing the properties of a medium (except vacuum) writes

$$\begin{pmatrix}\text{Velocity of light}\\ \text{in vacuum}\end{pmatrix}=\begin{pmatrix}\text{Velocity of light}\\ \text{in medium}\end{pmatrix}$$

This formula is
(A) dimensionally correct
(B) dimensionally incorrect
(C) numerically incorrect
(D) dimensionally and numerically correct

2. The position of a particle moving along the y-axis is given as $y = At^2 - Bt^3$ where y is measured in metre and t in second. Then
(A) $[A] = LT^{-2}$ (B) $[B] = LT^{-3}$
(C) $\dfrac{[A^3]}{[B^2]} = L$ (D) $\dfrac{[B^3]}{[A^2]} = L$

3. Dimensional analysis cannot be used to derive formulae
(A) containing trigonometrical functions
(B) containing exponential functions
(C) containing logarithmic functions
(D) containing dimensionless quantities

4. Which of the following is (are) dimensionless?
(A) Refractive index
(B) Poisson's ratio
(C) Universal gravitational constant
(D) Specific gravity

5. The equation of the stationary wave is $y = 2A\sin\left(\dfrac{2\pi ct}{\lambda}\right)\cos\left(\dfrac{2\pi x}{\lambda}\right)$. Which of the following statement(s) is/are correct?
(A) The unit of ct is same as that of λ
(B) The unit of x is same as that of λ
(C) The unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$
(D) The unit of c/λ is same as that of x/λ

6. Which of the following can be expressed as dynecm^{-2}?
(A) Pressure
(B) Longitudinal stress
(C) Young's modulus of elasticity
(D) Viscosity

7. The pair(s) having same dimensions is/are
(A) Torque and work
(B) Angular momentum and work
(C) Energy and Young's modulus
(D) Light year and wavelength

8. The pairs of physical quantities that possess same dimensions is/are
(A) Renold's number and coefficient of friction
(B) Curie and frequency of light wave
(C) Latent heat and gravitational potential
(D) Planck constant and torque

9. If dimensions of length are expressed as $G^x c^y h^z$, where G, c and h are the universal gravitational constant, speed of light and Planck constant respectively, then
(A) $x = \dfrac{1}{2},\ y = \dfrac{1}{2}$ (B) $x = \dfrac{1}{2},\ z = \dfrac{1}{2}$
(C) $y = -\dfrac{3}{2},\ z = \dfrac{1}{2}$ (D) $y = \dfrac{1}{2},\ z = \dfrac{3}{2}$

10. Choose the correct statement(s)
(A) A dimensionally correct equation may be correct
(B) A dimensionally correct equation may be incorrect
(C) A dimensionally incorrect equation may be correct
(D) A dimensionally incorrect equation must be incorrect

11. Out of the following unit(s), the one(s) measuring energy is/are
(A) kWhr (B) $(\text{volt})^2(\text{sec})(\text{ohm})^{-1}$
(C) $(\text{pascal})(\text{foot})^2$ (D) $(\text{weber})(\text{ampere})$

12. Consider three quantities; $x = \dfrac{E}{B}$, $y = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$ and $z = \dfrac{l}{CR}$. Here l is the length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings
(A) x, y have the same dimensions
(B) x, z have the same dimensions
(C) y, z have the same dimensions
(D) None of the three pairs have the same dimensions

13. Dimensions of light year are the same as those of
(A) leap year (B) wavelength
(C) radius of gyration (D) propagation constant

14. Which of the following is/are not a unit of time?
(A) parsec (B) light year
(C) micron (D) second

15. Choose the pairs of physical quantities, which have identical dimensions.
(A) Impulse and linear momentum
(B) Planck constant and angular momentum
(C) Moment of inertia and moment of force
(D) Young's modulus and pressure

16. The dimensions of energy per unit volume are the same as those of
(A) work (B) stress
(C) pressure (D) modulus of elasticity

17. $\left(\frac{B^2}{\mu_0}\right)$ has the same dimensional formula as that of (B is the magnetic field induction and μ_0 is absolute permeability of free space)
(A) energy density
(B) magnetic energy per unit volume
(C) magnetic intensity
(D) intensity of magnetisation

18. Which of the following is/are correct?
(A) The term science is derived from a Latin verb which means 'to know'
(B) The term physics in derived from a Greek word which means 'nature'
(C) L is the symbol of unit of a physical quantity
(D) Physics is an exact science because it is based on the experimental measurements

19. In international system of units, there are seven base quantities whose units are defined. Which physical quantities do not have a prefix with their units?
(A) Amount of substance
(B) Thermodynamic temperature
(C) Luminous intensity
(D) Mass

20. Which of the following is/are the units of mass?
(A) kgf (B) metric ton
(C) quintal (D) amu

21. Which of the following can be expressed as Nm^{-2}?
(A) Energy density of electric field
(B) Bulk modulus of elasticity
(C) Pressure of 1 mm mercury column
(D) Compressional stress

22. Which of the following physical quantities have dimensions zero in mass, 2 in length and -2 in time?
(A) Latent heat
(B) Potential energy per unit mass
(C) Gravitational potential
(D) Spring constant

23. Which of the following is/are true regarding significant figures?
(A) All non-zero digits are significant
(B) The zeros appearing in the middle of a number are significant, while those at the end of a number without a decimal point are ambiguous
(C) The powers of 10 are counted while counting the number of significant figures
(D) Greater the number of significant figures in a measurements smaller is the percentage error

24. Which of the following cannot be considered to be the most accurate?
(A) 200 m (B) 20×10^1 m
(C) 2×10^2 m (D) Data insufficient

25. The focal length (f) of a curved mirror is related to the object distance u and the image distance v in accordance with the mathematical relation, $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$.
The maximum relative error in calculating the focal length f of the mirror is
(A) $\frac{\Delta f}{f^2}=\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}$
(B) $\frac{\Delta f}{f}=\frac{1}{\Delta u/u}+\frac{1}{\Delta v/v}$
(C) $\frac{\Delta f}{f}=\frac{uv}{u+v}\left(\frac{\Delta v}{v^2}+\frac{\Delta v}{v^2}\right)$
(D) $\frac{\Delta f}{f}=\frac{\Delta u}{u}+\frac{\Delta v}{v}+\frac{\Delta u}{u+v}+\frac{\Delta v}{u+v}$

26. In the equation, $y=a\cos\left(\frac{t}{p}-qx\right)$, where t represents time in second and x represents distance in metre. Which of the following statement(s) is/are false?
(A) The unit of x is same as that of q
(B) The unit of x is same as that of p
(C) The unit of t is same as that of q
(D) The unit of t is same as that of p

27. In the relation, $y=A\sin\left[\frac{2\pi}{\lambda}(ct-x)\right]$, where y and x are measured in metre. Which of the following statement(s) is/are false?
(A) The unit of λ is same as that of x and A
(B) The unit of λ is same as that of x but not of A
(C) The unit of c is same as that of x
(D) The unit of $(ct-x)$ is same as that of $\frac{2\pi}{\lambda}$

28. Identify the pair(s) having identical dimensions.
(A) Linear momentum and moment of force
(B) Planck constant and angular momentum
(C) Pressure and modulus of elasticity
(D) Work and torque

29. Which of the following is/are true?
(A) The order of magnitude of 501 is 3
(B) The order of magnitude of 499 is 2
(C) The order of 2^{30} is nearly 10^{90}
(D) The unit of reduction factor of a tangent galvanometer is ampere

30. The velocity, acceleration and force in two systems of units are related to each other as under

(i) $v' = \frac{\alpha^2}{\beta} v$ (ii) $a' = (\alpha\beta) a$

(iii) $F' = \left(\frac{1}{\alpha\beta}\right) F$

where, all the primed symbols belong to one system of units and the unprimed symbols belong to the other system of units. Here α and β are dimensionless constants. Which of the following is/are correct?
(A) The standard of length in each of these two systems are related to each other as $l' = \frac{\alpha^3}{\beta^3} l$
(B) The standard of mass in each of these two systems are related to each other as $m' = \left(\frac{1}{\alpha^2\beta^2}\right) m$
(C) The standard of time in each of these two systems are related to each other as $t' = \left(\frac{\alpha}{\beta^2}\right) t$
(D) The standard of momentum in each of these two systems are related to each other as $p' = \left(\frac{1}{\beta^3}\right) p$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** The light year and wavelength have dimensions of length.
Statement-2: Both light year and wavelength represent distances.

2. **Statement-1:** The fundamental units selected in a particular system are the velocity of light, $3\times10^8\ \text{ms}^{-1}$, acceleration due to gravity is $10\ \text{ms}^{-2}$ and the mass of proton is 1.67×10^{-27} kg.
Statement-2: The value of time in such a system is 3×10^7 s.

3. **Statement-1:** Surface energy stands for energy and not for energy per unit area.
Statement-2: Dimensional formula of surface energy is $\left[MT^{-2}\right]$.

4. **Statement-1:** The density of earth is given by $\rho = \frac{3g}{4\pi R_e G}$, where R_e stands for the radius of earth.
Statement-2: $g = \frac{GM}{R_e^2}$.

5. **Statement-1:** The value of 1 micron is equal to 10^{-6} m.
Statement-2: Micron is the unit for measuring microscopic distance.

6. **Statement-1:** The dimensional formula for electrostatic potential is $\left[ML^2T^{-3}A^{-1}\right]$.
Statement-2: Electrostatic potential is directly proportional to work done.

7. **Statement-1:** Distance travelled in a particular second has the dimensions same as that of distance.
Statement-2: Velocity is the displacement covered per unit time.

8. **Statement-1:** When percentage errors in the measurement of mass and velocity are 1% and 2% respectively, the percentage error in the measure of kinetic energy (E) is 5%.
Statement-2: The relative error in Kinetic Energy or $E = \frac{1}{2}mv^2$ is $\frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$.

9. **Statement-1:** The quantity $\frac{1}{\sqrt{\mu_o \varepsilon_o}}$ is numerically equal to velocity of light and has dimensional formula same as that of velocity.
Statement-2: μ_o is permeability of free space and ε_o is the permittivity of free space.

10. **Statement-1:** Specific gravity is measured by hygrometer.
Statement-2: Thermometer is an instrument which is used to measure the temperature of a body.

11. **Statement-1:** The error in the measurement of radius of the sphere is 0.3%. The permissible error in its surface area is 0.6%.
Statement-2: The permissible relative error is calculated by the formula $\frac{\Delta A}{A} = 4\frac{\Delta r}{r}$.

12. **Statement-1:** Pressure can be subtracted from the pressure gradient.
Statement-2: Only like quantities can be added or subtracted from each other.

13. **Statement-1:** Method of dimensions cannot be used for deriving formula containing trigonometrical ratios.
Statement-2: This is because trigonometrical ratios have no dimensions.

14. **Statement-1:** A fundamental quantity is the one that does not depend upon other quantities.
Statement-2: Length, mass and time are derived quantities.

15. **Statement-1:** In any mathematical relation between physical quantities the dimensions on both the sides must be the same.
Statement-2: The dimensions of a physical quantity are the powers raised on fundamental units to obtain the derived units of that physical quantity.

16. **Statement-1:** The unit used for measuring nuclear cross section is barn.
Statement-2: 1 barn $= 10^{-4}$ m^2

17. **Statement-1:** In the relation, $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$ where symbols have usual meanings, m represents linear mass density.
Statement-2: Linear mass density is $m = \frac{\text{Mass}}{\text{Volume}}$

18. **Statement-1:** Avogadro number is a dimensionless constant.
Statement-2: It is number of atoms in 1 gram mole.

19. **Statement-1:** The time period of a simple pendulum is given by the formula, $T = 2\pi\sqrt{\frac{l}{g}}$.
Statement-2: According to the principle homogeneity of dimensional formula, only that formula is correct, where the dimensions of left hand side is equal to the dimensional formula of right hand side.

20. **Statement-1:** AU is a much bigger unit than Å.
Statement-2: 1 AU $= 1.5 \times 10^{11}$ m and 1Å $= 10^{-10}$ m

21. **Statement-1:** The mass of largest stone that can be moved by flowing river depends on velocity v of river, density ρ and acceleration due to gravity g.
Statement-2: It can be shown that m is proportional to the sixth power of v.

22. **Statement-1:** Pressure has the dimensions same as that of energy density.
Statement-2: Energy density $= \frac{\text{Energy}}{\text{Volume}} =$ Pressure

23. **Statement-1:** The graph between P and Q is a straight line, when $\frac{P}{Q} =$ constant.
Statement-2: The straight line graph means the slope of the graph is constant.

24. **Statement-1:** If $x = \frac{a^{\ell} b^{m}}{c^{n}}$, then $\frac{\Delta x}{x} = \ell\frac{\Delta a}{a} + m\frac{\Delta b}{b} + n\frac{\Delta c}{c}$, where $\frac{\Delta a}{a}$, $\frac{\Delta b}{b}$ and $\frac{\Delta c}{c}$ are the fractional errors in the values of a, b and c, respectively.
Statement-2: The above relation is valid only when $\Delta a \ll a$, $\Delta b \ll b$ and $\Delta c \ll c$.

25. **Statement-1:** If $x = \frac{a^{\ell} b^{m}}{c^{n}}$, then $\frac{\Delta x}{x} = \ell\frac{\Delta a}{a} + m\frac{\Delta b}{b} - n\frac{\Delta c}{c}$, where Δa, Δb and Δc are the increments in the values of a, b and c, respectively.
Statement-2: The above relation is valid only when $\Delta a \ll a$, $\Delta b \ll b$ and $\Delta c \ll c$.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

The following set of quantities have been observed to possess the same dimensional formulae.

A : Velocity, speed, distance covered in nth second.

B : Acceleration, retardation, gravitational _____.

C : Work, energy, heat, internal energy, moment of _____.

D : Force, thrust, weight, energy _____.

E : Surface tension, surface _____, spring constant.

F : Pressure, stress, _____ density, Young's modulus, Bulk modulus.

Based on the above information answer the following questions.

1. In B, the blank space is best filled with
(A) field (B) potential
(C) intensity (D) force

2. In C, the blank space is best filled with
(A) torque (B) force
(C) momentum (D) velocity

3. In D, the blank space is best filled with
(A) density (B) rate
(C) gradient (D) None of these

4. In E, the blank space is best filled with
(A) density (B) energy
(C) area (D) None of these

5. In F, the blank space is best filled with
(A) energy (B) number
(C) mass (D) force

Comprehension 2

The vander Waals equation is $\left(p+\frac{a}{V^2}\right)(V-b)=RT$, where p is pressure, V is molar volume and T is the temperature of the given sample of gas. R is called molar gas constant, a and b are called vander Wall constants.

Based on the above information answer the following questions.

6. The dimensional formula for b is same as that for
(A) p (B) V
(C) pV^2 (D) RT

7. The dimensional formula for a is same as that for
(A) V^2 (B) p
(C) pV^2 (D) RT

8. Which of the following does not possess the same dimensional formula as that for RT ?
(A) pV (B) pb
(C) $\frac{a}{V^2}$ (D) $\frac{ab}{V^2}$

9. The dimensional formula for $\frac{ab}{RT}$ is
(A) ML^5T^{-2} (B) $M^0L^3T^0$
(C) $ML^{-1}T^{-2}$ (D) $M^0L^6T^0$

10. The dimensional formula of RT is same as that of
(A) energy (B) force
(C) specific heat (D) latent heat

Comprehension 3

Dimensional analysis is a good method to find the dependency of a physical quantity on other physical quantities. Assume that the velocity of light c, universal gravitational constant G and the Plank's constant h be chosen as the fundamental units. Based on the above facts, answer the following questions.

11. In this new system, mass will have a dimensional formula given by
(A) $c^{\frac{1}{2}}G^{-\frac{1}{2}}h^{\frac{1}{2}}$ (B) $c^{-\frac{3}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$
(C) $c^{-\frac{5}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$ (D) None of these

12. In this new system, length will have a dimensional formula given by
(A) $c^{\frac{1}{2}}G^{-\frac{1}{2}}h^{\frac{1}{2}}$ (B) $c^{-\frac{3}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$
(C) $c^{-\frac{5}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$ (D) None of these

13. In this new system, time will be expressed as
(A) $c^{\frac{1}{2}}G^{-\frac{1}{2}}h^{\frac{1}{2}}$ (B) $c^{-\frac{3}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$
(C) $c^{-\frac{5}{2}}G^{\frac{1}{2}}h^{\frac{1}{2}}$ (D) None of these

Comprehension 4

Measurements always have uncertainties. If you measure the thickness of the cover of the book "*MECHANICS-I by RAHUL SARDANA*" using an ordinary ruler, then your measurement may be reliable only to the nearest millimeter and your result may be 2 mm (say). It would be absolutely wrong to state this result as 2.00 mm till we are not aware of the limitations of the measuring device. Based on the above facts, answer the following questions.

14. The percentage error in measuring a distance of about 50 cm with a meter stick having callibrations upto 1 mm is
(A) 0.1% (B) 0.2%
(C) 0.3% (D) 0.4%

15. The percentage error in measuring a mass of about 1 g with a balance having a least count 0.1 mg is
(A) 0.04% (B) 0.03%
(C) 0.02% (D) 0.01%

16. The percentage error in measuring a time interval of 4 minute with a stop watch having least count of 0.2 s is
(A) 5% (B) 0.06%
(C) 0.08% (D) 0.10%

Comprehension 5

The following set of quantities have been observed to possess the same dimensional formulae.

A : _____, Planck's constant and energy per unit _____.
B : Frequency, angular frequency, angular _____ and _____ gradient.
C : _____ field strength and _____ potential gradient.
D : _____ and latent heat.
E : _____ and strain.

Based on the above information answer the following questions.

17. In A, the blank space is best filled with
(A) momentum in both blanks
(B) energy in first blank and angular momentum in the second blank
(C) angular momentum in first blank and frequency in the second blank
(D) frequency in the first blank and angular momentum in the second blank

18. In B, the missing blank(s) are best filled as
(A) speed at both blanks
(B) speed at first blank, energy at second blank
(C) energy at first blank, speed at second blank
(D) energy at both blanks

19. In C, the blank space is best filled by
(A) electric at both
(B) electric at first, gravitational at second
(C) gravitational at first, electric at second
(D) gravitational at both

20. In D, the blank space is best filled by
(A) gravitational potential
(B) kinetic energy per unit mass
(C) gravitational potential energy per unit mass
(D) None of these

21. In E, the blank space is best filled by
(A) angle (B) velocity of light
(C) fine structure constant (D) Plank's constant

Comprehension 6

A particle is moving along a straight line and variable force F is acting on this F at any time t is given by

$$F = A + \frac{B}{t} + \frac{Ct}{D + t^2}$$

where A, B, C and D are constant. Based on the above facts, answer the following questions.

22. The dimensional formula of C is
(A) $[MLT^{-2}]$ (B) $[MLT^{-1}]$
(C) $[MLT^{-3}]$ (D) $[ML]$

23. The dimensions of B is equal to dimensions of
(A) Force (B) Power
(C) Linear momentum (D) Energy

24. Dimensions of $\left(\frac{1}{\sqrt{D}}\right)$ is equal to dimension of
(A) frequency (B) time
(C) length (D) force

Comprehension 7

Angular velocity of ceiling fan depends upon (i) applied voltage (V), (ii) resistance (R) of the coil of fan, (iii) mass (m) of fan and (iv) length (l) of the fan-blades. Given that the dimensional formula for voltage V is $ML^2T^{-3}A^{-1}$ and the resistance is defined as $R = \frac{V}{i}$, where i is the current flowing. Based on the above facts, answer the following questions.

25. Angular velocity of fan is proportional to
(A) $R^{-1/3}$ (B) $R^{-2/3}$
(C) $R^{-3/2}$ (D) None of these

26. By what factor angular velocity of fan will increase if applied voltage is increase 8 times

(A) 8 (B) 4
(C) 2 (D) 3

27. Length of blade of fan is increased from 10 inch to 14.4 inch keeping mass and coil-resistance same. How many times applied voltage needs to be increased as to keep angular velocity constant

(A) 1.44 times (B) 1.25 times
(C) 2.07 times (D) 1.2 times

Comprehension 8

The potential energy of a particle varies with distance x from a fixed origin as $U = \frac{A\sqrt{x}}{x+B}$, where A and B are constants. Based on the above facts, answer the following questions.

28. The dimension of AB is

(A) $ML^{5/2}T^{-2}$ (B) $ML^{3/2}T^{-2}$
(C) $ML^{1/2}T^{-2}$ (D) $ML^{7/2}T^{-2}$

29. The dimension of $\frac{A}{B^2}$ is

(A) $ML^{3/2}T^{-2}$ (B) $ML^{1/2}T^{-2}$
(C) $ML^{-1/2}T^{-2}$ (D) $ML^{-1/2}T^{2}$

30. The dimension of $\frac{A^2}{B}$ is

(A) $M^2L^2T^{-2}$ (B) $M^2L^2T^{-4}$
(C) $M^2L^4T^{-2}$ (D) $M^2L^{-4}T^{-4}$

Comprehension 9

If energy (E), velocity (V) and force (F) be taken as fundamental quantities. Based on the above facts, answer the following questions.

31. Dimensional formula for surface tension is

(A) E^1V^{-2} (B) $E^{-1}F^2$
(C) $E^{-1}VF^2$ (D) E^1V^2

32. Dimensional formula for angular momentum is

(A) $E^{-2}V^1F^1$ (B) $E^{-2}V^1F^{-1}$
(C) $E^2V^{-1}F^1$ (D) $E^2V^{-1}F^{-1}$

33. Dimensional formula for time is

(A) $EV^{-1}F^{-1}$ (B) EV^2F^{-1}
(C) $EV^{-1}F$ (D) EVF^2

Comprehension 10

A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$, where p is the static pressure, d is the density of water and E is the total energy of explosion. Based on the above facts, answer the following questions.

34. The value of a is

(A) $-\frac{5}{6}$ (B) $\frac{1}{2}$
(C) $-\frac{1}{2}$ (D) 1

35. The value of b is

(A) $-\frac{5}{6}$ (B) $\frac{1}{2}$
(C) $-\frac{1}{2}$ (D) 1

36. The value of c is

(A) $-\frac{5}{6}$ (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) 1

Comprehension 11

If Force (F), acceleration (A) and time (T) is considered to be fundamental quantities. Based on the above facts, answer the following questions.

37. The dimensions of torque are

(A) $FA^{-1}T^2$ (B) FA^2T^{-1}
(C) FAT^{-2} (D) FAT^2

38. The dimensions of velocity are

(A) AT (B) AT^2
(C) A^2T (D) FA^2T

39. The dimensions of momentum are

(A) FT^2 (B) F^2T
(C) FT (D) FT^{-1}

Comprehension 12

The distance travelled by a particle is given by

$$y = D\ln\left(1+\frac{At}{\lambda^2}\right) + B\sin\left(Cx^2\right)$$

here x and y are length, λ is wavelength and t is time. Based on the above facts, answer the following questions.

40. The dimensions of A and C are

(A) L^2T^{-1}, L^2 (B) L^2T^{-1}, L^{-2}

(C) $L^{-2}T^1$, L^{-2} (D) $L^{-2}T^{-1}$, L^2

41. Which pair has same dimensions

(A) A, B (B) B, C

(C) C, D (D) B, D

Comprehension 13

If pressure of a gas is given by

$$P = \frac{\alpha^2}{\beta}\sin\left(\frac{\alpha h}{k_B\theta}\right)$$

where h is the Plank's constant

k_B is the Boltzmann constant

θ is the absolute temperature

Based on the above facts, answer the following questions.

42. The dimensions of β are

(A) $[M^1L^0T^0]$ (B) $[M^0L^0T^0]$

(C) $[M^{-1}LT^0]$ (D) None of these

43. Which of the following is not dimensionless

(A) αt (B) $\frac{\beta P}{\alpha^2}$

(C) $\frac{h}{k\theta}$ (D) $\frac{BI}{L^2}$

(I : Moment of inertia, L : length and t : time)

Comprehension 14

The equation of state of gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

here, P is the pressure, V is the volume, T is the absolute temperature and a, b and R are constants. Based on the above facts, answer the following questions.

44. The dimensions of a are

(A) ML^5T^{-2}

(B) ML^5T^2

(C) MLT

(D) $M^2L^5T^{-2}$

45. The dimensions of b are

(A) MLT

(B) $M^0L^3T^0$

(C) ML^3T^0

(D) ML^3T^1

46. The dimensions of R are

(A) $ML^2T^{-2}K$

(B) $ML^2T^{-2}K^{-1}$

(C) $MLT^{-2}K^{-1}$

(D) $MLT^{-2}K$

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Match the following

Column-I	Column-II
(A) $\frac{R}{L}$	(p) Time
(B) CR	(q) Frequency
(C) $\frac{E}{B}$	(r) Speed
(D) $\frac{1}{\mu_0 \varepsilon_0}$	(s) Kinetic energy
	(t) Kinetic energy per unit mass

2. Match the following

Column-I	Column-II
(A) Stress	(p) Pressure
(B) Strain	(q) Energy density
(C) Modulus of elasticity	(r) Angle
(D) Torque	(s) Energy
	(t) Fine structure constant

3. Match the following

Column-I	Column-II
(A) Specific heat	(p) $ML^2T^{-2}K^{-1}$
(B) Boltzmann's constant	(q) $MT^{-3}K^{-4}$
(C) Wien's constant	(r) LK
(D) Stefan's constant	(s) $L^2T^{-2}K^{-1}$
	(t) $ML^2T^{-3}K^{-4}$

4. Match the units/dimensions in **COLUMN-I** with the physical quantities in **COLUMN-II**

Column-I	Column-II
(A) ML^2T^{-1}	(p) Impulse
(B) Js	(q) Planck's constant
(C) MLT^{-1}	(r) Angular momentum
(D) Energy per unit frequency	(s) Linear momentum
	(t) $\sqrt{2mE}$

(*Continued*)

5. Match the units given in **COLUMN-I** with the physical quantities in **COLUMN-II**

Column-I	Column-II
(A) Nm^{-2}	(p) Force constant
(B) Nm^{-1}	(q) Surface energy of a liquid
(C) Nm	(r) Stress
(D) kgs^{-2}	(s) Bulk modulus
	(t) Torque

6. Match **COLUMN-I** with **COLUMN-II**

Column-I	Column-II
(A) Dimensionless quantity	(p) Angle
(B) Young's modulus	(q) Mechanical equivalent of heat
(C) $Jcal^{-1}$	(r) $kgm^{-1}s^{-2}$
(D) pascal	(s) Thermal conductivity
	(t) Fine structure constant (α)

7. Match the units/dimensions in **COLUMN-I** with the physical quantities/expressions in **COLUMN-II**

Column-I	Column-II
(A) Jkg^{-1}	(p) $\frac{k_B T}{m}$
(B) $M^0L^2T^{-2}K^{-1}$	(q) Mean square velocity
(C) $M^0L^2T^{-2}$	(r) Latent heat
(D) JK^{-1}	(s) Specific heat
	(t) Entropy

8. Match the following

Column-I	Column-II
(A) Stress	(p) Pressure
(B) Strain	(q) Energy density
(C) Modulus of elasticity	(r) Angle
(D) Torque	(s) Energy
	(t) Renold's number (n_R)

(All symbols used have their standard meanings)

9. Some physical quantities are given in **COLUMN-I** and some possible SI units in which these quantities may be expressed are given in **COLUMN-II**. Match the physical quantities in **COLUMN-I** with the units in **COLUMN-II**.

Column-I	Column-II
(A) GM_eM_s G–universal gravitational constant, M_e–mass of the earth, M_s–mass of the sun.	(p) (volt) (coulomb) (metre)
(B) $\frac{3RT}{M}$ R–universal gas constant, T–absolute temperature, M–molar mass.	(q) (kilogram) $(\text{metre})^3$ $(\text{second})^{-2}$
(C) $\frac{F^2}{q^2B^2}$ F–force, q–charge, B–magnetic field.	(r) $(\text{metre})^2$ $(\text{second})^{-2}$
(D) $\frac{GM_e}{R_e}$ G–universal gravitational constant, M_e–mass of the earth, R_e–radius of the earth.	(s) (farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

10. Match the following

Column-I	Column-II
(A) Young's modulus	(p) L^2T^{-2}
(B) Gravitational potential	(q) $M^{-1}L^3T^{-2}$
(C) Latent heat	(r) $ML^{-1}T^{-2}$
(D) Gravitational constant	(s) ML^2T^{-2}

11. Match the following

Column-I	Column-II
(A) Coefficient of viscosity	(p) Dimensionless
(B) Strain	(q) Unitless
(C) Angle	(r) $ML^{-1}T^{-1}$
(D) Stress	(s) $ML^{-1}T^{-2}$

12. In the column given below:
C stands for capacitance
R stands for resistance
k stands for Boltzmann constant
c stands for speed of light
e stands for electronic charge
H stands for Henry

Column-I	Column-II
(A) $\frac{e^2}{2h\varepsilon_0 c}$	(p) Joule
(B) $\sqrt{\frac{R^2C^2}{\mu_0\varepsilon_0}}$	(q) Dimensionless
(C) k_BT	(r) m^{-1}
(D) $\frac{e^4m}{8\varepsilon_0^2h^3c}$	(s) Unitless

13. Match the physical quantities given in **COLUMN-I** with dimensional formula expressed in **COLUMN-II** in tabular form

Column-I	Column-II
(A) Coefficient of viscosity	(p) MLT^{-1}
(B) Angular momentum	(q) ML^2T^{-2}
(C) Torque	(r) LT^{-1}
(D) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$	(s) $ML^{-1}T^{-1}$

14. Match the physical quantities given in **COLUMN-I** with unit expressed in **COLUMN-II** in tabular form

Column-I	Column-II
(A) Pressure	(p) Watt
(B) Power	(q) Pascal
(C) Charge	(r) Hertz
(D) Frequency	(s) Coulomb

15. Match the following

Column-I	Column-II
(A) Young's modulus	(p) L^2T^{-2}
(B) Gravitational potential	(q) $M^{-1}L^3T^{-2}$
(C) Latent heat	(r) $ML^{-1}T^{-2}$
(D) Gravitational constant	(s) ML^2T^{-2}

16. Match the following

Column-I	Column-II
(A) Electric potential	(p) $ML^2T^{-2}A^{-2}$
(B) Resistance	(q) $ML^2T^{-3}A^{-1}$
(C) Capacitance	(r) $ML^2T^{-3}A^{-2}$
(D) Inductance	(s) $M^{-1}L^{-2}T^4A^2$

17. Match the following

Column-I	Column-II
(A) Coefficient of viscosity	(p) Dimensionless
(B) Strain	(q) Unitless
(C) Angle	(r) $ML^{-1}T^{-1}$
(D) Stress	(s) $ML^{-1}T^{-2}$

18. The Stefan-Boltzman's constant is not a fundamental constant and one can write it in terms of fundamental constant and written as $\sigma = ah^{\alpha}C^{\beta}G^{\gamma}k_B^{\delta}$, a is dimensionless constant, h is planck constant, C is speed of light, G is universal gravitational constant and k_B is Boltzman constant.

Column-I	Column-II
(A) α	(p) –2
(B) β	(q) –3
(C) γ	(r) 4
(D) δ	(s) 0

19. Match **COLUMN–I** with **COLUMN–II** in regard to dimensions of physical quantities mentioned in **COLUMN–I** and dimensions of physical quantities in **COLUMN–II**.

Column-I	Column-II
(A) Kinetic energy	(p) Pressure
(B) Energy density	(q) Torque
(C) Planck constant	(r) Angular momentum
(D) Efficiency of carnot cycle	(s) Longitudinal strain

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. The dimensional formula of $\frac{EJ^2}{M^5G^2}$, where E, M, J and G denote energy, mass, angular momentum and gravitational constant is $M^aL^bT^c$. Then find values of a, b, c.

2. The quantity $\frac{e^x}{2h\varepsilon_0 c}$ has dimensional formula $M^0L^0T^0$. Calculate x.

3. The rate of flow (V) of a liquid flowing through a pipe of radius r and pressure gradient p is given by Poiseuille's equation

$$V = \frac{\pi}{8}\frac{r^x}{\eta^z}p^y$$

Then calculate x, y, z.

4. A new system is formed such that it uses force, energy and velocity as fundamental quantities with units 20 N, 200 J and 5 ms^{-1}. Calculate the units of mass, length and time in this new system.

5. Consider a new system of units in which the unit of mass is α kg, unit of length is β m and that of time is γ s. The value of calorie in the new system is calculated to be $4.2\alpha^{-x}\beta^{-y}\gamma^z$. Find the values of x, y, z.

6. According to Kepler's Laws of planetary motion, the planets move around the sun in nearly circular orbits. Assuming that the period of rotation depends upon radius of the orbit (r), mass of sun (M) and Universal Gravitational Constant (G) as

$$T^a = 4\pi^2\frac{r^b}{M^cG^d}$$

Find a, b, c and d.

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1. **[Online April 2019]**

In SI units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is

(A) $AT^2M^{-1}L^{-1}$ (B) $AT^{-3}ML^{\frac{3}{2}}$

(C) $A^{-1}TML^3$ (D) $A^2T^3M^{-1}L^{-2}$

2. **[Online April 2019]**

If Surface tension (S), Moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be

(A) $S^{\frac{1}{2}}I^{\frac{3}{2}}h^{-1}$ (B) $S^{\frac{3}{2}}I^{\frac{1}{2}}h^0$

(C) $S^{\frac{1}{2}}I^{\frac{1}{2}}h^{-1}$ (D) $S^{\frac{1}{2}}I^{\frac{1}{2}}h^0$

3. **[Online April 2019]**

In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to

(A) 6.8% (B) 0.2%

(C) 3.5% (D) 0.7%

4. **[Online April 2019]**

In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is

(A) 2100 kgm^{-3} (B) 3100 kgm^{-3}

(C) 6400 kgm^{-3} (D) 1100 kgm^{-3}

5. **[Online April 2019]**

The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is

(A) 37.03 cm^2 (B) 37.0 cm^2

(C) 37.030 cm^2 (D) 37 cm^2

6. **[Online April 2019]**

In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units?

(A) $[M^{-2}L^{-2}T^6A^3]$ (B) $[M^{-1}L^{-2}T^4A^2]$

(C) $[M^{-2}L^0T^{-4}A^{-2}]$ (D) $[M^{-3}L^{-2}T^8A^4]$

7. **[Online April 2019]**
Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

(A) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (B) $\frac{\varepsilon_0}{\mu_0}$

(C) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (D) $\frac{\mu_0}{\varepsilon_0}$

8. **[Online January 2019]**
The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is

(A) 5.725 mm (B) 5.740 mm

(C) 5.755 mm (D) 5.950 mm

9. **[Online January 2019]**
Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to

(A) $\sqrt{\frac{Gh}{c^5}}$ (B) $\sqrt{\frac{c^3}{Gh}}$

(C) $\sqrt{\frac{Gh}{c^3}}$ (D) $\sqrt{\frac{hc^5}{G}}$

10. **[Online January 2019]**
The density of a material is SI units is $128\ \text{kgm}^{-3}$. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is

(A) 640 (B) 410

(C) 40 (D) 16

11. **[Online January 2019]**
The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures?

(A) $4264 \pm 81\ \text{cm}^3$ (B) $4300 \pm 80\ \text{cm}^3$

(C) $4260 \pm 80\ \text{cm}^3$ (D) $4264.4 \pm 81.0\ \text{cm}^3$

12. **[Online January 2019]**
The force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is

(A) $M^0L^2T^{-4}$ (B) M^2LT^{-4}

(C) MLT^{-2} (D) $M^2L^2T^{-2}$

13. **[Online January 2019]**
If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be

(A) $V^{-2}A^2F^{-2}$ (B) $V^{-2}A^2F^2$

(C) $V^{-4}A^2F$ (D) $V^{-4}A^{-2}F$

14. **[Online January 2019]**
The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure $5\ \mu$m diameter of a wire is

(A) 200 (B) 50

(C) 100 (D) 500

15. **[Online January 2019]**
Let l, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{1}{rcv}$ in SI units will be

(A) $[A^{-1}]$ (B) $[LA^{-2}]$

(C) $[LT^2]$ (D) $[LTA]$

16. **[2018]**
The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(A) 2.5% (B) 3.5%

(C) 4.5% (D) 6%

17. **[Online 2018]**
In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is

(A) 0.3150 cm (B) 0.2150 cm

(C) 0.4300 cm (D) 0.0430 cm

18. **[Online 2018]**
The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is

(A) $\frac{3}{2}\alpha$ (B) α

(C) $\frac{5}{2}\alpha$ (D) $\frac{2}{3}\alpha$

19. [Online 2018]
The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, $\hbar$ and c. Which of the following correctly gives the Planck length?

(A) $G\hbar^2c^3$ (B) $G^2\hbar c$

(C) $\left(\frac{G\hbar}{c^3}\right)^{1/2}$ (D) $G^{1/2}\hbar^2c$

20. [Online 2018]
The percentage errors in quantities P, Q, R and S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity $A=\frac{P^3Q^2}{\sqrt{RS}}$. The maximum percentage error in the value of A will be

(A) 6.5% (B) 8.5%

(C) 6% (D) 7.5%

21. [Online 2018]
The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible, the relative uncertainty in the mas of the earth is

(A) 6×10^{-2} (B) 10^{-2}

(C) 2×10^{-2} (D) 3×10^{-2}

22. [Online 2017]
Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be

(A) $[M]=[TC^{-2}h]$ (B) $[M]=[T^{-1}C^{-2}h^{-1}]$

(C) $[M]=[T^{-1}C^{-2}h]$ (D) $[M]=[T^{-1}C^{2}h]$

23. [Online 2017]
A physical quantity P is described by the relation $P=a^{1/2}b^2c^3d^{-4}$. If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be

(A) 25% (B) 12%

(C) 8% (D) 32%

24. [2016]
A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be

(A) 92 ± 2 s (B) 92 ± 5 s

(C) 92 ± 1.8 s (D) 92 ± 3 s

25. [Online 2016]
In the following 'I' refers to current and other symbols have their usual meaning. Choose the option that corresponds to the dimensions of electrical conductivity

(A) $M^{-1}L^{-3}T^3I$ (B) $M^{-1}L^{-3}T^3I^2$

(C) $M^{-1}L^3T^3I$ (D) $ML^{-3}T^{-3}I^2$

26. [Online 2016]
A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation $AD=C\ln(BD)$ holds true. Then which of the combination is not a meaningful quantity?

(A) $\frac{C}{BD}-\frac{AD^2}{C}$ (B) $A^2-B^2C^2$

(C) $\frac{A}{B}-C$ (D) $\frac{(A-C)}{D}$

27. [Online 2015]
If the capacitance of a nanocapacitor is measured in terms of a unit u made by combining the electronic charge e, Bohr radius a_0, Planck's constant h and speed of light c then

(A) $u=\frac{e^2c}{ha_0}$ (B) $u=\frac{e^2h}{ca_0}$

(C) $u=\frac{e^2a_0}{hc}$ (D) $u=\frac{hc}{e^2a_0}$

28. [Online 2015]
If electronic charge e, electron mass m, speed of light in vacuum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of

(A) $\left(\frac{hc}{me^2}\right)$ (B) $\left(\frac{h}{me^2}\right)$

(C) $\left(\frac{h}{ce^2}\right)$ (D) $\left(\frac{mc^2}{he^2}\right)$

29. [Online 2015]
A beaker contains a fluid of density ρ kgm^{-3}, specific heat S Jkg^{-1} °C^{-1} and viscosity η. The beaker is filled up to height h. To estimate the rate of heat transfer per unit area $\left(\frac{\dot{Q}}{A}\right)$ by convection when beaker is put on a hot plate, a student proposes that it should depend on η, $\left(\frac{S\Delta\theta}{h}\right)$ and $\left(\frac{1}{\rho g}\right)$ when $\Delta\theta$ (in °C) is the difference in the temperature between the bottom and top of the fluid. In that situation the correct option for $\left(\frac{\dot{Q}}{A}\right)$ is

(A) $\eta\frac{S\Delta\theta}{h}$ (B) $\eta\left(\frac{S\Delta\theta}{h}\right)\left(\frac{1}{\rho g}\right)$

(C) $\frac{S\Delta\theta}{\eta h}$ (D) $\left(\frac{S\Delta\theta}{\eta h}\right)\left(\frac{1}{\rho g}\right)$

30. [2014]

A student measured the length of a rod and wrote it as 3.50 cm . Which instrument did he use to measure it?

(A) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm .

(B) A meter scale.

(C) A vernier calliper where the 10 divisions in vernier scale. Matches with 9 division in main scale and main scale has 10 divisions in 1 cm .

(D) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm .

31. [2013]

Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then

(A) $[\varepsilon_0]=[M^{-1}L^2T^{-1}A]$

(B) $[\varepsilon_0]=[M^{-1}L^{-3}T^2A]$

(C) $[\varepsilon_0]=[M^{-1}L^{-3}T^4A^2]$

(D) $[\varepsilon_0]=[M^{-1}L^2T^{-1}A^{-2}]$

32. [2012]

Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is

(A) zero (B) 1%

(C) 3% (D) 6%

33. [2010]

The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(A) 4, 4, 2 (B) 5, 1, 2

(C) 5, 1, 5 (D) 5, 5, 2

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. [JEE (Advanced) 2017]

Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point of the surface of the expanding sphere is proportional to

(A) R (B) $\frac{1}{R}$

(C) R^3 (D) $R^{\frac{2}{3}}$

2. [JEE (Advanced) 2017]

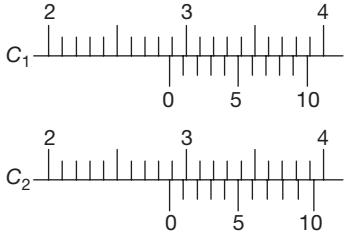
A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ s and he measures the depth of the well to be $L = 20$ m. Take the acceleration due to gravity $g = 10$ ms^{-2} and the velocity of sound is 300 ms^{-1} . Then the fractional error in the measurement, $\frac{\delta L}{L}$, is closest to

(A) 1% (B) 5%

(C) 3% (D) 0.2%

3. [JEE (Advanced) 2016]

There are two Vernier callipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the callipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other calliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two callipers are shown in the figure. The measured values (in cm) by callipers C_1 and C_2 respectively, are



(A) 2.87 and 2.86 (B) 2.87 and 2.87
(C) 2.87 and 2.83 (D) 2.85 and 2.82

4. [JEE (Advanced) 2013]

The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm . The 24^{th} division of the vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm
(C) 5.136 cm (D) 5.148 cm

5. [IIT-JEE 2012]

In the determination of Young's modulus $\left(Y=\frac{4MLg}{\pi ld^2}\right)$ by using Searle's method, a wire of length $L=2$ m and diameter $d=0.5$ mm is used. For a load $M=2.5$ kg , an extension $l=0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm . The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement is

(A) due to the errors in the measurements of d and l are the same.
(B) due to the error in the measurement of d is twice that due to the error in the measurement of l .
(C) due to the error in the measurement of l is twice that due to the error in the measurement of d .
(D) due to the error in the measurement of d is four times that due to the error in the measurement of l.

6. [IIT-JEE 2011]

The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(A) 0.9% (B) 2.4%
(C) 3.1% (D) 4.2%

7. [IIT-JEE 2010]

A vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier callipers, the least count is

(A) 0.02 mm (B) 0.05 mm
(C) 0.1 mm (D) 0.2 mm

8. [IIT-JEE 2008]

Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.

Least count for length $=0.1$ cm , Least count for time $=0.1$ s

Student	Length of the pendulum (cm)	Number of oscillations (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I , E_{II} and E_{III} are the percentage errors in g , i.e., $\left(\frac{\Delta g}{g}\times 100\right)$ for students I, II and III respectively

(A) $E_I=0$ (B) E_I is minimum
(C) $E_I=E_{II}$ (D) E_{II} is maximum

9. [IIT-JEE 2007]

A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1 kg . The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm . Take $g=9.8$ ms^{-2} (exact). The young's modulus obtained from the reading is close to

(A) $(2\pm 0.3)10^{11}$ Nm^{-2}
(B) $(2\pm 0.2)\times 10^{11}$ Nm^{-2}
(C) $(2\pm 0.1)\times 10^{11}$ Nm^{-2}
(D) $(2\pm 0.05)\times 10^{11}$ Nm^{-2}

10. [IIT-JEE 2007]

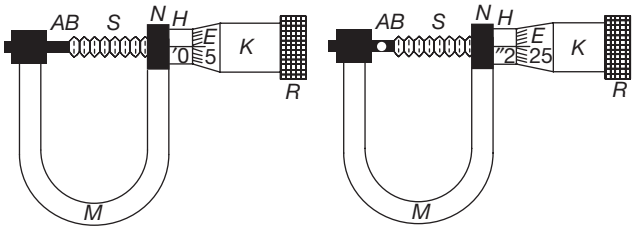
In the experiment to determine the speed of sound using a resonance column

(A) prongs of the tuning fork are kept in a vertical plane.
(B) prongs of the tuning fork are kept in a horizontal plane.

(C) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air.
(D) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air.

11. [IIT-JEE 2006]

The circular scale of a screw gauge has 50 divisions and pitch of 0.5 mm. Find the diameter of sphere. Main scale reading is 2.



(A) 1.2 mm (B) 1.25 mm
(C) 2.20 mm (D) 2.25 mm

12. [IIT-JEE 2006]

A student performs an experiment for determination of $g\left(=\frac{4\pi^2 l}{T^2}\right)$, $l \approx 1$ m, and he commits an error of Δl. For T he takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 s. For which of the following data, the measurement of g will be most accurate.

(A) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 20$
(B) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 50$
(C) $\Delta L = 0.5$, $\Delta T = 0.01$, $n = 20$
(D) $\Delta L = 0.1$, $\Delta T = 0.05$, $n = 50$

13. [IIT-JEE 2005]

Which of the following sets have different dimensions?
(A) Pressure, Young's modulus, Stress
(B) Emf, Potential difference, Electric Potential
(C) Heat, Work done, Energy
(D) Dipole moment, Electric flux, Electric field

14. [IIT-JEE 2004]

A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is

(A) 1 (B) 2
(C) 3 (D) 4

15. [IIT-JEE 2004]

In the relation, $P = \frac{\alpha}{\beta} e^{-\frac{\alpha Z}{k\theta}}$, P is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be

(A) $[M^0L^2T^0]$ (B) $[M^1L^2T^1]$
(C) $[M^1L^0T^{-1}]$ (D) $[M^0L^2T^{-1}]$

16. [IIT-JEE 2003]

A cube has a side of length 1.2×10^{-2} m. Calculate its volume

(A) 1.7×10^{-6} m^3 (B) 1.73×10^{-6} m^3
(C) 1.70×10^{-6} m^3 (D) 1.732×10^{-6} m^3

17. [IIT-JEE 2001]

A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

(A) resistance (B) charge
(C) voltage (D) current

18. [IIT-JEE 2000]

The dimension of $\left(\frac{1}{2}\right)\varepsilon_0 E^2$ (ε_0 : permittivity of free space; E : electric field) is

(A) MLT^{-1} (B) $ML^{-1}T^{-2}$
(C) MLT^{-2} (D) ML^2T^{-1}

19. [IIT-JEE 1995]

In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in $MKSQ$ system?

(A) $M^{-3}L^{-1}T^3Q^4$ (B) $M^{-3}L^{-2}T^4Q^4$
(C) $M^{-2}L^{-2}T^4Q^4$ (D) $M^{-3}L^{-2}T^4Q^1$

20. [IIT-JEE 1992]

A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn block A executes small oscillations. The time period of which is given by

(A) $2\pi\sqrt{\frac{M\eta}{L}}$ (B) $2\pi\sqrt{\frac{L}{M\eta}}$

(C) $2\pi\sqrt{\frac{ML}{\eta}}$ (D) $2\pi\sqrt{\frac{M}{\eta L}}$

21. [IIT-JEE 1991]

If L, R, C and V respectively represent inductance resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of

(A) current (B) $\frac{1}{\text{current}}$

(C) charge (D) $\frac{1}{\text{charge}}$

22. [IIT-JEE 1990]

If E, M, J and G respectively denote energy, mass, angular momentum and gravitational constant, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of

(A) length (B) angle

(C) mass (D) time

Multiple Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2019]

Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

(A) The dimension of force is L^{-3}

(B) The dimension of power is L^{-5}

(C) The dimension of linear momentum is L^{-1}

(D) The dimension of energy is L^{-2}

2. [JEE (Advanced) 2016]

A length-scale (l) depends on the permittivity (ε) of a dielectric material. Boltzmann's constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct?

(A) $l=\sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (B) $l=\sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(C) $l=\sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $l=\sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

3. [JEE (Advanced) 2016]

In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T=2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true?

(A) The error in the measurement of r is 10%

(B) The error in the measurement of T is 3.57%

(C) The error in the measurement of T is 2%

(D) The error in the determined value of g is 11%

4. [JEE (Advanced) 2015]

Planck's constant h, speed of light c and gravitational constant G are used to from a unit of length L and a unit of mass M. Then the correct option(s) is(are)

(A) $M\propto\sqrt{c}$ (B) $M\propto\sqrt{G}$

(C) $L\propto\sqrt{h}$ (D) $L\propto\sqrt{G}$

5. [JEE (Advanced) 2015]

In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are)

(A) $\mu_0 I^2=\varepsilon_0 V^2$ (B) $\varepsilon_0 I=\mu_0 V$

(C) $I=\varepsilon_0 cV$ (D) $\mu_0 cI=\varepsilon_0 V$

6. [JEE (Advanced) 2015]

Consider a vernier calliper in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then

(A) if the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm

(B) if the pitch of the screw gauge is twice the least count of the vernier calliper, the least count of the screw gauge is 0.05 mm

(C) if the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.01 mm.

(D) if the least count of the linear scale of the screw gauge is twice the least count of the Vernier calliper, the least count of the screw gauge is 0.005 mm.

7. [JEE (Advanced) 2010]
A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statement(s) is/are true?
(A) Error ΔT in measuring T, the time period, is 0.05 s
(B) Error ΔT in measuring T, the time period, is 1 s
(C) Percentage error in the determination of g is 5%
(D) Percentage error in the determination of g is 2.5%

8. [IIT-JEE 1998]
Let $[\varepsilon_0]$ denotes the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = Time and I = electric current, then
(A) $[\varepsilon_0] = M^{-1}L^{-3}T^2I$ (B) $[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$
(C) $[\mu_0] = MLT^{-2}I^{-2}$ (D) $[\mu_0] = ML^2T^{-1}I$

9. [IIT-JEE 1998]
The SI unit of the inductance, the henry can by written as
(A) $\frac{\text{weber}}{\text{ampere}}$ (B) $\frac{\text{volt-second}}{\text{ampere}}$
(C) $\frac{\text{joule}}{(\text{ampere})^2}$ (D) ohm-second

10. [IIT-JEE 1995]
The pairs of physical quantities that possess same dimensions is/are
(A) Reynold's number and coefficient of friction
(B) Curie and frequency of light wave
(C) Latent heat and gravitational potential
(D) Planck's constant and torque

11. [IIT-JEE 1992]
If the dimensions of length are expressed as $G^xc^yh^z$; where G, c and h are the universal gravitational constant, speed of light and Planck's constant respectively, then
(A) $x = \frac{1}{2}, y = \frac{1}{2}$ (B) $x = \frac{1}{2}, z = \frac{1}{2}$
(C) $y = \frac{1}{2}, z = \frac{3}{2}$ (D) $y = -\frac{3}{2}, z = \frac{1}{2}$

12. [IIT-JEE 1986]
The pair(s) having same dimensions is/are
(A) Torque and work
(B) Angular momentum and work
(C) Energy and Young's modulus
(D) Light year and wavelength

13. [IIT-JEE 1984]
L, C and R represent the physical quantities inductance, capacitance and resistance respectively. The combinations which have the dimensions of frequency are
(A) $\frac{1}{RC}$ (B) $\frac{R}{L}$
(C) $\frac{1}{\sqrt{LC}}$ (D) $\frac{C}{L}$

Linked Comprehension Type Problems

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

[JEE (Advanced) 2018]
In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, $[E]$ and $[B]$ stand for dimensions of electric and magnetic fields respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space, respectively. $[L]$ and $[T]$ are dimensions of length and time, respectively. All the quantities are given in SI units.

(There are two questions based on above paragraph, the question given below is one of them).

1. The relation between $[E]$ and $[B]$ is
(A) $[E] = [B][L][T]$
(B) $[E] = [B][L]^{-1}[T]$
(C) $[E] = [B][L][T]^{-1}$
(D) $[E] = [B][L]^{-1}[T]^{-1}$

2. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is
(A) $[\mu_0] = [\varepsilon_0][L]^2[T]^{-2}$
(B) $[\mu_0] = [\varepsilon_0][L]^{-2}[T]^2$
(C) $[\mu_0] = [\varepsilon_0]^{-1}[L]^2[T]^{-2}$
(D) $[\mu_0] = [\varepsilon_0]^{-1}[L]^{-2}[T]^2$

Comprehension 2

[JEE (Advanced) 2018]

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = \frac{x}{y}$. If the errors in x, y and z are Δx, Δy and Δz respectively, then $z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$.

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\frac{\Delta y}{y}$, is $1 \mp \left(\frac{\Delta y}{y}\right)$. The relative errors in independent variables are always added. So, the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$$

The above derivation makes the assumption that $\frac{\Delta x}{x} \ll 1$, $\frac{\Delta y}{y} \ll 1$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on above paragraph, the question given below is one of them)

3. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is $\Delta a\left(\frac{\Delta a}{a} \ll 1\right)$, then what is the error Δr in determining r?

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$

(C) $\frac{2\Delta a}{(1-a)^2}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

4. In an experiment, the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1 s. For $|x| \ll 1$, $\ln(1+x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ in s^{-1}, is

(A) 0.04 (B) 0.03

(C) 0.02 (D) 0.01

Comprehension 3

[JEE (Advanced) 2011]

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ω_p, which is called the plasma frequency.

To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

5. Taking the electronic charge as e and the permittivity as ε_0, use dimensional analysis to determine the correct expression for ω_p

(A) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\frac{m\varepsilon_0}{Ne}}$

(C) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

6. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}$ m^{-3}. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units

(A) 800 nm (B) 600 nm

(C) 300 nm (D) 200 nm

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

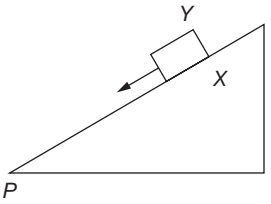
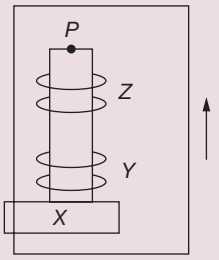
1. [IIT-JEE 2009]

COLUMN-II gives certain systems undergoing a process. **COLUMN-I** suggests changes in some of the parameters related to the system. Match the statements in **COLUMN-I** to the appropriate process(es) from **COLUMN-II**.

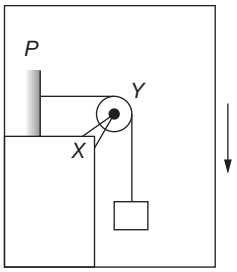
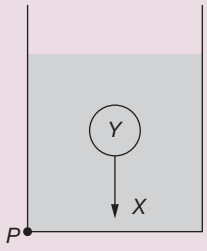
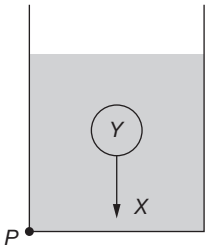
COLUMN-I	COLUMN-II
(A) The energy of the system is increased.	(p) System: A capacitor, initially uncharged. Process: It is connected to a battery.
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	(q) System: A gas in an adiabatic container fitted with an adiabatic piston. Process: The gas is compressed by pushing the piston.
(C) Internal energy of the system is converted into its mechanical energy.	(r) System: A gas in a rigid container. Process: The gas gets cooled due to colder atmosphere surrounding it.
(D) Mass of the system is decreased.	(s) System: A heavy nucleus, initially at rest. Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted.
	(t) System: A resistive wire loop. Process: The loop is placed in a time varying magnetic field perpendicular to its plane.

2. [IIT-JEE 2009]

COLUMN-II shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **COLUMN-I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) The force exerted by X on Y has a magnitude Mg.	(p) Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity. 
(B) The gravitational potential energy of X is continuously increasing.	(q) Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity. 

(*Continued*)

COLUMN-I	COLUMN-II
(C) Mechanical energy of the system $X + Y$ is continuously decreasing.	(r) A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity. 
(D) The torque of the weight of Y about point P is zero.	(s) A sphere Y of mass M is put in a non-viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid. 
	(t) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container. 

3. **[IIT-JEE 2007]**

Some physical quantities are given in **COLUMN-I** and some possible SI units in which these quantities may be expressed are given in **COLUMN-II**. Match the physical quantities in **COLUMN-I** with the units in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) GM_eM_s G – universal gravitational constant, M_e – mass of the earth, M_s – mass of the sun.	(p) (volt) (coulomb) (metre)
(B) $\frac{3RT}{M}$ R – universal gas constant, T – absolute temperature, M – molar mass.	(q) (kilogram) $(\text{metre})^3$ $(\text{second})^{-2}$
(C) $\frac{F^2}{q^2B^2}$ F – force, q – charge, B – magnetic field.	(r) $(\text{metre})^2$ $(\text{second})^{-2}$
(D) $\frac{GM_e}{R_e}$ G – universal gravitational constant, M_e – mass of the earth, R_e – radius of the earth.	(s) (farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. **[JEE (Advanced) 2019]**

An optical bench has 1.5 m long scale having four equal divisions in each cm . While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is …….

2. [JEE (Advanced) 2018]

A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} Nm^{-2} carries a load of mass m. The length of the wire with the load is 1 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is ……. (Take, $g = 10$ ms^{-2} and $\pi = 3.2$)

3. [JEE (Advanced) 2015]

The energy of a system as a function of time t is given as $E(t) = A^2\exp(-\alpha t)$, where $\alpha = 0.2$ s^{-1}. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is

4. [JEE (Advanced) 2014]

To find the distance d over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

5. [JEE (Advanced) 2014]

During Searle's experiment, zero of the vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20^{th} division of the vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45^{th} division of vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the vernier scale is 1×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Principle of Homogeneity: Verification)

1. 1 in mass, 0 in length, –2 in time
2. Yes
6. $M^0L^0T^0$
7. $m^{-3}L^{-2}T^4Q^4$
8. True
9. A : no units, B : m^2
10. No, $\tan\theta = \frac{v}{u}$

Test Your Concepts-II (Based on Principle of Homogeneity: Conversion)

1. $4.18\,\alpha^{-1}\beta^{-2}\gamma^2$ new units
2. 10^9 erg
3. 10 poise
4. 8 kg
5. 50 new units
6. 746 W
7. $p \propto \rho v^4 v^3$ and $\sigma \propto \frac{\rho v^3}{v}$
8. 500 new units of length
9. 10^4 kg
10. 1.4 kWm^{-2}

Test Your Concepts-III (Based on Principle of Homogeneity: Dependence)

1. $T = k\sqrt{\frac{r^3 d}{S}}$
2. $kp^{-\frac{5}{6}}d^{\frac{1}{2}}E^{\frac{1}{3}}$
3. $k\sqrt{\frac{m\ell}{f}}$
4. $k\left(\frac{W}{\eta r}\right)$
5. $k\left(\frac{r^4}{\eta}\right)p$
6. $k\left(\frac{\eta}{\rho d^2}\right)$
7. $k\left(\frac{S}{rgd}\right)$
9. $\rho g^4 v^6$
10. (a) $Kv^{-1}FT$
    (b) $KvFT$

Test Your Concepts-IV (Based on Errors, Significant Figures, Vernier Calliper and Screw Gauge)

1. $(4 \pm 0.65)\ \Omega$, $4\ \Omega \pm 16.25\%$
2. 8.1 $gcc^{-1} \pm 3.6\%$
3. $\frac{1}{10N}$ cm
4. 43.7 cm^3
5. 5, 3, 2, 3, 5, 2, 2
6. 7.36, 8.33, 9.44, 15.8, 7.37, 9.44, 15.8
7. (a) 11.4; (b) 2.4; (c) 44; (d) 108
8. (a) 0.01 cm; (b) 10.23 cm
9. 6.4 mm
10. Actual length will be 0.05 cm less than the measured length.

Single Correct Choice Type Questions

1. D	**2.** A	**3.** B	**4.** C	**5.** B	**6.** C	**7.** A	**8.** A	**9.** A	**10.** A
11. A	**12.** B	**13.** A	**14.** B	**15.** A	**16.** B	**17.** C	**18.** D	**19.** D	**20.** C
21. B	**22.** D	**23.** C	**24.** A	**25.** B	**26.** B	**27.** A	**28.** D	**29.** D	**30.** B
31. B	**32.** B	**33.** B	**34.** C	**35.** C	**36.** C	**37.** C	**38.** B	**39.** A	**40.** B
41. D	**42.** B	**43.** A	**44.** D	**45.** A	**46.** A	**47.** C	**48.** D	**49.** D	**50.** B
51. A	**52.** C	**53.** A	**54.** D	**55.** A	**56.** D	**57.** C	**58.** C	**59.** B	**60.** C

61. C	**62.** B	**63.** B	**64.** C	**65.** A	**66.** A	**67.** B	**68.** B	**69.** D	**70.** C
71. B	**72.** D	**73.** C	**74.** A	**75.** D	**76.** D	**77.** A	**78.** A	**79.** B	**80.** A
81. C	**82.** C	**83.** A	**84.** B	**85.** A	**86.** C	**87.** B	**88.** D	**89.** B	**90.** C
91. A	**92.** B	**93.** A	**94.** B	**95.** B	**96.** D	**97.** A	**98.** D	**99.** B	**100.** A
101. C	**102.** D	**103.** C	**104.** C	**105.** C	**106.** B	**107.** B	**108.** D	**109.** B	**110.** C
111. C	**112.** D	**113.** B	**114.** D	**115.** C	**116.** D	**117.** A	**118.** B	**119.** C	**120.** C
121. B	**122.** A	**123.** C	**124.** B	**125.** A	**126.** A	**127.** B	**128.** D	**129.** B	**130.** C
131. A	**132.** A	**133.** C	**134.** A	**135.** D	**136.** A	**137.** A	**138.** C	**139.** B	**140.** A
141. A	**142.** B	**143.** B	**144.** B	**145.** C	**146.** C	**147.** C	**148.** C	**149.** A	**150.** D
151. D	**152.** A	**153.** C	**154.** B	**155.** A	**156.** A	**157.** C	**158.** A	**159.** A	**160.** C
161. D									

Multiple Correct Choice Type Questions

1. A, C	**2.** A, B, C	**3.** A, B, C, D	**4.** A, B, D	**5.** C	**6.** A, B, C
7. A, D	**8.** A, B, C	**9.** B, C	**10.** A, B, C	**11.** A, B, D	**12.** A, B, C
13. B, C	**14.** A, B, C	**15.** A, B, D	**16.** B, C, D	**17.** A, B	**18.** A, B, D
19. A, B, C	**20.** B, C, D	**21.** A, B, C, D	**22.** A, B, C	**23.** A, B, D	**24.** B, C
25. A, C	**26.** A, B, C	**27.** B, C, D	**28.** B, C, D	**29.** A, B, D	**30.** A, B, C, D

Reasoning Based Questions

1. A	**2.** B	**3.** D	**4.** A	**5.** A	**6.** B	**7.** D	**8.** A	**9.** A	**10.** D
11. C	**12.** D	**13.** A	**14.** C	**15.** B	**16.** C	**17.** C	**18.** D	**19.** C	**20.** A
21. B	**22.** A	**23.** A	**24.** A	**25.** A					

Linked Comprehension Type Questions

1. A, C	**2.** B	**3.** C	**4.** B	**5.** A	**6.** B	**7.** C	**8.** C	**9.** D	**10.** A
11. A	**12.** B	**13.** C	**14.** B	**15.** D	**16.** C	**17.** C	**18.** A	**19.** A, D	**20.** A, B, C
21. A, C	**22.** B	**23.** C	**24.** A	**25.** A	**26.** B	**27.** D	**28.** A	**29.** C	**30.** B
31. B	**32.** D	**33.** A	**34.** A	**35.** B	**36.** C	**37.** D	**38.** A	**39.** C	**40.** B
41. D	**42.** C	**43.** C	**44.** A	**45.** B	**46.** B				

Matrix Match/Column Match Type Questions

1. A → (q)	B → (p)	C → (r)	D → (t)
2. A → (p, q)	B → (r, t)	C → (p, q)	D → (s)
3. A → (s)	B → (p)	C → (r)	D → (q)
4. A → (q, r)	B → (q)	C → (p, s, t)	D → (q)
5. A → (r, s)	B → (p, q)	C → (t)	D → (p, q)
6. A → (p, t)	B → (r)	C → (q)	D → (r)
7. A → (r)	B → (s)	C → (p, q, r)	D → (t)

8.	A → (p, q)	B → (r, t)	C → (p, q)	D → (s)
9.	A → (p, q)	B → (r, s)	C → (r, s)	D → (r, s)
10.	A → (r)	B → (p)	C → (p)	D → (q)
11.	A → (r)	B → (p, q)	C → (p)	D → (s)
12.	A → (q, s)	B → (r)	C → (p)	D → (r)
13.	A → (s)	B → (p)	C → (q)	D → (r)
14.	A → (q)	B → (p)	C → (s)	D → (r)
15.	A → (r)	B → (p)	C → (p)	D → (q)
16.	A → (q)	B → (r)	C → (s)	D → (p)
17.	A → (r)	B → (p, q)	C → (p)	D → (s)
18.	A → (q)	B → (p)	C → (s)	D → (r)
19.	A → (q)	B → (p)	C → (r)	D → (s)

Integer/Numerical Answer Type Questions

1. $a=0,\ b=0,\ c=0$	**2.** $x=2$	**3.** $x=4,\ y=1,\ z=1$
4. 8 kg, 10 m and 2 s	**5.** $x=1,\ y=1,\ z=2$	**6.** $a=2,\ b=3,\ c=1,\ d=1$

ARCHIVE: JEE MAIN

1. D	**2.** D	**3.** A	**4.** C	**5.** B	**6.** D	**7.** C	**8.** C	**9.** A	**10.** C
11. C	**12.** B	**13.** C	**14.** A	**15.** A	**16.** C	**17.** B	**18.** A	**19.** C	**20.** A
21. C	**22.** C	**23.** D	**24.** A	**25.** B	**26.** A, D	**27.** C	**28.** C	**29.** A	**30.** C
31. C	**32.** D	**33.** B							

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. A	**2.** A	**3.** C	**4.** B	**5.** C	**6.** C	**7.** D	**8.** B	**9.** B	**10.** A
11. A	**12.** D	**13.** D	**14.** D	**15.** A	**16.** A	**17.** D	**18.** B	**19.** B	**20.** D
21. B	**22.** B								

Multiple Correct Choice Type Problems

1. A, C, D	**2.** B, D	**3.** A, B, D	**4.** A, C, D	**5.** A, C
6. B, C	**7.** A, C	**8.** B, C	**9.** A, B, C, D	**10.** A, B, C
11. B, D	**12.** A, D	**13.** A, B, C		

Linked Comprehension Type Questions

1. C	**2.** D	**3.** B	**4.** C	**5.** C	**6.** B

Matrix Match/Column Match Type Questions

1. A → (p, q, s, t)	B → (q)	C → (s)	D → (s)
2. A → (p, t)	B → (q, s, t)	C → (p, r, t)	D → (q)
3. A → (p, q)	B → (r, s)	C → (r, s)	D → (r, s)

Integer/Numerical Answer Type Questions

1. 1.39	**2.** 3	**3.** ±4%	**4.** 3	**5.** 4

CHAPTER

3 Vectors

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

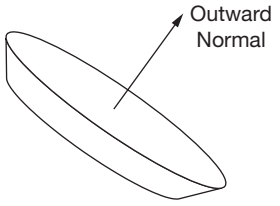
(a) Vector Fundamentals and Types
(b) Triangle, Parallelogram and Polygon Law of Vector Addition
(c) Position Vector and Vector Subtraction
(d) Vector Resolution
(e) Dot Product and Cross Product
(f) Scalar Triple Product and Vector Triple Product

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION

Physical quantities having magnitude, direction and obeying laws of vector algebra are called **vectors**. Magnitude of a vector is also called its **modulus**.

EXAMPLE:
Displacement, velocity, acceleration, momentum, force, impulse, weight, thrust, torque, angular momentum, angular velocity.



The physical quantity 'AREA' is a vector with its direction along the outward normal to the surface.

Conceptual Note(s)

(a) "If you love hot pizzas, then it does not imply that everything hot is a pizza".
(b) Similarly if a physical quantity has magnitude and direction both, then it does not always imply that it is a vector. For it to be a vector the third condition of obeying laws of vector algebra has to be satisfied.

Example: The physical quantity CURRENT has both magnitude and direction but is still a scalar as it disobeys the laws of vector algebra.

ILLUSTRATION 1

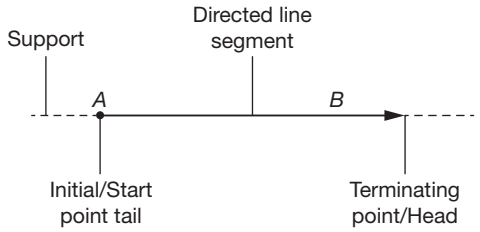
We can order events in time, and there is a sense of time, distinguishing past, present and future. Is therefore time a vector?

SOLUTION

Time always flow from past to present and then to future so a direction can be assigned to time. However as its direction is unique it is not to be specified. As direction is not to be specified so time cannot be a vector though it has direction. Also it does not obey laws of vector algebra.

GEOMETRICAL DEFINITION

A directed line segment is called a **vector,** denoted as $\overrightarrow{AB}$ (read as AB vector). In printing we can denote $\overrightarrow{AB}$ as $\boldsymbol{AB}$ (bold letters). Even if you have to write BA vector the arrow head above would always point from left to right i.e. $\overrightarrow{BA}$.



But mathematically $\overrightarrow{AB} = -\overrightarrow{BA}$

Also, let $\vec{A}$ and $\vec{B}$ be two vectors,

then $|\vec{A}| = A =$ magnitude of $\vec{A} =$ length of $\vec{A}$

$|\vec{B}| = B =$ magnitude of $\vec{B}$ = length of $\vec{B}$

Also note that if we have to compare vectors then we only compare their magnitudes. So,

$|\vec{A}| > |\vec{B}|$ and not $\vec{A} > \vec{B}$

Conceptual Note(s)

Every vector $(\overrightarrow{AB})$ has the following three characteristics

(a) Length: The length of $\overrightarrow{AB}$ is denoted by $|\overrightarrow{AB}|$ or simply AB.

(b) Support: The line of unlimited length of which AB is segment is called the support of vector $\overrightarrow{AB}$.

(c) Sense: The sense of $\overrightarrow{AB}$ is from $A \to B$ and that of $\overrightarrow{BA}$ will be from $B \to A$. The sense of directed line segment is from its initial point to the terminal point.

TYPES OF VECTOR

Equal Vectors

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction. Geometrically if head of one vector coincides will the head of other and so do the tails coincide then the vectors are said to be equal.

If $\vec{A} = \vec{B}$, then $A = B$, always.

But if, $A = B$ doesn't always imply $\vec{A} = \vec{B}$.

Problem Solving Technique(s)

A vector can always be transported parallel to its original direction.

Parallel Vector

Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when

(a) both have same direction.
(b) magnitude of one is scalar multiple of magnitude of the other

EXAMPLE:

$\vec{B} = 2\vec{A}$, i.e., Magnitude of $\vec{B}$ is twice the magnitude of $\vec{A}$ and both have same direction.

In general if $\vec{A} = k\vec{B}$ (where k is any constant) i.e. magnitude of $\vec{A}$ is k times that of $\vec{B}$, then both are **parallel** if $k > 0$.

Anti-parallel Vectors

Two vectors $\vec{A}$ and $\vec{B}$ are said to be antiparallel when

(a) both have opposite direction.
(b) magnitude of one is a scalar multiple of the magnitude of the other.

EXAMPLE:

$\vec{B} = -2\vec{A}$ Magnitude of $\vec{B}$ is twice the magnitude of $\vec{A}$ and both have opposite direction.

In general if $\vec{A} = k\vec{B}$ (where k is any constant) i.e. magnitude of $\vec{A}$ is k times that of $\vec{B}$, then both are **antiparallel** if $k > 0$.

Collinear Vectors

When the vectors under consideration can share the same support or have a common support then the considered vectors are collinear.

$\vec{A}$ ⟶

$\vec{B} = 2\vec{A}$ ⟶

$\vec{C} = -3\vec{A}$ ⟵

Common support shared by $\vec{A}$, $\vec{B}$ and $\vec{C}$ ----------

Zero Vector $(\vec{0})$

A vector having zero magnitude and arbitrary direction (not known to us) is a **zero vector**.

Properties of $\vec{0}$

(a) $\vec{A}+(-\vec{A})=\vec{0}$

(b) If $\vec{A}+\vec{B}=0 \Rightarrow \vec{A}=-\vec{B}$

(c) $k\,\vec{0}=\vec{0}$ [read as k times $\vec{0}$ equals $\vec{0}$]
{where, k is any scalar}

(d) $0\,\vec{A}=\vec{0}$ [read as zero times $\vec{A}$ equals zero vector]

(e) $\vec{A}+\vec{0}=\vec{0}+\vec{A}=\vec{A}$

Unit Vector

A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap/A carat/A hat). Unit vector for $\vec{B}$ is $\hat{B}$.

$$|\hat{A}|=1,\ |\hat{B}|=1,\ |\hat{C}|=1,\ |\hat{x}|=1,\ |\hat{y}|=1$$

Since, $\hat{A}=\dfrac{\vec{A}}{A}$

$\Rightarrow \vec{A}=A\hat{A}$

$\vec{B} = B\,\hat{B}$

Vector has two essential ingredients: Magnitude, Direction

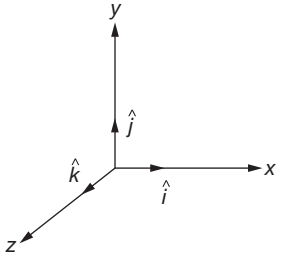
Thus, we conclude that unit vector gives us the direction.

Conceptual Note(s)

No units are to be attached (like newton (N), $msec^{-1}$, metre etc.) with a unit vector i.e., unit vector is dimensionless physical quantity.

Orthogonal Unit Vectors/Base Vectors

$\hat{i}$, $\hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a **Right Handed Triad**. It is a coordinate system such that when we Curl the fingers of right hand from x to y, then we must get the direction of z along thumb



from y to z, then we must get the direction of x along thumb
from z to x, then we must get the direction of y along thumb

$$\hat{i}=\frac{\vec{x}}{x}$$

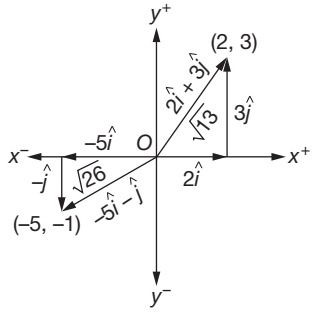
$$\Rightarrow \vec{x}=x\hat{i}$$

$$\hat{j}=\frac{\vec{y}}{y}$$

$$\Rightarrow \vec{y}=y\hat{j}$$

$$\hat{k}=\frac{\vec{z}}{z}$$

$$\Rightarrow \vec{z}=z\hat{k}$$



Graphical representation of vectors

EXAMPLE:

(a) A vector of 3 unit along x axis is $\vec{x}=3\hat{i}$.

(b) A vector of magnitude 6 along $-x$ axis is $\vec{x}=6(-\hat{i})=-6\hat{i}$.

(c) A vector of magnitude 5 along $-z$ axis is $\vec{x}=5(-\hat{k})=-5\hat{k}$.

Also, $(0,0)\equiv 0\hat{i}+0\hat{j}\equiv\vec{0}$, $(2,3)\equiv 2\hat{i}+3\hat{j}$

$(-5,-1)\equiv -5\hat{i}-\hat{j}$.

Fixed Vector

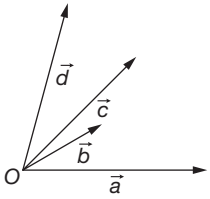
Fixed vector is that vector whose initial point or tail is fixed. Its also called **localised vector**.

EXAMPLE:
(a) Position vector
(b) Displacement vector

Free Vector

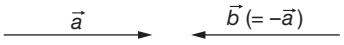
Free vector is that vector whose initial point or tail is not fixed. Its also known as **non localised vector**.

EXAMPLE:
Velocity vector of a particular moving along a straight line is a free vector.



Negative Vector

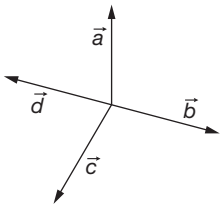
A vector is said to be negative of a given vector if its magnitude is the same as that of the given vector but direction is reversed. e.g. The negative vector of a vector $\vec{a}$ is denoted by $-\vec{a}$.



If $\vec{b}$ is negative of $\vec{a}$, then $\vec{b} = -\vec{a}$.

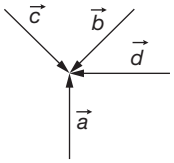
Co-initial Vectors

Co-initial vectors are those vectors which have the same initial point.



Co-terminus Vectors

Co-terminus vectors are those vectors which have common terminating point.



Coplanar Vector

Vectors are said to be coplanar if they occur in same or common plane. e.g. $\vec{A}$ and $\vec{B}$ and $\vec{C}$. Remember that any two vectors always lie in the same plane.

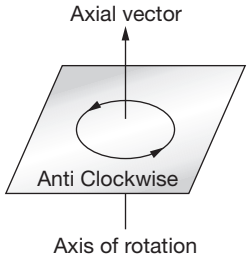
Polar Vector

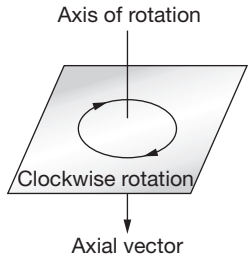
Vectors producing straight line linear effect are called **polar vectors** e.g. force, momentum, velocity, displacement.

Axial Vectors/Pseudo Vectors: (Virtual, Imaginary)

The rotational effect of a polar vector gives rise to a new vector called **axial vector** (acting along axis of rotation).

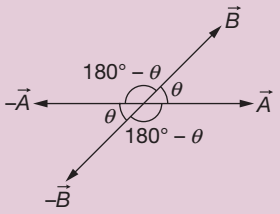
EXAMPLE:
Rotational effect of force (polar vector) is torque (an axial vector).
Rotational effect of linear momentum (a polar vector) is angular momentum (an axial vector).



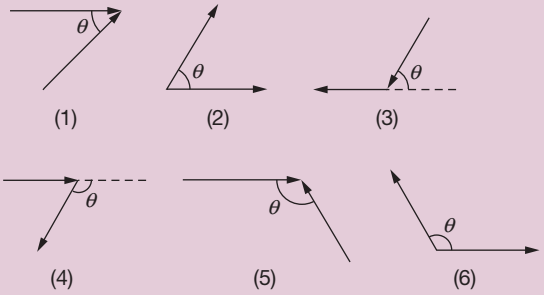


Conceptual Note(s)

(a) Whenever angle between two vectors is to be taken we must make sure that either their heads coincide or their tails coincide.



i.e., if two vectors have their heads coinciding or their tails coinciding, then internal angle is the angle between two vectors (whether acute or obtuse), as in (1), (2), (5) & (6).
If head coincides with tail then external supplementary angle (whether acute or obtuse) is the angle between the two vectors, as in (3) & (4).



(b) If angle between $\vec{A}$ and $\vec{B}$ is θ, then

(i) angle between $-\vec{A}$ and $\vec{B}$ is $(180° - \theta)$

(ii) angle between $\vec{A}$ and $-\vec{B}$ is $(180° - \theta)$

(iii) angle between $-\vec{A}$ and $-\vec{B}$ is θ

ILLUSTRATION 2

Let $\vec{A} = 2\hat{i} + \hat{j}$, $\vec{B} = 3\hat{j} - \hat{k}$ and $\vec{C} = 6\hat{i} - 2\hat{k}$, then find $\vec{A} - 2\vec{B} + 3\vec{C}$. Also calculate its magnitude.

SOLUTION

$$\vec{A} - 2\vec{B} + 3\vec{C} = (2\hat{i} + \hat{j}) - 2(3\hat{j} - \hat{k}) + 3(6\hat{i} - 2\hat{k})$$

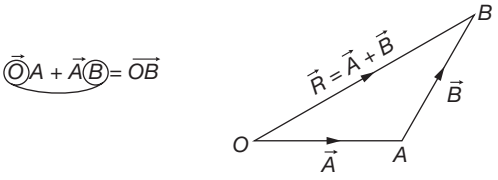
$$\vec{A} - 2\vec{B} + 3\vec{C} = 2\hat{i} + \hat{j} - 6\hat{j} + 2\hat{k} + 18\hat{i} - 6\hat{k}$$

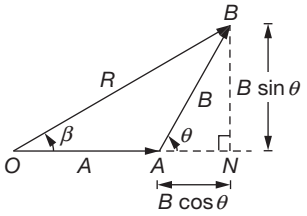
$$|\vec{A} - 2\vec{B} + 3\vec{C}| = \sqrt{(20)^2 + (-5)^2 + (-4)^2} = \sqrt{441} = 21$$

TRIANGLE LAW OF VECTOR ADDITION OF TWO VECTORS

If two non zero vectors are represented by the two sides of a triangle taken in same order then the resultant is given by the closing side of triangle in opposite order. i.e.,

$$\vec{R} = \vec{A} + \vec{B}$$





Magnitude of Resultant Vector

In ΔABN,

$$\cos\theta = \frac{AN}{B} \quad \Rightarrow \quad AN = B\cos\theta \qquad \left| \qquad \sin\theta = \frac{BN}{B} \quad \Rightarrow \quad BN = B\sin\theta \right.$$

In ΔOBN, we have $OB^2 = ON^2 + BN^2$

$$\Rightarrow \quad R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$\Rightarrow \quad R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow \quad R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$\Rightarrow \quad R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of Resultant Vector

If θ is angle between $\vec{A}$ and $\vec{B}$, then

$$|\vec{B} + \vec{A}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}\,.$$

If the resultant, $\vec{R}$ makes an angle β with $\vec{A}$, then in ΔOBN, then

$$\tan\beta = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Conceptual Note(s)

Resultant $\vec{R}$ of two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ is

$$R = |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and β is the angle made by the resultant $\vec{R}$ with $\vec{A}$, then $\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$

ILLUSTRATION 3

Can the magnitude of the resultant of two equal vectors be equal to the magnitude of either of the vectors? Explain your answer.

SOLUTION

Let the two vectors be $\vec{A}$ and $\vec{B}$ inclined at an angle θ, then

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

According to the question, we have been given

$$R = A = B$$

$$\Rightarrow \quad A^2 = A^2 + A^2 + 2A^2\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120^\circ$$

So, this is possible, when two vectors of equal magnitude are inclined to each other at an angle of 120°.

ILLUSTRATION 4

The sum of magnitude of two forces (in newton) acting at a point is 18 and the magnitude of their resultant (in newton) is 12. If resultant is at 90° with force of smaller magnitude, what are magnitudes of forces?

SOLUTION

METHOD I

Let F_1 and F_2 be the two forces. Let one of them, F_1 (say) is of smaller magnitude.

Now $F_1 + F_2 = 18$ …(1)

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} = 12 \quad \ldots(2)$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \to \infty \qquad \{\because \alpha = 90^\circ\}$$

$$\Rightarrow \quad F_1 + F_2\cos\theta = 0 \quad \ldots(3)$$

$$\Rightarrow \quad (18 - F_2) + F_2\cos\theta = 0$$

$$\Rightarrow \quad F_2(1 - \cos\theta) = 18 \quad \ldots(4)$$

Subtracting equation (2) from equation (1), we get

$$2F_1F_2(1 - \cos\theta) = (18)^2 - (12)^2 = 180 \quad \ldots(5)$$

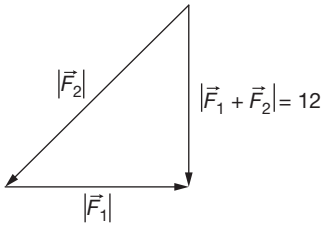
Dividing (5) by (4), we have

$$2F_1 = 10$$

$$\Rightarrow \quad F_1 = 5\text{ N} \text{ and } F_2 = 13\text{ N}$$

METHOD II

Since, $(\vec{F}_1 + \vec{F}_2)$ is resultant of $\vec{F}_1$ and $\vec{F}_2$, so from Triangle Law, we have drawn the diagram from which we get



$$F_2^2 = F_1^2 + (12)^2$$

$$\Rightarrow \quad F_2^2 - F_1^2 = 144$$

$$\Rightarrow \quad (F_1 + F_2)(F_2 - F_1) = 144 \quad \ldots(1)$$

Since, $F_1 + F_2 = 18$ …(2)

So, (1) gives $F_2 - F_1 = 8$ …(3)

Solving (2) and (3), we get

$$F_1 = 5\text{ N} \text{ and } F_2 = 13\text{ N}$$

ILLUSTRATION 5

The x and y components of vector $\vec{A}$ are 4 m and 6 m respectively. The x and y components of vector $\vec{A}+\vec{B}$ are 10 m and 9 m respectively. Calculate the

(a) x and y components of vector $\vec{B}$.

(b) length of vector $\vec{B}$.

(c) angle which $\vec{B}$ makes with x-axis.

SOLUTION

(a) $A_x = 4$ and $A_y = 6$ …(1)

$$\vec{A}+\vec{B} = \left(A_x\hat{i}+A_y\hat{j}\right)+\left(B_x\hat{i}+B_y\hat{j}\right)$$

$$\Rightarrow \quad \vec{A}+\vec{B} = \left(A_x+B_x\right)\hat{i}+\left(A_y+B_y\right)\hat{j}$$

So, $A_x+B_x = 10$ and $A_y+B_y = 9$ …(2)

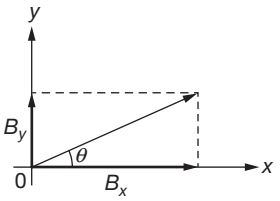
From equation sets (1) and (2), we get

$B_x = 6$ and $B_y = 3$

So, $\vec{B} = 6\hat{i}+3\hat{j}$

(b) $|\vec{B}| = \sqrt{6^2+3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$ m

(c) Let $\vec{B}$ make an angle θ with x-axis, then from the diagram



$$\tan\theta = \frac{B_y}{B_x} = \frac{3}{6} = \frac{1}{2}$$

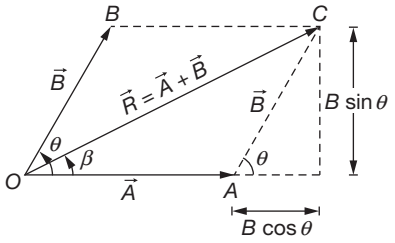
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right), \text{ with } x\text{-axis}$$

PARALLELOGRAM LAW OF VECTOR ADDITION OF TWO VECTORS

If two non zero vector are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

Magnitude of Resultant Vector

Since, $R^2 = ON^2 + CN^2$



$$\Rightarrow \quad R^2 = (OA+AN)^2 + CN^2$$

$$\Rightarrow \quad R^2 = A^2+B^2+2AB\cos\theta$$

$$\Rightarrow \quad \boxed{R = |\vec{R}| = |\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}}$$

Direction of Resultant Vector

$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A+B\cos\theta}$$

Conceptual Note(s)

Triangle Law and Parallelogram Law, both are equivalent. Since we know that a vector can be transported parallel to its original direction, so their equivalency holds good and is shown below.

TRIANGLE LAW PARALLELOGRAM LAW

ILLUSTRATION 6

If $\vec{A} = 4\hat{i}-3\hat{j}$ and $\vec{B} = 6\hat{i}+8\hat{j}$ then find the magnitude and direction of $\vec{A}+\vec{B}$.

SOLUTION

$$\vec{A}+\vec{B} = 4\hat{i}-3\hat{j}+6\hat{i}+8\hat{j} = 10\hat{i}+5\hat{j}$$

$$|\vec{A}+\vec{B}| = \sqrt{(10)^2+(5)^2} = 5\sqrt{5}$$

$$\tan\theta = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

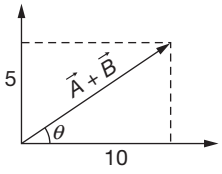
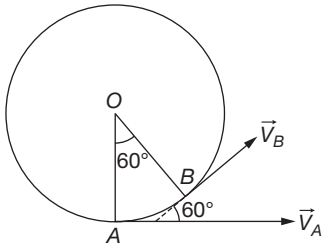


ILLUSTRATION 7

In figure, a particle is moving in a circle of radius r centred at O with constant speed v. What is the change in velocity in moving from A to B? Given $\angle AOB = 60°$.



SOLUTION

Change in velocity $= \Delta\vec{v} = \vec{v}_B - \vec{v}_A$

Since $|\vec{v}_B - \vec{v}_A| = \sqrt{v_B^2 + v_A^2 - 2v_A v_B \cos\theta}$

$$\Rightarrow \quad |\vec{v}_B - \vec{v}_A| = \sqrt{v^2 + v^2 - 2v^2\cos(60°)}$$

$$\Rightarrow \quad |\vec{v}_B - \vec{v}_A| = \sqrt{v^2 + v^2 - 2v^2\left(\frac{1}{2}\right)} = v$$

ILLUSTRATION 8

If the sum of two unit vectors is a unit vector, then find the magnitude of difference of these two vectors.

SOLUTION

Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum and difference of these two vectors be represented by

$$\vec{n}_s = \hat{n}_1 + \hat{n}_2 \text{ and } \vec{n}_d = \hat{n}_1 - \hat{n}_2$$

$$\Rightarrow \quad n_s^2 = n_1^2 + n_2^2 + 2n_1 n_2\cos\theta = 1+1+2\cos\theta$$

Since it is given that n_s is also a unit vector, therefore we have

$$1 = 1+1+2\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

The difference of the two vectors is given by

$$\vec{n}_d = \hat{n}_1 - \hat{n}_2$$

$$\Rightarrow \quad n_d^2 = n_1^2 + n_2^2 - 2n_1 n_2\cos\theta = 1+1-2\cos(120°)$$

$$\Rightarrow \quad n_d^2 = 2-2\left(-\frac{1}{2}\right) = 2+1 = 3$$

$$\Rightarrow \quad n_d = \sqrt{3}$$

ILLUSTRATION 9

Consider two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ. If $|\vec{A}| = |\vec{B}| = R$, then prove that

(a) $|\vec{A}+\vec{B}| = 2R\cos\left(\frac{\theta}{2}\right)$

(b) $|\vec{A}-\vec{B}| = 2R\sin\left(\frac{\theta}{2}\right)$

SOLUTION

(a) Since $|\vec{A}| = |\vec{B}| = 1$

$$\Rightarrow \quad |\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow \quad |\vec{A}+\vec{B}| = \sqrt{R^2 + R^2 + 2R^2\cos\theta} = \sqrt{2}R\sqrt{1+\cos\theta}$$

Since $1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad |\vec{A}+\vec{B}| = 2R\cos\left(\frac{\theta}{2}\right)$$

(b) Similarly,

$$|\vec{A}-\vec{B}| = \sqrt{R^2 + R^2 - 2R^2\cos\theta} = \sqrt{2}R\sqrt{1-\cos\theta}$$

Since $1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad |\vec{A}-\vec{B}| = 2R\sin\left(\frac{\theta}{2}\right)$$

TRIANGLE INEQUALITY

Since a vector cannot have a resultant more than the maximum value and less than the least value of the resultant, so we have

$$R_{\min} \le R \le R_{\max}$$

$$\Rightarrow \quad A - B \le R \le A + B$$

If $A > B$, then $A - B \le |\vec{A} + \vec{B}| \le A + B$

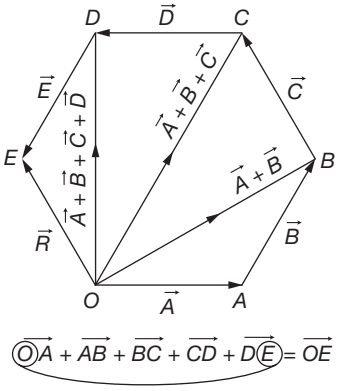
$$|\vec{A}| - |\vec{B}| \le |\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$$

$$|\vec{A}| - |\vec{B}| \le |\vec{A} + \vec{B}| \ \& \ |\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$$

{called Triangle Inequality}

POLYGON LAW OF VECTOR ADDITION

If a number of non zero vectors are represented by the $(n-1)$ sides of an n-sided polygon then the resultant



is given by the closing side or the n^{th} side of the polygon taken in opposite order. So,

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

PROPERTIES OF VECTOR ADDITION

(a) Vector Addition is Commutative

For any two vectors $\vec{a}$ and $\vec{b}$, we have

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

(b) Vector Addition is Associative

For any three vector $\vec{a}$, $\vec{b}$ and $\vec{c}$, we have

$$\vec{b} + (\vec{c} + \vec{a}) = \vec{c} + (\vec{a} + \vec{b}) = \vec{a} + (\vec{b} + \vec{c})$$

(c) Existence of Additive Identify

For every vector $\vec{a}$, we have

$$\vec{a} + \vec{0} = \vec{0} + \vec{a}$$

i.e., Adding $\vec{0}$ i.e., null vector to any vector (say $\vec{a}$) does not changes the magnitude of $\vec{a}$ as well as its direction, because $\vec{0}$ has zero magnitude and has an arbitrary/indeterminate direction.

(d) Existence of Additive Inverse

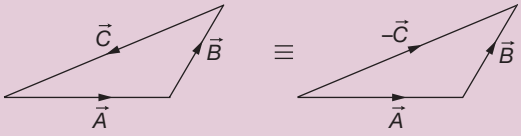
For a given vector $\vec{a}$, there exists a vector $(-\vec{a})$ such that

$$\vec{a} + (-\vec{a}) = \vec{0}$$

The vector $(-\vec{a})$ is called the additive inverse of $\vec{a}$ and vice versa.

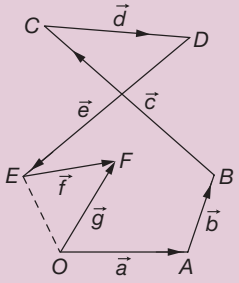
Problem Solving Technique(s)

(a) If the vectors form a closed *n* sided polygon with all the sides in the same order then the resultant is $\vec{0}$.



Addition of Vectors

(b) To find the sum of any number of vectors we must represent the vectors by the directed line segments with the terminal point of the previous vectors as the initial point of the next vector, then the lines segment joining the initial point of the first vector to the terminal



point of the last vector will represent the sum of the vectors. Such that

(c) If the terminal of the last vector coincides with the initial point of the first vector, then its sum will be zero.

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} + \vec{g} = \vec{0}$$

MULTIPLICATION OF A VECTOR BY A SCALAR

If m by any scalar and $\vec{a}$ be any vector then $m\vec{a}$ is defined as a vector whose magnitude is $|m|$ times that of the magnitude of $\vec{a}$ and whose direction is same as of $\vec{a}$ if m is positive and will be opposite to that of $\vec{a}$ if m is negative.

PROPERTIES OF MULTIPLICATION OF VECTOR BY A SCALAR

(a) If $m = 0$ (ZERO), then $m\vec{a} = 0$ and not $\vec{0}$.

(b) If m and n are two scalars and $\vec{a}$ be any vector in space, then

$$m(n\vec{a}) = n(m\vec{a}) = mn(\vec{a})$$

(c) Vector multiplication is distributive over scalar addition

$$\vec{a}(m+n) = m\vec{a} + n\vec{a}$$

where, m, n are any two scalars and $\vec{a}$ be any vector in space.

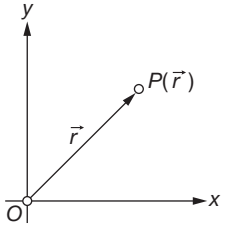
(d) Scalar multiplication is distributive over vector addition. Let $\vec{a}$ and $\vec{b}$ are any two vectors and m be any scalar, then

$$m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$$

POSITION VECTOR

If a point O is fixed as the origin is space (or plane) and P is any point, then $\overrightarrow{OP}$ is called the position vector of P with respect to O. So, position vector of a point P, from a convenient fixed origin O is a vector drawn from O to P.

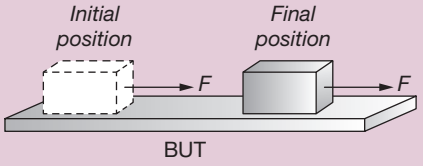
If it is said that P is any point in space, then it means that $\overrightarrow{OP}$ is a position vector or we can say that position vector of point P is $\vec{r}$ (say) with respect to some conveniently fixed origin O.



Conceptual Note(s)

A vector is an independent entity and can be transported any where keeping its Line of Action (LOA) parallel to the same straight line, magnitude and sense unchanged. i.e., A vector $\vec{a}$ will remain $\vec{a}$ till its magnitude and direction is retained.

EXAMPLE: In many examples, when a force F is applied on the block such that the block displaces from its original position, the force F applied on block remains same even after displacement w.r.t. the same frame of reference.

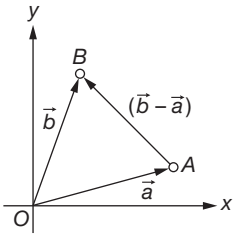


A position vector is a fixed quantity with its initial point as origin.

TO FIND $\overrightarrow{AB}$ IF THE POSITION VECTORS OF POINTS A AND B ARE KNOWN

Let $\vec{a}$ and $\vec{b}$ are position vectors of points A and B in plane (or space)

$\therefore \quad \overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$



Now, in ΔOAB, according to Triangular Law of Vector addition, we have

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\Rightarrow \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

(1) (2)

$$\Rightarrow \quad \overrightarrow{AB} = \begin{pmatrix}\text{Position Vector}\\ \text{of } B\end{pmatrix} - \begin{pmatrix}\text{Position Vector}\\ \text{of } A\end{pmatrix}$$

SUBTRACTION OF VECTORS

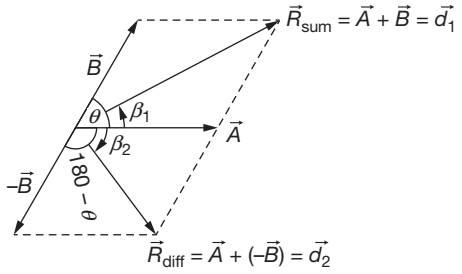
Since, $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

So, from above we conclude that subtraction of vectors is actually finding the resultant of $\vec{A}$ and $-\vec{B}$ OR $\vec{B}$ and $-\vec{A}$.

Also, $|\vec{A}+\vec{B}|=\sqrt{A^2+B^2+2AB\cos\theta}$ and

$$\tan\beta_1=\frac{B\sin\theta}{A+B\cos\theta}$$

Since angle between $\vec{A}$ and $\vec{B}$ is θ, so angle between, $\vec{A}$ and $-\vec{B}$ is $(180-\theta)$.



$$\Rightarrow\quad |\vec{A}-\vec{B}|=\sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

Since, $\cos(180-\theta)=-\cos\theta$

$$\Rightarrow\quad |\vec{A}-\vec{B}|=\sqrt{A^2+B^2-2AB\cos\theta}$$

$$\tan\beta_1=\frac{B\sin\theta}{A+B\cos\theta}$$

and $\tan\beta_2=\dfrac{B\sin(180-\theta)}{A+B\cos(180-\theta)}$

But $\sin(180-\theta)=\sin\theta$ and $\cos(180-\theta)=-\cos\theta$

$$\Rightarrow\quad \tan\beta_2=\frac{B\sin\theta}{A-B\cos\theta}$$

Significance of Vector Addition and Subtraction

If two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ then

$\vec{R}_{sum}=\vec{A}+\vec{B}=\vec{d_1}=$ Bigger Diagonal of parallelogram

$\vec{R}_{diff}=\vec{A}-\vec{B}=\vec{d_2}=$ Smaller Diagonal of parallelogram

So, $\vec{d_1}=\vec{A}+\vec{B}$ ...(1)

$\vec{d_2}=\vec{A}-\vec{B}$ ...(2)

Adding (1) & (2)

$$\vec{A}=\frac{\vec{d_1}+\vec{d_2}}{2}=\frac{1}{2}\left(\vec{d_1}+\vec{d_2}\right)$$

Subtracting (1) & (2)

$$\vec{B}=\frac{\vec{d_1}-\vec{d_2}}{2}=\frac{1}{2}\left(\vec{d_1}-\vec{d_2}\right)$$

RECTANGULAR COMPONENTS IN 2-D SPACE

Consider a vector $\vec{R}$ inclined to the x-axis at angle θ. If $\vec{R}_x$ and $\vec{R}_y$ are the components (projections) of $\vec{R}$ along x-axis and y-axis then by parallelogram law of vectors

$$\vec{R}=\vec{R}_x+\vec{R}_y \qquad \ldots(1)$$

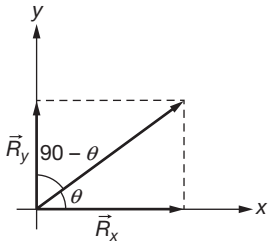
Since $\vec{R}_x=R_x\hat{i}$ and $\vec{R}_y=R_y\hat{j}$

$$\Rightarrow\quad \vec{R}=R_x\hat{i}+R_y\hat{j} \qquad \ldots(2)$$

Further

$$R_x=R\cos\theta \qquad \ldots(3)$$

$$R_y=R\sin\theta \qquad \ldots(4)$$



Since, $\vec{R}=R_x\hat{i}+R_y\hat{j}$

$$\Rightarrow\quad \vec{R}=R\left(\cos\theta\hat{i}+\sin\theta\hat{j}\right)$$

$$\Rightarrow\quad R\hat{R}=R\left(\cos\theta\hat{i}+\sin\theta\hat{j}\right) \qquad \left\{\because \vec{R}=R\hat{R}\right\}$$

$$\Rightarrow\quad \hat{R}=\cos\theta\hat{i}+\sin\theta\hat{j} \qquad \left\{|\hat{R}|=1\right\}$$

Squaring (3) and (4) and adding, we get

$$R_x^2+R_y^2=R^2\cos^2\theta+R^2\sin^2\theta$$

$$\Rightarrow \quad R_x^2 + R_y^2 = R^2$$

$$\Rightarrow \quad R = \sqrt{R_x^2 + R_y^2}$$

So, $\cos\theta = \dfrac{R_x}{R}$ {using (3)}

$$\Rightarrow \quad \cos\left(\begin{array}{c}\text{Angle which}\\ \vec{R}\text{ makes}\\ \text{with } x\text{-axis}\end{array}\right) = \frac{R_x}{R} = \frac{\left(\begin{array}{c}\text{Component}\\ \text{of vector}\\ \text{along } x\text{-axis}\end{array}\right)}{\left(\begin{array}{c}\text{Magnitude}\\ \text{of vector}\end{array}\right)}$$

Similarly $\sin\theta = \dfrac{R_y}{R}$ {using (4)}

$$\Rightarrow \quad \cos(90-\theta) = \frac{R_y}{R}$$

$$\Rightarrow \quad \cos\left(\begin{array}{c}\text{Angle which}\\ \vec{R}\text{ makes}\\ \text{with } x\text{-axis}\end{array}\right) = \frac{R_y}{R} = \frac{\left(\begin{array}{c}\text{Component}\\ \text{of vector}\\ \text{along } y\text{-axis}\end{array}\right)}{\left(\begin{array}{c}\text{Magnitude}\\ \text{of vector}\end{array}\right)}$$

If $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

Then $\left.\begin{aligned} R_x &= A_x + B_x + C_x \\ R_y &= A_y + B_y + C_y \end{aligned}\right\}$ in 2 D space

$$\tan\beta = \frac{R_y}{R_x} = \frac{A_y + B_y + C_y}{A_x + B_x + C_x}$$

where β is angle $\vec{R}$ makes with x-axis

Also, $\left.\begin{aligned} R_x &= A_x + B_x + C_x \\ R_y &= A_y + B_y + C_y \\ R_z &= A_z + B_z + C_z \end{aligned}\right\}$ 3D space

Problem Solving Technique(s)

A vector inclined at an angle θ to x-axis in 1st and 4th Quadrant OR at an angle θ with $-x$ axis in 2nd and 3rd Quadrant is

$x\ominus$ $y\oplus$	$x\oplus$ $y\oplus$
$x\ominus$ $y\ominus$	$x\oplus$ $y\ominus$

$$\vec{r}_{\text{1st quad}} = \vec{r}_1 = r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

$$\vec{r}_{\text{2nd quad}} = \vec{r}_2 = r\left(-\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

$$\vec{r}_{\text{3rd quad}} = \vec{r}_3 = -r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

$$\vec{r}_{\text{4th quad}} = \vec{r}_4 = r\left(\cos\theta\hat{i} - \sin\theta\hat{j}\right)$$

ILLUSTRATION 10

Construct a vector of magnitude 6 unit making an angle of 30° with y-axis.

SOLUTION

30° with y-axis $\Rightarrow$ 60° with x-axis.

Since, $\vec{R} = R\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

where, θ is the angle which $\vec{R}$ makes with x-axis

$$\Rightarrow \quad \vec{R} = 6\left(\cos 60\hat{i} + \sin 60\hat{j}\right)$$

$$\Rightarrow \quad \vec{R} = 3\left(1\hat{i} + \sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \vec{R} = 3\hat{i} + 3\sqrt{3}\hat{j}$$

ILLUSTRATION 11

A particle undergoes three successive displacements in a plane. The first time it moves 4 m south-west the second time 5 m east and the third time 6 m in direction 60° north of east. Draw a vector diagram and determine total displacement of particle from starting point.

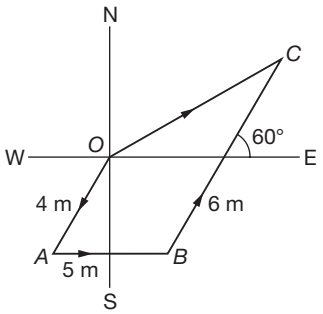
SOLUTION

$$\overrightarrow{OA} = 4\cos(180° + 45°)\hat{i} + 4\sin(180° + 45°)\hat{j}$$

$\overrightarrow{OA}$ makes $(180° + 45°)$ angle with positive x-axis and the same with positive y-axis so

x-component of $\overrightarrow{OA} = 4\cos(180° + 45°)$

y-component of $\overrightarrow{OA} = 4\sin(180° + 45°)$



$$\Rightarrow \quad \overrightarrow{OA} = -2\sqrt{2}\hat{i} - 2\sqrt{2}\hat{j}$$

$$\overrightarrow{AB} = 5\cos 0°\hat{i} = 5\hat{i}$$

$$\overrightarrow{BC} = 6\cos 60°\hat{i} + 6\sin 60°\hat{j} = 3\hat{i} + 3\sqrt{3}\hat{j}$$

$$\Rightarrow \quad \overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC}$$

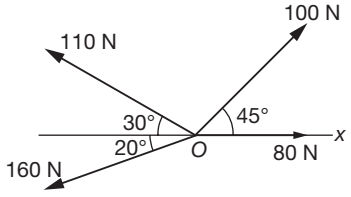
$$\Rightarrow \quad \overrightarrow{OC} = \left(-2\sqrt{2}\hat{i} - 2\sqrt{2}\hat{j}\right) + 5\hat{i} + \left(3\hat{i} + 3\sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \overrightarrow{OC} = \left(-2\sqrt{2} + 5 + 3\right)\hat{i} + \left(-2\sqrt{2} + 3\sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \overrightarrow{OC} = +5{\cdot}17\hat{i} + 2.37\hat{j}$$

ILLUSTRATION 12

Four coplanar forces act on a body at point O as shown in diagram by use of rectangular component find direction and magnitude of resultant force.



SOLUTION

The vectors and their components are as follows

Magnitude of resultant vector	*x* component of resultant vector	*y* component of resultant vector
80	80	0
100	$110\cos 45° = 71$	$100\sin 45° = 71$
110	$-110\cos 30° = -95$	$110\sin 30° = 55$
160	$-160\cos 20° = -150$	$-160\sin 20° = -55$

$$R_x = 81 + 71 - 95 - 150 = -94 \text{ N}$$

$$R_y = 0 + 71 + 55 - 55 = 71 \text{ N}$$

Magnitude of resultant is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(94)^2 + (71)^2} = 118 \text{ N}$$

$$\tan\alpha = \frac{71}{94}$$

$$\Rightarrow \quad \alpha = 37°$$

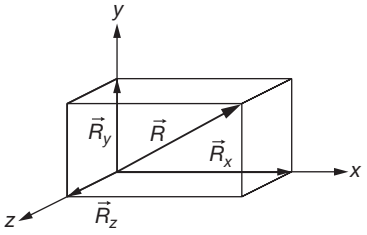
So resultant is 118 N at $180 - 37 = 143°$.

RECTANGULAR COMPONENTS IN 3-D SPACE

In 3-D space, we have

$$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z$$

$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$



If $\vec{R}$ makes an angle α with x axis, β with y axis and γ with z axis, then

$$\cos\alpha = \frac{R_x}{R} \qquad \cos\beta = \frac{R_y}{R} \qquad \cos\gamma = \frac{R_z}{R}$$

$$\Rightarrow \quad \cos\alpha = \frac{R_x}{R} = \frac{R_x}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = l$$

$$\Rightarrow \quad \cos\beta = \frac{R_y}{R} = \frac{R_y}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = m$$

$$\Rightarrow \quad \cos\gamma = \frac{R_z}{R} = \frac{R_z}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = n$$

where l, m, n are called **Direction Cosines** of the vector $\vec{R}$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{R_x^2 + R_y^2 + R_z^2}{R_x^2 + R_y^2 + R_z^2} = 1$$

Conceptual Note(s)

(a) If l, m, n are called Direction Cosines of the vector, then $l^2+m^2+n^2=1$

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

(b) In 3-D space a vector of magnitude r making an angle α with x-axis, β with y-axis and γ with z-axis can thus be written as

$$\vec{r}=r\left[(\cos\alpha)\hat{i}+(\cos\beta)\hat{j}+(\cos\gamma)\hat{k}\right]$$

ILLUSTRATION 13

A bird moves with velocity of $20\ \text{ms}^{-1}$ in the direction making angle $60°$ with eastern line and $60°$ with vertically upward. Represent the velocity vector in rectangular form.

SOLUTION

Velocity vector $\vec{v}$ makes angle α, β and γ with x-, y- and z- axis respectively

$\therefore \quad \alpha=60°$ and $\gamma=60°$

Since, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \quad \cos^2 60°+\cos^2\beta+\cos^2 60°=1$

$\Rightarrow \quad \left(\frac{1}{2}\right)^2+\cos^2\beta+\left(\frac{1}{2}\right)^2=1$

$\Rightarrow \quad \cos^2\beta=1-\frac{1}{2}$

$\Rightarrow \quad \cos\beta=\frac{1}{\sqrt{2}}$

Since $\vec{v}=v\cos\alpha\hat{i}+v\cos\beta\hat{j}+v\cos\gamma\hat{k}$

$\Rightarrow \quad \vec{v}=20\times\frac{1}{2}\hat{i}+20\times\frac{1}{\sqrt{2}}\hat{j}+20\times\frac{1}{2}\hat{k}$

$\Rightarrow \quad \vec{v}=10\hat{i}+10\sqrt{2}\hat{j}+10\hat{k}$

Test Your Concepts-I

Based on Addition, Subtraction and Resolution

(Solutions on page H.41)

1. Can three vectors not in one plane give a zero resultant? Can four vectors do?

2. The x and y components of vector $\vec{A}$ are 4 m and 6 m respectively. The x and y components of vector $\vec{A}+\vec{B}$ are 10 m and 9 m respectively. Calculate for the vector $\vec{B}$ the following:
(a) its x and y components,
(b) its length and
(c) the angle its makes with x-axis.

3. If $\vec{A}=4\hat{i}-3\hat{j}$ and $\vec{B}=6\hat{i}+8\hat{j}$, obtain the scalar magnitude and directions of $\vec{A}$, $\vec{B}$, $(\vec{A}+\vec{B})$, $(\vec{A}-\vec{B})$ and $(\vec{B}-\vec{A})$.

4. A particle has a displacement of 12 m towards east and 5 m towards north and 6 m vertically upwards. Find the magnitude of the sum of these displacements.

5. Show that if two vectors are equal in magnitude, their vector sum and difference are at right angles.

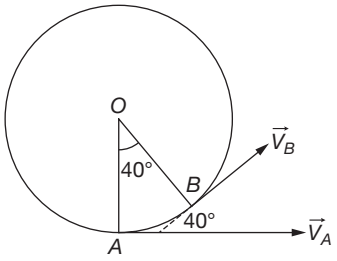
6. Establish the following vector inequalities:
(a) $|\vec{a}-\vec{b}|\le|\vec{a}|+|\vec{b}|$
(b) $|\vec{a}-\vec{b}|\ge|\vec{a}|-|\vec{b}|$
When does the equality sign apply?

7. Find the magnitude and direction of the resultant of following forces acting on a particle: $\vec{F}_1=3\sqrt{2}$ Kgf due north-east, $\vec{F}_2=6\sqrt{2}$ Kgf due south-east and $\vec{F}_3=\sqrt{2}$ Kgf due north-west.

8. What does the statement $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$ imply?

9. Two forces F_1 and F_2 acting at a point have a resultant R_1. If F_2 is doubled, the new resultant R_2 is at right angles to F_1. Prove that R_1 and F_2 have the same magnitude.

10. In figure, a particle is moving in a circle of radius r centred at O with constant speed v. What is the change in velocity in moving from A to B? Given $\angle AOB=40°$.



11. Prove that the resultant of two vectors of equal magnitude is equally inclined to either of the two vectors.

12. Consider two unit vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ. Prove that

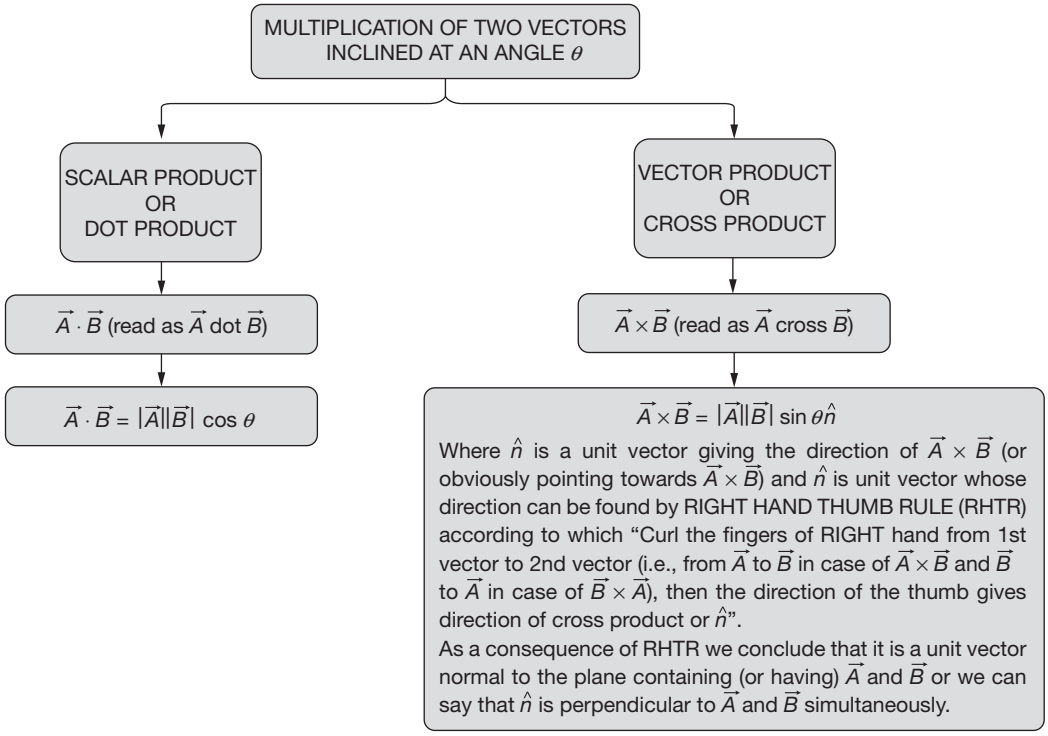
(a) $|\vec{A}+\vec{B}| = 2\cos\left(\frac{\theta}{2}\right)$

(b) $|\vec{A}-\vec{B}| = 2\sin\left(\frac{\theta}{2}\right)$

13. Can the magnitude of the resultant of two equal vectors be equal to the magnitude of either of the vectors? Explain your answer.

14. Prove that the vector $\vec{r} = \hat{i}+\hat{j}+\hat{k}$ is equally inclined to all the three axis.

VECTOR MULTIPLICATION OF 2 VECTORS


MULTIPLICATION OF TWO VECTORS INCLINED AT AN ANGLE θ
SCALAR PRODUCT OR DOT PRODUCT
VECTOR PRODUCT OR CROSS PRODUCT
$\vec{A}\cdot\vec{B}$ (read as $\vec{A}$ dot $\vec{B}$)
$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$
$\vec{A}\times\vec{B}$ (read as $\vec{A}$ cross $\vec{B}$)
$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\hat{n}$
Where $\hat{n}$ is a unit vector giving the direction of $\vec{A}\times\vec{B}$ (or obviously pointing towards $\vec{A}\times\vec{B}$) and $\hat{n}$ is unit vector whose direction can be found by RIGHT HAND THUMB RULE (RHTR) according to which "Curl the fingers of RIGHT hand from 1st vector to 2nd vector (i.e., from $\vec{A}$ to $\vec{B}$ in case of $\vec{A}\times\vec{B}$ and $\vec{B}$ to $\vec{A}$ in case of $\vec{B}\times\vec{A}$), then the direction of the thumb gives direction of cross product or $\hat{n}$".
As a consequence of RHTR we conclude that it is a unit vector normal to the plane containing (or having) $\vec{A}$ and $\vec{B}$ or we can say that $\hat{n}$ is perpendicular to $\vec{A}$ and $\vec{B}$ simultaneously.

DOT PRODUCT

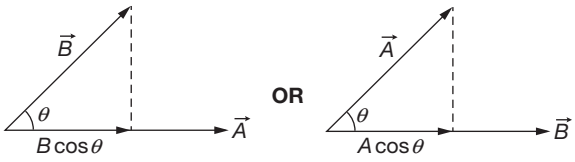
Mathematically, dot product is defined as the product of the magnitudes of the vectors and cosine of the angle between the vectors. So,

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

GEOMETRICAL INTERPRETATION

Since $\vec{A}\cdot\vec{B} = A(B\cos\theta)$

$\Rightarrow \quad \vec{A}\cdot\vec{B} = A\left(\text{projection of } \vec{B} \text{ along } \vec{A}\right)$



$$\Rightarrow \quad B\cos\theta = \frac{\vec{A}\cdot\vec{B}}{A}$$

$$\vec{B}\cdot\vec{A} = B(A\cos\theta)$$

$$\Rightarrow \quad \vec{B}\cdot\vec{A} = B \times (\text{Projection of } A \text{ along } B)$$

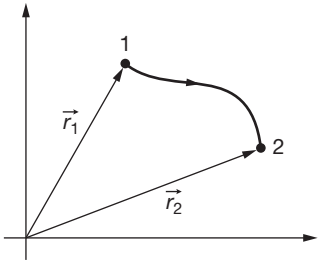
$$\Rightarrow \quad A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B}$$

PHYSICAL INTERPRETATION

Work done is dot product of force with displacement, so we have

$$W = \vec{F}\cdot\Delta\vec{r}$$

$$W = \vec{F}\cdot\left(\vec{r_2} - \vec{r_1}\right) \quad \text{where } \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$



Properties of Dot Product

(a) Dot product is commutative in nature

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

(b) Dot product is distributive with respect to sum

$$\vec{A}\cdot\left(\vec{B}\pm\vec{C}\right) = \vec{A}\cdot\vec{B}\pm\vec{A}\cdot\vec{C}$$

(c) If $\theta = 0°$ i.e. Vectors are parallel then $\vec{A}\cdot\vec{B} = AB\cos 0 = AB$ i.e., when the vectors are parallel $\vec{A}\cdot\vec{B}$ is just the simple product of the magnitudes of $\vec{A}$ and $\vec{B}$.

(d) Dot product of vector with itself is equal to the square of the magnitude of the vector

$$\vec{A}\cdot\vec{A} = (A)(A)\cos 0$$

$$\Rightarrow \quad \vec{A}\cdot\vec{A} = A^2$$

(e) If $\theta = 180°$ i.e., Vectors are antiparallel, then

$$\vec{A}\cdot\vec{B} = AB(-1) \qquad \{\because (\cos 180° = -1)\}$$

$$\vec{A}\cdot\vec{B} = -AB$$

i.e., If two vectors are antiparallel then their dot product equals the negative of simple product of magnitudes of vectors.

(f) If $\theta = 90°$ i.e., vectors are perpendicular

$$\vec{A}\cdot\vec{B} = AB(0) \qquad \vec{A}\cdot\vec{B} = 0 \text{ i.e.,}$$

Vectors perpendicular $\Leftrightarrow$ Dot product $= 0$

(g) $\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0$

Therefore, in general, $\hat{i}\cdot\hat{i} = 1,\ \hat{j}\cdot\hat{j} = 1,\ \hat{k}\cdot\hat{k} = 1$

(h) $\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos 90°$

$$\Rightarrow \quad \hat{i}\cdot\hat{j} = (1)(1)(0) = 0$$

Therefore, in general $\hat{i}\cdot\hat{j} = 0,\ \hat{j}\cdot\hat{k} = 0,\ \hat{k}\cdot\hat{i} = 0$ or $\hat{j}\cdot\hat{i} = 0,\ \hat{k}\cdot\hat{j} = 0,\ \hat{i}\cdot\hat{k} = 0$

(i) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ are any two vectors, then

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\Rightarrow \quad \vec{A}\cdot\vec{B} = A_x\hat{i}\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) +$$

$$A_y\hat{j}\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) + A_z\hat{k}\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\Rightarrow \quad \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

(j) Since $\vec{A}\cdot\vec{B} = AB\cos\theta$

and $-1 \le \cos\theta \le 1$

$$\Rightarrow \quad \vec{A}\cdot\vec{B} \text{ is maximum at } \cos\theta = 1 (\text{i.e., } \theta = 0°)$$

$$\vec{A}\cdot\vec{B} \text{ is minimum at } \cos\theta = -1 (\text{i.e., } \theta = 180°)$$

$$\Rightarrow \quad (\vec{A}\cdot\vec{B})_{\min} \le \vec{A}\cdot\vec{B} \le (\vec{A}\cdot\vec{B})_{\max}$$

$$\Rightarrow \quad -AB \le \vec{A}\cdot\vec{B} \le AB$$

(k) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ and θ is $\angle$ between $\vec{A}$ & $\vec{B}$.

Then $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$ where, $\vec{A}\cdot\vec{B} = AB\cos\theta$

(l) $|\vec{A}+\vec{B}|^2=(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B})$

$|\vec{A}-\vec{B}|^2=(\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})$

(m) $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=\vec{A}\cdot\vec{A}-\vec{B}\cdot\vec{B}+\cancel{\vec{B}\cdot\vec{A}}-\cancel{\vec{A}\cdot\vec{B}}$

$\Rightarrow\ (\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=A^2-\vec{B}$

$\{\because\ \vec{A}\cdot\vec{A}=A^2$ and $\vec{B}\cdot\vec{B}=B^2\}$

(n) Similarly

$$|\vec{A}+\vec{B}+\vec{C}|^2=(\vec{A}+\vec{B}+\vec{C})\cdot(\vec{A}+\vec{B}+\vec{C})$$

$$\Rightarrow\ |\vec{A}+\vec{B}+\vec{C}|^2=\vec{A}\cdot\vec{A}+\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{A}+\vec{B}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}+\vec{C}\cdot\vec{B}+\vec{C}\cdot\vec{C}$$

$$\Rightarrow\ |\vec{A}+\vec{B}+\vec{C}|^2=A^2+B^2+C^2+2(\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A})$$

(o) If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$, has l_1, m_1, n_1 as direction cosines

If $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, has l_2, m_2, n_2 as direction cosines

Also, $\cos\theta=\dfrac{A_xB_x+A_yB_y+A_zB_z}{AB}$

$$\Rightarrow\ \cos\theta=\frac{A_xB_x}{AB}+\frac{A_yB_y}{AB}+\frac{A_zB_z}{AB}$$

$$\Rightarrow\ \cos\theta=\left(\frac{A_x}{A}\right)\left(\frac{B_x}{B}\right)+\left(\frac{A_y}{A}\right)\left(\frac{B_y}{B}\right)+\left(\frac{A_z}{A}\right)\left(\frac{B_z}{B}\right)$$

$$\Rightarrow\ \boxed{\cos\theta=l_1l_2+m_1m_2+n_1n_2}$$

Conceptual Note(s)

(a) Angle between two vectors $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$ is

$$\cos\theta=\frac{A_xB_x+A_yB_y+A_zB_z}{\sqrt{A_x^2+A_y^2+A_z^2}\sqrt{B_x^2+B_y^2+B_z^2}}$$

(b) If $\langle l_1 \quad m_1 \quad n_1\rangle$ and $\langle l_2 \quad m_2 \quad n_2\rangle$ are direction cosines of $\vec{A}$ and $\vec{B}$, then $\cos\theta=l_1l_2+m_1m_2+n_1n_2$.

(c) If two vectors are perpendicular then

$$l_1l_2+m_1m_2+n_1n_2=0$$

(d) Also

$$\frac{d}{dt}(\vec{A}\cdot\vec{B})=\vec{A}\cdot\frac{d\vec{B}}{dt}+\vec{B}\cdot\frac{d\vec{A}}{dt}=\left(\frac{d\vec{A}}{dt}\right)\cdot\vec{B}+\left(\frac{d\vec{B}}{dt}\right)\cdot\vec{A}$$

ILLUSTRATION 14

Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that $\vec{A}=\vec{B}+\vec{C}$ and their magnitude are 5, 4, 3. Find angle between $\vec{A}$ and $\vec{C}$.

SOLUTION

As $\vec{A}=\vec{B}+\vec{C}$

$\Rightarrow\ \vec{B}=\vec{A}-\vec{C}$

Now $\vec{B}\cdot\vec{B}=(\vec{A}-\vec{C})\cdot(\vec{A}-\vec{C})$

$$B^2=A^2+C^2-2AC\cos\theta$$

$$\Rightarrow\ \cos\theta=\frac{A^2+C^2-B^2}{2AC}=\frac{5^2+3^2-4^2}{2\times5\times3}=\frac{18}{30}=0.6$$

$$\theta=\cos^{-1}(0.6)$$

ILLUSTRATION 15

For what value of a are the vectors $\vec{A}=a\hat{i}-2\hat{j}+\hat{k}$ and $\vec{B}=2a\hat{i}+a\hat{j}-4\hat{k}$ perpendicular to each other?

SOLUTION

Now $\vec{A}$ and $\vec{B}$ are $\perp$ to each other

$$\Rightarrow\ \vec{A}\cdot\vec{B}=0$$

$$\Rightarrow\ \vec{A}\cdot\vec{B}=(a\hat{i}-2\hat{j}+\hat{k})\cdot(2a\hat{i}+a\hat{j}-4\hat{k})$$

$$\Rightarrow\ \vec{A}\cdot\vec{B}=2a^2-2a-4=0$$

$\{\because\ \hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1,\ \hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0\}$

$$\Rightarrow\ a^2-a-2=0$$

$$\Rightarrow\ a^2-2a+a-2=0$$

$$\Rightarrow\ a(a-2)+(a-2)=0$$

$$\Rightarrow\ (a+1)(a-2)=0$$

$$\Rightarrow\ a=-1,\ a=2$$

ILLUSTRATION 16

Find the component of $\vec{a}=2\hat{i}+3\hat{j}$ along the direction of vector $\hat{i}+\hat{j}$.

SOLUTION

Given component of $\vec{a}$ in the direction of $\vec{b}$ is

$$\vec{r} = (a\cos\theta)\hat{b} = \left(\frac{\vec{a}\cdot\vec{b}}{b}\right)\hat{b} = \left(\frac{\vec{a}\cdot\vec{b}}{b^2}\right)\vec{b}$$

$$\Rightarrow \quad \vec{r} = \frac{\left(2\hat{i}+3\hat{j}\right)\cdot\left(\hat{i}+\hat{j}\right)}{\left|\sqrt{(1)^2+(1)^2}\right|^2}\left(\hat{i}+\hat{j}\right) = \frac{5}{2}\left(\hat{i}+\hat{j}\right)$$

So, component vector of $\vec{a}$ along $\vec{b}$ is $\vec{r} = \frac{5}{2}\left(\hat{i}+\hat{j}\right)$.

Test Your Concepts-II

Based on Dot Product

(Solutions on page H.43)

1. Express the scalar product of two vectors in terms of their rectangular components.
2. Two billiard balls are rolling on a flat table. One has the velocity components $v_x = 1\,\text{ms}^{-1}$, $v_y = \sqrt{3}\,\text{ms}^{-1}$ and the other has components $v_x' = 2\,\text{ms}^{-1}$ and $v_y' = 2\,\text{ms}^{-1}$. If both the balls start moving from the same point, what is the angle between their paths?
3. If $\vec{A} = 4\hat{i}+6\hat{j}-3\hat{k}$ and $\vec{B} = -2\hat{i}-5\hat{j}+7\hat{k}$, find the
   (a) direction cosines of $\vec{A}$ and $\vec{B}$.
   (b) angle between $\vec{A}$ and $\vec{B}$.
4. Show that the vectors $\vec{A} = 3\hat{i}-2\hat{j}+\hat{k}$, $\vec{B} = \hat{i}-3\hat{j}+5\hat{k}$ and $\vec{C} = 2\hat{i}+\hat{j}-4\hat{k}$ form a right angle triangle.
5. Find the projection of $\vec{A} = 10\hat{i}+8\hat{j}-6\hat{k}$ along $\vec{r} = 5\hat{i}+6\hat{j}+9\hat{k}$.
6. Obtain the scalar product of the vectors $(6, 2, 3)$, $(2, -9, 6)$ and also the angle between them.
7. Show that, for a vector $\vec{u}$ of constant magnitude, we have $\vec{u}\cdot\frac{d\vec{u}}{dt} = 0$.
8. Forces acting on a particle have magnitudes 5 N, 3 N and 1 N and act in the directions of the vectors $6\hat{i}+2\hat{j}+3\hat{k}$, $3\hat{i}-2\hat{j}+6\hat{k}$, $2\hat{i}-3\hat{j}-6\hat{k}$ respectively. These remain constant while the particle is displaced from the point $A(2, -1, -3)$ to $B(5, -1, -1)$. Find the work done by the forces, the unit of length being 1 m.

CROSS PRODUCT OR VECTOR PRODUCT

Mathematically,

$$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$$

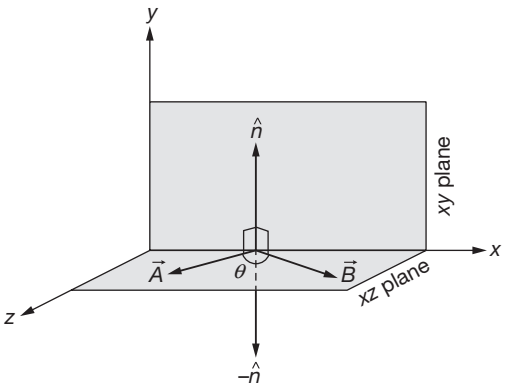
Direction of $\hat{n}$ is given by RHTR (stated earlier).

$\hat{n}$ indicates direction of $\vec{A}\times\vec{B}$ and $-\hat{n}$ indicates direction of $\vec{B}\times\vec{A}$.

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

So, cross product is Non-commutative in nature.

$$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$$



$\Rightarrow \quad |\vec{A}\times\vec{B}| = AB\sin\theta$

$\Rightarrow \quad |\vec{B}\times\vec{A}| = BA\sin\theta$

$\therefore \quad |\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$

Also, $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$, where, $\hat{n}$ indicates direction of $\vec{A}\times\vec{B}$

Conceptual Note(s)

(a) $|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$

(b) $\hat{n}$ is a new vector perpendicular to $\vec{A}$ as well as $\vec{B}$.

$\hat{n}\cdot\vec{A} = 0$

$\{$Perpendicular vectors have dot product $= 0\}$

$\hat{n}\cdot\vec{B} = 0$

Hence, $(\vec{A}\times\vec{B})$ is a new vector perpendicular to $\vec{A}$ as well as $\vec{B}$

(c) $\hat{n}$ is perpendicular to $\vec{A}$ as well as $\vec{B}$

(where $\hat{n}$ indicates direction of $\vec{A}\times\vec{B}$)

So, $\vec{A}\times\vec{B}$ is a new vector $\perp$ to $\vec{A}$ as well as $\vec{B}$.

$\Rightarrow \quad (\vec{A}\times\vec{B})\cdot\vec{A} = 0$

$\Rightarrow \quad (\vec{B}\times\vec{A})\cdot\vec{A} = 0$

$\Rightarrow \quad (\vec{A}\times\vec{B})\cdot\vec{B} = 0$

$\Rightarrow \quad (\vec{B}\times\vec{A})\cdot\vec{B} = 0$

GEOMETRICAL INTERPRETATION OF CROSS PRODUCT

Half of magnitude of cross product equals the area of triangle with adjacent sides $\vec{A}$ and $\vec{B}$.

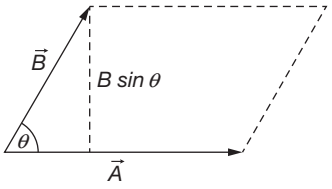
$$\text{Area} = \frac{1}{2}(A)(B\sin\theta) = \frac{1}{2}(AB\sin\theta)$$

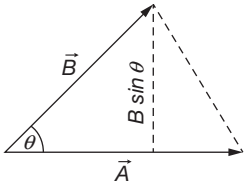
$$\Rightarrow \quad \text{Area of triangle} = \frac{1}{2}|\vec{A}\times\vec{B}|$$

$$\begin{pmatrix}\text{Area of}\\ \text{parallelogram}\end{pmatrix} = (\text{Base})\times\begin{pmatrix}\text{perpendicular}\\ \text{distance between}\\ \text{parallel sides}\end{pmatrix}$$

$$\begin{pmatrix}\text{Area of}\\ \text{parallelogram}\end{pmatrix} = A(B\sin\theta) = AB\sin\theta$$

Also, Area of parallelogram $= |\vec{A}\times\vec{B}| = \dfrac{1}{2}|\vec{d}_1\times\vec{d}_2|$





Conceptual Note(s)

So, half of the modulus of cross product equals the area of the triangle with adjacent sides $\vec{A}$ and $\vec{B}$ and magnitude of cross product equals the area of the parallelogram with adjacent sides $\vec{A}$ and $\vec{B}$.

PHYSICAL INTERPRETATION

(a) The torque due to a force $\vec{F}$ acting at a point with position vector $\vec{r}$ about the origin is $\vec{\tau} = \vec{r}\times\vec{F}$. The torque due to a force $\vec{F}$ acting at a point with position vector $\vec{r}$ about another point having position vector $\vec{r}_0$ is $\vec{\tau} = (\vec{r} - \vec{r}_0)\times\vec{F}$.

τ (read as tau) i.e., Torque (in Physics)

$\vec{r}$ is the distance of point of application of force from axis of rotation (A.O.R.) and F is force.

τ is also called the MOMENT OF FORCE.

(b) $\vec{L} = \vec{r}\times\vec{p}$

$\vec{L}$ = Angular momentum also called MOMENT OF LINEAR MOMENTUM

$\vec{p}$ = Linear momentum = $m\vec{v}$

$\vec{p} = m\vec{v}$

Since, Mass is a scalar therefore momentum can be read as mass times velocity

$$\vec{L} = \vec{r}\times(m\vec{v}) = m(\vec{r}\times\vec{v}) = m\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ v_x & v_y & v_z\end{vmatrix}$$

(c) $\vec{v} = \vec{\omega}\times\vec{r}$, where $\vec{v}$ is the linear velocity of a particle moving in a circle of radius vector $\vec{r}$ with angular velocity $\vec{\omega}$.

(d) Magnetic force experienced by a charge q entering a magnetic field with a velocity $\vec{v}$ is given by

$$\vec{F}_{\text{magnetic}} = q(\vec{v}\times\vec{B})$$

Properties of Vector Product/Cross Product

(a) Vector Product is Non commutative i.e.

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

$$\Rightarrow\quad (\vec{A}\times\vec{B})+(\vec{B}\times\vec{A})=\vec{0}$$

i.e., cross product is position sensitive.

(b) Cross product is distributive with respect to sum i.e.

$$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

(c) If $\vec{A}$ and $\vec{B}$ are parallel i.e. $\theta=0°$, then

$$\vec{A}\times\vec{B}=AB\sin 0\hat{n}$$

$$\Rightarrow\quad \vec{A}\times\vec{B}=\vec{0}$$

Vectors parallel $\Leftrightarrow$ cross-product equal to $\vec{0}$

(d) If $\vec{A}$ and $\vec{B}$ are antiparallel i.e. $\theta=180°$

$$\vec{A}\times\vec{B}=AB\sin(180°)\hat{n}$$

$$\Rightarrow\quad \vec{A}\times\vec{B}=\vec{0} \qquad \{\because \sin 180°=0\}$$

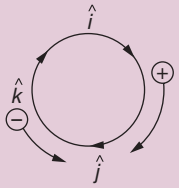
Vectors antiparallel $\Leftrightarrow$ cross product equal to $\vec{0}$

(e) $\vec{A}\times\vec{A}=AA\sin 0\hat{n}$

$$\vec{A}\times\vec{A}=\vec{0}$$

i.e., cross product of vector with itself is $\vec{0}$

(f) $\hat{i}\times\hat{i}=\vec{0},\ \hat{j}\times\hat{j}=\vec{0},\ \hat{k}\times\hat{k}=\vec{0}$



(g) $\hat{i}\times\hat{j}=\hat{k},\ \hat{k}\times\hat{i}=\hat{j},\ \hat{j}\times\hat{k}=\hat{i}$

For a Right handed triad system, curl fingers of your right hand from

x to *y*, thumb gives direction of *z*
y to *z*, thumb gives direction of *x*
z to *x*, thumb gives direction of *y*

(h) If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k},\ \vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, then,

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}_{3\times3}$$

$$\Rightarrow\quad \vec{A}\times\vec{B}=\hat{i}\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}_{2\times2}-\hat{j}\begin{vmatrix}A_x & A_z\\ B_x & B_z\end{vmatrix}_{2\times2}+\hat{k}\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}_{2\times2}$$

$$\Rightarrow\quad \vec{A}\times\vec{B}=\hat{i}(A_yB_z-B_yA_z)-\hat{j}(A_xB_z-A_zB_x)+\hat{k}(A_xB_y-B_xA_y)$$

Conceptual Note(s)

(a) $\vec{A}\times\vec{B}+\vec{C}\times\vec{A}\neq\vec{A}\times(\vec{B}+\vec{C})$

Instead, $\vec{A}\times\vec{B}+\vec{C}\times\vec{A}=\vec{A}\times\vec{B}-\vec{A}\times\vec{C}$

$$\Rightarrow\quad \vec{A}\times\vec{B}+\vec{C}\times\vec{A}=\vec{A}\times(\vec{B}-\vec{C})$$

Please note that Cross Product is always position sensitive. So be careful while changing the placement of vectors.

(b) Also we must know that

$$\frac{d}{dt}(\vec{A}\times\vec{B})=\vec{A}\times\frac{d\vec{B}}{dt}+\frac{d\vec{A}}{dt}\times\vec{B}=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$$

ILLUSTRATION 17

Find the area of triangle formed by tips of the vectors $\vec{a}=\hat{i}-\hat{j}-3\hat{k}$, $\vec{b}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}-\hat{j}+2\hat{k}$.

SOLUTION

Let *ABC* be the triangle formed by the tips of given vectors. Then

$$\overrightarrow{BA}=\vec{a}-\vec{b}=(\hat{i}-\hat{j}-3\hat{k})-(4\hat{i}-3\hat{j}+\hat{k})=-3\hat{i}+2\hat{j}-4\hat{k}$$

$$\overrightarrow{BC}=\vec{c}-\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})-(4\hat{i}-3\hat{j}+\hat{k})=-\hat{i}+2\hat{j}+\hat{k}$$

$$\Rightarrow\quad \overrightarrow{BA}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & 2 & -4\\ -1 & 2 & 1\end{vmatrix}$$

$$\Rightarrow\quad \overrightarrow{BA}\times\overrightarrow{BC}=\hat{i}(2+8)-\hat{j}(-3-4)+\hat{k}(-6+2)$$

$$\Rightarrow\quad \overrightarrow{BA}\times\overrightarrow{BC}=10\hat{i}+7\hat{j}-4\hat{k}$$

$$\Rightarrow\quad |\overrightarrow{BA}\times\overrightarrow{BC}|=\sqrt{(10)^2+(7)^2+(-4)^2}$$

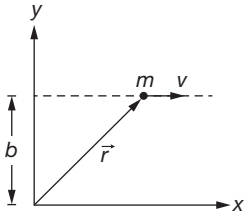
$$\Rightarrow\quad |\overrightarrow{BA}\times\overrightarrow{BC}|=\sqrt{165}=12.8$$

$$\Rightarrow\quad \text{Area of }\Delta ABC=\frac{1}{2}|\overrightarrow{BA}\times\overrightarrow{BC}|$$

$$\Rightarrow\quad \text{Area of }\Delta ABC=\frac{1}{2}\times12.8=6.4\text{ sq. unit}$$

ILLUSTRATION 18

If a particle of mass m is moving with constant velocity v parallel to x-axis in x-y plane as shown in figure, calculate its angular momentum with respect to origin at any time t.



SOLUTION

We know that

Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

As motion is in x-y plane $\{\therefore z = 0 \text{ and } p_z = 0\}$

$$\vec{L} = \hat{k}(xp_y - yp_x)$$

Here $x = vt$

$$y = b \qquad p_x = mv \qquad p_y = 0$$

$$\vec{L} = \hat{k}[vt \times 0 - bmv] = -(mbv)\hat{k}$$

From Result we can conclude that, if motion is in x-y plane angular momentum is always directed along z-axis i.e., angular momentum is always perpendicular to plane of motion.

ILLUSTRATION 19

Find the value of a for which the vectors $3\hat{i} + 3\hat{j} + 9\hat{k}$ and $\hat{i} + a\hat{j} + 3\hat{k}$ are parallel.

SOLUTION

Let $\vec{A} = 3\hat{i} + 3\hat{j} + 9\hat{k}$

$$\vec{B} = \hat{i} + a\hat{j} + 3\hat{k}$$

Now $\vec{A} \parallel \vec{B}$ so we have

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = k$$

$$\Rightarrow \quad \frac{3}{1} = \frac{3}{a} = \frac{9}{3} = k$$

$$\Rightarrow \quad 9a = 9$$

$$\Rightarrow \quad a = 1$$

CONFUSION?

Whether the vectors $\vec{A}$ and $\vec{B}$ are parallel or antiparallel then in both the cases

$$\vec{A} \times \vec{B} = \vec{0}$$

HOW TO REMOVE THE CONFUSION?

If $\vec{A} = k\vec{B}(k > 0)$, then vectors are parallel and if $\vec{A} = -k\vec{B}(k > 0)$, then vectors are antiparallel.

OR

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

If $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = +k(k > 0)$, **then** $\vec{A}$ parallel to $\vec{B}$,

else if $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = -k(k > 0)$, **then** $\vec{A}$ antiparallel to $\vec{B}$.

OR

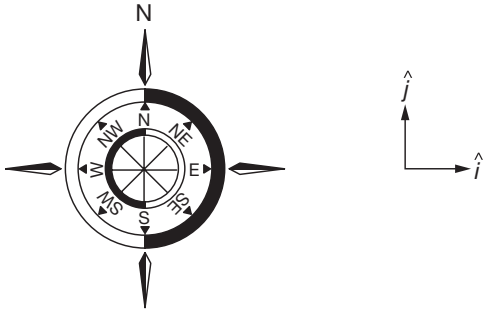
Find $\cos\theta$ i.e. angle between two vectors $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$

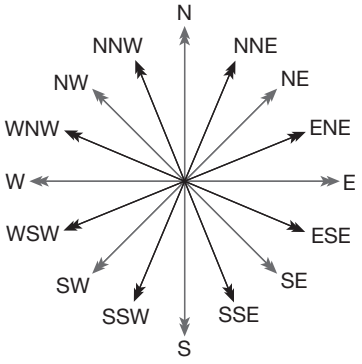
If $\cos\theta = 1$, **then** vectors are parallel

If $\cos\theta = -1$, **then** vectors are antiparallel

DIRECTIONS

The example below indicates the method to read and express directions.





NNE: North of North East.
ENE: East of North East.
ESE: East of South East.
SSE: South of South East.
SSW: South of South West.
WSW: West of South West.
WNW: West of North West.
NNW: North of North West.

(a) NW OR SW OR SE OR NE means 45° with either of the axis.

(b) $30°NW$ means 30° towards the north of west

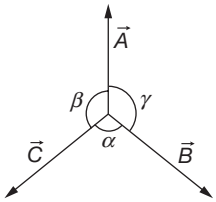
(c) $35°SW$ means 35° towards the south of west.

LAMI'S THEOREM

Statement

In any ΔABC with sides $\vec{a}$, $\vec{b}$, $\vec{c}$, if α, β and γ be the respective angles containing the sides, then

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$



OR

For any triangle the ratio of the sine of the angle containing the side to the length of the side is a constant.

Proof

For a triangle whose three sides are in the same order we establish the Lami's Theorem in the following manner. For the triangle shown all three sides are in same order, so

$$\vec{a}+\vec{b}+\vec{c}=\vec{0} \qquad \ldots(1)$$

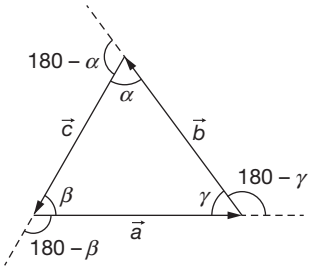
$$\Rightarrow \quad \vec{a}+\vec{b}=-\vec{c} \qquad \ldots(2)$$

Pre-multiplying both sides by $\vec{a}$

$$\vec{a}\times(\vec{a}+\vec{b})=-\vec{a}\times\vec{c}$$

$$\Rightarrow \quad \vec{0}+\vec{a}\times\vec{b}=-\vec{a}\times\vec{c}$$

$$\Rightarrow \quad \vec{a}\times\vec{b}=\vec{c}\times\vec{a} \qquad \ldots(3)$$



Pre-multiplying both sides of (2) by $\vec{b}$

$$\vec{b}\times(\vec{a}+\vec{b})=\vec{b}\times\vec{c}$$

$$\Rightarrow \quad \vec{b}\times\vec{a}+\vec{b}\times\vec{b}=-\vec{b}\times\vec{c}$$

$$\Rightarrow \quad -\vec{a}\times\vec{b}=-\vec{b}\times\vec{c}$$

$$\Rightarrow \quad \vec{a}\times\vec{b}=\vec{b}\times\vec{c} \qquad \ldots(4)$$

From (3) and (4), we get

$$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$$

Taking magnitude, we get

$$|\vec{a}\times\vec{b}|=|\vec{b}\times\vec{c}|=|\vec{c}\times\vec{a}|$$

$$\Rightarrow \quad ab\sin(180-\gamma)=bc\sin(180-\alpha)=ca\sin(180-\beta)$$

$$\Rightarrow \quad ab\sin\gamma=bc\sin\alpha=ca\sin\beta$$

Dividing throughout by abc, we have

$$\Rightarrow \quad \frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

SCALAR TRIPLE PRODUCT (STP)

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors, then the scalar triple product can be written as

$$(\vec{a}\times\vec{b})\cdot\vec{c} \text{ i.e., } (\vec{a} \text{ CROSS } \vec{b}) \text{ DOT } \vec{c}$$

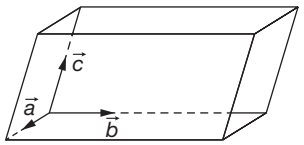
It can also be written as $[\vec{a} \quad \vec{b} \quad \vec{c}]$

Conceptual Note(s)

Scalar Triple Product (STP) is a scalar product and thus has no direction.

GEOMETRICAL INTERPRETATION OF SCALAR TRIPLE PRODUCT

Geometrically, the scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ represents the volume of parallelopiped whose coterminus edges $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a right handed system of vectors.



PROPERTIES OF SCALAR TRIPLE PRODUCT

(a) Cyclic Permutation of $\vec{a}$, $\vec{b}$ and $\vec{c}$ does not change the value of the scalar triple product

$$[\vec{a}\ \vec{b}\ \vec{c}]=[\vec{b}\ \vec{c}\ \vec{a}]=[\vec{c}\ \vec{a}\ \vec{b}]$$

(b) An antisymmetric or acyclic permutation changes sign only and not magnitude

$$[\vec{a}\ \vec{b}\ \vec{c}]=-[\vec{a}\ \vec{c}\ \vec{b}]=-[\vec{b}\ \vec{a}\ \vec{c}]$$

Conceptual Note(s)

The position of dot and cross can be interchanged keeping the cyclic order same. With such combinations, 12 different combinations are possible.

(c) Scalar Triple Product in Components Form

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors, then, $\vec{a}=a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$, $\vec{b}=b_x\hat{i}+b_y\hat{j}+b_z\hat{k}$ and $\vec{c}=c_x\hat{i}+c_y\hat{j}+c_z\hat{k}$

$$\Rightarrow\quad (\vec{a}\times\vec{b})\cdot\vec{c}=\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

(d) For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ and scalar λ, we have

$$[\lambda\vec{a}\ \ \vec{b}\ \ \vec{c}]=[\vec{a}\ \ \lambda\vec{b}\ \ \vec{c}]=[\vec{a}\ \ \vec{b}\ \ \lambda\vec{c}]=\lambda[\vec{a}\ \ \vec{b}\ \ \vec{c}]$$

(e) If $[\vec{a}\ \ \vec{b}\ \ \vec{c}]=0$, then atleast two of the three vectors are collinear, equal or parallel.

(f) If $[\vec{a}\ \ \vec{b}\ \ \vec{c}]=0$, then the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

ILLUSTRATION 20

Prove that the four points $(4\hat{i}+5\hat{j}+\hat{k})$, $-(\hat{j}+\hat{k})$, $(3\hat{i}+9\hat{j}+4\hat{k})$ and $4(-\hat{i}+\hat{j}+\hat{k})$ are coplanar.

SOLUTION

For four points (i.e., three vectors) to be collinear, let us first find three vectors by taking one point as the origin. So, let the origin be at the first vector, then

$$\vec{A}=(-\hat{j}-\hat{k})-(4\hat{i}+5\hat{j}+\hat{k})=-4\hat{i}-6\hat{j}-2\hat{k}$$

$$\vec{B}=(3\hat{i}+9\hat{j}+4\hat{k})-(4\hat{i}+5\hat{j}+\hat{k})=-\hat{i}+4\hat{j}+3\hat{k}$$

and

$$\vec{C}=(-4\hat{i}+4\hat{j}+4\hat{k})-(4\hat{i}+5\hat{j}+\hat{k})=-8\hat{i}-\hat{j}+3\hat{k}$$

For these to be coplanar, we must have $\vec{A}\cdot(\vec{B}\times\vec{C})=0$

Now, $\vec{A}\cdot(\vec{B}\times\vec{C})=\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$

$$\Rightarrow\quad \vec{A}\cdot(\vec{B}\times\vec{C})=-4(12+3)+6(-3+24)-2(1+32)$$

$$\Rightarrow\quad \vec{A}\cdot(\vec{B}\times\vec{C})=-60+126-66$$

$$\Rightarrow\quad \vec{A}\cdot(\vec{B}\times\vec{C})=0$$

Hence, we must say that the points or the vectors are coplanar.

VECTOR TRIPLE PRODUCT (VTP)

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors, then the vector triple product is

$$(\vec{a}\times\vec{b})\times\vec{c}\ \text{ or }\ \vec{a}\times(\vec{b}\times\vec{c})$$

and is defined as

$$(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$$

OR $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

i.e., $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

Read as "$\vec{a}$ cross $\vec{b}$ cross $\vec{c}$ equals $\vec{a}$ dot $\vec{c}$ times $\vec{b}$ minus $\vec{a}$ dot $\vec{b}$ times $\vec{c}$".

Problems Solving Technique(s)

If $\vec{a}\times(\vec{b}\times\vec{c})$ can also be written as $❶\times(❷\times❸)$, then it is defined as

$$❶\times(❷\times❸)=(❶\cdot❸)❷-(❶\cdot❷)❸$$

Conceptual Note(s)

If $\vec{r}=\vec{a}\times(\vec{b}\times\vec{c})$, then $\vec{r}$ is a vector perpendicular to $\vec{a}$ and lies in the plane of $\vec{b}\times\vec{c}$.

Test Your Concepts-III

Based on Cross Product, Scalar and Vector Triple Product

(Solutions on page H.44)

1. If a force $\vec{F}=(3\hat{i}+5\hat{j}-2\hat{k})$ N acts at a point defined by $(7\hat{i}-2\hat{j}+5\hat{k})$ m, find the torque:
(a) about the origin, and
(b) about the point $(0,-1,0)$.

2. Express the vector product of two vectors in terms of their rectangular components.

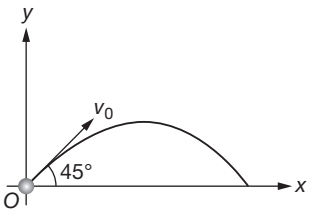
3. Prove that $|\vec{a}\times\vec{b}|=\sqrt{a^2b^2-(\vec{a}\cdot\vec{b})^2}$

4. Show that $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=-2(\vec{a}\times\vec{b})$ and use this result to find the area of a parallelogram whose diagonals are $\hat{i}-2\hat{j}-3\hat{k}$ and $2\hat{i}-3\hat{j}+2\hat{k}$.

5. If $\vec{L}$ and $\vec{L}'$, are two length vectors, what physical quantity does $[\vec{L}\times\vec{L}']$ represent?

6. If $\vec{A}\times\vec{B}=\vec{0}$ and $\vec{A}\cdot\vec{B}=0$, does it imply that one of the vectors $\vec{A}$ or $\vec{B}$ must be a null vector?

7. The particle of mass m is projected at $t=0$ from a point O on the ground with speed v_0 at an angle 45° to the horizontal as shown in figure. Compute the magnitude and direction of the angular momentum of the particle about the point O at position $\vec{r}=\left(\frac{0.7v_0^2}{g}\right)\hat{i}+\left(\frac{0.2v_0^2}{g}\right)\hat{j}$ when the velocity of the particle is $\vec{v}=(0.7v_0)\hat{i}-(0.3v_0)\hat{j}$.



8. The force on a positively charged particle is given by $\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}$. In a certain space there is a magnetic field B along z-axis and an electric field along y-axis. A positively charged particle is projected into this space. Find the direction and magnitude of minimum velocity so that it may pass on undeviated.

9. Find the moment of force $\vec{F}=\hat{i}+\hat{j}+\hat{k}$ acting at point $(-2,3,4)$ about the point $(1,2,3)$.

10. Prove that $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}$.

SOLVED PROBLEMS

PROBLEM 1

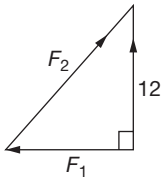
The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at $90°$ with the force of smaller magnitude, what are the magnitudes of forces?

SOLUTION

Let F_1 and F_2 be the two forces where $F_1 < F_2$

Here, $F_1 + F_2 = 18$ …(1)

The statement to the question can be diagrammatically drawn as shown in Figure.



So, the diagram clearly shows that 12 is the resultant of $\vec{F}_1$ and $\vec{F}_2$ (taken in same order). Also we observe that 12 i.e., resultant is perpendicular to $\vec{F}_1$. So, from the diagram, by using Pythagoras theorem, we get

$$F_2^2 = 144 + F_1^2$$

$$\Rightarrow \quad F_2^2 - F_1^2 = 144$$

$$\Rightarrow \quad (F_2 - F_1)(F_2 + F_1) = 144$$

$$\Rightarrow \quad (F_2 - F_1)(18) = 144$$

$$\Rightarrow \quad F_2 - F_1 = 8 \qquad \ldots(2)$$

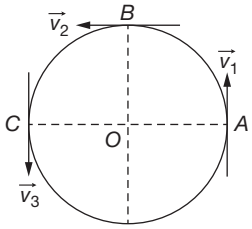
So, from (1) and (2), we get

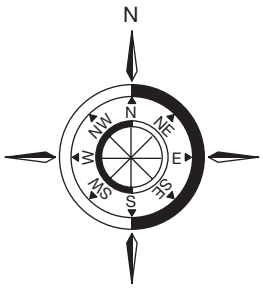
$$F_1 = 5 \text{ and } F_2 = 13$$

PROBLEM 2

A body is moving with uniform speed v in a horizontal circle in anticlockwise direction starting from A as shown in figure. Calculate the change in velocity in

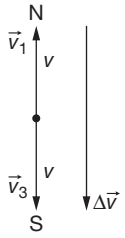
(a) half revolution
(b) first quarter revolution.





SOLUTION

Change in velocity $= \Delta\vec{v} = \vec{v}_f - \vec{v}_i$



(a) Now, for half revolution.

If $\vec{v}_3 = \vec{v}$ and $\vec{v}_1 = -\vec{v}$

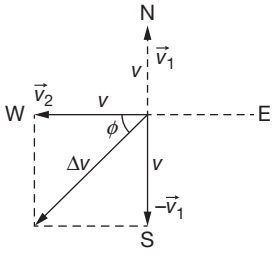
as their magnitudes are equal but directions opposite.

$$\Rightarrow \quad \Delta\vec{v} = \vec{v}_3 - \vec{v}_1 = \vec{v} - (-\vec{v}) = 2\vec{v}$$

$\Delta\vec{v} = 2v$ directed south.

(b) For, quarter revolution

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ and $\theta = 90°$



$$\Delta\vec{v} = \sqrt{v^2 + v^2 + 2vv\cos 90°} = \sqrt{2}v$$

and $\phi = \tan^{-1}\left(\frac{v}{v}\right) = 45°$

So, $\Delta\vec{v} = (\sqrt{2})v$ south-west

PROBLEM 3

If vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ have magnitudes 8, 15 and 17 unit and $\vec{A} + \vec{B} = \vec{C}$, find the angle between $\vec{A}$ and $\vec{B}$.

SOLUTION

$$\vec{A} + \vec{B} = \vec{C}$$

Taking magnitude both sides

$$|\vec{A}+\vec{B}|=|\vec{C}|$$

{Magnitudes of equal vectors is also equal}

Squaring both sides, we get

$$|\vec{A}+\vec{B}|^2=|\vec{C}|^2$$

$$\Rightarrow \quad (\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B})=\vec{C}\cdot\vec{C} \qquad \{\because \vec{A}\cdot\vec{A}=|\vec{A}|^2\}$$

$$\Rightarrow \quad A^2+B^2+2AB\cos\theta=C^2$$

$$\Rightarrow \quad \cos\theta=\frac{C^2-A^2-B^2}{2AB}$$

$$\Rightarrow \quad \cos\theta=\frac{17^2-\{(15)^2+(8)^2\}}{2\times15\times8}$$

$$\Rightarrow \quad \cos\theta=0$$

$$\Rightarrow \quad \theta=90^\circ$$

PROBLEM 4

If the unit vectors $\hat{a}$ and $\hat{b}$ are inclined at angle θ prove that $|\vec{a}-\vec{b}|=2\sin\left(\frac{\theta}{2}\right)$.

SOLUTION

Since we know that,

$$|\hat{a}-\hat{b}|^2=|\hat{a}|^2+|\hat{b}|^2-2|\hat{a}||\hat{b}|\cos\theta$$

$$\Rightarrow \quad |\hat{a}-\hat{b}|^2=1-\cos\theta-\cos\theta+1 \qquad \{\because |\hat{a}|=|\hat{b}|=1\}$$

$$\Rightarrow \quad |\hat{a}-\hat{b}|^2=2-2\cos\theta$$

$$\Rightarrow \quad |\hat{a}-\hat{b}|^2=2(1-\cos\theta)=2\times2\sin^2\left(\frac{\theta}{2}\right)=4\sin^2\left(\frac{\theta}{2}\right)$$

Taking square root both sides.

$$|\hat{a}-\hat{b}|=2\sin\left(\frac{\theta}{2}\right)$$

PROBLEM 5

(a) Find the area of the parallelogram determined by the vectors $\vec{a}=3\hat{i}+2\hat{j}$ and $\vec{b}=2\hat{j}-4\hat{k}$.

(b) Find the area of the triangle whose vertices are $(1,-1,-3)$, $(4,-3,1)$ and $(3,-1,2)$.

SOLUTION

(a) Vector area of parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\times\vec{b}=\left(3\hat{i}+2\hat{j}\right)\times\left(2\hat{j}-4\hat{k}\right)$$

$$\Rightarrow \quad \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&2&0\\0&2&-4\end{vmatrix}$$

$$\Rightarrow \quad \vec{a}\times\vec{b}=\hat{i}[-8-0]-\hat{j}[-12-0]+\hat{k}[6-0]$$

$$\Rightarrow \quad \vec{a}\times\vec{b}=-8\hat{i}+12\hat{j}+6\hat{k}$$

$\therefore$ Magnitude of the area of parallelogram

$$|\vec{a}\times\vec{b}|=\left|-8\hat{i}+12\hat{j}+6\hat{k}\right|$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|=\sqrt{64+144+36}=2\sqrt{61}$$

(b) Position vectors of the vertices A, B and C of the triangle ABC are $\vec{a}=\left(\hat{i}-\hat{j}-3\hat{k}\right)$,

$$\vec{b}=\left(4\hat{i}-3\hat{j}+\hat{k}\right) \text{ and } \vec{c}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)$$

$$\overrightarrow{BA}=\vec{a}-\vec{b}=-3\hat{i}+2\hat{j}-4\hat{k}$$

and $\overrightarrow{BC}=\vec{c}-\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$

Hence, vector area of triangle $=\frac{1}{2}(\overrightarrow{BA}\times\overrightarrow{BC})$

$$\Rightarrow \quad \text{Area}=\frac{1}{2}\left(-3\hat{i}+2\hat{j}-4\hat{k}\right)\times\left(-\hat{i}+2\hat{j}+\hat{k}\right)$$

$$\Rightarrow \quad \text{Area}=\frac{1}{2}\left(10\hat{i}+7\hat{j}-4\hat{k}\right)$$

Magnitude of area of the triangle is

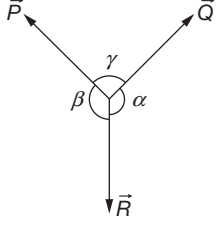
$$|\text{Area}|=\frac{1}{2}\sqrt{100+49+16}=\frac{1}{2}\sqrt{165} \text{ square units}$$

PROBLEM 6

A particle is in equilibrium under the action of three forces. Prove that each force bears a constant ratio with the sine of angle between the other two.

SOLUTION

Let $\vec{P}$, $\vec{Q}$ and $\vec{R}$ be the three forces acting at a point O. Since particle is in equilibrium.



$$\vec{P}+\vec{Q}+\vec{R}=0 \qquad \ldots(1)$$

$$\Rightarrow \quad \left(\vec{P}+\vec{Q}\right)=-\vec{R} \qquad \ldots(2)$$

Taking cross product, with $\vec{P}$ on both sides of (2)

$$\vec{P}\times\left(\vec{P}+\vec{Q}\right)=-\vec{P}\times\vec{R}$$

$$\Rightarrow \quad \vec{P}\times\vec{P}+\vec{P}\times\vec{Q}=\vec{R}\times\vec{P}$$

$$\Rightarrow \quad \vec{P}\times\vec{Q}=\vec{R}\times\vec{P} \qquad \ldots(3)$$

Similarly taking cross product with $\vec{Q}$ on both sides of (3), we get

$$\vec{P}\times\vec{Q}=\vec{Q}\times\vec{R} \qquad \ldots(4)$$

From (3) and (4)

$$\vec{P}\times\vec{Q}=\vec{Q}\times\vec{R}=\vec{R}\times\vec{P}$$

$$\Rightarrow \quad PQ\sin\gamma=QR\sin\alpha=RP\sin\beta$$

Dividing both sides by PQR, we get

$$\frac{PQ\sin\gamma}{PQR}=\frac{QR\sin\alpha}{PQR}=\frac{RP\sin\beta}{PQR}$$

$$\Rightarrow \quad \frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$$

The Lami's Theorem is very useful in the chapters to come. So please understand the theorem carefully so that you can apply it at the place required.

PROBLEM 7

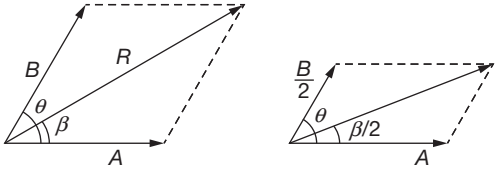
Two vectors of equal magnitude are inclined at an angle θ. When one of them is halved, the angle which the resultant makes with the other is also halved. Find θ.

SOLUTION

Let the two vectors be $\vec{A}$ and $\vec{B}$, inclined at an angle θ. Also we have been given that $|\vec{A}|=|\vec{B}|$. If β be the angle which the resultant (of $\vec{A}$ and $\vec{B}$) makes with $\vec{A}$, then

$$\tan\beta=\frac{B\sin\theta}{B+B\cos\theta}$$

$$\Rightarrow \quad \tan\beta=\frac{\sin\theta}{1+\cos\theta} \qquad \ldots(1)$$



When B is halved, then still the angle between A and $\frac{B}{2}$ is θ. However, the new resultant is now inclined to A at an angle $\frac{\beta}{2}$. So, we get

$$\tan\left(\frac{\beta}{2}\right)=\frac{\frac{B}{2}\sin\theta}{A+\frac{B}{2}\cos\theta}$$

Since $|\vec{A}|=|\vec{B}|$, so

$$\tan\left(\frac{\beta}{2}\right)=\frac{\sin\theta}{2+\cos\theta} \qquad \ldots(2)$$

Now, from concepts of trigonometric functions, we have

$$\tan\beta=\frac{2\tan\left(\frac{\beta}{2}\right)}{1-\tan^2\left(\frac{\beta}{2}\right)}$$

$$\Rightarrow \quad \frac{\sin\theta}{1+\cos\theta}=\frac{2\left(\frac{\sin\theta}{2+\cos\theta}\right)}{1-\frac{\sin^2\theta}{(2+\cos\theta)^2}}$$

$$\Rightarrow \quad \frac{\cancel{\sin\theta}}{1+\cos\theta}=\frac{2\left(\cancel{\sin\theta}\right)(2+\cos\theta)}{4+\cos^2\theta+4\cos\theta-\sin^2\theta}$$

$$\Rightarrow \quad 4+\cos^2\theta+4\cos\theta-\sin^2\theta=$$
$$(4+2\cos\theta)(1+\cos\theta)$$

$$\Rightarrow \quad 4+\cos^2\theta+4\cos\theta-\sin^2\theta=$$
$$4+2\cos^2\theta+6\cos\theta-\cos^2\theta-2\cos\theta-\sin^2\theta=0$$

$$\Rightarrow \quad -2\cos\theta-\left(\sin^2\theta+\cos^2\theta\right)=0$$

$$\Rightarrow \quad -2\cos\theta-1=0 \qquad \left\{\because \ \sin^2\theta+\cos^2\theta=1\right\}$$

$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$

$$\Rightarrow \quad \theta=120^\circ$$

PROBLEM 8

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

SOLUTION

Given that $|\vec{a}|=|\vec{b}|=|\vec{c}|$ and the three vectors are mutually perpendicular. So,

$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$$

Let θ_1 be the angle between $\vec{a}$ and $\vec{a}+\vec{b}+\vec{c}$. Then

$$\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=|\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\theta_1$$

$$\Rightarrow\quad \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=a|\vec{a}+\vec{b}+\vec{c}|\cos\theta_1$$

$$\Rightarrow\quad a^2=a|\vec{a}+\vec{b}+\vec{c}|\cos\theta_1$$

$$\Rightarrow\quad \cos\theta_1=\frac{a}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \ldots(1)$$

Similarly, if θ_2 be the angle between $\vec{b}$ and $\vec{a}+\vec{b}+\vec{c}$ and θ_3 is the angle between $\vec{c}$ and $\vec{a}+\vec{b}+\vec{c}$, then

$$\cos\theta_2=\frac{b}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \ldots(2)$$

$$\cos\theta_3=\frac{c}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \ldots(3)$$

Now, let us find the value of $|\vec{a}+\vec{b}+\vec{c}|$. Since we know that dot product of the vector with itself is equal to the square of its magnitude, so

$$(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=|\vec{a}+\vec{b}+\vec{c}|^2$$

$$\Rightarrow\quad \vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{c}+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=|\vec{a}+\vec{b}+\vec{c}|^2$$

Since $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$, so we get

$$a^2+b^2+c^2=|\vec{a}+\vec{b}+\vec{c}|^2$$

Again $|\vec{a}|=|\vec{b}|=|\vec{c}|$, so $|\vec{a}+\vec{b}+\vec{c}|^2=3a^2$

$$\Rightarrow\quad |\vec{a}+\vec{b}+\vec{c}|=\sqrt{3}a \qquad \ldots(4)$$

From (1), (2), (3) and (4), we get

$$\cos\theta_1=\frac{a}{|\vec{a}+\vec{b}+\vec{c}|}=\frac{a}{\sqrt{3}a}=\frac{1}{\sqrt{3}}$$

$$\cos\theta_2=\frac{b}{|\vec{a}+\vec{b}+\vec{c}|}=\frac{b}{\sqrt{3}b}=\frac{1}{\sqrt{3}}$$

$$\cos\theta_3=\frac{c}{|\vec{a}+\vec{b}+\vec{c}|}=\frac{c}{\sqrt{3}c}=\frac{1}{\sqrt{3}}$$

Since $\cos\theta_1=\cos\theta_2=\cos\theta_3=\frac{1}{\sqrt{3}}$

$$\Rightarrow\quad \theta_1=\theta_2=\theta_3=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Hence the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

PROBLEM 9

For a vector $\vec{a}$ in 3-D space, find the value of the expression $|\hat{i}\times\vec{a}|^2+|\hat{j}\times\vec{a}|^2+|\hat{k}\times\vec{a}|^2$

SOLUTION

Let the vector $\vec{a}$ make angle α, β and γ with x, y and z axis respectively. Then

$$|\hat{i}\times\vec{a}|^2=|\hat{i}||\vec{a}|\sin^2\alpha$$

$$\Rightarrow\quad |\hat{i}\times\vec{a}|^2=a^2\sin^2\alpha$$

Similarly

$$|\hat{j}\times\vec{a}|^2=a^2\sin^2\beta \text{ and } |\hat{k}\times\vec{a}|^2=a^2\sin^2\gamma$$

Now $|\hat{i}\times\vec{a}|^2+|\hat{j}\times\vec{a}|^2+|\hat{k}\times\vec{a}|^2=$

$$a^2(\sin^2\alpha+\sin^2\beta+\sin^2\gamma) \qquad \ldots(1)$$

Now since

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$$\Rightarrow\quad (1-\sin^2\alpha)+(1-\sin^2\beta)+(1-\sin^2\gamma)=1$$

$$\Rightarrow\quad \sin^2\alpha+\sin^2\beta+\sin^2\gamma=2 \qquad \ldots(2)$$

From (1) and (2) we get

$$|\hat{i}\times\vec{a}|^2+|\hat{j}\times\vec{a}|^2+|\hat{k}\times\vec{a}|^2=2a^2$$

PROBLEM 10

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=5$, then find the value of $(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$.

SOLUTION

Since $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \quad \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow \quad \vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}=-\vec{c}\cdot\vec{c}$

$\Rightarrow \quad \vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}=-c^2$ …(1)

Similarly $\vec{a}+\vec{c}=-\vec{b}$

$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{b}=-b^2$ …(2)

Again $\vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow \quad \vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}=-a^2$ …(3)

Adding (1), (2) and (3), we get

$$2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=-\left(a^2+b^2+c^2\right)$$

$$\Rightarrow \quad \left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=-\frac{1}{2}\left(3^2+4^2+5^2\right)=-25$$

$\Rightarrow \quad \left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=-25$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A particle starts from rest at the origin with a constant acceleration $\vec{a} = 2\hat{i} + 8\hat{j} - 6\hat{k}$ ms^{-2}. Its position at $t = 5$ s is

(A) $\left(25\hat{i} + 100\hat{j} - 75\hat{k}\right)$ m (B) $\left(25\hat{i} - 100\hat{j} - 75\hat{k}\right)$ m

(C) $\left(100\hat{i} - 25\hat{j} + 75\hat{k}\right)$ m (D) $\left(25\hat{i} - 100\hat{j} + 75\hat{k}\right)$ m

2. Three forces $\vec{F}_1 = \left(3\hat{i} + 2\hat{j} - \hat{k}\right)$N, $\vec{F}_2 = \left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$N and $\vec{F}_3 = A\left(\hat{i} + \hat{j} - \hat{k}\right)$N act simultaneously on a particle. In order that the particle remains in equilibrium, the value of A should be

(A) –6 (B) 6

(C) 9 (D) –9

3. The magnitude of vector product of two non zero vectors $\vec{A}$ and $\vec{B}$ is zero. The scalar product of $\vec{A}$ and $\vec{A} + \vec{B}$ is equal to

(A) zero (B) AB

(C) A^2 (D) $A^2 + AB$

4. The magnitude of resultant of two equal forces is equal to either of two forces. The angle between forces is

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

5. The condition under which the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are parallel is

(A) $\vec{a} \perp \vec{b}$ (B) $|\vec{a}| = |\vec{b}|$

(C) $\vec{a} \neq \vec{b}$ (D) $\vec{a} \parallel \vec{b}$

6. The two vectors $\vec{A}$ and $\vec{B}$ that are parallel to each other are

(A) $\vec{A} = 3\hat{i} + 6\hat{j} + 9\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

(B) $\vec{A} = 3\hat{i} - 6\hat{j} + 9\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

(C) $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

(D) $\vec{A} = 3\hat{i} + 6\hat{j} - 9\hat{k}$, $\vec{B} = \hat{i} - 2\hat{j} - 3\hat{k}$

7. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular if

(A) $\vec{A}$ and $\vec{B}$ are perpendicular to each other

(B) $\vec{A}$ and $\vec{B}$ are perpendicular but their magnitudes are arbitrary

(C) $|\vec{A}| = |\vec{B}|$ and their directions are arbitrary

(D) $\vec{A} \perp \vec{B}$ and $|\vec{A}| = |\vec{B}|$

8. Three forces 9, 12 and 15 N acting at a point are in equilibrium. The angle between 9 N and 15 N is

(A) $\cos^{-1}\left(\frac{3}{5}\right)$ (B) $\cos^{-1}\left(\frac{4}{5}\right)$

(C) $\pi - \cos^{-1}\left(\frac{3}{5}\right)$ (D) $\pi - \cos^{-1}\left(\frac{4}{5}\right)$

9. The condition that $(\vec{a} \cdot \vec{b})^2 = a^2 b^2$ is satisfied when

(A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \neq \vec{b}$

(C) $\vec{a} \cdot \vec{b} = 1$ (D) $\vec{a} \perp \vec{b}$

10. The sides of the triangle representing three force vectors are in the ratio 2000 : 1732 : 1000. The angles of the triangle (in degrees) are

(A) 70, 65, 45 (B) 80, 55, 45

(C) 90, 60, 30 (D) 90, 61, 39

11. A force $\vec{F} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right)$N acts at a point $(7,3,1)$. The torque about the origin (0, 0, 0) will be

(A) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (B) $14\hat{i} + 38\hat{j} - 16\hat{k}$

(C) $14\hat{i} - 38\hat{j} - 16\hat{k}$ (D) $14\hat{i} + 38\hat{j} + 16\hat{k}$

12. A vector $\vec{C} = \vec{A} - \vec{B}$ has a magnitude equal to $A + B$, the angle between $\vec{A}$ and $\vec{B}$ is

(A) zero (B) $\frac{\pi}{2}$

(C) π (D) 2π

13. The vector having module equal to 3 and perpendicular to two vectors $\vec{A} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 3\hat{k}$ is

(A) $\pm\left(2\hat{i} - \hat{j} - 2\hat{k}\right)$ (B) $\pm\left(3\hat{i} + \hat{j} - 2\hat{k}\right)$

(C) $-\left(3\hat{i} - \hat{j} - 3\hat{k}\right)$ (D) $\left(3\hat{i} - \hat{j} - 3\hat{k}\right)$

14. The vector sum of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is zero. If $\hat{i}$ and $\hat{j}$ are unit vectors in the directions of $\vec{a}$ and $\vec{b}$ respectively, then

(A) $\vec{c}$ is in the plane of $\hat{i}$ and $\hat{j}$.

(B) $\vec{c}$ is along $\left(\hat{i}\times\hat{j}\right)$.

(C) $\vec{c}$ is along $\hat{i}$.

(D) $\vec{c}$ is along $\hat{j}$.

15. 12 coplanar forces (all of equal magnitude) maintain a body in equilibrium, then the angle between any two adjacent forces is

(A) 15° (B) 30°

(C) 45° (D) 60°

16. If $\left|\vec{F}_1\times\vec{F}_2\right| = \vec{F}_1\cdot\vec{F}_2$, then $\left|\vec{F}_1+\vec{F}_2\right|$ is

(A) $\left|\vec{F}_1\right|+\left|\vec{F}_2\right|$

(B) $\sqrt{\left|\vec{F}_1\right|^2+\left|\vec{F}_2\right|^2}$

(C) $\sqrt{\left|\vec{F}_1\right|^2+\left|\vec{F}_2\right|^2+\frac{\left|\vec{F}_1\right|\left|\vec{F}_2\right|}{\sqrt{2}}}$

(D) $\sqrt{\left|\vec{F}_1\right|^2+\left|\vec{F}_2\right|^2+\sqrt{2}\left|\vec{F}_1\right|\left|\vec{F}_2\right|}$

17. The value of p so that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+p\hat{j}+5\hat{k}$ are coplanar should be

(A) 16 (B) –4

(C) 4 (D) –8

18. $ABCD$ is a quadrilateral. Forces $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ act at the point. Their resultant is

(A) $2\overrightarrow{AB}$ (B) $2\overrightarrow{DA}$

(C) $2\overrightarrow{BC}$ (D) $2\overrightarrow{BA}$

19. Which of the following group of forces will not accelerate a body?

(A) 5 N, 10 N, 12 N (B) 5 N, 10 N, 16 N

(C) 8 N, 10 N, 20 N (D) 7 N, 5 N, 15 N

20. Consider the two vectors $\vec{A}$ and $\vec{B}$. The magnitude of their sum, i.e. $|\vec{A}+\vec{B}|$, if $|\vec{A}|>|\vec{B}|$

(A) is equal to $|\vec{A}|+|\vec{B}|$

(B) cannot be less than $|\vec{A}|+|\vec{B}|$

(C) cannot be greater than $|\vec{A}|+|\vec{B}|$

(D) must be equal to $|\vec{A}|-|\vec{B}|$

21. Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are such that $|\vec{P}|=|\vec{Q}|$, $|\vec{R}|=\sqrt{2}\,|\vec{P}|$ and $\vec{P}+\vec{Q}+\vec{R}=\vec{0}$. The angles between $\vec{P}$ and $\vec{Q}$, $\vec{Q}$ and $\vec{R}$ and $\vec{P}$ and $\vec{R}$ are respectively (in degrees)

(A) 90, 135, 135 (B) 90, 45, 45

(C) 45, 90, 90 (D) 45, 135, 135

22. If $\hat{A}$ is a unit vector. The value $\hat{A}\cdot\frac{d\hat{A}}{dt}=$

(A) 0 (B) 1

(C) $\frac{\pi}{2}$ (D) π

23. If four non zero vectors satisfy $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ with $|\vec{a}|\neq|\vec{d}|$ and $|\vec{b}|\neq|\vec{c}|$, then

(A) $(\vec{a}-\vec{d})$ and $(\vec{b}-\vec{c})$ are perpendicular

(B) $(\vec{a}-\vec{d})$ and $(\vec{b}-\vec{c})$ are parallel

(C) $(\vec{a}-\vec{d})$ must equal $(\vec{b}-\vec{c})$

(D) $(\vec{a}-\vec{d})$ must equal $-(\vec{b}-\vec{c})$

24. If two non parallel vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude, then the vectors $(\vec{A}-\vec{B})$ and $(\vec{A}+\vec{B})$ will be

(A) parallel to each other

(B) parallel but oppositely directed

(C) perpendicular to each other

(D) inclined at an angle $\theta<90°$

25. A point of application of force $\vec{F}=-5\hat{i}+3\hat{j}+2\hat{k}$ is moved from $\vec{r}_1=2\hat{i}+7\hat{j}+4\hat{k}$ to $\vec{r}_2=5\hat{i}+2\hat{j}+4\hat{k}$. The work done will be

(A) –22 unit (B) 22 unit

(C) –30 unit (D) +30 unit

26. If vectors $\vec{a}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+x\hat{k}$ are to be perpendicular to each other, the value of x should be

(A) 2 (B) –2

(C) 3 (D) –3

27. When $t=0$, a particle at (1, 0, 0), moves towards (4, 4, 12) with a constant speed of 65 ms^{-1}. The position of the particle is measured in meter and the time in second. Assume constant velocity, the position of the particle for $t=2$ s is

(A) $\left(30\hat{i}-120\hat{j}+40\hat{k}\right)$ m (B) $\left(40\hat{i}+31\hat{j}-120\hat{k}\right)$ m

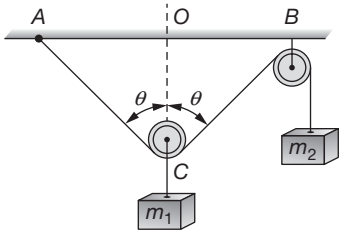
(C) $\left(13\hat{i}-40\hat{j}+12\hat{k}\right)$ m (D) $\left(31\hat{i}+40\hat{j}+120\hat{k}\right)$ m

28. The condition under which vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ should be at right angles to each other is

(A) $\vec{a} \neq \vec{b}$ (B) $\vec{a}\cdot\vec{b}=0$

(C) $|\vec{a}|=|\vec{b}|$ (D) $\vec{a}\cdot\vec{b}=1$

29. In the arrangement shown in figure, the instantaneous velocities of masses m_1 and m_2 are v_1 and v_2, respectively and $\angle ACB = 2\theta$ at the instant, then



(A) $\theta=\cos^{-1}\left(\frac{v_1}{2v_2}\right)$ (B) $\theta=\cos^{-1}\left(\frac{v_2}{2v_1}\right)$

(C) $\theta=\tan^{-1}\left(\frac{v_1}{2v_2}\right)$ (D) $\theta=\sin^{-1}\left(\frac{v_1}{v_2}\right)$

30. One vertex of a parallelopiped is at the point (1, –1, –2) of rectangular cartesian coordinates. If three adjacent vertices are at (0, 1, 3), (3, 0, –1) and (1, 4, 1), the volume of the parallelopiped is

(A) 400 unit (B) 80 unit

(C) 40 unit (D) 120 unit

31. $|\vec{A}\times\vec{B}|^2+|\vec{A}\cdot\vec{B}|^2$ is equal to

(A) $(\vec{A}+\vec{B})^2$ (B) $(\vec{A}-\vec{B})^2$

(C) A^2+B^2 (D) A^2B^2

32. In equation $\vec{F}=q(\vec{v}\times\vec{B})$, the quantity $\vec{F}$

(A) is perpendicular to $\vec{v}$ only

(B) is perpendicular to $\vec{B}$ only

(C) is perpendicular to both $\vec{v}$ and $\vec{B}$

(D) is perpendicular to q and $\vec{B}$

33. A particle moves from point (1, 0, 2.5) to the point $(-2, 3, 4)$ m when a force $\vec{F}=(\hat{i}+4\hat{k})$ N acts on it. The work done on it is

(A) 6 J (B) 30 J

(C) 3 J (D) 9 J

34. The vector that is added to $(\hat{i}-5\hat{j}+2\hat{k})$ and $(3\hat{i}+6\hat{j}-7\hat{k})$ to give a unit vector along the x-axis is

(A) $3\hat{i}+\hat{j}+5\hat{k}$ (B) $\hat{i}+3\hat{j}+5\hat{k}$

(C) $-3\hat{i}-\hat{j}+5\hat{k}$ (D) $3\hat{i}+\hat{j}-5\hat{k}$

35. Forces of 1 N and 2 N act along the lines $x=0$ and $y=0$. The equation of the line along which the resultant lies is given by

(A) $y-2x=0$ (B) $2y-x=0$

(C) $y+x=0$ (D) $y-x=0$

36. The radius vector of a point is $\vec{r}=(\hat{i}-2\hat{j}+3\hat{k})$ m and a force $\vec{F}=(4\hat{i}+5\hat{j})$ acts at that point. The moment of the force in Nm is

(A) $-15\hat{i}+12\hat{j}+13\hat{k}$ (B) $15\hat{i}-12\hat{j}+13\hat{k}$

(C) $-15\hat{i}-12\hat{j}+13\hat{k}$ (D) $15\hat{i}+12\hat{j}+13\hat{k}$

37. The area of a triangle bounded by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is

(A) $\frac{1}{2}|\vec{a}+\vec{b}+\vec{c}|$

(B) $\frac{1}{2}|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|$

(C) $\frac{1}{6}\left|\left[(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})+(\vec{a}\times\vec{b})\right]\right|$

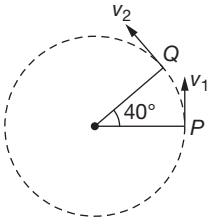
(D) $\frac{1}{2}|(\vec{a}\cdot\vec{b})+(\vec{b}\cdot\vec{c})+(\vec{c}\cdot\vec{a})|$

38. If $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-2\hat{k}$, the cosine of angle θ between them is equal to

(A) $\frac{5}{14}$ (B) $\frac{1}{7}$

(C) $\frac{3}{14}$ (D) $\frac{1}{14}$

39. A particle is moving in a circle of radius r centred at O with constant speed v. The change in velocity in moving from P to $Q(\angle POQ = 40°)$ is



(A) $2v\cos(40°)$ (B) $2v\sin(40°)$

(C) $2v\cos(20°)$ (D) $2v\sin(20°)$

40. Following set of forces act on a body. In which case the resultant cannot be zero?
(A) 10 N, 10 N, 20 N (B) 10 N, 10 N, 10 N
(C) 10 N, 20 N, 20 N (D) 10 N, 20 N, 40 N

41. $\frac{d}{dt}(\vec{A}\times\vec{B})$
(A) $\vec{A}\times\frac{d\vec{B}}{dt}+\vec{B}\times\frac{d\vec{A}}{dt}$ (B) $\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$
(C) $-\vec{A}\times\frac{d\vec{B}}{dt}-\frac{d\vec{A}}{dt}\times\vec{B}$ (D) $\vec{0}$

42. How many minimum number of coplanar vectors having different magnitudes can be added to give zero resultant.
(A) 2 (B) 3
(C) 4 (D) 5

43. How many minimum number of vectors in different planes can be added to give zero resultant?
(A) 2 (B) 3
(C) 4 (D) 5

44. If none of the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are zero and if $\vec{A}\times\vec{B}=0$ and $\vec{B}\times\vec{C}=0$, the value of $\vec{A}\times\vec{C}$ is
(A) unity (B) zero scalar
(C) zero vector (D) $AC\cos\theta$

45. In a watch the average angular velocity is maximum for the tip of
(A) second's hand (B) minute's hand
(C) hour's hand (D) equal for all hands

46. The volume of a parallelopiped bounded by vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be obtained from the expression
(A) $(\vec{A}\cdot\vec{B})\times\vec{C}$ (B) $(\vec{A}\times\vec{B})\cdot\vec{C}$
(C) $(\vec{A}\cdot\vec{C})\times\vec{B}$ (D) $(\vec{A}\times\vec{B})\times\vec{C}$

47. Pick up the axial vector
(A) force (B) acceleration
(C) linear momentum (D) torque

48. Pick out the only vector quantity
(A) pressure
(B) impulse
(C) gravitational potential
(D) coefficient of friction

49. Which of the following quantities is a scalar
(A) electric field
(B) electrostatic potential
(C) magnetic moment
(D) acceleration due to gravity

50. A body of mass 2 kg is constrained to move along the Y-direction. When a force of $2\hat{i}+5\hat{j}+7\hat{k}$ newton acts on it and the body is displaced through 10 m, the kinetic energy gained by the body is
(A) 50 J (B) 100 J
(C) 150 J (D) 350 J

51. If $\vec{a}$ denotes a unit vector along an incident light ray, $\vec{b}$ a unit vector along refracted ray into a medium having refractive index x (relative to first medium) and $\vec{c}$ a unit vector normal to boundary of two media and directed towards first medium, then law of refraction is
(A) $\vec{a}\cdot\vec{c}=x(\vec{b}\cdot\vec{c})$ (B) $\vec{a}\times\vec{c}=x(\vec{c}\times\vec{b})$
(C) $\vec{a}\times\vec{c}=x(\vec{b}\times\vec{c})$ (D) $x(\vec{a}\times\vec{c})=(\vec{b}\times\vec{c})$

52. The maximum and minimum magnitudes of the resultant of two given vectors are 17 unit and 7 unit respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is
(A) 14 (B) 16
(C) 18 (D) 13

53. A vector of magnitude $5\sqrt{3}$ unit and another vector of magnitude 10 unit are inclined to each other at an angle of $30°$. The magnitude of their vector product is
(A) $5\sqrt{3}$ unit (B) 10 unit
(C) $25\sqrt{3}$ unit (D) 50 unit

54. What is the value of linear velocity, if $\vec{\omega}=3\hat{i}-4\hat{j}+\hat{k}$ and $\vec{r}=5\hat{i}-6\hat{j}+6\hat{k}$;
(A) $6\hat{i}+2\hat{j}-3\hat{k}$ (B) $6\hat{i}-2\hat{j}+8\hat{k}$
(C) $4\hat{i}-13\hat{j}+6\hat{k}$ (D) $-18\hat{i}-13\hat{j}+2\hat{k}$

55. Which (one or more) of the following quantities is a vector?
(A) Pressure (B) Power
(C) Current (D) Angular momentum

56. The position vector of a particle is, $\vec{r}=(a\cos\omega t)\hat{i}+(a\sin\omega t)\hat{j}$. The velocity of the particle is
(A) parallel to position vector
(B) perpendicular to position vector
(C) directed towards the origin
(D) directed away from the origin

57. Given that $A=B=C$. If $\vec{A}+\vec{B}=\vec{C}$, then the angle between $\vec{A}$ and $\vec{C}$ is θ_1. If $\vec{A}+\vec{B}+\vec{C}=0$, then the angle between $\vec{A}$ and $\vec{C}$ is θ_2. What is the relation between θ_1 and θ_2?

(A) $\theta_1 = \theta_2$ (B) $\theta_1 = \frac{\theta_2}{2}$

(C) $\theta_1 = 2\theta_2$ (D) None of these

58. The magnitudes of the X and Y components of $\vec{P}$ are 7 and 6. Also, the magnitudes of the X and Y components $\vec{P}+\vec{Q}$ are 11 and 9 respectively. What is the magnitude of Q ?

(A) 5 (B) 6
(C) 8 (D) 9

59. Which of the following statements is false?
(A) Mass, speed and energy are scalar quantities
(B) Momentum, force and torque are vector quantities
(C) Distance is a scalar quantity but displacement is a vector quantity
(D) A vector has only magnitude, whereas a scalar has both magnitude and direction

60. The sum of the magnitudes of two vectors is 18 and the magnitude of their resultant is 12. If the resultant is perpendicular to one of the vectors, then what are the magnitudes of the two vectors?
(A) 5, 13 (B) 6, 12
(C) 7, 11 (D) 8, 10

61. When a mass is rotating in a plane about a fixed point its angular momentum is directed along
(A) the radius
(B) the tangent to the orbit
(C) the axis of rotation
(D) line at an angle of $45°$ to the axis of rotation

62. The momentum of a particle is $\vec{p} = 2\left[(\cos t)\hat{i} + (\sin t)\hat{j}\right]$. What is the angle between the force $\vec{F}$ acting on the particle and the momentum $\vec{p}$?

(A) $45°$ (B) $90°$
(C) $135°$ (D) $180°$

63. Three forces of magnitudes 30, 60 and P newton acting at a point are in equilibrium. If the angle between the first two is $60°$, the value of P is

(A) $25\sqrt{2}$ (B) $30\sqrt{3}$
(C) $30\sqrt{6}$ (D) $30\sqrt{7}$

64. The acceleration of a particle is given by $\vec{a} = \left[2\hat{i} + 6t\,\hat{j} + \frac{2\pi^2}{9}\cos\left(\frac{\pi t}{3}\right)\hat{k}\right]\text{ms}^{-2}$. The initial conditions are $\vec{r}(0) = x(0)\hat{i} + y(0)\hat{j} + z(0)\hat{k} = \vec{0}$, $\vec{v}(0) = \left(2\hat{i} + \hat{j}\right)\text{ms}^{-1}$ where

$$\vec{v}(0) = \frac{d\vec{r}(0)}{dt} = \frac{dx(0)}{dt}\hat{i} + \frac{dy(0)}{dt}\hat{j} + \frac{dz(0)}{dt}\hat{k}$$

The position vector at $t = 2$ s is

(A) $\left(3\hat{i} + 8\hat{j} + 10\hat{k}\right)$ m (B) $\left(8\hat{i} + 10\hat{j} + 3\hat{k}\right)$ m
(C) $\left(10\hat{i} + 3\hat{j} + 8\hat{k}\right)$ m (D) $\left(3\hat{i} + 10\hat{j} + 8\hat{k}\right)$ m

65. Pick out the only scalar quantity
(A) power (B) electric field
(C) magnetic moment (D) average velocity

66. Angular displacement is
(A) a scalar (B) a vector
(C) either (D) neither

67. If the vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other, then the positive value of a is
(A) 3 (B) 2
(C) 1 (D) 0

68. A particle of mass $m = 5$ unit is moving with a uniform speed $v = 3\sqrt{2}$ unit in the XOY plane along the line $y = x + 4$. The magnitude of the angular momentum of the particle about the origin is

(A) 60 unit (B) $40\sqrt{2}$ unit
(C) ZERO (D) 7.5 unit

69. Consider three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$. Which of the following is independent of choice of coordinate system?

(A) $\left(\vec{P} + \vec{Q} + \vec{R}\right)$ (B) $\left(P_x + Q_x + R_x\right)\hat{i}$
(C) $\left(P_x\hat{i} + Q_y\hat{j} + R_z\hat{k}\right)$ (D) All of these

70. Force acting on a particle is $\left(2\hat{i} + 3\hat{j}\right)$ N. Work done by this force is zero, when a particle is moved on the line $3y + kx = 5$. Here value of k is

(A) 2 (B) 4
(C) 6 (D) 8

71. A wheel of radius r is rolling on a horizontal surface. At $t = 0$, T is the top most point on the wheel then magnitude of displacement of point T when wheel has completed one-fourth rolling is

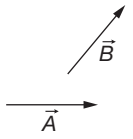
(A) $\sqrt{2}r$ (B) $2r$
(C) $r\sqrt{\left(\frac{\pi}{2} - 1\right)^2 + 1}$ (D) $r\sqrt{\left(\frac{\pi}{2} + 1\right)^2 + 1}$

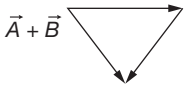
72. The direction of resultant of $\vec{a}$ and $\vec{b}$ remains same when magnitudes of $\vec{a}$ and $\vec{b}$ are increased by m units and n units respectively then

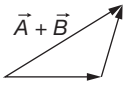
(A) $\frac{a}{b}=\frac{m}{n}$ (B) $\frac{a}{b}=\frac{n}{m}$

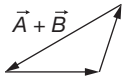
(C) $\frac{a}{b}=\frac{m+n}{m-n}$ (D) $\frac{a}{b}=\frac{m-n}{m+n}$

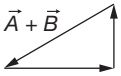
73. Vectors $\vec{A}$ and $\vec{B}$ are shown in figure then diagram of $\vec{A}+\vec{B}$ is



(A) 

(B) 

(C) 

(D) 

74. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is

(A) 120° (B) 60°

(C) 90° (D) 150°

75. If $\vec{a}$ and $\vec{b}$ are two unit vectors and $\vec{R}=\vec{a}+\vec{b}$ and also if $|\vec{R}|=R$, then

(A) $R<0$ (B) $R>2$

(C) $0\le R\le 2$ (D) R must be 2

76. If $\vec{a}$ and $\vec{b}$ are two sides of a parallelogram and $\vec{c}$ and $\vec{d}$ are the diagonals, then

(A) $c^2+d^2=a^2+b^2$ B) $c^2+d^2=2(a^2+b^2)$

(C) $c^2-d^2=a^2-b^2$ (D) $c^2-d^2=2(a^2-b^2)$

77. In Problem 76, if the angle between $\vec{a}$ and $\vec{b}$ is θ, then $\cos\theta$ is equal to

(A) $\frac{(a^2+b^2)}{4cd}$ (B) $\frac{(a^2-b^2)}{4cd}$

(C) $\frac{(c^2-d^2)}{4ab}$ (D) $\frac{(c^2+d^2)}{4ab}$

78. If magnetic force on a charge is given by $q(\vec{V}\times\vec{B})$ and a charged particle is projected in a magnetic field $(2\hat{i}+2\hat{j}+2\hat{k})$ tesla. The acceleration of the particle at an instant is $(x\hat{i}+2\hat{j}-6\hat{k})$ ms^{-2}, then of x equals

(A) 4 (B) 2

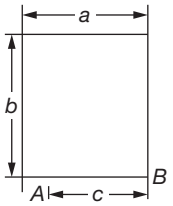
(C) 3 (D) 1

79. There are two vectors $\vec{A}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=\hat{i}+2\hat{j}-2\hat{k}$, then vector component of $\vec{A}$ along $\vec{B}$ is

(A) $\frac{2\hat{i}+2\hat{j}-2\hat{k}}{9}$ (B) $\frac{2}{3}(2\hat{i}+\hat{j}+\hat{k})$

(C) $\frac{2}{3}(\hat{i}+2\hat{j}-2\hat{k})$ (D) None of these

80. A billiards ball is at point A on a billiards table whose dimensions are shown in figure.

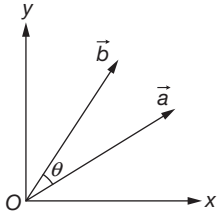


At what angle should the ball be struck so that it should rebound from two cushions and go into pocket B? Assume that in striking the cushion, the ball's direction of motion changes according to the law of reflection of light from a mirror, i.e., the angle of reflection equals the angle of incidence.

(A) $\frac{a-2c}{b}$ (B) $\frac{2a-c}{b}$

(C) $\frac{2a-c}{2b}$ (D) $\frac{a}{b}-\frac{a}{2c}$

81. For the vectors $\vec{a}$ and $\vec{b}$ shown in figure, $\vec{a}=\sqrt{3}\hat{i}+\hat{j}$ and $|\vec{b}|=10$ units while $\theta=23°$, then the value of $R=|\vec{a}+\vec{b}|$ is nearly



(A) 12 (B) 13

(C) 14 (D) 15

82. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular if

(A) $\vec{A}$ and $\vec{B}$ are perpendicular to each other and $|\vec{A}|=|\vec{B}|$

(B) $\vec{A}$ and $\vec{B}$ are perpendicular to each other

(C) $\vec{A}$ and $\vec{B}$ are perpendicular but their magnitudes are arbitrary

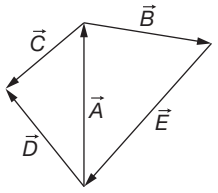
(D) $|\vec{A}|=|\vec{B}|$ and their directions are arbitrary

83. A particle moves in x-y plane. The position vector of particle at any time t is $\vec{r}=\left\{(2t)\hat{i}+(2t^2)\hat{j}\right\}$ m. The rate of change of θ at time $t=2$ s, where θ is the angle which its velocity vector makes with positive x-axis) is

(A) $\frac{2}{17}$ rads^{-1} (B) $\frac{1}{14}$ rads^{-1}

(C) $\frac{4}{7}$ rads^{-1} (D) $\frac{6}{5}$ rads^{-1}

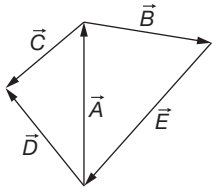
84. If figure, $\vec{E}$ equals



(A) $\vec{A}$ (B) $\vec{B}$

(C) $\vec{A}+\vec{B}$ (D) $-(\vec{A}+\vec{B})$

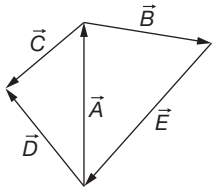
85. In figure, $\vec{D}-\vec{C}$ equals



(A) $\vec{A}$ (B) $-\vec{A}$

(C) $\vec{B}$ (D) $-\vec{B}$

86. In figure, $\vec{E}+\vec{D}-\vec{C}$ equals



(A) $\vec{A}$ (B) $-\vec{A}$

(C) $\vec{B}$ (D) $-\vec{B}$

87. The linear velocity of a rotating body is given by $\vec{v}=\vec{\omega}\times\vec{r}$, where ω is the angular velocity and $\vec{r}$ is the radius vector. The angular velocity of a body $\omega=\vec{\omega}=\hat{i}-2\hat{j}+2\hat{k}$ and their radius vector $\vec{r}=4\hat{j}-3\hat{i}$, then $|\vec{v}|$ is

(A) $\sqrt{29}$ units (B) 31 units

(C) $\sqrt{37}$ units (D) $\sqrt{41}$ units

88. Which of the following sets of displacements might be capable of returning a car to its starting point?

(A) 4, 6, 8 and 15 km (B) 10, 30, 50 and 120 km

(C) 5, 10, 30 and 50 km (D) 50, 50, 75 and 200 km

89. The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ may be

(a) Greater than AB (b) Equal to AB

(c) Less than AB (d) Equal to Zero

(A) a, b, c (B) b, c, d

(C) a, c, d (D) a, b, d

90. If $\vec{A}=\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{B}=3\hat{i}+6\hat{j}+2\hat{k}$, then the vector in the direction of $\vec{A}$ and having same magnitude as $|\vec{B}|$, is

(A) $\frac{7}{3}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ (B) $7\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

(C) $\frac{3}{7}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ (D) $\frac{7}{9}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

91. The area of the triangle whose vertices are $A(1,-1,2)$, $B(2,1,-1)$ and $C(3,-1,2)$ is

(A) 26 (B) $7\sqrt{13}$

(C) $\sqrt{13}$ (D) 8

92. An engine exerts a force $\vec{F}=\left(20\hat{i}-3\hat{j}+5\hat{k}\right)N$ and moves with velocity $\vec{v}=\left(6\hat{i}+20\hat{j}-3\hat{k}\right)$ ms^{-1}. The power of the engine (in watt) is

(A) 45 (B) 75

(C) 20 (D) 10

93. The maximum and the minimum magnitudes of the resultant of two given vectors are 17 unit and 7 unit respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is

(A) 14 (B) 16
(C) 18 (D) 13

94. For two vectors $\vec{a}$ and $\vec{b}$, if $\vec{R}=\vec{a}+\vec{b}$ and $\vec{S}=\vec{a}-\vec{b}$, if $2|\vec{R}|=|\vec{S}|$, when $\vec{R}$ is perpendicular to $\vec{a}$, then

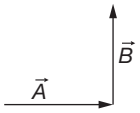
(A) $\frac{a}{b}\sqrt{\frac{3}{7}}$ (B) $\frac{a}{b}=\sqrt{\frac{7}{3}}$
(C) $\frac{a}{b}=\sqrt{\frac{1}{5}}$ (D) $\frac{a}{b}=\sqrt{\frac{5}{1}}$

95. Which one is correct?
(A) Resultant of two vectors of unequal magnitude can be zero
(B) Resultant of three non-coplanar vectors of equal magnitude can be zero
(C) Resultant of three coplanar vectors is always zero
(D) Minimum number of non-coplanar vectors whose resultant can be zero is four

96. If $\vec{r}=bt^2\hat{i}+ct^3\hat{j}$, where b and c are positive constant, the time at which velocity vector makes an angle $\theta=60°$ with positive y-axis is

(A) $\frac{c}{b}$ (B) $\frac{2b}{3\sqrt{3}c}$
(C) $\frac{2c}{\sqrt{3}b}$ (D) $\frac{2b}{\sqrt{3}c}$

97. Two vectors $\vec{A}$ and $\vec{B}$ are shown in the figure



Then $\vec{A}-\vec{B}$ is given by

(A) (B)
(C) (D) None of these

98. Resultant of two vectors $\vec{A}$ and $\vec{B}$ is $\vec{R}$. Now magnitude of vector is doubled keeping direction same, then magnitude of the resultant becomes $|\vec{B}|$. Angle between vector $\vec{A}$ and $\vec{B}$ is $120°$. Then magnitude of $\vec{A}$ is equal to

(A) $|\vec{B}|$ (B) $2|\vec{B}|$
(C) $\frac{|\vec{B}|}{2}$ (D) $4|\vec{B}|$

99. If $\vec{R}$ is the resultant of the vectors $\vec{A}$ and $\vec{B}$. The ratio of minimum value of $|\vec{R}|$ to the maximum value of $|\vec{R}|$ is $\frac{1}{4}$. Then $\frac{A}{B}$ may be

(A) $\frac{4}{1}$ (B) $\frac{2}{1}$
(C) $\frac{3}{5}$ (D) $\frac{1}{4}$

100. If a vector $\left(2\hat{i}+3\hat{j}+8\hat{k}\right)$ is perpendicular to the vector $-4\hat{i}+4\hat{j}+\alpha\hat{k}$, then the value of α is

(A) -1 (B) $\frac{1}{2}$
(C) $\frac{-1}{2}$ (D) 1

101. The unit vector parallel to the resultant of the vectors $\vec{A}=4\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{B}=-\hat{i}+3\hat{j}-8\hat{k}$ is

(A) $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$ (B) $\frac{1}{7}\left(3\hat{i}+6\hat{j}+2\hat{k}\right)$
(C) $\frac{1}{49}\left(3\hat{i}+6\hat{j}+2\hat{k}\right)$ (D) $\frac{1}{49}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$

102. Following sets of three forces act on a body. Whose resultant cannot be zero?
(A) 10 , 10, 10 (B) 10, 10, 20
(C) 10, 20, 20 (D) 10, 20, 40

103. A vector $\vec{F}_1$ is along the positive x-axis. If its vector product with another vector $\vec{F}_2$ is zero then $\vec{F}_2$ may be

(A) $4\hat{j}$ (B) $-\left(\hat{i}+\hat{j}\right)$
(C) $\left(\hat{i}+\hat{k}\right)$ (D) $-4\hat{i}$

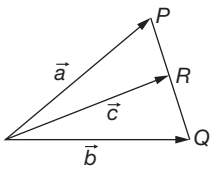
104. The component of $3\hat{i}+4\hat{j}$ along $\hat{i}+\hat{j}$

(A) $\frac{\hat{i}+\hat{j}}{2}$ (B) $\frac{3}{2}(\hat{i}+\hat{j})$
(C) $\frac{5}{2}\left(\hat{i}+\hat{j}\right)$ (D) $\frac{7}{2}\left(\hat{i}+\hat{j}\right)$

105. A vector of length l is turned through the angle θ about its tail. The change in the position vector of its head is

(A) $l\sin\left(\frac{\theta}{2}\right)$ (B) $2l\sin\left(\frac{\theta}{2}\right)$
(C) $2l\cos\left(\frac{\theta}{2}\right)$ (D) $l\cos\left(\frac{\theta}{2}\right)$

106. Figure shows three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, where R is the mid point of PQ. Then which of the following relation is correct?



(A) $\vec{a}+\vec{b}=2\vec{c}$ (B) $\vec{a}+\vec{b}=\vec{c}$

(C) $\vec{a}-\vec{b}=2\vec{c}$ (D) $\vec{a}-\vec{b}=\vec{c}$

107. A unit vector perpendicular to both vectors $\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ is

(A) $\hat{i}+\hat{j}$ (B) $\dfrac{-\hat{i}+\hat{k}}{\sqrt{2}}$

(C) $\hat{j}+\hat{k}$ (D) $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$

108. The adjacent sides of a parallelogram are represented by co-initial vectors $2\hat{i}+3\hat{j}$ and $\hat{i}+4\hat{j}$. The area of the parallelogram is

(A) 5 units along z-axis (B) 5 units in x-y plane

(C) 3 units in x-z plane (D) 3 units in y-z plane

109. Given that $\vec{A}+\vec{B}+\vec{C}=0$, out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

(A) 30°, 60°, 90° (B) 45°, 45°, 90°

(C) 45°, 60°, 90° (D) 90°, 135°, 135°

110. If a particle is moving on an elliptical path given by $\vec{r}=b\cos(\omega t)\hat{i}+a\sin(\omega t)\hat{j}$, then radial acceleration along $\vec{r}$ is

(A) $\omega\vec{r}$ (B) $\omega^2\vec{r}$

(C) $-\omega^2\vec{r}$ (D) None of these

111. Two forces of magnitudes 30, 60 and P Newton acting at a point are in equilibrium. If the angle between the first two is $60°$, the value of P is

(A) $25\sqrt{2}$ (B) $30\sqrt{3}$

(C) $30\sqrt{6}$ (D) $30\sqrt{7}$

112. The angle made by the vector $4\hat{i}-3\hat{j}+5\hat{k}$ with z-axis is

(A) 30° (B) 45°

(C) 90° (D) 120°

113. Given $|\vec{A}_1|=2$, $|\vec{A}_2|=3$ and $|\vec{A}_1+\vec{A}_2|=3$ then $(\vec{A}_1+2\vec{A}_2)\cdot(3\vec{A}_1-4\vec{A}_2)$ is

(A) –64 (B) 60

(C) –62 (D) None of these

114. Vector $\vec{A}$ makes equal angles with x, y and z-axis. Value of its components (in terms of magnitude of $\vec{A}$) will be

(A) $\dfrac{A}{\sqrt{3}}$ (B) $\dfrac{A}{\sqrt{2}}$

(C) $\sqrt{3}A$ (D) $\dfrac{\sqrt{3}}{A}$

115. The angle made by the vector $A=\hat{i}+\hat{j}$ with x-axis is

(A) 90° (B) 45°

(C) 22.5° (D) 30°

116. If a unit vector is represented by $0.5\hat{i}+0.8\hat{j}+c\hat{k}$, then the value of c is

(A) 1 (B) $\sqrt{0.11}$

(C) $\sqrt{0.01}$ (D) $\sqrt{0.39}$

117. The component of vector $A=2\hat{i}+3\hat{j}$ along the vector $\hat{i}+\hat{j}$ is

(A) $\dfrac{5}{\sqrt{2}}$ (B) $10\sqrt{2}$

(C) $5\sqrt{2}$ (D) 5

118. There are two force vectors, one of 5 N and other of 12 N at what angle the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively

(A) 0°, 180° and 90° (B) 0°, 90° and 180°

(C) 0°, 90° and 90° (D) 180°, 0° and 90°

119. If $\vec{A}=4\hat{i}-3\hat{j}$ and $\vec{B}=6\hat{i}+8\hat{j}$ then magnitude and direction of $\vec{A}+\vec{B}$ will be

(A) 5, $\tan^{-1}\left(\dfrac{3}{4}\right)$ (B) $5\sqrt{5}$, $\tan^{-1}\left(\dfrac{1}{2}\right)$

(C) 10, $\tan^{-1}(5)$ (D) 25, $\tan^{-1}\left(\dfrac{3}{4}\right)$

120. Two forces 3 N and 2 N are at an angle θ such that the resultant is R. The first force is now increased to 6 N and the resultant become $2R$. The value of θ is

(A) $30°$ (B) $60°$
(C) $90°$ (D) $120°$

121. If $\vec{A}\times\vec{B}=\vec{C}$, then which of the following statements is wrong?
(A) $\vec{C}\perp\vec{A}$ (B) $\vec{C}\perp\vec{B}$
(C) $\vec{C}\perp(\vec{A}+\vec{B})$ (D) $\vec{C}\perp(\vec{A}\times\vec{B})$

122. The velocity of a particle is $\vec{v}=6\hat{i}+2\hat{j}-2\hat{k}$. The component of the velocity of a particle parallel to vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ in vector form is
(A) $6\hat{i}+2\hat{j}+2\hat{k}$ (B) $2\hat{i}+2\hat{j}+2\hat{k}$
(C) $\hat{i}+\hat{j}+\hat{k}$ (D) $6\hat{i}+2\hat{j}-2\hat{k}$

123. Two forces of magnitude 7 newton and 5 newton act on a particle at an angle θ to each other. The minimum magnitude of the resultant force is
(A) 5 newton (B) 8 newton
(C) 12 newton (D) 2 newton

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. Which of the following will not depend on orientation of frame of reference
(A) a scalar
(B) a vector
(C) the magnitude of a vector
(D) component of a vector

2. If $\vec{P}\times\vec{Q}=\vec{R}$, $\vec{Q}\times\vec{R}=\vec{P}$ and $\vec{R}\times\vec{P}=\vec{Q}$, then
(A) $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are coplanar.
(B) angle between $\vec{P}$ and $\vec{Q}$ may be less than $90°$.
(C) $(\vec{P}+\vec{Q}+\vec{R})$ cannot be equal to zero.
(D) $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are mutually perpendicular.

3. A particle is moving towards east with velocity $10\ \text{ms}^{-1}$. It suddenly turns to left and continues to move with velocity v now. Change in its velocity $\Delta\vec{v}$
(A) cannot be less than $10\ \text{ms}^{-1}$ in magnitude
(B) will be directed $\theta\,NW$, where θ will depend upon magnitude of v
(C) will be directed $\theta\,NE$, where θ will depend upon magnitude of v
(D) None of these

4. For two vectors $\vec{A}$ and $\vec{B}$ which of the following relations are not commutative
(A) $\vec{A}+\vec{B}$ (B) $\vec{A}-\vec{B}$
(C) $\vec{A}\times\vec{B}$ (D) $\vec{A}\cdot\vec{B}$

5. Four forces acting on a particle keep it at rest. Then
(A) the forces must be coplanar.
(B) the forces cannot be coplanar.
(C) the forces may or may not be coplanar.
(D) if three of these forces are coplanar, so must be the fourth.

6. A vector will not change when
(A) frame of reference is translated
(B) frame of reference is rotated
(C) vector is translated parallel to itself
(D) vector is rotated

7. If a vector $\vec{A}$ has magnitude A and $\vec{n}$ is a unit vector in the direction of $\vec{A}$, then which of the following are correct
(A) $A=\dfrac{\vec{A}}{\vec{n}}$ (B) $\vec{n}=\dfrac{\vec{A}}{A}$
(C) $1=\dfrac{|\vec{A}|}{A}$ (D) $\vec{A}=A\vec{n}$

8. If $\vec{P}\times\vec{Q}=\vec{R}$ then which of the following statements is/are correct?
(A) $\vec{R}$ is perpendicular to $(\vec{P}+\vec{Q})$
(B) $\vec{R}$ is perpendicular to $\vec{P}\cdot\vec{Q}$
(C) $\vec{R}$ is perpendicular to $\vec{P}$
(D) $\vec{P}$ is perpendicular to $\vec{Q}\times\vec{R}$

9. Mark the correct statement/s.

(A) the magnitude of the vector $\left(3\hat{i}+4\hat{j}\right)$ is 5

(B) a force $\left(3\hat{i}+4\hat{j}\right)$ N acting on a particle causes a displacement of $5\hat{j}$ metres. The work done by the force is 25 J

(C) if $\vec{a}$ and $\vec{b}$ represent two adjacent sides of a parallelogram, $|\vec{a}\times\vec{b}|$ gives the area of that parallelogram

(D) a force has magnitude 20 N. Its component in a direction making an angle of $60°$ with the force is $10\sqrt{3}$ N

10. The angle that the vector $\vec{A}=2\hat{i}+3\hat{j}$ makes with y-axis is

(A) $\tan^{-1}\left(\frac{3}{2}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$ (D) $\sin^{-1}\left(\frac{2}{3}\right)$

11. The momentum of a particle is given by $\vec{p}=2\left[(\sin t)\hat{i}-(\cos t)\hat{j}\right]$ kgms^{-1}. Select the correct options

(A) momentum $\vec{p}$ of the particle is always perpendicular to $\vec{F}$

(B) momentum $\vec{p}$ of the particle is always parallel to $\vec{F}$

(C) magnitude of momentum remain constant

(D) None of these

12. Mark the correct statement(s)

(A) displacement is a polar vector.

(B) angular displacement is a polar vector.

(C) displacement is an axial vector.

(D) angular displacement is an axial vector.

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT-2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.

Statement-2: Both $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ lie in the plane containing $\vec{A}$ and $\vec{B}$, but $\vec{A}\times\vec{B}$ lies perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

2. **Statement-1:** Angle between $\hat{i}+\hat{j}$ and $\hat{i}$ is $45°$.

Statement-2: $\hat{i}+\hat{j}$ is equally inclined to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is $90°$.

3. **Statement-1:** If θ be the angle between $\vec{A}$ and $\vec{B}$, then $\tan\theta=\frac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}}$.

Statement-2: $\vec{A}\times\vec{B}$ is perpendicular to $\vec{A}+\vec{B}$.

4. **Statement-1:** If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is $90°$.

Statement-2: $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.

5. **Statement-1:** Vector product of two vectors represents an axial vector.

Statement-2: If $\vec{v}$ = instantaneous velocity, $\vec{r}$ = radius vector and $\vec{\omega}$ = angular velocity, then $\vec{\omega}=\vec{v}\times\vec{r}$.

6. **Statement-1:** Minimum number of non-equal vectors in a plane required to give zero resultant is three.

Statement-2: If $\vec{A}+\vec{B}+\vec{C}=\vec{0}$, then they must lie in one plane.

7. **Statement-1:** Vector addition of two vectors $\vec{A}$ and $\vec{B}$ is commutative.

Statement-2: $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.

8. **Statement-1:** $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$.

Statement-2: Dot product of two vectors is commutative.

9. **Statement-1:** $\vec{\tau}=\vec{r}\times\vec{F}$ and $\vec{\tau}\neq\vec{F}\times\vec{r}$.

Statement-2: Cross product of vectors is commutative.

10. **Statement-1:** A negative acceleration of a body is associated with a slowing down of a body.
Statement-2: Acceleration is vector quantity.

11. **Statement-1:** A physical quantity cannot be called as a vector if its magnitude is zero.
Statement-2: A vector has both, magnitude and direction.

12. **Statement-1:** The sum of two vectors can be zero.
Statement-2: The vectors cancel each other, when they are equal and opposite.

13. **Statement-1:** Two vectors are said to be like vectors if they have same direction but different magnitude.
Statement-2: Vector quantities do not have specific direction.

14. **Statement-1:** The scalar product of two vectors can be zero.
Statement-2: If two vectors are perpendicular to each other, their scalar product will be zero.

15. **Statement-1:** Multiplying any vector by a scalar is a meaningful operations.
Statement-2: Taking dot product of a scalar and a vector is meaningless.

16. **Statement-1:** A null vector is a vector whose magnitude is zero and direction is arbitrary.
Statement-2: A null vector does not exist.

17. **Statement-1:** If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that either of the vectors $\vec{A}$ and $\vec{B}$ must be a null vector.
Statement-2: Null vector is a vector with zero magnitude.

18. **Statement-1:** The cross product of a vector with itself is a null vector.
Statement-2: The cross product of two vectors results in a vector quantity.

19. **Statement-1:** The minimum number of non coplanar vectors whose sum can be zero, is four.
Statement-2: The resultant of two vectors of unequal magnitude can be zero.

20. **Statement-1:** If $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{C}$, then $\vec{A}$ may not always be equal to $\vec{C}$.
Statement-2: The dot product of two vectors involves cosine of the angle between the two vectors.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I

Consider a parallelogram whose adjacent sides are given by $\vec{a}=2\vec{m}+\vec{n}$ and $\vec{b}=\vec{m}-2\vec{n}$, where $\vec{m}$ and $\vec{n}$ are unit vectors inclined at an angle of $60°$. Based on this information, answer the following questions.

1. One diagonal of the parallelogram is represented by the vector
(A) $\vec{m}+\vec{n}$ (B) $3\vec{m}+\vec{n}$
(C) $3\vec{m}-\vec{n}$ (D) $-\vec{m}+2\vec{n}$

2. The other diagonal of the parallelogram is represented by the vector
(A) $\vec{n}-3\vec{m}$ (B) $\vec{m}+3\vec{n}$
(C) $-\vec{m}-3\vec{n}$ (D) $\vec{m}-\vec{n}$

3. The length of the diagonal found in PROBLEM 1 is
(A) $\sqrt{7}$ units (B) $\sqrt{10}$ units
(C) $\sqrt{12}$ units (D) $\sqrt{13}$ units

4. The length of the diagonal found in PROBLEM 2 is
(A) $\sqrt{7}$ units (B) $\sqrt{10}$ units
(C) $\sqrt{12}$ units (D) $\sqrt{13}$ units

5. The area of the parallelogram is
(A) $2\sqrt{3}$ sq. units
(B) $5\sqrt{3}$ sq. units
(C) $\frac{5}{2}\sqrt{3}$ sq. units
(D) $\frac{5}{4}\sqrt{3}$ sq. units

6. If the angle between the diagonals is θ. Then
(A) $\theta=\cos^{-1}\left(\frac{2}{\sqrt{91}}\right)$ (B) $\theta=\cos^{-1}\left(\frac{1}{\sqrt{91}}\right)$
(C) $\theta=\cos^{-1}\left(\frac{7}{\sqrt{91}}\right)$ (D) $\theta=\cos^{-1}\left(\frac{4}{\sqrt{91}}\right)$

Comprehension 2

One of the principles of addition of vector is the parallelogram law which says that "if two vectors $\vec{a}$ and $\vec{b}$ are acting at point O, then the resultant of these two vectors is obtained by completing a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$. The resultant is the diagonal of the parallelogram passing through the same point O." Based on this information, answer the following questions.

7. The diagonal $\vec{p}$ of parallelogram passing through point O is

(A) $\vec{a}-\vec{b}$ (B) $\vec{a}+\vec{b}$

(C) $\vec{a}+\sqrt{2}\vec{b}$ (D) $\vec{b}+\sqrt{2}\vec{a}$

8. The other diagonal $\vec{q}$ of the parallelogram is

(A) $\vec{a}-\vec{b}$ (B) $\vec{a}+\vec{b}$

(C) $\vec{a}+\sqrt{2}\vec{b}$ (D) $\vec{b}+\sqrt{2}\vec{a}$

9. The diagonals $\vec{p}$ and $\vec{q}$ will be perpendicular to each other if

(A) $|\vec{a}|=|\vec{b}|$ (B) $|\vec{a}|=2|\vec{b}|$

(C) $|\vec{b}|=2|\vec{a}|$ (D) None of these

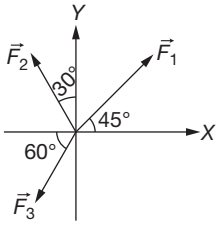
10. Angle between $\vec{a}+\vec{b}$ and $\vec{a}\times\vec{b}$ is

(A) 0° (B) 45°

(C) 60° (D) 90°

Comprehension 3

Three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ act on a particle as shown in the figure. If $|\vec{F}_1|=5\sqrt{2}$ N, $|\vec{F}_2|=10$ N and $|\vec{F}_3|=20$ N. Based on this information, answer the following questions.



11. X component of $\vec{F}_2$ is

(A) 5 N (B) –5 N

(C) $5\sqrt{3}$ N (D) $-5\sqrt{3}$ N

12. Y component of $\vec{F}_3$ is

(A) $10\sqrt{3}$ N (B) $-10\sqrt{3}$ N

(C) –10 N (D) 10 N

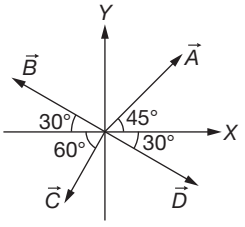
13. $\vec{F}_1$ is written as

(A) $\left(5\hat{i}-5\hat{j}\right)$ N (B) $\left(-5\hat{i}+5\hat{j}\right)$ N

(C) $\left(5\hat{i}+5\hat{j}\right)$ N (D) $5\sqrt{2}\left(\hat{i}+\hat{j}\right)$ N

Comprehension 4

Four vectors are shown in the figure where $|\vec{A}|=5\sqrt{2}$ m, $|\vec{B}|=10$ m, $|\vec{C}|=10$ m and $|\vec{D}|=10$ m. Based on this information, answer the following questions.



14. (B_x+D_x) is equal to

(A) $20\sqrt{3}$ m (B) $-10\sqrt{3}$ m

(C) zero (D) 10 m

15. (A_x+C_x) is equal to

(A) zero (B) 10 m

(C) –5 m (D) $-5\sqrt{3}$ m

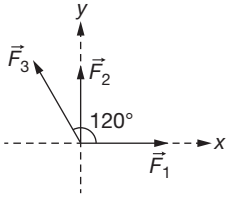
16. $\left(A_y+B_y+C_y+D_y\right)$ is equal to

(A) $5\left(1-\sqrt{3}\right)$ m (B) $5(-1)$ m

(C) $5\sqrt{3}$ m (D) $10\left(\sqrt{3}-1\right)$ m

Comprehension 5

Three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are acting on a body as shown in figure.



If $F_1=10$ N, $F_2=10\sqrt{3}$ N and $F_3=20$ N.

Based on this information, answer the following questions.

17. External force required to keep block at rest is (in newton)

(A) $20\sqrt{3}\hat{j}$ (B) $-20\sqrt{3}\hat{j}$

(C) $20\sqrt{2}\hat{i}$ (D) $20\hat{i}-20\sqrt{3}\hat{j}$

18. $\vec{F}_1 + \vec{F}_2 - \vec{F}_3$ is given by

(A) 20 newton in x-direction

(B) 20 newton in y-direction

(C) $20\sqrt{3}$ newton in y-direction

(D) -20 newton in x-direction

19. $\vec{F}_2 + \vec{F}_3 - \vec{F}_1$ is given by

(A) 20 newton at angle $120°$ with x-axis

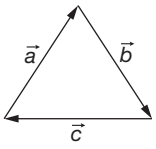
(B) 40 newton at angle $120°$ with x-axis

(C) $20\sqrt{3}$ newton at angle $150°$ with x-axis

(D) $40\sqrt{3}$ newton at angle $150°$ with x-axis

Comprehension 6

Three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ form a triangle taken in same order. Based on this information, answer the following questions.



20. $\vec{a} + \vec{b} - \vec{c} =$ _____

(A) $-2\vec{a}$ (B) $-2\vec{b}$

(C) $-2\vec{c}$ (D) zero

21. $(\vec{a} \times \vec{b}) \cdot \vec{c} =$ _____

(A) abc (B) $-abc$

(C) -1 (D) 0

22. $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} =$ _____

(A) $-b^2$ (B) $-a^2$

(C) $-c^2$ (D) zero

23. If triangle is equilateral, then angle between $\vec{a}$ and $\vec{b}$ is

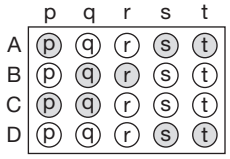
(A) $60°$ (B) $120°$

(C) $150°$ (D) $90°$

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:



1. Match the following

Column-I	Column-II
(A) $\vec{A} \cdot \vec{B}$	(p) Defined
(B) $(\vec{A} \cdot \vec{B})\vec{C}$	(q) Not Defined
(C) $(\vec{A} \cdot \vec{B}) \cdot \vec{C}$	(r) Scalar
(D) $(\vec{A} \times \vec{B}) \times \vec{C}$	(s) Vector

2. Match the following

Column-I	Column-II
(A) $\vec{A} \times \vec{B}$	(p) Zero Scalar
(B) $(\vec{A} \times \vec{B}) \cdot \vec{A}$	(q) Perpendicular to $\vec{A}$
(C) $\vec{A} \times \vec{A}$	(r) Zero Vector
(D) $\vec{0} \times \vec{A}$	(s) Perpendicular to $\vec{B}$

3. Match the following

Column-I	Column-II
(A) Parallel Vectors	(p) $\lvert\vec{A}\times\vec{B}\rvert$
(B) Perpendicular Vectors	(q) $\left(\frac{\vec{A}\cdot\vec{B}}{B^2}\right)\vec{B}$
(C) $\vec{A}\times\vec{B}+\vec{C}\times\vec{A}=\vec{0}$	(r) $\vec{A}\times\vec{B}=\vec{0}$
(D) Area of Parallelogram	(s) $\vec{B}=\vec{C}+k\vec{A}$
(E) Component Vector of $\vec{A}$ along $\vec{B}$	(t) $\vec{A}\cdot\vec{B}=0$

4. Match the following

Column-I	Column-II
(A) Resultant of two ordered vectors	(p) Zero Vector
(B) Dot Product	(q) Always Negative
(C) $\vec{A}\cdot\vec{B}=0$ and $\lvert\vec{A}\times\vec{B}\rvert=0$	(r) Triangle Law
(D) $\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{A}\times\vec{C})+\vec{C}\times(\vec{A}\times\vec{B})$	(s) Commutative in nature
(E) Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ taken in same order form a closed polygon, then $\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}$ is	(t) $\lvert\vec{A}\rvert=0$ OR $\lvert\vec{B}\rvert=0$

5. Match the following

Column-I	Column-II
(A) $\lvert\vec{A}+\vec{B}\rvert=\lvert\vec{A}-\vec{B}\rvert$	(p) $\lvert\vec{A}\rvert=\lvert\vec{B}\rvert$
(B) $\vec{A}-\vec{B}=\vec{A}+\vec{B}$	(q) $2(\vec{B}\times\vec{A})$
(C) $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$	(r) $\vec{B}=\vec{0}$
(D) $(\vec{A}+\vec{B})\perp(\vec{A}-\vec{B})$	(s) $\vec{A}\perp\vec{B}$
	(t) $\vec{A}\cdot\vec{B}=0$

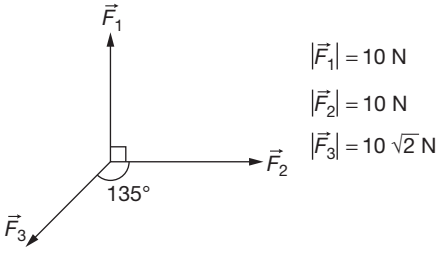
6. Match the following

Column-I	Column-II
(A) Resultant of two vectors of equal magnitude	(p) $\vec{A}\cdot\frac{d\vec{A}}{dt}=0$
(B) For a vector $\vec{A}$ of fixed magnitude and direction	(q) will be zero
(C) For any vector $\vec{A}$	(r) lies along the angle bisector of two vectors
(D) Resultant of n vectors of equal magnitude each inclined at an angle $\frac{2\pi}{n}$ with the preceding vector	(s) $\vec{A}\times\frac{d\vec{A}}{dt}=\vec{0}$

7. If $\vec{R}=\vec{a}+\vec{b}$ and $\vec{S}=\vec{a}-\vec{b}$ also θ is angle between $\vec{a}$ and $\vec{b}$.

Column-I	Column-II
(A) R^2+S^2	(p) $\vec{R}$ is perpendicular to $\vec{a}$
(B) R^2-S^2	(q) $2(a^2+b^2)$
(C) $\frac{R}{S}$	(r) $4\vec{a}\cdot\vec{b}$
(D) $R<S$	(s) $\tan\left(\frac{\theta}{2}\right)$ if $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert$

8. Three coplanar forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are acting simultaneously on a particle. **COLUMN-I** contains different operation between forces and **COLUMN-II** contains their magnitude. Match them

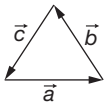
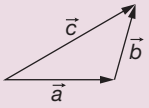
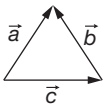
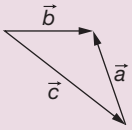


Column-I	Column-II
(A) $\vec{F}_1+\vec{F}_2+\vec{F}_3$	(p) 0
(B) $\vec{F}_1+\vec{F}_2-\vec{F}_3$	(q) 20
(C) $\vec{F}_1-\vec{F}_2+\vec{F}_3$	(r) $20\sqrt{2}$
(D) $\vec{F}_3\cdot(\vec{F}_1+\vec{F}_2)$	(s) 200

9. If $\vec{A}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{B}=\hat{i}+2\hat{j}+2\hat{k}$ then

Column-I	Column-II
(A) $\lvert\vec{A}\times\vec{B}\rvert$	(p) $\sqrt{11}$
(B) $\lvert\vec{A}-\vec{B}\rvert$	(q) 6
(C) $\vec{A}\cdot\vec{B}$	(r) $\sqrt{35}$
(D) $\lvert\vec{A}+\vec{B}\rvert$	(s) $\sqrt{90}$

10. **COLUMN-I** contains vector diagram of three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and **COLUMN-II** contains vector equation. Match them

Column-I	Column-II
(A) 	(p) $\vec{a}-(\vec{b}+\vec{c})=0$
(B) 	(q) $\vec{b}-\vec{c}=\vec{a}$
(C) 	(r) $\vec{a}+\vec{b}=-\vec{c}$
(D) 	(s) $\vec{a}+\vec{b}=\vec{c}$

11. In vector algebra match the vector operations with conditions under which operation is possible.

Column-I	Column-II
(A) $\vec{A}+\vec{B}=\vec{A}-\vec{B}$	(p) Not possible
(B) $\lvert\vec{A}+\vec{B}\rvert=\lvert\vec{A}-\vec{B}\rvert$	(q) $\theta=90°$
(C) $\lvert\vec{A}\times\vec{B}\rvert=\lvert\vec{A}\cdot\vec{C}\rvert$	(r) $B=0$
(D) $\vec{A}\times\vec{B}=\vec{A}\cdot\vec{C}$	(s) $\theta=45°$

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. A particle has a displacement of 12 m towards east, 3 m towards north and 4 m vertically upwards. Find the resultant magnitude of these displacements. Give your answer in cm.

2. At what angle, in degree, two forces of magnitude $A+B$ and $A-B(A>B)$ act, so that their resultant is $\sqrt{3A^2+B^2}$?

3. Two equal vectors act at a point. The square of their resultant is 3 times their product. Find the angle, in degree, between the vectors.

4. Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A}+\vec{B}=\vec{C}$. Also $A^2+B^2=C^2$. Find the angle between $\vec{A}$ and $\vec{B}$, in degree.

5. Given that $\lvert\vec{v}_1+\vec{v}_2\rvert=\lvert\vec{v}_1-\vec{v}_2\rvert$. Calculate the angle between $\vec{v}_1$ and $\vec{v}_2$, in degree.

6. The resultant of two forces $\vec{F}_1$ and $\vec{F}_2$ is P. If $\vec{F}_2$ is reversed, then the new resultant is Q. Find the value of $\dfrac{P^2+Q^2}{F_1^2+F_2^2}$.

7. Calculate the x and y components of a vector in first quadrant having magnitude same as that of $\vec{A}=7\hat{i}+24\hat{j}$ and parallel to $\vec{B}=3\hat{i}+4\hat{j}$.

8. Find $(y-x)$ such that the vectors $\vec{a}=x\hat{i}+4\hat{j}-8\hat{k}$ and $\vec{b}=2\hat{i}+y\hat{j}+4\hat{k}$ are collinear.

9. If a vector $\vec{P}$ makes angles α, β and γ with respect to X, Y and Z axes of a rectangular coordinate system, then find the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$.

10. One vertex of a parallelopiped is at the point $(1, -1, -2)$ of rectangular Cartesian coordinates. If three adjacent vertices are at $(0, 1, 3)$, $(3, 0, -1)$ and $(1, 4, 1)$. Calculate the volume of the parallelopiped.

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1. **[Online April 2019]**

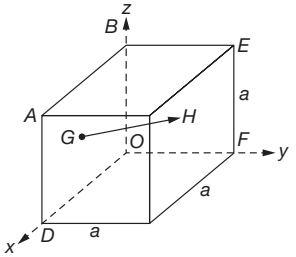
Let $|\vec{A}_1|=3$, $|\vec{A}_2|=5$ and $|\vec{A}_1+\vec{A}_2|=5$. The value of $(2\vec{A}_1+3\vec{A}_2)\cdot(3\vec{A}_1-2\vec{A}_2)$ is

(A) –106.5 (B) –118.5

(C) –99.5 (D) –112.5

2. **[Online January 2019]**

In the cube of side a shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be



(A) $\frac{1}{2}a(\hat{j}-\hat{i})$ (B) $\frac{1}{2}a(\hat{i}-\hat{k})$

(C) $\frac{1}{2}a(\hat{j}-\hat{k})$ (D) $\frac{1}{2}a(\hat{k}-\hat{i})$

3. **[Online January 2019]**

Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A}+\vec{B})$ is n times the magnitude of $(\vec{A}-\vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is

(A) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$ (B) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

(C) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$ (D) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

4. **[Online January 2019]**

Two forces P and Q, of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is

(A) 30° (B) 90°

(C) 60° (D) 120°

5. **[Online January 2019]**

A particle is moving along a circular path with a constant speed of $10\ \text{ms}^{-1}$. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(A) $10\ \text{ms}^{-1}$ (B) Zero

(C) $10\sqrt{3}\ \text{ms}^{-1}$ (D) $10\sqrt{2}\ \text{ms}^{-1}$

6. **[Online 2018]**

Let $\vec{A}=(\hat{i}+\hat{j})$ and $\vec{B}=(2\hat{i}-\hat{j})$. The magnitude of a coplanar vector such that $\vec{A}\cdot\vec{C}=\vec{B}\cdot\vec{C}=\vec{A}\cdot\vec{B}$, is given by

(A) $\sqrt{\frac{20}{9}}$ (B) $\sqrt{\frac{5}{9}}$

(C) $\sqrt{\frac{9}{12}}$ (D) $\sqrt{\frac{10}{9}}$

7. **[Online 2015]**

A vector $\vec{A}$ is rotated by a small angle $\Delta\theta$ radians $(\Delta\theta \ll 1)$ to get a new vector $\vec{B}$. In that case $|\vec{B}-\vec{A}|$ is

(A) 0 (B) $|\vec{A}|\left(1-\frac{\Delta\theta^2}{2}\right)$

(C) $|\vec{A}|\Delta\theta$ (D) $|\vec{B}|\Delta\theta-|\vec{A}|$

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Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2018]

Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a\left(\cos\omega t\hat{i} + \sin\omega t\hat{j}\right)$, where a is a constant and $\omega = \frac{\pi}{6}$ rads^{-1}. If $|\vec{A} + \vec{B}| = \sqrt{3}|\vec{A} - \vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is _____.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Addition, Subtraction and Resolution)

1. No, Yes

2. (a) $B_x = 6$ and $B_y = 3$

(b) $3\sqrt{5}$ m

(c) $\tan^{-1}\left(\frac{1}{2}\right)$, with x-axis

3.

	$\vec{A}$	$\vec{B}$	$\vec{A}+\vec{B}$	$\vec{A}-\vec{B}$	$\vec{B}-\vec{A}$
Magnitude	5	10	$5\sqrt{5}$	$5\sqrt{5}$	$5\sqrt{5}$
Direction (θ with x-axis)	$\tan^{-1}\left(\frac{3}{4}\right)$ Below x-axis	$\tan^{-1}\left(\frac{4}{3}\right)$ Above x-axis	$\tan^{-1}\left(\frac{1}{2}\right)$ Above x-axis	$\tan^{-1}\left(\frac{11}{2}\right)$ Below negative x-axis	$\tan^{-1}\left(\frac{11}{2}\right)$ Above x-axis

4. $\sqrt{205}$ m

7. $8\hat{i}-2\hat{j}$; $2\sqrt{17}$ kgf

8. $\vec{a}$ and $\vec{b}$ are parallel

10. $2v\sin(20°)$

13. Yes

Test Your Concepts-II (Based on Dot Product)

1. $A_xB_x + A_yB_y + A_zB_z$

2. 15°

3. (a) $\vec{A}\left(\frac{4}{\sqrt{61}}, \frac{6}{\sqrt{61}}, -\frac{3}{\sqrt{61}}\right)$, $\vec{B}\left(-\frac{2}{\sqrt{78}}, -\frac{5}{\sqrt{78}}, \frac{7}{\sqrt{78}}\right)$

(b) 31.3°

5. $\frac{44}{\sqrt{142}}$

6. 12; $\cos^{-1}\left(\frac{12}{77}\right)$

8. $\frac{177}{7}$ J

Test Your Concepts-III (Based on Cross Product, Scalar and Vector Triple Product)

1. (a) $-21\hat{i}+29\hat{j}+41\hat{k}$

(b) $-23\hat{i}+29j+38\hat{k}$

2. $\hat{i}\left(A_yB_z - A_zB_y\right) - \hat{j}\left(A_xB_z - A_zB_x\right) + \hat{k}\left(A_xB_y - A_yB_x\right)$

4. $\frac{1}{2}\sqrt{234}$ sq units

5. Area, with direction as the outward normal.

6. Yes

7. $\frac{7mv_0^2}{20g}$, along negative z-axis.

8. $v = \frac{E}{B}$, along $+x$ axis or $-x$ axis

9. $4\left(\hat{j}-\hat{k}\right)$

Single Correct Choice Type Questions

1. A	**2.** A	**3.** D	**4.** B	**5.** D	**6.** A	**7.** C	**8.** C	**9.** A	**10.** C
11. A	**12.** C	**13.** A	**14.** A	**15.** B	**16.** D	**17.** C	**18.** D	**19.** A	**20.** C
21. A	**22.** A	**23.** B	**24.** C	**25.** C	**26.** B	**27.** D	**28.** C	**29.** B	**30.** C
31. D	**32.** C	**33.** C	**34.** C	**35.** B	**36.** A	**37.** C	**38.** D	**39.** D	**40.** D
41. B	**42.** B	**43.** C	**44.** C	**45.** A	**46.** B	**47.** D	**48.** B	**49.** B	**50.** A
51. C	**52.** D	**53.** C	**54.** D	**55.** D	**56.** B	**57.** B	**58.** A	**59.** D	**60.** A
61. C	**62.** B	**63.** D	**64.** B	**65.** A	**66.** B	**67.** A	**68.** A	**69.** A	**70.** A
71. D	**72.** A	**73.** B	**74.** A	**75.** C	**76.** B	**77.** C	**78.** A	**79.** A	**80.** C
81. A	**82.** D	**83.** A	**84.** D	**85.** A	**86.** D	**87.** A	**88.** A	**89.** B	**90.** A
91. C	**92.** A	**93.** D	**94.** A	**95.** D	**96.** C	**97.** D	**98.** C	**99.** C	**100.** C
101. A	**102.** D	**103.** D	**104.** D	**105.** B	**106.** A	**107.** B	**108.** A	**109.** D	**110.** C
111. D	**112.** B	**113.** A	**114.** A	**115.** B	**116.** B	**117.** A	**118.** A	**119.** B	**120.** D
121. D	**122.** B	**123.** D							

Multiple Correct Choice Type Questions

1. A, B, C	**2.** C, D	**3.** B, C	**4.** B, C	**5.** C, D	**6.** A, B, C
7. B, C, D	**8.** A, C	**9.** A, C	**10.** B, C	**11.** A, C	**12.** A, D

Reasoning Based Questions

1. A	**2.** A	**3.** B	**4.** B	**5.** C	**6.** B	**7.** B	**8.** A	**9.** C	**10.** B
11. D	**12.** A	**13.** C	**14.** A	**15.** B	**16.** C	**17.** B	**18.** B	**19.** C	**20.** A

Linked Comprehension Type Questions

1. C	**2.** B	**3.** A	**4.** D	**5.** C	**6.** D	**7.** B	**8.** A	**9.** A	**10.** D
11. B	**12.** B	**13.** C	**14.** C	**15.** A	**16.** A	**17.** B	**18.** A	**19.** B	**20.** C
21. D	**22.** A	**23.** B							

Matrix Match/Column Match Type Questions

1. A → (p, r)	B → (p, s)	C → (q)	D → (p, q, s)	
2. A → (q, s)	B → (p)	C → (r)	D → (r)	
3. A → (r)	B → (t)	C → (s)	D → (p)	E → (q)
4. A → (r, s)	B → (s)	C → (t)	D → (p)	E → (q)
5. A → (s, t)	B → (r)	C → (q)	D → (p)	
6. A → (r)	B → (p)	C → (s)	D → (p)	
7. A → (q)	B → (r)	C → (s)	D → (p)	
8. A → (p)	B → (r)	C → (q)	D → (s)	
9. A → (s)	B → (p)	C → (q)	D → (r)	
10. A → (r)	B → (s)	C → (p)	D → (q)	
11. A → (r)	B → (q)	C → (s)	D → (p)	

Integer/Numerical Answer Type Questions

1. 1300	**2.** 60	**3.** 60	**4.** 90	**5.** 90
6. 2	**7.** $x = 15$ and $y = 20$	**8.** 2	**9.** 2	**10.** 40

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1. B	**2.** A	**3.** B	**4.** D	**5.** A	**6.** B	**7.** C

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Integer/Numerical Answer Type Questions

1. 2

CHAPTER

4 Kinematics I

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Rest, Motion and Position
(b) Distance Displacement
(c) Average Speed and Average Velocity
(d) Instantaneous Speed and Instantaneous Velocity
(e) Average and Instantaneous Acceleration
(f) Uniformly Acceleration Motion
(g) Variable Accelerated Motion
(h) Graphical Interpretation and Graphs
(i) Vertical Motion Under Gravity
(j) Motion in a Plane
(k) Relative Motion in One Dimension
(l) Relative Motion in Two Dimensions
(m) Distance of Closest Approach between Moving Bodies
(n) River-Swimmer Problems
(o) Aeroplane-Wind Problems
(p) Rain-Man-Wind Problems

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

RECTILINEAR MOTION AND MOTION UNDER GRAVITY

INTRODUCTION TO CLASSICAL MECHANICS

The branch of Physics dealing with motion of particles or bodies in space and time is called **Mechanics.** As long as the velocity of the moving bodies is small in comparison to the velocity of light (c), the linear dimensions and the time intervals remain invariable in all Reference Frames (platform(s) from where motion is being observed), i.e., they do not depend on choice of reference frame. Mechanics dealing with such like motion (also called as Non-Relativistic motion) is called as **Classical Mechanics**. However, when the bodies move with speeds comparable to the speed of light (called as relativistic speeds), then the part of Physics dealing with such like motion(s) is called **Relativistic Mechanics**. An interesting fact about relativistic mechanics is that it is more general and reduces to classical mechanics for the case of small (non relativistic) velocities. In this chapter, we shall be describing and studying motion in terms of space and time while ignoring the causes that produce motion. This particular part of Classical Mechanics is called **Kinematics**. Furthermore, in this part of chapter, we shall be limiting ourselves to the motion in one dimension and two dimensions i.e., motion along a straight line, also called as **Rectilinear Motion** and **planar motion**. From our everyday experience, we observe that actually motion represents a continuous change in the position of an object. In Physics, we can divide motion into three categories

(a) Translational Motion (studying now).
EXAMPLE: A car moving down a highway.

(b) Rotational Motion (to study in Rotational Dynamics).
EXAMPLE: Spinning i.e., rotation of earth about its own axis.

(c) Vibrational Motion (to study in Simple Harmonic Motion).
EXAMPLE: Back and forth (or to and fro) motion of a pendulum (or a spring).

In the Chapters to came, we shall be discussing the branch of Classical Mechanics called **DYNAMICS**, where we shall be studying the motion along with the cause producing it.

CONCEPT OF POINT OBJECT (PARTICLE MODEL)

In our study of translational motion, we shall be using the concept of **Point Object** also called as the **Particle Model.** A body is considered as a point object depending upon the nature of the motion followed by the body. In general, an object is regarded a point-object when it travels large distances in comparison to its own size and dimensions.

Also, in planetary motion, the bodies under consideration can be regarded as point objects when distances of separation are very large.

EXAMPLE:
The planets revolving around the sun may be considered as point objects.

CONCEPT OF REFERENCE FRAME

A reference frame is a platform from where a physical phenomena, such as motion, is being observed. Reference frames are mainly of two types

(a) Inertial (Non-Accelerated frames)
(b) Non-Inertial (Accelerated frames)

The detailed discussion about both these frames follows in the next chapter. As of now, for this chapter, the reference frame is a frame (fixed or moving with constant velocity) i.e., non accelerating frame. The best convenient reference frame for this chapter would be the **Ground Frame**. Please note here, that I said "the best convenient" which has convenience, but not accuracy attached to it. For practical purposes, we regard earth as an inertial frame (though it is accelerating), hence I said convenient and not accurate.

State of Rest and Motion

A particle is said to be in the state of rest when it does not change its position w.r.t. surroundings with the passage of time.

A particle is said to be in the state of motion when it changes its position w.r.t. surroundings with the passage of time.

Rest and Motion, these two terms are not absolute i.e., complete in themselves. These terms are relative terms i.e., a body can be in the state of rest and motion simultaneously, depending upon the relative observer viewing motion.

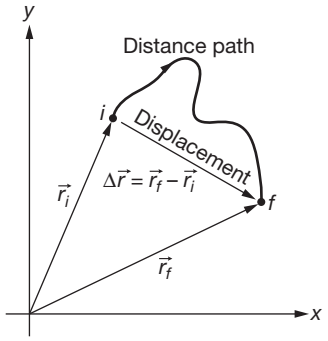
EXAMPLE:
Four persons sitting in the moving car are at rest w.r.t. an observer sitting in the car, whereas the same four persons are in motion w.r.t. a stationary observer viewing them from the ground.

Position of a Particle

The position of a particle is the location of the particle in space at a certain moment of time. In One-Dimensional Motion the position of the particle is denoted by x or y or z. However in 2-Dimensional and 3-Dimensional motion it is denoted by $\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

DISTANCE AND DISPLACEMENT (RELATIVE POSITION VECTOR)

The length of the actual path followed between the initial and the final points in the motion is called **Distance**. Its SI unit is metre and cgs unit is cm.



The change in position vector of a particle going from initial position (say i) to final position $(\text{say } f)$ is called **Displacement or Relative Position Vector of the particle**.
It is denoted by $\Delta\vec{r}$, such that

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

Conceptual Note(s)

In rectilinear motion of a particle, the displacement is denoted by $\Delta x = s = x_{\text{final}} - x_{\text{initial}}$

In 2-Dimensional or 3-Dimensional motion, the displacement is denoted by $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

PROPERTIES OF DISPLACEMENT

(a) It is a vector quantity.
(b) It has units same as that of distance.
(c) It is independent of the choice of origin.
(d) It is unique (one and only one) for any kind of motion between two points.
(e) It is always concealing about the actual track followed by the particle's motion between any two points.
(f) It can be positive, negative and even be zero.
(g) The magnitude of the displacement is always less than or equal to the distance for particle's motion between two points i.e.,

$$0 \le |\text{Displacement}| \le \text{Distance}$$

(h) A body may have finite distance travelled for zero displacement.

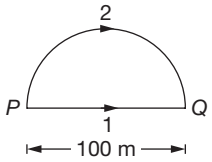
Conceptual Note(s)

(a) For Rectilinear (straight line) motion, always make sure that the selected Cartesian Co-ordinate axis coincides with the line followed by the particle.
(b) In other words Displacement may also be defined as the vector drawn from the initial point to the final point in the motion of the particle.
(c) Please keep in mind that distance may or may not be equal to the magnitude of the displacement.
(d) In general, we have $0 \le \dfrac{|\text{Displacement}|}{\text{Distance}} \le 1$

ILLUSTRATION 1

Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).

(a) Find the distance travelled by Ram and Shyam.
(b) Find the displacement of Ram and Shyam.



SOLUTION

(a) Distance travelled by Ram $= 100$ m
Distance travelled by Shyam $= \pi(50\text{ m}) = 50\pi$ m
(b) Displacement of Ram $= 100$ m
Displacement of Shyam $= 100$ m

ILLUSTRATION 2

The position x (in metre) of a particle varies with time t (in second) as $t = \sqrt{x} + 3$. Calculate the

(a) displacement of the particle from $t = 0$ to $t = 3$ s
(b) displacement of the particle from $t = 3$ to $t = 6$ s
(c) distance travelled and displacement from $t = 0$ to $t = 6$ s

SOLUTION

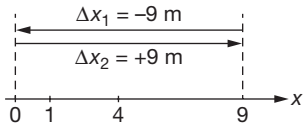
Since $t = \sqrt{x} + 3$

$$\Rightarrow \quad x = (t-3)^2 \qquad \ldots(1)$$

So, we get x as a function of t. Let us draw a table for x and t from $t = 0$ to $t = 6$. Then we get

t	0	1	2	3	4	5	6
x	9	4	1	0	1	4	9

This table gives us the answers to the three parts.
So, for (a), we have $\Delta x_1 = -9$ m
For (b), we have $\Delta x_2 = +9$ m
For (c), we have,
Distance $= |-9\text{ m}| + |9\text{ m}| = 18$ m and
Displacement = zero.



AVERAGE SPEED AND AVERAGE VELOCITY

Average speed of a particle, in a given interval of time is the ratio of the total distance travelled by the particle to the total time taken. It is a scalar quantity,

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{s_1 + s_2 + s_3 + ...}{t_1 + t_2 + t_3 + ...}$$

Average velocity of the particle is the ratio of the displacement of the particle to the time taken. It is a vector quantity.

In one dimensional motion, we have

$$v_{av} = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

In two or three dimensional motion, we have

$$\vec{v}_{av} = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Please note that, average speed may or may not be equal to the magnitude of average velocity. So,

Average Speed $\neq |\vec{v}_{av}|$ (sometimes may be equal)

CONCEPT OF AVERAGE SPEED

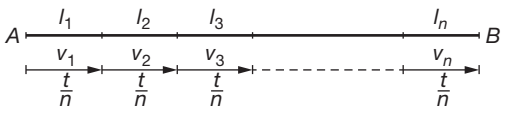
Average speed is simply defined as the total distance travelled by a body per unit total time taken to complete the motion.

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

For Interval Divided in Equal Time Parts

Let a particle go from A to B in a time t (say). Now if we say that for time $\frac{t}{n}$, the particle had a velocity v_1, for next $\frac{t}{n}$ it had a velocity v_2 and for the nth such equal time interval it had a velocity v_n (as shown). Let the corresponding distances travelled in each respective interval be $l_1, l_2, l_3, ..., l_n$. Then

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$



$$\Rightarrow \quad v_{av} = \frac{l_1 + l_2 + l_3 + ... + l_n}{\frac{t}{n} + \frac{t}{n} + \frac{t}{n} + ... + \frac{t}{n}(n \text{ times})}$$

$$\Rightarrow \quad v_{av} = \frac{v_1\left(\frac{t}{n}\right) + v_2\left(\frac{t}{n}\right) + v_3\left(\frac{t}{n}\right) + ... + v_n\left(\frac{t}{n}\right)}{t}$$

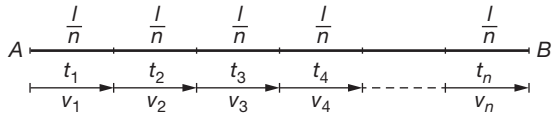
$$\Rightarrow \quad v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_n}{n}$$

So, we observe that when an interval is divided into n equal time parts, then the "average speed is the simple average of the speeds in the respective intervals".

For Interval Divided in Equal Length Parts

Consider an interval of length l, divided equally in n equal length parts. Let a particle travel first length part with a velocity v_1, second with a velocity v_2 and so on the nth length part with a velocity v_n (as shown). Let t_1, t_2, …, t_n be the respective time taken by the particle to cover the respective interval. Then

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$



$$\Rightarrow \quad v_{av} = \frac{\frac{l}{n} + \frac{l}{n} + ...n \text{ times}}{t_1 + t_2 + ... + t_n} = \frac{l}{t_1 + t_2 + ... + t_n}$$

$$\Rightarrow \quad v_{av} = \frac{l}{\frac{l}{nv_1} + \frac{l}{nv_2} + ... + \frac{l}{nv_n}}$$

$$\Rightarrow \quad \frac{1}{v_{av}} = \frac{\frac{1}{v_1} + \frac{1}{v_2} + ... + \frac{1}{v_n}}{n}$$

So, we observe that when an interval is divided into n equal length parts, then the **"reciprocal of the average speed is equal to the average of the reciprocals of the speeds in respective intervals".**

Problem Solving Technique(s)

So, we observe that

(a) when an interval is divided into n equal time parts, then the "average speed is the simple average of the speeds in the respective intervals".

(b) when an interval is divided into n equal length parts, then the "reciprocal of the average speed is equal to the average of the reciprocals of the speeds in respective intervals".

(c) **Time Average Speed:** When particle moves with different uniform speed v_1, v_2, v_3 ... etc. in different time intervals t_1, t_2, t_3, ... etc. respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}}$$

$$v_{av} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

(d) **Distance Average Speed:** When a particle describes different distances d_1, d_2, d_3, ... with different time intervals t_1, t_2, t_3, ... with speeds v_1, v_2, v_3, ... respectively, then the speed of particle averaged over the total distance can be given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...}$$

$$v_{av} = \frac{d_1 + d_2 + d_3 + ...}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3} + ...}$$

(e) If speed is continuously changing with time then

$$v_{av} = \frac{\int v\,dt}{\int dt}$$

(f) When a particle moves with a speed v_1 for half the time and with a speed v_2 for the remaining half of the time, then

$$v_{av} = \frac{v_1 + v_2}{2}.$$

(g) When a particle moves the first half of a distance with a speed v_1 and the second half of the distance with a speed v_2, then

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

(h) Similarly, when a particle covers one-third distance at speed v_1, next one third with a speed v_2 and the last one third at speed v_3, then

$$v_{av} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

ILLUSTRATION 3

Calculate the average speed and the average velocity in the following cases mentioned.

CASE-1: For a train that travels from one station to another at a uniform speed of $40\ \text{kmh}^{-1}$ and returns to first station at a speed of $60\ \text{kmh}^{-1}$.

CASE-2: For a man who walks at a speed of $1\ \text{ms}^{-1}$ for the first one minute and then runs at a speed of $3\ \text{ms}^{-1}$ for the next one minute along a straight track.

CASE-3: For a man who walks 720 m at a uniform speed of $2\ \text{ms}^{-1}$, then runs at a uniform speed of $4\ \text{ms}^{-1}$ for 5 minute and then again walks at a speed of $1\ \text{ms}^{-1}$ for 3 minutes.

(Please consider all uniform speeds to be the average speeds in respective intervals).

SOLUTION

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

CASE-1: $v_{av} = \dfrac{2v_1 v_2}{v_1 + v_2} = \dfrac{(2)(40)(60)}{40 + 60} = 48\ \text{kmh}^{-1}$

$$\text{Average Velocity} = \vec{v}_{av} = \vec{0}$$

$\{\because$ train returns to its station$\}$

CASE-2: $v_{av} = \dfrac{v_1 + v_2}{2} = \dfrac{1+3}{2} = 2\ \text{ms}^{-1}$

CASE-3: $v_{av} = \dfrac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$

where $s_1 = 720$ m and

$$t_1 = \frac{s_1}{v_1} = 360\ \text{s} = 6\ \text{minute}$$

$$s_2 = (4)(5)(60) = 1200\ \text{m},\ t_2 = 300\ \text{s}$$

$$s_3 = (1)(3)(60) = 180 \text{ m}, \; t_3 = 180 \text{ s}$$

$$\Rightarrow \quad v_{av} = \frac{720 + 1200 + 180}{360 + 300 + 180} = \frac{2100}{840}$$

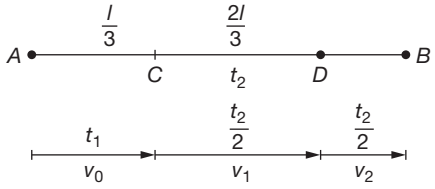
$$\Rightarrow \quad v_{av} = \frac{210}{84} = \frac{10}{4} = 2.5 \text{ ms}^{-1}$$

ILLUSTRATION 4

A particle traversed one third the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with a velocity v_2 for the remaining half of time. Assuming motion to be rectilinear, find the mean velocity of the particle averaged over the whole time of motion.

SOLUTION

Let us first draw a pictorial representation to the problem.



For *AC*

$$\frac{l}{3} = v_0 t_1$$

$$\Rightarrow \quad t_1 = \frac{l}{3v_0} \qquad \ldots(1)$$

For *CB*

$$\frac{2l}{3} = CD + DB$$

$$\Rightarrow \quad \frac{2l}{3} = v_1\left(\frac{t_2}{2}\right) + v_2\left(\frac{t_2}{2}\right)$$

$$\Rightarrow \quad t_2 = \frac{4l}{3(v_1 + v_2)} \qquad \ldots(2)$$

Since, average velocity is defined as

$$v_{av} = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\frac{l}{3} + \frac{2l}{3}}{t_1 + t_2}$$

$$\Rightarrow \quad v_{av} = \frac{3v_0(v_1 + v_2)}{4v_0 + v_1 + v_2}$$

INSTANTANEOUS SPEED AND INSTANTANEOUS VELOCITY

The speed of a particle at a particular instant of time is called the **instantaneous speed**. It is a scalar quantity.

$$v_{\text{instantaneous}} = v_{\text{ins}} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \dot{x}$$

The velocity of a particle at a particular instant of time is called the **instantaneous velocity**, denoted by $\vec{v}$. So,

$$\vec{v}_{\text{instantaneous}} = \vec{v}_{\text{ins}} = \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt} = \dot{\vec{x}}$$

(a) Please note that, in Physics, dot on a physical quantity indicates its derivative w.r.t. time e.g.

$$\vec{F} = \frac{d\vec{p}}{dt} = \dot{\vec{p}}, \; \vec{v} = \frac{d\vec{x}}{dt} = \dot{\vec{x}}, \; \vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}}$$

Also, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2} = \ddot{\vec{x}}$

So, $\vec{a} = \dot{\vec{v}} = \ddot{\vec{x}}$

(b) The magnitude of the instantaneous velocity is always equal to the instantaneous speed i.e., $v = |\vec{v}| = \left|\frac{d\vec{x}}{dt}\right| \neq \frac{dx}{dt}$, because in general $d\vec{x} \neq dx$.

Acceleration ($\vec{a}$)

The rate of change of velocity with time is called acceleration.

(a) Average Acceleration $= \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

(b) Instantaneous Acceleration is

$$\vec{a}_{\text{ins}} = \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \dot{\vec{v}}$$

So, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$

$$\Rightarrow \quad \vec{a} = \dot{\vec{v}} = \ddot{\vec{x}}$$

Conceptual Note(s)

(a) In 3-Dimensional space

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

then $\vec{v} = \frac{d\vec{r}}{dt} = \hat{i}\left(\frac{dx}{dt}\right) + \hat{j}\left(\frac{dy}{dt}\right) + \hat{k}\left(\frac{dz}{dt}\right)$

$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

and $\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d\vec{v}}{dt}$

$\Rightarrow \quad \vec{a} = \hat{i}\left(\frac{d^2x}{dt^2}\right) + \hat{j}\left(\frac{d^2y}{dt^2}\right) + \hat{k}\left(\frac{d^2z}{dt^2}\right)$

$\Rightarrow \quad \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

(b) $\left|\frac{d\vec{v}}{dt}\right|$ is the magnitude of total acceleration. While $\frac{d|\vec{v}|}{dt}$ represents the time rate of change of speed (called the tangential acceleration, a component of total acceleration) as $|\vec{v}| = v$.

(c) These two are equal in case of one dimensional motion (without change in direction).

(d) In case of uniform circular motion speed remains constant while velocity changes.

Hence, $\frac{d|\vec{v}|}{dt} = 0$ while $\left|\frac{d\vec{v}}{dt}\right| \neq 0$

(e) $\frac{d|\vec{v}|}{dt} \neq 0$ implies that speed of particle is not constant. Velocity cannot remain constant if speed is changing. Hence, $\left|\frac{d\vec{v}}{dt}\right|$ cannot be zero in this case. So, it is not possible to have $\left|\frac{d\vec{v}}{dt}\right| = 0$ while $\frac{d|\vec{v}|}{dt} \neq 0$.

FACTORS AFFECTING ACCELERATION OF A BODY

The acceleration changes, when the velocity of the body changes. The change in velocity may be due to any of factors listed below.

(a) If the magnitude of velocity changes but direction remains the same.

(b) If the direction of the velocity changes but magnitude remains the same.

(c) If both magnitude and direction of the velocity change.

Conceptual Note(s)

(a) A particle moving with uniform velocity has zero acceleration i.e. it neither changes in magnitude nor its direction.

(b) A particle moves with uniform acceleration if rate of change of velocity is constant.

(c) A body is subjected to Retardation or Deceleration when acceleration acts opposite to velocity.

(d) In case of a body subjected to retardation, we take

(i) a as negative, if velocity is taken as positive.

(ii) a as positive, if velocity is taken as negative.

(e) In case of a body subjected to acceleration, we take

(i) a as positive, if velocity is taken as positive.

(ii) a as negative, if velocity is taken as negative.

(f) However, a body at rest will be in accelerated motion irrespective of the sign of acceleration.

(g) For a body moving in straight line with uniform acceleration the average acceleration and instantaneous value of acceleration have same value.

(h) If a particle has an acceleration a_1 for a time t_1 and an acceleration a_2 for a time t_2, then average acceleration is $a_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$

ILLUSTRATION 5

A body moves along a straight line. Its distance x from a point on its path at a time t after passing that point, is given by $x_t = 8t^2 - 3t^3$ where x_t is in meter and t in second. Find

(a) the instantaneous velocity at $t = 1$ s

(b) instant and position at which the body is at rest

(c) the acceleration at $t = 4$ s

(d) the average velocity and average speed during the interval $t = 0$ s to $t = 4$ s

SOLUTION

$$x = 8t^2 - 3t^3 \qquad \ldots(1)$$

$$\Rightarrow \quad v = \frac{dx}{dt} = 16t - 9t^2 \qquad \ldots(2)$$

$\Rightarrow \quad a = 16 - 18t$ …(3)

(a) $v|_{t=1\text{ s}} = 16 - 9 = 7\ \text{ms}^{-1}$

(b) The body is at rest, when $v = 0$

$\Rightarrow \quad 16t - 9t^2 = 0$

$\Rightarrow \quad t = 0\ ,\ t = \dfrac{16}{9}\ \text{s}$

So, the particle is initially at rest. However at $t = 0$, $a = -18\ \text{ms}^{-2}$ which accelerates the particle to move.

$$x = 0 \text{ and } x = 8\left(\frac{16}{9}\right)^2 - 3\left(\frac{16}{9}\right)^3 = 8.43\ \text{m}$$

(c) $a|_{t=4\text{ s}} = 16 - 18(4) = -56\ \text{ms}^{-2}$

(d) Average Velocity $= \dfrac{x|_{t=4} - x|_{t=0}}{4 - 0}$

$\Rightarrow$ Average Velocity $= \dfrac{8(4)^2 - 3(4)^3 - 0}{4}$

$\Rightarrow$ Average Velocity $= -16\ \text{ms}^{-1}$

For calculating the average speed we must first calculate the zeros of velocity i.e., the times at which the particle is at rest (or momentarily at rest i.e., at the point of reversal of motion).
In this problem, we observe that

$v = 0$ at $t = 0$ (initially) and

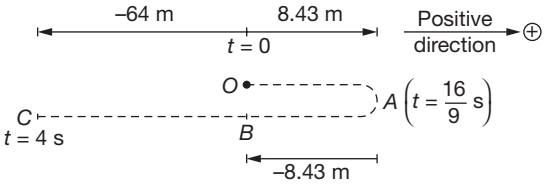
$v = 0$ at $t = \dfrac{16}{9}$ s {point of reversal of motion}

	Initial	Point of Zero Velocity	Final	Afterwards
t	0	$\frac{16}{9}$ s	4 s	$t > 4$
$x = 8t^2 - 3t^3$	0	8.43 m	–64 m	–
$v = 16t - 9t^2$	0	0	$-80\ \text{ms}^{-1}$	–
$a = 16 - 18t$	$16\ \text{ms}^{-2}$	$-16\ \text{ms}^{-2}$	$-56\ \text{ms}^{-2}$	–
Nature of Motion	Accelerating in Positive Direction	Accelerating in Negative Direction	Accelerating in Negative Direction	Accelerating

So, from the table, we observe that the particle reverses its direction of motion at $t = \dfrac{16}{9}$ s as shown. Hence average speed is

$$v_{av} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad v_{av} = \frac{8.43 + |-8.43| + |-64|}{4}$$



$\Rightarrow \quad v_{av} = 20.21\ \text{ms}^{-1}$

Alternatively,

$$\text{Average Speed} = \frac{\left|\int_0^{\frac{16}{9}} v\,dt\right| + \left|\int_{\frac{16}{9}}^{4} v\,dt\right|}{4} = 20.21\ \text{ms}^{-1}$$

CHECK YOURSELF!

ILLUSTRATION 6

A particle travels along a straight line with a velocity $v = (12 - 3t^2)\ \text{ms}^{-1}$, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Calculate the

(a) acceleration when $t = 4$ s
(b) displacement from $t = 0$ to $t = 10$ s and
(c) distance the particle travels from $t = 0$ to $t = 10$ s.

SOLUTION

(a) $v = 12 - 3t^2$ …(1)

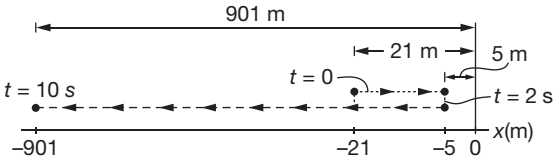
$$\Rightarrow \quad a = \frac{dv}{dt} = -6t\Big|_{t=4} = -24\ \text{ms}^{-2}$$

(b) Further $v = \dfrac{dx}{dt}$

$$\Rightarrow \quad \int_{-10}^{x} dx = \int_1^t v\,dt = \int_1^t (12 - 3t^2)\,dt$$

$\Rightarrow \quad x + 10 = 12t - t^3 - 11$

$\Rightarrow \quad x = 12t - t^3 - 21$



So, $x\big|_{t=0} = -21$ m and $x\big|_{t=10} = -901$ m

$\Rightarrow \quad \Delta x = -901 - (-21) = -880$ m

(c) From equation (1), we have

$v = 0$ when $12 - 3t^2 = 0$

$\Rightarrow \quad t = 2$ s

So, $x\big|_{t=2} = 12(2) - (2)^3 - 21 = -5$ m

$\Rightarrow$ Total Distance

$(x_{\text{total}}) = (21 - 5) + (901 - 5) = 912$ m

Test Your Concepts-I

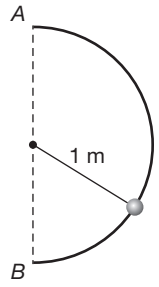
Based on Displacement, Velocity, Acceleration, Average Speed and Velocity

(Solutions on page H.65)

1. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)$ ms^{-2}, where t is in seconds. If $x = 1$ m and $v = 2$ ms^{-1} when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

2. A particle moves in a straight line with a uniform acceleration a. Initial velocity of the particle is zero. Find the average velocity of the particle in first s metre.

3. In one second a particle goes from point A to point B moving in a semicircle (see figure). Find the magnitude of average velocity.



4. A particle is moving along a straight line such that its position from a fixed point is $x = (12 - 15t^2 + 5t^3)$ m, where t is in seconds. Determine the total distance travelled by the particle from $t = 1$ s to $t = 3$ s. Also, find the average speed of the particle during this time interval.

5. The position (x) of a particle, in metre, moving along the x-axis depends on the time t, in seconds as $x = ct^2 - bt^3$, where $c = 3$ units and $b = 2$ units. Calculate the

(a) units of c and b.

(b) time taken by the particle to reach its maximum positive x value.

(c) the distance travelled and the displacement of the particle from $t = 0$ to $t = 4$ s.

(d) the velocity and acceleration at $t = 0$, 1, 2, 3 and 4 second.

6. The position of a particle along a straight line is given by $x = (t^3 - 9t^2 + 15t)$ m, here t is in second. Determine its maximum acceleration and maximum velocity during the time interval $0 \le t \le 10$ s.

7. At time $t = 0$, the position vector of a particle moving in the x-y plane is $5\hat{i}$ m. At time $t = 0.02$ s, its position vector has become $5.1\hat{i} + 0.4\hat{j}$ m. Determine the magnitude of the average velocity (v_{av}) during this interval and the angle θ made by the average velocity with the positive x-axis.

8. A particle travels along a straight line path such that in 4 s it moves from an initial position $x_A = -8$ m to a position $x_B = +3$ m. Then in another 5 s it moves from x_B to $x_C = -6$ m. Determine the

particle's average velocity and average speed during the 9 s time interval.

9. A particle moves along a horizontal path such that its velocity is given by $v=(3t^2-6t)\ \text{ms}^{-1}$, where t is the time in seconds. If it is initially located at the origin O, determine the
- **(a)** distance travelled by the particle during the time interval $t=0$ to $t=3.5$ s
- **(b)** particle's average velocity and average speed during this time interval.

10. The position of a particle along a straight line is given by $x=(1.5t^3-13.5t^2+22.5t)$ m, where t is in seconds. Determine the position of the particle when $t=6$ s and the total distance it travels during the 6 s time interval.

11. The velocity of a particle moving in a straight line decreases at the rate of $3\ \text{ms}^{-1}$ per meter of displacement at an instant when the velocity is $10\ \text{ms}^{-1}$. Calculate the acceleration of the particle at this instant.

UNIFORMLY ACCELERATED MOTION SYSTEMS

Consider a particle moving in a straight line with constant acceleration a. If u is the initial velocity, v is the velocity at time t, s is the displacement $(=\Delta x)$ in time t and s_n^{th} is the displacement in the n^{th} second of motion, then equations governing the motion of such a particle are

$$v=u+at$$

$$s=\Delta x=ut+\frac{1}{2}at^2$$

$$v^2-u^2=2as$$

$$s=\frac{u+v}{2}t$$

$$s_n^{\text{th}}=u+\frac{1}{2}a(2n-1)$$

For constant acceleration, $\vec{a}$, the equations of motion are written as

$$\vec{v}=\vec{u}+\vec{a}t$$

$$\vec{s}=\vec{u}t+\frac{1}{2}\vec{a}t^2$$

$$v^2-u^2=2\vec{a}\cdot\vec{s} \quad \text{OR} \quad \vec{v}\cdot\vec{v}-\vec{u}\cdot\vec{u}=2\vec{a}\cdot\vec{s}$$

$$\vec{s}=\left(\frac{\vec{u}+\vec{v}}{2}\right)t$$

where $\vec{u}$ = initial velocity of the particle

$\vec{v}$ = final velocity of the particle at time t

$\vec{s}=\Delta\vec{r}$ is the displacement of the particle

Conceptual Note(s)

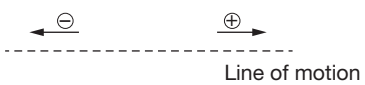
- **(a)** All the above equations of motion are to be applied only when the motion is uniformly accelerated.
- **(b)** For applying the above equations greater care has to be taken about the direction of the vector quantities involved.
- **(c)** Please note that this nth second has a duration of 1 second. So, $s_{n\text{th}}$ is the distance travelled (or the displacement) in 1 sec, hence

$$[s_{n\text{th}}]=LT^{-1}$$

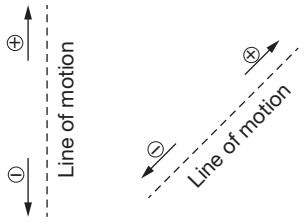
DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION

In straight line motion, all the vectors (position, displacement, velocity and acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector. For example

- **(a)** If a particle is moving in a horizontal line along x-axis, then, the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.



(b) For vertical or inclined motion, upward direction can be taken as positive and downward as negative.



For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e., the gravitational pull of the earth. Hence acceleration for this type of motion will always be $a = -g$ i.e., $a = -9.8\ \text{ms}^{-2}$ (negative sign, because the force and acceleration are directed downwards. If we select upward direction as positive).

Conceptual Note(s)

(a) If acceleration is in same direction as velocity, then speed of the particle increases.

(b) If acceleration is in opposite direction to the velocity then speed decreases i.e., the particle slows down. This situation is known as retardation.

ILLUSTRATION 7

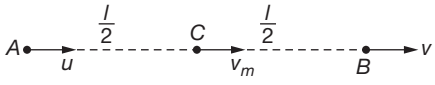
A point moving with constant acceleration from A to B in a straight line AB has velocities u and v at A and B respectively.

(a) Find its velocity at C, the midpoint of A and B.

(b) Find the ratio $\frac{v}{u}$, if time taken from A to C is twice the time to go from C to B.

SOLUTION

(a) Let v_m be the velocity at C, the midpoint of A and B.



For *AC*

$$v_m^2 - u^2 = 2a\left(\frac{l}{2}\right)$$

$$\Rightarrow \quad v_m^2 - u^2 = al \qquad \ldots(1)$$

For *CB*

$$v^2 - v_m^2 = 2a\left(\frac{l}{2}\right)$$

$$\Rightarrow \quad v^2 - v_m^2 = al \qquad \ldots(2)$$

Equating (1) and (2), we get

$$v_m = \sqrt{\frac{u^2 + v^2}{2}} \qquad \ldots(3)$$

(b) Since $t_{A\to C} = 2t_{C\to B}$

Let $t_{C\to B} = t$, then $t_{A\to C} = 2t$

$$\Rightarrow \quad \frac{l}{2} = \left(\frac{u + v_m}{2}\right)2t \qquad \{\text{for } AC\} \qquad \ldots(4)$$

$$\Rightarrow \quad \frac{l}{2} = \left(\frac{v_m + v}{2}\right)t \qquad \{\text{for } CB\} \qquad \ldots(5)$$

$$\Rightarrow \quad u + v_m = \frac{v_m + v}{2}$$

$$\Rightarrow \quad 2u + 2v_m = v_m + v$$

$$\Rightarrow \quad v_m = v - 2u$$

$$\Rightarrow \quad v_m^2 = v^2 + 4u^2 - 4uv$$

Since, $v_m = \sqrt{\frac{u^2 + v^2}{2}}$

$$\Rightarrow \quad \frac{u^2 + v^2}{2} = v^2 + 4u^2 - 4uv$$

$$\Rightarrow \quad u^2 + v^2 = 2v^2 + 8u^2 - 8uv$$

$$\Rightarrow \quad v^2 - 8uv + 7u^2 = 0$$

$$\Rightarrow \quad v^2 - 7uv - uv + 7u^2 = 0$$

$$\Rightarrow \quad v(v - 7u) - u(v - 7u) = 0$$

$$\Rightarrow \quad (v - u)(v - 7u) = 0$$

As motion is an accelerated motion

So, $v - u \neq 0$

$$\Rightarrow \quad v = 7u$$

$$\Rightarrow \quad \frac{v}{u} = 7$$

ILLUSTRATION 8

A body starts with an initial velocity of $10\ \text{ms}^{-1}$ and moves along a straight line path with constant acceleration. When the velocity of the body is $50\ \text{ms}^{-1}$ the acceleration is reversed in direction. Find the velocity of the particle as it reaches the starting point.

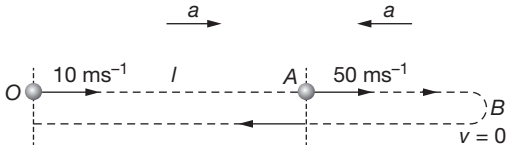
SOLUTION

For $OA(=l)$

$$(50)^2-(10)^2=2al$$

$$\Rightarrow \quad al=1200 \qquad \ldots(1)$$

Now, when the particle just attains the velocity of $50\ \text{ms}^{-1}$ at A, acceleration is reversed in direction i.e., it starts acting as retardation for $50\ \text{ms}^{-1}$ which will first be reduced to zero after travelling a distance AB and then the particle will again be accelerated from B to A with the same acceleration. So, velocity of the particle again at A will be $50\ \text{ms}^{-1}$, but in opposite direction.



From A to O

$$v^2-(50)^2=2al$$

$$\Rightarrow \quad v^2-2500=2(1200)$$

$$\Rightarrow \quad v^2=4900$$

$$\Rightarrow \quad v=70\ \text{ms}^{-1} \qquad \{\text{in the opposite direction}\}$$

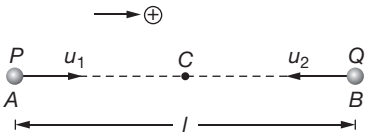
So, $v=70\ \text{ms}^{-1}$ from A to O

ILLUSTRATION 9

Two particles P and Q move in a straight line AB towards each other. P starts from A with a velocity u_1 and an acceleration a_1. Q starts from B with velocity u_2 and an acceleration a_2. They pass from each other at midpoint of AB and arrive at other ends of AB with equal velocities. Prove that $(u_1+u_2)(a_1-a_2)=8(a_1u_2-a_2u_1)$.

SOLUTION

Let the particle P reach C in time t, then the particle Q must reach C also in time t.



For P

$$\frac{l}{2}=u_1t+\frac{1}{2}a_1t^2 \qquad \ldots(1)$$

For Q

$$-\frac{l}{2}=-u_2t+\frac{1}{2}(-a_2)t^2$$

$$\Rightarrow \quad \frac{l}{2}=u_2t+\frac{1}{2}a_2t^2 \qquad \ldots(2)$$

From (1) and (2), we get (by subtracting)

$$(u_2-u_1)t+\frac{1}{2}(a_2-a_1)t^2=0$$

$$\Rightarrow \quad t=2\left(\frac{u_2-u_1}{a_1-a_2}\right) \qquad \ldots(3)$$

Now, substitute this value of t from (3) in any of equations (1) or (2), we get

$$\frac{l}{2}=u_1\left[2\left(\frac{u_2-u_1}{a_1-a_2}\right)\right]+\frac{1}{2}a_1\left[2\left(\frac{u_2-u_1}{a_1-a_2}\right)\right]^2$$

$$\Rightarrow \quad \frac{l}{2}=2\left(\frac{u_2-u_1}{a_1-a_2}\right)\left[u_1+a_1\left(\frac{u_2-u_1}{a_1-a_2}\right)\right]$$

$$\Rightarrow \quad l=\frac{4(u_2-u_1)}{(a_1-a_2)^2}\left(\cancel{u_1a_1}-u_1a_2+a_1u_2-\cancel{a_1u_1}\right)$$

$$\Rightarrow \quad l=\frac{4(u_2-u_1)}{(a_1-a_2)^2}(a_1u_2-a_2u_1) \qquad \ldots(4)$$

Again, after reading the question carefully, we observe that P reaches B and Q reaches A with equal velocities, say v.
Then, for P, we have

$$v^2-u_1^2=2a_1l \qquad \ldots(5)$$

For Q, we have

$$v^2-u_2^2=2a_2l \qquad \ldots(6)$$

$$\Rightarrow \quad u_2^2-u_1^2=2(a_1-a_2)l$$

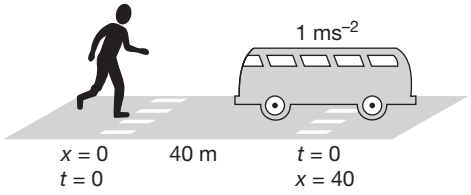
Substituting value of l, from (4), we get

$$u_2^2 - u_1^2 = 2(a_1 - a_2)\frac{4(u_2 - u_1)}{(a_1 - a_2)^2}(a_1u_2 - a_2u_1)$$

$$\Rightarrow \quad (u_1 + u_2)(a_1 - a_2) = 8(a_1u_2 - a_2u_1)$$

ILLUSTRATION 10

A man is standing 40 m behind the bus. Bus starts with $1\ \text{ms}^{-2}$ constant acceleration and also at the same instant the man starts moving with constant speed $9\ \text{ms}^{-1}$. Find the time taken by man to catch the bus.



SOLUTION

Let the man catches the bus at time t.

For bus

$$x = x_0 + ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad x = 40 + 0(t) + \frac{1}{2}(1)t^2$$

$$\Rightarrow \quad x = 40 + \frac{t^2}{2} \qquad \ldots(1)$$

For man

$$x = 9t \qquad \ldots(2)$$

From (1) and (2), we get

$$40 + \frac{t^2}{2} = 9t$$

$$\Rightarrow \quad t^2 - 18t + 80 = 0$$

$$\Rightarrow \quad t^2 - 10t - 8t + 80 = 0$$

$$\Rightarrow \quad (t - 8)(t - 10) = 0$$

$$\Rightarrow \quad t = 8\ \text{s} \quad \text{OR} \quad t = 10\ \text{s}$$

ILLUSTRATION 11

A car starts moving rectilinearly, first with an acceleration of $5\ \text{ms}^{-2}$ (initial velocity zero), then moves uniformly and finally decelerating at the same rate till it stops. The total time of journey is 25 s. The average speed during the interval is $72\ \text{kmh}^{-1}$. Calculate the time for which the car was in uniform motion.

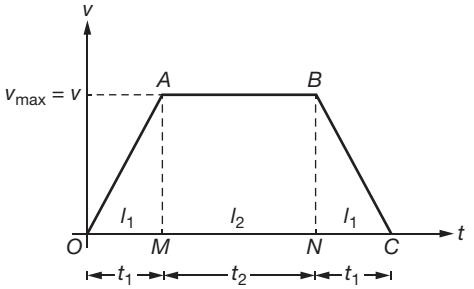
SOLUTION

Since the car accelerates from zero to v and decelerates from v to zero at the same rate of $5\ \text{ms}^{-2}$, so it must travel equal distances during the accelerated and the decelerated interval in equal times.

If t be the total time of journey, then

$$t = 25 = t_1 + t_2 + t_1$$

$$\Rightarrow \quad 25 = 2t_1 + t_2 \qquad \ldots(1)$$



Furthermore, the average speed

$$v_{av} = 72\ \text{kmh}^{-1} = 20\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_{av} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}} = \frac{l_1 + l_2 + l_1}{25}$$

$$\Rightarrow \quad 2l_1 + l_2 = (20)(25) = 500$$

$$\Rightarrow \quad 2l_1 + l_2 = 500 \qquad \ldots(2)$$

where $l_1 = \frac{5}{2}t_1^2$ $\qquad \ldots(3)$

Also, for OA, we have

$$v = 5t_1 \qquad \ldots(4)$$

For AB, we have

$$l_2 = vt_2 = 5t_1t_2 \qquad \ldots(5)$$

$$\Rightarrow \quad 2\left(\frac{5}{2}t_1^2\right) + 5t_1t_2 = 500 \qquad \{\text{substituting in (2)}\}$$

$$\Rightarrow \quad 5t_1^2 + 5t_1(25 - 2t_1) = 500$$

$$\Rightarrow \quad t_1^2 + t_1(25 - 2t_1) = 100$$

$$\Rightarrow \quad t_1^2 - 2t_1^2 + 25t_1 = 100$$

$\Rightarrow \quad t_1^2 - 25t_1 + 100 = 0$

$\Rightarrow \quad t_1^2 - 20t_1 - 5t_1 + 100 = 0$

$\Rightarrow \quad t_1(t_1 - 20) - 5(t_1 - 20) = 0$

$\Rightarrow \quad (t_1 - 5)(t_1 - 20) = 0$

$\Rightarrow \quad t_1 = 5 \text{ s} \quad \text{OR} \quad t_1 = 20 \text{ s}$

But if $t_1 = 20$ s, then total time becomes greater than 25 s which is impossible. So,

$t_1 = 5$ s

$\Rightarrow \quad t_2 = 15$ s

Hence the car was in uniform motion for 15 s.

ILLUSTRATION 12

A police inspector in a car is chasing a pickpocket an a straight road. The car is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the car is at a distance d away and the motorcycle starts with a constant acceleration a. Show that the pick pocket will be caught if $v \geq \sqrt{2ad}$.

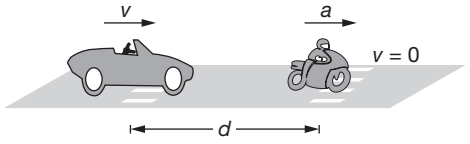
SOLUTION

Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2 \qquad \ldots(1)$$

During this interval the car travels a distance

$$s + d = vt \qquad \ldots(2)$$



From (1) and (2), we get

$$\frac{1}{2}at^2 + d = vt$$

$\Rightarrow \quad at^2 - 2vt + 2d = 0$

$\Rightarrow \quad t = \dfrac{v \pm \sqrt{v^2 - 2ad}}{a}$

The pickpocket will be caught if t is real and positive. This will be possible if

$v^2 \geq 2ad$

$\Rightarrow \quad v \geq \sqrt{2ad}$

REACTION TIME

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

ILLUSTRATION 13

A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving car at a speed of 5.4 kmh^{-1} and the brakes cause a deceleration of 6 ms^{-2}, find the distance travelled by the car after he sees the need to put the brakes.

SOLUTION

During the reaction time of 0.20 s, the car continues to move with a speed of 54 kmh^{-1} i.e. 15 ms^{-1}.
So, distance travelled by car during this time is

$s_1 = (15)(0.2) = 3$ m

Now, when brakes are applied, the distance s_2 travelled by the car is

$0^2 - (15)^2 = 2(-6)s_2$

$\Rightarrow \quad s_2 = \dfrac{225}{12} \text{ m} = 18.75 \text{ m}$

So, total distance covered by the car is

$s = s_1 + s_2 = 3 + 18.75$

$\Rightarrow \quad s = 21.75$ m

Test Your Concepts-II

Based on Constant Acceleration

(Solutions on page H.67)

1. A particle moving in a straight line with constant acceleration travels a distance x, y and z during the pth, qth and rth second respectively. Prove that

$$(q-r)x+(r-p)y+(p-q)z=0$$

2. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t seconds, evaluate

(a) the maximum velocity reached and
(b) the total distance travelled.

3. A particle starts moving from the position of rest under a constant acceleration. If it travels a distance x in t sec, what distance will it travel in next t sec?

4. In a car race, car A takes time t less than car B and passes the finishing point with a velocity v more than the velocity with which car B passes the point. Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2, show that $v=t\sqrt{a_1a_2}$.

5. Two cars start off to race with velocities v_1 and v_2 and travel in a straight line with uniform accelerations a_1 and a_2. If the race ends in a dead heat, prove that the length of the course is

$$\frac{2(v_1-v_2)(v_1a_2-v_2a_1)}{(a_1-a_2)^2}$$

6. A car moving with constant acceleration covered the distance between two points 60 m apart in 6 s. Its speed as it passes the second point was 15 ms^{-1}.

(a) What was the speed at the first point?
(b) What was the acceleration?
(c) At what prior distance from the first was the car at rest?

7. A train stopping at two stations 4 km apart takes 4 minute on the journey from one station to the other. Assuming that it first accelerates with a uniform acceleration x and then that of uniform retardation y, prove that $\frac{1}{x}+\frac{1}{y}=2$.

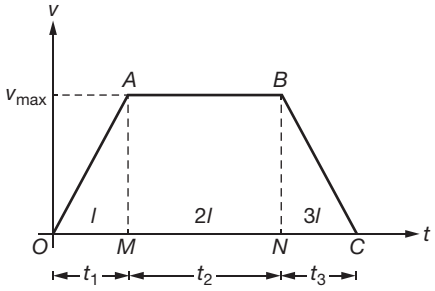
8. To stop a car, first you require a certain reaction time to begin braking and then the car slows under the constant braking deceleration. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 kmh^{-1} and 24.4 m when its initial speed is 48.3 kmh^{-1}. What is

(a) your reaction time and
(b) the magnitude of the deceleration?

9. A particle, starting from rest moves in a straight line with constant acceleration. After time t_0 the acceleration changes its direction without any change in its magnitude. Determine the time t from the beginning of motion in which the particle returns to the initial position.

10. The velocity v of a particle moving in a straight line varies with its displacement x as $v=(\sqrt{4+4x})$ ms^{-1}. Displacement of particle at time $t=0$ is $x=0$. Find displacement of particle at time $t=2$ s.

11. A particle starts from rest and traverses a distance l with uniform acceleration, then moves uniformly over a further distance $2l$ and finally it comes to rest after moving a further distance $3l$ under uniform retardation. Assuming the entire path to be a straight line find the ratio of the average speed over the journey to the maximum speed on its way.



12. An α particle travels along the inside of straight hollow tube, 2 m long, of a particle accelerator. Under uniform acceleration, how long is the particle in the tube if it enters at a speed of 1000 ms^{-1} and leaves at 9000 ms^{-1}. What is its acceleration during this interval?

13. A sports car passing a police check-point at $60\ \text{kmh}^{-1}$ immediately started slowing down uniformly until its speed was $40\ \text{kmh}^{-1}$. It continued to move at this speed until it was passed by a police car 1 km from the check-point. This police car had started from rest at the check-point at the same instant as the sports car had passed the check-point. The police car had moved with constant acceleration until it had passed the sports car. Assuming that the time taken by the sports car in slowing down from $60\ \text{kmh}^{-1}$ to $40\ \text{kmh}^{-1}$ was equal to the time that it travelled at constant speed before passed by the police car, find

(a) the time taken by the police car to reach the sports car,

(b) the speed of the police car at the instant when it passed the sports car,

(c) the time measured from the check-point when the speeds of the two cars were equal.

14. Two railway stations *A* and *B* are 50 km apart and are served by electric trains which can decelerate at $5\ \text{kmh}^{-1}$ per second, and accelerate $3\ \text{kmh}^{-1}$ per second. The maximum speed is $90\ \text{kmh}^{-1}$. There are twelve intermediate stations all more than a km apart. Find the least time which can be taken to made the journey from *A* to *B*

(a) by a fast non-stop train and

(b) by a slow train which stops $\frac{1}{2}$ minute at every station.

15. A truck starts from rest with an acceleration of $1.5\ \text{ms}^{-2}$ while a car 150 m behind starts from rest with an acceleration of $2\ \text{ms}^{-2}$. How long will it take before both the truck and car side by side, and how much distance is travelled by each?

16. Two cars travelling towards each other on a straight road at velocity $10\ \text{ms}^{-1}$ and $12\ \text{ms}^{-1}$ respectively. When they are 150 m apart, both drivers apply their brakes and each car decelerates at $2\ \text{ms}^{-2}$ until it stops. How far apart will they be when they have both come to a stop?

17. Two motor cars start from *A* simultaneously and reach *B* after 2 hours. The first car travelled half the distance at a speed of $30\ \text{kmh}^{-1}$ and the other half at a speed of $60\ \text{kmh}^{-1}$. The second car covered the entire distance with a constant acceleration. At what instant of time, the speeds of both the vehicles is the same? Will one of them overtake the other enroute?

18. A point travelling along a straight line traversed one third the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

19. A man runs at a speed of $4\ \text{ms}^{-1}$ to overtake a standing bus. When he is 6 m behind the door $(\text{at } t = 0)$, the bus moves forward and continues with a constant acceleration of $1.2\ \text{ms}^{-2}$. How long does it take for the man to go to the door? If he was initially 10 m behind the door, can he catch the bus?

20. Two cars *A* and *B* start off to a race on a straight path with initial velocities $8\ \text{ms}^{-1}$ and $5\ \text{ms}^{-1}$ respectively. Car *A* moves with uniform acceleration of $1\ \text{ms}^{-2}$ and *B* moves with uniform acceleration of $1.1\ \text{ms}^{-2}$. If both the cars reach the winning post together, find the length of the track. Also find which of the two cars was ahead 10 s before the finish.

21. A car starts from rest and moves with a constant acceleration of $1.5\ \text{ms}^{-2}$ until it achieves a velocity of $25\ \text{ms}^{-1}$. It then travels with constant velocity for 60 seconds. Determine the average speed and the total distance travelled.

22. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 14 m above the ground. If the elevator can accelerate at $0.2\ \text{ms}^{-2}$, decelerate at $0.1\ \text{ms}^{-2}$ and reach maximum speed of $2.5\ \text{ms}^{-1}$, determine the shortest time to make the lift, starting from rest and ending at rest.

23. The driver of a car wishes to pass a truck that is travelling at a constant speed of $20\ \text{ms}^{-1}$. Initially, the car is also travelling at $20\ \text{ms}^{-1}$. Initially, the vehicles are separated by 25 m, and the car pulls back into the truck's lane after it is 25 m ahead of the truck. The car is 5 m long, and the truck is 20 m long. The car's acceleration is a constant $0.6\ \text{ms}^{-2}$.

(a) How much time is required for the car to pass the truck?

(b) What distance does the car travel during this time?

(c) What is the final speed of the car?

24. A train of length $l = 348$ m starts moving rectilinearly with constant acceleration $a = 3\ \text{cms}^{-2}$. After 30 second a ball is dropped by the driver (event 1) and after 60 second another ball is dropped by the guard (event 2). How and at what constant velocity v should a driver drive his car parallel to the train so that he observes both the events to occur at the same point. Neglect the length of the car.

25. A particle moves rectilinearly with constant acceleration. The displacement, measured from a convenient fixed position, is 2 m at time $t = 0$ and is zero when $t = 10$ s. If the particle reverses its direction of motion at $t = 6$ s, determine the acceleration a and the velocity v when $t = 10$ s.

EQUATIONS OF MOTION FOR VARIABLE ACCELERATION

CASE-1: When acceleration a of the particle is a function of time

Since acceleration of a particle is a function of time, i.e., $a = f(t)$

$$\Rightarrow \quad \frac{dv}{dt} = f(t) \qquad \{\text{by definition}\}$$

$$\Rightarrow \quad dv = f(t)dt$$

Integrating both sides within suitable limits, we have

$$\int_u^v dv = \int_0^t f(t)dt$$

$$\Rightarrow \quad v = u + \int_0^t f(t)dt$$

CASE-2: When acceleration a of the particle is a function of distance

Since acceleration of a particle is a function of distance, i.e., $a = f(x)$

$$\Rightarrow \quad \frac{dv}{dt} = f(x) \qquad \{\text{by definition}\}$$

Multiply and divide the L.H.S. by dx and rearrange, we get

$$\frac{dv}{dx} \cdot \frac{dx}{dt} = f(x)$$

Integrating both sides within suitable limits, we have

$$\int_u^v vdv = \int_{x_0}^x f(x)dx$$

$$\Rightarrow \quad \frac{v^2}{2} - \frac{u^2}{2} = \int_{x_0}^x f(x)dx$$

$$\Rightarrow \quad v^2 = u^2 + 2\int_{x_0}^x f(x)dx$$

CASE-3: When acceleration a of the particle is a function of velocity

Since the particle is a function of velocity, i.e., $a = f(v)$

$$\Rightarrow \quad \frac{dv}{dt} = f(v) \qquad \{\text{by definition}\}$$

$$\Rightarrow \quad dt = \frac{dv}{f(v)}$$

Integrating both sides within suitable limits, we have

$$\int_0^t dt = \int_u^v \frac{dv}{f(v)}$$

$$\Rightarrow \quad t = \int_u^v \frac{dv}{f(v)}$$

In this case we shall get v as a function of time i.e., $v(t)$

Otherwise

$$v\frac{dv}{dx} = f(v)$$

Rearranging and integrating both sides within suitable limits, we have

$$\int_{x_0}^x dx = \int_u^v \frac{vdv}{f(v)}$$

$$\Rightarrow \quad x - x_0 = \int_u^v \frac{v\,dv}{f(v)}$$

$$\Rightarrow \quad x = x_0 + \int_u^v \frac{v\,dv}{f(v)}$$

In this case we shall get x as a function of velocity i.e., $x(v)$

ILLUSTRATION 14

An object starts from rest at $t = 0$ and accelerates at a rate given by $a = 6t$. What is

(a) its velocity and
(b) its displacement at any time t?

SOLUTION

As acceleration as a function of time is

$$a = 6t$$

$$\Rightarrow \quad \frac{dv}{dt} = 6t$$

$$\Rightarrow \quad dv = 6t\,dt$$

$$\Rightarrow \quad \int_0^v dv = 6\int_0^t t\,dt$$

$$\Rightarrow \quad v = 6\frac{t^2}{2} = 3t^2$$

$$\Rightarrow \quad v = 3t^2$$

Now $v = \frac{dx}{dt}$

$$\Rightarrow \quad \frac{dx}{dt} = 3t^2$$

$$\Rightarrow \quad dx = 3t^2 dt$$

$$\Rightarrow \quad \int_{x_1}^{x_2} dx = 3\int_0^t t\,dt$$

$$\Rightarrow \quad x_2 - x_1 = \Delta x = \frac{3t^3}{3}\Big|_0^t$$

$$\Rightarrow \quad x_2 - x_1 = \Delta x = t^3$$

$$\Rightarrow \quad \Delta x = t^3$$

ILLUSTRATION 15

For a particle moving along x-axis, velocity is given as a function of time as $v = 2t^2 + \sin t$. At $t = 0$, particle is at origin. Find the position and acceleration as a function of time. Also calculate initial speed and acceleration of the particle.

SOLUTION

$$v = 2t^2 + \sin t$$

$$\Rightarrow \quad \frac{dx}{dt} = 2t^2 + \sin t$$

$$\Rightarrow \quad \int_0^x dx = \int_0^t (2t^2 + \sin t)\,dt$$

$$\Rightarrow \quad x = \frac{2}{3}t^3 - \cos t + 1$$

Since $v = 2t^2 + \sin t$

$$\Rightarrow \quad a = \frac{dv}{dt} = 4t + \cos t$$

So, initial speed is $v|_{t=0} = u = 2(0)^2 + \sin(0) = 0$ and Initial Acceleration is $a|_{t=0} = 4(0) + \cos(0) = 1\ \text{ms}^{-2}$

ILLUSTRATION 16

A particle is subjected to an acceleration $a = \alpha t + \beta t^2$, where α and β are positive constants. The position and velocity of the particle at $t = 0$ are x_0 and v_0 respectively. Find the expression for the velocity (v) and position (x) of the particle at time t.

SOLUTION

Since $a = \alpha t + \beta t^2$ …(1)

$$\Rightarrow \quad \frac{dv}{dt} = \alpha t + \beta t^2$$

$$\Rightarrow \quad dv = \alpha t dt + \beta t^2 dt$$

Integrating, we get

$$\Rightarrow \quad \int_{v_0}^v dv = \alpha\int_0^t t dt + \beta\int_0^t t^2 dt$$

$$\Rightarrow \quad v - v_0 = \alpha\frac{t^2}{2} + \beta\frac{t^3}{3}$$

$$\Rightarrow \quad v = v_0 + \frac{\alpha t^2}{2} + \frac{\beta t^3}{3} \qquad \ldots(2)$$

Since $v = \frac{dx}{dt}$

$$\Rightarrow \quad \frac{dx}{dt} = v_0 + \frac{\alpha t^2}{2} + \frac{\beta t^3}{3}$$

$$\Rightarrow \quad dx = v_0 dt + \frac{\alpha t^2}{2} dt + \frac{\beta t^3}{3} dt$$

Integrating, we get

$$\Rightarrow \quad \int_{x_0}^{x} dx = v_0 \int_0^t dt + \frac{\alpha}{2} \int_0^t t^2 dt + \frac{\beta}{3} \int_0^t t^3 dt$$

$$\Rightarrow \quad x - x_0 = v_0 t + \frac{\alpha t^3}{6} + \frac{\beta t^4}{12}$$

$$\Rightarrow \quad x = x_0 + v_0 t + \frac{\alpha t^3}{6} + \frac{\beta t^4}{12} \qquad \ldots(3)$$

So, equations (2) and (3) give us the required expression for v and x.

ILLUSTRATION 17

For a particle moving along x-axis, acceleration is given as $a = x$. Find the position as a function of time if at $t = 0$, $x = 1$ m and $v = 1$ ms^{-1}

SOLUTION

$$a = x$$

Since, $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad v\frac{dv}{dx} = x$$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^x x\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{x^2}{2}$$

$$\Rightarrow \quad v = x$$

$$\Rightarrow \quad \frac{dx}{dt} = x$$

$$\Rightarrow \quad \int_1^x \frac{dx}{x} = \int_0^t dt$$

$$\Rightarrow \quad \log_e x\big|_1^x = t$$

$$\Rightarrow \quad \log_e x = t$$

$$\Rightarrow \quad x = e^t$$

ILLUSTRATION 18

For a particle moving along x-axis, acceleration is given as $a = 2v^2$. If the speed of the particle is v_0 at $x = 0$, find speed as a function of x.

SOLUTION

$$a = 2v^2$$

$$\Rightarrow \quad \frac{dv}{dt} = 2v^2$$

$$\Rightarrow \quad \frac{dv}{dx} \times \frac{dx}{dt} = 2v^2$$

$$\Rightarrow \quad v\frac{dv}{dx} = 2v^2$$

$$\Rightarrow \quad \frac{dv}{dx} = 2v$$

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = \int_0^x 2dx$$

$$\log_e v\big|_{v_0}^{v} = (2x)\big|_0^x$$

$$\Rightarrow \quad \log_e\left(\frac{v}{v_0}\right) = 2x$$

$$\Rightarrow \quad v = v_0 e^{2x} \qquad \left\{\because \text{ If } \log_e x = \alpha \Rightarrow x = e^{\alpha}\right\}$$

ILLUSTRATION 19

For a particle moving along x-axis, acceleration is given as $a = v$. Find the position as a function of time if at $t = 0$, $x = 0$ and $v = 1$ ms^{-1}.

SOLUTION

$$a = v$$

$$\Rightarrow \quad \frac{dv}{dt} = v$$

$$\Rightarrow \quad \int_1^v \frac{dv}{v} = \int_0^t dt$$

Since, $\int \frac{dv}{v} = \log_e v$

$$\Rightarrow \quad \log_e v\big|_1^v = t\big|_0^t$$

$$\Rightarrow \quad \log_e\left(\frac{v}{1}\right) = t$$

$$\Rightarrow \quad v = e^t$$

$$\Rightarrow \quad \frac{dx}{dt} = e^t$$

$$\Rightarrow \quad dx = e^t dt$$

$$\Rightarrow \quad \int_0^x dx = \int_0^t e^t dt$$

$$\Rightarrow \quad x = e^t - e^0$$

$$\Rightarrow \quad x = e^t - 1$$

ILLUSTRATION 20

The motion of a body is given by the equation $\frac{dv(t)}{dt} = 6 - 3v(t)$, when $v(t)$ is the speed in ms^{-1} and t in seconds. If the body was at rest at $t = 0$; Then test the correctness of the following result

(a) the terminal speed is $2\ \text{ms}^{-1}$

(b) the magnitudes of initial acceleration is $6\ \text{ms}^{-2}$

(c) the speed varies with time as $v(t) = 2(1 - e^{-3t})\ \text{ms}^{-1}$

(d) the speed is $1\ \text{ms}^{-1}$ when the acceleration is half the initial value

SOLUTION

(a) The terminal speed is the speed at which acceleration is zero i.e.,

$$\frac{dv(t)}{dt} = 0$$

$$\Rightarrow \quad v = 2\ \text{ms}^{-1}$$

(b) Since initially body is at rest hence

$$v(t) = v(0) = 0$$

$$\Rightarrow \quad \left.\frac{dv}{dt}\right|_{t=0} = 6\ \text{ms}^{-2}$$

(c) $\frac{dv}{6 - 3v} = dt$

Integrating both sides

$$\int_0^v \frac{dv}{6 - 3v} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{3}\log_e |6 - 3v| \Big|_0^v = t\Big|_0^t$$

$$\Rightarrow \quad \log_e\left(\frac{6 - 3v}{6}\right) = -3t$$

$$\Rightarrow \quad 6 - 3v = 6e^{-3t}$$

$$\Rightarrow \quad v = 2(1 - e^{-3t})\ \text{ms}^{-1}$$

(d) For acceleration to be half the initial value i.e.,

$$a = \frac{6}{2} = 3\ \text{ms}^{-2}$$

$$\Rightarrow \quad 3 = 6 - 3v(t)$$

$$\Rightarrow \quad v(t) = 1\ \text{ms}^{-1}$$

ILLUSTRATION 21

The velocity of a particle moving in the positive direction of x-axis varies as $v = \alpha\sqrt{x}$, where α is a positive constant. Assuming that at moment $t = 0$, the particle was located at the point $x = 0$. Find

(a) the time dependence of the velocity and the acceleration of the particle.

(b) the mean velocity of the particle averaged over the time that the particle takes to cover first s meters of the path.

SOLUTION

METHOD I

(a) $v = \alpha\sqrt{x}$

Since $a = \frac{dv}{dt} = \frac{d}{dt}(\alpha\sqrt{x})$

$$\Rightarrow \quad a = \frac{\alpha}{2\sqrt{x}}\frac{dx}{dt} = \frac{\alpha}{2\sqrt{x}}v \qquad \left\{\because \frac{dx}{dt} = v = \alpha\sqrt{x}\right\}$$

$$\Rightarrow \quad a = \left(\frac{\alpha}{2\sqrt{x}}\right)(\alpha\sqrt{x}) = \frac{\alpha^2}{2} = \text{constant}$$

Further, we have

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{\alpha^2}{2}$$

$$\Rightarrow \quad dv = \frac{\alpha^2}{2}dt$$

$$\Rightarrow \quad \int_0^v dv = \frac{\alpha^2}{2}\int_0^t dt$$

$$\Rightarrow \quad v = \left(\frac{\alpha^2}{2}\right)t$$

(b) Since $\langle v\rangle = \dfrac{\int_0^t v\,dt}{\int_0^t dt} = \dfrac{\text{Total Distance Travelled}}{\text{Total Time Taken}} = \dfrac{s}{t}$

To cover a distance s in time t with constant acceleration $a = \dfrac{\alpha^2}{2}$, we have

$$s = \frac{1}{2}\left(\frac{\alpha^2}{2}\right)t^2$$

$$\Rightarrow \quad t = \frac{2\sqrt{s}}{\alpha}$$

$$\Rightarrow \quad \langle v\rangle = v_{av} = \frac{s}{\left(\dfrac{2\sqrt{s}}{\alpha}\right)} = \frac{\alpha\sqrt{s}}{2}$$

METHOD II

Since $v = \alpha\sqrt{x}$, so we get

$$v^2 = \alpha^2 x$$

Comparing this equation with the equation of motion i.e.,

$$v^2 = u^2 + 2as$$

we get

$$u = 0 \text{ and } a = \frac{\alpha^2}{2}$$

Hence the motion is uniformly accelerated with zero initial velocity $(u = 0)$ and constant acceleration $\left(a = \dfrac{\alpha^2}{2}\right)$.

(a) Since $v = at$

$$\Rightarrow \quad v = \frac{\alpha^2 t}{2}$$

$$\Rightarrow \quad a = \frac{\alpha^2}{2} = \text{constant}$$

(b) $s = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{\alpha^2}{2}t^2$

$$\Rightarrow \quad t = \frac{2\sqrt{s}}{\alpha} = \text{ time taken to cover first } s \text{ metre}$$

$$\Rightarrow \quad v_{av} = \langle v\rangle = \frac{s}{t} = \frac{s}{\dfrac{2\sqrt{s}}{\alpha}}$$

$$\Rightarrow \quad v_{av} = \frac{\alpha\sqrt{s}}{2}$$

ILLUSTRATION 22

A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant. After what time and distance will the particle will be at rest?

SOLUTION

$v = v_0$ at $t = 0$ and $a = -\alpha\sqrt{v}$

Time taken by particle to come to rest:

$$a = \frac{dv}{dt} = -\alpha\sqrt{v}$$

$$\Rightarrow \quad \int_{v_0}^{0} -\frac{dv}{\alpha\sqrt{v}} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{\alpha}\left|\frac{v^{-1/2+1}}{-1/2+1}\right|_{v_0}^{0} = |t|_0^t$$

$$\Rightarrow \quad t = -\frac{1}{\alpha}\left(\frac{v^{1/2}}{1/2}\right)\Bigg|_{v_0}^{0}$$

$$\Rightarrow \quad t = -\frac{1}{\alpha}\left|0 - \frac{\sqrt{v_0}}{1/2}\right| = \frac{2}{\alpha}\sqrt{v_0}$$

Distance travelled before coming to rest:

$$a = v\frac{dv}{ds} = -\alpha\sqrt{v}$$

$$\Rightarrow \quad -\frac{v\,dv}{\alpha\sqrt{v}} = ds$$

$$\Rightarrow \quad -\int_{v_0}^{0}\frac{1}{\alpha}\sqrt{v}\,dv = \int_S^0 ds$$

$$\Rightarrow \quad -\frac{1}{\alpha}\left(\frac{v^{3/2}}{3/2}\right)\Bigg|_{v_0}^{0} = |s|_s^0$$

$$\Rightarrow \quad -\frac{1}{\alpha}\left[0-\frac{2}{3}|v_0|^{3/2}\right]=0-s$$

$$\Rightarrow \quad s=\frac{2}{3\alpha}v_0^{3/2}$$

ILLUSTRATION 23

The velocity of a particle travelling along a straight line is $v=v_0-kx$, where k is constant. If $x=0$ when $t=0$, determine the position and acceleration of the particle as a function of time.

SOLUTION

Since $v=\frac{dx}{dt}$

$$\Rightarrow \quad \int_0^t dt=\int_0^x \frac{dx}{v_0-kx}$$

$$\Rightarrow \quad t\Big|_0^t=-\frac{1}{k}\ln(v_0-kx)\Big|_0^x$$

$$\Rightarrow \quad t=\frac{1}{k}\ln\left(\frac{v_0}{v_0-kx}\right)$$

$$\Rightarrow \quad e^{kt}=\frac{v_0}{v_0-kx}$$

$$\Rightarrow \quad x=\frac{v_0}{k}(1-e^{-kt})$$

Now, $v=\frac{dx}{dt}=\frac{d}{dt}\left[\frac{v_0}{k}(1-e^{-kt})\right]$

$$\Rightarrow \quad v=v_0e^{-kt}$$

Since, we know that

$$a=\frac{dv}{dt}=\frac{d}{dt}(v_0e^{-kt})$$

$$\Rightarrow \quad a=-kv_0e^{-kt}$$

Conceptual Note(s)

(a) For a particle having zero initial velocity if $v\propto t$, $s\propto t^2$ and $v^2\propto s$ then acceleration of particle must be constant i.e., particle is moving rectilinearly with uniform acceleration.

(b) For a particle having zero initial velocity if $s\propto t^{\alpha}$, where $\alpha>2$, then particle's acceleration increases with time.

(c) For a particle having zero initial velocity if $s\propto t^{\alpha}$, where $\alpha<0$, then particle's acceleration decreases with time.

Problem Solving Technique(s)

When acceleration of particle is not constant, we go for basic equations of velocity and acceleration, i.e.,

(a) $\vec{v}=\frac{d\vec{x}}{dt}$ or sometimes $\vec{v}=\frac{d\vec{r}}{dt}$

(b) $\vec{a}=\frac{d\vec{v}}{dt}$

(c) $d\vec{x}=d\vec{r}=\vec{v}dt$

(d) $d\vec{v}=\vec{a}dt$

For one dimensional motion, above relations can be written as under.

(a) $v=\frac{dx}{dt}$

(b) $a=\frac{dv}{dt}=v\frac{dv}{dx}$

(c) $dx=vdt$

(d) $dv=adt$ or $vdv=adx$

Test Your Concepts-III

Based on Variable Acceleration

(Solutions on page H.73)

1. A particle starts from rest and travels along a straight line with an acceleration $a=\left(30-\frac{v}{5}\right)\ \text{ms}^{-2}$ where v is in ms^{-1}. Determine the time when the velocity of the particle is $v=30\ \text{ms}^{-1}$.

2. The acceleration (a) of a particle moving in a straight line varies with its displacement (s) as $a=2s$. The velocity of the particle is zero at zero displacement. Find the corresponding velocity-displacement equation.

3. The velocity of a particle travelling along a straight line is $v=v_0-kx$, where k is constant. Initially, it is observed that the particle is at the origin of the coordinate system, then determine the position and acceleration of the particle as a function of time.

4. The retardation of a particle moving in a straight line is proportional to its displacement (proportionality constant being 4). Find the total distance covered by the particle till it comes to rest. Given that velocity of particle is v_0 at zero displacement.

5. The acceleration (a) of a particle travelling along a straight line varies with distance x as $a=(8-2x)\ \text{ms}^{-2}$, where x is in meters. Assuming that the particle starts from rest from the origin of the coordinate system, calculate the velocity of the particle at $x=2$ m and the position of the particle when the velocity is maximum. Also calculate the maximum velocity.

6. The acceleration of a particle travelling along a straight line is $a=\frac{k}{v}$, where k is a constant. If $x=0$, $v=v_0$ initially, determine the velocity of the particle as a function of time t.

7. The motion of a body falling from rest in a resisting medium is described by the equation given as $\frac{dv}{dt}=A-Bv$; where A and B are constants. Find the initial acceleration and the velocity at which the acceleration is zero. Find the velocity at any instant of time.

8. A particle is moving along a straight line such that its speed is defined as $v=\left(-4x^2\right)\ \text{ms}^{-1}$, where x is in meters. If $x=2$ m when $t=0$, determine the velocity and acceleration as functions of time.

9. The acceleration of a particle is given by $a(t)=(3-2t)$.

(a) Find the initial speed v_0 such that the particle will have the same x-coordinate at $t=5$ s as it had at $t=0$.

(b) What will be the velocity at $t=5$ s?

10. The acceleration of a particle travelling along a straight line is $a=\left(5e^t\right)\ \text{ms}^{-2}$, where t is in seconds. If $v=0$, $x=0$ when $t=0$, determine the velocity and displacement of the particle as a function of t.

11. A particle moves along a straight line with a velocity $v=(200x)\ \text{mms}^{-1}$, where x is in millimeters. Determine the acceleration of the particle at $x=2000$ mm. How long does the particle take to reach this position if $x=500$ mm when $t=0$?

12. A particle has acceleration varying with time t as $\vec{a}=\left(2t\hat{i}+3t^2\hat{j}\right)\ \text{ms}^{-2}$. If the particle is initially at rest, find the velocity of the particle at time $t=2$ s.

13. When a particle is projected vertically upwards with an initial velocity of v_0, it experiences an acceleration $a=-\left(g+kv^2\right)$, where g is the acceleration due to gravity, k is a constant and v is the velocity of the particle. Determine the maximum height reached by the particle.

14. If the velocity v of a particle moving along a straight line decreases linearly with its displacement from $20\ \text{ms}^{-1}$ to a value approaching zero at $x=30$ m, determine the acceleration of the particle when $x=15$ m and show that the particle never reaches the 30 m displacement.

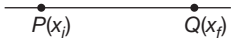
15. A particle is moving along a straight line with the acceleration $a=\left(12t-3\sqrt{t}\right)\ \text{ms}^{-2}$, where t is in seconds. Determine the velocity and the position of the particle as a function of time. Assume that, initially the particle starts from rest from $x_0=15$ m.

GRAPHICAL INTERPRETATION OF SOME QUANTITIES

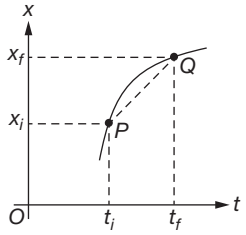
Average Velocity

If a particle passes a point $P(x_i)$ at time $t = t_i$ and reaches $Q(x_f)$ at a later time instant $t = t_f$, its average velocity in the interval PQ is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = (\text{Slope of chord } PQ)$$



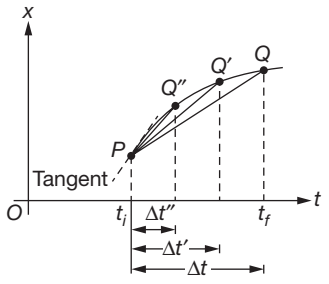
This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points P and Q on the x-t graph.



Instantaneous Velocity

Consider the motion of the particle between the two points P and Q on the x-t graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ $(\Delta t, \Delta t', \Delta t'', ...)$ get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line $(PQ, PQ', PQ''.....)$.

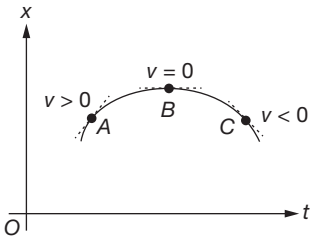
As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P.



As $\Delta t \to 0$, $v_{av}\left(= \frac{\Delta x}{\Delta t}\right) \to v_{\text{inst.}}$

Geometrically, as $\Delta t \to 0$, chord $PQ \to$ tangent at P.

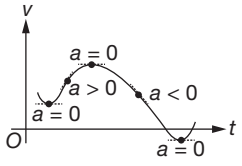
Hence the instantaneous velocity at P is the slope of the tangent at P in the x-t graph.



When the slope of the x-t graph is positive, v is positive (as at the point A in figure). At C, v is negative because the tangent has negative slope. The instantaneous velocity at point B (turning point) is zero as the slope is zero.

Instantaneous Acceleration

The derivative of velocity with respect to time is the slope of the tangent in velocity time $(v$-$t)$ graph.



MOTION WITH UNIFORM VELOCITY

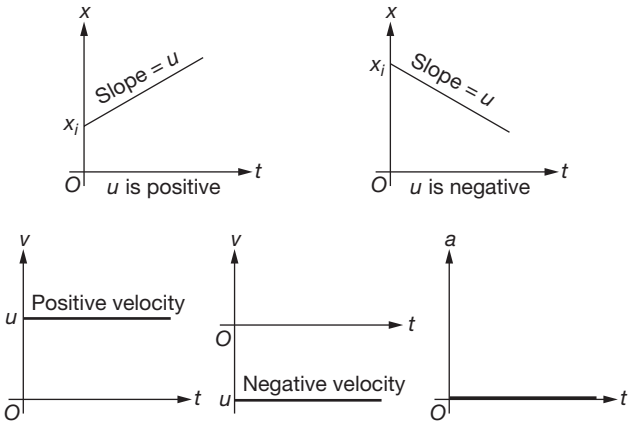
Consider a particle moving along x-axis with uniform velocity u starting from the point $x = x_i$ at $t = 0$.

Equations of x, v, a are

1. $x(t) = x_i + ut$ **2.** $v(t) = u$ **3.** $a(t) = 0$

So, x-t graph is a straight line of slope u through x_i

As velocity is constant, v-t graph is a horizontal line and a-t graph coincides with time axis because $a = 0$ at all-time instants.



Conceptual Note(s)

Graph Identification

(a) Straight Line Equation, Graph, Slope (+ve, –ve, zero slope)

If θ is the angle at which a straight line is inclined to the positive direction of x-axis and $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$.

If θ is $90°$, m does not exist, but the line is parallel to the y-axis.

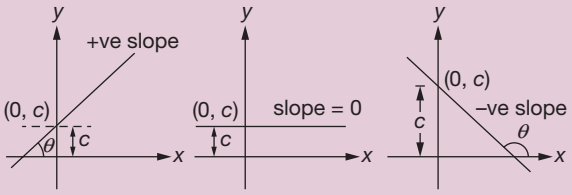
If $\theta = 0$, then $m = 0$ and the line is parallel to the x-axis.

For a straight line, the average slope is equal to instantaneous slope, because a straight line has constant slope.

Equation of Line: Slope-Intercept Form

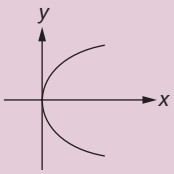
$y = mx + c$ is the equation of a straight line whose slope is m and which makes an intercept c on the y-axis i.e., the line cuts the y-axis at $(0, +c)$.

$$m = \text{slope} = \tan\theta = \frac{dy}{dx}$$

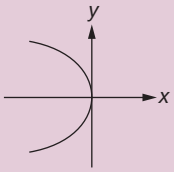


(b) Parabolic Curve-Equation and Graphs

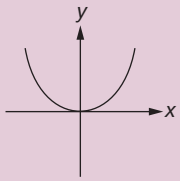
(i) $y^2 = kx$



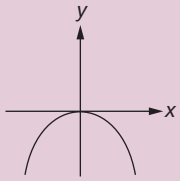
(ii) $y^2 = -kx$



(iii) $x^2 = ky$



(iv) $x^2 = -ky$



Where k is a positive constant.

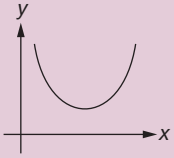
(c) Equation of Parabola

CASE-1:

$$y = ax^2 + bx + c$$

For $a > 0$

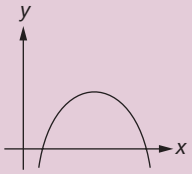
The nature of the parabola will be, like that of the nature of $x^2 = ky$



CASE-2:

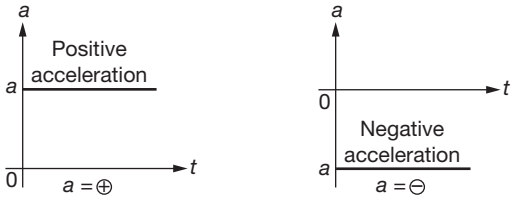
$a < 0$

The nature of the parabola will, like that of the nature of $x^2 = -ky$.

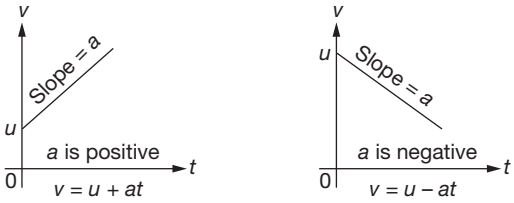


GRAPHS IN UNIFORMLY ACCELERATED MOTION ($a \neq 0$)

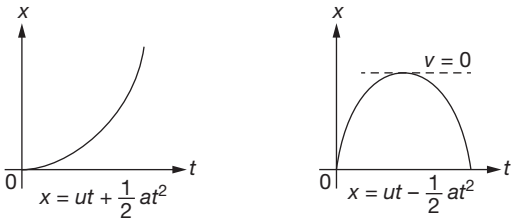
(a) If acceleration a is constant, then a-t graph is a horizontal line as shown.



(b) For constant acceleration, v is a linear polynomial in terms of t. Hence v-t graph is a straight line of slope a as shown.



(c) For constant acceleration, x is a quadratic polynomial in terms of t. Hence x-t graph is a parabola.



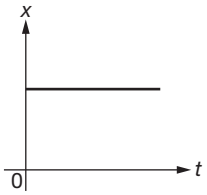
INTERPRETATION OF SOME MORE GRAPHS

Position vs Time Graph

(a) Zero Velocity: As position of particle is fixed at all the times, so the body is at rest.

$$\text{Slope} = \frac{dx}{dt} = \tan\theta = \tan 0° = 0$$

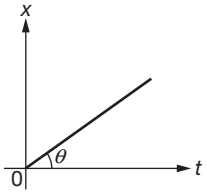
$\Rightarrow$ Velocity of particle is zero



(b) Uniform Velocity: Here $\tan\theta$ is constant so,

$$\tan\theta = \frac{dx}{dt} = \text{constant}$$

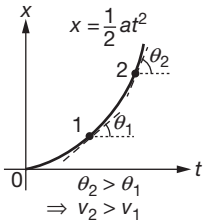
$\Rightarrow$ Velocity of particle is constant.



(c) Non Uniform Velocity (Increasing with Time): In this case, as time is increasing, θ is also increasing.

$\Rightarrow$ $\frac{dx}{dt} = \tan\theta$ also increases

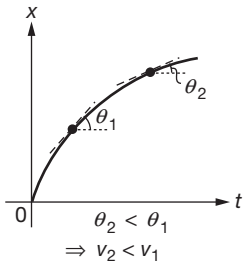
Hence, velocity of particle is increasing.



(d) Non Uniform Velocity (Decreasing with Time): In this case, as time increases, θ decreases.

$\Rightarrow$ $\frac{dx}{dt} = \tan\theta$ also decreases

Hence, velocity of particle is decreasing.



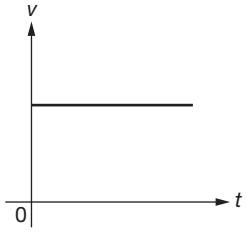
Velocity vs Time Graph

(a) Zero Acceleration: Velocity is constant

$\Rightarrow \quad \tan\theta = 0$

$\Rightarrow \quad \frac{dv}{dt} = 0$

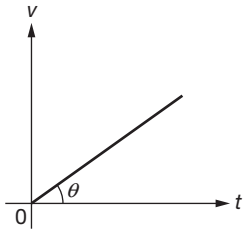
Hence, acceleration is zero i.e., v-t graph is a straight line parallel to time axis.



(b) Uniform Acceleration: $\tan\theta$ is constant

$\Rightarrow \quad \frac{dv}{dt} = \text{constant}$

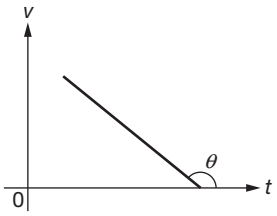
i.e., v-t graph is a straight line with positive slope.



(c) Uniform Retardation: Since $\theta > 90°$, so $\tan\theta$ is constant and negative.

$\Rightarrow \quad \frac{dv}{dt} = \text{negative constant}$

i.e., v-t graph is a straight line with negative slope.

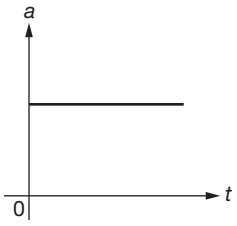


Acceleration vs Time Graph

(a) Constant Acceleration: $\tan\theta = 0$

$\Rightarrow \quad \frac{da}{dt} = 0$

Hence acceleration is constant i.e., a-t graph is a straight line parallel to time axis.

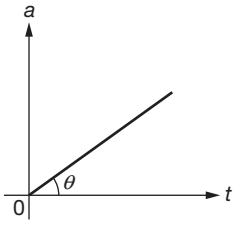


(b) Uniformly Increasing Acceleration: θ is constant.

$0° < \theta < 90° \quad \Rightarrow \quad \tan\theta > 0$

$\Rightarrow \quad \frac{da}{dt} = \tan\theta = \text{constant} > 0$

Hence, acceleration is uniformly increasing with time i.e., a-t graph is a straight line with positive slope.



(c) Uniformly Decreasing Acceleration: Since $\theta > 90°$

$\Rightarrow$ $\tan\theta$ is constant and negative

$\Rightarrow$ $\dfrac{da}{dt}$ = negative constant

Hence, acceleration is uniformly decreasing with time i.e., a-t graph is a straight line with negative slope.

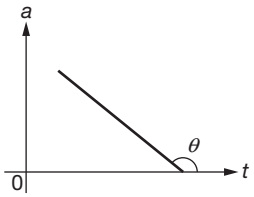
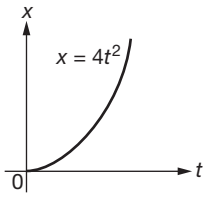


ILLUSTRATION 24

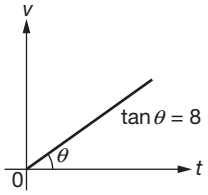
The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



SOLUTION

$x = 4t^2$

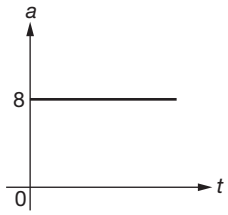
$\Rightarrow$ $v = \dfrac{dx}{dt} = 8t$



Hence, velocity-time graph is a straight line having slope i.e., $\tan\theta = 8$

$a = \dfrac{dv}{dt} = 8$

Hence, acceleration is constant throughout and is equal to $8\ \text{ms}^{-2}$.



Conceptual Note(s)

(a) For uniformly accelerated motion $(a \neq 0)$, x-t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.

(b) For uniformly accelerated motion $(a \neq 0)$, v-t graph is a straight line whose slope gives the acceleration of the particle.

(c) In general, the slope of tangent in x-t graph is velocity and the slope of tangent in v-t graph is the acceleration.

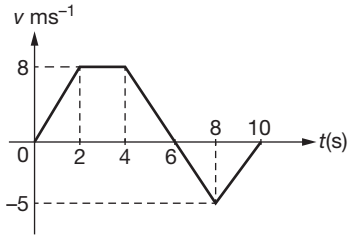
(d) The area under a-t graph gives the change in velocity.

(e) The area between the v-t graph gives the distance travelled by the particle, if we take all areas as positive.

(f) Area under v-t graph gives displacement, if areas below the t-axis are taken negative.

ILLUSTRATION 25

For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?



SOLUTION

Distance travelled = Area under v-t graph (taking all areas as positive)

$$\begin{pmatrix}\text{Distance}\\ \text{travelled}\end{pmatrix}=\begin{pmatrix}\text{Area of}\\ \text{trapezium}\end{pmatrix}+\begin{pmatrix}\text{Area of}\\ \text{triangle}\end{pmatrix}$$

$$\Rightarrow \quad \text{Distance travelled} = \frac{1}{2}(2+6)(8)+\frac{1}{2}(4)(5)$$

$$\Rightarrow \quad \text{Distance travelled} = 32+10 = 42 \text{ m}$$

Displacement = Area under v-t graph (taking areas below time axis as negative)

$$\text{Displacement} = \begin{pmatrix}\text{Area of}\\ \text{trapezium}\end{pmatrix}-\begin{pmatrix}\text{Area of}\\ \text{triangle}\end{pmatrix}$$

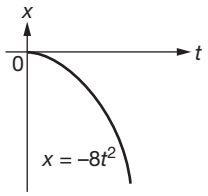
$$\Rightarrow \quad \text{Displacement} = \frac{1}{2}(2+6)(8)-\frac{1}{2}(4)(5)$$

$$\Rightarrow \quad \text{Displacement} = 32-10 = 22 \text{ m}$$

Hence, distance travelled is 42 m and displacement is 22 m

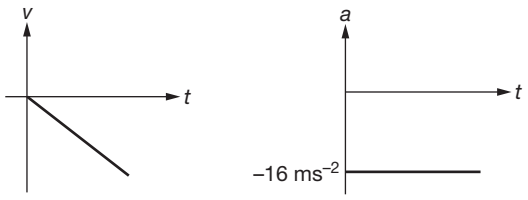
ILLUSTRATION 26

The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



SOLUTION

Taking upward direction as positive so, downward direction is taken as negative.



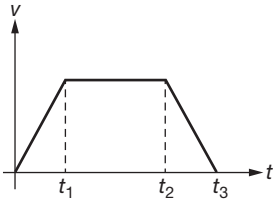
$$x = -8t^2$$

$$\Rightarrow \quad v = \frac{dx}{dt} = -16t$$

$$\Rightarrow \quad a = -16 \text{ ms}^{-2}$$

ILLUSTRATION 27

Draw displacement-time and acceleration-time graph for the given velocity-time graph.



SOLUTION

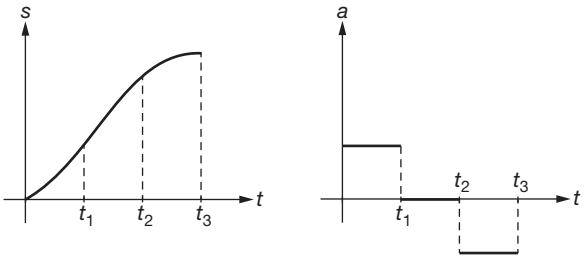


ILLUSTRATION 28

The speed of a train increases at a constant rate α from zero to v and then remains constant for an interval and finally decreases to zero at a constant rate β. If l is the total distance travelled by the train then find the total time taken to complete the journey. At what value of v is the time of travel shortest? What is the value of the shortest time?

SOLUTION

Let us first draw v-t graph showing the situation discussed.

For accelerated Motion (O to A)

$$v_{\max} = v = \alpha t_1$$

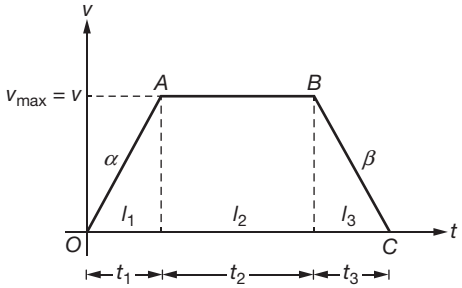
$$\Rightarrow \quad t_1 = \frac{v}{\alpha} \qquad \ldots(1)$$

Also, $l_1 = \dfrac{v^2}{2\alpha}$ $\quad \{\because\ v^2 - 0^2 = 2\alpha l_1\}$ …(2)

For Uniform Motion (*A* to *B*)

$$l_2 = vt_2$$

$$\Rightarrow \quad t_2 = \frac{l_2}{v} \qquad \ldots(3)$$



For Decelerated Motion (*B* to *C*)

$$0 = v + (-\beta)t_3$$

$$\Rightarrow \quad t_3 = \frac{v}{\beta} \qquad \ldots(4)$$

Also, $l_3 = \dfrac{v^2}{2\beta}$ $\quad \{\because\ 0^2 - v^2 = 2(-\beta)l_3\}$ …(5)

If t be the total time of journey, then

$$t = t_1 + t_2 + t_3$$

$$\Rightarrow \quad t = \frac{v}{\alpha} + \frac{l_2}{v} + \frac{v}{\beta}$$

$$\Rightarrow \quad t = \frac{v}{\alpha} + \frac{l - (l_1 + l_3)}{v} + \frac{v}{\beta} \qquad \{\because\ l_1 + l_2 + l_3 = l\}$$

$$\Rightarrow \quad t = \frac{v}{\alpha} + \frac{l}{v} - \left(\frac{\frac{v^2}{2\alpha}}{v} + \frac{\frac{v^2}{2\beta}}{v}\right) + \frac{v}{\beta} \qquad \{\text{using (2) and (5)}\}$$

$$\Rightarrow \quad t = \frac{v}{\alpha} + \frac{l}{v} - \frac{v}{2\alpha} - \frac{v}{2\beta} + \frac{v}{\beta}$$

$$\Rightarrow \quad t = \frac{l}{v} + \frac{v}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad \ldots(6)$$

For t to be MINIMUM, we must have

$$\frac{dt}{dv} = 0$$

$$\Rightarrow \quad -\frac{l}{v^2} + \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = 0$$

$$\Rightarrow \quad v = \sqrt{\frac{2l\alpha\beta}{\alpha + \beta}} \qquad \ldots(7)$$

So, $t_{MIN} = \dfrac{l}{\sqrt{\frac{2l\alpha\beta}{\alpha+\beta}}} + \dfrac{1}{2}\sqrt{\dfrac{2l\alpha\beta}{\alpha+\beta}}\left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right)$

{Substitute (7) in (6)}

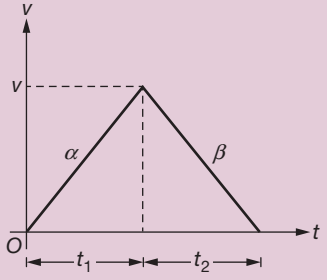
$$\Rightarrow \quad t_{MIN} = \sqrt{\frac{l(\alpha+\beta)}{2\alpha\beta}} + \sqrt{\frac{l\alpha\beta}{2(\alpha+\beta)}}\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$\Rightarrow \quad t_{MIN} = \sqrt{\frac{l(\alpha+\beta)}{2\alpha\beta}} + \sqrt{\frac{l(\alpha+\beta)}{2\alpha\beta}}$$

$$\Rightarrow \quad t_{MIN} = 2\sqrt{\frac{l(\alpha+\beta)}{2\alpha\beta}} = \sqrt{\frac{2l(\alpha+\beta)}{\alpha\beta}}$$

$$\Rightarrow \quad t_{MIN} = \sqrt{2l\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)}$$

Please note that this time equals the time of journey when the train just accelerated to attain a velocity v and immediately decelerated to zero velocity after covering a total distance l. Then



$$t_{MIN} = t_1 + t_2$$

$$\Rightarrow \quad t_{MIN} = \frac{v}{\alpha} + \frac{v}{\beta}$$

$$\Rightarrow \quad t_{MIN} = v\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad \ldots(1)$$

Also, $l = \dfrac{v^2}{2\alpha} + \dfrac{v^2}{2\beta}$

$$\Rightarrow \quad v=\sqrt{\frac{2l\alpha\beta}{\alpha+\beta}}$$

Substituting in (1), we get

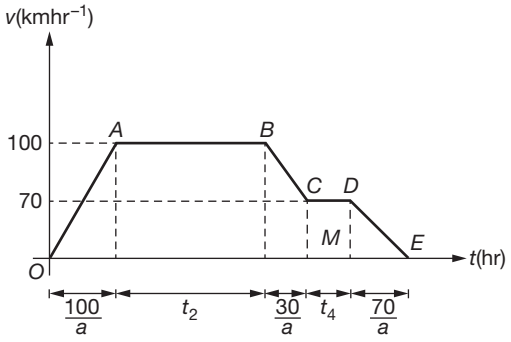
$$t_{MIN}=\sqrt{2l\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)}$$

ILLUSTRATION 29

Two road rally checkpoints A and B are located on the same highway and are 12 km apart. The speed limits for the first 8 km and the last 4 km of the section of highway are 100 kmh^{-1} and 70 kmh^{-1}, respectively. Drivers must stop at each checkpoint and the specified time between points A and B is $\frac{25}{3}$ min. Knowing that a driver accelerates and decelerates at the same constant rate, determine the magnitude of his acceleration if he travels at the speed limit as much as possible.

SOLUTION

Total time taken by the driver to go from A to B is $\frac{25}{3}$ min. We must note here that a driver first accelerates with acceleration a (say) for time t_1, moves with uniform velocity of 100 kmh^{-1} for some time t_2 (say) and retards to a speed of 70 kmh^{-1} with deceleration a (again) for a time t_3 (say) to travel a total distance of 8 km. Just after he completes 8 km, he maintains his speed at 70 kmh^{-1} for a time t_4 (say) and then finally retards with deceleration a (again) for a time t_5 (say) to complete a further distance of 4 km. As per this discussion, the v-t plot for the situation is shown here.



Since $t_1+t_2+t_3+t_4+t_5=\frac{25}{3}$ min

$$\Rightarrow \quad \frac{100}{a}+t_2+\left(\frac{100-70}{a}\right)+t_4+\left(\frac{70}{a}\right)=\frac{25}{3}\text{ min}$$

$$\Rightarrow \quad t_2+t_4+\frac{200}{a}=\frac{25}{3}\text{ min}$$

$$\Rightarrow \quad t_2+t_4+\frac{200}{a}=\frac{25}{180}\text{ hr} \qquad \ldots(1)$$

Further, area under the v-t graph gives displacement, so

$$\text{Area}(OABCM)=8\text{ km}$$

$$\frac{1}{2}\left(\frac{100}{a}\right)(100)+100t_2+\frac{1}{2}\left(\frac{30}{a}\right)(100+70)=8$$

$$\Rightarrow \quad \frac{5000}{a}+100t_2+\frac{2550}{a}=8$$

$$\Rightarrow \quad \frac{7550}{a}+100t_2=8 \qquad \ldots(2)$$

Also, $\text{Area}(CDE)=4$ km

$$\Rightarrow \quad 70t_4+\frac{1}{2}\left(\frac{70}{a}\right)(70)=4\text{ km}$$

$$\Rightarrow \quad \frac{2450}{a}+70t_4=4 \qquad \ldots(3)$$

From (2), (3), we get

$$t_2=\frac{1}{100}\left(8-\frac{7550}{a}\right)\text{ and }t_4=\frac{1}{70}\left(4-\frac{2450}{a}\right)$$

Substituting in (1), we get

$$\frac{1}{100}\left(8-\frac{7550}{a}\right)+\frac{1}{70}\left(4-\frac{2450}{a}\right)+\frac{200}{a}=\frac{25}{180}$$

$$\Rightarrow \quad \frac{8}{100}-\frac{7550}{100a}+\frac{4}{70}-\frac{2450}{70a}+\frac{200}{a}=\frac{25}{180}$$

$$\frac{96}{700}-\frac{75.5}{a}-\frac{35}{a}+\frac{200}{a}=\frac{25}{180}$$

$$\Rightarrow \quad \frac{89.5}{a}=\frac{11}{6300}$$

$$\Rightarrow \quad a=51259\text{ kmhr}^{-2}$$

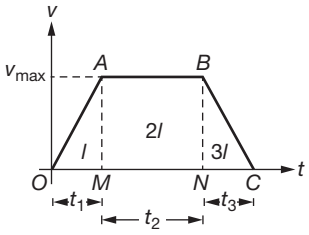
$$\Rightarrow \quad a=3.96\text{ ms}^{-2}$$

ILLUSTRATION 30

A particle starts from rest and traverses a distance l with uniform acceleration, then moves uniformly over a further distance $2l$ and finally comes to rest after moving a further distance $3l$ under uniform retardation. Assuming entire motion to be rectilinear motion, find the ratio of average speed over the journey to the maximum speed on its way.

SOLUTION

$$v_{av} = \frac{\text{Total Distance}}{\text{Total Time}}$$



$$\Rightarrow \quad v_{av} = \frac{l + 2l + 3l}{t_1 + t_2 + t_3}$$

For OA, $l = \frac{1}{2} t_1 v_{max}$

$$\Rightarrow \quad t_1 = \frac{2l}{v_{max}}$$

For AB, $2l = v_m t_2$

$$\Rightarrow \quad t_2 = \frac{2l}{v_{max}}$$

For BC, $3l = \frac{1}{2} t_3 v_{max}$

$$\Rightarrow \quad t_3 = \frac{6l}{v_{max}}$$

$$\Rightarrow \quad v_{av} = \frac{6l}{\left(\frac{10l}{v_{max}}\right)}$$

Hence, $\frac{v_{av}}{v_{max}} = \frac{3}{5}$

ILLUSTRATION 31

A train starts from station A with uniform acceleration α for some distance and then goes with uniform retardation β for some more distance to come to rest at station B. The distance between station A and B is 4 km and the train takes 4 minute to complete this journey, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$, where α and β are in $\text{km}(\text{min})^{-2}$.

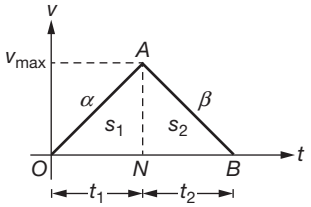
SOLUTION

Since, $s_1 + s_2 = 4$ km

and $t_1 + t_2 = 4$ min {given}

Also, $t_1 = \frac{v_{max}}{\alpha}$ and

$$t_2 = \frac{v_{max}}{\beta}$$



Adding we get,

$$t_1 + t_2 = v_{max}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \quad \ldots(1)$$

Now $s_1 = \frac{1}{2} v_{max} t_1$ …(2)

and $s_2 = \frac{1}{2} v_{max} t_2$ …(3)

Adding (2) and (3) we get,

$$v_{max} = 2 \text{ km}(\text{min})^{-1}$$

$$\therefore \quad \frac{1}{\alpha} + \frac{1}{\beta} = 2 \text{ km}(\text{min})^{-2}$$

INTERPRETATION OF GRAPHS OF VARIOUS TYPES OF MOTION

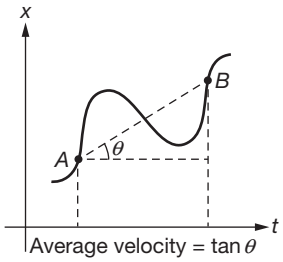
Interpretation of *x*-*t* Graph

(a) The line on the graph representing $x(t)$ is called a **World-Line**.

(b) A point on the World Line is called an **Event**.

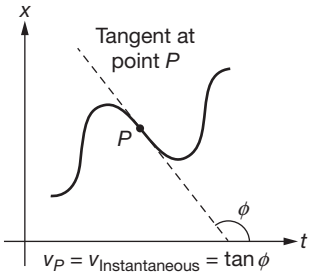
(c) The average slope of the World Line gives the average velocity. Just join the two points with a line and then find the angle which this line (joining the points) makes with the x-axis. If this angle is θ, then

$$\text{Average Velocity} = v_{av} = \tan\theta$$



(d) The slope of an event gives the instantaneous velocity. Just draw a tangent to the curve at a point P and then find the angle which this tangent makes with the x-axis. If this angle is ϕ, then the instantaneous velocity at the point P is

$$v_P = v_{\text{instantaneous}} = \tan\phi$$



Interpretation of *v*-*t* Graph

(a) The average slope of a curve in $\vec{v}(t)$ graph gives average acceleration.

(b) The instantaneous slope of a curve at a point in $\vec{v}(t)$ graph gives the instantaneous acceleration.

(c) The area under a curve in $\vec{v}(t)$ graph gives the displacement of the body.

(d) v-t graph can also be used to find the average velocity of the body, because

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\text{Area under } v\text{-}t \text{ graph}}{\text{Time Interval}}$$

Interpretation of *a*-*t* Graph

(a) The area under a curve in a-t graph gives the change in the velocity of the body. So

$$\text{Area under } a\text{-}t \text{ graph} = \Delta v = v_f - v_i$$

(d) a-t graph can also be used to find the average acceleration of the body, because

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{\text{Area under } a\text{-}t \text{ graph}}{\text{Time Interval}}$$

Interpretation to Other Graphs

(a) Area under a curve in a-x graph $= \dfrac{v^2 - u^2}{2}$

(b) Slope of v^2-x graph gives twice the acceleration

i.e. $\dfrac{d}{dx}(v^2) = 2a$

(c) From v-x graph we can also find the instantaneous acceleration of a particle. Suppose we have to find the acceleration a_P at a point P on the v-x curve. To calculate a_P, we follow the steps given below.

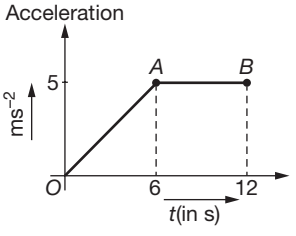
STEP-1: First find the velocity v_P of the particle at the point P.

STEP-2: Then calculate the instantaneous slope $\left(\dfrac{dv}{dx}\right)_P = \tan\theta$ of the curve at the same point.

STEP-3: Finally, the acceleration at the point P is the product of values calculated in Steps 1 and 2 i.e. $a_P = v_P\left(\dfrac{dv}{dx}\right)_P$.

ILLUSTRATION 32

For an airplane to take-off it accelerates according to the graph shown and takes 12 s to take-off from the rest position. Calculate the distance travelled by the airplane from $t = 0$ to $t = 12$ s.



SOLUTION

From O → A we have acceleration varying linearly with time hence

$$a-0=\frac{5}{6}(t-0)$$

$$\Rightarrow \quad a=\frac{5}{6}t$$

$$\Rightarrow \quad \frac{dv}{dt}=\frac{5}{6}t$$

$$\Rightarrow \quad \int_0^{v_1} dv=\frac{5}{6}\int_0^t t\,dt$$

$$\Rightarrow \quad v_1=\frac{5}{12}t^2$$

Further $v_1=\dfrac{dx_1}{dt}=\dfrac{5}{12}t^2$

$$\Rightarrow \quad \int_0^{x_1} dx_1=\frac{5}{12}\int_0^6 t^2dt$$

$$\Rightarrow \quad x_1=\frac{5}{36}t^3\Big|_0^6$$

$$\Rightarrow \quad x_1=\frac{5}{36}(6^3)=30 \text{ m}$$

For A $\rightarrow$ B the acceleration is constant and have a value of $5\ \text{ms}^{-2}$. Now velocity at point A is

$$v_1=\frac{5}{12}(36)=15\ \text{ms}^{-1}$$

$$\Rightarrow \quad x_2=v_1(6)+\frac{1}{2}(5)(6)^2$$

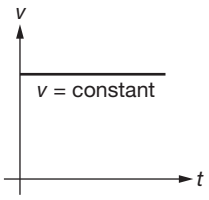
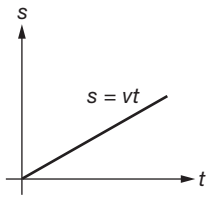
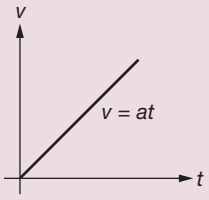
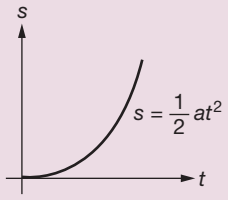
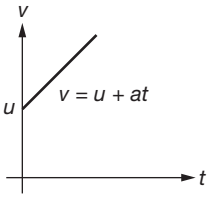
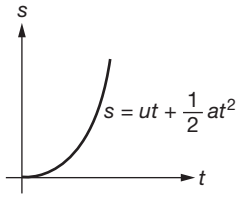
$$\Rightarrow \quad x_2=180 \text{ m}$$

So, total distance travelled is

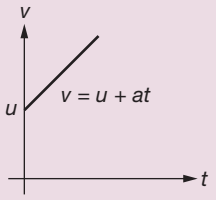
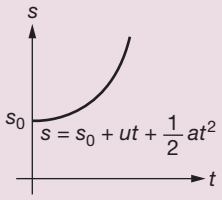
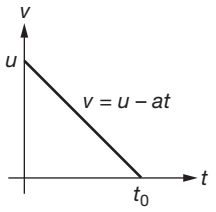
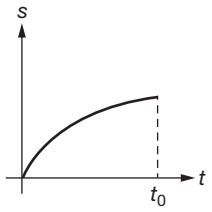
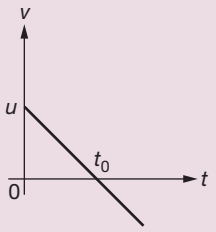
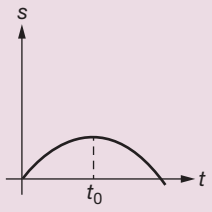
$$x=30+180=210 \text{ m}$$

GRAPHS

Now, let us plot v-t and s-t graphs of some standard results. To draw the following graphs assume that the particle has got either a one-dimensional motion with uniform velocity or with constant acceleration.

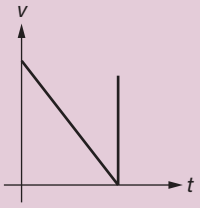
S.No.	Situation	*v-t* Graph	*s-t* Graph	Interpretation
1.	Uniform motion			(i) Slope of s-t graph i.e. v is constant. (ii) In s-t graph $s=0$ at $t=0$.
2.	Uniformly accelerated motion with $u=0$ and $s=0$ at $t=0$			(i) $u=0$, i.e., $v=0$ at $t=0$. (ii) $u=0$, i.e., slope of s-t graph at $t=0$, should be zero. (iii) a or slope of v-t graph is constant.
3.	Uniformly accelerated motion with $u\neq 0$ but $s=0$ at $t=0$			(i) $u\neq 0$, i.e., v or slope of s-t graph at $t=0$ is not zero. (ii) v or slope of s-t graph gradually goes on increasing.

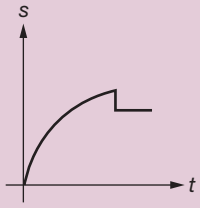
(*Continued*)

S.No.	Situation	*v-t* Graph	*s-t* Graph	Interpretation
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$			(i) $s = s_0$ at $t = 0$
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of *s-t* graph at $t = 0$ gives u. (ii) Slope of *s-t* graph at $t = t_0$ becomes zero. (iii) In this case u can't be zero.
6.	Uniformly retarded then accelerated in opposite direction.			(i) At time $t = t_0$, $v = 0$ or slope of *s-t* graph is zero. (ii) In *s-t* graph slope or velocity first decreases then increases with opposite sign.

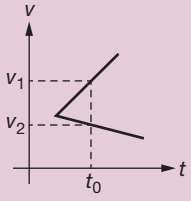
Conceptual Note(s)

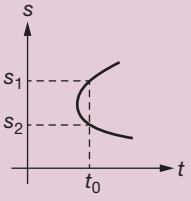
(a) Slopes of *v-t* or *s-t* graphs can never be infinite at any point, because infinite slope of *v-t* graph means infinite acceleration. Similarly, infinite slope of *s-t* graph means infinite velocity. Hence, the following graphs are not possible:





(b) At one time, two values of velocity or displacement are not possible. Hence, the following graphs are not acceptable:





(c) Different values of displacements in *s-t* graph corresponding to given *v-t* graph can be calculated just by calculating areas under *v-t* graph. There is no need of using equations like $v = u + at$, etc.

ILLUSTRATION 33

A particle starting from rest undergoes an acceleration that increases linearly with time. Estimate the particle's velocity n second after the start and the distance moved by the particle in these n second. Draw the graph showing the variation of acceleration, velocity and distance with time.

SOLUTION

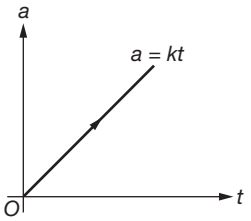
$$a = kt$$

$$\Rightarrow \quad \frac{dv}{dt} = kt$$

$$\Rightarrow \quad dv = ktdt$$

$$\Rightarrow \quad \int_0^v dv = k\int_0^t t\,dt$$

$$\Rightarrow \quad v = \frac{kt^2}{2} = \frac{kn^2}{2}$$



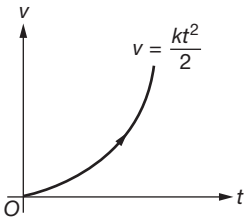
Further $v = \frac{dx}{dt}$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{kt^2}{2}$$

$$\Rightarrow \quad dx = \frac{kt^2}{2}dt$$

$$\Rightarrow \quad \int_0^x dx = \frac{k}{2}\int_0^t t^2 dt$$

$$\Rightarrow \quad x = \frac{kt^3}{3} = \frac{kn^3}{6}$$



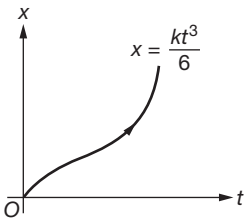
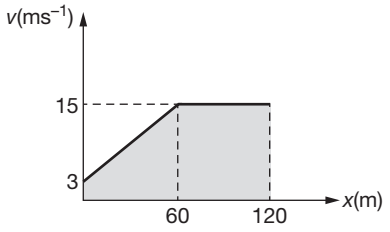


ILLUSTRATION 34

The velocity-displacement (v-x) graph describing the motion of a motorcycle is shown in figure. Construct the a-x graph of the motion and determine the time needed for the motorcycle to reach the position $x = 120$ m.



SOLUTION

For $0 < x \le 60$ m, we have

$$v = \frac{12}{60}x + 3 = 3 + \frac{x}{5}$$

$$\frac{dv}{dt} = \left(\frac{1}{5}\right)\frac{dx}{dt} = \frac{1}{5}(v) = \frac{1}{5}\left(3 + \frac{x}{5}\right) = \frac{3}{5} + \frac{x}{25} \quad \ldots(1)$$

$$\Rightarrow \quad a = \frac{3}{5} + \frac{x}{25}$$

i.e., the a-x graph is a straight line, where at $x = 0$, $a = \frac{3}{5}$ ms^{-2} $= 0.6$ ms^{-2} and at $s = 60$ m, $a = 3$ ms^{-2}

For $s > 60$ m, we have $v =$ constant

$$\Rightarrow \quad a = 0$$

The corresponding a-x graph is shown here

From equation (1), we have

$$\frac{dv}{dt} = \frac{v}{5}$$

$$\Rightarrow \quad \int_3^v \frac{dv}{v} = \frac{1}{5}\int_0^t dt$$

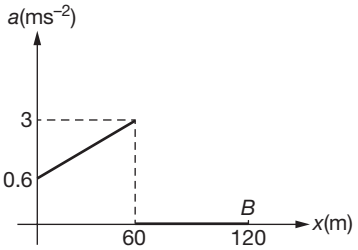
$$\Rightarrow \quad \ln\left(\frac{v}{3}\right) = \frac{t}{5}$$

$$\Rightarrow \quad v = 3e^{t/5}$$

$$\Rightarrow \quad \int_0^{60} dx = 3\int_0^{t_1} e^{t/5}dt \qquad \left\{\because\ v = \frac{dx}{dt}\right\}$$

$$\Rightarrow \quad 60 = 15\left(e^{t_1/5} - 1\right)$$

$$\Rightarrow \quad t_1 = 8 \text{ s}$$



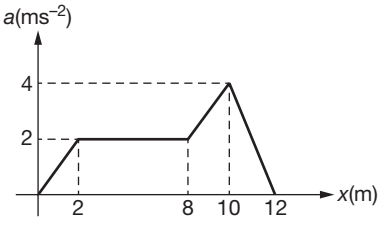
Time taken to travel next 60 m with speed 15 ms^{-1} will be t_2 given by

$$t_2 = \frac{60}{15} = 4 \text{ s}$$

$\therefore$ Total time $= t_1 + t_2 = 8 + 4 = 12$ s

ILLUSTRATION 35

The acceleration-displacement $(a - x)$ graph of a particle moving in a straight line is as shown. Assume the particle to start from rest, find the velocity of the particle when displacement of the particle is, 12 m.



SOLUTION

Since $a = \dfrac{dv}{dt}$

$\Rightarrow \quad v\dfrac{dv}{dx} = a$

$\Rightarrow \quad vdv = adx$

$\Rightarrow \quad \displaystyle\int_0^v vdv = \int adx$

$\Rightarrow \quad \dfrac{v^2}{2} = \text{area under } a\text{-}x \text{ graph}$

$\Rightarrow \quad v = \sqrt{2(\text{area under } a\text{-}x \text{ graph})}$

The area A under the a-x graph is given by

$$A = \frac{1}{2}(2)(2) + (6)(2) + \frac{1}{2}(2+4)(2) + \frac{1}{2}(2)(4)$$

$\Rightarrow \quad A = 2 + 12 + 6 + 4 = 24 \text{ m}^2\text{s}^{-2}$

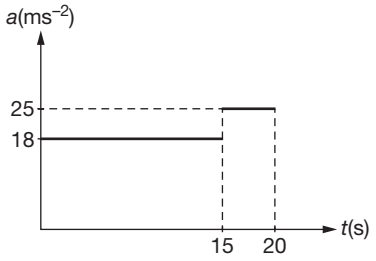
$\Rightarrow \quad v = \sqrt{2 \times 24} = 4\sqrt{3} \text{ ms}^{-1}$

Test Your Concepts-IV

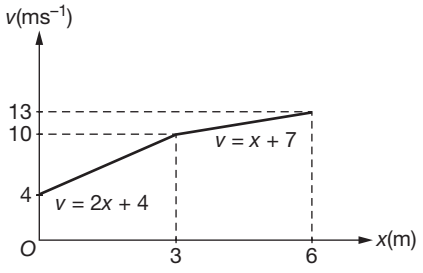
Based on Graph

(Solutions on page H.77)

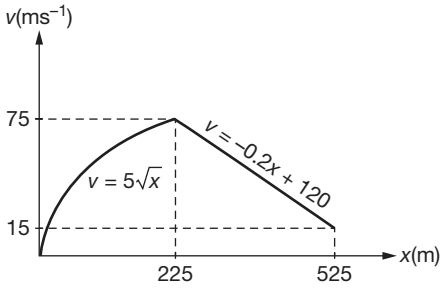
1. A two stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage burns out and the second stage ignites. Plot the v-t and s-t graphs which describe the two-stage motion of the missile for $0 \le t \le 20$ s.



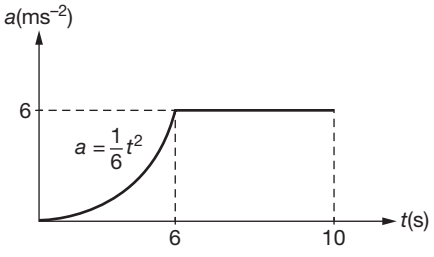
2. The particle moves rectilinearly with the velocity described by the graph. Construct the acceleration-displacement graph.



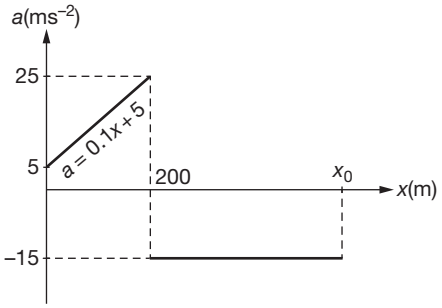
3. The car is moving along a straight road with the speed described by the v-x graph. Construct the a-x graph.



4. A motorcyclist starting from rest travels along a straight road and for $0 < t \le 10$ s, it has an acceleration as shown. Draw the v-t graph that describes the motion and find the distance travelled in 10 s.

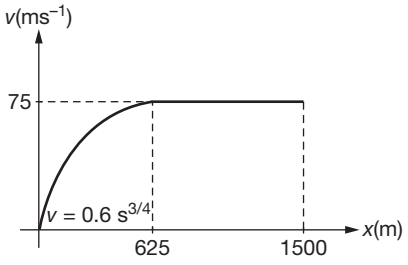


5. The dragster starts from rest and travels along a straight track with an acceleration-deceleration described by the graph. Construct the *v*-*x* graph for $0 \le x \le x_0$ and determine the distance x_0 travelled before the dragster again comes to rest.

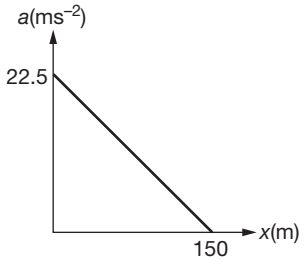


6. A particle travels along a curve defined by the equation $x = (t^3 - 3t^2 + 2t)$ m, where t is in seconds. Draw the *x*-*t*, *v*-*t* and *a*-*t* graphs for the particle for $0 \le t \le 3$ s.

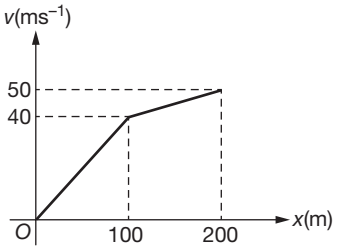
7. A truck is travelling along the straight line with a velocity described by the graph. Construct the *a*-*x* graph for $0 \le x \le 1500$ m.



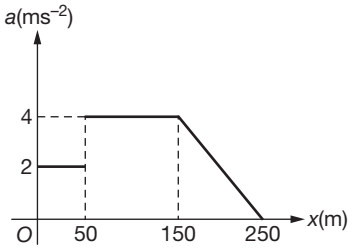
8. The jet plane starts from rest from $x = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has travelled 60 m.



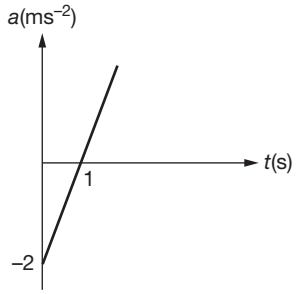
9. The *v*-*x* graph for an airplane travelling on a straight run-way is shown. Determine the acceleration of the plane at $x = 50$ m and $x = 150$ m. Draw the *a*-*x* graph.



10. Starting from rest from $x = 0$, a boat travels in a straight line with an acceleration as shown by the *a*-*x* graph. Determine the boat's speed at $x = 40$ m, 90 m and 200 m.



11. The acceleration of particle moving rectilinearly, varies with time as shown.

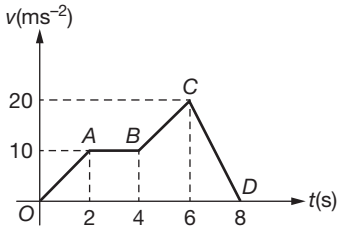


(a) Find an expression for velocity in terms of *t*.

(b) Calculate the displacement of the particle in the interval from $t = 2$ s to $t = 4$ s. Assume that $v = 0$ at $t = 0$.

12. Acceleration-time graph of a particle moving in a straight line is shown in figure. Velocity of particle at time $t = 0$ is $2\ ms^{-1}$. Find velocity at the end of fourth second.

13. Velocity-time graph of a particle moving in a straight line is shown in figure. Plot the corresponding displacement-time graph of the particle if at time $t = 0$, displacement $s = 0$.



VERTICAL MOTION UNDER GRAVITY

Vertical motion under gravity, is the case of motion of a particle moving rectilinearly under the influence of gravity. To solve problems involving motion of a particle under gravity, we make use of the following steps for the sake of convenience and accuracy.

STEP-1

Take initial direction of motion of the particle as positive.

FOR EXAMPLE

(a) If a particle is dropped from a tower, then we can take downward direction (the direction of initial motion) as positive.

(b) If a particle is thrown vertically upwards from a tower, then we can take upward direction (the direction of initial motion) as positive.

STEP-2

The quantities (i.e. u, v, a and s) along the initial direction of motion will be taken as positive whereas opposite to the initial direction of motion are taken as negative.

FOR EXAMPLE

(a) For the particle dropped from the tower of height h, we have taken downward direction (the direction of initial motion) as positive. So, in this case we have

$u = 0,\ v = +v,\ a = +g$ and $s = +h$

as all are in the downward direction.

(b) For the particle thrown vertically upwards from a tower of height h, we have taken upward direction (the direction of initial motion) as positive. So, in this case, since

- initial velocity u is upwards, so $u = +u$,
- acceleration due to gravity is downwards, so $a = -g$,
- particle eventually reaches the ground, so the displacement of the particle is downwards and hence $s = \Delta y = -h$.

STEP-3

Now apply the equations of rectilinear motion of the particle i.e.

$$v = u + at, \qquad s = ut + \frac{1}{2}at^2$$

$$v^2 - u^2 = 2as, \qquad s = \frac{u+v}{2}t$$

$$s_n^{th} = u + \frac{1}{2}a(2n-1)$$

and substituting the values as calculated in **Step 2**, to get the desired solution to the problems as discussed below.

FOR EXAMPLE

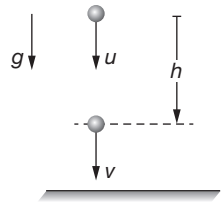
(a) For a body dropped $(u = 0)$ from a height h, the equation of motion are

$$v = gt, \qquad h = \frac{1}{2}gt^2, \qquad v = \sqrt{2gh}$$

Initial velocity in case of dropping is zero.

(b) For a body thrown downward with initial velocity u from a height h, the equations of motion are

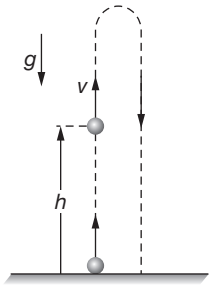
$$v = u + gt, \quad h = ut + \frac{1}{2}gt^2, \quad v = \sqrt{u^2 + 2gh}$$



Take downwards as positive

(c) For a body launched up from the ground, with initial velocity u, the equations of motion before the particle attains maximum height are

$$v = u - gt, \quad h = ut - \frac{1}{2}gt^2, \quad v = \sqrt{u^2 - 2gh}$$



Take upwards as positive

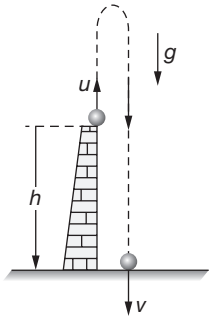
When the particle returns to the ground, then $s = 0$ and hence we have

$$0 = ut - \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \text{Time of Flight} = \frac{2u}{g}$$

(d) For a body launched up from a tower of height h, taking upward direction as positive, then we have

$$s = -h, \quad u = +u, \quad a = -g$$



So, the equations of motion are written as

$$-h = ut + \frac{1}{2}(-g)t^2$$

$$v = u + (-g)t$$

and

$$v^2 - u^2 = 2(-g)(-h)$$

ILLUSTRATION 36

A ball is thrown upwards from the ground with an initial speed of u. At two instants of time, having an interval of 6 s, the ball is at a height of 80 m from the ground. Find u. Take $g = 10\ \text{ms}^{-2}$.

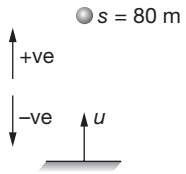
SOLUTION

METHOD I

Consider the upward direction as positive, then

$$u = +u\ \text{ms}^{-1},\ a = g = -10\ \text{ms}^{-2} \text{ and } s = +80\ \text{m}$$

Since, $s = ut + \frac{1}{2}at^2$, so we get



$$80 = ut - 5t^2$$

$$\Rightarrow \quad 5t^2 - ut + 80 = 0$$

$$\Rightarrow \quad t = \frac{u + \sqrt{u^2 - 1600}}{10}$$

and $\dfrac{u - \sqrt{u^2 - 1600}}{10}$

Now it is given that these two instants, when the ball is at 80 m from the ground have an interval of 6 s in between. So,

$$\frac{u + \sqrt{u^2 - 1600}}{10} - \frac{u - \sqrt{u^2 - 1600}}{10} = 6$$

$$\Rightarrow \quad \frac{\sqrt{u^2 - 1600}}{5} = 6$$

$$\Rightarrow \quad \sqrt{u^2 - 1600} = 30$$

$$\Rightarrow \quad u^2 - 1600 = 900$$

$$\Rightarrow \quad u^2 = 2500$$

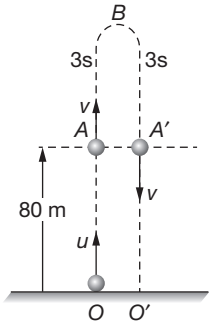
$$\Rightarrow \quad u = \pm 50\ \text{ms}^{-1}$$

$$\Rightarrow \quad |u| = 50\ \text{ms}^{-1}$$

METHOD II

Since $t_{A \to A'} = 6\,\text{s}$

$$\Rightarrow \quad t_{A \to B} = t_{B \to A'} = 3\,\text{s}$$



For A → B

$$0 = v + (-10)(3)$$

$$\Rightarrow \quad v = 30\ \text{ms}^{-1}$$

For O → A

$$v^2 - u^2 = 2(-10)(80)$$

$\Rightarrow \quad 900 - u^2 = -1600$

$\Rightarrow \quad u^2 = 900 + 1600$

$\Rightarrow \quad u = \sqrt{2500} = 50 \text{ ms}^{-1}$

ILLUSTRATION 37

A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 ms^{-1} from the ground along the same line. Find out the position where two particle will meet?

SOLUTION

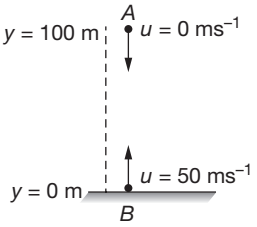
Let the upward direction be taken as positive.
Let the particles meet at a distance y from the ground.

For particle *A*

$y_0 = +100 \text{ m}$

$u = 0$

$a = -10 \text{ ms}^{-2}$



Since $y = y_0 + ut + \frac{1}{2}at^2$

$\Rightarrow \quad y = 100 + 0(t) - \frac{1}{2} \times 10 \times t^2$

$\Rightarrow \quad y = 100 - 5t^2 \qquad \ldots(1)$

For particle *B*

$y_0 = 0 \text{ m}$

$u = +50 \text{ ms}^{-1}$

$a = -10 \text{ ms}^{-2}$

$\Rightarrow \quad y = 50(t) - \frac{1}{2} \times 10 \times t^2$

$\Rightarrow \quad y = 50t - 5t^2 \qquad \ldots(2)$

According to the problem

$50t - 5t^2 = 100 - 5t^2$

$\Rightarrow \quad t = 2 \text{ s}$

Putting $t = 2 \text{ s}$ in equation (1), we get

$y = 100 - 20 = 80 \text{ m}$

Hence, the particles will meet at a height 80 m above the ground.

ILLUSTRATION 38

A particle is dropped from a tower is found to travel 45 m in the last second of its journey. Calculate the height of the tower.

SOLUTION

Let the total time of journey be n seconds. Then

$s_{n^{\text{th}}} = u + \frac{a}{2}(2n - 1)$

$\Rightarrow \quad 45 = 0 + \frac{10}{2}(2n - 1)$

$\Rightarrow \quad n = 5 \text{ s}$

So, 5^{th} second is the last second of motion. Hence height of tower is given by

$h = \frac{1}{2}gt^2$

$\Rightarrow \quad h = \frac{1}{2}(10)(5)^2$

$\Rightarrow \quad h = 125 \text{ m}$

ILLUSTRATION 39

A particle is projected vertically upwards. Prove that it will be at $\frac{3}{4}$ of its greatest height at time which are in the ratio $1:3$.

SOLUTION

For the particle projected upwards with an initial velocity u, the greatest height attained is

$H = \frac{u^2}{2g}$

Let t be the time when the particle is at a height

$h = \frac{3H}{4} = \frac{3}{4}\left(\frac{u^2}{2g}\right)$

Using the formula $s = ut + \frac{1}{2}gt^2$, we get

$\frac{3}{4}\left(\frac{u^2}{2g}\right) = ut + \frac{1}{2}gt^2$

$\Rightarrow \quad t^2 - \frac{2u}{g}t + \frac{6u^2}{8g^2} = 0$

Solving for t, we have

$t = \frac{\frac{2u}{g} \pm \sqrt{\left(\frac{4u^2}{g^2} - \frac{3u^2}{g^2}\right)}}{2} = \frac{u}{g} \pm \frac{u}{2g}$

Taking negative sign, $t_1 = \frac{u}{2g}$

Taking only positive sign, $t_2 = \frac{3u}{2g}$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{\left(\frac{u}{2g}\right)}{\left(\frac{3u}{2g}\right)} = 1:3$$

ILLUSTRATION 40

A body projected vertically upwards from the top of a tower reaches the ground in t_1 second. If it is projected vertically downwards from the same point with same speed, it reaches the ground in t_2 second. If it is just dropped from the top, it reaches the ground in t second. Prove that $t = \sqrt{t_1 t_2}$.

SOLUTION

Let h be the height of the tower. Taking downward direction as positive, we get

for the body projected upward, $h = -ut_1 + \frac{1}{2}gt_1^2$ ...(1)

for the body projected downward, $h = ut_2 + \frac{1}{2}gt_2^2$ ...(2)

for the freely falling body, $h = 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$ ...(3)

Multiplying equation (1) by t_2, we get

$$ht_2 = -ut_1t_2 + \frac{1}{2}gt_1^2t_2$$

Multiplying equation (2) by t_1, we get

$$ht_1 = ut_1t_2 + \frac{1}{2}gt_2^2t_1$$

Adding (1) and (2), we get

$$h(t_1 + t_2) = \frac{1}{2}gt_1^2t_2 + \frac{1}{2}gt_2^2t_1 = \frac{1}{2}gt_1t_2(t_1 + t_2)$$

$$\Rightarrow \quad h = \frac{1}{2}gt_1t_2 \qquad \ldots(4)$$

$$\Rightarrow \quad \frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \quad \left\{\because \ h = \frac{1}{2}gt^2 \text{ from equation (3)}\right\}$$

$$\Rightarrow \quad t^2 = t_1t_2$$

$$\Rightarrow \quad t = \sqrt{t_1t_2}$$

ILLUSTRATION 41

A stone is dropped from a balloon going up with a uniform velocity of $5\ \text{ms}^{-1}$. If the balloon was $60\ \text{m}$ high when the stone was dropped, find the height of balloon when the stone hits the ground. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad -60 = 5(t) + \frac{1}{2}(-10)t^2$$

$$\Rightarrow \quad -60 = 5t - 5t^2$$

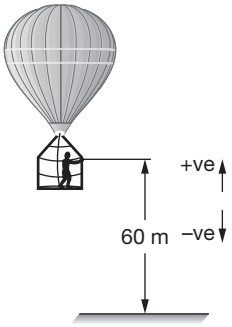
$$\Rightarrow \quad 5t^2 - 5t - 60 = 0$$

$$\Rightarrow \quad t^2 - t - 12 = 0$$

$$\Rightarrow \quad t^2 - 4t + 3t - 12 = 0$$

$$\Rightarrow \quad (t-4)(t+3) = 0$$

$$\Rightarrow \quad t = 4\ \text{s}$$



Height of balloon from ground at this instant is

$$h = 60 + (4)(5)$$

$$\Rightarrow \quad h = 80\ \text{m}$$

Conceptual Note(s)

As the particle is detached from the balloon it is having the same velocity as that of balloon, but it is moving under the influence of gravity, so $a = -g$ (Because, g is acting downwards and we have taken upward direction as positive).

ILLUSTRATION 42

A particle is projected vertically upwards from a point A on the ground. It takes t_1 second to reach a point B at a height h from A but still continues to move up. If it takes further t_2 second from B to ground again, then show that

(a) $h = \frac{1}{2}gt_1t_2$

(b) maximum height reached is $\frac{g(t_1 + t_2)^2}{8}$ and

(c) the velocity of the particle at a height $\frac{h}{2}$ is $\frac{g}{2}\left(t_1^2 + t_2^2\right)^{1/2}$.

SOLUTION

(a) Let the particle be projected upwards with a velocity u. Suppose t_1 and t_2 be the times taken for the motion from A to B and for the motion from B to C and then from C to A respectively.

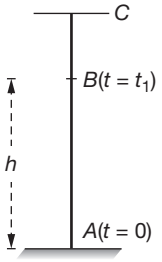
$$\therefore \quad \text{Time of flight} = (t_1 + t_2) = \left(\frac{2u}{g}\right)$$

$$\Rightarrow \quad u = \frac{g}{2}(t_1 + t_2)$$

For the motion AB,

$$h = ut_1 - \frac{1}{2}gt_1^2$$

$$\Rightarrow \quad h = \frac{g}{2}(t_1 + t_2)t_1 - \frac{1}{2}gt_1^2 = \frac{1}{2}gt_1t_2$$



(b) Maximum height reached

$$AC = H = \frac{u^2}{2g} = \frac{1}{2g}\left[\frac{g}{2}(t_1 + t_2)\right]^2$$

$$\Rightarrow \quad AC = \frac{g}{8}(t_1 + t_2)^2$$

(c) Let v be the velocity at a height $\frac{h}{2}$, then

$$v^2 = u^2 - 2g\left(\frac{h}{2}\right) = u^2 - gh$$

$$\Rightarrow \quad v^2 = \frac{g^2}{4}(t_1 + t_2)^2 - g \times \frac{1}{2}gt_1t_2$$

$$\Rightarrow \quad v^2 = \frac{g^2}{4}\left[(t_1 + t_2)^2 - 2t_1t_2\right]$$

$$\Rightarrow \quad v^2 = \frac{g^2}{4}(t_1^2 + t_2^2)$$

$$\Rightarrow \quad v = \frac{g}{2}\sqrt{t_1^2 + t_2^2}$$

ILLUSTRATION 43

A ball is dropped from a height of 80 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 11.2 s $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$.

SOLUTION

Just Before First Collision

Time taken by the ball to fall through a height of 80 m is obtained as follows:

$$u = 0$$

$$h = \frac{1}{2}gt^2$$

$$80 = \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2 \times 80}{10}} = 4 \text{ s}$$

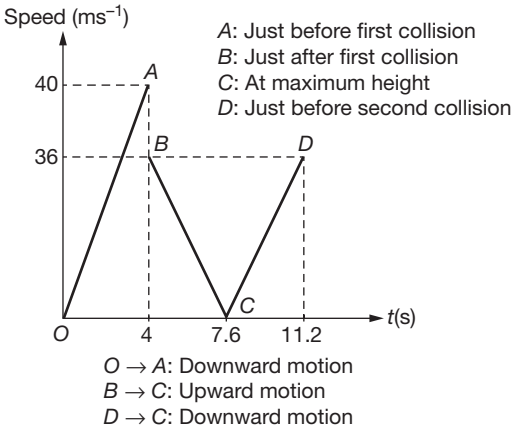
Now, $v(t) = gt$

$$\Rightarrow \quad v(4) = 10 \times 4 = 40 \text{ ms}^{-1}$$

From time $t = 0$ to $t = 4$ s, $v(t) = gt = 10t$

$$\Rightarrow \quad v(t) \propto t$$

In this duration speed increases linearly with time t from 0 to 40 ms^{-1} during the downward motion of the ball and this speed-time variation has been shown by straight line OA in figure.



Just After First Collision

At first collision with the floor

$$\text{Speed lost by ball} = \frac{1}{10}\times 40 = 4 \text{ ms}^{-1}$$

Thus, the ball rebounds with a speed of $40-4=36 \text{ ms}^{-1}$. For the further upward motion, the speed at any instant t is given by

$$v(t) = v(0) - gt = 36 - 10\times t$$

Now, the speed decreases linearly with time and becomes zero after time

$$t = \frac{36}{10} = 3.6 \text{ s}$$

Thus, the ball reaches the highest point again after time $t = 4+3.6 = 7.6$ s from the start. Straight line BC represents the speed-time graph for this upward motion.

Just Before Second Collision

At highest point, speed of ball is zero. It again starts falling. At any instant t its speed is given by

$$v(t) = 10t$$

Again, the speed of the ball increases linearly with time t from 0 to 36 ms^{-1} (initial speed of the previous upward motion) in the next time-interval of 3.6 s. Total time taken from the start is $t = 4+3.6+3.6 = 11.2$ s. This part of motion has been shown by straight line CD.

ILLUSTRATION 44

A ball is thrown upward with an initial velocity of 100 ms^{-1}.

(a) Calculate the time taken by the ball to return to the point of launch.

(b) Draw velocity-time graph for the ball and find from the graph

(i) the maximum height attained by the ball and

(ii) height of the ball after 15 s.

Take $g = 10 \text{ ms}^{-2}$.

SOLUTION

(a) Taking upward direction as positive, we get

$$u = 100 \text{ ms}^{-1},\ g = -10 \text{ ms}^{-2}$$

At highest point i.e., the point of reversal of velocity, $v = 0$

Since $v = u + gt$

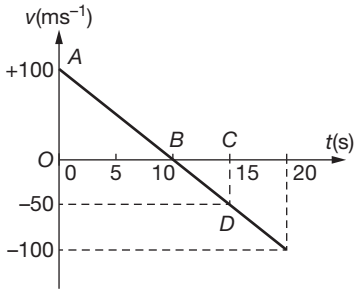
$$\Rightarrow \quad 0 = 100 - 10\times t$$

So, time taken to reach highest point, is

$$t = \frac{100}{10} = 10 \text{ s}$$

The ball will return to the ground at $T = 2t = 20$ s.

(b) Corresponding velocity-time graph of the ball is shown in figure



(i) Maximum height (H) attained by the ball is equal to the Area of ΔAOB

$$\Rightarrow \quad H = \frac{1}{2}\times 10\times 100 = 500 \text{ m}$$

(ii) Height attained after 15 s is

h = Area of ΔAOB + Area of ΔBCD

$$\Rightarrow \quad h = 500 + \frac{1}{2}(15-10)\times(-50)$$

$$\Rightarrow \quad h = 500 - 125 = 375 \text{ m}$$

ILLUSTRATION 45

A stone is dropped from the top of a cliff of height h. n second later, a second stone is projected downwards from the same cliff with a vertically downward velocity u. Show that the two stones will reach the bottom of the cliff together, if $8h(u-gn)^2 = gn^2(2u-gn)^2$. What can you say about the limiting value of n.

SOLUTION

The time taken by the first stone to reach the bottom of the cliff is $\sqrt{\frac{2h}{g}}$

According to the given problem, the second stone is projected n second later. The two stones will reach the bottom together, if the second covers the same distance in time $t = \sqrt{\frac{2h}{g}} - n$

Since, $h = ut + \frac{1}{2}gt^2$

Hence, $h = u\left(\sqrt{\frac{2h}{g}} - n\right) + \frac{1}{2}g\left(\sqrt{\frac{2h}{g}} - n\right)^2$

$\Rightarrow \quad h = u\sqrt{\frac{2h}{g}} - un + \frac{1}{2}g\left(\frac{2h}{g} - 2n\sqrt{\frac{2h}{g}} + n^2\right)$

$\Rightarrow \quad h = (u - gn)\sqrt{\frac{2h}{g}} + \left(h - un + \frac{n^2 g}{2}\right)$

$\Rightarrow \quad 2h = (u - gn)\sqrt{\frac{2h}{g}} + \frac{n^2 g}{2} - un$

Solving it, we get

$$h = \frac{gn^2}{8}\left(\frac{(2u - gn)}{(u - gn)}\right)^2$$

$\Rightarrow \quad 8h(u - gn)^2 = gn^2(2u - gn)^2$

Hence, the two stones will reach the bottom together, if the above condition is satisfied. Also, since

$t = \sqrt{\frac{2h}{g}} - n$, so we must have $t \geq 0$

$\Rightarrow \quad \sqrt{\frac{2h}{g}} - n \geq 0$

$\Rightarrow \quad n \leq \sqrt{\frac{2h}{g}}$

So the limiting value of n is $n_{\text{MAX}} = \sqrt{\frac{2h}{g}}$

ILLUSTRATION 46

Two bodies are thrown vertically upward, with the same initial velocity of 98 ms^{-1} but 4 s apart. How long after the first one is thrown will they meet?

SOLUTION

Let $y_{\max}$ be the maximum height at which the velocity of first body reduces to zero.

Using the equation $v^2 = u^2 + 2gs$, we have

$$0 = (98)^2 - 2 \times (9.8) \times y_{\max}$$

$\Rightarrow \quad y_{\max} = \frac{(98)^2}{2 \times (9.8)} = 490$ m

If t be the time taken by the body to reach the height 980 m, then from the equation $v = u + gt$, we have

$$0 = 98 - (9.8)t$$

$\Rightarrow \quad t = 10$ s

Now the second body which was thrown after 4 sec has been moving upward for 6 sec. The velocity v acquired by this body is given by

$$v = 98 - (9.8) \times 6 = 39.2 \text{ ms}^{-1}$$

The height y reached by the body is

$$y = (98 \times 6) - \frac{1}{2}(9.8) \times (6)^2 \qquad \left\{\because s = ut + \frac{1}{2}gt^2\right\}$$

$\Rightarrow \quad y = 588 - 176.4 = 411.6$ m

At this moment, the distance between the first and second body is

$$\Delta y = 490 - 411.6 = 78.4 \text{ m}$$

Now the two bodies will meet during the downward journey of the first and the upward journey of the second. Let the two bodies meet after a time t measured from above moment. The first body is coming down (initial velocity zero) and let it covers a distance y in t second. The second body is moving upward and covers a distance $(78.4 - y)$ in t second. Using $y = ut + \frac{1}{2}gt^2$ for two bodies, we have

$$y = \frac{1}{2} \times 9.8 \times t^2$$

and $(78.4 - y) = (39.2)t - \frac{1}{2} \times 9.8 \times t^2$

Solving the two equations, we get

$$t = 2 \text{ s}$$

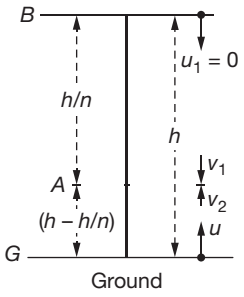
Hence the two bodies shall meet $10 + 2 = 12$ s after the first body is thrown.

ILLUSTRATION 47

A particle is dropped from the top of a tower h m high and at the same moment another particle is projected upwards from the bottom. They meet when the upper one has descended a distance $\frac{h}{n}$. Show that the velocities of the two when they meet are in the ratio $2:(n-2)$ and that the initial velocity of the particle projected up is $\sqrt{\frac{ngh}{2}}$.

SOLUTION

The situation is shown in figure.



Let the two particles meet at A after a time t
For first particle,

$$\frac{h}{n} = \frac{1}{2}gt^2 \qquad \ldots(1)$$

For second particle,

$$\left(h - \frac{h}{n}\right) = ut - \frac{1}{2}gt^2 \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$h = ut$$

$$\Rightarrow \quad t = \frac{h}{u} \qquad \ldots(3)$$

Substituting the value of t from equation (3) in equation (1), we get

$$\frac{h}{n} = \frac{1}{2}g\left(\frac{h}{u}\right)^2$$

$$\Rightarrow \quad u^2 = \frac{1}{2}ngh$$

$$\Rightarrow \quad u = \sqrt{\frac{ngh}{2}} \qquad \ldots(4)$$

Velocity v_1 of first particle at A is given by

$$v_1^2 = u_1^2 + 2g\left(\frac{h}{n}\right) = 0 + 2g\left(\frac{h}{n}\right) \qquad \ldots(5)$$

Velocity v_2 of second particle at A is given by

$$v_2^2 = u^2 - 2g\left(h - \frac{h}{n}\right)$$

$$\Rightarrow \quad v_2^2 = \frac{1}{2}ngh - 2gh\left(1 - \frac{1}{n}\right)$$

$$\Rightarrow \quad v_2^2 = gh\left[\frac{n}{2} - 2\left(1 - \frac{1}{n}\right)\right]$$

$$\Rightarrow \quad v_2^2 = gh\left(\frac{n}{2} - 2 + \frac{2}{n}\right) = gh\left(\frac{n^2 - 4n + 4}{2n}\right)$$

$$\Rightarrow \quad v_2^2 = gh\frac{(n-2)^2}{2n} \qquad \ldots(6)$$

Dividing equation (5) by equation (6), we get

$$\frac{v_1^2}{v_2^2} = \frac{\frac{2gh}{n}}{gh\frac{(n-2)^2}{2n}} = \frac{4gh}{gh(n-2)^2}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{2}{n-2}$$

ILLUSTRATION 48

A parachutist after bailing out falls 50 m, without friction. When the parachute opens, he decelerates downwards with $2\ \text{ms}^{-2}$. He reaches the ground with a speed of $3\ \text{ms}^{-1}$.

(a) How long is the parachutist in the air?
(b) At what height did he bail out?

SOLUTION

For fall without friction, i.e., free fall $h = 50$ m, $u = 0$, $g = 9.8\ \text{ms}^{-2}$ and $t = t_1$

Using $h = ut + \frac{1}{2}gt^2$, we have

$$50 = 0 + \frac{1}{2} \times 9.8 \times t_1^2$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{50 \times 2}{9.8}} = 3.16\ \text{s}$$

The velocity after falling through 50 m may be obtained by using the formula

$$v^2 = u^2 + 2gh$$

$$\Rightarrow \quad v^2 = 0 + 2 \times 9.8 \times 50$$

$$\Rightarrow \quad v = \sqrt{(2 \times 9.8 \times 50)}$$

$$\Rightarrow \quad v = 31.3\ \text{ms}^{-1}$$

Taking downward direction as positive, the initial velocity is $31.3\ \text{ms}^{-1}$ and final velocity is $3\ \text{ms}^{-1}$.

Acceleration $a = -2\ \text{ms}^{-2}$

Let time be t_2. Now using $v = u + at$, we have

$$3 = 31.3 - 2t_2$$

$$\Rightarrow \quad t_2 = \frac{28.3}{2} = 14.15\ \text{s}$$

If distance travelled be h, then $h = ut$ where $u = \frac{31.3+3}{2} = \frac{34.3}{2} = 17.15\ \text{ms}^{-1}$ and $t = 14.15\ \text{s}$

$$\Rightarrow \quad h = 17.15 \times 14.15 = 242.7\ \text{m}$$

Now, total time $= t = 3.19 + 14.15 = 17.34\ \text{s}$

and total height $= h = 50 + 242.7 = 292.7\ \text{m}$

Test Your Concepts-V

Based on Motion Under Gravity

(Solutions on page H.81)

1. Can you think of examples where velocity of a particle is
- **(a)** in opposite direction to the acceleration
- **(b)** zero but its acceleration is not zero
- **(c)** perpendicular to the acceleration

2. As a body is projected to a high altitude above the earth's surface, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0\left[\frac{R^2}{(R+y)^2}\right]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth and the positive direction is measured upward. If $g_0 = 9.81\ \text{ms}^{-2}$ and $R = 6356\ \text{km}$, determine the minimum initial velocity at which a projectile should be shot vertically from the earth's surface so that it does not return to the earth.

3. A particle is projected vertically upwards and t second after another particle is projected upwards with the same initial velocity. Prove that the particles will meet after a lapse of $\left(\frac{t}{2} + \frac{u}{g}\right)$ second from the instant of projection of the first particle. What are the velocities of the particles when they meet?

4. A ball is projected vertically upwards with a velocity of $100\ \text{ms}^{-1}$. Find the speed of the ball at half the maximum height. Take $g = 10\ \text{ms}^{-2}$.

5. A particle is projected vertically upwards from the ground at time $t = 0$ and reaches a height h at $t = T$. Show that the greatest height of the particle is $\frac{(gT^2 + 2h)^2}{8gT^2}$.

6. A rocket is fired vertically upwards with a net acceleration of $4\ \text{ms}^{-2}$ and initial velocity zero. At $t = 5\ \text{s}$ its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and returns to ground. Plot velocity-time and displacement-time graphs for the complete journey. Take $g = 10\ \text{ms}^{-2}$.

7. A stone is dropped from the top of a tall cliff and n second later another stone is thrown vertically downwards with a velocity $u\ \text{ms}^{-1}$. How far below the top of the cliff will the second stone overtake the first?

8. A body of mass m is thrown straight up with a velocity u_0. Find the velocity u' with which the body comes down if the air drag equals cu^2 where c is a constant and u is the velocity of the body.

9. To test the quality of a tennis ball it is dropped onto the floor from a height of 4 m and it rebounds to a height of 2 m. If the ball is in contact with the floor for a duration of 12 ms. Calculate the average acceleration during that contact? Take $g = 9.8\ \text{ms}^{-2}$.

10. A rocket is fired vertically from rest and ascends with constant net vertical acceleration of $300\ \text{msec}^{-2}$ for 1 minute. Its fuel is then all used up and it continues as a free particle in the gravitational field of the earth. Find
- **(a)** maximum height reached
- **(b)** the total time elapsed from take-off until the rocket strikes the earth.

11. A football is kicked vertically upward from the ground and a student gazing out of the window sees it moving upwards past her at $5\ \text{ms}^{-1}$. The window is 15 m above the ground. Air resistance may be ignored.

(a) How high does the football go above ground?
(b) How much time does it take to go from the ground to its highest point?

Take $g = 10\ \text{ms}^{-2}$?

12. Two balls of same mass are shot upward one after another at an interval of 2 second along the same vertical line with the same initial velocity of $39.2\ \text{msec}^{-1}$. Find the height at which they collide. Take $g = 9.8\ \text{msec}^{-2}$.

13. Two bodies are projected vertically upwards from one point with the same initial velocity v_0. The second body is projected t_0 second after the first. After how much time will the bodies meet?

14. A stone is dropped from the top of a tower. When it crosses a point 5 m below the top, another stone is dropped from a point 25 m below the top. Both stones reach the bottom of the tower simultaneously. Find the height of the tower. Take $g = 10\ \text{ms}^{-2}$.

15. An elevator without a ceiling is ascending with a constant speed of $10\ \text{ms}^{-1}$. A boy on the elevator throws a ball directly upward, from a height of 2 m above the elevator floor, just as the elevator floor is 28 m above the ground. The initial speed of the ball with respect to the elevator is $20\ \text{ms}^{-1}$.

(a) What maximum height above the ground does the ball reach?
(b) How long does the ball take to return to the elevator floor?

Take $g = 10\ \text{ms}^{-2}$

16. A ball is projected vertically upwards with a velocity of $24.5\ \text{msec}^{-1}$ from the bottom of a tower. A boy, who is standing at the top of the tower is unable to catch the ball when it passes him in the upward direction. But the ball again reaches him after 3 second when it is falling and then he catches it. Find what was the velocity of the ball when the boy caught it and find also the height of the tower.

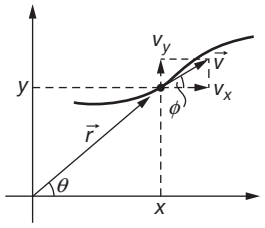
17. Two particles begin a free fall from rest from the same height, 1 s apart. How long after the first particle begins to fall will the two particles be 10 m apart?

Take $g = 10\ \text{ms}^{-2}$.

MOTION IN A PLANE AND RELATIVE VELOCITY

MOTION IN A PLANE: AN INTRODUCTION

Any type of planar motion can be resolved into two rectangular rectilinear motion i.e., two mutually perpendicular independent motions resolved along x and y axis (since x and y components do not have any dependence in each other). Consider a particle to be moving along a curve C in x-y plane. Let it be at a point $P(x, y)$ which has a position vector $\vec{r}$ at any particular instant of time t. Let this position vector make an angle θ with position x-axis as shown in Figure.



From Figure, we get

$$x = r\cos\theta \text{ and } y = r\sin\theta$$

Also, $r = \sqrt{r^2 + y^2}$ and $\tan\theta = \dfrac{y}{x}$

Let $\vec{v}$ be the velocity of the particle at point P and let ϕ be the angle made by $\vec{v}$ with x-axis. So, again we get

$$v_x = v\cos\phi$$

$$v_y = v\sin\phi$$

$$v = \sqrt{v_x^2 + v_y^2} \text{ and } \tan\phi = \frac{v_y}{v_x}$$

A similar and identical treatment can also be done for the acceleration components a_x and a_y. Further, the equations of motion in vector form are

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$v^2 - u^2 = 2\vec{a}.\vec{r}$$

$$\vec{r} = \left(\frac{\vec{u} + \vec{v}}{2}\right)t$$

x-component of equations of motion	y-components of equations of motion
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$
$x = u_x t + \frac{1}{2}a_x t^2$	$y = u_y t + \frac{1}{2}a_y t^2$
$v_x^2 - u_x^2 = 2a_x x$	$v_y^2 - u_y^2 = 2a_y y$
$x = \left(\frac{u_x + v_x}{2}\right)t$	$y = \left(\frac{u_y + v_y}{2}\right)t$

The most significant thing about these types of motion is that they are independent of each other.

ILLUSTRATION 49

Calculate the displacement, when a particle is displaced

(a) 5 m due North and then 12 m due East.

(b) from point 1 to 2 having position vector $\vec{r}_1 = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$ m and $\vec{r}_2 = \left(6\hat{i} - 6\hat{j} + 16\hat{k}\right)$ m.

Also find the magnitude in both the cases.

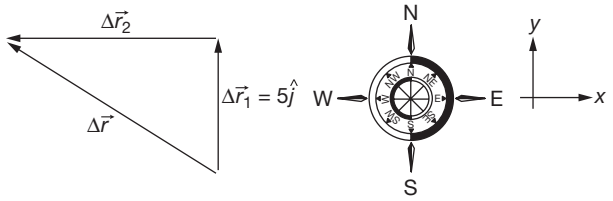
SOLUTION

(a) $\Delta\vec{r}_1 = 5\hat{j}$

$\Delta\vec{r}_2 = 12\left(-\hat{i}\right)$

$\Rightarrow \quad \Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 = \left(5\hat{j} - 12\hat{i}\right)$ m

$\Rightarrow \quad |\Delta\vec{r}| = \sqrt{25 + 144} = 13$ m



(b) $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{r}_f - \vec{r}_i$

$\Rightarrow \quad \Delta\vec{r} = \left(6\hat{i} - 6\hat{j} + 16\hat{k}\right) - \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$

$\Rightarrow \quad \Delta\vec{r} = \left(4\hat{i} - 3\hat{j} + 12\hat{k}\right)$ m

$$\Rightarrow \quad |\Delta\vec{r}| = \sqrt{(4)^2 + (3)^2 + (12)^2} = 13 \text{ m}$$

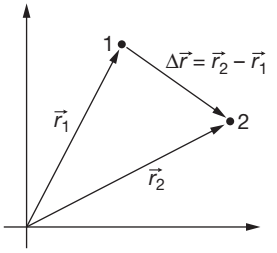


ILLUSTRATION 50

A particle travels along the parabolic path given by $y = bx^2$. If its component of velocity along the y-axis is $v_y = ct^2$, determine the x and y components of the particle's acceleration. Assume b and c to be positive constants.

SOLUTION

Since $v_y = ct^2$

$$\Rightarrow \quad \frac{dy}{dt} = ct^2$$

$$\Rightarrow \quad dy = ct^2 dt$$

$$\Rightarrow \quad \int_0^y dy = \int_0^t ct^2 dt$$

$$\Rightarrow \quad y = \frac{c}{3}t^3 \qquad \ldots(1)$$

Substituting y from (1) in $y = bx^2$, we get

$$\frac{c}{3}t^3 = bx^2$$

$$\Rightarrow \quad x = \sqrt{\frac{c}{3b}}t^{3/2}$$

Thus, the x component of the particle's velocity can be determined by taking the time derivative of x

$$\Rightarrow \quad v_x = \dot{x} = \frac{dx}{dt} = \frac{d}{dt}\left[\sqrt{\frac{c}{3b}}t^{3/2}\right] = \frac{3}{2}\sqrt{\frac{c}{3b}}t^{1/2}$$

Now again to calculate x and y components of acceleration, we use

$$a_x = \frac{dv_x}{dt} = \dot{v}_x \text{ and } a_y = \frac{dv_y}{dt} = \dot{v}_y$$

$$\Rightarrow \quad a_x = \dot{v}_x = \frac{d}{dt}\left(\frac{3}{2}\sqrt{\frac{c}{3b}}t^{1/2}\right) = \frac{3}{4}\sqrt{\frac{c}{3b}}t^{-1/2} = \frac{3}{4}\sqrt{\frac{c}{3b}}\frac{1}{\sqrt{t}}$$

$$\Rightarrow \quad a_y = \dot{v}_y = \frac{d}{dt}(ct^2) = 2ct$$

ILLUSTRATION 51

The position of a particle is $\vec{r} = (3t^3 - 2t)\hat{i} - (4\sqrt{t} + t)\hat{j} + (3t^2 - 2)\hat{k}$ m, where t is in seconds. Determine the magnitude of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[(3t^3 - 2t)\hat{i} - (4\sqrt{t} + t)\hat{j} + (3t^2 - 2)\hat{k}\right]$$

$$\Rightarrow \quad \vec{v} = \left[(9t^2 - 2)\hat{i} - \left(\frac{2}{\sqrt{t}} + 1\right)\hat{j} + (6t)\hat{k}\right] \text{ ms}^{-1}$$

$$\Rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[(9t^2 - 2)\hat{i} - \left(\frac{2}{\sqrt{t}} + 1\right)\hat{j} + (6t)\hat{k}\right] \text{ ms}^{-1}$$

$$\Rightarrow \quad \vec{a} = \left[(18t)\hat{i} + t^{-3/2}\hat{j} + 6\hat{k}\right] \text{ ms}^{-2}$$

When $t = 2$ s,

$$\vec{v} = \left((9(2^2) - 2)\hat{i} - (2(2^{-1/2}) + 1)\hat{j} + 6(2)\hat{k}\right) \text{ ms}^{-1}$$

$$\Rightarrow \quad \vec{v} = \left(34\hat{i} - 2.414\hat{j} + 12\hat{k}\right) \text{ ms}^{-1}$$

So, the magnitude of the particle's velocity is

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{34^2 + (-2.414)^2 + 12^2}$$

$$\Rightarrow \quad |\vec{v}| = 36.1 \text{ ms}^{-1}$$

Acceleration

When $t = 2$ s,

$$\vec{a} = \left[18(2)\hat{i} + 2^{-3/2}\hat{j} + 6\hat{k}\right] \text{ ms}^{-2}$$

$$\Rightarrow \quad \vec{a} = \left[36\hat{i} + 0.3536\hat{j} + 6\hat{k}\right] \text{ ms}^{-2}$$

So, the magnitude of the particle's acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{36^2 + (0.3536)^2 + 6^2} = 36.5 \text{ ms}^{-2}$$

ILLUSTRATION 52

The velocity of a particle is $\vec{v} = \left[3\hat{i} + (6 - 2t)\hat{j}\right] \text{ ms}^{-1}$, where t is in seconds. If $\vec{r} = \vec{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

The position $\vec{r}$ of the particle can be determined by integrating the kinematic equation $\vec{v} = \frac{d\vec{r}}{dt}$ (i.e., $d\vec{r} = \vec{v}dt$) and then using the initial condition i.e., at $t = 0$, $\vec{r} = \vec{0}$ for solving the integral.

$$d\vec{r} = \vec{v}dt$$

$$\Rightarrow \int_0^r d\vec{r} = \int_0^t \left[3\hat{i} + (6-2t)\hat{j}\right]dt$$

$$\Rightarrow \vec{r} = \left[3t\hat{i} + (6t - t^2)\hat{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$\vec{r}|_{t=1\text{ s}} = 3(1)\hat{i} + \left[6(1) - 1^2\right]\hat{j} = \left(3\hat{i} + 5\hat{j}\right)\text{ ms}^{-1}$$

$$\vec{r}|_{t=3\text{ s}} = 3(3)\hat{i} + \left[6(3) - 3^2\right]\hat{j} = \left(9\hat{i} + 9\hat{j}\right)\text{ ms}^{-1}$$

Thus, the displacement of the particle is

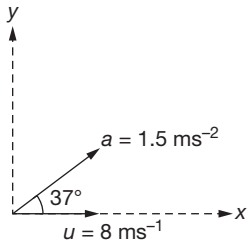
$$\Delta\vec{r} = \vec{r}|_{t=3\text{ s}} - \vec{r}|_{t=1\text{ s}}$$

$$\Rightarrow \Delta\vec{r} = \left(9\hat{i} + 9\hat{j}\right) - \left(3\hat{i} + 5\hat{j}\right)$$

$$\Rightarrow \Delta\vec{r} = \left(6\hat{i} + 4\hat{j}\right)\text{ m}$$

ILLUSTRATION 53

A particle is moving in the x-y plane with constant acceleration of 1.5 ms^{-2} that makes an angle of $37°$ with the x-axis. At $t = 0$ the particle is at the origin and its velocity is 8 ms^{-1} along the x-axis. Find the velocity and the position of the particle at $t = 4$ s. Given $\sin(37°) = 0.6$



SOLUTION

Let us first calculate the components of constant acceleration. If a_x and a_y be the respective acceleration components along x and y-axis then

$$a_x = (1.5\text{ ms}^{-2})(\cos 37°) = (1.5\text{ ms}^{-2}) \times \frac{4}{5} = 1.2\text{ ms}^{-2}$$

and $a_y = (1.5\text{ ms}^{-2}) \times \frac{3}{5} = 0.9\text{ ms}^{-2}$

At $t = 0$, $u_x = 8\text{ ms}^{-1}$, $u_y = 0$

$a_x = 1.2\text{ ms}^{-2}$, $a_y = 0.9\text{ ms}^{-2}$

$x = 0$ and $y = 0$

The x-component of the velocity at time $t = 4$ s is given by

$$v_x = u_x + a_x t$$

$$\Rightarrow v_x = 8\text{ ms}^{-1} + (1.2\text{ ms}^{-2})(4\text{ s})$$

$$\Rightarrow v_x = 8\text{ ms}^{-1} + 4.8\text{ ms}^{-1} = 12.8\text{ ms}^{-1}$$

The y-component of velocity at $t = 4$ s is given by

$$v_y = u_y + a_y t$$

$$\Rightarrow v_y = 0 + (0.9\text{ ms}^{-2})(4\text{ s}) = 3.6\text{ ms}^{-1}$$

The velocity of the particle at $t = 4$ s is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(12.8\text{ ms}^{-1})^2 + (3.6\text{ ms}^{-1})^2}$$

$$v = 13.3\text{ ms}^{-1}$$

Assume that the velocity makes an angle θ with the x-axis, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{3.6\text{ ms}^{-1}}{12.8\text{ ms}^{-1}} = \frac{9}{32}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{9}{32}\right)$$

The x-coordinate at $t = 4$ s is given by

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\Rightarrow x = (8)(4) + \frac{1}{2}(1.2)(4)^2$$

$$\Rightarrow x = 32\text{ m} + 9.6\text{ m} = 41.6\text{ m}$$

The y-coordinate at $t = 4$ s is given by

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow y = \frac{1}{2}(0.9)(4)^2 \qquad \{u_y = 0\}$$

$$\Rightarrow y = 7.2\text{ m}$$

Thus, the coordinates of the particle at 4 s are (41.6 m, 7.2 m).

ILLUSTRATION 54

A particle is moving in a plane with velocity given by $\vec{v} = u_0\hat{i} + [a\omega\cos(\omega t)]\hat{j}$. If the particle is at the origin at $t = 0$,

(a) calculate the trajectory of the particle

(b) find its distance from the origin at time $\left(\frac{3\pi}{2\omega}\right)$.

SOLUTION

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\Rightarrow v_x = \frac{dx}{dt} = u_0$$

$$\Rightarrow dx = u_0 dt$$

$$\Rightarrow \int_0^x dx = u_0\int_0^t dt$$

$$\Rightarrow x = u_0 t \quad \ldots(1)$$

$$v_y = \frac{dy}{dt} = a\omega\cos(\omega t)$$

$$\Rightarrow \int_0^y dy = a\omega\int_0^t \cos(\omega t)\,dt$$

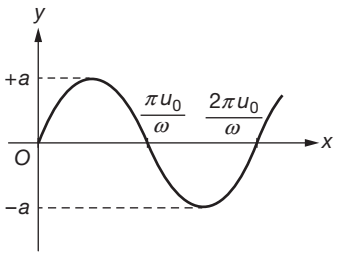
$$\Rightarrow y = a\omega\left(\frac{\sin(\omega t)}{\omega}\Big|_0^t\right)$$

$$\Rightarrow y = a\sin(\omega t) \quad \ldots(2)$$

(a) From (1), $t = \frac{x}{u_0}$. Substituting in (2), we get

$$y = a\sin\left(\frac{\omega x}{u_0}\right) \qquad \{\text{Equation of Trajectory}\}$$

This is the desired trajectory and it is a sine curve as shown in figure.



(b) For $t = \left(\frac{3\pi}{2\omega}\right)$ from equations (1) and (2), we get

$$x = u_0\left(\frac{3\pi}{2\omega}\right) \text{ and } y = -a$$

So, distance from the origin at $t = \left(\frac{3\pi}{2\omega}\right)$ will be

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{3\pi u_0}{2\omega}\right)^2 + a^2}$$

$$\Rightarrow \quad r = \sqrt{a^2 + \frac{9\pi^2 u_0^2}{4\omega^2}}$$

ILLUSTRATION 55

A particle of mass 2 kg has a velocity of $2\ \text{ms}^{-1}$. A constant force of 4 N acts on the particle for 1 s in a direction perpendicular to its initial velocity. Find the velocity and displacement of the particle at $t = 1$ s.

SOLUTION

Let the velocity of the particle be along x-axis. Then

$$\vec{u} = 2\hat{i}$$

So, force on the particle is

$$\vec{F} = 4\hat{j}$$

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}}{m} = 2\hat{j}$$

Since $\vec{a}$ = constant, so we can use

$$\vec{v} = \vec{u} + \vec{a}t \text{ and } \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\Rightarrow \quad \vec{v} = 2\hat{i} + (2t)\hat{j}$$

$$\Rightarrow \quad \vec{v}|_{t=1\text{ s}} = 2\hat{i} + 2\hat{j}$$

So, the situation is shown in Figure (a).

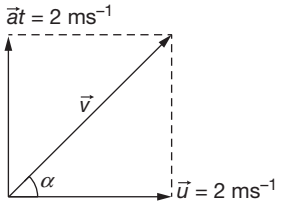


Figure (a)

If $\vec{v}$ makes an angle α with x-axis, then

$$\alpha = \tan^{-1}\frac{|\vec{a}t|}{|\vec{u}|} = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = 45°$$

Thus, velocity of the particle at the end of 1 s is $2\sqrt{2}$ ms^{-1} at an angle of 45° with its initial velocity. The situation is again shown in Figure (b).

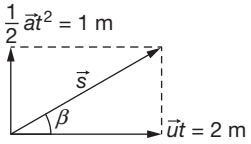


Figure (b)

Since $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\Rightarrow \quad \vec{s} = (2t)\hat{i} + \frac{1}{2}(2)t^2\hat{j}$

$\Rightarrow \quad \vec{s}|_{t=1\text{ s}} = 2\hat{i} + \hat{j}$

$\Rightarrow \quad |\vec{s}| = \sqrt{5}$ m

If β is the angle which $\vec{s}$ makes with x-axis, then

$$\beta = \tan^{-1}\left(\frac{\left|\frac{1}{2}\vec{a}t^2\right|}{|\vec{u}t|}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

So, displacement of the particle at the end of 1 s is $\sqrt{5}$ m at an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ from its initial velocity.

ILLUSTRATION 56

A particle leaves the origin with an initial velocity $\vec{v} = (3\hat{i})$ ms^{-1} and a constant acceleration $\vec{a} = \left(-1\hat{i} - 0.5\hat{j}\right)$ ms^{-2}. When the particle reaches its maximum x coordinate, what are

(a) its velocity and
(b) its position vector?

SOLUTION

(a) $u_x = 3$ ms^{-1}

$a_x = -1$ ms^{-2} and $a_y = -0.5$ ms^{-2}

Since $x = u_x t + \frac{1}{2}a_x t^2$

For x to be MAXIMUM, we have

$$\frac{dx}{dt} = 0$$

$$\Rightarrow \quad u_x + \frac{1}{2}a_x(2t) = 0$$

$$\Rightarrow \quad u_x + a_x t = 0 \qquad \ldots(1)$$

$$\Rightarrow \quad t = -\frac{u_x}{a_x}$$

$$\Rightarrow \quad t = -\frac{3}{(-1)} = 3 \text{ s}$$

Please note that this happens to be the time when the x motion of the particle is reversed i.e., this time is the time when the x component of velocity should be zero and equation (1) shows that. At this instant

$$v_x = 0$$

and $v_y = u_y + a_y t = 0 - 0.5 \times 3 = -1.5$ ms^{-1}

$$\Rightarrow \quad \vec{v} = \left(-1.5\hat{j}\right) \text{ ms}^{-1}$$

(b) $x = u_x t + \frac{1}{2}a_x t^2 = 3 \times 3 + \frac{1}{2}(-1)(3)^2 = 4.5$ m

$y = u_y t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}(0.5)(3)^2 = -2.25$ m

$$\Rightarrow \quad \vec{r} = \left(4.5\hat{i} - 2.25\hat{j}\right) \text{ m}$$

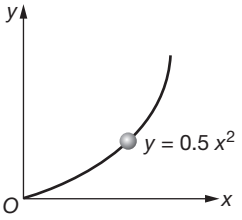
Test Your Concepts-VI

Based on Planar Motion

(Solutions on page H.85)

1. A particle starts from the origin at $t = 0$ with a velocity of $8\hat{j}$ ms^{-1} and moves in the xy plane with a constant acceleration of $\left(4\hat{i} + 2\hat{j}\right)$ ms^{-2}. At the instant the particle's x coordinate is 32 m, what are
(a) its y-coordinate and
(b) its speed?

2. A particle starts from the origin of the coordinates along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x-axis is $v_x = (5t)$ ms^{-1}, where t is in second, determine the particle's distance from the origin O and the magnitude of acceleration when $t = 1$ s.

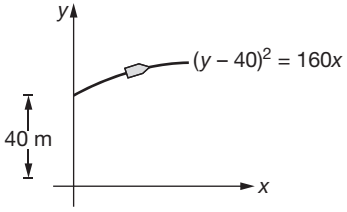


3. A point moves the plane x-y according to the law $x = k\sin(\omega t)$ and $y = k[1-\cos(\omega t)]$ where k and ω are positive constants. Find the distance traversed by the particle during time t.

4. A particle moves in the plane according to the law $x = kt$, $y = kt(1-\alpha t)$, when k and α are positive constants, and t is the time. Find
- **(a)** the equation of the particle's trajectory $y(x)$
- **(b)** the velocity v and the acceleration a of the point as a function of time.

5. The speed of a particle moving in a plane is equal to the magnitude of its instantaneous velocity, $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$. Show that the rate of change of the speed is $\dfrac{dv}{dt} = \dfrac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$.

6. As soon as a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y-40)^2 = 160x$, where the coordinates are measured in meters. Assuming that the component of velocity in the vertical direction is constant at $v_y = 180\ \text{ms}^{-1}$, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



7. Velocity and acceleration of a particle at time $t = 0$ are $\vec{u} = \left(2\hat{i} + 3\hat{j}\right)\ \text{ms}^{-1}$ and $\vec{a} = \left(4\hat{i} + 2\hat{j}\right)\ \text{ms}^{-2}$ respectively. Find the velocity and displacement of particle at $t = 2$ s.

8. A balloon is ascending at the rate $v = 12\ \text{kmh}^{-1}$ and is being carried horizontally by the wind blowing at $v_w = 20\ \text{kmh}^{-1}$. If a ballast bag is dropped from the balloon at the instant $h = 50$ m, determine the time required for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground? $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$.

RELATIVE MOTION

Motion is a combined property of the object under study as well as the observer. It is always relative because there is no such thing like absolute motion or absolute rest. *Motion is always defined with respect to an observer or a reference frame*.

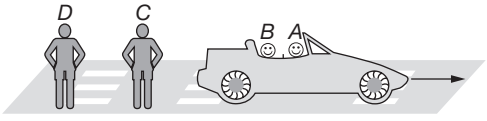
Reference Frame

The motion of a particle is described by using kinematical quantities such as velocity, acceleration. However, these quantities are dependent upon the state of motion as seen by the observer. So, to understand the concept of relative motion it becomes mandatory for us to talk about or introduce the concept of a "reference frame".

Reference frame is an axis system from which motion is observed along with a clock attached to the axis, to measure time. Reference frame can be stationary or moving.

In layman language a Reference Frame is a platform from where the observer observes motions and takes measurements with respect to it.

Suppose there are two persons A and B sitting in a car moving at constant speed. Two stationary persons C and D observe them from the ground.



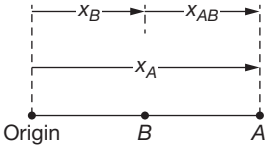
Here B appears to be moving for C and D, but at rest for A. Similarly C appears to be at rest for D but moving backward for A and B.

RELATIVE MOTION IN ONE DIMENSION

Relative Position

It is the position of a particle w.r.t. observer. In general if position of A w.r.t. origin is x_A and that of B w.r.t. origin is x_B, then position of A w.r.t B (i.e. B is now the new origin) is denoted by x_{AB} and is given by

$$x_{AB} = x_A - x_B$$



and v_{AB} = velocity of A w.r.t. $B = \dfrac{dx_{AB}}{dt} = \dfrac{d}{dt}(x_A - x_B)$

$$\Rightarrow \quad v_{AB} = \frac{dx_A}{dt} - \frac{dx_B}{dt} = v_A - v_B$$

So, for particles A and B moving in the same direction

$$v_{AB} = |v_A - v_B|$$

and for particles A and B moving in the opposite direction

$$v_{AB} = v_A + v_B$$

Relative Velocity

Relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest. In other words, it is the velocity with which A appears to move as seen by B considering itself to be at rest.

Conceptual Note(s)

(a) All velocities are relative and have no significance unless observer is specified. However, when we say velocity of A, what we mean is, velocity of A w.r.t. ground which is assumed to be at rest.
(b) Velocity of an object w.r.t. itself is always zero.
(c) Velocity of A and B must always be measured from the same reference.

ILLUSTRATION 57

Two trains, each having a speed of $30\ \text{kmhr}^{-1}$ are headed towards each other on the same straight track. A bird that can fly at $60\ \text{kmhr}^{-1}$ flies off one train, when they are 60 km apart and heads directly for the other train. On reaching the other train, it flies directly back to the first and so on.

(a) How many trips can the bird make from one train to the other before they crash?
(b) What is the total distance the bird travels?

SOLUTION

The relative velocity of one train relative to the other is $60\ \text{kmhr}^{-1}$ and as the distance between the trains is 60 km, the two trains will crash after 1 hr.

(a) Now, the velocity of bird with respect to train towards which it is moving will be $v = 90\ \text{kmhr}^{-1}$.
So, the time taken by bird for first trip is $t_1 = \left(\dfrac{60}{90}\right) = \dfrac{2}{3}$ hr and in this time the trains have moved towards each other $\left(\dfrac{2}{3}\right) \times 60 = 40$ km, so the remaining distance $= 60 - 40 = 20$ km.
So, the time taken by bird for second trip, $t_2 = \dfrac{20}{90} = \left(\dfrac{2}{3^2}\right)$ hr
Proceeding in the same way time taken by the bird for nth trip, $t_n = \left(\dfrac{2}{3^n}\right)$hr
Now, if the bird makes n trips till the train crashes,

$$t_1 + t_2 + t_3 + ... + t_n = 1 \text{ hr}$$

$$\Rightarrow \quad \frac{2}{3} + \frac{2}{3^2} + ... + \frac{2}{3^n} = 1 \text{ hr}$$

$$\Rightarrow \quad \frac{2}{3}\left[1 + \frac{1}{3} + \frac{1}{3^2} + ... + \frac{1}{3^{n-1}}\right] = 1 \text{ hr}$$

$$\Rightarrow \quad \frac{2}{3}\left[\frac{1 - \left(\frac{1}{3}\right)^n}{1 - \left(\frac{1}{3}\right)}\right] = 1 \text{ hr}$$

$$\Rightarrow \quad 1 - \left(\frac{1}{3}\right)^n = 1$$

$$\Rightarrow \quad \left(\frac{1}{3}\right)^n = 0$$

$$\Rightarrow \quad n \to \infty$$

So, the bird will make infinite trips.

(b) As the speed of bird is $60\ \text{kmhr}^{-1}$ and the two trains crash after 1 hr., so the total distance travelled by the bird is the distance travelled by the bird in 1 hr. i.e.,

$$d = 60\left(\frac{\text{km}}{\text{hr}}\right) \times 1\ \text{hr} = 60\ \text{km}$$

RELATIVE ACCELERATION

It is the rate at which relative velocity is changing

$$a_{AB} = \frac{dv_{AB}}{dt} = \frac{dv_A}{dt} - \frac{dv_B}{dt} = a_A - a_B$$

Equations of motion when relative acceleration a_{rel} *is constant are*

$$v_{\text{rel}} = u_{\text{rel}} + a_{\text{rel}}t$$

$$s_{\text{rel}} = u_{\text{rel}}t + \frac{1}{2}a_{\text{rel}}t^2$$

$$v_{\text{rel}}^2 = u_{\text{rel}}^2 + 2a_{\text{rel}}s_{\text{rel}}$$

where u_{rel} is the initial relative velocity, v_{rel} is the final relative velocity i.e. relative velocity at some instant of time t and s_{rel} is the relative displacement of the particles.

EQUATIONS OF MOTION IN RELATIVE VELOCITY FORM

The kinematical equations of motion are also modified as follows

$$v_r = u_r + a_r t$$

$$s_r = u_r t + \frac{1}{2}a_r t^2$$

$$v_r{}^2 - u_r{}^2 = 2a_r s_r$$

where u_r is the initial relative velocity
v_r is the final relative velocity at time t
a_r is the relative constant acceleration
s_r is the relative separation at time t

ILLUSTRATION 58

When two particles A and B are at point O, A is moving with a constant velocity $50\ \text{ms}^{-1}$, while B is not moving. But B possesses a constant acceleration of $10\ \text{ms}^{-2}$. After how much time they will be at a distance of 125 m ?

SOLUTION

For particle A

$$u_A = 50\ \text{ms}^{-1}$$

$$a_A = 0\ \text{ms}^{-2}$$

For particle B

$$u_B = 0$$

$$a_B = 10\ \text{ms}^{-2}$$

So, initial velocity of A w.r.t. B

$$u_{AB} = u_A - u_B = 50\ \text{ms}^{-1}$$

and acceleration of A w.r.t. B

$$a_{AB} = a_A - a_B = -10\ \text{ms}^{-2}$$

The distance between A and B after time t is given by

$$s_{AB} = u_{AB}t + \frac{1}{2}a_{AB}t^2$$

$$\Rightarrow \quad 125 = 50t - 5t^2$$

$$\Rightarrow \quad t^2 - 10t + 25 = 0$$

$$\Rightarrow \quad (t-5)^2 = 0$$

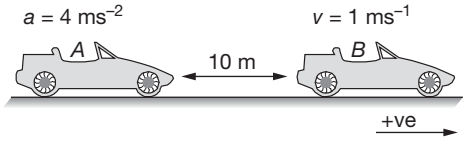
$$\Rightarrow \quad t = 5\ \text{s}$$

ILLUSTRATION 59

Car A and car B start moving simultaneously in the same direction along the line joining them. Car A with a constant acceleration $a = 4\ \text{ms}^{-2}$, while car B moves with a constant velocity $v = 1\ \text{ms}^{-1}$. At time $t = 0$, car A is 10 m behind car B. Find the time when car A overtakes car B.

SOLUTION

Given $u_A = 0$, $u_B = 1\ \text{ms}^{-1}$, $a_A = 4\ \text{ms}^{-2}$ and $a_B = 0$

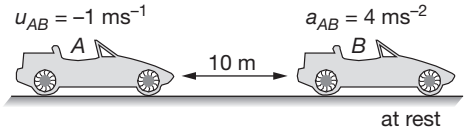


Assuming car B to be at rest, we have

$$u_{AB} = u_r = u_A - u_B = 0 - 1 = -1\ \text{ms}^{-1}$$

$$a_{AB} = a_r = a_A - a_B = 4 - 0 = 4\ \text{ms}^{-2}$$

Now, the problem can be assumed in simplified form as follows:



Substituting the proper values in equation

$$s_r = u_r t + \frac{1}{2} a_r t^2$$

$$\Rightarrow \quad 10 = -t + \frac{1}{2}(4)(t)^2$$

$$\Rightarrow \quad 2t^2 - t - 10 = 0$$

$$\Rightarrow \quad t = \frac{1 \pm \sqrt{1+80}}{4} = \frac{1 \pm \sqrt{81}}{4}$$

$$\Rightarrow \quad t = \frac{1 \pm 9}{4}$$

$$\Rightarrow \quad t = 2.5\ \text{s} \text{ and } -2\ \text{s}$$

Ignoring the negative value, the desired time is 2.5 s

Problem Solving Technique(s)

This problem can also be solved without using the concept of relative motion as under,
At the time when *A* overtakes *B*,

$$s_A = s_B + 10$$

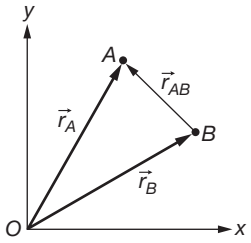
$$\Rightarrow \quad \frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$$

$$\Rightarrow \quad 2t^2 - t - 10 = 0$$

Which on solving gives $t = 2.5$ second and –2 second, the same as we found above.

RELATIVE MOTION IN TWO DIMENSION

Let $\vec{r}_A$ be the position of A with respect to O i.e. position vector of the point A is $\vec{r}_A$.
Similarly let $\vec{r}_B$ be the position of B with respect to O i.e. position vector of the point B is $\vec{r}_B$.



Then the position of A with respect to B is $\vec{r}_{AB}$ given by

$$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$$

(The vector sum $\vec{r}_A - \vec{r}_B$ can be done by Δ law of addition or resolution method).

Now the velocity of A with respect to B is $\vec{v}_{AB}$, which is the rate at which position of A with respect to B changes i.e.

$$\vec{v}_{AB} = \frac{d}{dt}(\vec{r}_{AB})$$

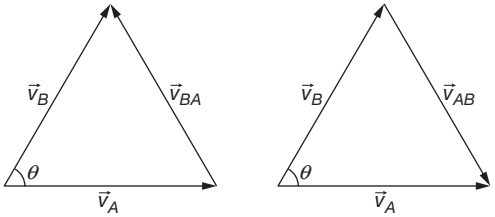
$$\Rightarrow \quad \frac{d}{dt}(\vec{r}_{AB}) = \frac{d}{dt}(\vec{r}_A) - \frac{d}{dt}(\vec{r}_B) = \frac{d\vec{r}_A}{dt} - \frac{d\vec{r}_B}{dt} = \vec{v}_A - \vec{v}_B$$

$$\Rightarrow \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

So, $\vec{v}_{AB} = \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B =$ velocity of A relative to B

Similarly

$$\vec{v}_{BA} = \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = \text{velocity of } B \text{ relative to } A$$



If $\vec{v}_A$ and $\vec{v}_B$ are inclined to each other at angle θ, then

$$|\vec{v}_{AB}| = |\vec{v}_{BA}| = \sqrt{{v_A}^2 + {v_B}^2 - 2v_2 v_B \cos\theta}$$

and $\vec{v}_{AB} = -\vec{v}_{BA}$

Problem Solving Technique(s)

(a) Since $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$$\Rightarrow \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B + \vec{v}_P - \vec{v}_P$$

$$\Rightarrow \quad \vec{v}_{AB} = (\vec{v}_A - \vec{v}_P) - (\vec{v}_B - \vec{v}_P) = \vec{v}_{AP} - \vec{v}_{BP}$$

$$\Rightarrow \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \vec{v}_{AP} - \vec{v}_{BP} = \vec{v}_{AO} - \vec{v}_{BO}$$

i.e., relative velocity is simplify independent of the velocity of the observer (O) analysing the motion, as long as both the particles A and B have their velocities specified w.r.t. the same observer.

By default $\vec{v}_A$ and $\vec{v}_B$ are specified with respect to the ground or with respect to a stationary observer at the ground.

(b) If $\Delta\vec{r}$ is the relative separation between any two particles at time Δt, then

$$\Delta\vec{r} = \vec{v}_r \Delta t$$

$$\Rightarrow \quad |\Delta\vec{r}| = \left(\sqrt{{v_A}^2 + {v_B}^2 - 2v_A v_B \cos\theta}\right)\Delta t$$

(c) A similar treatment is also given when we are asked to deal with the relative acceleration $\vec{a}_{AB}$ or $\vec{a}_{BA}$

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = \text{Acceleration of } A \text{ relative to } B$$

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = \text{Acceleration of } B \text{ relative to } A$$

Similarly

$$|\vec{a}_{AB}| = |\vec{a}_{BA}| = \sqrt{a_A^2 + a_B^2 - 2a_A a_B \cos\theta}$$

But here we must keep one thing in mind that if we are asked to calculate the acceleration of a ball falling freely with respect to another ball projected vertically upwards, then relative acceleration is zero (and not 2g). This is due to the fact that both the balls (either the ball falling freely or the one launched upward) move under the influence of gravity which acts vertically downwards for both.

Similarly, the acceleration of A with respect to B is $\vec{a}_{AB}$, which is the rate at which velocity of A with respect to B changes i.e.

$$\vec{a}_{AB} = \frac{d}{dt}(\vec{v}_{AB})$$

$$\Rightarrow \quad \frac{d}{dt}(\vec{v}_{AB}) = \frac{d}{dt}(\vec{v}_A) - \frac{d}{dt}(\vec{v}_B) = \frac{d\vec{v}_A}{dt} - \frac{d\vec{v}_B}{dt} = \vec{a}_A - \vec{a}_B$$

$$\Rightarrow \quad \vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

If $\vec{a}_{AB}$ is acceleration of A relative to B and $\vec{a}_{BA}$ is acceleration of B relative to A, then

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B \text{ and } \vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

$$\Rightarrow \quad \vec{a}_{AB} = -\vec{a}_{BA}$$

RELATIVE MOTION FOR BODIES MOVING INDEPENDENTLY

For two bodies moving independently in the same direction, relative velocity is $v_r = v_A \sim v_B = |v_A - v_B|$ ($\sim$ sign indicates positive difference between v_A and v_B)

This relative velocity can be the relative velocity of approach or of separation depending upon the location of the bodies and the magnitudes of their velocities.

$v_r = v_A - v_B$, is the relative velocity of approach if $v_A > v_B$ and A is following (or approaching) B

Relative separation between A and B decreases with time

Whereas the same v_r becomes relative velocity of separation when B is behind A.

Relative separation between A and B increases with time

For two bodies moving independently in opposite direction

$$v_r = v_A + v_B$$

is the relative velocity of approach till the bodies do not meet and after they meet, the same value is the relative velocity of separation (or receding).

Bodies approaching and relative separation decreasing with time

Bodies receding and relative separation increasing with time

RELATIVE MOTION FOR BODIES MOVING DEPENDENTLY

Dependent motion is the case when a person is moving on a conveyer belt or on the surface of a moving carriage train or an escalator.

In such cases, we have to first understand that "if a velocity of the person walking on a moving surface is given, then this velocity is the velocity of the person with respect to the surface (i.e. his ground) on which he is moving and not with respect to the stationary ground."
Let's discuss and understand this through the following situations.

Situation 1

Consider a train T moving with a velocity v_T. Let a person P move with a velocity v (say) on this train along the direction of the train, then the velocity of the person with respect to his ground (which is the train) is v, so

$$v_{PT} = v \text{, where } v_{PT} = v_P - v_T$$

$$\Rightarrow \quad v_P = v_T + v_{PT}$$

which simply means that the velocity of the person with respect to a stationary observer A at the ground is

$$v_P = v_T + v$$

Situation 1a

If the observer A also starts moving on the ground in the direction of motion of the train, then he sees the person to be moving with a velocity

$$v_{PA} = (v_T + v) - v_A$$

Situation 1b

If the observer A also starts moving on the ground in the direction opposite to the motion of the train, then he sees the person to be moving with a velocity

$$v_{PA} = (v_T + v) + v_A$$

Situation 2

Similarly if the person P moves with a velocity v (say) on this train opposite to the direction of the train, then the velocity of the person with respect to his ground (which is the train) is v, so

$$v_{PT} = -v \text{, where } v_{PT} = v_P - v_T$$

(The direction of motion of train is taken as positive)

$$\Rightarrow \quad v_P = v_T + v_{PT}$$

which simply means that the velocity of the person with respect to a stationary observer at the ground is

$$v_P = v_T - v$$

Situation 2a

If the observer A also starts moving on the ground in the direction of motion of the train, then he sees the person to be moving with a velocity

$$v_{PA} = (v_T - v) - v_A$$

Situation 2b

If the observer A also starts moving on the ground in the direction opposite to the motion of the train, then he sees the person to be moving with a velocity

$$v_{PA} = (v_T - v) + v_A$$

Situation 3

If a person (P) is moving with a velocity v_P on a stationary escalator (E), then with respect to the ground (G), we have

$$v_{PG} = v_{PE} = v_P$$

Situation 3a

If a person (P) is moving with a velocity v up an escalator (E) which is also moving up with a velocity v_E, then with respect to the ground (G), we have

$$v_{PG} = v_{PE} + v_{EG}$$

Since, we must take a note here that $v_{PE} = v$, so velocity of the person with respect to ground or a stationary observer A on the ground is

$$v_P = v_E + v$$

Situation 3b

If a person (P) is moving with a velocity v down an escalator (E) which is moving up with a velocity v_E, then with respect to the ground (G), we have

$$v_{PG} = v_{PE} + v_{EG}$$

Taking direction of motion of the escalator as positive, we note here that $v_{PE} = -v$ and $v_{EG} = +v_E$, so velocity of the person with respect to ground or a stationary observer A on the ground is

$$v_P = v_E - v$$

ILLUSTRATION 60

Consider a collection of a large number of particles each with speed v. The direction of velocity is randomly distributed in the collection. Show that the magnitude of the relative velocity between a pair of particles averaged over all the pairs in the collection is greater that v.

SOLUTION

As shown in figure, let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of any two particles and θ be the angle between them. As each particle has speed v, so

$$|\vec{v}_1| = |\vec{v}_2| = v$$

The magnitude of relative velocity $\vec{v}_{21}$ of particle 2 with respect to 1 is given by

$$v_{21} = \sqrt{|-\vec{v}_1|^2 + |\vec{v}_2|^2 + 2|-\vec{v}_1||\vec{v}_2|\cos(180^\circ - \theta)}$$

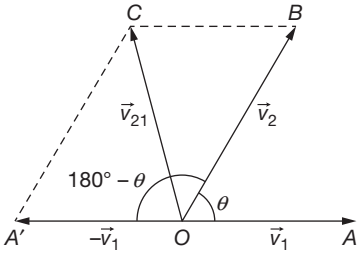
$$\Rightarrow \quad v_{21} = \sqrt{v^2 + v^2 - 2(v)(v)\cos\theta} = \sqrt{2v^2(1-\cos\theta)}$$

Since $1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad v_{21} = \sqrt{2v^2 \times 2\sin^2\left(\frac{\theta}{2}\right)} = 2v\sin\left(\frac{\theta}{2}\right)$$

As the velocities of the particles are randomly distributed, so θ can vary from 0 to 2π. If $\langle v_{21} \rangle$ is the magnitude of the average velocity when averaged over all pairs, then

$$\langle v_{21} \rangle = \frac{\int_0^{2\pi} 2v\sin\left(\frac{\theta}{2}\right)d\theta}{\int_0^{2\pi} d\theta} = \frac{-2v\left[\frac{\cos\left(\frac{\theta}{2}\right)}{\left(\frac{1}{2}\right)}\right]_0^{2\pi}}{\theta\Big|_0^{2\pi}}$$



$$\Rightarrow \quad \langle v_{21} \rangle = \frac{-4v\left[\cos\left(\frac{\theta}{2}\right)\right]_0^{2\pi}}{2\pi - 0} = -\frac{2v}{\pi}(\cos\pi - \cos 0)$$

$$\Rightarrow \quad \langle v_{21} \rangle = -\frac{2v}{\pi}(-1-1) = \frac{4v}{\pi} = 1.273v$$

$$\Rightarrow \quad \langle v_{21} \rangle > v$$

ILLUSTRATION 61

Two trains, each having a speed of 30 kmhr^{-1} are headed towards each other on the same straight track. A bird that can fly at 60 kmhr^{-1} flies off one train, when they are 60 km apart and heads directly for the other train. On reaching the other train, it flies directly back to the first and so on.

(a) How many trips can the bird make from one train to the other before they crash?

(b) What is the total distance the bird travels?

SOLUTION

The relative velocity of one train relative to the other is 60 kmhr^{-1} and as the distance between the trains is 60 km, the two trains will crash after 1 hr.

(a) Now, the velocity of bird with respect to train towards which it is moving will be $v = 90$ kmhr^{-1}. So, the time taken by bird for first trip is

$$t_1 = \left(\frac{60}{90}\right) = \frac{2}{3} \text{ hr}$$

and in this time the trains have moved towards each other $\left(\frac{2}{3}\right) \times 60 = 40$ km, so the remaining distance $= 60 - 40 = 20$ km.

So, the time taken by bird for second trip,

$$t_2 = \frac{20}{90} = \left(\frac{2}{3^2}\right) \text{ hr}$$

Proceeding in the same way time taken by the bird for nth trip, $t_n = \left(\frac{2}{3^n}\right)$hr

Now, if the bird makes n trips till the train crashes,

$$t_1 + t_2 + t_3 + ... + t_n = 1 \text{ hr}$$

$$\Rightarrow \quad \frac{2}{3} + \frac{2}{3^2} + ... + \frac{2}{3^n} = 1 \text{ hr}$$

$$\Rightarrow \quad \frac{2}{3}\left[1+\frac{1}{3}+\frac{1}{3^2}+....+\frac{1}{3^{n-1}}\right]=1 \text{ hr}$$

$$\Rightarrow \quad \frac{2}{3}\left[\frac{1-\left(\frac{1}{3}\right)^n}{1-\left(\frac{1}{3}\right)}\right]=1 \text{ hr}$$

$$\Rightarrow \quad 1-\left(\frac{1}{3}\right)^n=1$$

$$\Rightarrow \quad \left(\frac{1}{3}\right)^n=0$$

$$\Rightarrow \quad n\to\infty$$

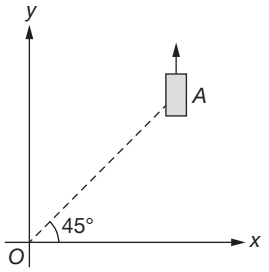
So, the bird will make infinite trips.

(b) As the speed of bird is 60 kmhr^{-1} and the two trains crash after 1 hr., so the total distance travelled by the bird is the distance travelled by the bird in 1 hr. i.e.,

$$d=60\left(\frac{\text{km}}{\text{hr}}\right)\times 1 \text{ hr}=60 \text{ km}$$

ILLUSTRATION 62

On a frictionless horizontal surface, assumed to be x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $\left(\sqrt{3}-1\right) \text{ ms}^{-1}$. At a particular instant, when the line OA makes an angle of $45°$ with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.

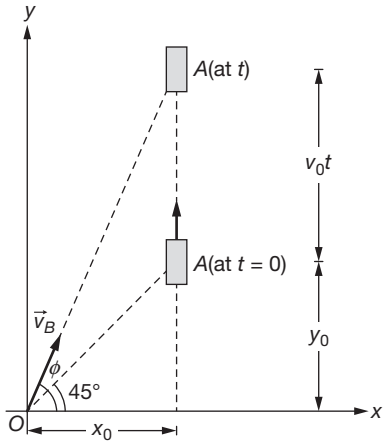


(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi=\frac{4\theta}{3}$.

SOLUTION

(a) Since the motion of the ball $(\text{say } B)$ is observed from the frame of the trolley A, so in this problem we must deal with $\vec{v}_{BA}$ i.e., velocity of ball B with respect to the trolley A.



Now $\vec{v}_{BA}=\vec{v}_B-\vec{v}_A$ ...(1)

Since ϕ is the angle made by the velocity of the ball with the x-axis, to hit the trolley, so

$$\vec{v}_B=v_B\left(\cos\phi\hat{i}+\sin\phi\hat{j}\right) \quad ...(2)$$

Also, θ is the angle made by the velocity vector of the ball with the x-axis in the frame of the trolley B, so

$$\vec{v}_{BA}=v_{BA}\left(\cos\theta\hat{i}+\sin\theta\hat{j}\right) \quad ...(3)$$

Also, we are given that $\vec{v}_A=v_A\hat{j}=\left(\sqrt{3}-1\right)\hat{j} \text{ ms}^{-1}$

Substituting all in equation (1), we get

$$v_{BA}\left(\cos\theta\hat{i}+\sin\theta\hat{j}\right)=\left(v_B\cos\phi\right)\hat{i}+\left(v_B\sin\phi-v_A\right)\hat{j} \quad ...(4)$$

$$\Rightarrow \quad v_{BA}\cos\theta=v_B\cos\phi \quad ...(5)$$

and $v_{BA}\sin\theta=v_B\sin\phi-v_A$ ...(6)

So, $\tan\theta=\dfrac{v_B\sin\phi-v_A}{v_B\cos\phi}$ ...(7)

Now, let the ball B strike the trolley A at time t, then the y coordinate of the ball as well as the trolley have to be the same, so

$$(y)_{\text{Ball at } t} = (y)_{\text{Trolley at time } t}$$

$$\Rightarrow \quad (v_B \sin\phi)t = y_0 + v_0 t \qquad \ldots(8)$$

Also, we observe that

$$(v_B \cos\phi)t = x_0 \qquad \ldots(9)$$

Since we know observe that $x_0 = y_0$, so from (8) and (9), we get

$$(v_B \sin\phi)t = (v_B \cos\phi)t + v_0 t$$

$$\Rightarrow \quad v_B \sin\phi - v_0 = v_B \cos\phi \qquad \ldots(10)$$

Using (10) and (7), we get

$$\tan\theta = 1$$

$$\Rightarrow \quad \theta = 45°$$

Also, we could have arrived the result without these calculations. Just keep in mind that for the ball (B) to hit the trolley A, relative velocity of the ball B with respect to the trolley A must be directed along OA. So, $\vec{v}_{BA}$ makes an angle of 45° with the x-axis. Hence $\theta = 45°$.

(b) Since $\phi = \dfrac{4\theta}{3} = \dfrac{4(45°)}{3} = 60°$

So, from (7), we get

$$\tan 45° = \frac{v_B \sin(60°) - (\sqrt{3}-1)}{v_B \cos(60°)}$$

$$v_B\left(\frac{\sqrt{3}}{2}\right) - \sqrt{3} + 1 = \frac{v_B}{2}$$

$$\Rightarrow \quad v_B\left(\frac{\sqrt{3}-1}{2}\right) = \sqrt{3} - 1$$

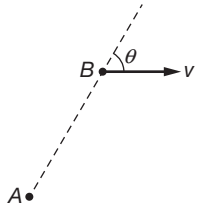
$$\Rightarrow \quad v_B = 2 \text{ ms}^{-1}$$

VELOCITY OF APPROACH/SEPARATION IN TWO DIMENSION

Relative velocity of approach of two bodies is the relative velocity of the bodies along the line joining the two bodies. If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

ILLUSTRATION 63

Particle A is at rest and particle B is moving with constant velocity v as shown in the figure at $t = 0$. Find their velocity of separation.



SOLUTION

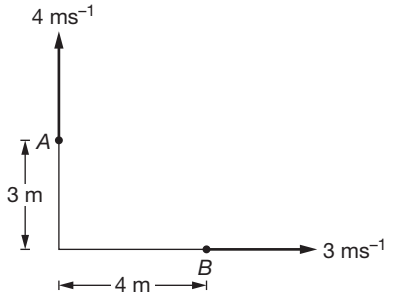
$$v_{BA} = v_B - v_A = v$$

v_{sep} = component of v_{BA} along line $AB = v\cos\theta$

ILLUSTRATION 64

Particles A and B are moving as shown in the figure at $t = 0$. Find their velocity of separation

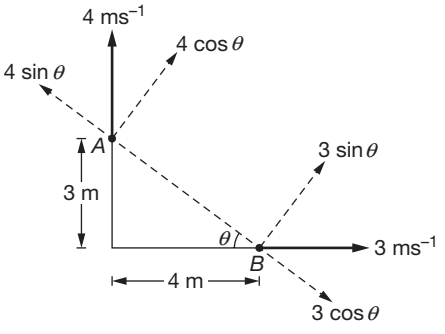
(a) at $t = 0$ **(b)** at $t = 1$ s



SOLUTION

(a) $\tan\theta = \dfrac{3}{4}$

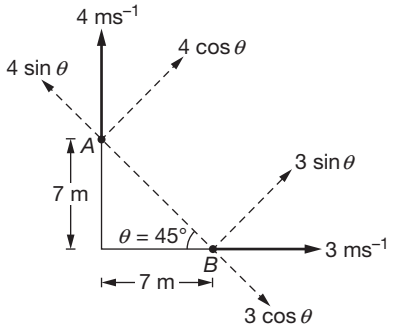
v_{sep} = relative velocity along line AB



$$\Rightarrow \quad v_{sep} = 3\cos\theta + 4\sin\theta$$

$$\Rightarrow \quad v_{sep} = 3\left(\frac{4}{5}\right) + 4\left(\frac{3}{5}\right) = \frac{24}{5} = 4.8 \text{ ms}^{-1}$$

(b) $\theta = 45°$



v_{sep} = relative velocity along line AB

$$\Rightarrow \quad v_{sep} = 3\cos\theta + 4\sin\theta$$

$$\Rightarrow \quad v_{sep} = 3\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{\sqrt{2}}\right) = \frac{7}{\sqrt{2}} \text{ ms}^{-1}$$

ILLUSTRATION 65

Four persons K, L, M and N are initially at the four corners of a square of side a. Each person now moves with a uniform speed v in such a way that K moves always directly towards L, L directly towards M, M directly towards N and N directly towards K. Find the path-equation followed by the persons. Also calculate the time taken by them to meet.

SOLUTION

The symmetry of configuration implies that each person follows similar trajectories and perform similar motion. They, therefore meet at the same time at the centre O of the square. Since they are always directing their velocities to the next neighbours, therefore they must follow a curved path as shown in the figure. The persons are always located at the corners of the square which rotates and shrinks in size and ultimately collapses to zero.

Path Equation Followed by Persons

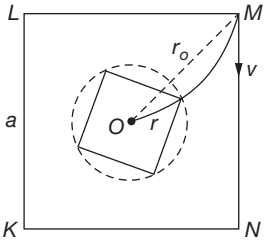
Let $\vec{r}$ be the position vector of point M at any instant t. The velocity $\vec{v}$ of the person M always makes fixed angle $\varphi = 45°$ with $\vec{r}$

In time dt, the person travels a distance $ds = vdt$ …(1)

Further in ΔPQM,

$$\Rightarrow \quad (ds)^2 = (dr)^2 + (rd\theta)^2 \qquad \ldots(2)$$

where, dr = distance moved by particle along radial direction $rd\theta$ = distance moved by particle along tangential direction (normal to $\vec{r}$)



Further in ΔPQM,

$$\Rightarrow \quad \tan 45° = \frac{rd\theta}{dr}$$

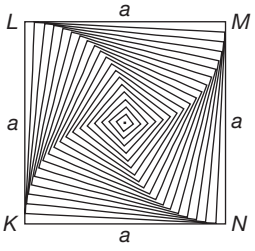
$$\Rightarrow \quad dr = rd\theta \qquad \ldots(3)$$

$$\Rightarrow \quad \int_{\frac{a}{\sqrt{2}}}^{r} \frac{dr}{r} = \int_{0}^{\theta} d\theta$$

$$\Rightarrow \quad \theta = \log_e r\Big|_{\frac{a}{\sqrt{2}}}^{r}$$

$$\Rightarrow \quad r \times \frac{\sqrt{2}}{a} = e^{\theta}$$

$$\Rightarrow \quad r = \frac{a}{\sqrt{2}} e^{\theta} \qquad \{\textbf{Equation of Trajectory}\}$$



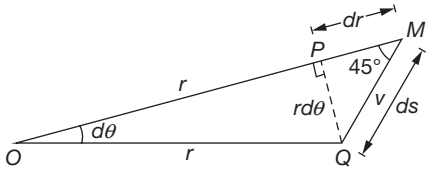
where θ is the angle measured from M along the line OM. Putting (3) in (2), generates

$$\Rightarrow \quad (ds)^2 = 2(dr)^2$$

$$\Rightarrow \quad ds = \sqrt{2}dr$$

$$\Rightarrow \quad \frac{ds}{dt} = \sqrt{2}\frac{dr}{dt}$$

$$\Rightarrow \quad v = \sqrt{2}\frac{dr}{dt}$$



$$\Rightarrow \quad \sqrt{2}\int_0^{\frac{a}{\sqrt{2}}} dr = v\int_0^T dt$$

$$\Rightarrow \quad \sqrt{2}\left(\frac{a}{\sqrt{2}} - 0\right) = v(T - 0)$$

$$\Rightarrow \quad T = \frac{a}{v}$$

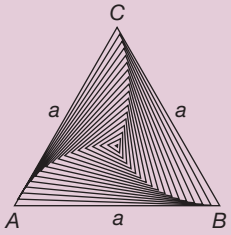
Conceptual Note(s)

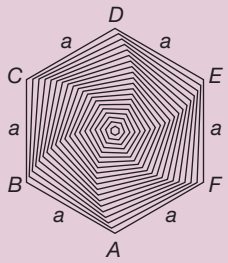
The above analysis can be applied to any other configuration where moving object makes a fixed angle ϕ with $\vec{r}$

Then equation of trajectory will be $r = r_o e^{\frac{\theta}{\tan\phi}}$

Time taken by the objects to meet $T = \dfrac{r_o}{v\cos\phi}$

where, T is the time in which object moves from original radial distance r_0 to $r = 0$, with a uniform speed v.





e.g., For equilateral triangle $\phi = 30°$, $r_o = \dfrac{a}{\sqrt{3}}$

$$\therefore \quad r = \frac{a}{\sqrt{3}}e^{\sqrt{3}\theta} \text{ and } T = \frac{a/\sqrt{3}}{v\cos 30} = \frac{2a}{3v}$$

e.g. For hexagon, $\phi = 60°$, $r_0 = a$

$$r = ae^{\frac{\theta}{\sqrt{3}}} \text{ and } T = \frac{a}{v\cos 60} = \frac{2a}{v}$$

Shortcut Method to Find Only the Time Taken

If the particles are located at the sides of an n sided symmetrical polygon with each side a and each particle moves towards the other, then

$$T = \frac{\text{Initial Separation}}{\text{Relative Velocity of Approach}}$$

$$\Rightarrow \quad T = \frac{a}{v - v\cos\left(\frac{2\pi}{n}\right)}$$

$$\Rightarrow \quad T = \frac{a}{v\left[1 - \cos\left(\frac{2\pi}{n}\right)\right]}$$

$$\Rightarrow \quad T = \frac{a}{2v\sin^2\left(\frac{\pi}{n}\right)}$$

For triangle $n = 3$ $\Rightarrow$ $T = \dfrac{2a}{3v}$

For square, $n = 4$ $\Rightarrow$ $T = \dfrac{a}{v}$

For hexagon, $n = 6$ $\Rightarrow$ $T = \dfrac{2a}{v}$

WHERE TO APPLY THE CONCEPT OF RELATIVE MOTION?

The concept of relative motion can be applied to two and three dimensional motion also. The problems involving the concept of relative motion mainly belong to the following categories.

(a) Distance of closest approach (i.e., minimum distance) between two moving bodies
(b) River-Boat problems or River-Swimmer problems
(c) Aeroplane-Wind problems
(d) Rain-Man problems
(e) Relative motion in the case of projectiles (discussed in projectile motion)

CATEGORY 1: DISTANCE OF CLOSEST APPROACH BETWEEN TWO MOVING BODIES

METHOD I (Calculus Method)

For calculating the distance of closest approach (i.e., the minimum distance between two moving bodies, we first calculate the distance r between them at any instant of time t in the light of some conditions given with the problem. Then we have to minimise r for calculating its minimum value. Mathematically, for r to be MINIMUM, we have

$$\frac{dr}{dt} = 0$$

So we find or calculate the time at which r attains a minimum value. Then this value of t is put in r to get $r_{\min}$.

METHOD II (Relative Velocity Method)

This method can be applied using the following steps.

STEP-1

First calculate $\vec{v}_{BA}$ i.e. velocity of B as seen by A (i.e. velocity of B as seen by A, when A considers itself to be at rest).

You can also calculate $\vec{v}_{AB}$ i.e. velocity of A as seen by B (i.e. velocity of A seen by B, when B considers itself to be at rest).

STEP-2

Find the direction of $\vec{v}_{BA}$ and then draw a line along the direction of $\vec{v}_{BA}$.

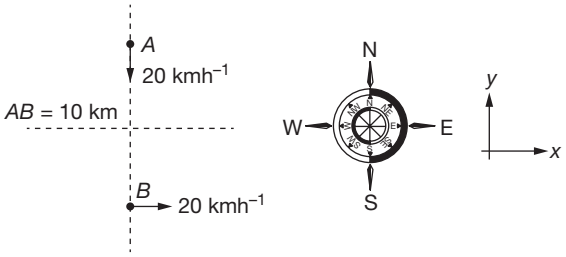
STEP-3

Then perpendicular distance between A (considering it to be at rest) and the line along which $\vec{v}_{BA}$ is directed is the distance of closest approach or the minimum separation between the two moving bodies.

For this see Illustrative Example below.

ILLUSTRATION 66

Ship A is moving towards south with a speed of $20\ \text{kmh}^{-1}$. Another ship B is moving towards east with a speed of $20\ \text{kmh}^{-1}$. At a certain instant the ship B is due south of ship A and is at a distance of $10\ \text{km}$ from ship A. Find the shortest distance between the ships and the time after which they are closest to each other.

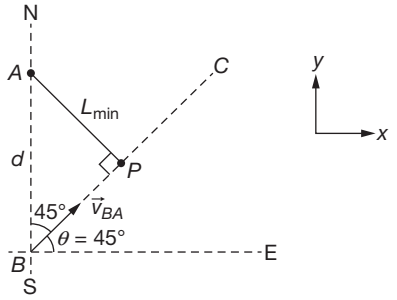


SOLUTION

If we analyse the positions of ships at different instants, we find that the separation decreases at first, becomes minimum at a particular instant of time and then increases.

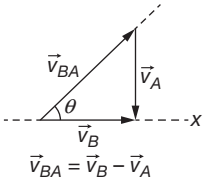
METHOD I: Using the Concept of Relative Velocity

Attach your reference frame with ship A and then analyse the motion of ship B. For you the ship A will be at rest and the ship B will appear to be moving with relative velocity $\vec{v}_{BA} = \left(20\hat{i} + 20\hat{j}\right)\ \text{kmhr}^{-1}$ along line BC.



Motion of A relative to B

For A, B will appear to be moving with relative velocity $\vec{v}_{BA}$ of magnitude $20\sqrt{2}\ \text{kmhr}^{-1}$ at an angle of $45°$ with east (i.e., $+x$ axis). There is a point P on the line of relative motion of B where the separation $AB\left(=L_{\min}\right)$ becomes perpendicular to the line of relative motion. At this instant the separation is minimum.



For the triangle ABP,

$$\sin 45° = \frac{L_{\min}}{d}$$

$$\Rightarrow \quad L_{\min} = \frac{d}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ km}$$

And the time taken is given by

$$|\vec{v}_{BA}| = \sqrt{|\vec{v}_A|^2 + |\vec{v}_B|^2} = 20\sqrt{2} \text{ kmh}^{-1}$$

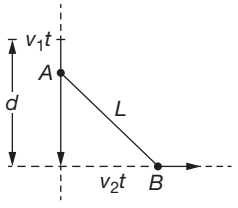
$$t = \frac{BP}{|\vec{v}_{BA}|} = \frac{d\cos 45°}{20\sqrt{2}} = \frac{10 \times \frac{1}{\sqrt{2}}}{20\sqrt{2}} = \frac{1}{4} \text{ hr} = 15 \text{ min}$$

$$\tan\theta = \frac{|\vec{v}_A|}{|\vec{v}_B|} = 1$$

$$\Rightarrow \quad \theta = 45°$$

METHOD II: Using Calculus

We have to find the time after which the separation is minimum, so let us write separation as a function of time and then minimize the function.



At time t the distances moved by A and B are v_1t and v_2t respectively.

Their separation L after time t is given by

$$L^2 = (d - v_1 t)^2 + v_2^2 t^2 \qquad \ldots(1)$$

Differentiating w.r.t. time

$$2L\frac{dL}{dt} = 2(d - v_1 t)(-v_1) + 2v_2^2 t$$

When the separation L is minimum, $\frac{dL}{dt} = 0$

$$\Rightarrow \quad (v_1^2 + v_2^2)t = v_1 d$$

$$\Rightarrow \quad t = \frac{v_1 d}{(v_1^2 + v_2^2)}$$

$$\Rightarrow \quad t = \frac{(10)(20)}{(20^2 + 20^2)}$$

$$\Rightarrow \quad t = \frac{1}{4} \text{ hr} = 15 \text{ min}$$

Now the minimum separation as given by equation (1) is

$$L_{\min} = \sqrt{\left(10 - 20 \times \frac{1}{4}\right)^2 + 20^2 \times \frac{1}{4^2}}$$

$$\Rightarrow \quad L_{\min} = 5\sqrt{2} \text{ km}$$

ILLUSTRATION 67

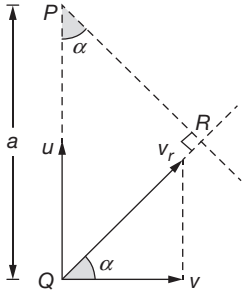
The distance between two moving particles P and Q at any time is a. If v_r be their relative velocity and if u and v be the components of v_r along and perpendicular to PQ, then show that their closest distance is $\frac{av}{v_r}$ and that the time that elapses before they arrive at their nearest distance is $\frac{au}{v_r^2}$.

SOLUTION

METHOD I

Assuming P to be at rest, particle Q is moving with velocity v_r in the direction shown in figure. Components of v_r along and perpendicular to PQ are u and v respectively. In the figure

$$\sin\alpha = \frac{u}{v_r}, \quad \cos\alpha = \frac{v}{v_r}$$



(i) The closest distance between the particles is PR. So,

$$s_{\min} = PR = PQ\cos\alpha = (a)\left(\frac{v}{v_r}\right)$$

$$\Rightarrow \quad s_{\min} = \frac{av}{v_r}$$

(ii) Time after which they arrive at their nearest distance is

$$t = \frac{QR}{v_r} = \frac{(PQ)\sin\alpha}{v_r} = \frac{(a)\left(\frac{u}{v_r}\right)}{v_r} = \frac{au}{v_r^2}$$

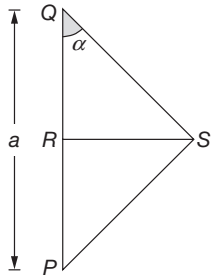
METHOD II

In time t, particle P will travel a distance $PR = ut$ relative to Q along PQ and a distance $RS = vt$ perpendicular to PQ. S is the position of P after time t relative to Q. The distance x between them after time t is given by

$$x^2 = QR^2 + RS^2 = (a - ut)^2 + v^2t^2$$

$$x^2 = (u^2 + v^2)t^2 - 2aut + a^2$$

$$x^2 = v_r^2t^2 - 2aut + a^2 \qquad \{\because u^2 + v^2 = v_r^2\}$$



This can also be written as

$$x^2 = v_r^2\left(t - \frac{au}{v_r^2}\right)^2 + \frac{a^2v^2}{v_r^2} \qquad \ldots(1)$$

From equation (1) it is clear that x is minimum at $t = \frac{au}{v_r^2}$ and the minimum value of x is

$$x_{\min} = \frac{av}{v_r}$$

RELATIVE MOTION IN RIVER FLOW (ONE-DIMENSIONAL APPROACH)

If a swimmer (s) can swim relative to river (r) with velocity $\vec{v}_{sr}$ and river is flowing relative to ground with velocity $\vec{v}_r$, velocity of swimmer relative to the ground is $\vec{v}_s$ then

$$\vec{v}_{sr} = \vec{v}_s - \vec{v}_r$$

$$\Rightarrow \quad \vec{v}_s = \vec{v}_{sr} + \vec{v}_r$$

If $\vec{v}_r = \vec{0}$, then $\vec{v}_s = \vec{v}_{sr}$

Conceptual Note(s)

If $\vec{v}_r = \vec{0}$, then $\vec{v}_s = \vec{v}_{sr}$

In other words, velocity of man in still water = velocity of man w.r.t. river.

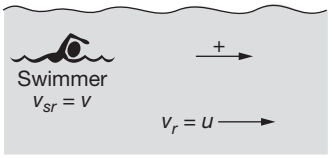
So, whenever a problem says that velocity of swimmer in still water is 5 kmhr^{-1}, then we must take $\vec{v}_{sr} = 5$ kmhr^{-1}.

RIVER PROBLEM IN ONE DIMENSION

Velocity of river is u and velocity of man in still water is v.

CASE-1

Man swimming downstream (along the direction of river flow) In this case



Observer at ground sees the swimmer moving with velocity $v_s = (v + u)$

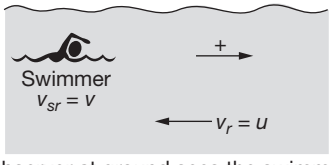
Velocity of river $v_r = +u$

Velocity of man w.r.t. river $v_{sr} = +v$

Now $v_s = v_{sr} + v_r = u + v$

CASE-2

Man swimming upstream (opposite to the direction of river flow). In this case



Observer at ground sees the swimmer moving with velocity $v_s = (v - u)$

Velocity of river $v_r = -u$

Velocity of man w.r.t. river $v_{sr} = +v$

Now $v_s = v_{sr} + v_r = (v - u)$

ILLUSTRATION 68

A swimmer capable of swimming with velocity v relative to water jumps in a flowing river having velocity u. The man swims a distance d downstream and returns to the original position. Find out the time taken in complete motion.

SOLUTION

$$\text{Total time} = \begin{pmatrix}\text{Time of}\\ \text{swimming}\\ \text{downstream}\end{pmatrix} + \begin{pmatrix}\text{Time of}\\ \text{swimming}\\ \text{upstream}\end{pmatrix}$$

$$t = t_{\text{down}} + t_{\text{up}} = \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$

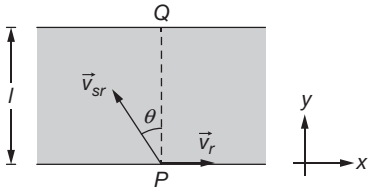
CATEGORY 2: RIVER-BOAT PROBLEMS OR RIVER-SWIMMER PROBLEMS

While solving problems related to river boat or river swimmer we come across the following terms

(a) Absolute velocity of swimmer/boatman $\vec{v}_s$ or $\vec{v}_b$ (absolute means velocity relative to ground)

(b) Absolute velocity of river current $(\vec{v}_r)$ and

(c) Velocity of swimmer/boatman with respect to the river $\vec{v}_{sr}$ or $\vec{v}_{br}$



Since, $\vec{v}_{sr} = \vec{v}_s - \vec{v}_r$

$\Rightarrow \quad \vec{v}_s = \vec{v}_{sr} + \vec{v}_r \qquad \ldots(1)$

Observing the situation shown in the diagram below, we arrive at some standard results and their special cases.

Consider a river of width l across which a swimmer wants to swim. Let $\vec{v}_{sr}$ be the relative velocity of swimmer with respect to river current and let $\vec{v}_{sr}$ make an angle θ with PQ shown in the figure. Let us assume the flow of river to be along the positive x-axis.

Since from (1), we get

$$\vec{v}_s = \vec{v}_{sr} + \vec{v}_r$$

$$\Rightarrow (v_s)_x = (v_{sr})_x + (v_r)_x \quad \Rightarrow (v_s)_y = (v_{sr})_y + (v_r)_y$$

$$\Rightarrow (v_s)_x = (-v_{sr}\sin\theta) + v_r \quad \Rightarrow (v_s)_y = v_{sr}\cos\theta + 0$$

$$\Rightarrow (v_s)_x = v_r - v_{sr}\sin\theta \ldots(2) \quad \Rightarrow (v_s)_y = v_{sr}\cos\theta \ldots(3)$$

If t is the time taken by the swimmer to cross the river, then

$$t = \frac{l}{(v_s)_y}$$

$$\Rightarrow \quad t = \frac{l}{v_{sr}\cos\theta} \qquad \ldots(4)$$

Further if x is the drift (displacement along x axis) when he reaches the opposite bank, then

$$x = (v_b)_x\, t$$

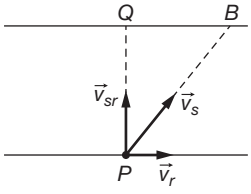
$$\Rightarrow \quad x = \text{Drift} = (v_r - v_{sr}\sin\theta)\frac{l}{v_{sr}\cos\theta} \qquad \ldots(5)$$

CONDITION FOR THE SWIMMER TO CROSS THE RIVER IN THE MINIMUM POSSIBLE TIME

Since $t = \dfrac{l}{v_{sr}\cos\theta}$

For t to be MINIMUM, $\cos\theta$ must be MAXIMUM i.e., $\cos\theta = 1$

$$\Rightarrow \quad \theta = 0^0$$



i.e., The relative velocity of swimmer with respect to river must be perpendicular to the river current.

As a result of this the swimmer will be drifted to the point B.
So,

$$AB = \text{Drift} = v_r t = \left(\frac{v_r}{v_{sr}}\right) l \qquad \ldots(6)$$

CONDITION FOR ZERO DRIFT OR CONDITION TO REACH THE OPPOSITE POINT

Since we calculated the drift value to be x given by

$$x = (v_r - v_{sr}\sin\theta)\frac{l}{v_{sr}\cos\theta}$$

For the above condition to be met, we must have

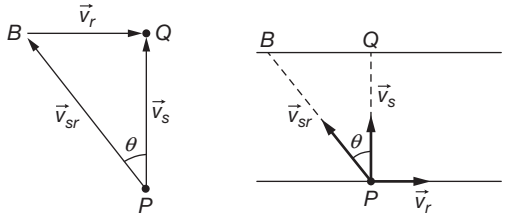
$$x = 0$$

$$\Rightarrow \quad v_r = v_{sr}\sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{v_r}{v_{sr}}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{v_r}{v_{sr}}\right) \qquad \ldots(7)$$

A diagramatic representation is given in support of the above mathematical argument.



Let $\vec{v}_{sr}$ be the velocity of swimmer relative to river current, $\vec{v}_r$ the river current, then the swimmer wants to swim in a manner such that he reaches the point just opposite to the point from where he started. For such a thing to happen he must have his relative velocity directed in a direction opposite to the river current at an angle θ to the line PQ (say).
Further since

$$\vec{v}_{sr} = \vec{v}_s - \vec{v}_r \quad \Rightarrow \quad \vec{v}_s = \vec{v}_{sr} + \vec{v}_r$$

So, $\sin\theta = \dfrac{BQ}{PB} = \dfrac{v_r}{v_{sr}} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\dfrac{v_r}{v_{sr}}\right)$

So, we conclude that the swimmer reaches Q when he directs his velocity relative to river at an angle $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{sr}}\right)$, upstream from PQ.

Also we observe that if $v_r > v_{sr}$ then the swimmer would never reach the point Q.

Condition for Minimum Drift

For minimising the drift, we must have

$$\frac{dx}{d\theta} = 0$$

$$\Rightarrow \quad \frac{d}{d\theta}\left[(v_r - v_{sr}\sin\theta)\frac{l}{v_{sr}\cos\theta}\right] = 0$$

$$\Rightarrow \quad \frac{d}{d\theta}\left[\left(\frac{v_r}{v_{sr}}\sec\theta - \tan\theta\right) l\right] = 0$$

$$\Rightarrow \quad \frac{v_r}{v_{sr}}(\sec\theta\tan\theta) - \sec^2\theta = 0$$

$$\Rightarrow \quad \frac{v_r}{v_{sr}}(\tan\theta) - \sec\theta = 0$$

$$\Rightarrow \quad \frac{v_r}{v_{sr}}\frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{v_{sr}}{v_r}\right)$$

If we are asked to calculate the angle with the horizontal, then we get

$$\text{Total angle} = \frac{\pi}{2} + \theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_{sr}}{v_r}\right)$$

Also, we conclude that the drift x can be minimised only if $v_{sr} < v_r$.

ILLUSTRATION 69

A swimmer can swim at the rate of $5\ \text{kmh}^{-1}$ in still water. A 1 km wide river flows at the rate of $3\ \text{kmh}^{-1}$. The swimmer wishes to swim across the river directly opposite to the starting point.

(a) Along what direction must the swimmer swim?
(b) What should be his resultant velocity?
(c) How much time will he take to cross the river?

SOLUTION

(a) The velocity of swimmer with respect to river $v_{sr} = 5\ \text{kmh}^{-1}$, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The angle of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_r}{v_{sr}}\right) = 90° + \sin^{-1}\left(\frac{v_r}{v_{sr}}\right)$$

$$\theta = 90° + \sin^{-1}\left(\frac{3}{5}\right) = 90° + 37°$$

$\theta = 127°$ w.r.t. the river flow or $37°$ w.r.t. perpendicular in backward direction

(b) Resultant velocity will be

$$v_s = \sqrt{v_{sr}^2 - v_r^2} = \sqrt{5^2 - 3^2} = 4\ \text{kmh}^{-1}$$

Along the direction perpendicular to the river flow.

(c) Time taken to cross the

$$t = \frac{d}{\sqrt{v_{sr}^2 - v_r^2}} = \frac{1\ \text{km}}{4\ \text{kmh}^{-1}} = \frac{1}{4}\ \text{h} = 15\ \text{min}$$

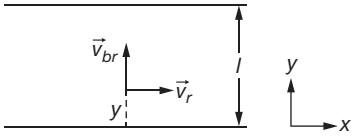
ILLUSTRATION 70

The current velocity of river grows in proportion to the distance from its bank and reaches the maximum value v_0 in the middle. Near the banks the velocity is zero. A boat is moving along the river in such a manner that it is always perpendicular to the current. The speed of the boat in still water is u. Find the distance through which the boat crossing the river will be carried away by the current if the width of the river is l. Also determine the trajectory of the boat.

SOLUTION

Given that $|\vec{v}_{br}| = v_y = \frac{dy}{dt} = u$ …(1)

$$|\vec{v}_r| = v_x = \frac{dx}{dt} = \left(\frac{2v_0}{l}\right)y \quad \ldots(2)$$



From equations (1) and (2) we get

$$\frac{dy}{dx} = \frac{ul}{2v_0 y}$$

$$\Rightarrow \int_0^y y\,dy = \frac{ul}{2v_0}\int_0^x dx$$

$$\Rightarrow y^2 = \frac{ulx}{v_0}$$

At $y = \frac{l}{2}$, $x = \frac{lv_0}{4u}$

$$\Rightarrow x_{\text{net}} = 2x = \frac{lv_0}{2u}$$

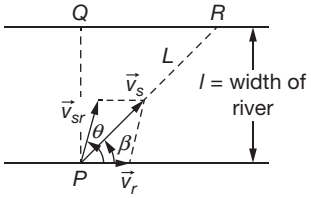
CONDITION WHEN THE BOATMAN CROSSES THE RIVER ALONG THE SHORTEST ROUTE

Here we have to discuss this condition in the light of two cases.

CASE-1: When the velocity of the swimmer with respect to river (v_{sr}) is greater than river current (v_r).

CASE-2: When the velocity of the swimmer with respect to river (v_{sr}) is less than river current (v_r).

For the sake of ease we re-assign symbols to v_{sr} and v_r as v and u respectively.



Let us further assume that $\vec{v}_{sr}$ (i.e., v) makes angle θ with the river current v_r (i.e., u) and $\vec{v}_s$ makes angle β with the river current. Then,

$$\tan\beta = \frac{v_{sr}\sin\theta}{v_r + v_{sr}\cos\theta} = \frac{v\sin\theta}{u + v\cos\theta} \quad \ldots(1)$$

Total length of the path is $PR = L$. Further

$$L = PR = \frac{l}{\sin\beta} = l\,\text{cosec}\,\beta$$

$$\Rightarrow \quad L = l\sqrt{1+\cot^2\beta}$$

$$\Rightarrow \quad L = l\sqrt{1+\left(\frac{u+v\cos\theta}{v\sin\theta}\right)^2}$$

$$\Rightarrow \quad L = \frac{l}{v\sin\theta}\sqrt{u^2\sin^2\theta+u^2+v^2\cos^2\theta+2uv\cos\theta}$$

$$\Rightarrow \quad L = \frac{l}{v\sin\theta}\sqrt{u^2+v^2+2uv\cos\theta}$$

For L to be MINIMUM or L^2 to be MINIMUM

$$\frac{d}{d\theta}(L^2)=0$$

$$\Rightarrow \quad \frac{d}{d\theta}\left[\frac{l^2}{v^2}\left(\frac{u^2+v^2+2uv\cos\theta}{\sin^2\theta}\right)\right]=0$$

$$\Rightarrow \quad \frac{\sin^2\theta(-2uv\sin\theta)-(u^2+v^2+2uv\cos\theta)(2\sin\theta\cos\theta)}{\sin^4\theta}=0$$

$$\Rightarrow \quad uv\sin^2\theta+(u^2+v^2+2uv\cos\theta)\cos\theta=0$$

$$\Rightarrow \quad uv(1-\cos^2\theta)+(u^2+v^2+2uv\cos\theta)\cos\theta=0$$

$$\Rightarrow \quad uv-uv\cos^2\theta+(u^2+v^2)\cos\theta+2uv\cos^2\theta=0$$

$$\Rightarrow \quad uv\cos^2\theta+(u^2+v^2)\cos\theta+uv=0$$

$$\Rightarrow \quad \cos\theta=-\frac{(u^2+v^2)\pm\sqrt{(u^2+v^2)^2-4u^2v^2}}{2uv}$$

$$\Rightarrow \quad \cos\theta=-\frac{(u^2+v^2)\pm\sqrt{(u^2+v^2)^2}}{2uv}$$

$$\Rightarrow \quad \cos\theta=-\frac{(u^2+v^2)\pm(u^2+v^2)}{2uv}$$

Either

$$\cos\theta=\frac{-u^2-v^2+u^2-v^2}{2uv}$$

$$\Rightarrow \quad \cos\theta=-\frac{v}{u} \qquad \ldots(2)$$

Equation (2) holds good only when $v<u$

Further put (2) and (3) one by one in (1), then

For $v<u$ we have

$$\cos\theta=-\frac{v}{u}$$

or

$$\cos\theta=\frac{-u^2-v^2-u^2+v^2}{2uv}$$

$$\Rightarrow \quad \cos\theta=-\frac{u}{v} \qquad \ldots(3)$$

Equation (3) holds good only when $v<u$

For $u<v$ we have

$$\cos\theta=-\frac{u}{v}$$

$$\Rightarrow \quad \tan\beta=\frac{v\sqrt{1-\cos^2\theta}}{u+v\cos\theta} \qquad \Rightarrow \quad \tan\beta=\frac{v\sqrt{1-\cos^2\theta}}{u+v\cos\theta}$$

$$\Rightarrow \quad \tan\beta=\frac{v\sqrt{1-\frac{v^2}{u^2}}}{u+v\left(-\frac{v}{u}\right)} \qquad \Rightarrow \quad \tan\beta=\frac{v\sqrt{1-\frac{u^2}{v^2}}}{u+v\left(-\frac{u}{v}\right)}$$

$$\Rightarrow \quad \tan\beta=\frac{v}{\sqrt{u^2-v^2}} \qquad \Rightarrow \quad \tan\beta\to\infty$$

$$\Rightarrow \quad \beta=\tan^{-1}\left(\frac{v}{\sqrt{u^2-v^2}}\right) \qquad \Rightarrow \quad \beta=\frac{\pi}{2}$$

So, we conclude that the direction of shortest route followed by the swimmer is at right angles to the river current when velocity of swimmer with respect to river current (v) is greater then river current (v_r) or u and for $v<u$ (the opposite case) it is in a direction $\beta=\tan^{-1}\left(\frac{v}{\sqrt{u^2-v^2}}\right)$

CATEGORY 3: AEROPLANE-WIND PROBLEMS

These problems proceed the same way as we have done for river-swimmer problems or river-boat problems. Here $\vec{v}_{sr}$ is just replaced by $\vec{v}_{aw}$ (velocity of aircraft with respect to wind), where $\vec{v}_{aw}=\vec{v}_a-\vec{v}_w$.

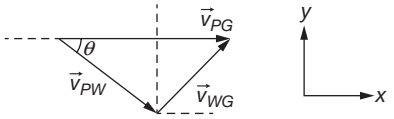
ILLUSTRATION 71

A plane moves in windy weather due east while the pilot points the plane somewhat south of east. The wind is blowing at 50 kmhr^{-1} directed 30° east of north, while the plane moves at 200 kmhr^{-1} relative to the wind. What is the velocity of the plane relative to the ground and what is the direction in which the pilot points the plane?

SOLUTION

Three vectors $\vec{v}_{PG}$, $\vec{v}_{PW}$ and $\vec{v}_{WG}$ are involved. We have to find the magnitude of $\vec{v}_{PG}$ and the direction of $\vec{v}_{PW}$ (i.e., θ). Let us first relate the three vectors using the concept of relative velocity. We can write

$$\begin{pmatrix}\text{Velocity of}\\ \text{plane}\\ \text{w.r.t. wind}\end{pmatrix}=\begin{pmatrix}\text{Velocity of}\\ \text{plane}\\ \text{w.r.t. ground}\end{pmatrix}-\begin{pmatrix}\text{Velocity of}\\ \text{wind}\\ \text{w.r.t. ground}\end{pmatrix}$$



$$\Rightarrow \quad \vec{v}_{PW} = \vec{v}_{PG} - \vec{v}_{WG}$$

$$\Rightarrow \quad \vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$$

Here we have two unknown quantities $(\vec{v}_{PG}$ and $\theta)$, so we should not relate the magnitudes of these vectors using triangle law. In such situations we resolve the vectors into components on the coordinate system and then solve the equations for both axes $(x$ and $y)$.

For the y components

$$v_{PG,\,y} = v_{PW,\,y} + v_{WG,\,y}$$

i.e., $0 = -(200)\sin\theta + (50\cos 30°)$

Solving, we get

$$\sin\theta = \frac{\sqrt{3}}{8};\ \theta = \sin^{-1}(0.216) = 12°$$

For the x components

$$v_{PG,\,x} = v_{PW,\,x} + v_{WG,\,x}$$

$$v_{PG} = (200\cos\theta) + (50\sin 30°)$$

$$\Rightarrow \quad v_{PG} = 221 \text{ kmh}^{-1}$$

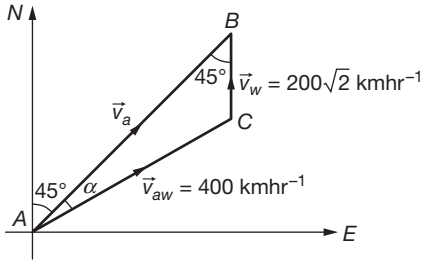
ILLUSTRATION 72

An aircraft flies at 400 kmhr^{-1} in still air. A wind of $200\sqrt{2}$ kmhr^{-1} is blowing from the south. The pilot wishes to travel from A to a point B north east of A. Find the direction he must steer and time of his journey if $AB = 1000$ km.

SOLUTION

METHOD I

Given that $v_w = 200\sqrt{2}$ kmhr^{-1}, $v_{aw} = 400$ kmhr^{-1} and $\vec{v}_a$ should be along AB or in north-east direction. Thus, the direction of $\vec{v}_{aw}$ should be such as the resultant of $\vec{v}_w$ and $\vec{v}_{aw}$ is along AB or in north-east direction.



Let $\vec{v}_{aw}$ makes an angle α with AB as shown in figure. Applying sine law in triangle ABC, we get

$$\frac{AC}{\sin 45°} = \frac{BC}{\sin\alpha}$$

$$\Rightarrow \quad \sin\alpha = \left(\frac{BC}{AC}\right)\sin 45° = \left(\frac{200\sqrt{2}}{400}\right)\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\Rightarrow \quad \alpha = 30°$$

Therefore, the pilot should steer in a direction at an angle of $(45° + \alpha)$ or $75°$ from north towards east.

Further, $\dfrac{|\vec{v}_a|}{\sin(180° - 45° - 30°)} = \dfrac{400}{\sin 45°}$

$$\Rightarrow \quad |\vec{v}_a| = \frac{\sin 105°}{\sin 45°} \times (400) \text{ kmh}^{-1}$$

$$\Rightarrow \quad |\vec{v}_a| = \left(\frac{\cos 15°}{\sin 45°}\right)(400) \text{ kmh}^{-1}$$

$$|\vec{v}_a| = \left(\frac{0.9659}{0.707}\right)(400) \text{ kmh}^{-1} = 546.47 \text{ kmh}^{-1}$$

So, the time of journey from A to B is

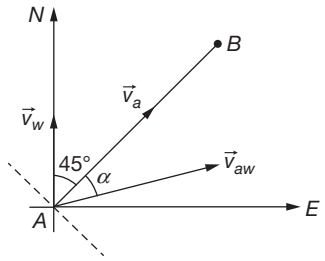
$$t = \frac{AB}{|\vec{v}_a|} = \frac{1000}{546.47} \text{ hr}$$

$$\Rightarrow \quad t = 1.83 \text{ hr}$$

METHOD II

Suppose a vector $\vec{C}$ is a vector sum of two vectors $\vec{A}$ and $\vec{B}$ and the direction of $\vec{C}$ is given to us. Let the vector $\vec{C}$ be directed along a line PQ. Then $\vec{A} + \vec{B}(= \vec{C})$ should be along the line PQ, i.e., the sum of components of $\vec{A}$ and $\vec{B}$ perpendicular to line PQ must be zero. Similarly, if $\vec{C} = \vec{A} - \vec{B}$ and $\vec{C}$ is directed along the line PQ, then the sum of components of $\vec{A}$ and $-\vec{B}$ perpendicular to line PQ must be zero.

For example, if $\vec{v}_a$ has to be along AB and we know that $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$. Therefore, sum of components of $\vec{v}_{aw}$ and $\vec{v}_w$ perpendicular to line AB (shown as dotted) should be zero.



So, $|\vec{v}_{aw}|\sin\alpha = |\vec{v}_w|\sin 45°$

$$\Rightarrow \quad \sin\alpha = \frac{|\vec{v}_w|}{|\vec{v}_{aw}|}\sin 45°$$

$$\Rightarrow \quad \sin\alpha = \left(\frac{200\sqrt{2}}{400}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$$

$$\Rightarrow \quad \alpha = 30°$$

Now, $|\vec{v}_a| = |\vec{v}_{aw}|\cos\alpha + |\vec{v}_w|\cos 45°$

$$\Rightarrow \quad |\vec{v}_a| = (400)\cos 30° + (200\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad |\vec{v}_a| = (400)\frac{\sqrt{3}}{2} + 200$$

$$\Rightarrow \quad |\vec{v}_a| = 346.47 + 200 = 546.47 \text{ kmhr}^{-1}$$

Hence, the time of journey from A to B is given by

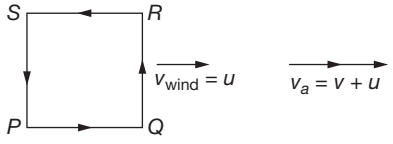
$$t = \frac{AB}{|\vec{v}_a|} = \frac{1000}{546.47} = 1.83 \text{ hr}$$

ILLUSTRATION 73

Find the time an aeroplane having velocity v, takes to fly around a square with side l if the wind is blowing at a velocity u along one side of the square.

SOLUTION

Velocity of aeroplane while flying from P to Q is



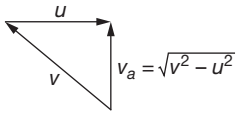
$$v_a = v + u$$

$$\Rightarrow \quad t_{PQ} = \frac{l}{v+u}$$

Velocity of aeroplane while flying from Q to R is

$$v_a = \sqrt{v^2 - u^2}$$

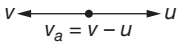
$$\Rightarrow \quad t_{QR} = \frac{l}{\sqrt{v^2 - u^2}}$$



Velocity of aeroplane while flying from R to S is

$$v_a = v - u$$

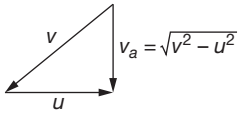
$$\Rightarrow \quad t_{RS} = \frac{l}{v-u}$$



Velocity of aeroplane while flying from S to P is

$$v_a = \sqrt{v^2 - u^2}$$

$$\Rightarrow \quad t_{SP} = \frac{l}{\sqrt{v^2 - u^2}}$$



Total time

$$t = t_{PQ} + t_{QR} + t_{RS} + t_{SP}$$

$$\Rightarrow \quad t = \frac{l}{v+u} + \frac{l}{\sqrt{v^2-u^2}} + \frac{l}{v-u} + \frac{l}{\sqrt{v^2-u^2}}$$

$$\Rightarrow \quad t = \frac{2a}{v^2-u^2}\left(v + \sqrt{v^2-u^2}\right)$$

CATEGORY 4: RAIN-MAN PROBLEMS

In such problems we come across the following terminology according to which

(a) $\vec{v}_r$ is the absolute velocity of rain.

(b) $\vec{v}_m$ is the absolute velocity of man/cyclist/motorist/observer.

(c) $\vec{v}_w$ is the absolute velocity of wind (if it is blowing) and

(d) $\vec{v}_{rm}$ is the velocity of rain with respect to man or the velocity of rain which appears to the man.

For dealing with such like problems, we have two options

Option A: When no wind is blowing.

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m \quad \text{or} \quad \vec{v}_{rm} = \vec{v}_r + (-\vec{v}_m)$$

So, while dealing with the problems in which rain and man are there but no wind exists, then to calculate the direction of $\vec{v}_{rm}$, we simply reverse the direction of man's velocity $(-\vec{v}_m)$ and then find the resultant of $-\vec{v}_m$ and $\vec{v}_r$ i.e., $-\vec{v}_r + (-\vec{v}_m)$ to get the direction and magnitude of $\vec{v}_{rm}$. It's the Best Trick! Try following and see the results.

Option B: When wind is blowing.

$$\vec{v}_{rm} = (\vec{v}_r + \vec{v}_w) - \vec{v}_m \text{ or } \vec{v}_{rm} = \vec{v}_r + \vec{v}_w + (-\vec{v}_m)$$

So, while dealing with the problems involving rain, man and the blowing wind we first calculate the resultant of rain and wind i.e., net velocity of rain under the inference of wind $\vec{v}_r + \vec{v}_w = \vec{v}_{\text{net rain}}$. The man sees this rain falling with a velocity $\vec{v}_{rm} = \vec{v}_{\text{net rain}} - \vec{v}_m$

$$\Rightarrow \quad \vec{v}_{rm} = (\vec{v}_r + \vec{v}_w) - \vec{v}_m$$

$$\Rightarrow \quad \vec{v}_{rm} = (\vec{v}_r + \vec{v}_w) + (-\vec{v}_m)$$

Again, we reverse the direction of velocity of man and then find the resultant of $\vec{v}_{\text{net rain}}$ and $-\vec{v}_m$ to get $\vec{v}_{rm}$ with magnitude and direction.

So, to deal with problems involving rain, man and wind we just reverse the direction of $\vec{v}_m$ i.e., make it $-\vec{v}_m$ and then find the resultant of $\vec{v}_r$, $\vec{v}_w$ and $-\vec{v}_m$ i.e., $\vec{v}_r + \vec{v}_w + (-\vec{v}_m)$.

ILLUSTRATION 74

A standing man, observes rain falling with velocity of $20\ \text{ms}^{-1}$ at an angle of $30°$ with the vertical.

(a) Find the velocity with which the man should move so that rain appears to fall vertically to him.

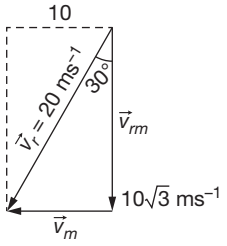
(b) Now if he further increases his speed, rain again appears to fall at $30°$ with the vertical. Find his new velocity.

SOLUTION

(a) $\vec{v}_m = -v\hat{i}$ (let)

$$\vec{v}_r = -10\hat{i} - 10\sqrt{3}\hat{j}$$

$$\vec{v}_{rm} = -(10 - v)\hat{i} - 10\sqrt{3}\hat{j}$$



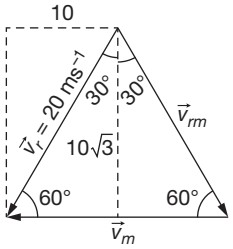
$$\Rightarrow \quad -(10 - v) = 0 \quad \text{(for vertical fall, horizontal component must be zero)}$$

$$\Rightarrow \quad v = 10\ \text{ms}^{-1}$$

(b) $\vec{v}_r = -10\hat{i} - 10\sqrt{3}\hat{j}$

$\vec{v}_m = -\vec{v}_x\hat{i}$

$\vec{v}_{rm} = -(10 - v_x)\hat{i} - 10\sqrt{3}\hat{j}$



Angle with the vertical $= 30°$

$$\Rightarrow \quad \tan 30° = \frac{10 - v_x}{-10\sqrt{3}}$$

$$\Rightarrow \quad v_x = 20\ \text{ms}^{-1}$$

ILLUSTRATION 75

To a man walking at the rate of $3\ \text{kmh}^{-1}$ the rain appears to fall vertically. When he increases his speed to $6\ \text{kmh}^{-1}$ it appears to meet him at an angle of $45°$ will vertical. Find the speed of rain.

SOLUTION

Let $\hat{i}$ and $\hat{j}$ be the unit vectors along the horizontal and vertical directions respectively.

Let velocity of rain be $\vec{v}_r = a\hat{i} + b\hat{j}$ …(1)

Then speed of rain will be

$$|\vec{v}_r| = \sqrt{a^2 + b^2} \quad \text{…(2)}$$

In the first case $\vec{v}_m$ = velocity of man $= 3\hat{i}$

$\Rightarrow \quad \vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\hat{i} + b\hat{j}$

It seems to be in vertical direction. Hence, the horizontal component must be zero.

$\Rightarrow \quad a - 3 = 0$

$\Rightarrow \quad a = 3$

In the second case $\vec{v}_m = 6\hat{i}$

$\Rightarrow \quad \vec{v}_{rm} = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$

This seems to be at 45° with the vertical

$\Rightarrow \quad \tan(45°) = \dfrac{|v_y|}{|v_x|} = \dfrac{|b|}{3}$

$\Rightarrow \quad |b| = 3$

Therefore, from equation (2) speed of rain is

$$|\vec{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2} \text{ kmh}^{-1}$$

Test Your Concepts-VII

Based on Relative Velocity

(Solutions on page H.86)

1. A river 400 m wide is flowing at a rate of 2 ms^{-1}. A boat is sailing at a velocity of 10 ms^{-1} with respect to the water, in a direction perpendicular to the river.
   (a) Find the time taken by the boat to reach the opposite bank.
   (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank?

2. Snow is falling vertically at a constant speed of 8 ms^{-1}. At what angle from the vertical do the snow flakes appear to be falling as viewed by the driver of a car travelling on a straight, level road with a speed of 50 kmh^{-1}?

3. Show that the direction of shortest route is at right angles to the river when the velocity of boat with respect to water v is greater than that of the river velocity u and in opposite case it is $\tan^{-1}\left(\dfrac{v}{\sqrt{u^2 - v^2}}\right)$.

4. A ship A is travelling due east at 10 $kmhr^{-1}$ and at 9 am is 30 km south-west of another ship B. If B travels at 15 $kmhr^{-1}$ so as to intercept A calculate the
   (a) direction in which B must travel.
   (b) time it takes when the interception takes place.

5. A motorboat going downstream overcame a raft at point A. One hour later it turned back and met the raft again at a distance 6 km from point A. Find the river velocity.

6. An elevator car whose floor to ceiling distance is equal to 2.7 m starts ascending with constant acceleration 1.2 ms^{-2}. 2 second after the start, a bolt begins falling from the ceiling of the car. Find the
   (a) time after which bolt hits the floor of the elevator.
   (b) net displacement and distance travelled by the bolt, with respect to earth.
   (Take $g = 9.8$ ms^{-2})

7. A particle is moving in a circle of radius r centred at O with constant speed v. Calculate the change in velocity in moving from A to B when $\angle AOB = 40°$.

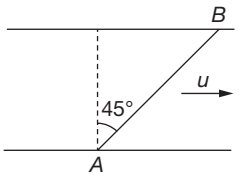
8. Two ships A and B are 10 km apart on a line running south to north. Ship A farther north is streaming west at 20 $kmhr^{-1}$ and ship B is streaming north at 20 $kmhr^{-1}$. What is their distance of closest approach and how long do they take to reach it?

9. Car A has an acceleration of 2 ms^{-2} due east and car B, 4 ms^{-2} due north. What is the acceleration of car B with respect to car A?

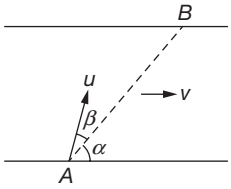
10. A bullet train A starts from rest at $t = 0$ and travels along a straight track with a constant acceleration of 6 ms^{-2} until it reaches a speed of 80 ms^{-1}. Afterwards it maintains this speed. Also, when $t = 0$, another bullet train B located 6000 m down on a parallel track is travelling towards A at a constant speed of 60 ms^{-1}. Determine the distance travelled by train A when they pass each other.

11. A body is thrown up in a lift with a velocity u relative to the lift. If the time of flight is found to be t then find the acceleration of the lift.

12. A man wants to reach point B on the opposite bank of a river flowing at a speed as shown in the figure. What minimum speed relative to water should the man have so that the can reach point B? In which direction should he swim?



13. A launch plies between two points A and B on the opposite banks of a river always following the line AB. The distance S between points A and B is 1200 m. The velocity of the river current $v = 1.9\ \text{ms}^{-1}$ is constant over the entire width of the river. The line AB makes an angle $\alpha = 60°$ with the direction of the current. With what velocity u and at what angle β to the line AB should the launch move to cover the distance AB and back in a time $t = 5$ min? The angle β remains the same during the passage from A to B and from B to A.



14. A man wishes to cross a river of width 120 m by a motorboat. His rowing speed in still water is $3\ \text{ms}^{-1}$ and his maximum walking speed is $1\ \text{ms}^{-1}$. The river flows with velocity of $4\ \text{ms}^{-1}$.

(a) Find the path which he should take to get to the point directly opposite to his starting point in the shortest time.

(b) Also, find the time which he takes to reach his destination.

15. A ball is thrown vertically upward from the 12 m level in an elevator shaft with an initial velocity of $18\ \text{ms}^{-1}$. At the same instant an open platform type elevator passes the 5 m level, moving upward with a constant velocity of $2\ \text{ms}^{-1}$. Find

(a) when and where the ball will hit the elevator

(b) the relative velocity of the ball with respect to the elevator when the ball hits the elevator.

16. A swimmer heads directly across a river, swimming at $1.6\ \text{ms}^{-1}$ relative to still water. He arrives at a point 40 m downstream from the point directly across the river, which is 80 m wide.

(a) What is the speed of the river current?

(b) What is the swimmer's speed relative to the shore?

(c) In what direction should the swimmer head so as to arrive at the point directly opposite to the starting point?

17. A cyclist, riding at a speed V, overtakes a pedestrian who can move at a speed not greater than v, the two travelling along parallel tracks at a distance d apart. Show that if the cyclist rings his bell when at a distance less that $\frac{V}{v}d$, he may safely maintain his speed and keep to his course regardless of the behaviour of the pedestrian.

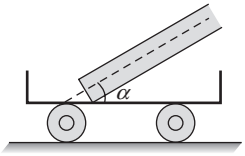
18. A swimmer wishes to cross a 500 m wide river flowing at $5\ \text{kmh}^{-1}$. His speed with respect to water is $3\ \text{kmh}^{-1}$.

(a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river.

(b) Find the shortest possible time to cross the river.

(c) Assuming that the swimmer has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.

19. A pipe which can be rotated in a vertical plane is mounted on a cart. The cart moves uniformly along a horizontal path with a velocity $v_1 = 2\ \text{ms}^{-1}$. At what angle α to the horizontal should the pipe be placed so that drops of rain falling vertically with a velocity $v_2 = 6\ \text{ms}^{-1}$ move parallel to the walls of the pipe without touching them.



20. Two mirrors, mounted vertically, are made to move towards each other with a speed *v* each. A particle that can bounce back between the two mirrors starts from one mirror when the mirrors are *d* apart. On reaching the second mirror, it bounces back and so on. If the particle keeps on travelling at a constant speed of 3 v, how many trips can it make before the mirrors run into each other? What total distance does it cover?

SOLVED PROBLEMS

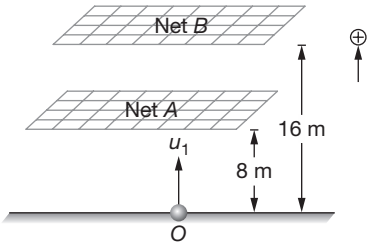
PROBLEM 1

A particle is projected vertically with a speed of 32.9 ms^{-1} from a point O. It passes successively through two horizontal thin nets at height 8 m and 16 m respectively, above O. Find the greatest height above O that the particle attains net and its speed on reaching back to O. Assume that each thin net has a mesh which is broad enough to allow the ball to cross the net but witch half the velocity with which the ball enters.

SOLUTION

Take upward direction as positive. In the upward journey, let v_1 be the velocity of the particle just before it enters the net A. Then

$$v_1^2 = u_1^2 - 2gh = (39.2)^2 - 2\times 9.8\times 8$$



Solving it, we get $v_1 = 37.15$ ms^{-1}

Velocity of the particle after crossing the net is $\frac{v_1}{2}$. So

$$\frac{v_1}{2} = 18.575 \text{ ms}^{-1}$$

Let v_2 be the velocity of the particle just before it enters the net B. Then

$$v_2^2 = (18.575)^2 - 2\times 9.8\times 8$$

Solving it, we get $v_2 = 13.72$ ms^{-1}

Velocity of the particle after it crosses the net B is

$$v_2 = \frac{13.72}{2} = 6.86 \text{ ms}^{-1}$$

Let h be the height reached above the net B where velocity of particle becomes zero. Then

$$(0)^2 = (6.86)^2 - 2\times 9.8\times h$$

$$\Rightarrow \quad h = 2.4 \text{ m}$$

$\therefore$ Total height above O is $16 + 2.4 = 18.4$ m

In the return journey, the velocity of the particle at the time of reaching net B is given by

$$v_3^2 = (0)^2 + 2gh$$

$$\Rightarrow \quad v_3 = \sqrt{2gh} = \sqrt{2\times 9.8\times 2.4}$$

Velocity after crossing net B is $\frac{1}{2}\sqrt{2\times 9.8\times 2.4}$

Velocity at net A is $\sqrt{\left(\frac{2\times 9.8\times 2.4}{4}\right) + 2\times 9.8\times 8}$

$$(v_A)_{\text{down}} = \sqrt{9.8\times 17.2}$$

Velocity after crossing the net A is $\frac{1}{2}\sqrt{9.8\times 17.2}$

Let the velocity at O be v_4. Then

$$v_4^2 = \frac{(9.8\times 17.2)}{4} + 2\times 9.8\times 8$$

$$\Rightarrow \quad v_4 = 14.1 \text{ ms}^{-1}$$

PROBLEM 2

Find the trajectory of the particle of mass m acted upon by a force $\vec{F} = \cos t\hat{i} + \sin t\hat{j}$. At $t = 0$, the position and velocity of particle are $x_0\hat{i}$ and $v_0\hat{j}$ respectively.

SOLUTION

Since, $\vec{F} = m\frac{d^2\vec{r}}{dt^2} = \cos t\hat{i} + \sin t\hat{j}$ {Given}

Integrating both sides w.r.t. t, we have

$$\int\left(m\frac{d^2\vec{r}}{dt^2}\right)dt = \int(\cos t\hat{i} + \sin t\hat{j})dt$$

$$\Rightarrow \quad m\frac{d\vec{r}}{dt} = \sin t\hat{i} + (-\cos t)\hat{j} + c_1 \qquad \ldots(1)$$

At time $t = 0$, we have $\ldots(2)$

$$mv_0\hat{j} = -\hat{j} + c_1 \qquad \{\text{putting } t = 0 \text{ in equation (1)}\}$$

$$\Rightarrow \quad c_1 = (mv_0 + 1)\hat{j}$$

Substituting value of c_1 in equation (1) we get

$$m\frac{d\vec{r}}{dt} = \sin t\hat{i} + (-\cos t)\hat{j} + (mv_0 + 1)\hat{j}$$

$$\Rightarrow \quad m\frac{d\vec{r}}{dt} = \sin t\hat{i} + (mv_0 + 1 - \cos t)\hat{j}$$

Integrating again both sides w.r.t. t, we have

$$\int\left(m\frac{d\vec{r}}{dt}\right)dt = \int(\sin t\hat{i} + (mv_0 + 1 - \cos t)\hat{j})\,dt$$

$$\Rightarrow \quad m\vec{r} = (-\cos t)\hat{i} + (mv_0 t + t - \sin t)\hat{j} + c_2 \qquad \ldots(3)$$

Again at time $t = 0$, $\vec{r} = x_0\hat{i}$, thus equation (3) gives us

$$c_2 = (mx_0 + 1)\hat{i}$$

Substituting value of c_2 in equation (3), we get

$$m\vec{r} = (mx_0 + 1 - \cos t)\hat{i} + (mv_0 t + t - \sin t)\hat{j}$$

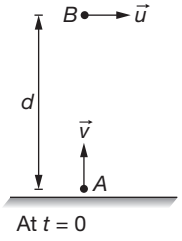
$$\Rightarrow \quad \vec{r} = \frac{1}{m}\left[(mx_0 + 1 - \cos t)\hat{i} + (mv_0 t + t - \sin t)\hat{j}\right] \qquad \ldots(4)$$

Comparing equation (4) with $\vec{r} = x\hat{i} + y\hat{j}$

$$\therefore \quad x = x_0 + \left(\frac{1 - \cos t}{m}\right) \text{ and } y = v_0 t + \left(\frac{t - \sin t}{m}\right)$$

PROBLEM 3

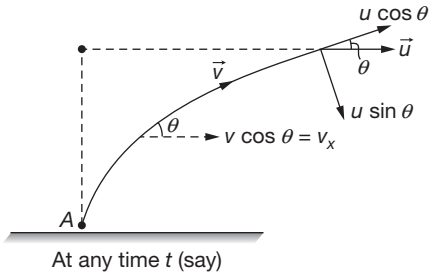
A sitting cat in a field suddenly sees a standing dog. To save its life the cat runs away in a straight horizontal line with constant velocity $\vec{u}$. Without any time lag the dog starts with velocity $\vec{v}$ constant in magnitude and always directed towards the cat to catch it. Initially, $\vec{v}$ and $\vec{u}$ are perpendicular and separation between the cat and the dog is d. Find the time after which the dog catches the cat.



SOLUTION

Initially the particles are at A and B, a distance d apart and their velocities are perpendicular to each other.

The velocity $\vec{u}$ is constant in magnitude and direction while the velocity $\vec{v}$ changes continuously in direction and is always aimed at B. Let, at any instant t, $\vec{v}$



makes an angle θ with $\vec{u}$. Then resolving $\vec{u}$ into two components

(a) $u\cos\theta$, parallel to $\vec{v}$, and

(b) $u\sin\theta$, perpendicular to $\vec{v}$.

Therefore, relative velocity of approach of A towards B is

$$(v - u\cos\theta) = -\frac{dx}{dt}$$

(Negative sign indicates that distance between particles decreases with time)

$$\Rightarrow \quad -dx = (v - u\cos\theta)dt \qquad \ldots(1)$$

Integrating both sides of equation (1),

$$\Rightarrow \quad -\int_d^0 dx = \int_0^t (v - u\cos\theta)\,dt$$

$$\Rightarrow \quad d = \int_0^t v\,dt - \int_0^t u\cos\theta dt \qquad \ldots(2)$$

Since $u < v$, Therefore, $u\cos\theta < v$ and hence, they meet.

Also, the horizontal distance travelled by both of them is the same, when they meet.

Therefore, $\Delta x = \int v_x\,dt$

$$\Rightarrow \quad ut = \int_0^t v\cos\theta dt$$

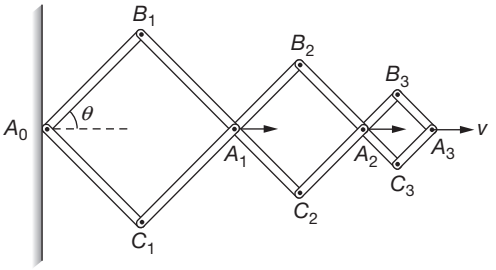
$$\Rightarrow \quad d = v\int_0^t dt - u\int_0^t \cos\theta\,dt$$

$$\Rightarrow \quad d = vt - u\left(\frac{ut}{v}\right)$$

$$\Rightarrow \quad t = \frac{vd}{v^2 - u^2}$$

PROBLEM 4

A hinged construction consists of three rhombus with ratio of sides $3:2:1$ (as shown in figure). Vertex A_3 moves in a horizontal direction with velocity v. Determine velocities of vertices A_1, A_2 and B_2 at instant when angle of construction is $45°$.



SOLUTION

Let A_0 be the Origin of the whole system shown in the diagram. Let

$$A_0B_1 = B_1A_1 = 3k$$

$$A_1B_2 = B_2A_2 = 2k$$

$$A_2B_3 = B_3A_3 = k$$

Hence, $A_0A_1 = 3K\cos\theta + 3K\cos\theta = 6K\cos\theta = x_{A_1}$

$$A_0A_2 = A_0A_1 + A_1A_2 = 6K\cos\theta + 4K\cos\theta$$

$$\Rightarrow \quad x_{A_2} = 10K\cos\theta$$

$$A_0A_3 = A_0A_2 + A_2A_3 = 10K\cos\theta + 2K\cos\theta$$

$$\Rightarrow \quad x_{A_3} = 12K\cos\theta$$

Similarly,

$$x_{B_2} = 6K\cos\theta + 2K\cos\theta = 8K\cos\theta,$$

and

$$y_{B_2} = 2K\sin\theta$$

According to Problem, $\dfrac{dx_{A_3}}{dt} = v$

$$\Rightarrow \quad -12K\sin\theta\frac{d\theta}{dt} = v$$

$$\Rightarrow \quad -K\sin\theta\frac{d\theta}{dt} = \frac{v}{12}$$

Therefore,

$$v_{A_2} = \frac{dx_{A_2}}{dt} = -10K\sin\theta\left(\frac{d\theta}{dt}\right) = 10\left(\frac{v}{12}\right) = \frac{5}{6}v$$

$$\Rightarrow \quad v_{A_1} = \frac{dx_{A_1}}{dt} = -6K\sin\theta\left(\frac{d\theta}{dt}\right) = 6\left(\frac{v}{12}\right) = \frac{v}{2}$$

For B_2 :

$$\left(v_{B_2}\right)_x = \frac{dx_{B_2}}{dt} = -8K\sin\theta\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \quad \left(v_{B_2}\right)_y = \frac{dy_{B_2}}{dt} = 2K\cos\theta\left(\frac{d\theta}{dt}\right)$$

Now, $\left(v_{B_2}\right)^2 = \left(v_{B_2}\right)_x^2 + \left(v_{B_2}\right)_y^2$

$$\Rightarrow \quad \left(v_{B_2}\right)^2 = 64\left(K\sin\theta\frac{d\theta}{dt}\right)^2 + 4\cos^2\theta\left(K\frac{d\theta}{dt}\right)^2$$

$$\Rightarrow \quad \left(v_{B_2}\right)^2 = 64\left(\frac{v}{12}\right)^2 + 4\cos^2\theta\left(K\frac{d\theta}{dt}\right)$$

$$\Rightarrow \quad \left(v_{B_2}\right)^2 = 64\left(\frac{v}{12}\right)^2 + 4\cos^2\theta\left(\left(\frac{v}{12}\right)^2\frac{1}{\sin^2\theta}\right)$$

$$\Rightarrow \quad \left(v_{B_2}\right)^2 = \left(\frac{v}{12}\right)^2\left(64 + 4\cot^2\theta\right)$$

$$\Rightarrow \quad v_{B_2} = \frac{v}{12}\sqrt{64 + 4\cot^2\theta}$$

Now when θ is $45°$, then

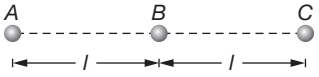
$$\Rightarrow \quad v_{B_2} = \frac{v}{12}\sqrt{68} = \frac{\sqrt{17}}{6}v$$

PROBLEM 5

At the initial moment three points A, B, and C are on a horizontal straight line at equal distance from one another. Point A begins to move vertically upwards with a constant velocity v and point C vertically downward without any initial velocity at a constant acceleration a. How should the point B move vertically for all the three points to be constantly on one straight line? The points begin to move simultaneously.

SOLUTION

Initially, at $t = 0$, all the particles are equidistant from each other.



Let at time t the particle A goes to A', B goes to B' and C goes to C'

$$AA' = vt$$

$$BB' = u_B t + \frac{1}{2}a_B t^2$$

$$CC' = \frac{1}{2}at^2$$

In similar triangles AOA', BOB' and COC' we have

$$\frac{vt}{l-x} = \frac{u_B t + \frac{1}{2}a_B t^2}{x} = \frac{\frac{1}{2}at^2}{l+x} \quad \ldots(1)$$

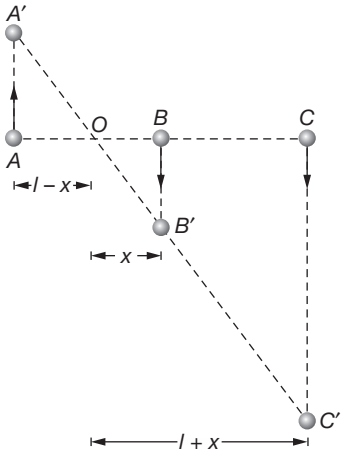
$$\Rightarrow \quad \frac{2v}{l-x} = \frac{at}{l+x}$$

$$\Rightarrow \quad x = \left(\frac{at-2v}{at+2v}\right)l \quad \ldots(2)$$

Put (2) in (1) we get

$$u_B t + \frac{1}{2}a_B t^2 = \left(\frac{-v}{2}\right)t + \frac{1}{2}\left(\frac{a}{2}\right)t^2$$

i.e., $u_B = -\frac{v}{2}$ and $a_B = \frac{a}{2}$



Hence, for all the particles to be on one straight line the particle at B must move with an initial velocity $\frac{v}{2}$ in the upward direction and an acceleration $\frac{a}{2}$ in the downward direction.

PROBLEM 6

Two boats, A and B, move away from a buoy anchored at middle of the river along mutually perpendicular straight lines. The boat A along the river and the boat B across the river. Having moved off an equal distance from the buoy the boats returned. Find the ratio of the times of motion of boats $\frac{\tau_A}{\tau_B}$ if the velocity of each boat w.r.t. water is $\eta = 1.2$ times greater than the stream velocity.

SOLUTION

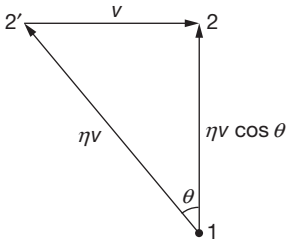
Let, d is the distance moved by boats, A and B, away from boys in direction of motion (or perpendicular to) stream velocity when they go from point 1 to 2.

v is the velocity of stream

Therefore, ηv is the velocity of the boat.

For Boat A:

$$\begin{pmatrix}\text{Total time}\\ \text{taken by}\\ \text{Boat A}\end{pmatrix} = \begin{pmatrix}\text{Time taken}\\ \text{downstream}\\ \text{by Boat A}\end{pmatrix} + \begin{pmatrix}\text{Time taken}\\ \text{upstream}\\ \text{by Boat A}\end{pmatrix}$$



$$\Rightarrow \quad t_A = t_{\text{down}} + t_{\text{up}}$$

$$\Rightarrow \quad t_A = \frac{d}{\eta v + v} + \frac{d}{\eta v - v}$$

$$\Rightarrow \quad t_A = \frac{2d\eta}{v(\eta^2 - 1)}$$

For Boat B: Time taken by Boat B in going from 1 to 2

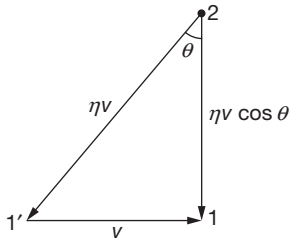
$$t_{1\to 2} = \frac{d}{\eta v \cos\theta} \quad \ldots(1)$$

and $\sin\theta = \frac{v}{\eta v} = \frac{1}{\eta}$

$$\Rightarrow \quad \cos\theta = \sqrt{1 - \frac{1}{\eta^2}}$$

Substituting respective values in (1),

$$\Rightarrow \quad t_{1\to 2} = \frac{d}{v\sqrt{\eta^2 - 1}}$$



Time taken by boat B to return from 2 to 1

$$\Rightarrow \quad t_{2\to1} = \frac{d}{v\sqrt{\eta^2 - 1}}$$

$$\Rightarrow \quad t_B = t_{1\to2} + t_{2\to1} = \frac{2d}{v\sqrt{\eta^2 - 1}}$$

$$\Rightarrow \quad \frac{t_A}{t_B} = \frac{\eta}{\sqrt{\eta^2 - 1}} = 1.8$$

PROBLEM 7

A particle of mass 10^{-2} kg is moving along the positive x-axis under the influence of force $F(x) = -\frac{k}{2x^2}$, $k = 10^{-2}$ Nm^2. At $t = 0$, it is at $x = 1$ m and its velocity is $v = 0$

(a) Find its velocity when it reaches $x = 0.5$ m.
(b) Time at which it reaches $x = 0.25$ m.

SOLUTION

(a) $F(x) = -\frac{k}{2x^2}$ {Given}

$$\Rightarrow \quad mv\frac{dv}{dx} = -\frac{k}{2x^2}$$

$$\Rightarrow \quad mvdv = -\frac{k}{2x^2}dx \qquad \ldots(1)$$

Integrating (1) within suitable limits

$$\Rightarrow \quad m\int_0^v vdv = -\frac{k}{2}\int_{1.0}^{0.5} x^{-2}dx$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{k}{2}\left[\frac{1}{0.5} - 1\right]$$

$$\Rightarrow \quad v^2 = +1$$

$$\Rightarrow \quad v = \pm 1 \text{ ms}^{-1}$$

Since the particle is going from $x = 1$ m to $x = 0.5$ m, hence, its velocity is directed along the negative x-axis.

$$\Rightarrow \quad \vec{v} = (-1)\hat{i}$$

(b) $v^2 = \frac{k}{m}\left[\frac{1}{x} - 1\right]$

$$\Rightarrow \quad -v = \sqrt{\frac{1-x}{x}} \text{ {directed towards negative axis}}$$

$$\Rightarrow \quad -\frac{dx}{dt} = \sqrt{\frac{1-x}{x}} \qquad \ldots(2)$$

Integrating relation (2) within suitable limits

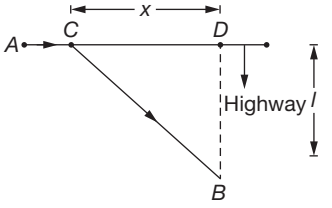
$$\Rightarrow \quad \int_0^t dt = -\int_1^{\frac{1}{4}} \sqrt{\frac{x}{1-x}}\,dx$$

Solving, by substituting $x = \sin^2\theta$, we get

$$t = \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right) \text{ s}$$

PROBLEM 8

From point A located on a highway one has to get by car as soon as possible to point B located in the field at a distance l from highway. It is known that car moves in the field η times slower than on highway. At what distance from D one must turn off the highway?



SOLUTION

Total Time = $t_{A\to C} + t_{C\to B}$

$$\Rightarrow \quad t = \frac{AC}{v} + \frac{CB}{\frac{v}{\eta}}$$

$$\Rightarrow \quad t = \frac{AD - x}{v} + \frac{\sqrt{x^2 + l^2}}{\frac{v}{\eta}}$$

The car has to reach B as soon as possible

$$\therefore \quad \frac{dt}{dx} = 0$$

$$\Rightarrow \quad t = -\frac{1}{v} + \frac{1}{\frac{v}{\eta}} \times \frac{2x}{2\sqrt{x^2 + l^2}}$$

$$\Rightarrow \quad \frac{\eta^2 x^2}{x^2 + l^2} = 1$$

$$\Rightarrow \quad x = \frac{l}{\sqrt{\eta^2 - 1}}$$

PROBLEM 9

A particle moves in x-y plane with velocity $\vec{v} = k_1\hat{i} + k_2 x\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axes, k_1 and k_2 are constants. Initially the particle is at origin (0, 0). Find

(a) equation of particles trajectory $y(x)$,
(b) curvature radius of trajectory of particle as a function of x.

SOLUTION

(a) $\vec{v} = k_1\hat{i} + k_2 x\hat{j}$ {Given}

$$\vec{v}_x = k_1\hat{i} \qquad \vec{v}_y = k_2 x\hat{j}$$

$$\frac{dx}{dt} = k_1 \qquad \frac{dy}{dt} = k_2 x = k_1 k_2 t$$

$$\int_0^x dx = \int_0^t k_1 dt \qquad \int_0^y dy = k_1 k_2 \int_0^t t\, dt$$

$$x = k_1 t \quad \ldots(1) \qquad y = \frac{k_1 k_2}{2} t^2 \quad \ldots(2)$$

Eliminating t from the above set of two equations, we have

$$y = \left(\frac{k_2}{2k_1}\right) x^2 \quad \ldots(3)$$

Equation (1) sounds much more like the equation of the parabola

(b) Let R be the radius of the parabolic equation,

$$\Rightarrow \quad R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left(\frac{d^2y}{dx^2}\right)}$$

$$\Rightarrow \quad R = \frac{\left[1 + \left(\frac{k_2 x}{k_1}\right)^2\right]^{\frac{3}{2}}}{\left(\frac{k_2}{k_1}\right)}$$

$$\Rightarrow \quad R = \frac{k_1}{k_2}\left[1 + \left(\frac{k_2 x}{k_1}\right)^2\right]^{\frac{3}{2}}$$

PROBLEM 10

A helicopter takes off along the vertical with an acceleration $a = 3\ \text{ms}^{-2}$ and zero initial velocity. In a certain time t_1, the pilot switches off the engine. At the point of take off, the sound dies away in time $t_2 = 30\ \text{s}$. Determine the velocity v of the helicopter at the moment when the engine is switched off; assuming that the velocity of sound, $c = 320\ \text{ms}^{-1}$.

SOLUTION

Since, $u = \text{ms}^{-1}$, $a = 3\ \text{ms}^{-2}$, $c = 320\ \text{ms}^{-1}$ and $t_2 = 30\ \text{s}$ {Given}

Velocity of helicopter at time t_1 is

$$v = 0 + 3t_1$$

$$\Rightarrow \quad v = 3t_1$$

Distance traversed by helicopter in time t_1 is

$$s = 0 + \frac{1}{2} \times 3 \times t_1^2$$

$$\Rightarrow \quad s = \frac{3}{2} t_1^2$$

Now, $t_2 = t_1 + \frac{s}{c}$ {Given}

$$\Rightarrow \quad 30 = t_1 + \frac{3}{2}\frac{t_1^2}{320}$$

$$\Rightarrow \quad 3t_1^2 + 640\ t_1 - 19200 = 0$$

$$\Rightarrow \quad t_1 = \frac{-640 \pm \sqrt{(640)^2 + 4\times 3\times 19200}}{6}$$

$$\Rightarrow \quad t_1 = \frac{-640 \pm 800}{6}$$

As time cannot be negative,

$$\Rightarrow \quad t_1 = \frac{-640 + 800}{6} = \frac{160}{6} = \frac{80}{3}\ \text{s}$$

$$\Rightarrow \quad v = 3t_1 = 3\times\frac{80}{3}$$

$$\Rightarrow \quad v = 80\ \text{ms}^{-1}$$

PROBLEM 11

A steel ball is dropped from a roof of a building. An observer standing in front of a window 1m high notes that the ball takes 0.1 sec to fall from the top to the bottom of the window. The ball continues to fall and makes a complete elastic collision with the ground, so that it rebounds with the same velocity with which it strikes the ground. The ball reappears at the bottom of the window 2 second after passing the bottom of the window on the way down, find the height of the building (Take $g = 10\ \text{ms}^{-2}$)

SOLUTION

Let v_1 and v_2 be the velocities of a ball at the top and bottom of the windows respectively. Let h_1 be the height of the building above the top of the window and h_2 that below the bottom of window, then

$$\text{Height of Building} = h_1 + \text{Height of Window} + h_2$$

$$\Rightarrow \quad \text{Height of Building} = (h_1 + 1 + h_2)\ \text{m}$$

For the path between the top and bottom of window,

We have $h = 1\ \text{m}$, $t = 0.1\ \text{s}$

$$a = g = 10\ \text{ms}^{-2}$$

$$h = v_1 t + \frac{1}{2}gt^2$$

$$\Rightarrow \quad 1 = v_1\times 0.1 + \frac{1}{2}\times(10)\times(0.1)^2$$

$$\Rightarrow \quad v_1 = 9.5\ \text{ms}^{-1}$$

Also from equation $v = u + at$

$$\Rightarrow \quad v_2 = v_1 + 10\times 0.1 = 9.5 + 1 = 10.5\ \text{ms}^{-1}$$

Also, from equation of motion $v^2 - u^2 = 2as$,

$$\Rightarrow \quad v_1^2 = 0 + 2\times 10\times h_1$$

$$\Rightarrow \quad h_1 = \frac{v_1^2}{2\times 10} = \frac{(9.5)^2}{2\times 10} = 4.51\ \text{m}$$

$$\Rightarrow \quad \text{Path above top of Window} = 4.51\ \text{m}$$

For the path below the bottom of window, the ball reappears at the bottom of window to second after passing the bottom on its way down. Hence, it takes a time of 1 s to fall from the bottom of window to the ground and rebounds to the same height in a further time of 1 s.

$$\Rightarrow \quad h_2 = v_2 t + \frac{1}{2}gt^2$$

$$\Rightarrow \quad h_2 = 10.5\times 1 + \frac{1}{2}\times 10\times 1^2$$

$$\Rightarrow \quad h_2 = 10.5 + 5$$

$$\Rightarrow \quad \text{Path below the bottom of window} = 15.5\ \text{m}$$

Hence, Height of the Building is $H = 4.51 + 1 + 15.5$

$$\Rightarrow \quad H = 21.01\ \text{m}$$

PROBLEM 12

A driver having a definite reaction time (i.e., the interval between the perception of a signal to stop and the application of brakes) is capable of stopping his car over a distance of 30 m on seeing a red traffic signal when the speed of the car is 72 kmh^{-1} and over a distance of 10 m when the speed of the car is 36 kmh^{-1}. Find the distance over which he can stop the car if it were running at a speed of 54 kmh^{-1}. Assume that his reaction time and the deceleration of the car remains same in all the three cases.

SOLUTION

Let t_0 is the reaction time, and a is the deceleration

During reaction time, the car travels at constant speed. If u is speed of car, the distance traversed in reaction time t_0 is ut_0, therefore for stopping distance s, the distance traversed with deceleration a is $(s - ut_0)$ and final velocity $= 0$

CASE-1

Initial speed $u_1 = 72\ \text{kmh}^{-1} = 20\ \text{ms}^{-1}$

$$s_1 = 30\ \text{m}$$

$$\Rightarrow \quad 0 = u_1^2 - 2a(30 - u_1 t_o)$$

$$\Rightarrow \quad u_1^2 = 2a(30 - 20u_1 t_o)$$

$$\Rightarrow \quad (20)^2 = 2a(30 - u_1 t_o) \qquad \ldots(1)$$

CASE-2

$$u_2 = 36 \text{ kmh}^{-1} = 10 \text{ ms}^{-1}, \; s_2 = 10 \text{ m}$$

$$\Rightarrow \quad 0 = u_2^2 - 2a(10 - u_2 t_o)$$

$$\Rightarrow \quad u_2^2 = 2a(10 - u_2 t_o)$$

$$\Rightarrow \quad (10)^2 = 2a(10 - 10t_o) \qquad \ldots(2)$$

Solving (1) and (2),

$$\Rightarrow \quad t_0 = 0.5 \text{ s and } a = 10 \text{ ms}^{-2} \qquad \{\text{Retardation}\}$$

CASE-3

$$u_3 = 54 \text{ kmh}^{-1} = 15 \text{ ms}^{-1}$$

$$s_3 = ?$$

$$\Rightarrow \quad 0 = (15)^2 - 2 \times 10 \times (s_3 - 15 \times 0.5)$$

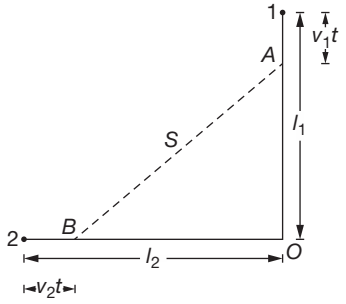
$$\Rightarrow \quad s_3 = 18.75 \text{ m}$$

PROBLEM 13

Two particles A and B move with constant velocity $\vec{v}_1$ and $\vec{v}_2$ along two mutually perpendicular straight lines towards intersection point O. At moment $t = 0$ particles were located at distances l_1 and l_2 respectively from O. How soon will the distance between particles be minimum and what is that minimum distance equal to?

SOLUTION

Let the separation between the particles be minimum at time t. Then



Since $OB = l_2 - v_2 t$ and $OA = l_1 - v_1 t$ and $AB^2 = OB^2 + OA^2$

$$\Rightarrow \quad s^2 = (l_1 - v_1 t)^2 + (l_2 - v_2 t)^2$$

For s to be minimum, $\frac{ds}{dt} = 0$ or $\frac{d}{dt}(s^2) = 0$

$$\Rightarrow \quad 2s\frac{ds}{dt} = 2(l_1 - v_1 t)(-v_1) + 2(l_2 - v_2 t)(-v_2) = 0$$

$$\Rightarrow \quad -l_1 v_1 + v_1^2 t - l_2 v_2 + v_2^2 t = 0$$

$$\Rightarrow \quad t = \frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2}$$

Minimum s is given by

$$s_{\min}^2 = \left[l - v_1 \left(\frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2} \right) \right]^2 + \left[l_2 - v_2 \left(\frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2} \right) \right]^2$$

$$\Rightarrow \quad s_{\min}^2 = \frac{(l_1 v_2 - l_2 v_1)^2}{v_1^2 + v_2^2}$$

$$\Rightarrow \quad s_{\min} = \frac{|l_1 v_2 - l_2 v_1|}{\sqrt{v_1^2 + v_2^2}}$$

PROBLEM 14

Two bodies move in the same straight line at the same instant of time from the same origin. The first body moves with a constant velocity of 40 ms^{-1} and second starts with a constant acceleration of 4 ms^{-2}. Find the time t that elapses before the second catches the first body. Find also the greater distance between them prior to it and the time at which this occurs.

SOLUTION

Let the two bodies meet after a time t. The distance travelled by both is the same

The distance travelled by first body is

$$s_1 = 40 \times t$$

The distance travelled by second body is

$$s_2 = 0 + \frac{1}{2}(4)t^2$$

Since $s_1 = s_2$

$$\Rightarrow \quad 40t = \frac{1}{2}(4)t^2$$

$$\Rightarrow \quad t = 20 \text{ s}$$

The distance between the two bodies goes on increasing as long as the velocity of the second body remains less than $40\ \text{ms}^{-1}$. The distance between them will be greatest prior to their meeting when the velocity of the second body becomes $40\ \text{ms}^{-1}$. Let t_1 be the time when this happens. Then

$$40 = 0 + 4t_1 \qquad \{\because\ v = u + at\}$$

$$\Rightarrow \quad t_1 = 10\ \text{s}$$

So the distance between them will be greatest after $t_1 = 10\ \text{s}$. Let x_1 and x_2 be the distances moved by the two bodies in $t_1 = 10\ \text{s}$, then

$$x_1 = (40 \times 10) + \frac{1}{2}(0)(10)^2$$

$$\Rightarrow \quad x_1 = 400\ \text{m}$$

and $x_2 = 0 + \frac{1}{2}(4)(10)^2$

$$\Rightarrow \quad x_2 = 200\ \text{m}$$

Greatest distance between them is $\Delta x = x_1 - x_2$

$$\Rightarrow \quad \Delta x = 400 - 200 = 200\ \text{m}$$

PROBLEM 15

A motor cycle and a car start from rest at the same place at the same time and travel in the same direction. The cycle accelerates uniformly at $1\ \text{ms}^{-2}$ upto a speed of $36\ \text{kmh}^{-1}$ and the car at $0.5\ \text{ms}^{-2}$ upto a speed of $54\ \text{kmh}^{-1}$. Calculate the time and distance at which the car overtakes the cycle.

SOLUTION

When the car overtakes the motor cycle, the two have travelled the same distance in the same time. Let the time taken be t second while the total distance travelled be x metre

For Motor Cycle: Maximum speed attained is

$$v_1 = 36\ \text{kmh}^{-1} = 10\ \text{ms}^{-1}$$

Its acceleration is $a_1 = 1\ \text{ms}^{-2}$. Let t_1 be the time taken to reach the maximum speed, then using $v = u + at$, we get

$$10 = 0 + (1)(t_1)$$

$$\Rightarrow \quad t_1 = 10\ \text{s}$$

The distance travelled before reaching the maximum speed is

$$x_1 = 0 + \frac{1}{2}(1)t_1^2 = \frac{1}{2}(1)(10)^2 = 50\ \text{m} \qquad \ldots(1)$$

The time during which the motor cyclist moves with constant speed is $(t - 10)\ \text{s}$

The distance travelled at constant speed is

$$x_2 = 10(t - 10) = (10t - 100)\ \text{m} \qquad \ldots(2)$$

Total distance $x = x_1 + x_2$

$$\Rightarrow \quad x = 50 + 10t - 100 = (10t - 50)\ \text{m} \qquad \ldots(3)$$

For Car: Maximum speed attained is

$$v_2 = 54\ \text{kmh}^{-1} = 15\ \text{ms}^{-1}$$

Its acceleration is $a_2 = 0.5\ \text{ms}^{-2}$. Let t_2 be the time taken to reach the maximum speed. Then

$$15 = 0 + (0.5)t_2$$

$$\Rightarrow \quad t_2 = \frac{15}{0.5} = 30\ \text{s}$$

The distance travelled before reaching the maximum speed is

$$x_3 = 0 + \frac{1}{2}(0.5)(30)^2 = 255\ \text{m} \qquad \ldots(4)$$

The time for which the car is moving at constant speed is $(t - 30)\ \text{s}$

The distance travelled at constant speed

$$x_4 = 15(t - 30) = 15t - 450 \qquad \ldots(5)$$

Total distance travelled is $x' = x_3 + x_4$

$$\Rightarrow \quad x' = 225 + 15t - 450 = 15t - 225 \qquad \ldots(6)$$

Equating equations (3) and (6)

$$10t - 50 = 15t - 225$$

$$\Rightarrow \quad 5t = 175$$

$$\Rightarrow \quad t = 35\ \text{s}$$

Substituting this value of t either in equation (3) or in equation (6), we get

$$x = x' = (10)(35) - 50 = 300\ \text{m}$$

PROBLEM 16

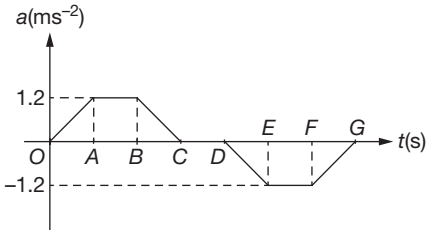
An airport shuttle train travels between two terminals that are 2.5 km apart. To maintain passenger comfort, the acceleration of the train is limited to $\pm 1.2\ \text{ms}^{-2}$ and the jerk, or rate of change of acceleration, is limited to $\pm 0.24\ \text{ms}^{-3}$. If the shuttle has a maximum speed of $30\ \text{kmh}^{-1}$, determine

(a) the shortest time for the shuttle to travel between the two terminals,

(b) the corresponding average velocity of the shuttle.

SOLUTION

(a) Corresponding a-t graph is as shown in figure



In the figure $t_{OA} = t_{BC} = t_{DE} = t_{FG} = \dfrac{1.2}{0.24} = 5$ s

Now let $t_{AB} = t_{EF} = t_1$ and $t_{CD} = t_2$

According to the problem, maximum speed is

$$v_{max} = v_C = 30 \text{ kmhr}^{-1} = \frac{25}{3} \text{ ms}^{-1}$$

Since $v_C =$ Area under a-t graph from O to C,

$$\Rightarrow \quad \frac{25}{3} = \frac{1}{2} \times 1.2 (t_1 + t_1 + 10)$$

$$\Rightarrow \quad t_1 = \frac{35}{18} \text{ s}$$

From O to A, we have, $a = 0.24t$

$$\Rightarrow \quad \int_0^v dv = \int_0^t 0.24t\,dt$$

$$\Rightarrow \quad v = 0.12t^2$$

$$\Rightarrow \quad v_A = (0.12)(5)^2 = 3 \text{ ms}^{-1}$$

Again, $\displaystyle\int_0^{x_{OA}} dx = 0.12 \int_0^5 t^2 dt$

$$\Rightarrow \quad x_{OA} = 5 \text{ m}$$

From A to B, we have

$a = 1.2 \text{ ms}^{-2} = \text{constant}$

$$\Rightarrow \quad v_B = v_A + at_{AB} = 3 + (1.2)\left(\frac{35}{18}\right) = \frac{16}{3} \text{ ms}^{-1}$$

$$x_{AB} = v_A t_{AB} + \frac{1}{2} a (t_{AB})^2$$

$$\Rightarrow \quad x_{AB} = (3)\left(\frac{35}{18}\right) + \frac{1}{2}(1.2)\left(\frac{35}{18}\right)^2 \cong 8.1 \text{ m}$$

From B to C, we have $a = 1.2 - 0.24t$

$$\Rightarrow \quad \int_{16/3}^v dv = \int_0^t (1.2 - 0.24t)\,dt$$

$$\Rightarrow \quad v = \frac{16}{3} + 1.2t - 0.12t^2$$

Also $\displaystyle\int_0^{x_{BC}} dx = \int_0^5 \left(\frac{16}{3} + 1.2t - 0.12t^2\right) dt$

$$\Rightarrow \quad x_{BC} = \frac{80}{3} + 0.6(5)^2 - 0.04(5)^3 = 36.67 \text{ m}$$

$$\Rightarrow \quad x_{OC} = x_{DG} = (5 + 8.1 + 36.67) = 49.77 \text{ m}$$

$$\Rightarrow \quad x_{OC} = x_{DG} \cong 50 \text{ m}$$

$$\Rightarrow \quad x_{OC} + x_{DG} = 100 \text{ m}$$

The remaining distance $(2500 - 100)$ m or 2400 m must have been travelled with a constant speed of $\dfrac{25}{3}$ ms^{-1}. Hence

$$t_2 = \frac{2400}{25/3} = 288 \text{ s}$$

(a) Total time

$$T = 2\left(5 + \frac{35}{18} + 5\right) + 288 = 312 \text{ s} = 5.2 \text{ minute}$$

(b) Average velocity,

$$v_{av} = \frac{2.5}{\frac{5.2}{60}} \text{ kmhr}^{-1} = 28.8 \text{ kmhr}^{-1}$$

PROBLEM 17

An open lift is moving upwards with velocity 10 ms^{-1}. It has an upward acceleration of 2 ms^{-2}. A ball is projected upwards with velocity 20 ms^{-1} relative to ground. Find

(a) time when ball again meets the lift.
(b) displacement of lift and ball at that instant.
(c) distance travelled by the ball upto that instant.

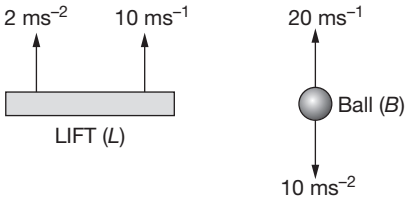
Take $g = 10$ ms^{-2}

SOLUTION

(a) At the time when ball again meets the lift,

$$s_L = s_B$$

$$\Rightarrow \quad 10t + \frac{1}{2} \times 2 \times t^2 = 20t - \frac{1}{2} \times 10t^2$$



Solving, we get

$$t = 0 \text{ and } t = \frac{5}{3} \text{ s}$$

So, ball will again meet the lift after $\frac{5}{3}$ s

(b) At this instant

$$s_L = s_B = 10\left(\frac{5}{3}\right) + \frac{1}{2}(2)\left(\frac{5}{3}\right)^2 = \frac{175}{9} \text{ m} = 19.4 \text{ m}$$

(c) For the ball u and g are antiparallel. Therefore we will first find t_0, the time when its velocity becomes zero.

Since $v = u + at$

$$\Rightarrow \quad 0 = u + (-g)t_0$$

$$\Rightarrow \quad t_0 = \frac{u}{g} = \frac{20}{10} = 2 \text{ s}$$

Since $t\left(= \frac{5}{3} \text{ s}\right) < t_0$, so the distance and displacement are equal i.e.,

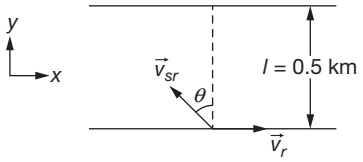
$$d = 19.4 \text{ m}$$

PROBLEM 18

A man can row a boat in still water at 3 kmh^{-1}. He can walk at a speed of 5 kmh^{-1} on the shore. The water in the river flows at 2 kmh^{-1}. If the man rows across the river and walks along the shore to reach the opposite point on the river bank, find the direction in which he should row the boat so that he could reach the opposite shore in the least possible time. Also calculate this time. The width of the river is 500 m.

SOLUTION

Suppose the boatman rows with velocity $\vec{v}_{sr}$ in the direction shown in figure



Given, $v_{sr} = 3 \text{ kmhr}^{-1}$, $v_r = 2 \text{ kmhr}^{-1}$

and v_w = walking speed $= 5 \text{ kmhr}^{-1}$

$$\vec{v}_s = \vec{v}_{sr} + \vec{v}_r$$

$$\Rightarrow \quad (v_s)_x = v_r - v_{sr}\sin\theta = 2 - 3\sin\theta$$

and $(v_s)_y = v_{sr}\cos\theta = 3\cos\theta$

Time taken to reach the other side

$$t_1 = \frac{l}{(v_s)_y} = \frac{0.5}{3\cos\theta} = \frac{1}{6\cos\theta}$$

Horizontal drift x is given by

$$x = (v_s)_x t_1 = (2 - 3\sin\theta)\frac{1}{6\cos\theta} = \frac{1}{3\cos\theta} - \frac{\tan\theta}{2}$$

Time to travel this distance by walking

$$t_2 = \frac{x}{v_w} = \frac{1}{15\cos\theta} - \frac{\tan\theta}{10} \qquad \ldots(1)$$

$$\text{Total time } t = t_1 + t_2 = \frac{1}{10}\left(\frac{7}{3\cos\theta} - \tan\theta\right) \qquad \ldots(2)$$

For time to be minimum, we have $\frac{dt}{d\theta} = 0$

$$\Rightarrow \quad \frac{7}{3}\sec\theta\tan\theta - \sec^2\theta = 0$$

$$\Rightarrow \quad \sin\theta = \frac{3}{7}$$

From equation (2), we get

$$t_{\min} = \frac{1}{10}\left(\frac{7}{3} \times \frac{7}{\sqrt{40}} - \frac{3}{\sqrt{40}}\right)$$

$$\Rightarrow \quad t_{\min} = 0.21 \text{ hour} = 756 \text{ s}$$

PROBLEM 19

The velocity of water current in a river changes with distance along the perpendicular to the river according to the function

$$u = \begin{cases} \frac{2u_0}{d}y & \text{for} \quad 0 \le y \le \frac{d}{2} \\ \frac{2u_0}{d}(d - y) & \text{for} \quad \frac{d}{2} \le y \le d \end{cases}$$

where u_0 is velocity at the mid-point of the river and d is width of the river. A boat travels from a point O on one bank of the river to the opposite bank and its steering angle is adjusted to keep its relative velocity perpendicular to the river current. Calculate the time in which the boat will reach the other bank. The velocity of the boat in still water is u_0.

SOLUTION

Denoting the river current by R and the boat by B. Let the boat steer making an angle θ with the river flow. Then

$$\vec{u}_R = u\hat{i}\ ,\ \vec{u}_B = u_0\left(\cos\theta\hat{i} + \sin\theta\hat{i}\right)$$

$$\vec{u}_{BR} = \vec{u}_B - \vec{u}_R = \left(u_0\cos\theta - u\right)\hat{i} + u_0\sin\theta\hat{j}$$

As the boat is steered perpendicular to flow, so we have

$$\left(u_{BR}\right)_x = 0$$

$$\Rightarrow \quad u_0\cos\theta - u = 0$$

$$\Rightarrow \quad \cos\theta = \frac{u}{u_0}$$

$$\Rightarrow \quad \left|\vec{u}_{BR}\right| = u_0\sin\theta = \mu_0\sqrt{1-\frac{u^2}{u_0^2}} = \sqrt{u_0^2 - u^2}$$

$$\Rightarrow \quad \frac{dy}{dt} = \sqrt{u_0^2 - u^2}$$

$$\Rightarrow \quad \int dt = \int \frac{dy}{\sqrt{u_0^2 - u^2}}$$

$$\Rightarrow \quad t = \int_0^{\frac{d}{2}} \frac{dy}{\sqrt{u_0^2 - \frac{4u_0^2}{d^2}y^2}} + \int_{\frac{d}{2}}^{d} \frac{dy}{\sqrt{u_0^2 - \frac{4u_0^2}{d^2}(d-y)^2}}$$

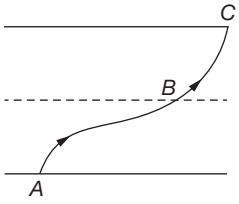
$$\Rightarrow \quad t = \frac{d}{2u_0}\int_0^{\frac{d}{2}} \frac{dy}{\sqrt{\left(\frac{d}{2}\right)^2 - y^2}} + \frac{d}{2u_0}\int_{\frac{d}{2}}^{d} \frac{dy}{\sqrt{\left(\frac{d}{2}\right)^2 - (d-y)^2}}$$

$$\Rightarrow \quad t = \frac{d}{2u_0}\left[\sin^{-1}\left(\frac{2y}{d}\right)\right]\Bigg|_0^{\frac{d}{2}} + \frac{d}{2u_0}\left[\sin^{-1}\left(\frac{2(d-y)}{d}\right)\right]\Bigg|_{\frac{d}{2}}^{d}$$

$$\Rightarrow \quad t = \frac{d}{2u_0}\left(\frac{\pi}{2} - 0\right) + \frac{d}{2u_0}\left[0 - \left(\frac{\pi}{2}\right)\right] = \frac{\pi d}{2u_0}$$

PROBLEM 20

The current velocity of a river grows in proportion to the distance from its bank and reaches the maximum value v_0 in the middle. Near the banks the velocity is zero. A boat is moving along the river in such a manner that it is always perpendicular to current and the speed of the boat in still water is u. Find the distance through which the boat crossing the river will be carried away by the current if the width of river is l. Also determine the trajectory of boat.



SOLUTION

Let the river flow or the current flow be along x-axis. At any instant t, let the boat be at a distance y from the bank, then

$$v_x = k_y \qquad \ldots(1)$$

Also, we have been given in the problem, that

$$v_x = v_0 \text{ when } y = \frac{l}{2}$$

From (1), we get

$$v_0 = k\left(\frac{l}{2}\right)$$

$$\Rightarrow \quad k = \frac{2v_0}{l} \qquad \ldots(2)$$

Since $y = ut$ $\qquad \ldots(3)$

So, equation (1) becomes

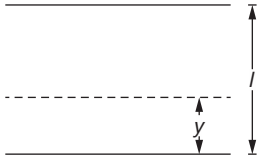
$$v_x = \left(\frac{2v_0}{l}\right)ut$$

$$\Rightarrow \quad \frac{dv_x}{dt} = a_x = \frac{2v_0u}{l} = \text{constant}$$

Since $x = u_xt + \frac{1}{2}a_xt^2$

$$\Rightarrow \quad x = 0 + \frac{1}{2}\left(\frac{2v_0u}{l}\right)t^2$$

$$\Rightarrow \quad x = \left(\frac{v_0u}{l}\right)t^2 \qquad \ldots(4)$$



From (3), $t = \frac{y}{u}$

$$\Rightarrow \quad x = \left(\frac{v_0 u}{lu^2}\right) y^2$$

$$\Rightarrow \quad x = \left(\frac{v_0}{lu}\right) y^2$$

$$\Rightarrow \quad y^2 = \left(\frac{lu}{v_0}\right) x \qquad \ldots(5)$$

This happens to be the equation of a parabola

Now, when $y = \frac{l}{2}$, we have, from (5)

$$\frac{l^2}{4} = \left(\frac{lu}{v_0}\right) x$$

$$\Rightarrow \quad x = \frac{lv_0}{4u}$$

Hence total drift is

$$\text{Drift} = 2x = \frac{2lv_0}{4u}$$

$$\Rightarrow \quad \text{Drift} = \frac{lv_0}{2u}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A particle has an initial velocity of 9 ms^{-1} due east and a constant acceleration of 2 ms^{-2} due west. The distance covered by the particle in the fifth second of its motion is

(A) 0 m (B) 0.5 m
(C) 2 m (D) 20 m

2. The velocity of a car travelling on a straight road is given by the equation $v = 9 + 8t - t^2$ where v is in metre per second and t in second. The instantaneous acceleration when $t = 5$ s is

(A) 2 ms^{-2} (B) 1 ms^{-2}
(C) –1 ms^{-2} (D) –2 ms^{-2}

3. A racing car travelling at a constant speed has to pass through a horizontal turn where the radius of curvature of the road is 200 m. If the normal acceleration of the car cannot exceed 0.8 g where $g = 10$ ms^{-2}, the maximum speed of the car without sliding can be

(A) 36 kmh^{-1} (B) 72 kmh^{-1}
(C) 144 kmh^{-1} (D) 174 kmh^{-1}

4. Ball A is dropped from the top of a tower of height H. At the same instant ball B is thrown vertically upwards from the ground. When the balls collide, they are moving in opposite directions and the speed of A is twice the speed of B. The height from the ground where the collision happens is

(A) $\frac{H}{4}$ (B) $\frac{H}{3}$
(C) $\frac{2H}{5}$ (D) $\frac{2H}{3}$

5. Wind is blowing at a harbour with a speed of 72 kmh^{-1} and the flag on the mast of a boat anchored at harbour flutters along the North-East direction. If the boat starts moving at a speed of 51 kmh^{-1} due North, the direction of the flag is (in approximation)

(A) towards east (B) towards west
(C) towards north (D) towards south

6. Two bodies begin to fall freely from the same height but the second falls T second after the first. The time (after which the first body begins to fall) when the distance between the bodies equals L is

(A) $\frac{T}{2}$ (B) $\frac{T}{2} + \frac{L}{gT}$
(C) $\frac{L}{gT}$ (D) $T + \frac{2L}{gT}$

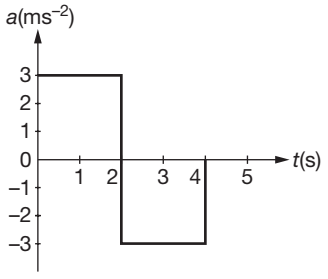
7. A particle is dropped from point A at a certain height from ground. It falls freely and passes through three points B, C and D with $BC = CD$. The time taken by the particle to move from B to C is 2 seconds and from C to D is 1 second. The time taken to move from A to B is

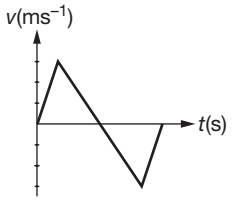
(A) 0.25 s (B) 0.5 s
(C) 0.75 s (D) 1.5 s

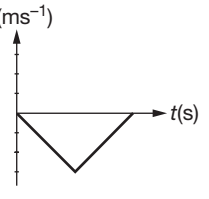
8. A juggler maintains four balls in motion, making each of them to rise a height of 20 m from his hand. The time interval that he should maintain, for the proper distance between them $g = 10$ ms^{-2} is

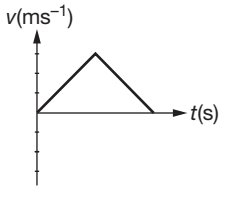
(A) 0.5 s (B) 1 s
(C) 2 s (D) 3 s

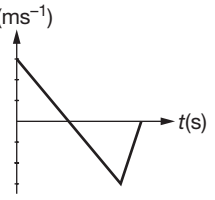
9. A particle starts from rest at time $t = 0$ and undergoes acceleration a as shown. The velocity as function of time during the interval 0 to 4 second is indicated in



(A)


(B)


(C)


(D)


10. The slopes of the windscreen of two motor cars are $\beta_1 = 30°$ and $\beta_2 = 15°$ respectively. The cars have velocities v_1 and v_2 in the horizontal direction. If the hailstones appear to the drivers to be bounced by the windscreen of their respective cars in the vertical direction then $\frac{v_1}{v_2}$ (assuming that hailstones were falling on the cars vertically) is

(A) 3 (B) 1

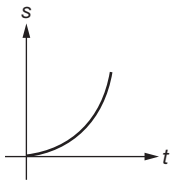
(C) $\frac{1}{3}$ (D) $\frac{1}{9}$

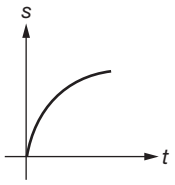
11. A person walks up a stationary escalator in time t_1. If he remains stationary on the escalator, then he reaches up in time t_2. The time it would take him to walk up the moving escalator is

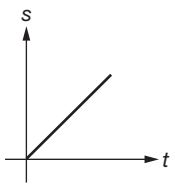
(A) $\frac{t_1 + t_2}{2}$ (B) $t_1 + t_2$

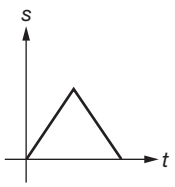
(C) $\sqrt{t_1 t_2}$ (D) $\frac{t_1 t_2}{t_1 + t_2}$

12. One stone is dropped from a tower from rest and simultaneously another stone is projected vertically upwards from the tower with some initial velocity. The graph of the distance, s between the two stones varies with time (t) as (before either stone hits the ground)

(A) 

(B) 

(C) 

(D) 

13. Wind is blowing from the south at $10\ \text{ms}^{-1}$ but to a cyclist it appears to be blowing from the east at $10\ \text{ms}^{-1}$. The cyclist has a velocity

(A) $10\hat{i} - 10\hat{j}$ (B) $10\hat{i} + 10\hat{j}$

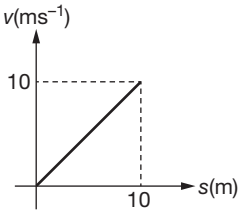
(C) $-10\hat{i} + 10\hat{j}$ (D) $-10\hat{i} - 10\hat{j}$

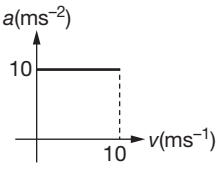
14. If a particle takes t second less and acquires a velocity of $v\ \text{ms}^{-1}$ more in falling through the same distance on two planets where the accelerations due to gravity are $2g$ and $8g$ respectively then

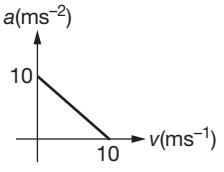
(A) $v = 4gt$ (B) $v = 5gt$

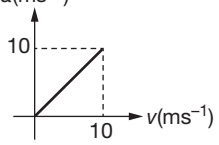
(C) $v = 2gt$ (D) $v = 16gt$

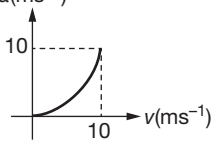
15. Velocity versus displacement graph of a particle moving in a straight line is shown in figure. Corresponding acceleration versus velocity graph will be



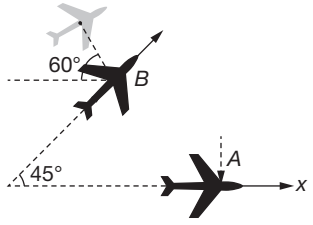
(A) 

(B) 

(C) 

(D) 

16. Passengers in the jet transport A flying east at a speed of $800\ \text{kmh}^{-1}$ observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the $45°$ north east direction, plane B appears to the passengers in A to be moving away from the transport at the $60°$ angle as shown. The true velocity of B is



(A) $586\ \text{kmh}^{-1}$ (B) $400\sqrt{2}\ \text{kmh}^{-1}$

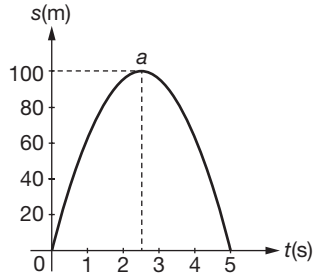
(C) $717\ \text{kmh}^{-1}$ (D) $400\ \text{kmh}^{-1}$

17. A body falling freely from a tower of height h covers a distance of $\frac{7}{16}h$ during the last second of its motion. Then the height of tower is (Take $g = 10\ \text{ms}^{-2}$)

(A) 60 m (B) 70 m

(C) 80 m (D) 90 m

18. The figure shows the displacement-time graph (a parabola) of a body. This graph indicates that the initial velocity, in ms^{-1} and acceleration, in ms^{-2} respectively are



(A) 80, 40 (B) 40, 80
(C) 80, 32 (D) 32, 16

19. Acceleration of a particle is a for a time t. It is followed immediately by a retardation of a for time $\frac{t}{2}$. Consider this as one cycle. If initial velocity of particle is zero, then the displacement of the particle after n such cycles in succession is

(A) $\frac{n(3n+4)}{8}at^2$ (B) $\frac{n(n+1)}{2}at^2$
(C) $\frac{(n^2+n+1)}{4}at^2$ (D) nat^2

20. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at the lowest position and has a speed u. The magnitude of the change in velocity as it reaches a position where the string is horizontal is

(A) $\sqrt{u^2-2gL}$ (B) $\sqrt{2gL}$
(C) $\sqrt{u^2-gL}$ (D) $\sqrt{2\left(u^2-gL\right)}$

21. For a particle moving rectilinearly the displacement x depends on time t as $x=at^3+bt^2+ct+d$. The ratio of its initial acceleration to its initial velocity depends
(A) only on a and b (B) only on b and c
(C) only on a and c (D) only on a

22. A self-propelled vehicle of mass m whose engine delivers constant power P has an acceleration $a=\frac{P}{mv}$ (assume that there is no friction). In order to increase its velocity from v_1 to v_2, the distance it has to travel will be

(A) $\frac{m}{3P}\left(v_2-v_1\right)$ (B) $\frac{m}{3P}\left(v_2^2-v_1^2\right)$
(C) $\frac{m}{3P}\left(v_2^3-v_1^3\right)$ (D) $\frac{3P}{m}\left(v_2^2-v_1^2\right)$

23. The motion of a body falling from rest in a resisting medium is described by the equation $\frac{dv}{dt}=a-bv$ where a and b are constants. The velocity at any time t is

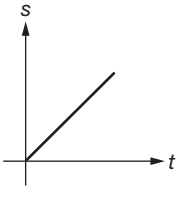
(A) $v_t=\frac{a}{b}\left(1-e^{-bt}\right)$ (B) $v_t=\frac{b}{a}e^{-bt}$
(C) $v_t=\frac{a}{b}\left(1+e^{-bt}\right)$ (D) $v_t=\frac{b}{a}e^{bt}$

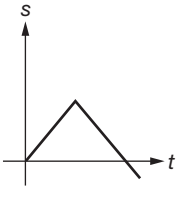
24. A parachutist drops freely from an airplane for 10 s before the parachute opens. He then descends with a uniform retardation of 2.5 ms^{-2}. If he bails out of the plane at a height of 2495 m and g is 10 ms^{-2}, his velocity on reaching the ground will be
(A) 5 ms^{-1} (B) 10 ms^{-1}
(C) 15 ms^{-1} (D) 20 ms^{-1}

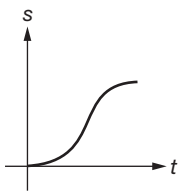
25. The nucleus of a helium atom travels along the axis of a straight hollow tube 4 m long. The tube is part of a particle accelerator. If the particle enters the tube with a speed of 1000 ms^{-1} and leaves at 9000 ms^{-1}, and assuming that the acceleration is uniform, the time the particle remains inside the tube is
(A) 4×10^{-3} s (B) 8×10^{-4} s
(C) 1×10^{-4} s (D) 8×10^{-3} s

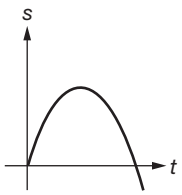
26. Two cars A and B start off to race with velocities 8 ms^{-1} and 4 ms^{-1} and travel in straight line with uniform accelerations 2 ms^{-2} and 4 ms^{-2} respectively. If they reach the final point at the same instant, then the length of the path is
(A) 24 m (B) 32 m
(C) 48 m (D) 16 m

27. A particle is moving in x-y plane with $y=\frac{x}{2}$ and $v_x=4-2t$. The displacement versus time graph of the particle would be

(A)


(B)


(C) 

(D) 

28. A ball is thrown vertically upwards. It was observed at a height h twice with a time interval Δt. The initial velocity of the ball is

(A) $\sqrt{8gh+g^2(\Delta t)^2}$ (B) $\sqrt{8gh+\left(\frac{g\Delta t}{2}\right)^2}$

(C) $\frac{1}{2}\sqrt{8gh+g^2(\Delta t)^2}$ (D) $\sqrt{8gh+4g^2(\Delta t)^2}$

29. A point moves in x-y plane according to the law $x=5\sin(6t)$ and $y=5(1-\cos(6t))$, where x and y are in metre. The distance traversed by the particle in $t=4$ s is

(A) 24 m (B) 48 m

(C) 96 m (D) 120 m

30. A particle moving with uniform acceleration along a straight line covers distances a and b in successive intervals of p and q second. The acceleration of the particle is

(A) $\frac{pq(p+q)}{2(bp-aq)}$ (B) $\frac{2(aq-bp)}{pq(p+q)}$

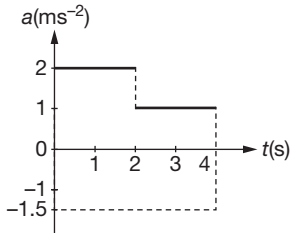
(C) $\frac{2(ap-bq)}{pq(p+q)}$ (D) $\frac{2(bp-aq)}{pq(p+q)}$

31. The motion of a particle is defined by $x=a\cos(\omega t)$ and $y=a\sin(\omega t)$. The acceleration of the particle is

(A) $a\omega$ (B) $a^2\omega$

(C) $a\omega^2$ (D) $\frac{a\omega^2}{2}$

32. The figure shows the acceleration versus time graph of a train. If it starts from rest, the distance it travels before it comes to rest is



(A) 30 metre (B) 26 metre

(C) 13 metre (D) 40 metre

33. A car A is going north east at 80 kmh^{-1} and another car B is going south east with a velocity of 60 kmh^{-1}. The velocity of A relative to B makes an angle with the north equal to

(A) $\tan^{-1}\left(\frac{2}{7}\right)$ (B) $\tan^{-1}\left(\frac{7}{2}\right)$

(C) $\tan^{-1}(7)$ (D) $\tan^{-1}\left(\frac{1}{7}\right)$

34. A particle is moving along a circular path of radius 3 metre in such a way that the distance travelled measured along the circumference is given by $s=\frac{t^2}{2}+\frac{t^3}{3}$. The acceleration of the particle when $t=2$ second is

(A) 1.3 ms^{-2} (B) 3 ms^{-2}

(C) 10 ms^{-2} (D) 13 ms^{-2}

35. A lead ball is dropped into a lake from the diving board 5 m above the water. In hits the water with a certain velocity and then sinks to the bottom with this same constant velocity. It reaches the bottom 5 s after it is dropped. The depth of the lake is $\left(\text{take } g=10\ \text{ms}^{-2}\right)$

(A) 10 m (B) 20 m

(C) 25 m (D) 40 m

36. A balloon starts rising from the ground with an acceleration of 1.25 ms^{-2}. After 8 s, a stone is released from the balloon. The stone will

(A) cover a distance of 40 m

(B) have a displacement of 50 m

(C) reach the ground in 4 s

(D) begin to move down after being released

37. The velocity acquired by a body when it falls through a height h is v. If it further falls through a height x $(x \ll h)$, the increase in velocity is approximately

(A) $\frac{vx}{2h}$ (B) $\frac{2v}{xh}$

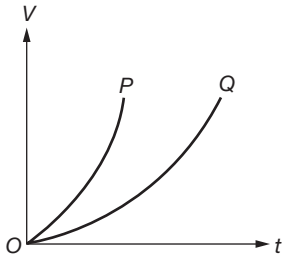
(C) $\frac{2vx}{h}$ (D) $\frac{v}{2xh}$

38. An armoured car 2 m long and 3 m wide is moving at 10 ms^{-1} when a bullet hits it in a direction making an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the length of the car as seen

by a stationary observer. The bullet enters one edge of the car at the corner and passes out at the diagonally opposite corner. Neglecting any interaction between the car and the bullet, the time for the bullet to cross the car is

(A) 0.20 s (B) 0.15 s
(C) 0.10 s (D) 0.50 s

39. A time-velocity graph of two vehicles P and Q starting from rest at the same time is given in the figure. The statement that can be deduced correctly from the graph is



(A) velocity of Q is greater than that of P
(B) acceleration of P is increasing at a slower rate than that of Q
(C) acceleration of Q is greater than that of P
(D) acceleration of P is greater than that of Q

40. A stone takes time t to fall through a height h. The increment in time when it falls further through a distance x $(x \ll h)$ is

(A) $\frac{xth}{2}$ (B) $\frac{xt}{2}$
(C) $\frac{xt}{2h}$ (D) $\frac{2h}{xt}$

41. The speed of an aeroplane at the instant it lands on a runway is $60\ \text{ms}^{-1}$. If the deceleration of the aeroplane is given as $a = -0.6 - 0.001\,v^2$, the distance that it covers on the runway before coming to a stop is

(A) 98 metre (B) 450 metre
(C) 973 metre (D) 1800 metre

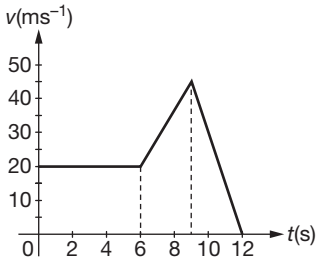
42. In a 100 metre race, a runner accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance at that velocity. If he finishes the race in 10.4 second, then his maximum velocity was

(A) $10\ \text{ms}^{-1}$ (B) $15\ \text{ms}^{-1}$
(C) $20\ \text{ms}^{-1}$ (D) $25\ \text{ms}^{-1}$

43. The position of a particle is given by $\vec{r} = a\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j} + bt\hat{k}$ where $\omega = \frac{2\pi}{T}$ and T is time period for one revolution of the particle following a helical path. The distance moved by the particle in one full turn of the helix is

(A) $\frac{4\pi}{\omega}\sqrt{a^2 + b^2\omega^2}$ (B) $\frac{2\pi}{\omega}\sqrt{a^2\omega^2 + b^2}$
(C) $\frac{2\pi}{\omega}\sqrt{a^2 + b^2\omega^2}$ (D) $\frac{4\pi}{\omega}\sqrt{a^2\omega^2 + b^2}$

44. The graph in the figure shows the velocity of a body plotted as a function of time. The distance covered by the body in the first 12 s is

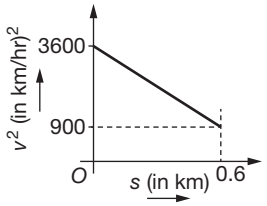


(A) 360 m (B) 240 m
(C) 285 m (D) 500 m

45. A horizontal wind is blowing with a velocity v towards north-east. A man starts running towards north with acceleration a. The time after which man will feel the wind blowing towards east is

(A) $\frac{v}{a}$ (B) $\frac{\sqrt{2}v}{a}$
(C) $\frac{v}{\sqrt{2}a}$ (D) $\frac{2v}{a}$

46. A graph between the square of the velocity of a particle and the distance s moved by the particle is shown in the figure. The acceleration of the particle in kilometre per hour square is



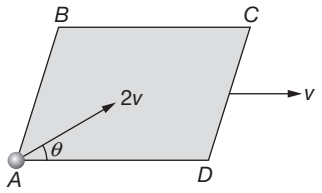
(A) 2250 (B) 225
(C) –2250 (D) –225

47. The speed of a body moving in a straight line changes from $25\ \text{ms}^{-1}$ to $10\ \text{ms}^{-1}$ in 3 s at a constant rate.
(A) during this time it travels 52.5 metre .
(B) its acceleration is $5\ \text{ms}^{-2}$.
(C) its acceleration is greatest at the beginning because it is going fastest at that time.
(D) the distance travelled during an interval of 3 second cannot be calculated from the given data.

48. A car covers the first half of the distance between two places at a speed of $40\ \text{kmh}^{-1}$ and the second half at $60\ \text{kmh}^{-1}$. The average speed of the car is
(A) $50\ \text{kmh}^{-1}$ (B) $42\ \text{kmh}^{-1}$
(C) $35\ \text{kmh}^{-1}$ (D) $48\ \text{kmh}^{-1}$

49. Rain, pouring down at an angle α with the vertical has a speed of $10\ \text{ms}^{-1}$. A girl runs against the rain with a speed of $8\ \text{ms}^{-1}$ and sees that the rain makes an angle β with the vertical, then relation between α and β is
(A) $\tan\alpha = \dfrac{8+10\sin\beta}{10\cos\beta}$ (B) $\tan\beta = \dfrac{8+10\sin\alpha}{10\cos\alpha}$
(C) $\tan\alpha = \tan\beta$ (D) $\tan\alpha = \cot\beta$

50. A smooth square platform $ABCD$ is moving towards right with a uniform speed v. A particle is projected from A with speed $2v$ making an angle θ with AD so that it strikes the point B. Then θ equals



(A) 30° (B) 45°
(C) 60° (D) 90°

51. An express elevator can accelerate or decelerate with values whose magnitudes are limited to $0.4g$. The elevator attains a maximum vertical speed of 400 metre per minute . The minimum time required by the elevator to start from rest from the 10th floor and to stop at the 30th floor, a distance 100 m apart is
(A) 1.67 s (B) 16.7 s
(C) 167 s (D) 1670 s

52. A swimmer crosses a flowing stream of width d to and fro in time t_1. The time taken to cover the same distance up and down the stream is t_2. If t_3 is the time the swimmer would take to swim a distance $2d$ in still water, then
(A) $t_3 = t_1 + t_2$ (B) $t_3^2 = t_1 t_2$
(C) $t_2^2 = t_1 t_3$ (D) $t_1^2 = t_2 t_3$

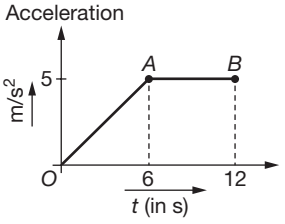
53. A bullet loses $\dfrac{1}{20}$ of its velocity in passing through a plank. The least number of planks required to stop the bullet is
(A) 10 (B) 11
(C) 12 (D) 23

54. A motorboat going down stream overcame a raft at a point A. 60 minute later it turned back and after some time passed the raft at a distance of 6 km from the point A. Assuming the duty of the engine to be constant, the flow velocity is
(A) $3\ \text{kmh}^{-1}$ (B) $4\ \text{kmh}^{-1}$
(C) $5\ \text{kmh}^{-1}$ (D) $6\ \text{kmh}^{-1}$

55. A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of $2\ \text{ms}^{-1}$. The angle α with the vertical at which the wind must screen be placed so that the rain drops, falling vertically downwards with velocity $6\ \text{ms}^{-1}$, strike the wind screen normally is
(A) $\sin^{-1}\left(\dfrac{1}{3}\right)$ (B) $\cos^{-1}(3)$
(C) $\tan^{-1}\left(\dfrac{1}{3}\right)$ (D) $\tan^{-1}(3)$

56. A train travelling at $72\ \text{kmh}^{-1}$ is checked by track repairs. It retards uniformly for 200 m covering next 400 m at constant speed and accelerates to $72\ \text{kmh}^{-1}$ in a further distance of 600 m. If the time at constant lower speed is equal to the sum of the times taken in retarding and accelerating, the total time taken is
(A) 140 s (B) 160 s
(C) 120 s (D) 100 s

57. For an airplane to take-off it accelerates according to the graph shown and takes 12 s to take-off from the rest position. The distance travelled by the airplane is



(A) 21 m (B) 210 m
(C) 2100 m (D) 120 m

58. A body with constant acceleration travels 2 metre in the first 2 second and 2.2 m in the next 4 second. The velocity at the end of the seventh second from the start shall be

(A) $0.1\ \text{ms}^{-1}$ (B) $0.2\ \text{ms}^{-1}$

(C) $0.5\ \text{ms}^{-1}$ (D) $1\ \text{ms}^{-1}$

59. A particle is released from rest from a tower of height $3h$. The ratio of times to fall equal heights h, i.e., $t_1 : t_2 : t_3$ is

(A) $3:2:1$

(B) $\sqrt{3}:\sqrt{2}:1$

(C) $1:(\sqrt{2}-1):(\sqrt{3}-\sqrt{2})$

(D) $9:4:1$

60. Two particles A and B are thrown up simultaneously from the edge of a cliff with initial speeds v and $2v$. Assuming that the particle A comes to rest immediately after striking the ground, the variation in relative position of the particle B with respect to the particle A with time, till both the stones strike the ground is plotted. This variation plot is

(A) only linear
(B) only parabolic
(C) first parabolic then linear
(D) first linear then parabolic

61. A dog is chasing a cat who is running along a straight line at constant speed u. The dog moves with a constant speed v, always heading towards the cat. Initially i.e. at $t = 0$, the velocities of dog and cat are perpendicular and the initial perpendicular distance between them is l. The dog catches the cat at

(A) $t = \dfrac{lv}{v^2 - u^2}$ for $v > u$

(B) $t = \dfrac{lv}{u^2 - v^2}$ for $u > v$

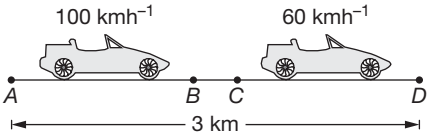
(C) $t = \dfrac{lv^2}{\sqrt{v^2 - u^2}}$ for $v > u$

(D) $t = \dfrac{lv^2}{\sqrt{u^2 - v^2}}$ for $u > v$

62. The acceleration is constant when the relationship between the

(A) position coordinate s and the square of the velocity v is linear
(B) position coordinate s and the velocity v is linear
(C) position coordinate and the reciprocal of the velocity v is linear
(D) square of the position coordinate s and the velocity v is linear

63. In travelling a distance of 3 kilometre between points A and D, a car is driven at $100\ \text{kmh}^{-1}$ from A to B for t second and at $60\ \text{kmh}^{-1}$ from C to D for t second. If the brakes are applied for 4 second between B and C to give the car a uniform deceleration, the value of t is



(A) 75.5 second (B) 45.5 second
(C) 65.5 second (D) 56.5 second

64. A car is travelling on a straight road. The maximum velocity the car can attain is $24\ \text{ms}^{-1}$. The maximum acceleration and deceleration it can attain are $1\ \text{ms}^{-2}$ and $4\ \text{ms}^{-2}$ respectively. The shortest time the car takes to start from rest and come to rest in a distance of 200 metre is

(A) 22.4 second (B) 33.6 second

(C) 11.2 second (D) 5.6 second

65. A person walks up a stalled escalator in 90 s. When standing on the same escalator, now moving, he is carried in 60 s. The time it would take him to walk up the moving escalator will be

(A) 36 s (B) 18 s

(C) 72 s (D) 27 s

66. Two particles, A and B move with constant velocities $\vec{v}_A$ and $\vec{v}_B$. Initially their radius vectors are $\vec{r}_A$ and $\vec{r}_B$. For the particles to collide the four vectors must be interrelated as

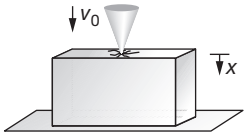
(A) $\vec{v}_A - \vec{v}_B = \vec{r}_A - \vec{r}_B$

(B) $\vec{v}_A = \vec{v}_B$ and $\vec{r}_A = \vec{r}_B$

(C) $\dfrac{\vec{v}_A + \vec{v}_B}{|\vec{v}_A + \vec{v}_B|} = \dfrac{\vec{r}_A + \vec{r}_B}{|\vec{r}_A + \vec{r}_B|}$

(D) $\dfrac{\vec{v}_B - \vec{v}_A}{|\vec{v}_B - \vec{v}_A|} = \dfrac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$

67. The cone falling with a speed v_0 strikes and penetrates the block of packing material. The acceleration of the cone after impact is $a = g - cx^2$, where c is a positive constant and x is the penetration distance. If the maximum penetration depth is x_m. Then c equals



(A) $\frac{2gx_m + v_0^2}{x_m^2}$ (B) $\frac{2gx_m - v_0^2}{x_m^2}$

(C) $\frac{6gx_m - 3v_0^2}{2x_m^3}$ (D) $\frac{6gx_m + 3v_0^2}{2x_m^3}$

68. Water drops fall at regular intervals from a roof. At an instant when a drop is about to leave the roof, the separations between 3 successive drops below the roof are in the ratio

(A) 1: 2 : 3 (B) 1 : 4 : 9

(C) 1 : 3 : 5 (D) 1 : 5 : 13

69. A particle has an initial velocity $\vec{u} = \left(6\hat{i} + 8\hat{j}\right)$ ms^{-1} and an acceleration of $\vec{a} = \left(0.8\hat{i} + 0.6\hat{j}\right)$ ms^{-2}. Its speed after 10 s is

(A) 20 ms^{-1} (B) 14 ms^{-1}

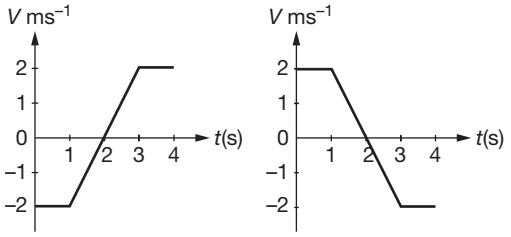
(C) $14\sqrt{2}$ ms^{-1} (D) 17 ms^{-1}

70. A car moves rectilinearly for 6 s with a velocity that varies with time as $|t-3|$ ms^{-1} where t is in second. The total distance moved by the car is

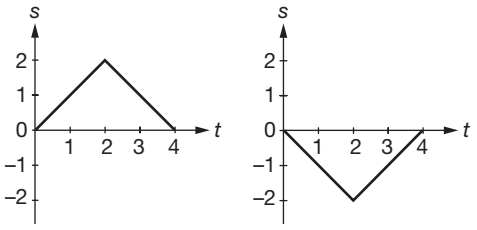
(A) 3 m (B) 6 m

(C) 9 m (D) 12 m

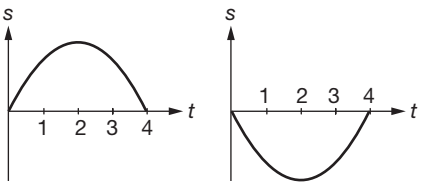
71. The V-t curves for two different particle motions are shown. The corresponding displacement time curves, taking $S = 0$ when $t = 0$ for both instances will be



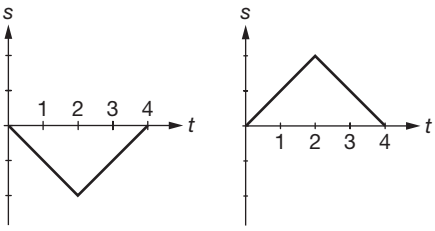
(A)



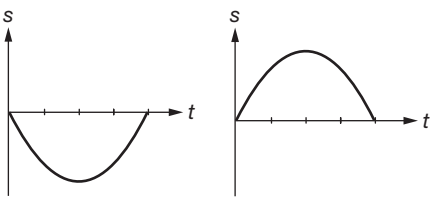
(B)



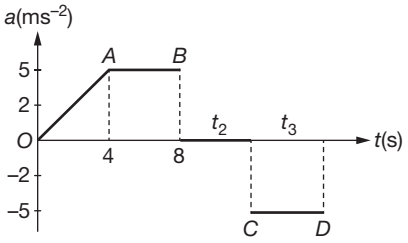
(C)



(D)



72. The acceleration of a train between two stations 2 kilometre apart is shown in the figure. The maximum speed of the train is



(A) 30 ms^{-1} (B) 60 ms^{-1}

(C) 90 ms^{-1} (D) 120 ms^{-1}

73. The acceleration is constant when the relationship between

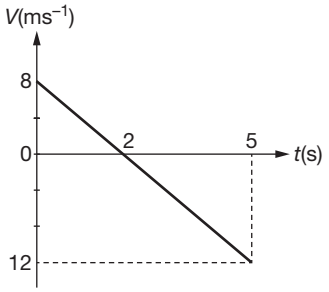
(A) the square of the position coordinate x and the velocity v is linear

(B) the position coordinate x and the reciprocal of the velocity is linear

(C) the position coordinate x and velocity v is linear

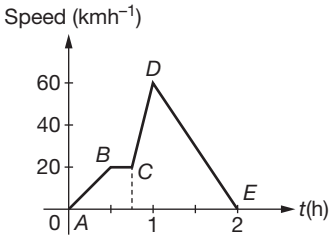
(D) the position coordinate x and the square of the velocity v is linear

74. A particle moves along a horizontal straight line with a velocity-time relationship as shown in figure. The total distance moved by the particle is



(A) 39 m (B) 13 m
(C) 26 m (D) 2.6 m

75. A train moves from one station to another in 2 hour, and its speed during the motion is shown in the graph. The maximum acceleration during the journey is



(A) 80 kmh^{-2} (B) 160 kmh^{-2}
(C) 40 kmh^{-2} (D) 60 kmh^{-2}

76. The wind appears to blow from the north to a man moving in the north-east direction. When he doubles his velocity the wind appears to move in the direction $\cot^{-1}(2)$ east of north. The actual direction of the wind is

(A) $\sqrt{2}v$ towards east (B) $\frac{v}{\sqrt{2}}$ towards west
(C) $\sqrt{2}v$ towards west (D) $\frac{v}{\sqrt{2}}$ towards east

77. A man drives a car from Y towards X at speed 60 kmh^{-1}. A car leaves station X for station Y every 10 min. The distance between X and Y is 60 km. The car travels at speed 60 kmh^{-1}. A man drives a car from Y towards X at speed 60 kmh^{-1}. If he starts at the moment when first car leaves station X. The number of cars he would meet on route is
(A) 5 (B) 7
(C) 10 (D) 20

78. A body falls from rest, in the last second of its fall, it covers half of the total distance. If g is 9.8 ms^{-2}, then the total time of its fall is (in second)
(A) 2 (B) $2+\sqrt{2}$
(C) $2-\sqrt{2}$ (D) $2\pm\sqrt{2}$

79. The motion of a body falling from rest in a resisting medium is described by the equation $\frac{dv}{dt}=A-Bv$ where A and B are constants. The velocity at any time t is
(A) $A(1-e^{Bt})$ (B) $ABte^{-t}$
(C) $\frac{A}{B}(1-e^{-Bt})$ (D) $AB^2(1-e^{-Bt})$

80. In a car race, car A takes t_0 time less to finish than car B and passes the finishing point with a velocity v_0 more than car B. The cars start from rest and travel with constant accelerations a_1 and a_2. Then the ratio $\frac{v_0}{t_0}$ is equal to
(A) $\frac{a_1+a_2}{2}$ (B) $\frac{a_2^2}{a_1}$
(C) $\frac{a_1^2}{a_2}$ (D) $\sqrt{a_1a_2}$

81. A 2 m wide car is moving with a uniform speed of 8 ms^{-1} along the edge of a straight horizontal road. A pedestrian starts to cross the road with a speed v when the car is 12 m away from him. The minimum value of v for the pedestrian to cross the road safely is
(A) $\frac{4}{3}\text{ms}^{-1}$ (B) $\frac{3}{4}\text{ms}^{-1}$
(C) $\frac{1}{2}\text{ms}^{-1}$ (D) $\frac{1}{6}\text{ms}^{-1}$

82. In PROBLEM 81, if θ is the angle made by the velocity of the pedestrian with the road then
(A) $\tan\theta=2$ (B) $\tan\theta=\frac{1}{6}$
(C) $\tan\theta=6$ (D) $\tan\theta=\frac{1}{2}$

83. In PROBLEM 81, the time to cross the moving vehicle safely is
(A) $\frac{24}{37}$ s (B) $\frac{37}{24}$ s
(C) $\frac{24}{49}$ s (D) $\frac{3}{2}$ s

84. A proton in a cyclotron moves in a circle of radius 0.8 metre at a speed of 10^7 ms^{-1}. The acceleration of the proton and acceleration due to gravity have a ratio of approximately

(A) 10^{10} (B) 10^{11}

(C) 10^{13} (D) 10^{14}

85. Two particles are released from the same height at an interval of 1 s. How long after the first particle begins to fall will the two particles be 10 m apart. $(g = 10\ \text{ms}^{-2})$

(A) 1.25 s (B) 1.5 s

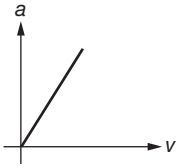
(C) 2 s (D) 2.5 s

86. A particle thrown down from the top of a tower takes time t_1 to reach the ground. It takes time t_2 if thrown from the same point with the same speed in the upward direction. The time taken by it to fall freely to the ground from the top of tower is

(A) $\frac{1}{2}(t_1 + t_2)$ (B) $\frac{1}{2}(t_1 - t_2)$

(C) $\frac{t_1 t_2}{t_1 + t_2}$ (D) $\sqrt{t_1 t_2}$

87. The acceleration-velocity graph of a particle moving rectilinearly is as shown in figure. Then slope of velocity-displacement graph must be



(A) increasing linearly
(B) decreasing linearly
(C) a constant
(D) increasing parabolically

88. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity v and other with a uniform acceleration a. If α is the angle between the lines of motion of two particles then the least value of relative velocity will be at time given by

(A) $\frac{v}{a}\sin\alpha$ (B) $\frac{v}{a}\cos\alpha$

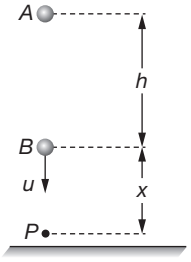
(C) $\frac{v}{a}\tan\alpha$ (D) $\frac{v}{a}\cot\alpha$

89. In PROBLEM 88, the least value of relative velocity is

(A) $v\sin\alpha$ (B) $v\cos\alpha$

(C) $v\tan\alpha$ (D) $v\cot\alpha$

90. A body dropped from a certain height attains the same velocity as another falling with an initial velocity u from a height h below the first body. If g is the acceleration due to gravity, then



(A) $u = \sqrt{gh}$ (B) $u = 2\sqrt{gh}$

(C) $u = \sqrt{2gh}$ (D) $u = \sqrt{\frac{gh}{2}}$

91. Two objects move uniformly toward each other. They get closer by 4 metre each second but when they move uniformly in the same direction, with the same speeds, they get 4 metre closer every 10 second. The speeds of the two objects are

(A) 2.2 ms^{-1} and 1.8 ms^{-1}

(B) 1.1 ms^{-1} and 1.8 ms^{-1}

(C) 22 ms^{-1} and 18 ms^{-1}

(D) 1.7 ms^{-1} and 2.1 ms^{-1}

92. A body falls freely under gravity. The distance travelled by it in the last second of its journey equals the distance travelled by it in the first three second of its free fall. The total time taken by the body to reach the ground is

(A) 5 s (B) 8 s

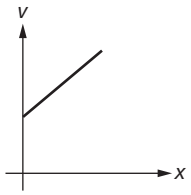
(C) 12 s (D) 15 s

93. Six persons are initially at the six corners of a hexagon of side a. Each person now moves with a uniform speed v in such a manner that 1 is always directed towards 2, 2 towards 3, 3 towards 4 and so on. The time after which they meet is

(A) $\frac{2a}{v}$ (B) $\frac{a}{v}$

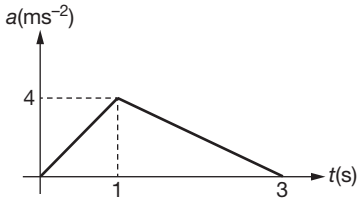
(C) $\frac{2a}{3v}$ (D) $\frac{a}{2v}$

94. Velocity versus displacement graph of a particle moving in a straight line is as shown in figure. The acceleration of the particle is



(A) constant
(B) increases parabolically with x
(C) increases linearly with x^2
(D) increases linearly with x

95. The acceleration time graph of a particle moving in a straight line is as shown in figure. The velocity of the particle at time $t = 0$ is $2\ \text{ms}^{-1}$. The velocity after 2 second will be



(A) $2\ \text{ms}^{-1}$ (B) $4\ \text{ms}^{-1}$
(C) $6\ \text{ms}^{-1}$ (D) $8\ \text{ms}^{-1}$

96. A ball is thrown vertically up with a speed of $20\ \text{ms}^{-1}$. It is caught on its way down $5\ \text{m}$ above the point from where it was thrown. The time lapse between the throw and the catch is

(A) 1.9 s (B) 3.8 s
(C) 8.3 s (D) 1.2 s

97. Two particles start moving from the same point along the same straight line. The first moves with constant velocity $2v$ and the second with constant acceleration a. During the time that elapses before the second catches the first, the greatest distance between the particles is

(A) $\dfrac{v^2}{a}$ (B) $\dfrac{v^2}{2a}$
(C) $\dfrac{2v^2}{a}$ (D) $\dfrac{v^2}{4a}$

98. Starting from rest a particle moves in a straight line with acceleration $a = [2 + |t - 2|]\ \text{ms}^{-2}$. Velocity of particle at the end of $4\ \text{s}$ will be

(A) $8\ \text{ms}^{-1}$ (B) $12\ \text{ms}^{-1}$
(C) $16\ \text{ms}^{-1}$ (D) $20\ \text{ms}^{-1}$

99. A body moving rectilinearly traversed one third of the total distance with a velocity $4\ \text{ms}^{-1}$. The remaining part of the distance was covered with a velocity $2\ \text{ms}^{-1}$ for half the time and with velocity $6\ \text{ms}^{-1}$ for the other half of time. The mean velocity averaged over the whole time of motion is

(A) $5\ \text{ms}^{-1}$ (B) $4.5\ \text{ms}^{-1}$
(C) $3.5\ \text{ms}^{-1}$ (D) $4\ \text{ms}^{-1}$

100. A, B, C and D are four collinear points such that $AB = BC = CD$. If the average value of velocities between A and B, C and D are $12\ \text{ms}^{-1}$ and $20\ \text{ms}^{-1}$ respectively, then the value of average velocity between B and C if the body moves with uniform acceleration throughout is

(A) $16\ \text{ms}^{-1}$ (B) $14\ \text{ms}^{-1}$
(C) $\left(1 + \sqrt{241}\right)\ \text{ms}^{-1}$ (D) $8\ \text{ms}^{-1}$

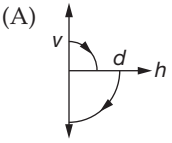
101. A target is made of two plates, one of wood and the other of iron. The thickness of the wooden plate is 4 cm and that of iron plate is 2 cm. A bullet fired goes through the wood first and then penetrates 1 cm into iron. A similar bullet fired with the same velocity from opposite direction goes through iron first and then penetrates 2 cm into wood. If a_1 and a_2 be the retardations offered to the bullet by wood and iron plates respectively then

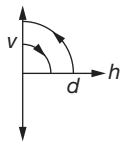
(A) $a_1 = 2a_2$ (B) $a_2 = 2a_1$
(C) $a_1 = a_2$ (D) Data Insufficient

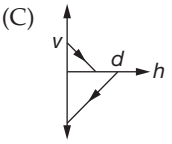
102. A ball is thrown from the top of a tower of height 80 metre with a horizontal velocity of $30\ \text{ms}^{-1}$. The velocity with which it strikes the level ground is

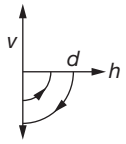
(A) $20\ \text{ms}^{-1}$ (B) $50\ \text{ms}^{-1}$
(C) $80\ \text{ms}^{-1}$ (D) $100\ \text{ms}^{-1}$

103. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $\dfrac{1}{2}d$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

(A) 

(B) 

(C) 

(D) 

104. A ball is dropped from the roof of a tower of height h. The total distance covered by it in the last second of its motion is equal to the distance covered by it in first three second. The value of h in meters is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 80 (B) 100
(C) 125 (D) 200

105. A stone falls freely from a point O. It passes through the points P, Q, R,.....such that OP, OQ, OR, ... are in geometric progression. Then velocities of stone at P, Q, R, ...are in

(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) logarithmic mean

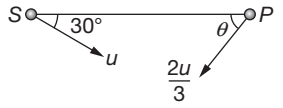
106. A particle projected vertically upwards attains a maximum height H. If the ratio of the times to attain a height h $(h < H)$ is $\frac{1}{3}$ then

(A) $4h = 3H$ (B) $3h = 4H$
(C) $3h = H$ (D) $4h = H$

107. The velocity of a boat in still water is η times less than the velocity of flow of the river $(\eta > 1)$. The angle with the stream direction at which the boat must move to minimise drifting is

(A) $\sin^{-1}\left(\frac{1}{\eta}\right)$ (B) $\cot^{-1}\left(\frac{1}{\eta}\right)$
(C) $\frac{\pi}{2} + \sin^{-1}\left(\frac{1}{\eta}\right)$ (D) $\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{\eta}\right)$

108. Sachin (S) hits a ball along the ground with a speed u in a direction which makes an angle $30°$ with the line joining him and the fielder Prem (P). Prem runs to intercept the ball with a speed $\frac{2u}{3}$. At what angle θ should he run to intercept the ball?



(A) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (B) $\sin^{-1}\left(\frac{2}{3}\right)$
(C) $\sin^{-1}\left(\frac{3}{4}\right)$ (D) $\sin^{-1}\left(\frac{4}{5}\right)$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A particle moves with an initial velocity v_0 and retardation αv, where v is its velocity at any time t. If $\log_e(2) = 0.7$, then which of the following statement(s) is/are correct?

(A) The particle will cover a total distance $\frac{v_0}{\alpha}$.

(B) The particle will come to rest after time $\frac{1}{\alpha}$.

(C) The particle will continue to move for a very long time.

(D) The velocity of the particle will become $\frac{v_0}{2}$ after time $\frac{7}{10\alpha}$.

2. Average velocity of a particle moving in a straight line, with constant acceleration a and initial velocity u and final velocity v in first t second is

(A) $\frac{u+v}{2}$ (B) $u + at$
(C) $\frac{1}{2}(u + at)$ (D) $u + \frac{1}{2}at$

3. The velocity of a particle moving along a straight line increases according to the linear law $v = v_0 + kx$, where k is a constant. Then

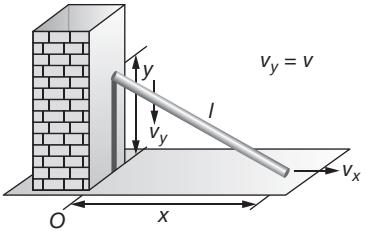
(A) the acceleration of the particle is $k(v_0 + kx)$.

(B) the particle takes a time $\frac{1}{k}\log_e\left(\frac{v_1}{v_0}\right)$ to attain a velocity v_1.

(C) velocity varies linearly with displacement with slope of velocity displacement curve equal to k.

(D) data is insufficient to arrive at a conclusion.

4. A rod of length l leans by its upper end against a smooth vertical wall, while its other end leans against the floor. The end that leans against the wall moves uniformly downward . Then the



(A) other end moves uniformly forward with speed v.

(B) other end moves with a speed whose value decreases with increase in y and vanishes at $y = 0$.

(C) other end moves with a speed whose value decreases with decrease in y and vanishes at $y = 0$.

(D) other end moves such that the ratio $\frac{v_x}{v}$ equals $\frac{y}{\sqrt{l^2 - y^2}}$.

5. For a particle moving in a plane, if $\vec{v}$ and $\vec{a}$ be the instantaneous velocity and acceleration, then rate of change of speed, $\frac{dv}{dt}$, of the particle equal(s)

(A) $|\vec{a}|$

(B) the component of $\vec{a}$ perpendicular to $\vec{v}$

(C) $\frac{\vec{a} \cdot \vec{v}}{v}$

(D) the projection of $\vec{a}$ along $\vec{v}$

6. The position of a particle travelling along x-axis is given by $x_t = t^3 - 9t^2 + 6t$ where x_t is in cm and t is in second. Then

(A) the body comes to rest firstly at $(3 - \sqrt{7})$ s and then at $(3 + \sqrt{7})$ s.

(B) the total displacement of the particle in travelling from the first zero of velocity to the second zero of velocity is zero.

(C) the total displacement of the particle in travelling from the first zero of velocity to the second zero of velocity is –74 cm.

(D) the particle reverses its velocity at $(3 - \sqrt{7})$s and then at $(3 + \sqrt{7})$ s and has a negative velocity for $(3 - \sqrt{7}) < t < (3 + \sqrt{7})$

7. A particle, starting from rest is first accelerated for time t_1 with constant acceleration a_1 and then stops in time t_2 with constant retardation a_2. Let v_1 be the average velocity in this case and s_1 the total displacement. Now, the same particle, starting again from rest is accelerated for the same time t_1 with constant acceleration $2a_1$ and finally comes to rest with constant retardation a_2 in time t_3. If v_2 is the average velocity in this case and s_2 the total displacement. Then

(A) $s_2 = 2s_1$ (B) $2s_1 < s_2 < 4s_1$

(C) $v_2 = 2v_1$ (D) $2v_1 < v_2 < 4v_1$

8. A particle moves with an initial velocity v_0 and retardation αv, where v is velocity at any instant t. Then

(A) the particle will cover a total distance $\frac{v_0}{\alpha}$.

(B) the particle continues to move for a long time span.

(C) the particle attains a velocity $\frac{1}{2}v_0$ at $t = \frac{1}{\alpha}$.

(D) the particle comes to rest at $t = \frac{1}{\alpha}$.

9. A particle moving along a straight line with uniform acceleration has velocities 7 ms^{-1} at A and 17 ms^{-1} at B. C is the mid-point of AB. Then

(A) the average velocity between C and B is 15 ms^{-1}

(B) the ratio of time to go from A to C and that from C to B is 3 : 2

(C) the velocity at C is 10 ms^{-1}

(D) the average velocity between A and C is 10 ms^{-1}

10. An aeroplane flies along a straight line from A to B with a speed v_0 and back again with the same speed v_0. A steady wind v is blowing. If $AB = l$ then

(A) total time for the trip is $\frac{2v_0 l}{v_0^2 - v^2}$, if wind blows along the line AB.

(B) total time for the trip is $\frac{2l}{\sqrt{v_0^2 - v^2}}$, if wind blows perpendicular to the line AB.

(C) total time for the trip decreases because of the presence of wind.

(D) total time for the trip increases because of the presence of wind.

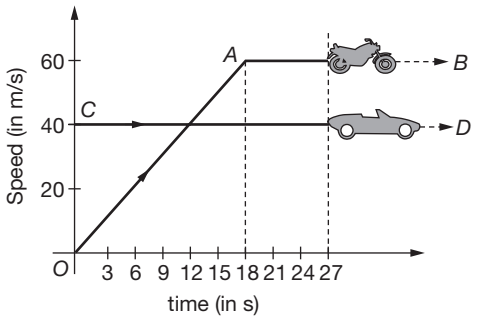
11. Acceleration of a particle which is at rest at $x = 0$ is $\vec{a} = (4 - 2x)\hat{i}$. Select the correct alternative(s)

(A) maximum speed of particle is 4 units

(B) particle further comes to rest at $x = 4$

(C) particle oscillates about $x = 2$
(D) all of the above

12. At the instant a motor bike starts from rest in a given direction, a car overtakes the motor bike, both moving in the same direction.



The speed time graphs for motor bike and car are represented by OAB and CD respectively. Then

(A) at $t = 18$ s the motor bike and car are 180 m apart.
(B) at $t = 18$ s the motor bike and car are 720 m apart.
(C) the relative distance between motor bike and car reduces to zero at $t = 27$ s and both are 1080 m far from origin.
(D) the relative distance between motor bike and car always remains same.

13. A car is moving rectilinearly on a horizontal path with acceleration a_0. A person sitting inside the car observes that an insect S is crawling up the screen with an acceleration a. If θ is the inclination of the screen with the horizontal, then the acceleration of the insect

(A) perpendicular to screen is $a_0 \tan\theta$
(B) perpendicular to screen is $a_0 \sin\theta$
(C) along the horizontal is $a_0 - a\cos\theta$
(D) parallel to screen is $a + a_0 \cos\theta$

14. A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.

(A) The particle comes to rest at $t = \dfrac{2\sqrt{v_0}}{\alpha}$.
(B) The particle will come to rest at infinity.
(C) The distance travelled by the particle is $\dfrac{2v_0^{3/2}}{\alpha}$.
(D) The distance travelled by the particle is $\dfrac{2}{3}\dfrac{v_0^{3/2}}{\alpha}$.

15. A car starts moving rectilinearly (initial velocity zero) first with an acceleration of $5\ \text{ms}^{-2}$ then uniformly and finally decelerating at the same rate till it stops. Total time of journey is 25 s and average velocity during the journey is $72\ \text{kmh}^{-1}$. Then

(A) total distance travelled by the car is 500 m.
(B) maximum speed attained during the journey is $25\ \text{ms}^{-1}$.
(C) car travels with uniform speed for 15 s.
(D) car accelerates for 5 s and decelerates also for 5 s.

16. For a moving body, which of the following statement(s) is/are true?

(A) If its speed changes but direction of motion does not change, its velocity may remain constant.
(B) If its speed changes, its velocity must change and it must have some acceleration.
(C) If its velocity changes, its speed must change and it must have some acceleration.
(D) If its velocity changes, its speed may or may not change, and it must have some acceleration.

17. A body is moving along a straight line. Its distance x_t from a point on its path at a time t after passing that point is given by $x_t = 8t^2 - 3t^3$, where x_t is in metre and t is in second.

(A) Average speed during the interval $t = 0$ s to $t = 4$ s is $20.21\ \text{ms}^{-1}$.
(B) Average velocity during the interval $t = 0$ s to $t = 4$ s is $-16\ \text{ms}^{-1}$.
(C) The body starts from rest and at $t = \dfrac{16}{9}$ s it reverses its direction of motion at $x_t = 8.43$ m from the start.
(D) It has an acceleration of $-56\ \text{ms}^{-2}$ at $t = 4$ s.

18. Let $\vec{r}$ be the radius vector of a particle in motion about some reference point and r its modulus. Similarly $\vec{v}$ be the velocity vector and v its modulus, then

(A) $|d\vec{r}| \neq dr$ (B) $v \neq \dfrac{dr}{dt}$
(C) $v = \dfrac{dr}{dt}$ (D) $v = \left|\dfrac{d\vec{r}}{dt}\right|$

19. Two particles P and Q move in a straight line AB towards each other. P starts from A with velocity u_1 and an acceleration a_1. Q starts from B with velocity u_2 and acceleration a_2. They pass each other at the midpoint of AB and arrive at the other ends of AB with equal velocities.

(A) They meet at midpoint at time $t = \dfrac{2(u_2 - u_1)}{(a_1 - a_2)}$.

(B) The length of path specified i.e. AB is $l = \dfrac{4(u_2 - u_1)(a_1u_2 - a_2u_1)}{(a_1 - a_2)^2}$.

(C) They reach the other ends of AB with equal velocities if $(u_2 + u_1)(a_1 - a_2) = 8(a_1u_2 - a_2u_1)$.

(D) They reach the other ends of AB with equal velocities if $(u_2 - u_1)(a_1 + a_2) = 8(a_2u_1 - a_1u_2)$.

20. Consider a body moving rectilinearly under the influence of constant acceleration $\vec{a}$. Let $\vec{v}$ denote the velocity of the body at any instant of time. Which of the following argument(s) is/are correct?

(A) Speed must decrease when $\vec{a}$ is negative.

(B) Speed must increase when $\vec{a}$ is negative for the body that starts from rest initially.

(C) Speed will increase when both $\vec{v}$ and $\vec{a}$ are both negative.

(D) Speed will decrease when $\vec{v}$ is negative and $\vec{a}$ positive.

21. The co-ordinate of the particle in x-y plane are given as $x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$. The motion of the particle is

(A) along a parabolic path

(B) non-uniformly accelerated

(C) along a straight line

(D) uniformly accelerated

22. Two particles A and B are located in x-y plane at points $(0, 0)$ and $(0, 4\text{ m})$. They simultaneously start moving with velocities $\vec{v}_A = 2\hat{j}$ ms^{-1} and $\vec{v}_B = 2\hat{i}$ ms^{-1}. Select the correct alternative(s)

(A) the distance between them is constant

(B) the distance between them first decreases and then increases

(C) time after which they are at minimum distance is 1 s

(D) the shortest distance between them is $2\sqrt{2}$ m

23. Consider a body moving rectilinearly with velocity v under the influence of an acceleration a. Which of the following statement(s) is/are correct?

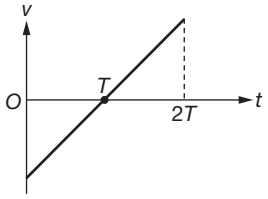
(A) The direction of a must have some correlation with the direction of v.

(B) a can be non-zero when $v = 0$

(C) a must be zero when $v = 0$

(D) a may be zero when $v \neq 0$

24. For a particle moving rectilinearly, the velocity-time $(\vec{v}\text{-}t)$ graph is plotted. Which of the following argument(s) is/are correct to explain the facts about its motion?



(A) The acceleration of the particle remains constant.

(B) The particle changes its direction of motion at some point.

(C) The initial and final speeds of the particle are the same.

(D) The displacement of the particle is zero.

25. Two stationary objects when seen by an observer that moves with a constant speed along the line joining them (the stationary objects) will

(A) move in the same direction

(B) move in opposite directions

(C) have the same velocity

(D) have the same speed

26. A particle moving along x-axis has its velocity (v) varying with x co-ordinate (x) as $v = \sqrt{x}$. Then

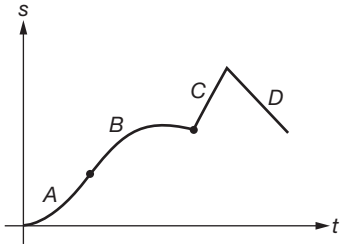
(A) initial velocity of particle is zero

(B) motion is uniformly accelerated

(C) acceleration of particle at $x = 2$ m is $\dfrac{1}{2}$ ms^{-2}

(D) acceleration of particle at $x = 4$ m is 1 ms^{-2}

27. Displacement time graph of a particle moving in a straight line is as shown in figure



(A) in region A acceleration is positive

(B) in region B acceleration is negative

(C) in region C motion is uniform

(D) in region D acceleration is negative

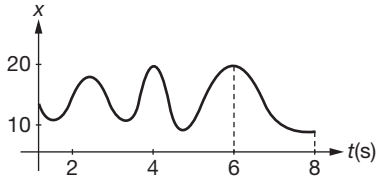
28. A particle moves on a straight line position at any time t is given by $x = x_0e^{-kt}$, where k is a constant. Select the correct statement(s).

(A) Distance moved is infinite.
(B) Distance moved during total motion is finite
(C) Average speed for total motion is zero.
(D) Average speed for total motion is infinite.

29. The motion of the body starting from rest is governed by the relation $\frac{dv}{dt} = -v^2 + 2v - 1$, where v is speed in ms^{-1} and t is time in second, then select the correct statement(s).

(A) Terminal velocity is $1\ \text{ms}^{-1}$.
(B) The magnitude of initial acceleration is $1\ \text{ms}^{-2}$.
(C) Instantaneous speed is $v = -\frac{1}{1+t}$.
(D) The speed is $1.5\ \text{ms}^{-1}$ when acceleration is one fourth of its initial value

30. In the figure is shown the position of a particle moving on the x-axis as a function of time. Then

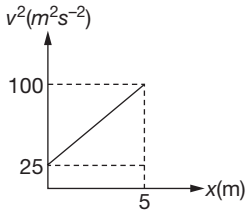


(A) the particle has come to rest for 6 times
(B) the maximum speed is at $t = 6$ s
(C) the velocity remain positive for $t = 0$ to $t = 6$ s
(D) the average velocity for the total period shown is negative

31. A particle moves with an initial velocity v_0 and retardation αv, where v is its velocity at any time t. Select the correct statement(s).

(A) The particle will cover a total distance $\frac{v_0}{\alpha}$
(B) The particle comes to rest after a time $\frac{1}{\alpha}$
(C) The particle will continue to move for a very long time
(D) The velocity of the particle will become $\frac{v_0}{2}$ after a time $\frac{1}{\alpha}$

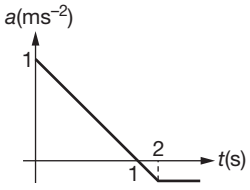
32. A particle starts moving rectilinearly from the origin along the x-axis. The graph between the square of speed and position of the particle is given in the figure. Select the correct statement(s).

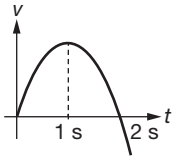


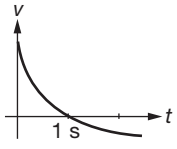
(A) Acceleration of the particle is $15\ \text{ms}^{-1}$ at $t = \frac{1}{2}$ s
(B) Acceleration of the particle is $7.5\ \text{ms}^{-1}$ at $t = 1$ s
(C) Acceleration of the particle is constant
(D) At $t = 1$ s, velocity of particle is $12.5\ \text{ms}^{-1}$

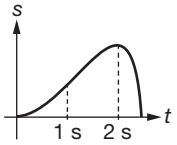
33. Mark the correct statement for a particle going on a straight line
(A) If the velocity and acceleration have opposite sign, the object is slowing down.
(B) If the position and velocity have opposite sign, the particle is moving towards the origin.
(C) If the velocity is zero at an instant, the acceleration should also be zero at that instant.
(D) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.

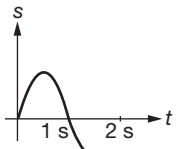
34. Acceleration vs time graph for a particle moving in straight line is as shown in figure. If particle starts from rest at $t = 0$, then which of the following curve is true for the same particle



(A)


(B)


(C)


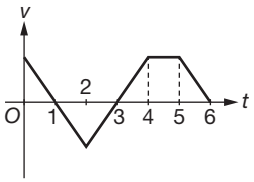
(D)


35. A train accelerates from rest for time t_1, at a constant acceleration α for distance x. Then it decelerates to rest at constant retardation β in time t_2 for distance y. Then

(A) $\frac{x}{y} = \frac{\beta}{\alpha}$ (B) $\frac{\beta}{\alpha} = \frac{t_1}{t_2}$

(C) $\frac{x}{y} = \frac{t_1}{t_2}$ (D) $x = y$

36. From v-t graph shown in figure. We can draw the following conclusion



(A) between $t = 1$ is to $t = 2$ s speed of particle is decreasing

(B) between $t = 2$ s to $t = 3$ s speed of particle is increasing

(C) between $t = 5$ to $t = 6$ s acceleration of particle is negative

(D) between $t = 0$ to $t = 4$ s particle changes, its direction of motion twice

37. Which of the following statement(s) is/are incorrect ?

(A) Distance and speed can never be negative

(B) Distance and speed may decrease or increase

(C) Distance is always greater than magnitude of displacement

(D) Speed is always greater than magnitude of velocity

38. A car accelerates from rest at a constant rate of $2\ \text{ms}^{-2}$ for some time. Then it retards (speed decrease) at a constant rate of $4\ \text{ms}^{-2}$ and comes to rest. It remains in motion for a time of 6 s.

(A) Its maximum speed is $8\ \text{ms}^{-1}$

(B) Its maximum speed is $6\ \text{ms}^{-1}$

(C) It travelled a total distance of 24 m

(D) It travelled a total distance of 18 m

39. A particle moves along a straight line and its velocity depends on time as $v = 4t - t^2$. Then for first 5 s, the

(A) Average velocity is $\frac{25}{3}\ \text{ms}^{-1}$

(B) Average speed is $10\ \text{ms}^{-1}$

(C) Average velocity is $\frac{5}{3}\ \text{ms}^{-1}$

(D) Acceleration is $4\ \text{ms}^{-2}$ at $t = 0$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** In a uniformly accelerated motion, acceleration time graph is straight line with positive slope.
Statement-2: Acceleration is rate of change of velocity.

2. **Statement-1:** A body having non-zero acceleration can have a constant velocity.
Statement-2: Acceleration is the rate of change of velocity.

3. **Statement-1:** Irrespective of the kind of motion possessed by a body, the body will always stay at rest in a reference frame attached to the body itself.
Statement-2: The relative velocity of a body with respect to itself is always zero.

4. **Statement-1:** The instantaneous velocity does not depend on instantaneous position vector.
Statement-2: The instantaneous velocity and average velocity of a particle are always same.

5. **Statement-1:** A balloon ascends from the surface of earth with constant speed. When it was at a height 50 m above the ground, a packet is dropped from it. To an observer on the balloon, the displacement of the packet, from the moment it is dropped to the moment it reaches the surface of earth, is 50 m.
Statement-2: Displacement (vector) depends upon the reference frame used to measure it.

6. **Statement-1:** A man who can swim at a speed v relative to water wants to cross the river of width d, flowing with speed u. He cannot reach a point P just opposite to him across the river, if $u > v$.
Statement-2: The time to reach the opposite point P across the river is $\frac{d}{\sqrt{v^2-u^2}}$ and if $u > v$ time will come out to be imaginary.

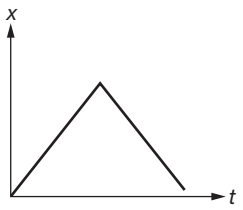
7. **Statement-1:** When a particle moves along a straight line magnitude of its average velocity is equal to its average speed over any time interval.
Statement-2: For one dimensional motion displacement and distance may or may not be equal.

8. **Statement-1:** If two particles, moving with constant velocities are to meet, the relative velocity must be along the line joining the two particles.
Statement-2: Relative velocity means motion of one particle as viewed from the other.

9. **Statement-1:** Two balls are dropped one after the other from a tall tower. The distance between them increases linearly with time (that elapses after the second ball is dropped and before the first hits ground).
Statement-2: Relative acceleration is zero, whereas relative velocity non-zero in the above situation.

10. **Statement-1:** $\left|\frac{d\vec{v}}{dt}\right| = \frac{d}{dt}|\vec{v}|$, where $\vec{v}$ has its usual meaning.
Statement-2: Acceleration is the rate of change of velocity.

11. **Statement-1:** $x-t$ graph, for a particle undergoing rectilinear motion, can be as shown in the figure.



Statement-2: Infinitesimal changes in velocity are physically possible only in infinitesimal time.

12. **Statement-1:** Area under velocity-time graph gives displacement.
Statement-2: Area under acceleration-time graph gives average velocity.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

Two particles A and B start from rest at the origin $x = 0$ and move along a straight line such that $a_A = (6t - 3)\ \text{ms}^{-2}$ and $a_B = (12t^2 - 8)\ \text{ms}^{-2}$, where t is in seconds. Based on the above facts, answer the following questions.

1. Total distance travelled by A at $t = 4$ s is
(A) 40 m (B) 41 m
(C) 42 m (D) 43 m

2. Total distance travelled by B at $t = 4$ s is
(A) 192 m (B) 184 m
(C) 196 m (D) 200 m

3. The distance between them at $t = 4$ s is
(A) 144 m (B) 148 m
(C) 152 m (D) 156 m

Comprehension 2

A particle initially at $x = 10$ m, starts moving along the positive x-axis with an initial velocity of 40 ms^{-1} under the influence of an acceleration of 10 ms^{-2} directed along the negative x direction. Based on this information, answer the following questions.

4. The particle reverses its direction of motion at time t_0 from the start. Then
(A) $t_0 = 2$ s (B) $t_0 = 4$ s
(C) $t_0 = 6$ s (D) $t_0 = 8$ s

5. The maximum x-coordinate of the particle is
(A) 30 m (B) 60 m
(C) 90 m (D) 120 m

6. The velocity of the particle (in ms^{-1}) at the origin of the coordinate system is
(A) $30\sqrt{2}$ along $+x$ direction
(B) $30\sqrt{2}$ along $-x$ direction
(C) $10\sqrt{7}$ along $+x$ direction
(D) $10\sqrt{7}$ along $-x$ direction

7. The time at which the particle crosses the origin is

(A) $(4-3\sqrt{2})$ s (B) $(4+3\sqrt{2})$ s

(C) $(4-\sqrt{7})$ s (D) $(4+\sqrt{7})$ s

Comprehension 3

Starting from rest, a particle moves rectilinearly along x-axis, under the influence of acceleration a which varies with time t (in second) as $a=(2t-4)$ ms^{-2}. Based on this information answer the following questions.

8. The particle comes to rest at time equal to

(A) 1 s (B) 2 s

(C) 3 s (D) 4 s

9. The maximum velocity of the particle is v_{max} at time t_0 (say). Then

(A) $v_{max}=2$ ms^{-1}, along $+x$ axis at time $t_0=2$ s

(B) $v_{max}=4$ ms^{-1}, along $-x$ axis at time $t_0=2$ s

(C) $v_{max}=2$ ms^{-1}, along $-x$ axis at time $t_0=4$ s

(D) $v_{max}=4$ ms^{-1}, along $+x$ axis at time $t_0=4$ s

10. The velocity-time graph of the particle is a

(A) Straight line passing through $(0, 0)$

(B) Straight line passing through $(2, 0)$

(C) Parabola having origin at $(4, 2)$

(D) Parabola having origin at $(2, 4)$

Comprehension 4

The motion of a body falling initially from rest in a resistive medium is described by the differential equation

$$\frac{dv}{dt}=6-3v$$

where v is the velocity of the body at any instant (in ms^{-1}). Based on the above facts, answer the following questions.

11. The initial acceleration is of the body is

(A) 6 ms^{-2} (B) 3 ms^{-2}

(C) 2 ms^{-2} (D) 18 ms^{-2}

12. The terminal velocity i.e., the velocity at which acceleration becomes zero is given by

(A) 6 ms^{-1} (B) 3 ms^{-1}

(C) 2 ms^{-1} (D) 18 ms^{-1}

13. The velocity at any time t is given by

(A) $v=e^{-3t}$ (B) $v=1-e^{-3t}$

(C) $v=\frac{1}{2}(1-e^{-3t})$ (D) $v=2(1-e^{-3t})$

14. The acceleration (a) of the body as the function of time is

(A) $a=2e^{-3t}$ (B) $a=3e^{-2t}$

(C) $a=6e^{-3t}$ (D) $a=6e^{-2t}$

Comprehension 5

A particle moves in the xy plane, such that at any instant t, its x and y coordinates are given by

$$x=a\sin(\omega t)$$

and

$$y=a[1-\cos(\omega t)]$$

where a and ω are constants. Based on the above facts, answer the following questions.

15. The magnitude of the velocity of the particle is

(A) $a\omega$

(B) $a\omega\sin(\omega t)$

(C) $a\omega\cos(\omega t)$

(D) $a\omega[\sin(\omega t)+\cos(\omega t)]$

16. The equation of trajectory followed by the particle is

(A) $x=\sqrt{2ay}$ (B) $x=\sqrt{2ay}\sqrt{1-\frac{y}{2a}}$

(C) $x=4ay^2$ (D) None of these

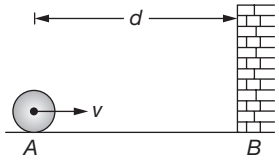
17. The magnitude of the acceleration of the particle is

(A) $a\omega$ (B) $a\omega^2$

(C) $\frac{a\omega^2}{2}$ (D) $\frac{a\omega^2}{3}$

Comprehension 6

A small ball is pushed with a speed v from A. It moves on a smooth surface and collides with the wall at B at distance d from A. During impact loses one third of its velocity.



Based on the above facts, answer the following questions.

18. The average velocity of the ball during its motion from A to B and back to A will be

(A) zero (B) $\frac{v}{2}$

(C) $\frac{v}{3}$ (D) $\frac{2v}{3}$

19. The total time taken by the ball in moving from A to B and back to A is T. Then T equals

(A) $\frac{2d}{v}$ (B) $\frac{3d}{v}$

(C) $\frac{4d}{v}$ (D) $\frac{5d}{2v}$

20. The average speed during the journey from A to B and back to A is

(A) $\frac{v}{5}$ (B) $\frac{2v}{5}$

(C) $\frac{3v}{5}$ (D) $\frac{4v}{5}$

Comprehension 7

A car accelerates from rest with $2\ \text{ms}^{-2}$ on a straight track and then it comes to rest applying its brakes. The total distance travelled by the car is 100 m in 20 s. Based on the above facts, answer the following questions.

21. The maximum speed attained by the car is

(A) $5\ \text{ms}^{-1}$ (B) $10\ \text{ms}^{-1}$

(C) $15\ \text{ms}^{-1}$ (D) $20\ \text{ms}^{-1}$

22. The duration for which the brakes were applied is

(A) 10 s (B) 5 s

(C) 15 s (D) 16 s

23. The maximum retardation given to the car is

(A) $2\ \text{ms}^{-2}$ (B) $\frac{2}{3}\ \text{ms}^{-2}$

(C) $\frac{4}{3}\ \text{ms}^{-2}$ (D) $\frac{5}{3}\ \text{ms}^{-2}$

24. The average speed of the car for the entire tenure of motion is

(A) $5\ \text{ms}^{-1}$ (B) $6\ \text{ms}^{-1}$

(C) $8\ \text{ms}^{-1}$ (D) $8.2\ \text{ms}^{-1}$

25. The distance covered during acceleration is

(A) 25 m (B) 15 m

(C) 10 m (D) 5 m

26. The distance covered during retardation is

(A) 85 m (B) 75 m

(C) 50 m (D) 25 m

Comprehension 8

A body falling from a height H hits an inclined plane in its path at a height $h(<H)$. As a result of this impact, the direction of the velocity of the body becomes horizontal. Based on the above facts, answer the following questions.

27. The value of $\frac{h}{H}$ for which the body will take the maximum time to reach the ground is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$

(C) $\frac{1}{2}$ (D) 2

28. The time taken by the body to hit the ground is

(A) $T = 2\sqrt{\frac{H}{g}}$ (B) $T = 4\sqrt{\frac{H}{g}}$

(C) $T = \sqrt{\frac{2H}{g}}$ (D) $T = \sqrt{\frac{3H}{g}}$

Comprehension 9

A body is projected from the ground vertically upwards. The body is observed to be at height h above the ground at two times t_1 and t_2 while ascending and descending respectively. Based on the above facts, answer the following questions.

29. The height h in terms of t_1 and t_2 is

(A) $h = gt_1t_2$ (B) $h = 2gt_1t_2$

(C) $h = \frac{1}{2}gt_1t_2$ (D) $h = \frac{1}{4}gt_1t_2$

30. The velocity of projection (u) must be

(A) $u = \frac{1}{2}g(t_1 + t_2)$ (B) $u = \frac{1}{4}g(t_1 + t_2)$

(C) $u = 2g(t_1 + t_2)$ (D) $u = 4g(t_1 + t_2)$

31. The maximum height (H) reached by the body is

(A) $H = \frac{1}{2}gt_1^2$ (B) $H = \frac{1}{2}gt_2^2$

(C) $H = \frac{1}{8}g(t_1 + t_2)^2$ (D) $H = \frac{1}{4}g(t_1 + t_2)^2$

32. The velocity (v) of the body at height $\frac{h}{2}$ is

(A) $v = \frac{1}{4}gt_1$ (B) $v = \frac{1}{4}gt_2$

(C) $v = \frac{1}{2}g\sqrt{t_1^2 + t_2^2}$ (D) $v = \frac{1}{4}g\sqrt{t_1t_2}$

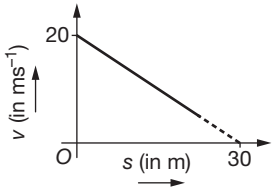
33. The velocity of the particle at half the maximum height.

(A) $\frac{g}{2}\sqrt{t_1^2+t_2^2}$ (B) $\frac{g}{4}\left(t_1^2+t_2^2\right)$

(C) $\frac{g}{2\sqrt{2}}\left(t_1+t_2\right)$ (D) $\frac{g}{\sqrt{2}}\left(t_1+t_2\right)$

Comprehension 10

If the velocity v of a particle moving along a straight line decreases linearly with its displacement s from $20\ \text{ms}^{-1}$ to a value approaching zero at $s=30$ m. Based on the above facts, answer the following questions.



34. Acceleration of the particle at $s=15$ m is

(A) $\frac{2}{3}\ \text{ms}^{-2}$ (B) $-\frac{2}{3}\ \text{ms}^{-2}$

(C) $\frac{20}{3}\ \text{ms}^{-2}$ (D) $-\frac{20}{3}\ \text{ms}^{-2}$

35. The time taken by the particle to reach the 30 m position is

(A) 1.5 s (B) 3 s

(C) 6 s (D) Infinite

Comprehension 11

Buses are going from one city to other and vice-versa. The buses start at regular interval of 5 minutes each from station A to B and station B to A each with same speed $60\ \text{kmh}^{-1}$. The distance between two stations is 30 km. A bus marked A starts from city A and finds buses approaching from opposite direction.

Based on the above facts, answer the following questions.

36. The time interval after which bus A will meet two buses coming from opposite side is

(A) 5 min (B) 2.5 min

(C) 1.25 min (D) 10 min

37. The distance travelled by bus A to meet two buses from opposite side is

(A) 2.5 km (B) 5 km

(C) 10 km (D) None of these

38. How many buses the bus A will meet on way from city A to city B?

(A) 6 (B) 5

(C) 12 (D) 11

Comprehension 12

A particle moves in positive x-direction according to law $x=12t-t^2$ m. where t time in second. (Take +ve x-direction as +ve).

Based on the above facts, answer the following questions.

39. Average velocity from $t=0$ to $t=8$ s is

(A) $4\ \text{ms}^{-1}$ (B) $6\ \text{ms}^{-1}$

(C) $8\ \text{ms}^{-1}$ (D) $-4\ \text{ms}^{-1}$

40. Average speed from $t=0$ to $t=8$ s is

(A) $4\ \text{ms}^{-1}$ (B) $5\ \text{ms}^{-1}$

(C) $6\ \text{ms}^{-1}$ (D) $8\ \text{ms}^{-1}$

41. Average acceleration from $t=0$ s to $t=8$ s is

(A) $-\frac{1}{4}\ \text{ms}^{-2}$ (B) $+\frac{1}{4}\ \text{ms}^{-2}$

(C) $-\frac{1}{2}\ \text{ms}^{-2}$ (D) $-2\ \text{ms}^{-2}$

Comprehension 13

A particle starts from rest from the origin with a time varying acceleration $a=(2t-4)$, where t is in seconds and a in ms^{-2}. Assuming the particle to move rectilinearly.

Based on the above facts, answer the following questions.

42. Particle comes to rest (after a time) at

(A) 1 s (B) 4 s

(C) 3 s (D) 2 s

43. The speed of the particle moving in negative direction is maximum at time t. Then t equals

(A) 3 s (B) 4 s

(C) 2 s (D) 1 s

44. The distance travelled by the particle from the start to the moment when it comes to rest is

(A) $\frac{16}{3}$ m (B) 2 m

(C) 3 m (D) None of these

Comprehension 14

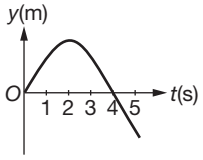
A person standing on the roof of a building throws a ball vertically upward at an instant $t=0$. The ball leaves his

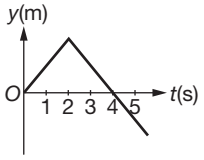
hand with an upward speed $20\ \text{ms}^{-1}$ and it is then in free fall. The ball rises to a certain height and then moves down. On its way down, the ball just misses to hit the roof of the building and keeps falling towards the earth. The ball hits earth at $t = 5$ s. Considering that

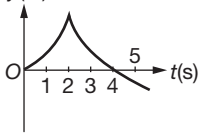
(i) the vertically upward direction is the positive y-direction
(ii) the position of ball at $t = 0$ is the origin
(iii) the ball does not rebound and comes to rest at the same place where it hits earth and
(iv) air resistance is negligible, answer these questions. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

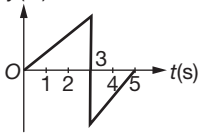
Based on the above facts, answer the following questions.

45. Position-time graph for the given motion of the ball is

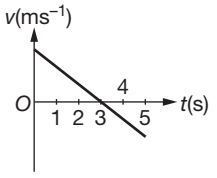
(A) 

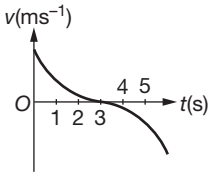
(B) 

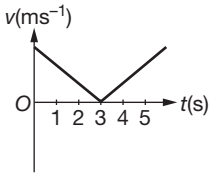
(C) 

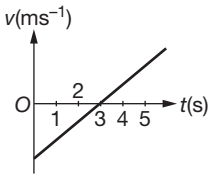
(D) 

46. Velocity of the ball will vary with time as

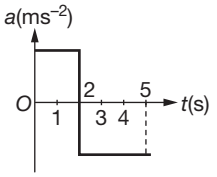
(A) 

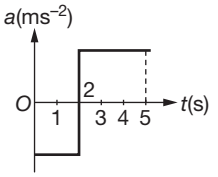
(B) 

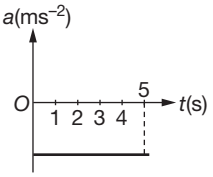
(C) 

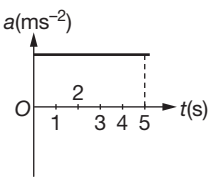
(D) 

47. Acceleration of the ball will vary with time as

(A) 

(B) 

(C) 

(D) 

Comprehension 15

The position of a particle is moving along the x-axis depends on the time according to the equation $x = 6t^2 - t^3$, where x is in metre and t in seconds.

Based on the above facts, answer the following questions.

48. Time at which velocity of the particle is maximum along positive direction of x-axis is
(A) 1 s (B) 2 s
(C) 3 s (D) 4 s

49. Distance travelled by the particle during time interval $t = 3$ s to $t = 5$ s is
(A) 2 m (B) 5 m
(C) 12 m (D) 10 m

50. Average speed of the particle during time interval $t = 0$ s to $t = 6$ s is
(A) $2\ \text{ms}^{-1}$ (B) Zero
(C) $4\ \text{ms}^{-1}$ (D) None of these

Comprehension 16

A particle is moving along x-axis and its initial velocity is $27\ \text{ms}^{-1}$. The acceleration of particle is given by the relation $a = (-6t)\ \text{ms}^{-2}$, where t is in seconds. At $t = 0$ particle is at $x = 0$.

Based on the above facts, answer the following questions.

51. The velocity of particle, when it travels 26 m is
(A) $21\ \text{ms}^{-1}$ (B) $15\ \text{ms}^{-1}$
(C) $24\ \text{ms}^{-1}$ (D) $18\ \text{ms}^{-1}$

52. Maximum value of velocity along positive x-direction is
(A) $35\ \text{ms}^{-1}$ (B) $33\ \text{ms}^{-1}$
(C) $27\ \text{ms}^{-1}$ (D) $30\ \text{ms}^{-1}$

53. Maximum value of displacement along positive x-direction is
(A) 54 m (B) 27 m
(C) 120 m (D) None of these

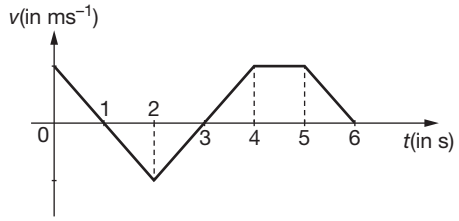
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	●	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. From the v-t graph shown in figure, match the quantities in **COLUMN-I** to their respective conclusions in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) between $t = 0$ and $t = 1$ s	(p) $v = 0$
(B) between $t = 1$ s and $t = 2$ s	(q) $a = 0$
(C) between $t = 2$ s and $t = 3$ s	(r) $v \neq 0$
(D) between $t = 3$ s and $t = 4$ s	(s) $a \neq 0$
(E) between $t = 4$ s and $t = 5$ s	(t) accelerated
(F) between $t = 5$ s and $t = 6$ s	(u) decelerated
(G) at $t = 1$ s and at $t = 3$ s	

2. For a particle moving rectilinearly, the x varies with t as per the equation $x = -5t^2 + 20t + 10$, where x is in metre and t is in second.

COLUMN-I	COLUMN-II
(A) Average speed, in ms^{-1}, from $t = 0$ to $t = 4$ s	(p) 20
(B) Average velocity, in ms^{-1}, from $t = 0$ to $t = 4$ s	(q) 10
(C) Acceleration, in ms^{-2}, at $t = 4$ s	(r) Zero
(D) Speed, in ms^{-1}, at $t = 4$ s	(s) –4
	(t) None of these

(*Continued*)

3. Match the quantities in **COLUMN-I** with the corresponding expressions in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Velocity	(p) $\frac{d\vec{v}}{dt}$
(B) Tangential acceleration	(q) $\frac{d\vec{r}}{dt}$
(C) Acceleration	(r) $\frac{d\lvert\vec{v}\rvert}{dt}$
(D) Instantaneous speed	(s) $\frac{d^2\vec{r}}{dt^2}$
	(t) $\left\lvert\frac{d\vec{v}}{dt}\right\rvert$
	(u) None of these

4. A particle moves such that its x coordinate is related to the time t by the relation $t = \sqrt{x} + 3$, where x is in metre, t is in second. Based on this information, match the values in **COLUMN-I** (in SI units) to their respective quantities for the particles motion given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 0	(p) Acceleration at $t = 5$ s.
(B) 2	(q) Average speed from $t = 0$ to $t = 6$ s.
(C) 3	(r) Velocity at the point of reversal of motion.
(D) 18	(s) Total distance travelled from $t = 0$ to $t = 6$ s.
	(t) Displacement from $t = 0$ to $t = 6$ s.

5. A man can row a boat with $4\ \text{kmh}^{-1}$ in still water. The man wishes to cross the river of width 4 km having a water current of $2\ \text{kmhr}^{-1}$. To cross the river with zero drift he swims making at an angle α degree with the current flow taking a time t_1 minutes to cross the river.

Now he wishes to cross the river in the shortest time t_2 minutes making an angle β degree with the river flow.

Further he takes a time t_3 minutes to row 2 km upstream and then downstream back to the start point. Assuming all the cases to be independent of each other, the man to start from the river bank from the same point in the first two cases and from the mid-point of the river in the third case, match the quantities in **COLUMN-I** to the values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) α	(p) $40\sqrt{3}$
(B) β	(q) 60
(C) t_1	(r) 80
(D) t_2	(s) 90
(E) t_3	(t) 120
	(u) Zero

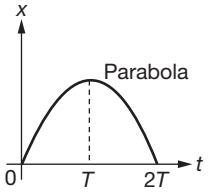
6. For one dimensional motion if v_{av} be the average speed, $\vec{v}_{av}$ be the average velocity, v_{inst} be the instantaneous speed, $\vec{v}_{\text{inst}}$ be the instantaneous velocity and v be the speed, then match the following

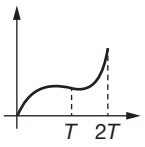
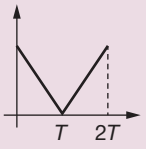
COLUMN-I	COLUMN-II
(A) $\vec{v}_{\text{inst}} = \vec{v}_{av}$	(p) for uniform motion in any direction
(B) $\lvert\vec{v}_{\text{inst}}\rvert = v$	(q) for uniform motion in given direction
(C) $v_{\text{inst}} = v_{av}$	(r) Always true
(D) $\lvert\vec{v}_{\text{inst}}\rvert < v$	(s) Never true

7. Match the following

COLUMN-I	COLUMN-II
(A) Motion of dropped ball	(p) Two dimensional motion
(B) Motion of a snake	(q) Three dimensional motion
(C) Motion of a bird	(r) One dimensional motion
(D) Earth	(s) Absolute rest

8. The displacement-time graph of a body moving on a straight line is given by



COLUMN-I	COLUMN-II
(A) Velocity – time graph	(p) 
(B) Acceleration-time graph	(q) 

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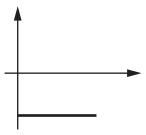
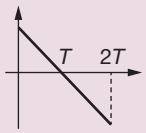
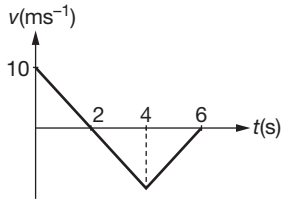
COLUMN-I	COLUMN-II
(C) Distance – time graph	(r) 
(D) Speed – time graph	(s) 

9. For the velocity-time graph shown in figure, in a time interval from $t = 0$ to $t = 6$ s, match the following



COLUMN-I	COLUMN-II
(A) Change in velocity	(p) $-\frac{5}{3}$ SI unit
(B) Average acceleration	(q) –20 SI unit
(C) Total displacement	(r) –10 SI unit
(D) Acceleration at $t = 3$ s	(s) –5 SI unit

10. A balloon rises up with constant net acceleration of $10\ \text{ms}^{-2}$. After 2 s a particle drops from the balloon. After further 2 s match the following $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

COLUMN-I	COLUMN-II
(A) Height of particle ground	(p) Zero
(B) Speed of particle	(q) 10 SI units
(C) Displacement of particle	(r) 40 SI units
(D) Acceleration of particle	(s) 20 SI units

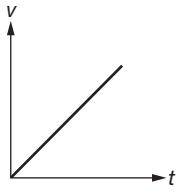
11. A body accelerates from rest for time t_1 at a constant rate α for distance x then it decelerates at constant rate β for time t_2 and covers distance y in this time and come at rest. If all quantities are in SI units, then match the following columns.

COLUMN-I	COLUMN-II
(A) $\frac{x}{y}$	(p) $\frac{t_1}{t_2}$
(B) $\frac{\alpha}{\beta}$	(q) $\frac{t_2}{t_1}$
(C) average speed for whole journey	(r) $\sqrt{\frac{2\alpha\beta}{\alpha+\beta}(x+y)}$
(D) Maximum speed attained in it whole journey	(s) $\sqrt{\frac{\alpha\beta}{\alpha+\beta}\left(\frac{x+y}{2}\right)}$

12. The equation of one dimensional motion of particle is described in **COLUMN-I.** At $t = 0$, particle is at origin and at rest. Match the **COLUMN-I** with the statements in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $x = (3t^2 + 2)$ m	(p) velocity of particle at $t = 1$ s is 8 ms^{-1}
(B) $v = 8t\ \text{ms}^{-1}$	(q) particle moves with uniform acceleration
(C) $a = 16t$	(r) particle moves with variable acceleration
(D) $v = 6t - 3t^2$	(s) particle will change its direction some time

13. v-t graph of a particle moving along positive direction x is shown in figure. Match the items in **COLUMN-I** with the respective answers in **COLUMN-II**.

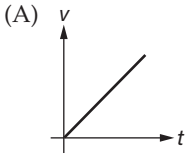
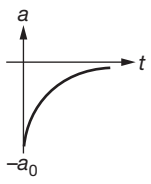
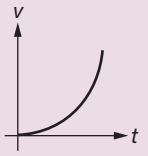
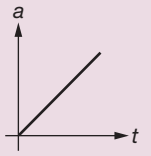
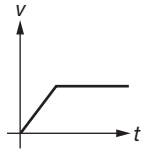
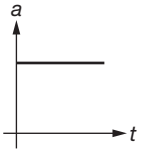
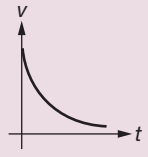
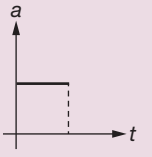


COLUMN-I	COLUMN-II
(A) a-x graph	(p) Parabola

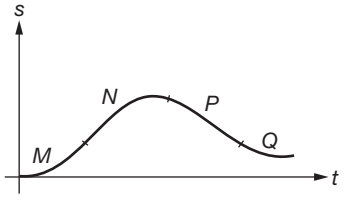
(*Continued*)

COLUMN-I	COLUMN-II
(B) v-x graph	(q) Circle
(C) a-t graph	(r) Straight line
(D) a-v graph	(s) Ellipse

14. Match the v-t graphs in **COLUMN-I** with the respective a-t graphs in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) 
(B) 	(q) 
(C) 	(r) 
(D) 	(s) 

15. Let us call a motion, A when velocity is positive and increasing. A^{-1} when velocity is negative and increasing. R when velocity is positive and decreasing and R^{-1} when velocity is negative and decreasing. Now match the following two tables for the given s-t graph

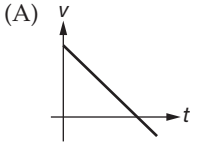
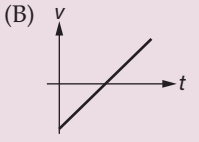
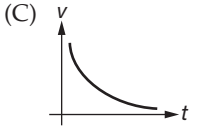
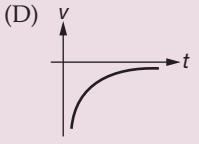


COLUMN-I	COLUMN-II
(A) M	(p) A^{-1}
(B) N	(q) R^{-1}
(C) P	(r) A
(D) Q	(s) R

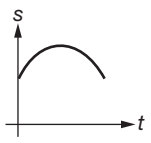
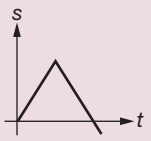
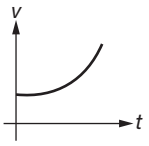
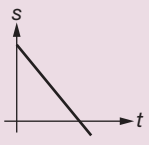
16. In the s-t equation $(s = 10 + 20t - 5t^2)$ match the following

COLUMN-I	COLUMN-II
(A) Distance travelled in 3 s	(p) –20 unit
(B) Displacement in 1 s	(q) 15 unit
(C) Initial acceleration	(r) 25 unit
(D) Velocity at 4 s	(s) –10 unit

17. The velocity time graphs for a particle moving along a straight line is given in each situation of **COLUMN–I**. Match the graph in **COLUMN–I** with corresponding statements in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) Speed of particle is continuously decreasing.
(B) 	(q) Magnitude of acceleration of particle is decreasing with time.
(C) 	(r) Direction of acceleration of particle does not change.
(D) 	(s) Magnitude of acceleration of particle does not change.
	(t) Acceleration is always opposite to the direction of velocity.

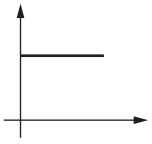
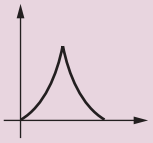
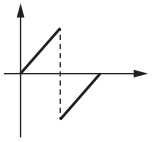
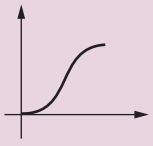
18. Match the statements in **COLUMN–I** with corresponding graphs in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Particle moving with constant speed.	(p) 
(B) Particle moving with increasing acceleration.	(q) 
(C) Particle moving with constant negative acceleration.	(r) 
(D) Particle moving with zero acceleration.	(s) 

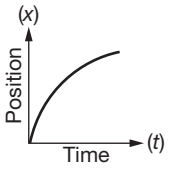
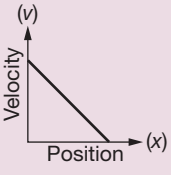
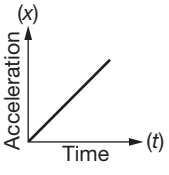
19. The motion of an object over time can often be communicated by graphs of its distance, velocity or acceleration with time. Different features of these graphs correspond to quantities of the motion. Match each quantity in the **COLUMN-I** with its graphical manifestation in the **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Distance travelled Δd	(p) Slope of a distance-time graph
(B) Velocity change Δv	(q) Slope of velocity-time graph
(C) Velocity v	(r) Area under a velocity-time graph
(D) Acceleration a	(s) Area under an acceleration-time graph.

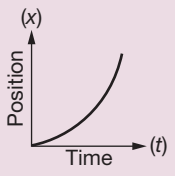
20. A particle is dropped vertically downward under gravity. Consider the downward direction as positive and the collision of the ball with the ground to be elastic, match the statements in **COLUMN–I** with corresponding graphs in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) The distance travelled by particle varies with time as	(p) 
(B) Velocity of particle changes with time as	(q) 
(C) Displacement of particle depends on time as	(r) 
(D) Dependency of acceleration on time is given by	(s) 

21. A particle is moving along x-direction in four ways. Different graphs is plotted in **COLUMN-I**.

COLUMN-I	COLUMN-II
(A) 	(p) Variable velocity
(B) 	(q) Positive acceleration
(C) 	(r) Negative acceleration

(*Continued*)

COLUMN-I	COLUMN-II
(D) 	(s) Constant speed

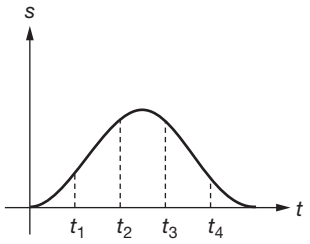
22. Match the following

COLUMN-I	COLUMN-II
(A) Constant positive acceleration	(p) Speed may increase
(B) Constant negative acceleration	(q) Speed may decrease
(C) Constant displacement	(r) Speed is zero
(D) Constant slope of a-t graph	(s) Speed must increase

23. Two ships A and B are 10 km apart on a line running from south to north. A is towards north of B and moving west with a speed of 20 kmh^{-1} while B is moving towards north with 20 kmh^{-1}. The distance of their closest approach in metres is l and the time in second taken to reach this position is t.

COLUMN-I	COLUMN-II
(A) l	(p) North-West
(B) t	(q) 7071
(C) $\vec{V}_{AB}$	(r) 900
(D) $\vec{V}_{BA}$	(s) South-East

24. A particle is moving in straight line and its displacement versus time graph is as shown in figure. **COLUMN-I** contains different instant and **COLUMN-II** contains values of acceleration and velocities at those instants. Match them



COLUMN-I	COLUMN-II
(A) t_1	(p) $a > 0$
(B) t_2	(q) $a < 0$
(C) t_3	(r) $v > 0$
(D) t_4	(s) $v < 0$
	(t) None

25. A particle is moving along x-axis. Its x-coordinate is varying with time as $x = -20t + 5t^2$. For the given equation, match the following columns.

COLUMN-I	COLUMN-II
(A) At what time particle changes its direction of motion	(p) 1 s
(B) At what time magnitude of velocity and acceleration are equal	(q) 2 s
(C) In how much time, distance travelled by the particle becomes 25 m	(r) 3 s
(D) In how much time the displacement of the particle becomes 15 m	(s) 4 s
	(t) None of the above

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. If the body covers equal displacements in successive intervals of time t_1, t_2 and t_3 then show that $\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{k}{t_1+t_2+t_3}$. Find k.

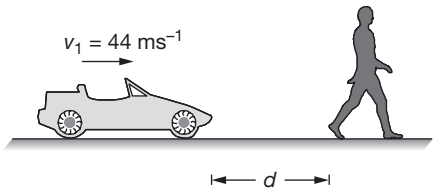
2. Between two stations a train accelerates uniformly at first, then moves with a constant speed and finally retards uniformly. If the ratios of the time taken are $1:8:1$ and the greatest speed attained by the train is $60\ \text{kmh}^{-1}$, find the average speed, in ms^{-1}, over the whole journey.

3. A sphere is fired downwards into a medium with an initial speed of $27\ \text{ms}^{-1}$. If it experiences a deceleration of $a=(-6t)\ \text{ms}^{-2}$, where t is in seconds, determine the distance, in metre, travelled before it stops.

4. A particle travels along a straight line such that in 2 s it moves from an initial position $x_A=+0.5$ m to a position $x_B=-1.5$ m. Then in another 4 s it moves from x_B to $x_C=+2.5$ m. Determine the particle's average speed, in ms^{-1}, during the 6 s time interval.

5. A particle moving with constant acceleration along a straight line covers the distance between two points 80 m apart in 10 s. Its speed as at passes second point is $18\ \text{ms}^{-1}$.
(a) What is its speed, in ms^{-1}, at the first point?
(b) What is its acceleration in ms^{-2}?
(c) At what prior distance, in m and time, in s from first point, the particle reverses its direction of motion?
(d) What is the total distance travelled, in m, during this 10 s?

6. Tests reveal that a normal driver takes about 0.75 s before he or she can react to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are travelling on a straight road at $44\ \text{ms}^{-1}$ and their cars can decelerate at $2\ \text{ms}^{-2}$, determine the shortest stopping distance (d) for each, in metre, from the moment they see the pedestrians.



7. A bus starts from rest with a constant acceleration of $5\ \text{ms}^{-2}$. At the same time a car travelling with a constant velocity of $50\ \text{ms}^{-1}$ overtakes and passes the bus. Find
(a) at what distance, in metre, will the bus overtake the car?
(b) how fast, in ms^{-1}, will the bus be travelling then?

8. A truck travelling along a straight road at a constant speed of $72\ \text{kmh}^{-1}$ passes a car at time $t=0$ moving much slower. At the instant the truck passes the car, the car starts accelerating at constant $1\ \text{msec}^{-2}$ and overtake the truck 0.6 km further down the road, from where the car moves uniformly. Find the distance between them, in metre, at time $t=50$ s from the start.

9. A train travelling at $72\ \text{kmh}^{-1}$ is checked by track repairs. It retards uniformly for 200 m, covering the next 400 m at constant speed and accelerates uniformly to $72\ \text{kmh}^{-1}$ in a further 600 m. If the time at the constant lower speed is equal to the sum of the times taken in retarding and accelerating, find the total time, in minutes, taken.

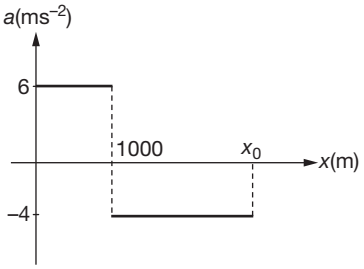
10. At the instant the traffic light turns green, a car starts with a constant acceleration of $2\ \text{ms}^{-2}$. At the same instant a truck travelling with a constant speed of $10\ \text{ms}^{-1}$, overtakes and passes the car. How far beyond the starting point, in m, with the car overtake the truck? How fast, in ms^{-1}, will the car be travelling at that instant?

11. A bullet fired into a fixed target looses half of its velocity after penetrating 3 cm. How much further, in cm, it will penetrate before coming to rest assuming that it faces constant resistance to motion.

12. A body travels 200 cm in the first two second and 220 cm in the next four second. What will be the velocity, in cms^{-1} at the end of seventh second from the start?

13. A particle moves with uniform acceleration a. If v_1, v_2 and v_3 be the average velocities in three successive intervals of time t_1, t_2 and t_3 respectively, then find the value of $\frac{(v_1-v_2)(t_3+t_2)}{(v_2-v_3)(t_2+t_1)}$.

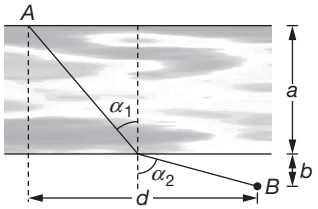
14. A sports car travels along a straight road with an acceleration-deceleration described by the graph. If the car starts from rest, determine the distance x_0, in m, the car travels until it stops.



15. A balloon rises from rest on the ground with constant acceleration $\frac{g}{8}$. A stone is dropped when the balloon has risen to a height H metre. The time taken by the stone to reach the ground is given by $x\sqrt{\frac{H}{g}}$. Find x.

16. As a train accelerates uniformly it passes successive kilometre marks while travelling at velocities of $2\ \text{ms}^{-1}$ and then $10\ \text{ms}^{-1}$. Determine the train's velocity, in ms^{-1}, when it passes the next kilometre mark and the time it takes, in s, to travel the 2 km distance.

17. From a point A on bank of a channel with still water a person must get to a point B on the opposite bank. All the distances are shown in figure. The person uses a boat to travel across the channel and then walks along the bank to point B. The velocity of the boat is v_1 and the velocity of the walking person is v_2. If $v_1 = 3\sqrt{3}\ \text{ms}^{-1}$, $\alpha_1 = 30°$ and $\alpha_2 = 60°$, then for what value of v_2, in ms^{-1}, the person takes a minimum time to go from A to B.



18. A person walks up a stationary 15 m long escalator in 90 s. When standing on the same escalator, now moving, the person is carried up in 60 s. How much time, in seconds, would it take that person to walk up the moving escalator? Does the answer depend on the length of the escalator?

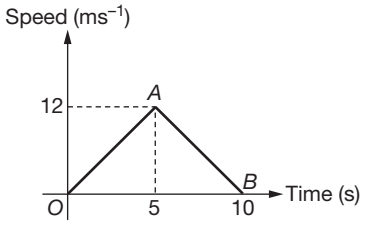
19. The driver of the train A moving with a speed of $144\ \text{kmh}^{-1}$ sights another train B 1 km ahead of him. The train B is moving with a uniform speed of $108\ \text{kmh}^{-1}$. The driver of the train A immediately applies brakes producing a constant retardation and just manages to avoid a collision. What is the retardation of the train A, in cms^{-2}? For how long is this retardation produced?

20. A sailor in a boat, which is going due east with a speed of $8\ \text{ms}^{-1}$ observes that a submarine is heading towards north at a speed of $12\ \text{ms}^{-1}$ and sinking at a rate of $2\ \text{ms}^{-1}$. The commander of submarine observes a helicopter ascending at a rate of $5\ \text{ms}^{-1}$ and heading towards west with $4\ \text{ms}^{-1}$. Find the actual speed of the helicopter and its speed with respect to boat, both in ms^{-1}.

21. The speed-time graph of a particle moving along a fixed direction is shown in figure. Obtain the distance travelled by the particle, in metre and the average speed of the particle in ms^{-1} between

(a) $t = 0$ to 10 s

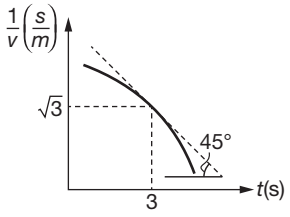
(b $t = 2$ to 6 s.



22. At the moment $t = 0$, a particle leaves the origin and moves in the positive direction of the x-axis. Its velocity varies with time as $v = v_0\left(1 - \frac{t}{5}\right)$ where $v_0 = 10\ \text{cms}^{-1}$ is the initial speed of the particle. The particle will be at the distance of 10 cm from the origin at three different instants. Find out the approximate time interval between the second and the third instant.

23. An athlete takes 2 s to reach his maximum speed of $36\ \text{kmh}^{-1}$. The magnitude of his average acceleration (in ms^{-2}) is .....

24. The diagram shows variation of $\frac{1}{v}$ with respect to time (where v is in ms^{-1}).



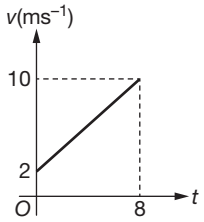
Find the instantaneous acceleration $\left(\text{in } \frac{m}{s^2}\right)$ of body at $t = 3$ s.

25. A ball is thrown upwards from the foot of a tower. The ball crosses the top of tower twice after an interval of 4 seconds and the ball reaches ground after 8 seconds, then the height of tower in meters is $(g = 10 \text{ ms}^{-2})$

26. An insect moves with a constant velocity v from one corner of a room to other corner which is opposite of the first corner along the largest diagonal of room. If the insect cannot fly and dimensions of room is $a \times a \times a$, then the minimum time in which the insect can move is $\frac{a}{v}$ times the square root of a number n, then n is equal to?

27. A body moves with constant acceleration covers 16 m and 24 m in successive intervals of 4 s and 2 s. Then its acceleration in ms^{-2} is .....

28. Figure shows the graph of velocity versus time for a particle going along x-axis. Initially at $t = 0$, particle is at $x = 3$ m. The position of the particle at $t = 2$ s (in m) is .....



29. A swimmer jumps from a bridge over a canal and swims 1 km up stream. After that first km, he passes a floating cork. He continues swimming for half an hour and then turns around and swims back to the bridge. The swimmer and the cork reach the bridge at the same time. Assuming the swimmer had been swimming at a constant speed, calculate how fast does the water in the canal flow in kmh^{-1}.

30. A car travelling at 60 kmh^{-1} over takes another car travelling at 42 kmh^{-1}. Assuming each car to be 5 m long. Find the time taken during the over take. (in sec)

31. A particle is moving on a straight line with constant retardation of 1 ms^{-2}. What is the average speed of the particle on the last two meters before it stops? (in ms^{-1})

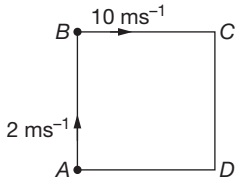
32. A bullet going with speed 16 ms^{-1} enters a concrete wall and penetrates a distance of 0.4 m before coming to rest. Then the time taken during the retardation is $x \times 10^{-2}$ s. Find x.

33. A baseball is moving at 25 ms^{-1} when it is struck by a bat and moves off in the opposite direction at 35 ms^{-1}. If the impact lasted 0.010 s, find the baseball's acceleration during the impact. (in kms^{-2})

34. A point moves with uniform acceleration and its initial speed and final speed are 2 ms^{-1} and 8 ms^{-1} respectively then, the space average of velocity (in ms^{-1}) over the distance moved is .....

35. A man is running with a speed 8 ms^{-1} constant in magnitude and direction passes under a lantern hanging at a height 10 m above the ground. Find the velocity which the edge of the shadow of the man's head moves over the ground with if his height is 2 m.

36. Two men P and Q are standing at corners A and B of square $ABCD$ of side 8 m. They start moving along the tank with constant speed 2 ms^{-1} and 10 ms^{-1} respectively. Find the time, in second, when they will meet for the first time.

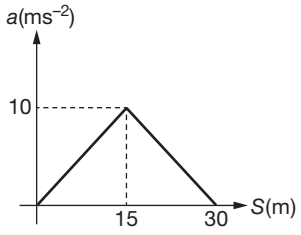


37. A particle starting from rest undergoes acceleration given by $a = |t - 2| \text{ ms}^{-2}$ where t is time in sec. Velocity of particle after 4 sec is .....

38. A ball is thrown upward from the edge of a cliff with an initial velocity of 6 ms^{-1}. How fast is it moving half second later? $(g = 10 \text{ ms}^{-2})$

39. A car goes from 20 to 30 kmh^{-1} in 1.5 s. At the same acceleration, how long will it take the car to go from 30 to 36.7 kmh^{-1}? (in sec)

40. The particle moves with rectilinear motion given the acceleration-displacement $(a\text{-}S)$ curve is shown in figure. If the initial velocity is 10 ms^{-1} then velocity of particle after particle has travelled 30 m divided by 5 is equal to.

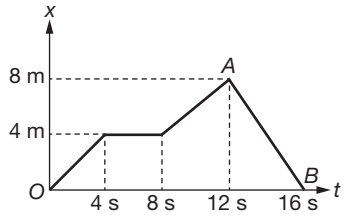


41. A police jeep is chasing a culprit going on a motor bike. The motor bike crosses a turning at a speed of $72\ \text{kmh}^{-1}$. The jeep follows it a speed of $90\ \text{kmh}^{-1}$ crossing the turning ten seconds later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike? (in km)

42. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of $1\ \text{ms}^{-2}$ and projection velocity in the vertical direction is $9.8\ \text{ms}^{-1}$. How far behind the boy will the ball fall on the car? (in m)

43. The speed of a motor launch with respect to the water is $v = 5\ \text{ms}^{-1}$, the speed of stream $u = 3\ \text{ms}^{-1}$. When the launch began travelled $3.6\ \text{km}$ up stream, turned about and caught up with the float. How long is it before the launch reaches the float again? (Find answer in hour).

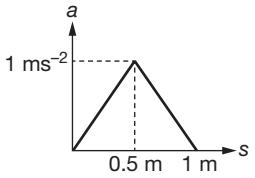
44. Figure shows the graph of the x-coordinate of a particle going along the x-axis as function of time.



The speed of particle at $t = 12.5\ \text{s}$ (in ms^{-1}) is .....

45. A particle moving in a straight line covers half the distance with speed of $3\ \text{ms}^{-1}$. The other half of the distance is covered in two equal time intervals with a speeds of $4.5\ \text{ms}^{-1}$ and $7.5\ \text{ms}^{-1}$, respectively. Find the average speed of the particle during this motion.

46. A body initially at rest moving along x-axis in such a way so that its acceleration displacement plot is as shown in figure. What will be the maximum velocity of particle in ms^{-1}.



ARCHIVE: JEE MAIN

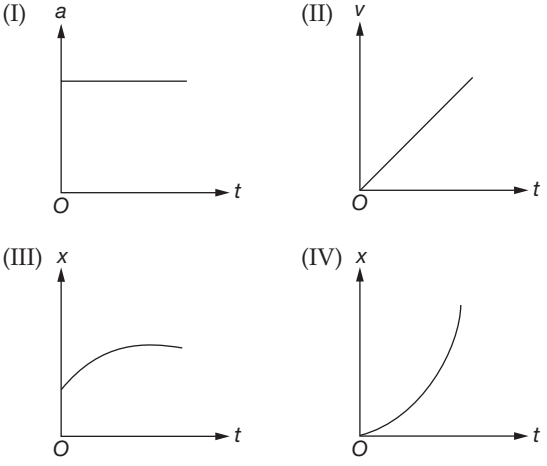
1. **[Online April 2019]**

Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}\ \text{kmhr}^{-1}$ where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of $80\ \text{km}$ east and $150\ \text{km}$ north of Ship A and is sailing towards west at $10\ \text{kmhr}^{-1}$. A will be at minimum distance B in

(A) 2.2 hr (B) 4.2 hr
(C) 3.2 hr (D) 2.6 hr

2. **[Online April 2019]**

A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)



(A) (I) (B) (I), (II), (III)
(C) (I), (II), (IV) (D) (II), (III)

3. [Online April 2019]
The stream of a river is flowing with a speed of 2 kmh^{-1}. A swimmer can swim at a speed of 4 kmh^{-1}. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?
(A) 60° (B) 90°
(C) 150° (D) 120°

4. [Online April 2019]
The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$?
(A) 50 (B) 100
(C) 40 (D) 25

5. [Online April 2019]
The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be
(A) $a + \frac{b^2}{4c}$ (B) $a + \frac{b^2}{3c}$
(C) $a + \frac{b^2}{2c}$ (D) $a + \frac{b^2}{c}$

6. [Online April 2019]
A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to
(A) 0.4 ms^{-1} (B) 0.7 ms^{-1}
(C) 0.3 ms^{-1} (D) 0.1 ms^{-1}

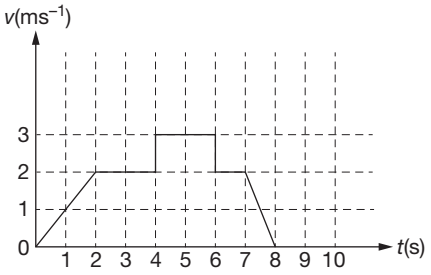
7. [Online April 2019]
A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$).
(A) $b^2\tau$ (B) $\frac{b^2\tau}{4}$
(C) $\frac{b^2\tau}{2}$ (D) $\frac{b^2\tau}{\sqrt{2}}$

8. [Online January 2019]
A particle is moving with a velocity $\vec{v} = k(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is
(A) $y^2 = x + \text{constant}$ (B) $y = x^2 + \text{constant}$
(C) $y^2 = x^2 + \text{constant}$ (D) $xy = \text{constant}$

9. [Online January 2019]
In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed v more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then v is equal to
(A) $\frac{a_1 + a_2}{2}t$ (B) $\frac{2a_1a_2}{a_1 + a_2}t$
(C) $\sqrt{2a_1a_2}t$ (D) $\sqrt{a_1a_2}t$

10. [Online January 2019]
The position co-ordinates of a particle moving in a 3-D coordinate system is given by $x = a\cos(\omega t)$, $y = a\sin(\omega t)$ and $z = a\omega t$. The speed of the particle is
(A) $2a\omega$ (B) $\sqrt{2}a\omega$
(C) $\sqrt{3}a\omega$ (D) $a\omega$

11. [Online January 2019]
A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle time $t = 5$ s ?



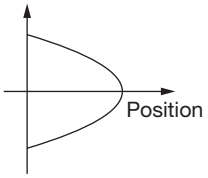
(A) 9 m (B) 6 m
(C) 10 m (D) 3 m

12. [Online January 2019]
A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction and (ii) in the opposite directions is
(A) $\frac{25}{11}$ (B) $\frac{5}{2}$
(C) $\frac{11}{5}$ (D) $\frac{3}{2}$

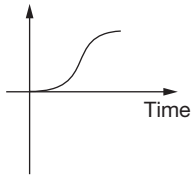
13. [2018]

All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up

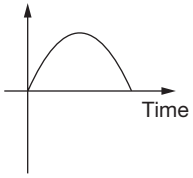
(A) Velocity



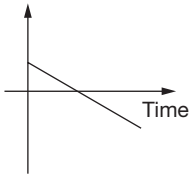
(B) Distance



(C) Position

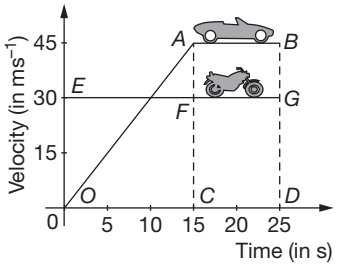


(D) Velocity



14. [Online 2018]

The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively



(A) 112.5 m and 15 s (B) 337.5 m and 25 s
(C) 225.5 m and 10 s (D) 112.5 m and 22.5 s

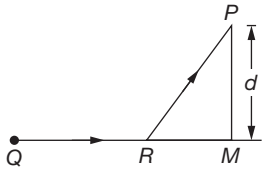
15. [Online 2018]

An automobile, travelling at 40 kmh^{-1}, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 kmh^{-1}, the minimum stopping distance, in metres, is (assume no skidding)

(A) 100 m (B) 75 m
(C) 160 m (D) 150 m

16. [Online 2018]

A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum?

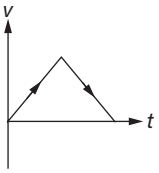


(A) $\frac{d}{2}$ (B) $\frac{d}{\sqrt{3}}$
(C) $\frac{d}{\sqrt{2}}$ (D) d

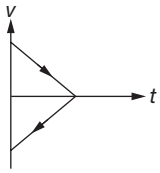
17. [2017]

A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity versus time?

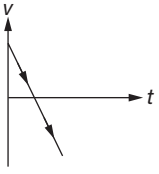
(A)



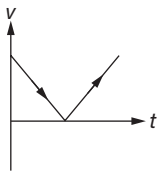
(B)



(C)



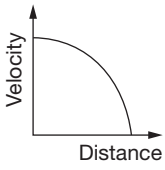
(D)



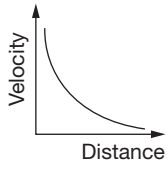
18. [Online 2017]

Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity?

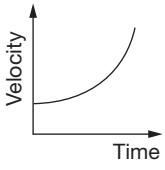
(A)



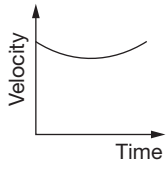
(B)



(C)

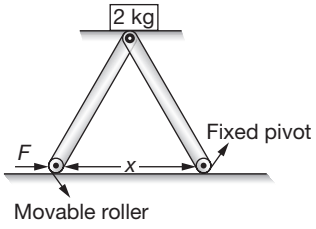


(D)



19. [Online 2017]

The machine as shown has 2 rods of length 1 m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot. As the roller goes back and forth, a 2 kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a

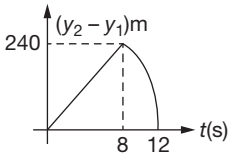


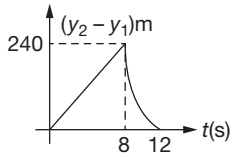
(A) speed which is $\frac{3}{4}^{\text{th}}$ of that of the roller when the weight is 0.4 m above the ground.
(B) constant speed.
(C) decreasing speed.
(D) increasing speed.

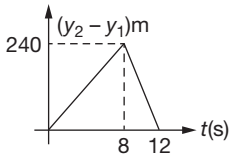
20. [Online 2017]
A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 ms^{-2} and the car has acceleration 4 ms^{-2}. The car will catch up with the bus after a time of

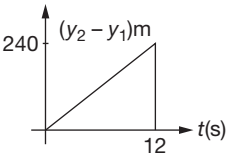
(A) $\sqrt{120}$ s (B) 15 s
(C) $10\sqrt{2}$ s (D) $\sqrt{110}$ s

21. [2015]
Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 ms^{-1} and 40 ms^{-1} respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ ms^{-2}$)
(The figures are schematic and not drawn to scale)

(A)


(B)


(C)


(D)


22. [2014]
From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is

(A) $gH = (n-2)u^2$ (B) $2gH = n^2u^2$
(C) $gH = (n-2)^2u^2$ (D) $2gH = nu^2(n-2)$

23. [2011]
An object moving with a speed of 6.25 ms^{-1}, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object, to come to rest, would be
(A) 1 s (B) 2 s
(C) 4 s (D) 8 s

24. [2010]
A particle is moving with velocity $\vec{v} = K\left(y\hat{i} + x\hat{j}\right)$, where K is a constant. The general equation for its path is

(A) $y^2 = x^2 + \text{constant}$ (B) $y = x^2 + \text{constant}$
(C) $y^2 = x + \text{constant}$ (D) $xy = \text{constant}$

25. [2009]
A particle has an initial velocity $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is

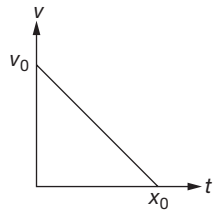
(A) 10 units (B) $7\sqrt{2}$ units
(C) 7 units (D) 8.5 units

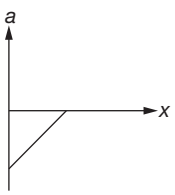
ARCHIVE: JEE ADVANCED

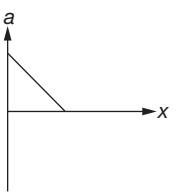
Single Correct Choice Type Problems

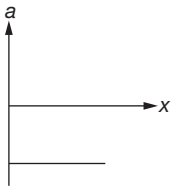
In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

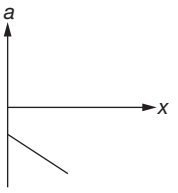
1. [IIT-JEE 2005]
The given graph shows the variation of velocity with displacement. Which one of the graph given below correctly represents the variation of acceleration with displacement?



(A) 

(B) 

(C) 

(D) 

2. [IIT-JEE 2004]

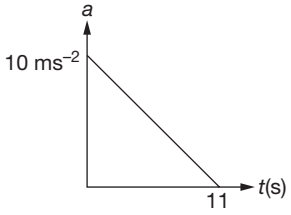
A small block slides without friction down an inclined plane starting from rest. Let S_n be the distance travelled from time $t = n-1$ to $t = n$. Then $\frac{S_n}{S_{n+1}}$ is

(A) $\frac{2n-1}{2n}$ (B) $\frac{2n+1}{2n-1}$

(C) $\frac{2n-1}{2n+1}$ (D) $\frac{2n}{2n+1}$

3. [IIT-JEE 2004]

A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be

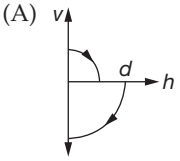


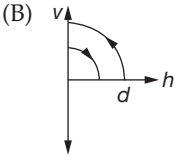
(A) 110 ms^{-1} (B) 55 ms^{-1}

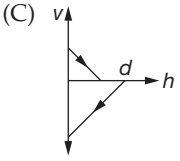
(C) 550 ms^{-1} (D) 660 ms^{-1}

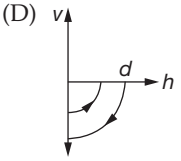
4. [IIT-JEE 2000]

A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $\frac{1}{2}d$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

(A) 

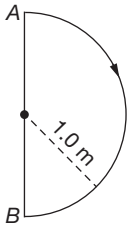
(B) 

(C) 

(D) 

5. [IIT-JEE 1999]

In 1.0 second a particle goes from point A to point B moving in a semi-circle of radius 1.0 metre (figure). The magnitude of the average velocity is

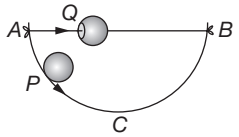


(A) 3.14 ms^{-1} (B) 2.0 ms^{-1}

(C) 1.0 ms^{-1} (D) ZERO

6. [IIT-JEE 1993]

A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of same mass as P is ejected from A at $t = 0$ along the horizontal string AB, with a speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B, then



(A) $t_P < t_Q$

(B) $t_P = t_Q$

(C) $t_P > t_Q$

(D) $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of chord } AB}$

7. [IIT-JEE 1988]

A boat which has a speed of $5\ \text{kmh}^{-1}$ in still water crosses a river of width 1 km along the shortest possible path in 15 minute. The velocity of the river water in kmh^{-1} is

(A) 1 (B) 3

(C) 4 (D) $\sqrt{41}$

8. [IIT-JEE 1983]

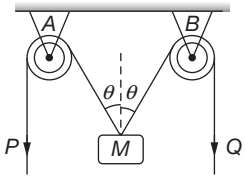
A river is flowing from west to east at a speed of 5 m per minute. A man on the south bank of the river, capable of swimming at 10 m per minute in still water wants to swim across the river to a point directly opposite in the shortest time. He should then swim in a direction

(A) 60° west of north (B) 30° east of north

(C) 30° west of north (D) 60° east of north

9. [IIT-JEE 1982]

In the arrangement shown in the figure, the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed



(A) $2U\cos\theta$ (B) $\dfrac{U}{\cos\theta}$

(C) $\dfrac{2U}{\cos\theta}$ (D) $U\cos\theta$

10. [IIT-JEE 1982]

A particle is moving Eastwards with a velocity of $5\ \text{ms}^{-1}$. In 10 s, the velocity changes to $5\ \text{ms}^{-1}$ Northwards. The average acceleration in this time is

(A) zero

(B) $\dfrac{1}{\sqrt{2}}\ \text{ms}^{-2}$ towards North-East

(C) $\dfrac{1}{\sqrt{2}}\ \text{ms}^{-2}$ towards North-West

(D) $\dfrac{1}{2}\ \text{ms}^{-2}$ towards North

Multiple Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [IIT-JEE 1999]

The co-ordinates of a particle moving in a plane are given by $x = a\cos(pt)$ and $y = b\sin(pt)$ where a, $b(<a)$ and p are positive constants of appropriate dimensions. Then

(A) the path of the particle is an ellipse.

(B) the velocity and acceleration of the particle are normal to each other at $t = \dfrac{\pi}{2p}$.

(C) the acceleration of the particle is always directed towards the focus.

(D) the distance travelled by the particle in time interval $t = 0$ to $t = \dfrac{\pi}{2p}$ is a.

2. [IIT-JEE 1993]

A particle of mass m moves on the x-axis as follows: it starts from rest at $t = 0$ from the point $x = 0$ and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate time $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle, then

(A) α cannot remain positive for all t in the interval $0 \le t \le 1$

(B) $|\alpha|$ cannot exceed 2 at any point in its path

(C) $|\alpha|$ must be ≥ 4 at some point or points in its path

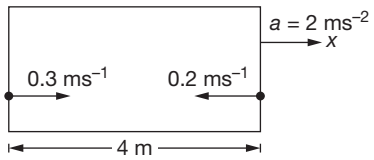
(D) α must change sign during the motion but no other assertion can be made with the information given

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

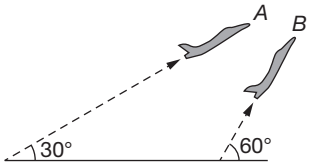
1. [JEE (Advanced) 2014]

A rocket is moving in a gravity free space with a constant acceleration of $2\ \text{ms}^{-2}$ along $+x$ direction (shown in figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of $0.3\ \text{ms}^{-1}$ relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of $0.2\ \text{ms}^{-1}$ from its right end relative to the rocket. The time in seconds when the two balls hit each.



2. [JEE (Advanced) 2014]

Airplanes A and B are flying with constant velocity in the same vertical plane at angles $30°$ and $60°$ with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



Assertion and Reasoning Type Problems

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [JEE (Advanced) 2008]

Statement-I: For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement-II: If the observer and the object are moving at velocities v_1 and v_2, respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $v_2 - v_1$.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Displacement, Velocity, Acceleration, Average Speed and Velocity)

1. $32\ \text{ms}^{-1}$, 67 m, 66 m

2. $\sqrt{\frac{as}{2}}$

3. $2\ \text{ms}^{-1}$

4. 30 m, $15\ \text{ms}^{-1}$

5. (a) ms^{-2}, ms^{-3}
(b) 1 s
(c) 82 m, –80 m
(d) at $t = 0$, $v = 0$ and $a = 6\ \text{ms}^{-2}$
at $t = 1$ s, $v = 0$ and $a = -6\ \text{ms}^{-2}$
at $t = 2$ s, $v = -12\ \text{ms}^{-1}$ and $a = -18\ \text{ms}^{-2}$
at $t = 3$ s, $v = -36\ \text{ms}^{-1}$ and $a = -30\ \text{ms}^{-2}$
at $t = 4$ s, $v = -72\ \text{ms}^{-1}$ and $a = -42\ \text{ms}^{-2}$

6. $135\ \text{ms}^{-1}$, $42\ \text{ms}^{-2}$

7. $20.6\ \text{ms}^{-1}$, 76°

8. $0.222\ \text{ms}^{-1}$, $2.22\ \text{ms}^{-1}$

9. (a) 14.125 m
(b) $1.75\ \text{ms}^{-1}$, $4.03\ \text{ms}^{-1}$

10. –27 m, 69 m

11. $-30\ \text{ms}^{-2}$

Test Your Concepts-II (Based on Constant Acceleration)

2. (a) $\left(\frac{\alpha\beta}{\alpha+\beta}\right)t$

(b) $\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$

3. $3x$

6. (a) $5\ \text{ms}^{-1}$
(b) $1.67\ \text{ms}^{-2}$
(c) 7.5 m

8. 0.74 s, $6.2\ \text{ms}^{-2}$

9. $3.45t_0$

10. 8 m

11. $\frac{3}{5}$

12. 4×10^{-4} s, $2\times10^{7}\ \text{ms}^{-2}$

13. (a) 80 s
(b) $90\ \text{kmh}^{-1}$
(c) 37 s

14. (a) 33.7 min
(b) 44.53 min

15. 24.5 s, 600 m

16. 89 m

17. $\frac{3}{4}$ h, $\frac{3}{2}$ h, No overtaking

18. $v_{av} = \frac{3v_0(v_1+v_2)}{4v_0+v_1+v_2}$

19. 2.27 s

20. 2280 m, Car A

21. $22.3\ \text{ms}^{-1}$, 1500 m

22. 20.6 s

23. (a) 15.8 s
(b) 391 m
(c) $29.5\ \text{ms}^{-1}$

24. $4\ \text{ms}^{-1}$ opposite to the direction of motion of the train.

25. $0.2\ \text{ms}^{-2}$, $0.8\ \text{ms}^{-1}$

Test Your Concepts-III (Based on Variable Acceleration)

1. $5\log_e\left(\frac{5}{4}\right)$ s

2. $v = \pm\sqrt{2}s$

3. $x = \frac{v_0}{k}(1-e^{-kt})$, $a = -kv_0e^{-kt}$

4. $s = 2v_0$

5. $\pm2\sqrt{6}\ \text{ms}^{-1}$, 4 m, $\pm4\sqrt{2}$ m

6. $v = \sqrt{v_0^2+2kt}$

7. $v = \frac{A}{B}(1-e^{-Bt})$

8. $v = -\frac{16}{(8t+1)^2}$, $a = \frac{256}{(8t+1)^3}$

9. (a) $\frac{5}{6}\ \text{ms}^{-1}$

(b) $-\frac{55}{6}\ \text{ms}^{-1}$

10. $v = 5(e^t-1)$, $x = 5(e^t-t-1)$

11. 80 kms^{-2}, 6.93 ms

12. $\left(4\hat{i}+8\hat{j}\right)\ \text{ms}^{-1}$

13. $h_{\max}=\frac{1}{2k}\log_e\left(\frac{g+kv_0^2}{g}\right)$

14. $-\frac{20}{3}\ \text{ms}^{-2}$

15. $v=\left(6t^2-2t^{3/2}\right)$, $x=\left(2t^3-\frac{4}{5}t^{5/2}+15\right)$

Test Your Concepts-IV (Based on Graph)

4. 114 m

5. 400 m

8. 46.47 ms^{-1}

10. 12.7 ms^{-1}, 22.8 ms^{-1}, 36.1 ms^{-1}

11. (a) $v=t^2-2t$

(b) 6.67 m

12. 10 ms^{-1}

Test Your Concepts-V (Based on Motion Under Gravity)

2. $\sqrt{2g_0R}$

3. $|v|=\frac{gt}{2}$

4. $10\sqrt{50}\ \text{ms}^{-1}$

7. $\frac{gn^2}{8}\left(\frac{gn-2u}{gn-u}\right)^2$

8. $\frac{u_0}{\sqrt{1+\frac{cu_0^2}{mg}}}$

9. $1.26\times10^3\ \text{ms}^{-2}$

10. (a) 216,000 metre

(b) 447.8 sec

11. $5\sqrt{13}\ \text{ms}^{-1}$

(a) 16.25 m

(b) 1.8 s

12. 73.302 metre

13. $\frac{v_0}{g}+\frac{t_0}{2}$

14. 45 m

15. (a) 75 m

(b) 4.1 s

16. 14.7 ms^{-1}, 19.6 metre

17. 1.5 s

Test Your Concepts-VI (Based on Planar Motion)

1. (a) 48 m

(b) $16\sqrt{2}\ \text{ms}^{-1}$

2. 4 m, 37.8 ms^{-2}

3. $k\omega t$

4. (a) $y=x-\alpha\frac{x^2}{k}$

(b) $v=k\sqrt{1+(1-2\alpha t)^2}$, $\vec{a}=-(2\alpha k)\hat{j}$

6. 201.25 ms^{-1}, 405 ms^{-2}

7. $\left(10\hat{i}+7\hat{j}\right)\ \text{ms}^{-1}$, $\left(12\hat{i}+10\hat{j}\right)$ m

8. 32 ms^{-1}

Test Your Concepts-VII (Based on Relative Velocity)

1. (a) 40 s

(b) 80 m

2. 60°

4. (a) 28°

(b) 1.48 hour

5. 3 kmhr^{-1}

6. (a) 0.7 s

(b) 1.3 m

7. $2v\sin(20°)$

8. $5\sqrt{2}$ km, 15 minutes

9. $2\sqrt{5}\ \text{ms}^{-2}$ at an angle of $\alpha=\tan^{-1}(2)$ NW

10. 3200 m

11. $a=\frac{2u-gt}{t}$

12. $\frac{u}{\sqrt{2}}$, at 45° perpendicular to stream flow

13. 8 ms^{-1}, 12°

14. (a) He should row at an angle $90°+\sin^{-1}\left(\frac{3}{5}\right)$ upstream.

(b) 160 s

15. (a) 3.65 s, 12.3 m
(b) 19.8 ms^{-1}

16. (a) 0.8 ms^{-1}
(b) 1.79 ms^{-1}
(c) 120° upstream

18. (a) $\frac{10}{\sin\theta}$ minute
(b) 10 minute
(c) $\frac{2}{3}$ km

19. $\tan^{-1}(3)$

20. Infinite, $\frac{3d}{2}$

Single Correct Choice Type Questions

1. B	**2.** D	**3.** C	**4.** D	**5.** A	**6.** B	**7.** B	**8.** B	**9.** C	**10.** A
11. D	**12.** C	**13.** B	**14.** A	**15.** C	**16.** C	**17.** C	**18.** C	**19.** A	**20.** D
21. B	**22.** C	**23.** A	**24.** A	**25.** B	**26.** C	**27.** D	**28.** C	**29.** D	**30.** D
31. C	**32.** B	**33.** D	**34.** D	**35.** D	**36.** C	**37.** A	**38.** A	**39.** D	**40.** C
41. C	**42.** A	**43.** B	**44.** C	**45.** C	**46.** C	**47.** A	**48.** D	**49.** B	**50.** C
51. B	**52.** D	**53.** B	**54.** A	**55.** D	**56.** C	**57.** B	**58.** A	**59.** C	**60.** D
61. A	**62.** A	**63.** C	**64.** A	**65.** A	**66.** D	**67.** D	**68.** C	**69.** C	**70.** C
71. D	**72.** A	**73.** D	**74.** C	**75.** B	**76.** D	**77.** B	**78.** B	**79.** C	**80.** D
81. A	**82.** C	**83.** B	**84.** C	**85.** B	**86.** D	**87.** C	**88.** B	**89.** A	**90.** C
91. A	**92.** A	**93.** A	**94.** D	**95.** D	**96.** B	**97.** C	**98.** B	**99.** D	**100.** C
101. B	**102.** B	**103.** A	**104.** C	**105.** B	**106.** A	**107.** C			

Multiple Correct Choice Type Questions

1. A, C, D	**2.** A, D	**3.** A, B, C	**4.** C, D	**5.** C, D
6. A, C, D	**7.** B, C	**8.** A, B	**9.** A, B, D	**10.** A, B, D
11. B, C	**12.** A, C	**13.** B, C	**14.** A, D	**15.** A, B, C, D
16. B, D	**17.** A, B, C, D	**18.** A, B, D	**19.** A, B, C	**20.** B, C, D
21. C, D	**22.** B, C, D	**23.** B, D	**24.** A, B, C, D	**25.** A, C, D
26. A, B, C	**27.** A, B, C	**28.** B, C	**29.** A, B, D	**30.** A, D
31. A, C	**32.** B, C, D	**33.** A, B, D	**34.** A, C	**35.** A, B, C
36. C, D	**37.** B, C, D	**38.** A, C	**39.** C, D	

Reasoning Based Questions

1. D	**2.** D	**3.** A	**4.** C	**5.** D	**6.** A	**7.** D	**8.** A	**9.** A	**10.** D
11. D	**12.** C								

Linked Comprehension Type Questions

1. B	**2.** D	**3.** C	**4.** B	**5.** C	**6.** B	**7.** B	**8.** D	**9.** B	**10.** D
11. A	**12.** C	**13.** D	**14.** C	**15.** A	**16.** B	**17.** B	**18.** A	**19.** D	**20.** D
21. B	**22.** C	**23.** B	**24.** A	**25.** A	**26.** B	**27.** C	**28.** A	**29.** C	**30.** A

31. C	**32.** C	**33.** C	**34.** D	**35.** D	**36.** B	**37.** A	**38.** D	**39.** A	**40.** B
41. C	**42.** B	**43.** C	**44.** D	**45.** A	**46.** A	**47.** C	**48.** B	**49.** C	**50.** B
51. C	**52.** C	**53.** A							

Matrix Match/Column Match Type Questions

1. A → (s, u)	B → (s, t)	C → (s, u)	D → (s, t)	E → (q, r)	F → (s, u)	G → (p, s)
2. A → (q)	B → (r)	C → (s)	D → (p)			
3. A → (q)	B → (r)	C → (p, s)	D → (t)			
4. A → (r, t)	B → (p)	C → (q)	D → (s)			
5. A → (t)	B → (s)	C → (p)	D → (q)	E → (r)		
6. A → (q)	B → (r)	C → (p, q)	D → (s)			
7. A → (r)	B → (p)	C → (q)	D → (p)			
8. A → (s)	B → (r)	C → (p)	D → (q)			
9. A → (r)	B → (p)	C → (r)	D → (s)			
10. A → (r)	B → (p)	C → (s)	D → (q)			
11. A → (p)	B → (q)	C → (s)	D → (r)			
12. A → (q)	B → (p, q)	C → (p, r)	D → (r, s)			
13. A → (r)	B → (p)	C → (r)	D → (r)			
14. A → (r)	B → (q)	C → (s)	D → (p)			
15. A → (r)	B → (s)	C → (p)	D → (q)			
16. A → (r)	B → (q)	C → (s)	D → (p)			
17. A → (r, s)	B → (r, s)	C → (p, q, r, t)	D → (p, q, r, t)			
18. A → (q, s)	B → (r)	C → (p)	D → (s)			
19. A → (r)	B → (s)	C → (p)	D → (q)			
20. A → (s)	B → (r)	C → (q)	D → (p)			
21. A → (p, r)	B → (p, r)	C → (p, q)	D → (p, q)			
22. A → (p, q)	B → (p, q)	C → (r)	D → (p, q)			
23. A → (q)	B → (r)	C → (s)	D → (p)			
24. A → (p, r)	B → (q, r)	C → (q)	D → (q, s)			
25. A → (q)	B → (r)	C → (r)	D → (p, r)			

Integer/Numerical Answer Type Questions

1. 3	**2.** 15	**3.** 54	**4.** 1	**5.** (a) 2 (b) 2 (c) 1, 1 (d) 82
6. 517, 616	**7.** (a) 1000 (b) 100	**8.** 300	**9.** 2	**10.** 100, 20
11. 1	**12.** 10	**13.** 1	**14.** 2500	**15.** 2
16. 14, 250	**17.** 9	**18.** 36	**19.** 5, 200	**20.** 13, 13
21. (a) 60, 6 (b) 10.8, 9	**22.** 2	**23.** 5	**24.** 3	**25.** 60
26. 5	**27.** 4	**28.** 9	**29.** 1	**30.** 2
31. 1	**32.** 5	**33.** 6	**34.** 5.6	**35.** 10
36. 3	**37.** 4	**38.** 1	**39.** 1	**40.** 4
41. 1	**42.** 2	**43.** 1	**44.** 2	**45.** 4
46. 1				

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1. D	**2.** C	**3.** D	**4.** A	**5.** B	**6.** B	**7.** C	**8.** C	**9.** D	**10.** B
11. A	**12.** C	**13.** B	**14.** D	**15.** C	**16.** B	**17.** C	**18.** A	**19.** C	**20.** C
21. A	**22.** D	**23.** B	**24.** A	**25.** B					

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Single Correct Choice Type Problems

1. A	**2.** C	**3.** B	**4.** A	**5.** B	**6.** A	**7.** B	**8.** C	**9.** B	**10.** C

Multiple Correct Choice Type Problems

1. A, B, C	**2.** A, D

Integer/Numerical Answer Type Questions

1. 2 or 8	**2.** 5

Assertion and Reasoning Type Problems

1. B

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CHAPTER

5 Kinematics II

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

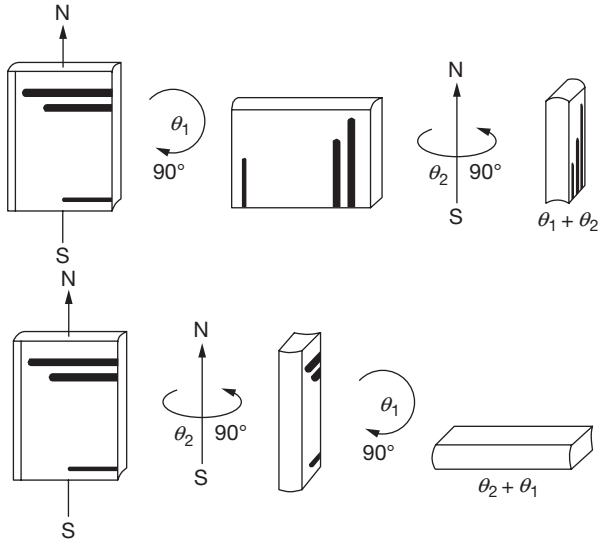
(a) Curvilinear Motion and Angular Parameters
(b) Angular, Centripetal and Tangential Acceleration
(c) Kinematics of Circular Motion
(d) Motion of Particle in a Curved Track
(e) Radius of Curvature
(f) Horizontal Projectile and Properties
(g) Oblique Projectile and Properties
(h) Relative Motion between two Projectiles
(i) Condition of Collision between two Projectiles
(j) Motion of Projectile Up and Down an Inclined Plane

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

CURVILINEAR MOTION

INTRODUCTION

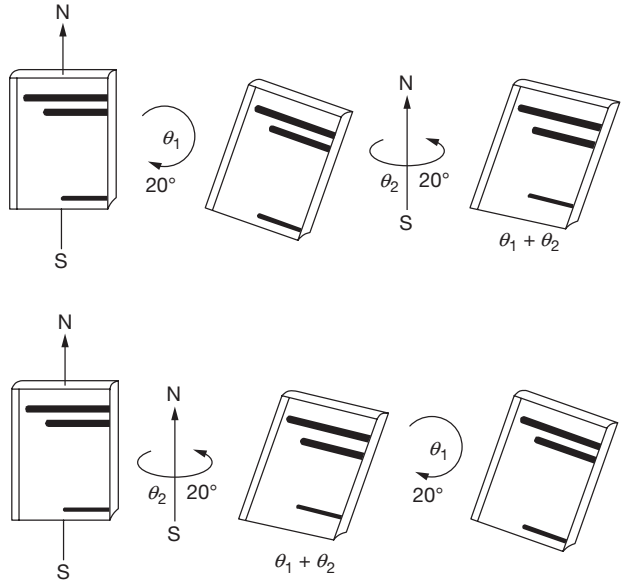
The linear displacement $(\vec{x})$, velocity $(\vec{v})$ and acceleration $(\vec{a})$ are vectors. Correspondingly, the angular counterparts may also be vectors, because in addition to the magnitude, we must also specify a direction for them, namely, the direction of **Axis of Rotation** in space. Let us first have our discussion for θ, which commonly is called Angular Displacement but is not. It should be called as angle traversed or just finite angle, because it is **not a vector**. So, **finite angle θ is not a vector**, because it does not add like vectors, as Illustrated by the successive rotation technique for the book, where we observe that $(\text{Rotation})_1 + (\text{Rotation})_2 \neq (\text{Rotation})_2 + (\text{Rotation})_1$.



A book rotated θ_1 (90° as shown about an axis at right angles to the page) and then θ_2 (90° as shown about a north-south axis) has a different final orientation that if rotated first through θ_2 and then θ_1. This property is called the non-commutivity of finite angles under addition: $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$.

This non-commutative property associated with finite rotation provides us a solid platform for calling it a non-vector. In the first case, we rotate the book through two successive rotations of $\frac{\pi}{2}$ each and we observe that $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$.

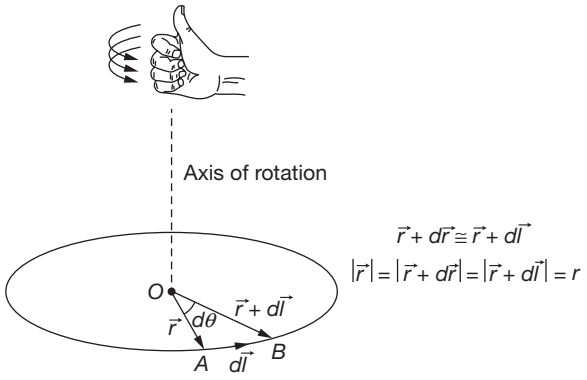
In the other case, if we rotate the book through successive rotations of $20°$, then too $\theta_1 + \theta_2$ would still differ from $\theta_2 + \theta_1$, but the difference would be smaller. As a matter of fact, as the two rotations are made smaller and smaller, the difference between the two sums disappears rapidly. So, if the infinitesimal angles are taken then we observe that the order of addition no longer affects the results. Hence **infinitesimal angular displacements** are **vector quantities**.



The lower group repeats the experiment for 20° displacements. We see here that $\theta_1 + \theta_2 \cong \theta_2 + \theta_1$. Also we observe that when $\theta_1, \theta_2 \to 0$, the final positions approach each other. Finite angles under addition tend to commute as the angles become very small. Infinitesimal angles do commute under addition, making it possible to treat them as vectors.

ANGULAR DISPLACEMENT, ANGULAR VELOCITY, ANGULAR AND CENTRIPETAL ACCELERATION

From the above discussion, we have observed that, the finite angle is not a vector, so **cannot be called as Angular Displacement**. However, infinitesimal angle $d\theta$ is a vector and hence is **called as Angular Displacement.**



Consider a particle moving in a circle of radius r. Let the particle go from point A to B such that Arc $AB = dl$. If $d\theta$ is the angle subtended by the arc at the centre of the circle, then $dl = rd\theta$. Since $d\theta$ is very small, so we can also say

$$\overset{\frown}{AB} = d\vec{l} = d\vec{r}$$

So, in triangle OAB, we can have

$$\vec{r} + d\vec{r} = \vec{r} + d\vec{l}$$

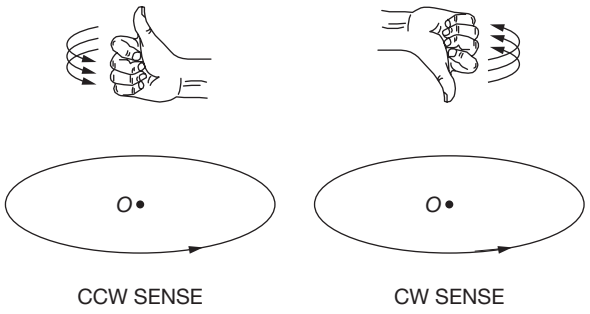
Since, both $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the radius of the same circle, so

$$|\overrightarrow{OA}| = |\overrightarrow{OB}| = r$$

$$|\vec{r}| = |\vec{r} + d\vec{r}| = |\vec{r} + d\vec{l}| = r \quad \ldots(1)$$

The direction of $d\vec{\theta}$ is given by Right Hand Thumb Rule, according to which, "curl the fingers of Right Hand in the sense of rotation, then thumb gives the direction of $d\vec{\theta}$ (angular displacement), $\vec{\omega}$ (angular velocity), $\vec{\alpha}$ (angular acceleration), $\vec{\tau}$ (torque) and $\vec{L}$ (angular momentum)".
So, in magnitude we have

$$dl = rd\theta$$



Direction of $d\vec{\theta}$, $\vec{\omega}$, $\vec{\alpha}$, $\vec{\tau}$ and $\vec{L}$ given by THUMB But vectorially, we observe that

$$d\vec{l} = d\vec{\theta} \times \vec{r} \qquad \ldots(2)$$

Angle between $d\vec{\theta}$ and $\vec{r}$ is $\frac{\pi}{2}$, so

$$|d\vec{\ell}| = d\ell = r\,d\theta \qquad \left\{\because \sin\left(\frac{\pi}{2}\right) = 1\right\}$$

Dividing both sides of (1) by dt, we get

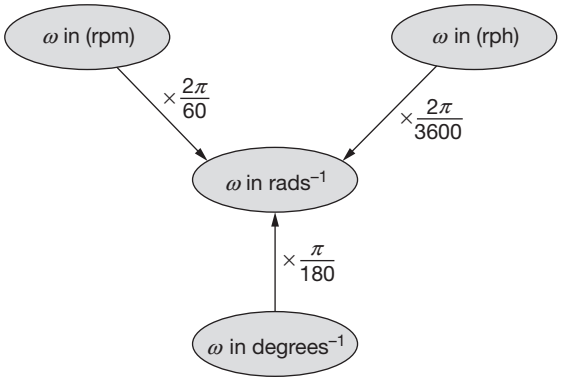
$$\frac{d\vec{l}}{dt} = \frac{d\vec{\theta}}{dt} \times \vec{r}$$

{Please note, it is dividing, not differentiating}

$$\Rightarrow \quad \vec{v} = \vec{\omega} \times \vec{r} \qquad \ldots(3)$$

where $\vec{\omega} = \frac{d\vec{\theta}}{dt}$ = Angular Velocity of the particle

$$\vec{v} = \frac{d\vec{l}}{dt} = \frac{d\vec{r}}{dt} = \text{ Velocity of the particle}$$



$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

In magnitude,

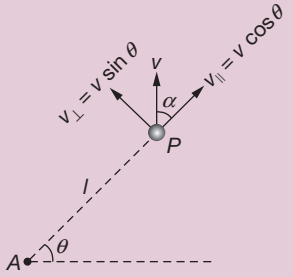
$$v = r\omega \qquad \ldots(4)$$

The $\vec{\omega}$, angular velocity of the particle is always to be measured in SI units of rads^{-1}. Generally, ω can also be expressed as rpm (revolution per minute), rph (revolution per hour), degrees^{-1}.

Conceptual Note(s)

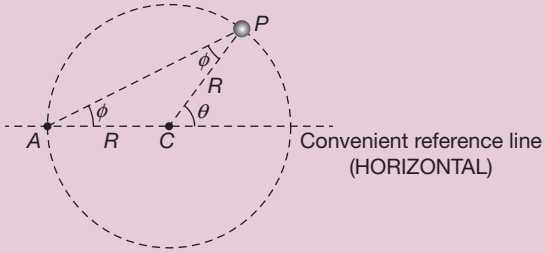
(a) Angular velocity is always to be defined with respect to the point from where the position vector of the point is drawn. Consider a particle having a velocity *v*, as shown. Let us calculate ω of the point *P* w.r.t. another arbitrarily chosen point *A* (say) or with respect to the observer at *A* is

$$\omega = \frac{v_\perp}{l} = \frac{v\sin\alpha}{l} = \frac{d\theta}{dt}$$



where θ is the angle made by *AB* at any instant with a convenient reference line (generally horizontal).

(b) Furthermore, the angular velocity of a particle can be different about different points. For that, let us consider a particle moving on the circumference of the circle, as shown.



Then the angular velocity of point P w.r.t. the observer at the centre of the circle C is

$$\omega_{PC} = \frac{d\theta}{dt}$$

Similarly, the angular velocity of the point P w.r.t. observer at the circumference of circle at A is

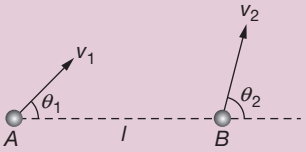
$$\omega_{PA} = \frac{d\phi}{dt}$$

In triangle ACP, since $\theta = 2\phi$

$$\Rightarrow \quad \omega_{PA} = \frac{d}{dt}\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(\frac{d\theta}{dt}\right) = \frac{1}{2}\omega_{PC}$$

$$\Rightarrow \quad \omega_{PC} = 2\omega_{PA}$$

(c) If two particles A and B are moving with velocities $\vec{v}_1$ and $\vec{v}_2$ in the directions shown in figure. Then $\frac{dl}{dt}$ = rate of change of distance between the two particles



$\frac{dl}{dt}$ = relative velocity between them along AB

$$\Rightarrow \quad \frac{dl}{dt} = v_2\cos\theta_2 - v_1\cos\theta_1$$

and $\omega_r = \omega_{BA}$ = angular velocity of B with respect to A or angular velocity of line AB, then

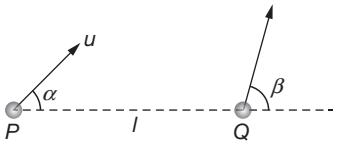
$$\omega_r = \omega_{BA} = \frac{|v_2\sin\theta_2 - v_1\sin\theta_1|}{l}$$

Magnitude of the relative angular velocity between them can also be given by

$$\omega_r = \frac{\text{Relative velocity perpendicular to } AB}{\text{Distance between them}}$$

ILLUSTRATION 1

The line joining P and Q is of constant length l and the velocities of P and Q are in directions inclined at angles α and β respectively with PQ. If u is the velocity of P, what is the angular speed of PQ?



SOLUTION

Let v be the velocity of Q. Then $u\cos\alpha = v\cos\beta$
Since l = constant

$$\Rightarrow \quad \frac{dl}{dt} = 0$$

$$\Rightarrow \quad v = \frac{u\cos\alpha}{\cos\beta}$$

Now angular speed of line PQ is given by

$$\omega = \frac{\text{Relative speed } \perp \text{ to } PQ}{l}$$

$$\Rightarrow \quad \omega = \frac{v\sin\beta - u\sin\alpha}{l} = \frac{\left(\frac{u\cos\alpha}{\cos\beta}\right)\sin\beta - u\sin\alpha}{l}$$

$$\Rightarrow \quad \omega = \frac{u(\tan\beta\cos\alpha - \sin\alpha)}{l}$$

ILLUSTRATION 2

Two points are moving with uniform velocities u and v along the perpendicular axes, OX and OY. The motion is directed toward O, the origin. When $t = 0$, they are at a distance a and b respectively from O.

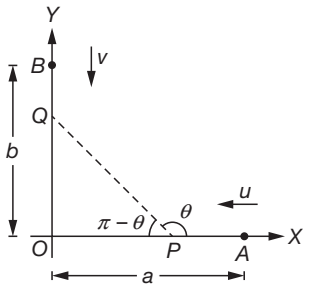
(a) Calculate the angular velocity of the line joining them at time t.

(b) Show that it is greatest, when $t = \frac{au + bv}{u^2 + v^2}$

SOLUTION

(a) At $t = 0$, the particles are at A and B respectively.

Let at time t, the particles are at P and Q respectively.



From figure, $OP = a - ut$ …(1)

and $OQ = b - vt$ …(2)

Let θ be the angle which the line PQ makes with the direction OX at time t. From ΔQOP.

$$\tan(\pi - \theta) = \frac{OQ}{OP} = \frac{b - vt}{a - ut}$$

$$\Rightarrow \quad -\tan\theta = \frac{b - vt}{a - ut}$$

$$\Rightarrow \quad \tan\theta = \left(\frac{b - vt}{ut - a}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{b - vt}{ut - a}\right) \qquad \ldots(3)$$

Differentiating equation (3), we get

$$\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{b - vt}{ut - a}\right)^2} \times \frac{(ut - a)(-v) - (b - vt)u}{(ut - a)^2}$$

$$\Rightarrow \quad \omega = \frac{d\theta}{dt} = \frac{av - bu}{(a - ut)^2 + (b - vt)^2}$$

(b) ω will be maximum when denominator is minimum, because numerator is constant.

Let, $(\text{Den})^r = S = (a - ut)^2 + (b - vt)^2$. For it to be MINIMUM, we have

$$\frac{dS}{dt} = 2(a - ut)(-u) + 2(b - vt)(-v) = 0$$

$$\Rightarrow \quad t = \frac{au + bv}{u^2 + v^2}$$

It can be seen that $\frac{d^2S}{dt^2} > 0$. Hence, S is minimum, or ω is maximum, at

$$t = \frac{au + bv}{u^2 + v^2}$$

ANGULAR, CENTRIPETAL, TANGENTIAL AND TOTAL ACCELERATION

Since we have now understood the concept of $\vec{\omega}$ so, let us now discuss about angular acceleration (α). Angular acceleration is the rate of change of angular velocity. Mathematically

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

The direction of $\vec{\alpha}$ is again given by Right Hand Thumb Rule, as discussed earlier.

If $\vec{a}$ be the acceleration of the particle, then

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Since $\vec{v} = \vec{\omega} \times \vec{r}$

$$\Rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

Using $\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{A} \times \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \times \vec{B}$

$$\Rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r}$$

$$\Rightarrow \quad \vec{a} = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r} \qquad \ldots(1)$$

The term $\vec{\omega} \times \vec{v}$ is the centripetal or radial acceleration of the particle and $\vec{\alpha} \times \vec{r}$ is the tangential acceleration of the particle. So,

$$\vec{a}_C = \vec{\omega} \times \vec{v} \qquad \ldots(2)$$

$$\vec{a}_T = \vec{\alpha} \times \vec{r} \qquad \ldots(3)$$

In magnitude, $a_C = v\omega$ and $a_T = r\alpha$

$$\Rightarrow \quad a_C = v\omega = r\omega^2 = \frac{v^2}{r} \qquad \ldots(4)$$

We could have prove this vectorially, by using

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Substituting in (2), we get

$$\vec{a}_C = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Using $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$, we get

$$\vec{a}_C = (\vec{\omega} \cdot \vec{r})\vec{\omega} - (\vec{\omega} \cdot \vec{\omega})\vec{r}$$

Now, since $\vec{\omega} \perp \vec{r}$, so $\vec{\omega} \cdot \vec{r} = 0$

$$\Rightarrow \quad \vec{a}_C = 0 - \omega^2\vec{r} \qquad \{\because \ \vec{\omega} \cdot \vec{\omega} = \omega^2\}$$

$$\Rightarrow \quad \vec{a}_C = -\omega^2 \vec{r} \qquad \ldots(5)$$

$$\Rightarrow \quad |\vec{a}_C| = a_C = r\omega^2 \qquad \{\because \ |\vec{r}| = r\}$$

Please note that from (5), we conclude that the centripetal acceleration is directed radially inwards (see the negative sign justifying this).

Also, if we had been asked to give the expressions for centripetal force and tangential force, then we would have just multiplied the above expressions with the mass of the particle, say *m*, to get

$\vec{F}_c = m\vec{a}_c = m(\vec{\omega} \times \vec{v}) = -\omega^2 \vec{r}$ and $\vec{F}_T = m\vec{a}_T = m(\vec{\alpha} \times \vec{r})$

Problem Solving Technique(s)

(a) For motion with uniform angular acceleration, we can use the following equations

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

$$\theta = \left(\frac{\omega + \omega_0}{2}\right) t$$

where, θ is the angle traversed (in radian) in time *t* (second)
ω_0 is the initial angular velocity (in rads^{-1}),
ω is the final angular velocity (in rads^{-1}) at time *t* and
α is the angular acceleration (in rads^{-2}).

(b) If α is not a constant, analyse the problem and feel free to use

$$\omega = \frac{d\theta}{dt} \text{ or } \alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$$

(c) The SI unit of θ is radian, ω is rads^{-1} and α is rads^{-2}.

(d) $\omega = \frac{2\pi}{T} = 2\pi f$, where *T* is the period of revolution and *f* is the frequency and $f = \frac{1}{T}$.

(e) Here too, the graphs interpret analogous to their fellows in Linear Kinematics, like
Slope in the θ-*t* graph gives **angular velocity** (ω).
Slope in the ω-*t* graph gives **angular acceleration** (α).
Area under a curve in ω-*t* graph gives the **angle traversed** (θ).

ILLUSTRATION 3

A solid body rotates with deceleration about a stationary axis with an angular deceleration $|\alpha| = k\sqrt{\omega}$, where ω is its angular velocity and k is a positive constant. Find the mean angular velocity of the body averaged over the whole time of rotation if at the initial moment of time its angular velocity was equal to ω_0.

SOLUTION

$$|\alpha| = k\sqrt{\omega}$$

$$\Rightarrow \quad -\frac{d\omega}{dt} = k\sqrt{\omega}$$

$$\Rightarrow \quad \int_{\omega_0}^{\omega} \frac{d\omega}{\sqrt{\omega}} = -\int_0^t k\,dt$$

$$\Rightarrow \quad \left(2\sqrt{\omega}\right)\Big|_{\omega_0}^{\omega} = -kt$$

$$\Rightarrow \quad \sqrt{\omega_0} - \sqrt{\omega} = \frac{kt}{2}$$

$$\Rightarrow \quad \omega = \left(\sqrt{\omega_0} - \frac{kt}{2}\right)^2 \qquad \ldots(1)$$

The body will stop when

$$\sqrt{\omega_0} - \frac{kt}{2} = 0$$

$$\Rightarrow \quad t = \frac{2\sqrt{\omega_0}}{k}$$

Now average angular velocity over this time interval is

$$\langle\omega\rangle = \frac{\int_0^{\frac{2\sqrt{\omega_0}}{k}} \omega\,dt}{\int_0^{\frac{2\sqrt{\omega_0}}{k}} dt} = \frac{\int_0^{\frac{2\sqrt{\omega_0}}{k}} \left(\sqrt{\omega_0} - \frac{kt}{2}\right)^2 dt}{\int_0^{\frac{2\sqrt{\omega_0}}{k}} dt} = \frac{\omega_0}{3}$$

ILLUSTRATION 4

The speed (v) of a particle moving in a circle of radius R varies with distance s as $v = ks$, where k is a positive constant. Calculate the total acceleration of the particle.

SOLUTION

$$v = ks$$

Since the particle is moving in a circle, so total acceleration, a is

$$a = \sqrt{a_C^2 + a_T^2}$$

where $a_C = \dfrac{v^2}{R}$ and $a_T = \dfrac{dv}{dt}$

$$\Rightarrow \quad a_C = \frac{k^2 s^2}{R} \text{ and } a_T = \frac{d}{dt}(ks) = k\left(\frac{ds}{dt}\right) = kv$$

$$\Rightarrow \quad a_C = \frac{k^2 s^2}{R} \text{ and } a_T = k(ks) = k^2 s \quad \left\{\because \frac{ds}{dt} = v = ks\right\}$$

$$\Rightarrow \quad a = \sqrt{\frac{k^4 s^4}{R^2} + k^4 s^2} = k^2 s\sqrt{1 + \frac{s^2}{R^2}}$$

ILLUSTRATION 5

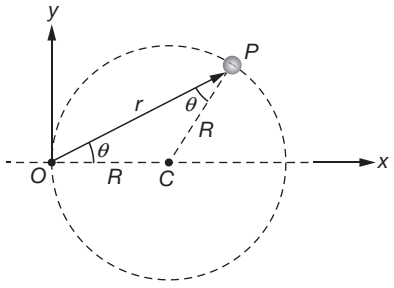
A particle P moves along a circle of radius R so that its radius vector $\vec{r}$, relative to the point O at the circumference rotates with constant angular velocity ω. Find the magnitude of the velocity of the particle and the direction of its total acceleration.

SOLUTION

In triangle OPC, we get from Lami's Theorem

$$\frac{r}{\sin(\pi - 2\theta)} = \frac{R}{\sin\theta}$$

$$\Rightarrow \quad \frac{r}{2\sin\theta\cos\theta} = \frac{R}{\sin\theta}$$



$$\Rightarrow \quad r = 2R\cos\theta \qquad \ldots(1)$$

Further we observe that

$$\vec{r} = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}$$

$$\Rightarrow \quad \vec{r} = \left(2R\cos^2\theta\right)\hat{i} + (2R\sin\theta\cos\theta)\hat{j}$$

Since $\vec{v} = \dfrac{d\vec{r}}{dt}$

$$\Rightarrow \quad \vec{v} = -\left(4R\cos\theta\sin\theta\frac{d\theta}{dt}\right)\hat{i} + \left(2R\cos(2\theta)\frac{d\theta}{dt}\right)\hat{j}$$

Since $\dfrac{d\theta}{dt} = \omega$

$$\Rightarrow \quad \vec{v} = -2R\omega\left[-\sin(2\theta)\hat{i} + \cos(2\theta)\hat{j}\right] \qquad \ldots(2)$$

$$\Rightarrow \quad |\vec{v}| = 2R\omega$$

Further, we know that $\vec{a} = \dfrac{d\vec{v}}{dt}$

$$\Rightarrow \quad \vec{a} = 4R\omega^2\left(\cos(2\theta)\hat{i} + \sin(2\theta)\hat{j}\right)$$

$$\Rightarrow \quad |\vec{a}| = 4R\omega^2$$

UNIT VECTORS ALONG THE RADIUS ($\hat{r}$) AND THE TANGENT ($\hat{t}$)

Consider a particle P moving in a circle of radius r centred at origin O. The angular position of the particle at some instant is say θ. Let us here define two unit vectors, one is $\hat{r}$ (called radial unit vector) which is along OP and the other is $\hat{t}$ (called the tangential unit vector) which is perpendicular to OP. Now, since

$$|\hat{r}| = |\hat{t}| = 1$$

We can write these two vectors as

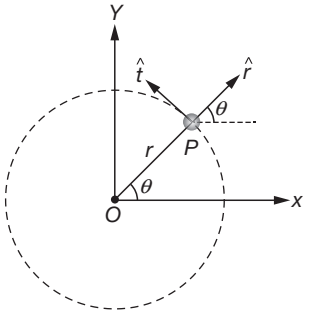
$$\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j} \text{ and } \hat{t} = -\sin\theta\hat{i} + \cos\theta\hat{j}$$

VELOCITY AND ACCELERATION OF PARTICLE IN CIRCULAR MOTION

The position vector of particle P at the instant shown in figure can be written as

$$\vec{r} = \overrightarrow{OP} = r\hat{r}$$

$$\Rightarrow \quad \vec{r} = r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$



The velocity of the particle can be obtained by differentiating $\vec{r}$ with respect to time t So, we get

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)\right]$$

$$\vec{v} = r\left[\left(-\sin\theta\frac{d\theta}{dt}\right)\hat{i} + \left(\cos\theta\frac{d\theta}{dt}\right)\hat{j}\right]$$

$$\Rightarrow \quad \vec{v} = r\omega\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right) \qquad \left\{\because \frac{d\theta}{dt} = \omega\right\} \qquad \ldots(1)$$

$$\Rightarrow \quad \vec{v} = (r\omega)\hat{t}$$

Thus, we observe that velocity of the particle is $r\omega$ along $\hat{t}$ or in tangential direction. Acceleration of the particle can be obtained by differentiating equation (1) with respect to time t. So, we get

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[r\omega\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)\right]$$

$$\Rightarrow \quad \vec{a} = r\left[\omega\frac{d}{dt}\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right) + \left(\frac{d\omega}{dt}\right)\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)\right]$$

$$\Rightarrow \quad \vec{a} = r\omega\left[-\cos\theta\left(\frac{d\theta}{dt}\right)\hat{i} - \sin\theta\left(\frac{d\theta}{dt}\right)\hat{j}\right] + r\left(\frac{d\omega}{dt}\right)\hat{t}$$

$$\Rightarrow \quad \vec{a} = -r\omega^2\left[\cos\theta\hat{i} + \sin\theta\hat{j}\right] + r\left(\frac{d\omega}{dt}\right)\hat{t}$$

$$\Rightarrow \quad \vec{a} = -r\omega^2\hat{r} + \frac{dv}{dt}\hat{t} \qquad \ldots(2)$$

Thus, acceleration of a particle moving in a circle has two components one is along $\hat{t}$ (along tangent) and the other along $-\hat{r}$ (or towards centre). In equation (2), the first term is called the tangential acceleration. (a_t) and the second term is called radial or centripetal acceleration (a_r). Thus,

$$a_t = \frac{dv}{dt} = \text{rate of change of speed}$$

and $a_r = a_c = r\omega^2 = \dfrac{v^2}{r} = v\omega \qquad \{\because v = r\omega\}$

Here, the two components are mutually perpendicular. Therefore, net acceleration of the particle will be

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{a_c^2 + a_T^2}$$

$$a = \sqrt{(r\omega^2)^2 + \left(\frac{dv}{dt}\right)^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

Conceptual Note(s)

(a) In uniform circular motion, speed (v) of the particle is constant, i.e., $\dfrac{dv}{dt} = 0$. Thus

$$a_t = 0 \text{ and } a = a_r = r\omega^2$$

(b) In accelerated circular motion, $\dfrac{dv}{dt}$ = positive, i.e., a_t is along $\hat{t}$ or tangential acceleration of particle is parallel to velocity $\vec{v}$ because $\vec{v} = r\omega\hat{t}$ and $\vec{a}_t = \dfrac{dv}{dt}\hat{t}$.

(c) In decelerated circular motion, $\dfrac{dv}{dt}$ = negative and hence, tangential acceleration is anti-parallel to velocity $\vec{v}$.

ILLUSTRATION 6

The speed of a particle moving in a circle of radius $r = 2$ m varies with time t as $v = t^2$, where t is in second and v in ms^{-1}. Find the radial, tangential and net acceleration at $t = 2$ s.

SOLUTION

Linear speed of particle at $t = 2$ s is $v = (2)^2 = 4\ \text{ms}^{-1}$

Radial acceleration $a_r = \dfrac{v^2}{r} = \dfrac{(4)^2}{2} = 8\ \text{ms}^{-2}$

The tangential acceleration is

$$a_t = \frac{dv}{dt} = 2t$$

So, tangential acceleration at $t = 2$ s is

$$a_t = (2)(2) = 4\ \text{ms}^{-2}$$

The net acceleration of particle at $t = 2$ s is

$$a = \sqrt{(a_C)^2 + (a_T)^2} = \sqrt{(8)^2 + (4)^2}$$

$$\Rightarrow \quad a = \sqrt{80}\ \text{ms}^{-2}$$

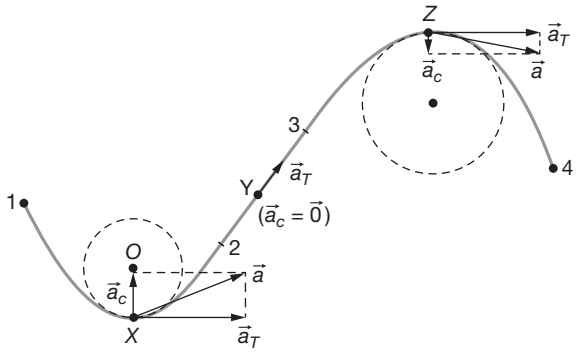
KINEMATICS OF MOTION OF PARTICLE IN A CURVED TRACK

Consider a curved track, 1234, having portions 12, 23 and 34 on which points X, Y and Z are taken respectively. The particle moving in a curved track has following accelerations.

(a) Centripetal acceleration $\vec{a}_C$, acting radially inwards.

(b) Tangential acceleration $\vec{a}_T$, acting tangentially.

So, at the points X and Z, on the curved track the particle has two accelerations $\vec{a}_C$ and $\vec{a}_T$.



At the point Y, $r \to \infty$, so $\vec{a}_C \to 0$, hence from 2 to 3, the particle just follows a straight track 23 under the influence of a single force $\vec{a}_T$.
So, we can say that for a particle moving in a curved track, net acceleration is given by

$$a = \sqrt{a_C^2 + a_T^2} \qquad \{\because \vec{a}_C \perp \vec{a}_T\}$$

$$\Rightarrow \quad a = \sqrt{(r\omega^2)^2 + (r\alpha)^2}$$

$$\Rightarrow \quad a = r\sqrt{\omega^4 + \alpha^2}$$

RADIUS OF CURVATURE

As observed, any curved track/path can be assumed to be made of a large number of circular arcs of variable radii. The radius of curvature at a point is the radius of the circular arc that suitably fits on the curve at that point.

Since $a_C = \dfrac{v^2}{r}$ where v is the tangential velocity and can be denoted by v_T.

$$\Rightarrow \quad a_C = \frac{v_T^2}{r}$$

$$\Rightarrow \quad r = \frac{v_T^2}{a_C} = \frac{v_\perp^2}{a_P}$$

where $v_T = v_\perp$ i.e., tangential velocity equals the velocity of the particle **perpendicular to the radius** of the curve and hence v_T can also be denoted by $v_\perp$ (read as v perpendicular) and $a_C = a_P$ i.e., centripetal acceleration equals the acceleration of the particle **parallel to the radius** of the curve and hence a_C can also be denoted by a_P.

Conceptual Note(s)

(a) On any curved path (not necessarily a circular one) the acceleration of the particle has two components a_T and a_N in two mutually perpendicular directions. Component of $\vec{a}$ along $\vec{v}$ is a_T and perpendicular to $\vec{v}$ is a_N or a_C. So, we have

$$|\vec{a}| = \sqrt{a_T^2 + a_C^2} = \sqrt{a_x^2 + a_y^2} \quad \{\text{For 2D motion}\}$$

where a_T = rate of change of speed $= \dfrac{dv}{dt} = \dfrac{d|\vec{v}|}{dt}$

$$\Rightarrow \quad a_T = \frac{d}{dt}\left(\sqrt{v_x^2 + v_y^2}\right) = \frac{v_x \dfrac{dv_x}{dt} + v_y \dfrac{dv_y}{dt}}{\sqrt{v_x^2 + v_y^2}}$$

$$\Rightarrow \quad a_T = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}} = \frac{\vec{a} \cdot \vec{v}}{v}$$

$$\Rightarrow \quad a_T = \text{component of } \vec{a} \text{ along } \vec{v}$$

(b) If a and a_T are known, a_C can easily be found using the relation

$$a = \sqrt{a_C^2 + a_T^2}$$

$$\Rightarrow \quad a_C = \sqrt{a^2 - a_T^2} = \sqrt{a_x^2 + a_y^2 - a_T^2}$$

$$\Rightarrow \quad a_C = \sqrt{\left(\frac{dv_x}{dt}\right)^2 + \left(\frac{dv_y}{dt}\right)^2 - \left(\frac{dv}{dt}\right)^2}$$

ILLUSTRATION 7

A balloon starts rising from the earth's surface. The ascension rate is constant and equal to v_0. Due to the wind, the balloon gathers the horizontal velocity component $v_x = ky$, where k is a constant and y is the height of ascent. Find the dependence of the following quantities on y.

(a) The horizontal drift of the balloon $x(y)$.
(b) The total tangential acceleration of the balloon.
(c) The normal acceleration of the balloon.

SOLUTION

(a) Since the balloon is ascending at a constant rate. So,

$$v_0 = \frac{dy}{dt}$$

$$\Rightarrow \quad dy = v_0 dt$$

$$\Rightarrow \quad y = v_0 t$$

Also, we have

$$v_x = \frac{dx}{dt} = ky$$

$$\Rightarrow \quad dx = kydt = kv_0 t\, dt \qquad \{\because\ y = v_0 t\}$$

Integrating, we get

$$x = kv_0\left(\frac{t^2}{2}\right)$$

$$\Rightarrow \quad x = k\frac{v_0}{2}\left(\frac{y}{v_0}\right)^2$$

$$\Rightarrow \quad x = \frac{1}{2}\frac{ky^2}{v_0}$$

This is the trajectory of the particle which is a parabola.

(b) For finding the tangential and normal accelerations, we require an expression for the velocity as a function of height y. So, we have

$$v_y = v_0 \text{ and } v_x = ky$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + k^2y^2}$$

Therefore tangential acceleration,

$$a_T = \frac{dv}{dt} = \frac{k^2y}{\sqrt{v_0^2 + k^2y^2}}\frac{dy}{dt} = \frac{k^2yv_0}{\sqrt{v_0^2 + k^2y^2}}$$

$$\Rightarrow \quad a_T = \frac{k^2y}{\sqrt{1 + \frac{k^2y^2}{v_0^2}}}$$

Now, the total acceleration a is, given by

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(\frac{dv_y}{dt}\right)^2 + \left(\frac{dv_x}{dt}\right)^2}$$

$$\Rightarrow \quad a = \frac{dv_x}{dt} = k\frac{dy}{dt} = kv_0 \qquad \left\{\because\ \frac{dv_y}{dt} = 0\right\}$$

(c) To find the normal acceleration, since we know that

$$a^2 = a_N^2 + a_T^2$$

$$\Rightarrow \quad a_N = \sqrt{a^2 - a_T^2} = \frac{kv_0}{\sqrt{1 + \left(\frac{ky}{v_0}\right)^2}}$$

Test Your Concepts-I

Based on Curvilinear Motion

(Solutions on page H.143)

1. The speed of a particle moving in a plane is equal to the magnitude of its instantaneous velocity, $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$. Show that the rate of change of speed can be expressed as $\frac{dv}{dt} = \frac{\vec{v}\cdot\vec{a}}{v}$ and use this result to explain why $\frac{dv}{dt}$ is equal to a_T, the component of $\vec{a}$ that is parallel to $\vec{v}$.

2. If a point moves along a circle with constant speed, prove that its angular speed about any point on the circle is half of that about the centre.

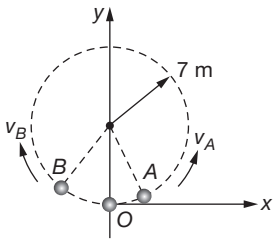
3. Calculate the angular speed of a particle which moves in a circle of radius 0.25 m with a linear speed of 2 ms^{-1}.

4. A particle moves in a circle of radius 0.25 m at a speed that uniformly increases. Find the angular acceleration of particle if its speed changes from 2 ms^{-1} to 4 ms^{-1} in 4 s.

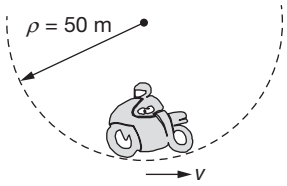
5. A car is travelling along a circular curve that has a radius of 50 m. If its speed is 16 ms^{-1} and is increasing uniformly at 8 ms^{-2}, determine the magnitude of its acceleration at this instant.

6. A boy whirls a stone in a horizontal circle of radius 0.5 m and at height 20 m above the level ground. The string breaks, and the stone flies off horizontally to strike the ground after travelling a horizontal distance of 10 m. Calculate the magnitude of the centripetal acceleration of the stone while in circular motion? $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$

7. Two particles A and B start from the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ ms^{-1} and $v_B = 1.5$ ms^{-1}, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens. $\left(\text{Take } \pi = \frac{22}{7}\right)$



8. Starting from rest, the motorcycle travels around the circular path of radius 50 m, at a speed $v = \frac{t^2}{5}$ ms^{-1}, where t is in seconds. Determine the magnitudes of the motorcycle's velocity and acceleration at the instant $t = 3$ s.



9. Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 ms^{-2} while rounding a track having a radius of curvature of 200 m.

10. An automobile is travelling on a horizontal circular curve having a radius of 800 m. If the acceleration of the automobile is 5 ms^{-2}, determine the constant speed at which the automobile is travelling.

11. A car travels along a horizontal circular curved road that has a radius of 600 m. If the speed is uniformly increased at a rate of 2000 kmh^{-2}, determine the magnitude of the acceleration at the instant the speed of the car is 60 kmh^{-1}.

12. When designing a highway curve it is required that cars travelling at a constant speed of 25 ms^{-1} must not have an acceleration that exceeds 3 ms^{-2}. Determine the minimum radius of curvature of the curve.

13. At a given instant, a car travels along a circular curved road with a speed of 20 ms^{-1} while decreasing its speed at the rate of 3 ms^{-2}. If the magnitude of the car's acceleration is 5 ms^{-2}, determine the radius of curvature of the road.

14. The wheel of a truck moving with velocity 5 ms^{-1} throws up mud particles from its rim. The diameter of the wheel is 3 m. Find the maximum height from ground to which a particle can reach? (Assume no sliding between the wheel and road).

PROJECTILE MOTION

PROJECTILE MOTION: AN INTRODUCTION

A particle launched with an initial velocity in any arbitrary direction and then allowed to move freely under the gravitational influence of the earth $(\vec{g})$ is called a **projectile**.

For studying the projectile motion the following assumptions must be kept in mind.

Assumption 1: The surface of the earth is more or less flat.

Assumption 2: Acceleration due to gravity $(\vec{g})$ has a constant value throughout the entire motion.

Assumption 3: No air drag is present.

EXAMPLES:

(a) A cricket ball thrown by a bowler.

(b) A football kicked by the player.

(c) Food packet dropped from an airplane and many more examples can be thought of as projectiles from everyday life.

Conceptual Note(s)

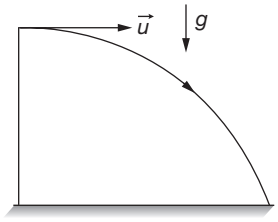
The path followed by a projectile is called its **trajectory**, which is a parabola.

TYPES OF PROJECTILE MOTION

We shall study three kinds of projectile motion.

Horizontal Projectile

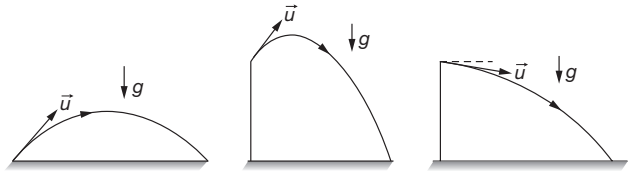
In this, the projectile is given an initial velocity directed along the horizontal and then it moves under the influence of gravity to follow a parabolic path.



(a)

Oblique Projectile

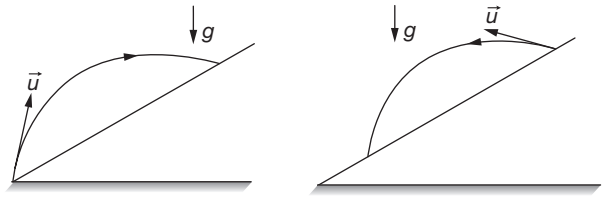
In this, the projectile is given an initial velocity making an angle θ with the horizontal (or the vertical) and it moves under the influence of gravity to follow a parabolic path.



(b) (c) (d)

Projectile on an Inclined Plane

In this, the projectile is given an initial velocity making an angle with the horizontal on an inclined plane and it again moves under the influence of gravity to follow a parabolic path.



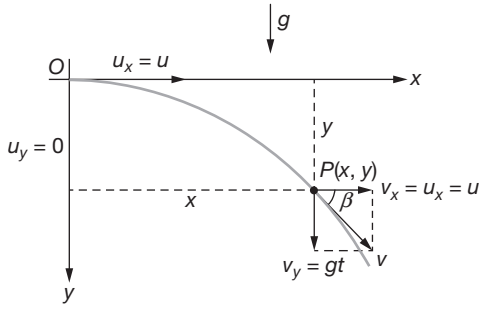
(e) (f)

We now discuss in detail the equation of trajectory, velocity any point P and at particular instant and position.

HORIZONTAL PROJECTILE

Equation of Trajectory

Consider a point $P(x, y)$ at time t.



Horizontal Motion (HM)

Since acceleration due to gravity acts along the vertical and hence, has got no component along the horizontal i.e., $a_x = 0$.

So, horizontal motion is a non-accelerated motion with uniform velocity.

$$\Rightarrow \quad x = u_x t = ut \qquad \ldots(1)$$

Vertical Motion (VM)

Also $a_y = g$ and $u_y = 0$

Since, $y = u_y t + \frac{1}{2} a_y t^2$

$$\Rightarrow \quad y = 0 + \frac{1}{2} g t^2$$

$$\Rightarrow \quad y = \frac{1}{2} g t^2 \qquad \ldots(2)$$

From (1), we get $t = \frac{x}{u}$

Substituting in (2), we get

$$y = \left(\frac{g}{2u^2} \right) x^2$$

which is the equation of a parabola.

Velocity at Any Instant (t)

Horizontal Motion (HM)

Since horizontal motion is non-accelerated,

$$\Rightarrow \quad v_x = u_x = u$$

Vertical Motion (VM)

Also, $v_y = u_y + a_y t$

$$\Rightarrow \quad v_y = 0 + gt$$

$$\Rightarrow \quad v_y = gt$$

Since, we have

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$\Rightarrow \quad \vec{v} = u\hat{i} + (gt)\hat{j}$$

$$\Rightarrow \quad |\vec{v}| = v = \sqrt{u^2 + g^2 t^2}$$

If β is the angle made by $\vec{v}$ with the horizontal, then

$$\tan \beta = \frac{v_y}{v_x} = \frac{gt}{u}$$

If h is the distance of the ground from the point of launch, T is the time taken to strike the ground and R is the range of the projectile when it hits the ground, then

$$h = \frac{1}{2} g T^2$$

$$\Rightarrow \quad T = \sqrt{\frac{2h}{g}} \text{ and } R = uT$$

$$\Rightarrow \quad R = u\sqrt{\frac{2h}{g}}$$

Deviation Suffered by a Horizontal Projectile in Time t

If gravity were absent, then the body launched with initial velocity u would continue to move horizontally forever. However in the presence of gravity it suffers a deviation, $\delta = y = \frac{1}{2} g t^2$. So,

$$\text{Deviation} = \delta = y = \frac{1}{2} g t^2 = \left(\frac{g}{2u^2} \right) x^2$$

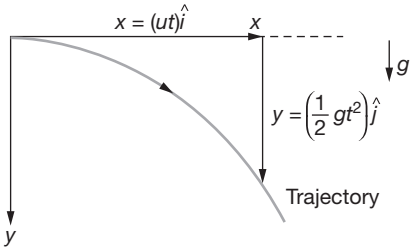


ILLUSTRATION 8

A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a hill 490 m high. Find

(a) the time taken by the projectile to reach the ground

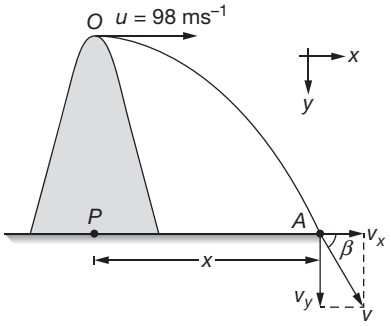
(b) the distance of the target from the hill and

(c) the velocity with which the projectile hits the ground. $\left(g = 9.8 \text{ ms}^{-2} \right)$.

SOLUTION

Here, it will be more convenient to choose x and y directions as shown in figure. So, let the downward direction be positive, then

$$u_x = 98 \text{ ms}^{-1},\ a_x = 0,\ u_y = 0 \text{ and } a_y = g$$



(a) At A, $y = 490$ m. So, applying

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 490 = 0 + \frac{1}{2}(9.8)t^2$$

$$\Rightarrow \quad t = 10 \text{ s}$$

(b) $PA = x = u_x t + \frac{1}{2} a_x t^2$

$$\Rightarrow \quad PA = (98)(10) + 0$$

$$\Rightarrow \quad PA = 980 \text{ m}$$

(c) $v_x = u_x = 98 \text{ ms}^{-1}$

$$v_y = u_y + a_y t = 0 + (9.8)(10) = 98 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{(98)^2 + (98)^2} = 98\sqrt{2} \text{ ms}^{-1}$$

and $\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1$

$$\Rightarrow \quad \beta = 45°$$

Thus, the projectile hits the ground with a velocity $98\sqrt{2}$ ms^{-1} at an angle of $\beta = 45°$ with horizontal as shown in figure.

ILLUSTRATION 9

Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at one point and moved with velocities $u_1 = 3 \text{ ms}^{-1}$ and $u_2 = 4 \text{ ms}^{-1}$ horizontally in opposite directions. Find the distance between the particles when their velocity vectors become mutually perpendicular.

SOLUTION

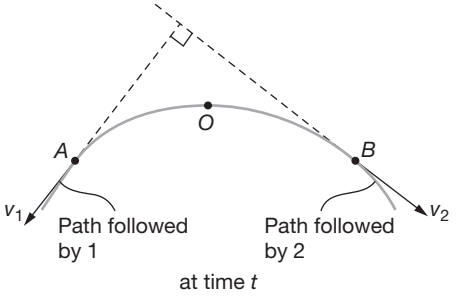
Let the velocities of the particles be at right angles to each other at time t. Then

$$\vec{v}_1 = \vec{u}_1 + \vec{g}t$$

$$\Rightarrow \quad \vec{v}_1 = 3\hat{i} + gt\hat{j}$$

$$\vec{v}_2 = \vec{u}_2 + \vec{g}t$$

$$\Rightarrow \quad \vec{v}_2 = -4\hat{i} + gt\hat{j}$$



For velocities to be mutually perpendicular, we have

$$\vec{v}_1 \cdot \vec{v}_2 = 0$$

$$\Rightarrow \quad -12 + g^2 t^2 = 0$$

$$\Rightarrow \quad t = \sqrt{\frac{12}{g^2}} = \frac{2\sqrt{3}}{g} \text{ s}$$

Separation between the particles is

$$x = (u_1 + u_2)t = 7\frac{2\sqrt{3}}{g}$$

$$\Rightarrow \quad x = \frac{14\sqrt{3}}{g} \text{ m}$$

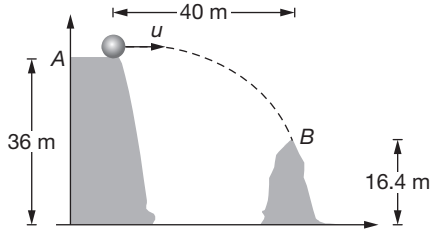
Test Your Concepts-II

Based on Horizontal Projectile

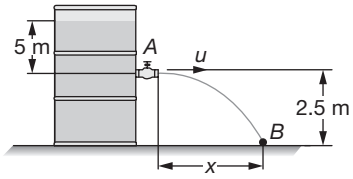
(Solutions on page H.144)

1. A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected. Take $g = 9.8\ \text{ms}^{-2}$.

2. With what minimum horizontal velocity u can a boy throw a ball at A and have it just clear the obstruction at B?



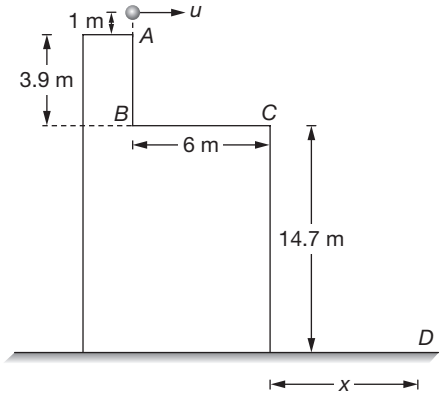
3. The velocity of the water jet discharging from the orifice (small hole in the tank) can be obtained from $u = \sqrt{2gh}$, where $h = 5$ m is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x, where it hits the surface. Take $g = 10\ \text{ms}^{-2}$.



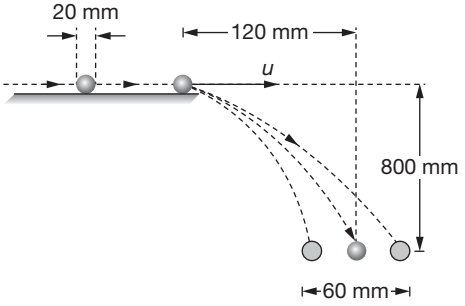
4. Determine the horizontal velocity u of a tennis ball launched from A at height of 2.25 m from the ground so that it just clears the net of height 1 m at B, a horizontal distance of 6.4 m from A. Also, find the horizontal distance from the net, where the ball strikes the ground. Take $g = 10\ \text{ms}^{-2}$.

5. A rock is thrown horizontally from top of a tower and hits the ground 4 s later. The line of sight from top of tower to the point where the rock hits the ground makes an angle of 30° with the horizontal. Calculate the horizontal launch velocity of the rock. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$.

6. Calculate the minimum velocity u along the horizontal such that the ball just clears the point C. Assume that the ball is launched by a man who holds the ball at a distance 1 m above A. Also find x, where the ball strikes the ground. Take $g = 9.8\ \text{ms}^{-2}$.



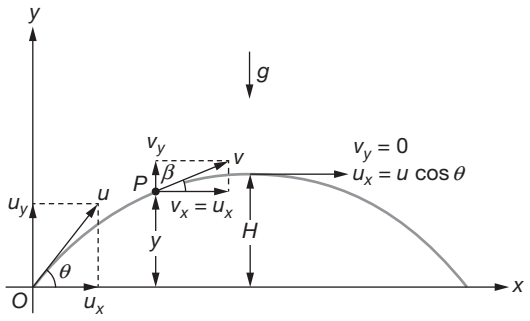
7. Ball bearings of diameter 20 mm leave the horizontal with a velocity of magnitude u and fall through the 60 mm diameter hole at a depth of 800 mm as shown. Calculate the permissible range of u which will enable the ball bearings to enter the hole. Take the dotted positions to represent the limiting conditions. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$



OBLIQUE PROJECTILE

Let a projectile be launched with an initial velocity u making an angle θ with horizontal then

$$u_x = u\cos\theta \text{ and } u_y = u\sin\theta$$



Equation of Trajectory

Consider a point $P(x, y)$ at time t.

Horizontal Motion (HM)

Since horizontal motion is non-accelerated

i.e., $a_x = 0$.

Also,

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\Rightarrow \quad x = (u\cos\theta)t \qquad \ldots(1)$$

Vertical Motion (VM)

$$y = u_y t + \frac{1}{2}a_y t^2$$

If we take upwards as positive, then

$$y = (u\sin\theta)t + \frac{1}{2}(-g)t^2 \qquad \{\because a_y = -g\}$$

$$\Rightarrow \quad y = (u\sin\theta)t - \frac{1}{2}gt^2 \qquad \ldots(2)$$

From (1), $t = \dfrac{x}{u\cos\theta}$ and put in (2), we get

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \qquad \ldots(3)$$

which is the equation of a parabola

Velocity at Any Instant (t)

Horizontal Motion (HM)

Since horizontal motion is non-accelerated. So,

$$v_x = u_x = u\cos\theta \qquad \ldots(4)$$

Vertical Motion (VM)

Since,

$$v_y = u_y + a_y t$$

$$\Rightarrow \quad v_y = u\sin\theta + (-g)t$$

$$\Rightarrow \quad v_y = u\sin\theta - gt \qquad \ldots(5)$$

From figure, we have

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\Rightarrow \quad \vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j} \qquad \ldots(6)$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

If β is the angle made by $\vec{v}$ with x-axis, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta} \qquad \ldots(7)$$

Velocity at Any Position at Vertical Height (h)

Horizontal Motion (HM)

Since horizontal motion is a non-accelerated motion, so

$$v_x = u_x = u\cos\theta \qquad \ldots(8)$$

Vertical Motion (VM)

$$v_y^2 - u_y^2 = 2a_y y$$

$$\Rightarrow \quad v^2\sin^2\beta = u^2\sin^2\theta + 2(-g)h$$

$$\Rightarrow \quad v_y = \sqrt{u^2\sin^2\theta - 2gh} \qquad \ldots(9)$$

Since,

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\Rightarrow \quad \vec{v} = (u\cos\theta)\hat{i} + (\sqrt{u^2\sin^2\theta - 2gh})\hat{j} \qquad \ldots(10)$$

$$\Rightarrow \quad |\vec{v}| = v = \sqrt{u^2 - 2gh}$$

If β is the angle made by $\vec{v}$ with x-axis, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{\sqrt{u^2\sin^2\theta - 2gh}}{u\cos\theta} \qquad \ldots(11)$$

Problem Solving Technique(s)

Projectile motion is a two dimensional motion with constant acceleration (g). So, we can use

$\vec{v} = \vec{u} + \vec{a}t$, $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$, etc. in projectile motion as well. Here

$$\vec{u} = u\cos\alpha\hat{i} + u\sin\alpha\hat{j} \text{ and } \vec{a} = -g\hat{j}$$

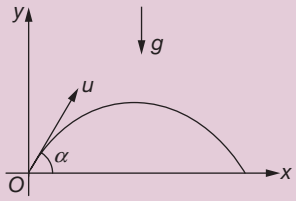
Now, suppose we want to find velocity at time t.

$$\vec{v} = \vec{u} + \vec{a}t = \left(u\cos\alpha\hat{i} + u\sin\alpha\hat{j}\right) - gt\hat{j}$$

$$\Rightarrow \quad \vec{v} = u\cos\alpha\hat{i} + \left(u\sin\alpha - gt\right)\hat{j}$$

Similarly displacement at time t will be,

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$



$$\vec{s} = \left(u\cos\alpha\hat{i} + u\sin\alpha\hat{j}\right)t - \frac{1}{2}gt^2\hat{j}$$

$$\vec{s} = ut\cos\alpha\hat{i} + \left(ut\sin\alpha - \frac{1}{2}gt^2\right)\hat{j}$$

Maximum Height (H)

Maximum vertical displacement of projection is called **Maximum Height**. At maximum height $v_y = 0$.

Since, $v_y^2 - u_y^2 = 2a_y y$

$$\Rightarrow \quad 0^2 - (u\sin\theta)^2 = 2(-g)H$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\theta}{2g}$$

Time of Flight (T)

Time taken by projectile to strike the ground. At $t = T$, $y = 0$ (because projectile returns to the ground). Since

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad 0 = (u\sin\theta)T + \frac{1}{2}(-g)T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\theta}{g} \qquad \{\because T \neq 0\}$$

Further, $T = t_{\text{ascent}} + t_{\text{descent}}$ and since no air drag exists, so,

$$t_{\text{ascent}} = t_{\text{descent}}$$

$$\Rightarrow \quad t_{\text{ascent}} = t_{\text{descent}} = \frac{T}{2} = \frac{u\sin\theta}{g}$$

Range (R)

Maximum horizontal distance travelled is called **Range**.

At $t = T$, $x = R$

Since $x = u_x t + \frac{1}{2}a_x t^2$

and $a_x = 0$ $\{\because$ horizontal motion is non-accelerated$\}$

$$\Rightarrow \quad R = (u\cos\theta)T$$

$$\Rightarrow \quad R = \frac{2}{g}(u\cos\theta)(u\sin\theta)$$

$$\Rightarrow \quad R = \frac{2}{g}\left(\begin{array}{c}\text{Horizontal}\\ \text{Component of}\\ \text{Initial Velocity}\end{array}\right)\left(\begin{array}{c}\text{Vertical}\\ \text{Component of}\\ \text{Initial Velocity}\end{array}\right)$$

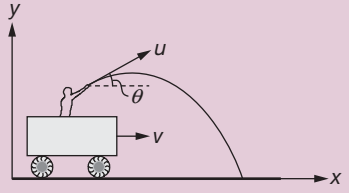
$$\Rightarrow \quad R = \frac{u^2\sin(2\theta)}{g}$$

Problem Solving Technique(s)

When the ball is projected from a body as per the cases discussed.

CASE-1: Body moving in same direction

Consider that a man is sitting in a trolley and the trolley is moving with velocity v in positive x direction as shown. Let the man projects a ball in the direction of trolley.



In this case, we have

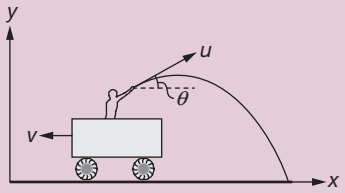
Horizontal component $u_x = u\cos\theta + v$

Vertical component $u_y = u\sin\theta$

$$\Rightarrow \quad R = \frac{2}{g}(u\cos\theta + v)(u\sin\theta)$$

CASE-2: Body moving in opposite direction

Consider that a man is sitting in a trolley and the trolley is moving with velocity v in negative x direction as shown. Let the man projects a ball in the direction of trolley.



In this case, we have

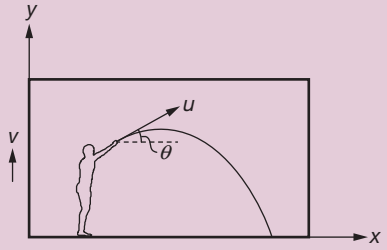
Horizontal component $u_x = u\cos\theta - v$

Vertical component $u_y = u\sin\theta$

$$\Rightarrow \quad R = \frac{2}{g}(u\cos\theta - v)(u\sin\theta)$$

CASE-3: Body moving up

Consider that a man is standing in a lift compartment which is moving up with velocity v as shown. Let the man project a ball in the direction shown.



In this case, we have

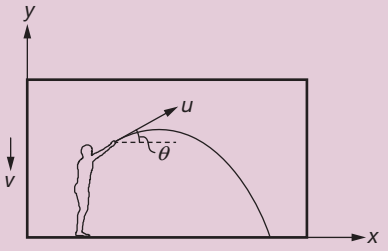
Horizontal component $u_x = u\cos\theta$

Vertical component $u_y = u\sin\theta + v$

$$\Rightarrow \quad R = \frac{2}{g}(u\cos\theta)(u\sin\theta + v)$$

CASE-4: Body moving down

Consider that a man is standing in a lift compartment which is moving down with velocity v as shown. Let the man project a ball in the direction shown.



In this case, we have

Horizontal component $u_x = u\cos\theta$

Vertical component $u_y = u\sin\theta - v$

$$\Rightarrow \quad R = \frac{2}{g}(u\cos\theta)(u\sin\theta - v)$$

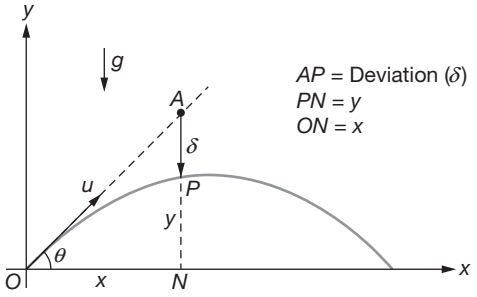
Deviation Suffered by an Oblique Projectile in Time t

Again we proceed with the same kind of thought process, where we assume gravity to be absent first. Then the body launched with initial velocity would continue to move forever along the line OA. However gravity is present, due to which it suffers a deviation δ from its actual track (in the absence of gravity). In triangle OAN.

$$\tan\theta = \frac{AN}{ON} = \frac{y+\delta}{x}$$

$$\Rightarrow \quad \delta = x\tan\theta - y \qquad \ldots(1)$$

Now since we know that $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$



$$\Rightarrow \quad \delta = (x\tan\theta) - \left(x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \right)$$

$$\Rightarrow \quad \delta = \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

Since, $x = (u\cos\theta)t$

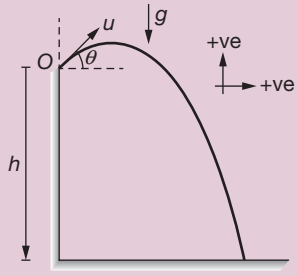
$$\Rightarrow \quad \delta = \frac{1}{2}gt^2$$

Hence deviation suffered by an oblique projectile in time t (or in terms of x, u and launch angle θ) is

$$\delta = \frac{1}{2}gt^2 = \frac{gx^2}{2u^2\cos^2\theta}$$

Problem Solving Technique(s)

CASE-1: Particle projected at an angle θ above the horizontal



The situation is shown in figure. In this case, we have

$$y = -h,\ u_x = u\cos\theta,\ u_y = u\sin\theta \text{ and } a_y = -g$$

For horizontal motion,

$$x = (u\cos\theta)t$$

For vertical motion,

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad gt^2 - (2u\sin\theta)t - 2h = 0$$

$$\Rightarrow \quad t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}}$$

CASE-2: Particle projected at an angle θ below the horizontal

The situation is shown in figure. In this case, we have

$$y = +h,\ u_x = u\cos\theta,\ u_y = +u\sin\theta \text{ and } a_y = +g$$

For horizontal motion,

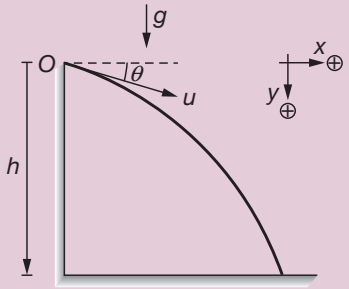
$$x = (u\cos\theta)t$$

For vertical motion,

$$+h = (u\sin\theta)t + \frac{1}{2}gt^2$$

$$\Rightarrow \quad gt^2 + (2u\sin\theta)t - 2h = 0$$

$$\Rightarrow \quad t = \frac{-u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}}$$



Here negative root should be excluded otherwise t would be negative.

ILLUSTRATION 10

A stone is thrown from the top of a tower of height 50 m with a velocity of 30 ms^{-1} at an angle of 30° above the horizontal. Find

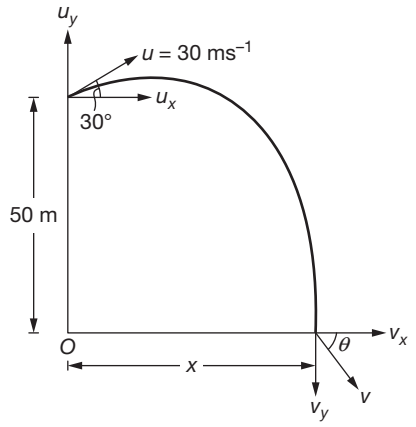
(a) the time during which the stone will be in air,
(b) the distance from the tower base to where the stone will hit the ground,
(c) the speed with which the stone will hit the ground,
(d) the angle formed by the trajectory of the stone with the horizontal at the point of hit.

SOLUTION

The situation is shown in figure

(a) Horizontal component of velocity

$$u_x = 30\cos(30°) = 25.98 \text{ ms}^{-1}$$



Vertical component of velocity

$$v_y = 30\sin(30°) = 15 \text{ ms}^{-1}$$

Let t be the time taken by the stone to reach the ground, i.e., the time during which the stone will be in air. Taking the upward direction as positive, we have

$$-50 = 15t - \frac{1}{2}\times 10 \times t^2 \qquad \left\{\because\ h = ut + \frac{1}{2}gt^2\right\}$$

Solving for t, we get

$$t = 5 \text{ s}$$

(b) The distance x, where the stone will hit the ground

$$x = u_x t = 25.98 \times 5 = 104.90 \text{ m}$$

(c) From figure, $v_x = u_x = 25.98$ ms^{-1}

and $v_y = -15 + (10\times 5) = 35$ ms^{-1}

Since $v = \sqrt{v_x^2 + v_y^2}$

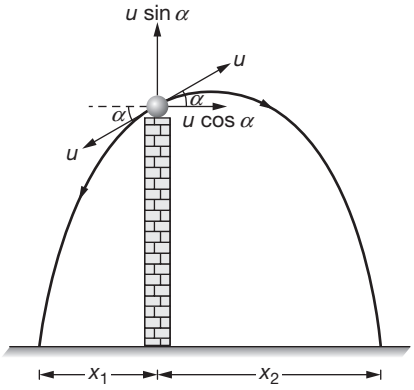
$$\Rightarrow \quad v = \sqrt{\left[(25.98)^2 + (35)^2\right]} = 43.6 \text{ ms}^{-1}$$

(d) $\tan\theta = \left(\dfrac{v_y}{v_x}\right)$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{35}{25.98}\right) \cong 53°$$

ILLUSTRATION 11

From a point at a height h above the horizontal ground a particle A is projected with a velocity u in an upward direction making an angle α with the horizon. Another particle B is projected with the same velocity u but in a downward direction exactly opposite to A. The two particles will strike the ground at a distance x apart. Find x.



SOLUTION

Taking upward direction as positive. Then

For particle A

Vertical displacement $= -h$ (downwards)

Vertical component of velocity $= u\sin\alpha$ (upwards)

and Acceleration $= -g$ (downwards)

$$\Rightarrow \quad -h = (u\sin\alpha)t_1 - \frac{1}{2}gt_1^2$$

$$\Rightarrow \quad \frac{1}{2}gt_1^2 - (u\sin\alpha)t_1 - h = 0$$

$$\Rightarrow \quad gt_1^2 - 2u\sin\alpha t_1 - 2h = 0$$

$$\Rightarrow \quad t_1 = \frac{2u\sin\alpha \pm \sqrt{4u^2\sin^2\alpha + 8gh}}{2g}$$

$$\Rightarrow \quad t = \frac{u\sin\alpha \pm \sqrt{u^2\sin^2\alpha + 2gh}}{g}$$

If we use negative sign, then time t_1 will be negative. So, dropping the negative sign, we get

$$t_1 = \frac{u\sin\alpha + \sqrt{u^2\sin^2\alpha + 2gh}}{g}$$

For particle *B*

We have Vertical Displacement $= -h$ (downwards) Vertical Component of Velocity $= -u\sin\alpha$ (downwards) and Acceleration $= -g$ (downwards)

$$\Rightarrow \quad -h = -(u\sin\alpha)t_2 - \frac{1}{2}gt_2^2$$

$$\Rightarrow \quad h = (u\sin\alpha)t_2 + \frac{1}{2}gt_2^2$$

$$\Rightarrow \quad gt_2^2 + 2(u\sin\alpha)t_2 - 2h = 0$$

Solving, we get

$$\Rightarrow \quad t_2 = \frac{-u\sin\alpha + \sqrt{u^2\sin^2\alpha + 2gh}}{g}$$

t_2 is the time taken by the particle to reach the ground

Now, distance travelled $x = x_1 - (-x_2) = x_1 + x_2$

$$\Rightarrow \quad x = (u\cos\alpha)t_1 + (u\cos\alpha)t_2$$

$$\Rightarrow \quad x = u\cos\alpha(t_1 + t_2)$$

$$\Rightarrow \quad x = u\cos\alpha\left(\frac{2}{g}\sqrt{u^2\sin^2\alpha + 2gh}\right)$$

$$\Rightarrow \quad x = \frac{2u\cos\alpha}{g}\sqrt{u^2\sin^2\alpha + 2gh}$$

ILLUSTRATION 12

A stone is projected from the ground with a velocity of $25\ \text{ms}^{-1}$. Two second later it just clears a wall $5\ \text{m}$ high. Find the angle of projection of the stone, the greatest height reached. How far beyond the wall the stone will again hit the ground. $g = 10\ \text{ms}^{-2}$.

SOLUTION

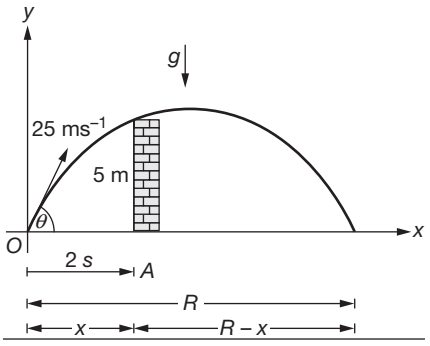
Since $y = u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad 5 = (u\sin\theta)t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \quad 5 = (25\sin\theta)2 - \frac{1}{2}(10)(2)^2$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30^\circ$$



Greatest height reached is $H = \frac{u^2\sin^2\theta}{2g}$

$$\Rightarrow \quad H = \frac{(25)^2\left(\frac{1}{4}\right)}{2(10)} = 7.81\ \text{m}$$

$$\text{Range} = R = \frac{u^2\sin(2\theta)}{g} = \frac{(25)^2\sin(60)}{10}$$

$$= \frac{(625)\sqrt{3}}{20} = 31.25\sqrt{3}\ \text{m}$$

Since $x = (u\cos\theta)t = (25)\left(\frac{\sqrt{3}}{2}\right)(2) = 25\sqrt{3}\ \text{m}$

So, $(R - x) = 6.25\sqrt{3}\ \text{m} = 10.8\ \text{m}$

ILLUSTRATION 13

Two seconds after the projection a projectile is travelling in a direction inclined at 30° to the horizon. After one more second it is travelling horizontally. Determine the magnitude and direction of its velocity.

SOLUTION

At $t = 2$ s

$$v\cos 30 = u\cos\theta \qquad \ldots(1)$$

Also,

$$v_y = u_y + a_y t$$

$$\Rightarrow \quad v\sin 30 = u\sin\theta - 20 \qquad \ldots(2)$$

After one more second, it is travelling horizontally i.e., it is at the maximum height. So, time of ascent is 3 s. Hence

$$\frac{u\sin\theta}{g} = 3$$

$$\Rightarrow \quad u\sin\theta = 30 \qquad \ldots(3)$$

From (2) and (3), we get

$$\frac{v}{2} = 30 - 20$$

$$\Rightarrow \quad v = 20 \text{ ms}^{-1}$$

Substituting in (1), we get

$$u\cos\theta = 10\sqrt{3} \qquad \ldots(4)$$

Squaring (3) and (4) and adding, we get

$$u^2 = 900 + 300$$

$$\Rightarrow \quad u = \sqrt{1200}$$

$$\Rightarrow \quad u = 20\sqrt{3} \text{ ms}^{-1}$$

From (1), we get

$$10\sqrt{3} = 20\sqrt{3}\cos\theta$$

$$\Rightarrow \quad \theta = 60°$$

So, the particle was launched with an initial velocity of $20\sqrt{3}$ ms^{-1} making an angle of $60°$ with the horizontal.

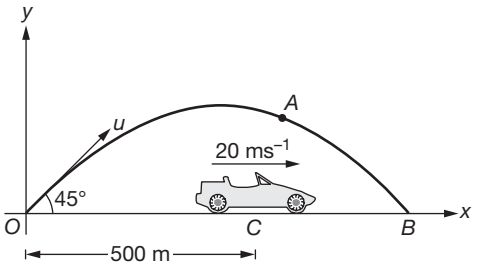
ILLUSTRATION 14

A gun kept on a horizontal straight road, is used to hit a car travelling along the same road away from the gun with a horizontal speed of 72 kmh^{-1}. The car is at a distance of 500 m from the gun when the gun is fired at an angle of $45°$ with the horizontal. Find the distance of the car from the gun when the shell hits it and the speed of the projection of the shell. (Take $g = 9.8$ ms^{-2} and $\sqrt{2} = 1.41$)

SOLUTION

For the shell to hit the car, time taken by the shell to fly from $O \to A \to B$ must equal the time taken by the car to go from C to B with a uniform speed of 20 ms^{-1}. So,

$$t_{\substack{\text{car from}\\ C\to B}} = t_{\substack{\text{shell from}\\ O\to A\to B}} = T = \frac{2u\sin(45°)}{g} = \frac{\sqrt{2}u}{g}$$



Also, we observe that when the shell hits the car at B, then

$$OC + CB = \text{Range}(R)$$

$$\Rightarrow \quad 500 + (20)T = \frac{u^2\sin(90°)}{g}$$

$$\Rightarrow \quad 500 + \frac{20\sqrt{2}u}{g} = \frac{u^2}{g}$$

$$\Rightarrow \quad u^2 - 20\sqrt{2}u - 500g = 0$$

$$\Rightarrow \quad u^2 - 20\sqrt{2}u - 4900 = 0$$

$$\Rightarrow \quad u = \frac{20\sqrt{2} + \sqrt{800 + 4(4900)}}{2}$$

$$\Rightarrow \quad u = \frac{20\sqrt{2} + 142.8}{2} = \frac{28.2 + 142.8}{2} = 85.5 \text{ ms}^{-1}$$

When the shell hits the car, distance of car from the gun is equal to the range

$$R = \frac{u^2}{g} = 746 \text{ m}$$

ILLUSTRATION 15

The velocity of a projectile when it is at the greatest height is $\sqrt{\frac{2}{5}}$ times its velocity when it is at half of its greatest height. Determine its angle of projection.

SOLUTION

Let the particle be projected with velocity u at an angle θ with the horizontal. Horizontal component of its velocity at all heights will be $u\cos\theta$.

At the greatest height, the vertical component of velocity is zero, so the resultant velocity is

$$v_1 = u\cos\theta$$

At half the greatest height during upward motion, we have

$$v_y = \sqrt{u_y^2 - 2a_y y}$$

$$\Rightarrow \quad v_y = \sqrt{u^2\sin^2\theta - 2(g)\frac{u^2\sin^2\theta}{4g}}$$

$$\Rightarrow \quad v_y = \frac{u\sin\theta}{\sqrt{2}}$$

Hence, resultant velocity at half of the greatest height is

$$v_2 = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + \frac{u^2\sin^2\theta}{2}}$$

Since, it is given that $\frac{v_1}{v_2} = \sqrt{\frac{2}{5}}$

$$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{u^2\cos^2\theta}{u^2\cos^2\theta + \frac{u^2\sin^2\theta}{2}} = \frac{2}{5}$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{2}\tan^2\theta} = \frac{2}{5}$$

$$\Rightarrow \quad 2 + \tan^2\theta = 5$$

$$\Rightarrow \quad \tan^2\theta = 3$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

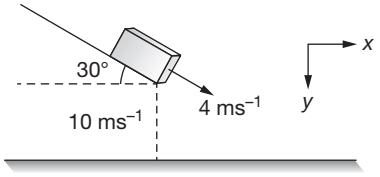
ILLUSTRATION 16

A person working on the roof of a house suddenly drops his hammer, which slides down the roof at a constant speed of $4\ \text{ms}^{-1}$. The roof makes an angle of 30° with the horizontal, and its lowest point is 10 m above the ground. What is the horizontal distance travelled by the hammer after it leaves the roof of the house before it hits the ground?

SOLUTION

$$u_x = 4\cos 30^\circ = 2\sqrt{3}\ \text{ms}^{-1}$$

$$u_y = 4\sin 30^\circ = 2\ \text{ms}^{-1}$$



Since $y = u_y + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad 10 = 2t + \frac{1}{2}(9.8)(t^2)$$

Solving these equations we get

$$t = 1.24\ \text{s}$$

and $x = u_x t = 4.29\ \text{m}$

ILLUSTRATION 17

The range of the rifle bullet is $1000\ \text{m}$, when θ is the angle of projection. If the bullet is fired with the same angle from a car travelling horizontally at a speed of $36\ \text{kmh}^{-1}$ towards the target, calculate the amount by which the range is increased.

SOLUTION

$$R = 1000 = \frac{2}{g}(u\cos\theta)(u\sin\theta) \qquad \ldots(1)$$

When the bullet is fired from a car travelling at a speed of $36\ \text{kmh}^{-1}\ (= 10\ \text{ms}^{-1})$ at the same angle, then the new horizontal component of the velocity of the bullet becomes

$$u_x' = u\cos\theta + 10$$

However, the y component remains unaltered. So, if R' is the new range, then

$$R' = \frac{2}{g}(u\cos\theta + 10)(u\sin\theta)$$

$$\Rightarrow \quad R' = \frac{2}{g}(u\cos\theta)(u\sin\theta) + \frac{20}{g}u\sin\theta$$

$$\Rightarrow \quad R' - R = \frac{20}{g}u\sin\theta \qquad \ldots(2)$$

From (1),

$$u = \sqrt{\frac{500g}{\sin\theta\cos\theta}}$$

$$\Rightarrow \quad R' - R = \frac{20}{g}\sqrt{\frac{500g}{\sin\theta\cos\theta}}\sin\theta$$

$$\Rightarrow \quad R' - R = 20\sqrt{\frac{500}{g}(\tan\theta)} = 20\sqrt{\frac{5000}{98}\tan\theta}$$

$$\Rightarrow \quad R' - R = 20\sqrt{\frac{2500}{49}\tan\theta}$$

$$\Rightarrow \quad R' - R = \frac{1000}{7}\sqrt{\tan\theta}\ \text{m}$$

ILLUSTRATION 18

It at any point on the path of a projectile its velocity be u and inclination be α. Show that the particle will move at right angles to the former direction after a time $t = \dfrac{u}{g\sin\alpha}$ when its velocity would be $v = u\cot\alpha$.

SOLUTION

The initial velocity of the particle is

$$\vec{u} = (u\cos\alpha)\hat{i} + (u\sin\alpha)\hat{j} \qquad \ldots(1)$$

At any time t, the velocity of the particle is $\vec{v}$. Then

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\Rightarrow \quad \vec{v} = (u\cos\alpha)\hat{i} + (u\sin\alpha - gt)\hat{j} \qquad \ldots(2)$$

Now, $\vec{v} \perp \vec{u}$, so we have

$$\vec{v}\cdot\vec{u} = 0$$

$$\Rightarrow \quad u^2\cos^2\alpha + u^2\sin^2\alpha - ugt\sin\alpha = 0$$

$$\Rightarrow \quad u^2 - ugt\sin\alpha = 0$$

$$\Rightarrow \quad t = \frac{u}{g\sin\alpha}$$

Further, from (2), we have

$$|\vec{v}| = \sqrt{u^2\cos^2\alpha + u^2\sin^2\alpha + g^2t^2 - 2ugt\sin\alpha}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{u^2 + g^2t^2 - 2ugt\sin\alpha}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{u^2 + g^2\frac{u^2}{g^2\sin^2\alpha} - 2ug\left(\frac{u}{g\sin\alpha}\right)\sin\alpha}$$

$$\Rightarrow \quad |\vec{v}| = u\sqrt{1 + \text{cosec}^2\alpha - 2}$$

$$\Rightarrow \quad |\vec{v}| = u\sqrt{\text{cosec}^2\alpha - 1}$$

$$\Rightarrow \quad |\vec{v}| = u\cot\alpha \qquad \{\because\ 1 + \cot^2\alpha = \text{cosec}^2\alpha\}$$

ILLUSTRATION 19

With what minimum speed must a particle be projected from origin so that it is able to pass through a given point $P(x,\ y)$?

SOLUTION

Let u and θ be the velocity and angle of projection respectively.

For the projectile to pass through $P(x, y)$

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

$$\Rightarrow \quad gx^2\tan^2\theta - 2xu^2\tan\theta + (gx^2 + 2yu^2) = 0$$

The projectile will pass through $P(x, y)$ if this equation (quadratic in $\tan\theta$) gives some real value of θ, i.e., its discriminant ≥ 0.

$$4x^2u^4 - 4gx^2(gx^2 + 2yu^2) \geq 0$$

$$u^4 - 2gyu^2 - g^2x^2 \geq 0$$

$$u^4 - 2gyu^2 + g^2y^2 \geq g^2x^2 + g^2y^2$$

$$(u^2 - gy)^2 \geq (x^2 + y^2)g^2$$

$$\Rightarrow \quad u \geq \sqrt{gy + g\sqrt{x^2 + y^2}}$$

So, the minimum value of u is $u_{\text{MIN}} = \sqrt{gy + g\sqrt{x^2 + y^2}}$.

ILLUSTRATION 20

Two shots are projected from a gun at the top of a hill with the same velocity u at angles of projection α and β respectively. If the shots strike the horizontal ground through the foot of the hill at the same point, show that the height h of the hill above the plane is given by

$$h = \frac{2u^2(1 - \tan\alpha\tan\beta)}{g(\tan\alpha + \tan\beta)^2}$$

SOLUTION

Let R be the horizontal range for both. Taking the point of projection as origin, the point struck is (R, h). This point satisfies the equations of both the trajectories.

$$-h = R\tan\alpha - \frac{1}{2}g\frac{R^2}{u^2\cos^2\alpha} \quad \ldots(1)$$

and $$-h = R\tan\beta - \frac{1}{2}g\frac{R^2}{u^2\cos^2\beta} \quad \ldots(2)$$

From these equations, we have

$$R\tan\alpha - \frac{1}{2}g\frac{R^2}{u^2\cos^2\alpha} = R\tan\beta - \frac{1}{2}g\frac{R^2}{u^2\cos^2\beta}$$

$$\Rightarrow \quad R(\tan\alpha - \tan\beta) = \frac{1}{2}g\frac{R^2}{u^2\cos^2\alpha} - \frac{1}{2}g\frac{R^2}{u^2\cos^2\beta}$$

$$\Rightarrow \quad \frac{2u^2}{g}(\tan\alpha - \tan\beta) = R(\sec^2\alpha - \sec^2\beta)$$

$$\Rightarrow \quad \frac{2u^2}{g}\cdot(\tan\alpha - \tan\beta) = R\left[(1+\tan^2\alpha) - (1+\tan^2\beta)\right]$$

$$\Rightarrow \quad \frac{2u^2}{g} = R(\tan\alpha + \tan\beta) \quad \ldots(3)$$

$$\Rightarrow \quad R = \frac{2u^2}{g(\tan\alpha + \tan\beta)}$$

Substituting the value of R in equation (1), we get

$$h = R\left(\frac{gR}{2u^2\cos^2\alpha} - \tan\alpha\right)$$

$$\Rightarrow \quad h = \frac{2u^2}{g(\tan\alpha+\tan\beta)}\left(\frac{\sec^2\alpha}{\tan\alpha+\tan\beta} - \tan\alpha\right)$$

$$\Rightarrow \quad h = \frac{2u^2}{g(\tan\alpha+\tan\beta)^2}(\sec^2\alpha - \tan^2\alpha - \tan\alpha\tan\beta)$$

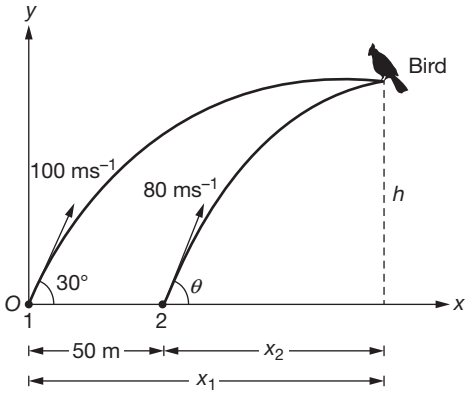
$$\Rightarrow \quad h = \frac{2u^2}{g(\tan\alpha+\tan\beta)^2}(1 - \tan\alpha\tan\beta)$$

ILLUSTRATION 21

Two boys simultaneously aim their guns at a bird sitting on a tower. The first boy releases his shot with a speed of $100\ \text{ms}^{-1}$ at an angle of projection of $30°$. The second boy is ahead of the first by a distance of 50 m and releases his shot with a speed of $80\ \text{ms}^{-1}$. How must he aim his gun so that both the shots hit the bird simultaneously? What is the distance of the foot of the tower from the two boys and the height of the tower? With what velocities and when do the two shots hit the bird?

SOLUTION

The situation is shown in figure



For first shot,

horizontal displacement $= x_1$

vertical displacement $= h$

For second shot,

horizontal displacement $= x_2$

vertical displacement $= h$

Here, $x_1 = x_2 + 50 \quad \ldots(1)$

The two shots will hit the bird simultaneously at a particular instant t, if the horizontal and vertical displacements of the two shots are as shown.

For first shot,

$$h = (100\sin 30°)t - \frac{1}{2}gt^2 \quad \ldots(2)$$

For second shot,

$$h = (80\sin\theta)t - \frac{1}{2}gt^2 \quad \ldots(3)$$

$$\Rightarrow \quad (100\sin 30°)t - \frac{1}{2}gt^2 = (80\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad \sin\theta = \frac{100\sin 30°}{80} = \frac{50}{80} = 0.625$$

$$\Rightarrow \quad \theta = \sin^{-1}(0.625) = 38°68'$$

Now, $x_1 = (100\cos 30°)t$ and $x_2 = 80\cos(38°68')t$

Since $x_1 = x_2 + 50$

$\Rightarrow (100\cos 30°)t = 50 + 80\cos(38°68')t$

$\Rightarrow t\left[100\left(\frac{\sqrt{3}}{2}\right) - 80\times 0.7806\right] = 50$

$\Rightarrow t[86.6025 - 62.4499] = 50$

$\Rightarrow t = \frac{50}{24.1526} = 2.07$ s

So, $x_1 = 100\cos 30°\times 2.07 = 179.27$ m and

$x_2 = 179.27 - 50 = 129.27$ m

Also, $h = 100\sin 30°t - \frac{1}{2}\times 9.8\times(2.07)^2$

$\Rightarrow h = (103.5 - 20.996) = 82.504$ m

ILLUSTRATION 22

A particle moves in the plane x-y with constant acceleration a directed along negative y-axis. The equation of motion of particle has the form $y = Ax - Bx^2$. A and B are positive constants. Find the velocity of the particle at the origin of the co-ordinates.

SOLUTION

METHOD I

Comparing the above given equation with the equation of the oblique projectile moving under the influence of the acceleration field $\vec{a}$.

$\Rightarrow y = x\tan\theta - \frac{ax^2}{2u^2\cos^2\theta}$

{Equation for Oblique Projectile}

$\Rightarrow A = \tan\theta$ and $B = \frac{a}{2u^2\cos^2\theta}$

$\Rightarrow A = \tan\theta$ and $\sec^2\theta = \frac{2u^2B}{a}$ …(1)

We know that

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \frac{2u^2B}{a} - A^2 = 1$ {using (1)}

$\Rightarrow u = \sqrt{(1+A^2)\frac{a}{2B}}$

METHOD II

$\frac{d^2y}{dt^2} = -a$; $\frac{d^2x}{dt^2} = 0$ {Given}

$\Rightarrow y = Ax - Bx^2$ …(1)

Differentiating both sides of equation (1) w.r.t. t,

$\Rightarrow \frac{dy}{dt} = A\frac{dx}{dt} - 2Bx\frac{dx}{dt}$ …(2)

Again differentiating both sides of equation (2) w.r.t. t,

$\Rightarrow \frac{d^2y}{dt^2} = A\frac{d^2x}{dt^2} - 2B\left[\left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2}\right]$

$\Rightarrow -a = 0 - 2B\left[\left(\frac{dx}{dt}\right)^2 + 0\right]$ {from given relations}

$\Rightarrow u_x^2 = \frac{a}{2B}$ …(3)

Also from (2),

$\Rightarrow \left.\frac{dy}{dt}\right|_{x=0} = u_y = Au_x = A\sqrt{\frac{a}{2B}}$ {at Origin}

$\Rightarrow u_y = A\sqrt{\frac{a}{2B}}$ $\{\because$ at Origin $x = 0\}$ …(4)

Now, $u^2 = u_x^2 + u_y^2$

$\Rightarrow u^2 = \frac{a}{2B} + A^2\frac{a}{2B}$ {using (3) & (4)}

$\Rightarrow u = \sqrt{(1+A^2)\frac{a}{2B}}$

RANGE, MAXIMUM HEIGHT AND TIME OF FLIGHT FOR COMPLIMENTARY ANGLES

For complimentary angles, say ϕ and $(90-\phi)$, let the corresponding ranges, maximum heights and times of flight be R_ϕ, $R_{90-\phi}$, H_ϕ, $H_{90-\phi}$ and T_ϕ, $T_{90-\phi}$, then

$R_\phi = \frac{2u^2\sin\phi\cos\phi}{g}$ and $R_{90-\phi} = \frac{2u^2\cos\phi\sin\phi}{g}$

$\Rightarrow R_\phi = R_{90-\phi} = \frac{2u^2\sin\phi\cos\phi}{g}$ …(1)

So, we must remember that for complimentary angles range is the same i.e.,

$$R_{1^\circ} = R_{89^\circ},\ R_{15^\circ} = R_{75^\circ} \text{ and so on}$$

Further more,

$$H_\phi = \frac{u^2 \sin^2 \phi}{2g} \qquad \ldots(2)$$

$$H_{90-\phi} = \frac{u^2 \sin^2 (90-\phi)}{2g} = \frac{u^2 \cos^2 \phi}{2g} \qquad \ldots(3)$$

From (2) and (3), we observe that

$$H_\phi + H_{90-\phi} = \frac{u^2}{2g} \qquad \ldots(4)$$

Also, from equations (1), (2) and (3), we observe that

$$R_\phi = R_{90-\phi} = 4\sqrt{H_\phi H_{90-\phi}} \qquad \ldots(5)$$

$$\Rightarrow \quad R_{1^\circ} = R_{89^\circ} = 4\sqrt{H_{1^\circ} H_{89^\circ}}$$

Finally,

$$T_\phi = \frac{2u \sin \phi}{g} \text{ and } T_{90-\phi} = \frac{2u \cos \phi}{g}$$

$$\Rightarrow \quad T_\phi T_{90-\phi} = \frac{4u^2 \sin\phi \cos\phi}{g^2} = \frac{2}{g}\left(\frac{2u^2 \sin\phi\cos\phi}{g}\right)$$

$$\Rightarrow \quad T_\phi T_{90-\phi} = \frac{2R_\phi}{g} = \frac{2R_{90-\phi}}{g} \qquad \ldots(6)$$

i.e., $T_{1^\circ} T_{89^\circ} = \dfrac{2R_{1^\circ}}{g} = \dfrac{2R_{89^\circ}}{g}$

Problem Solving Technique(s)

So, for complimentary angles ϕ and $90-\phi$, following points are worthnoting.

(a) $R_\phi = R_{90-\phi} = \dfrac{2u^2 \sin\phi\cos\phi}{g}$

(b) $H_\phi + H_{90-\phi} = \dfrac{u^2}{2g}$

(c) $R_\phi = R_{90-\phi} = 4\sqrt{H_\phi H_{90-\phi}}$

(d) $T_\phi T_{90-\phi} = \dfrac{2R_\phi}{g} = \dfrac{2R_{90-\phi}}{g}$

ILLUSTRATION 23

A shell bursts on contact with the ground and the fragments fly in all the directions with speeds upto $39.2\ \text{ms}^{-1}$. Show that a man 78.4 m away is in danger for $4\sqrt{2}$ s.

SOLUTION

Since $R = \dfrac{u^2 \sin(2\theta)}{g}$

$$\Rightarrow \quad 78.4 = \frac{(39.2)^2 \sin(2\theta)}{9.8}$$

$$\Rightarrow \quad \sin(2\theta) = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = 30^\circ \text{ or } 150^\circ$$

$$\Rightarrow \quad \theta = 15^\circ \text{ or } 75^\circ$$

i.e., the range will be same for these two complimentary angles of 15° and 75°. However, the time of flight will not be the same. Hence the person 78.4 m away will be in danger for a time t given by

$$t = t_{75^\circ} - t_{15^\circ}$$

$$\Rightarrow \quad t = \frac{2u}{g}(\sin(75^\circ) - \sin(15^\circ))$$

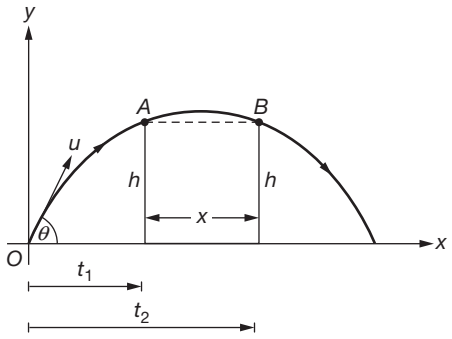
$$\Rightarrow \quad t = \frac{2(39.2)}{9.8}\left[2\sin\left(\frac{75^\circ+15^\circ}{2}\right)\sin\left(\frac{75^\circ-15^\circ}{2}\right)\right]$$

$$\Rightarrow \quad t = 16\sin(45^\circ)\sin(30^\circ)$$

$$\Rightarrow \quad t = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}\ \text{s}$$

TWO UNIQUE TIMES FOR WHICH PROJECTILE IS AT SAME HEIGHT

It has been observed that, there are two unique times at which the projectile is at same height $h(<H)$. Let us consider a projectile launched with initial velocity u, making an angle θ with the horizontal. Let the projectile be at a height $h(<H)$ at time t. Then we observe just by a qualitative argument that this is true for upward motion of the projectile and then for the downward motion, as shown.



Mathematically, at time t, we have

$$h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0 \qquad \ldots(1)$$

This equation happens to be a quadratic equation in t (with non-zero discriminant), hence has two unique roots t_1 and t_2. A very important observation which cannot be skipped here is that

$$t_1 + t_2 = \frac{2u\sin\theta}{g} = T \qquad \ldots(2)$$

$$t_1 t_2 = \frac{2h}{g} \qquad \ldots(3)$$

From the diagram we must understand that,

t_1 is the time taken by the projectile to fly from $O \to A$ and

t_2 is the time taken by the projectile to fly from $O \to A \to B$

So, time taken by the projectile to fly from A to B is

$$t = t_2 - t_1 \qquad \ldots(4)$$

Since $(t_2 - t_1)^2 = (t_2 + t_1)^2 - 4t_1t_2$

$$t = t_2 - t_1 = \sqrt{\frac{4u^2\sin^2\theta}{g^2} - \frac{8h}{g}} \qquad \ldots(5)$$

If, x be the horizontal separation between the two events, then

$$x = (u\cos\theta)t = (u\cos\theta)\sqrt{\frac{4u^2\sin^2\theta}{g^2} - \frac{8h}{g}} \qquad \ldots(6)$$

The following Illustration shows how to use equations (2), (3), (4), (5) and (6) to get the results efficiently.

ILLUSTRATION 24

A stone is projected from a point on the ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain a maximum height of $2h$ above the ground. If at the instant of projection the bird were to fly away horizontally with a uniform speed then calculate the ratio between the horizontal velocity of the bird and the stone, if the stone still hits the bird while descending.

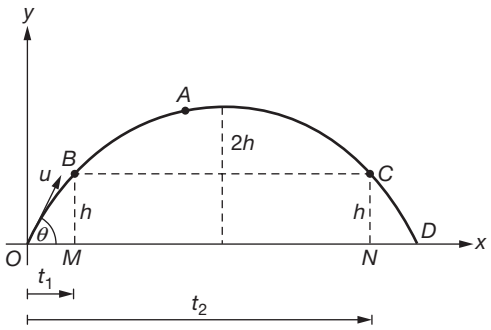
SOLUTION

$$H = 2h = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow \quad u\sin\theta = 2\sqrt{gh} \qquad \ldots(1)$$

Since $h = (u\sin\theta)t + \frac{1}{2}(-g)t^2$

$$\Rightarrow \quad h = 2\sqrt{gh}t - \frac{1}{2}gt^2$$



$$\Rightarrow \quad t^2 - 4\sqrt{\frac{h}{g}}t + \frac{2h}{g} = 0 \qquad \ldots(2)$$

This quadratic equation in t will have two roots, t_1 and t_2. So,

$$t_1 = \frac{4\sqrt{\frac{h}{g}} - \sqrt{\frac{16h}{g} - \frac{8h}{g}}}{2} = (2 - \sqrt{2})\sqrt{\frac{h}{g}} \qquad \ldots(3)$$

$$t_2 = \frac{4\sqrt{\frac{h}{g}} + \sqrt{\frac{16h}{g} - \frac{8h}{g}}}{2} = (2+\sqrt{2})\sqrt{\frac{h}{g}} \qquad \ldots(4)$$

So, time taken by the stone to fly from B to C is $(t_2 - t_1)$.

$$\Rightarrow \quad BC = (u\cos\theta)(t_2 - t_1) \qquad \ldots(5)$$

After reading the question carefully, we observe that at the instant of projection of stone the bird were to fly away horizontally with a uniform speed (say v_b) and the stone, during its downward motion (at C) still hits the bird. So, time taken by the bird to fly from B to C is the time taken by the stone to go from $O \to B \to A \to C$. Hence, for the bird, we have

$$BC = v_b t_2 \qquad \ldots(6)$$

From (5) and (6), we get

$$v_b t_2 = (u\cos\theta)(t_2 - t_1)$$

$$\Rightarrow \quad \frac{v_b}{u\cos\theta} = \frac{t_2 - t_1}{t_2} = \frac{2\sqrt{2}}{2+\sqrt{2}} = \frac{2}{\sqrt{2}+1}$$

RADIUS OF CURVATURE OF AN OBLIQUE PROJECTILE AT A POINT *P*

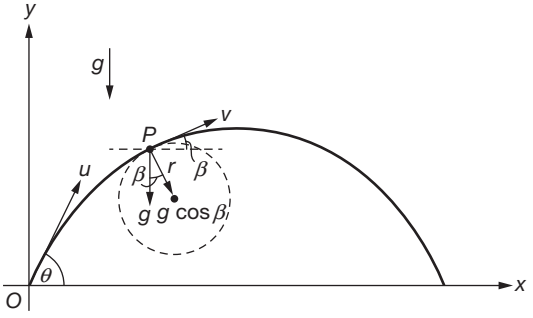
At Any Instant of Time (*t*)

Consider a projectile launched with initial velocity u, making an angle θ with the horizontal. Let the projectile be at a point P, at any instant t (say). Then we know that

$$r = \frac{v_T^2}{a_C} = \frac{v_\perp^2}{a_\parallel} = \frac{v^2}{g\cos\beta} \qquad \ldots(1)$$

where

$$v^2 = (u\sin\theta - gt)^2 + (u\cos\theta)^2 \qquad \{\text{studied earlier}\}$$



$$\Rightarrow \quad v^2 = u^2 + g^2t^2 - 2ugt\sin\theta$$

At Any Position at Vertical Height *h*(< *H*)

Here too everything remains the same i.e.,

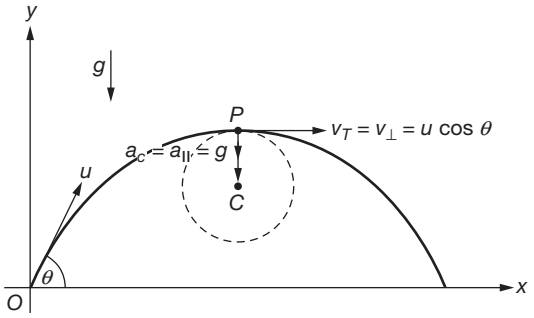
$$r = \frac{v_T^2}{a_C} = \frac{v_\perp^2}{a_\parallel} = \frac{v^2}{g\cos\beta}$$

But here, $v^2 = u^2 - 2gh$ {studied earlier}

At Maximum Height (*H*)

At the maximum height H, velocity is $u\cos\theta$.

$$\Rightarrow \quad v_T = v_\perp = u\cos\theta$$



Also, $a_C = a_\parallel = g$

$$\Rightarrow \quad r = \frac{v_T^2}{a_C} = \frac{v_\perp^2}{a_\parallel} = \frac{u^2\cos^2\theta}{g}$$

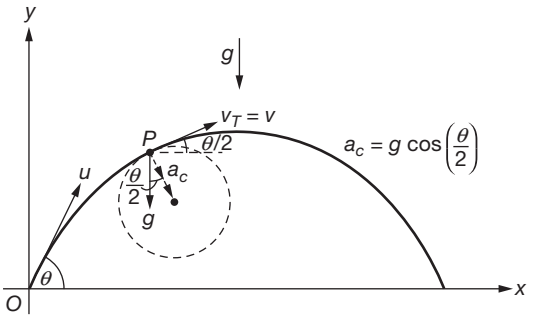
This result can also be obtained from cases discussed in (A) and (B), where we can put special values to get the desired result. (I hope this is a small exercise for you people).

ILLUSTRATION 25

A particle is projected with a speed u making an angle θ with the horizontal. Find the radius of curvature at the point where the particle velocity makes an angle $\frac{\theta}{2}$ with the horizontal. Assume the particle to move under gravity in the absence of any air drag.

SOLUTION

First of all, after reading the question carefully, we see that the particle will act as an oblique projectile following a parabolic path, as shown.



Since $r = \frac{v_T^2}{a_c} = \frac{v_\perp^2}{a_\parallel}$

$$\Rightarrow \quad r = \frac{v^2}{g\cos\left(\frac{\theta}{2}\right)} \qquad \ldots(1)$$

Since $v\cos\left(\frac{\theta}{2}\right) = u\cos\theta$

$\{\because$ Horizontal Motion is Non-Accelerated$\}$

$$\Rightarrow \quad v = \frac{u\cos\theta}{\cos\left(\frac{\theta}{2}\right)} \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$r = \frac{u^2\cos^2\theta}{g\cos^3\left(\frac{\theta}{2}\right)}$$

EQUATION OF TRAJECTORY OF AN OBLIQUE PROJECTILE IN TERMS OF RANGE

Since, we know that for a projectile launched with initial velocity u, making an angle θ with the horizontal, the equation of trajectory is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\Rightarrow \quad y = x\tan\theta\left[1 - \frac{gx^2}{2u^2\cos^2\theta(x\tan\theta)}\right]$$

$$\Rightarrow \quad y = x\tan\theta\left[1 - \frac{x}{\left(\frac{2u^2\sin\theta\cos\theta}{g}\right)}\right]$$

Since $R = \frac{2u^2\sin\theta\cos\theta}{g}$

$$\Rightarrow \quad y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

ILLUSTRATION 26

A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the magnitude and direction of the velocity of the ball.

SOLUTION

As the ball strikes the ground at a distance 14 m from the wall, the range is $R = (4+14)\text{ m} = 18\text{ m}$.

Since $y = x\tan\theta\left(1 - \frac{x}{R}\right)$ $\qquad \ldots(1)$

where, $x = 4\text{ m}$, $y = 4\text{ m}$ and $R = 18\text{ m}$.

$$\Rightarrow \quad 4 = 4\tan\theta\left(1 - \frac{4}{18}\right) = 4\tan\theta\left(\frac{7}{9}\right)$$

$$\Rightarrow \quad \tan\theta = \frac{9}{7}$$

$$\Rightarrow \quad \sin\theta = \frac{9}{\sqrt{130}} \text{ and } \cos\theta = \frac{7}{\sqrt{130}}$$

Again $R = \frac{2}{g}u^2\sin\theta\cos\theta$

$$\Rightarrow \quad R = \frac{2}{9.8}\times u^2 \times \frac{9}{\sqrt{130}} \times \frac{7}{\sqrt{130}}$$

$$\Rightarrow \quad u^2 = \frac{18\times 9.8\times\sqrt{130}\times\sqrt{130}}{2\times 9\times 7}$$

$$\Rightarrow \quad u^2 = \frac{98\times 13}{7} = 182$$

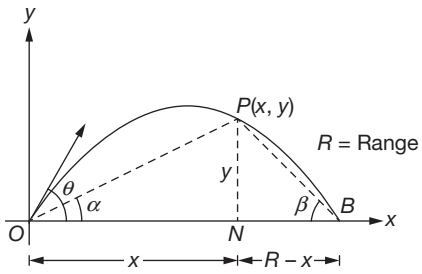
$$\Rightarrow \quad u = \sqrt{182}\text{ ms}^{-1} = 13.5\text{ ms}^{-1}$$

ILLUSTRATION 27

A particle is thrown over a triangle from one end of a horizontal base and after grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.

SOLUTION

The situation is shown in figure. In triangle NOP, $\tan\alpha = \frac{y}{x}$.



In triangle BPN, $\tan\beta = \frac{y}{R-x}$

$$\Rightarrow \quad \tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x}$$

$$\Rightarrow \quad \tan\alpha + \tan\beta = \frac{yR}{x(R-x)} \qquad \ldots(1)$$

Since, the equation of trajectory is $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$$\Rightarrow \quad \tan\theta = \frac{yR}{x(R-x)} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\tan\theta = \tan\alpha + \tan\beta$$

RELATIVE MOTION BETWEEN TWO PROJECTILES/MOTION OF ONE PROJECTILE AS SEEN FROM ANOTHER PROJECTILE

METHOD I

Since both the projectiles are moving under the influence of gravity, so acceleration of one projectile as seen from the other is

$$\vec{a} = \vec{g} - \vec{g} = \vec{0} \;\Rightarrow\; a_x\hat{i} + a_y\hat{j} = \vec{0}$$

$\Rightarrow \quad a_x = 0$	$a_y = 0$
$\Rightarrow \quad \frac{dv_x}{dt} = 0$	$\Rightarrow \quad \frac{dv_y}{dt} = 0$
$\Rightarrow \quad v_x = \text{constant} = k_1$ (say)	$\Rightarrow \quad v_y = \text{constant} = k_2$ (say)
$\Rightarrow \quad \frac{dx}{dt} = k_1$	$\Rightarrow \quad \frac{dy}{dt} = k_2$
$\Rightarrow \quad dx = k_1 dt$	$\Rightarrow \quad dy = k_2 dt$
$\Rightarrow \quad \int_0^x dx = k_1\int_0^t dt$	$\Rightarrow \quad \int_0^y dy = k_2\int_0^t dt$
$\Rightarrow \quad x = k_1 t \qquad \ldots(1)$	$\Rightarrow \quad y = k_2 t \qquad \ldots(2)$

$$\Rightarrow \quad \frac{y}{x} = \frac{k_2}{k_1} = k \qquad \text{(say)}$$

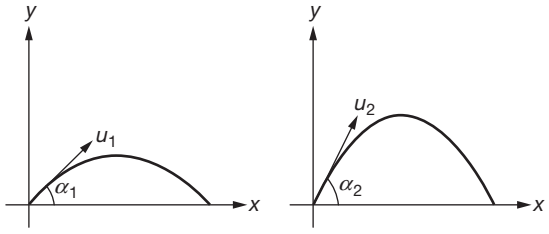
$\Rightarrow \quad y = kx$, equation of a straight line with slope $\frac{k_2}{k_1}$

So, motion of one projectile as seen from another projectile is a straight line passing through the origin (or the point of launch).

METHOD II

Let us now discuss the relative motion between two projectiles or the path observed by one projectile of the other. Suppose that two particles are projected from the ground with speeds u_1 and u_2 at angles α_1 and α_2 as shown in figure. Acceleration of both the particles is g downwards. So, relative acceleration between them is zero because

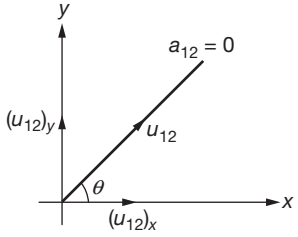
$$a_{12} = a_1 - a_2 = g - g = \text{zero}$$



i.e., the relative motion between the two particles is uniform. Now,

$$(u_1)_x = u_1\cos\alpha_1 \text{ and } (u_2)_x = u_2\cos\alpha_2$$

$$(u_1)_y = u_1\sin\alpha_1 \text{ and } (u_2)_y = u_2\sin\alpha_2$$



Therefore,

$$(u_{12})_x = (u_1)_x - (u_2)_x = u_1\cos\alpha_1 - u_2\cos\alpha_2$$

and $$(u_{12})_y = (u_1)_y - (u_2)_y = u_1\sin\alpha_1 - u_2\sin\alpha_2$$

$(u_{12})_x$ and $(u_{12})_y$ are the x and y components of relative velocity of 1 with respect to 2.

Hence, relative motion of 1 with respect to 2 is a straight line at an angle $\theta = \tan^{-1}\left[\dfrac{(u_{12})_y}{(u_{12})_x}\right]$ with positive x-axis.

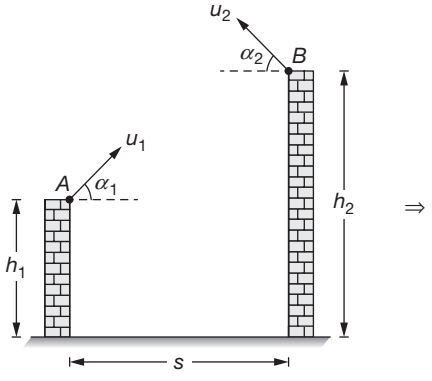
(a) If $(u_{12})_x = 0$, i.e., $u_1\cos\alpha_1 = u_2\cos\alpha_2$, then the relative motion is along y-axis or in vertical direction (as $\theta = 90°$).

(b) Similarly, if $(u_{12})_y = 0$, i.e., $u_1\sin\alpha_1 = u_2\sin\alpha_2$, then the relative motion is along x-axis or in horizontal direction (as $\theta = 0°$).

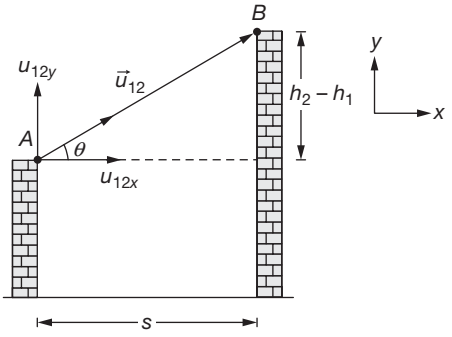
CONDITION OF COLLISION BETWEEN TWO PROJECTILES

Now it is clear that relative motion between two projectiles is uniform and the path of one projectile as observed by the other is a straight line. Let the particles be projected simultaneously from two different heights h_1 and h_2 with speeds u_1 and u_2 in the directions shown in figure. Then the particles collide in air if the relative velocity of 1 with respect to $2(\vec{u}_{12})$ is along line AB or the relative velocity of 2 with respect to $1(\vec{u}_{21})$ is along the line BA. Thus,

$$\tan\theta = \frac{(u_{12})_y}{(u_{12})_x} = \left(\frac{h_2 - h_1}{s}\right)$$



$\Rightarrow$



Here, $(u_{12})_y = u_1\sin\alpha_1 - u_2\sin\alpha_2$ and

$$(u_{12})_x = (u_1\cos\alpha_1) - (-u_2\cos\alpha_2)$$

$$\Rightarrow \quad (u_{12})_x = u_1\cos\alpha_1 + u_2\cos\alpha_2$$

If both the particles are initially at the same level $(h_1 = h_2)$, then for collision

$$(u_{12})_y = 0 \text{ or } u_1\sin\alpha_1 = u_2\sin\alpha_2$$

The time of collision of the two particles will be

$$t = \frac{AB}{|\vec{u}_{12}|} = \frac{AB}{\sqrt{(u_{12})_x^2 + (u_{12})_y^2}}$$

Further, the above conditions are not merely sufficient for collision to takes place. For example, the time of collision discussed above should be less than the time of collision of either of the particles with the ground.

ILLUSTRATION 28

Two guns situated at the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ ms^{-1} at some interval of time. One gun fires horizontally and the other fires upwards at an angle of $60°$ with the horizontal. The shots collide in air at a point P. Find

(a) the time interval between the firings and

(b) the coordinates of the point P. Take origin of coordinates system at the foot of the hill right below the muzzle and trajectory in x-y plane.

SOLUTION

Let the shot fired at $60°$ above horizontal reach point P in a time t. Then the shot fired horizontal must have taken a time $(t - \Delta t)$ to reach the point P, where Δt is the time log between firing of shots.

(a) Let they collide at the point $P(x, y)$, then

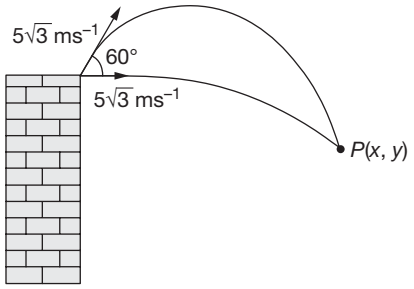
$$x = (5\sqrt{3}\cos 60) \quad t = 5\sqrt{3}(t - \Delta t)$$

$$\Rightarrow \quad t = 2t - 2\Delta t$$

$$\Rightarrow \quad t = 2\Delta t \qquad \ldots(1)$$

$$\Rightarrow \quad y = \left(-5\sqrt{3}\sin 60\right)\ t + \frac{1}{2}gt^2$$

$$\Rightarrow \quad y = \frac{1}{2}g(t - \Delta t)^2$$



$$\Rightarrow \quad -\frac{15}{2}t + 5t^2 = 5(t - \Delta t)^2$$

$$\Rightarrow \quad -\frac{3}{2}t + t^2 = \frac{t^2}{4}$$

$$\Rightarrow \quad -6t + 4t^2 = t^2$$

$$\Rightarrow \quad 3t^2 - 6t = 0$$

$$\Rightarrow \quad 3t(t - 2) = 0$$

$$\Rightarrow \quad t = 2 \text{ s} \qquad \{\because t \neq 0\}$$

So, the interval between the firings is

$$\Delta t = \frac{t}{2} = 1 \text{ s}$$

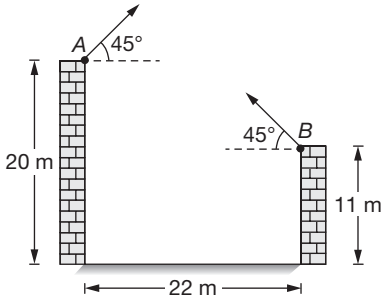
(b) $x = \left(5\sqrt{3}\cos 60\right)t = 5\sqrt{3}$ m

$$\Rightarrow \quad y = 5 \text{ m}$$

Hence the particles collide at $\left(5\sqrt{3}, 5\right)$ m.

ILLUSTRATION 29

Two particles are simultaneously thrown from the roofs of two high buildings as shown in figure. Their velocities are $v_A = 2$ ms^{-1} and $v_B = 14$ ms^{-1} respectively. Calculate the minimum distance between the particles in the process of their motion. Also find the time when they are at closest distance.



SOLUTION

Assuming A to be at rest

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = 0 \text{ as } \vec{a}_A = \vec{a}_B = g \text{ (downwards)}$$

Thus, the relative motion between them is uniform. Relative velocity of B with respect to A in vertical direction,

$$(u_{BA})_V = u_B \sin 45° - u_A \sin 45° = (14 - 2)\frac{1}{\sqrt{2}}$$

$$(u_{BA})_V = 6\sqrt{2} \text{ ms}^{-1}$$

Relative velocity of B with respect to A in horizontal direction,

$$(u_{BA})_H = u_B \cos 45° - (-u_A \cos 45°)$$

$$(u_{BA})_H = (14 + 2)\frac{1}{\sqrt{2}} = 8\sqrt{2} \text{ ms}^{-1}$$

Horizontal distance between A and B after time t

$$x = 22 - \left((u_{BA})_H\right)t = \left(22 - 8\sqrt{2}t\right) \text{ m}$$

and vertical distance between A and B after time t is

$$y = 9 - \left((u_{BA})_V\right)t = \left(9 - 6\sqrt{2}t\right) \text{ m}$$

Therefore, distance between them after time t is

$$r = \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad r^2 = x^2 + y^2 = \left(22 - 8\sqrt{2}t\right)^2 + \left(9 - 6\sqrt{2}t\right)^2$$

For r to be minimum $\frac{d}{dt}(r^2) = 0$

$$\Rightarrow \quad 2\left(22 - 8\sqrt{2}t\right)\left(-8\sqrt{2}\right) + 2\left(9 - 6\sqrt{2}t\right)\left(-6\sqrt{2}\right) = 0$$

$$\Rightarrow \quad 88 - 32\sqrt{2}t + 27 - 18\sqrt{2}t = 0$$

$$\Rightarrow \quad t = \frac{23}{10\sqrt{2}} \text{ s}$$

$$\Rightarrow \quad r_{\min} = \sqrt{x^2 + y^2} \text{ at time } t = \frac{23}{10\sqrt{2}} \text{ s}$$

Substituting the values, we get

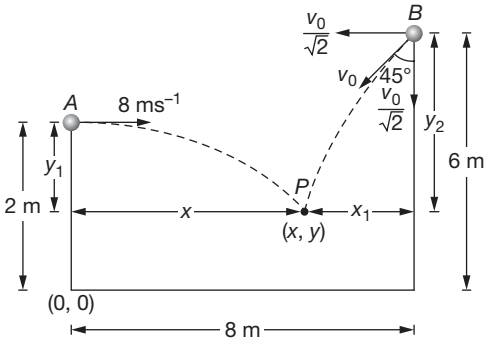
$$r_{\min} = 6 \text{ m}$$

ILLUSTRATION 30

From points A and B, at the respective heights of 2 m and 6 m, two bodies are thrown simultaneously towards each other; one is thrown horizontally with a velocity of 8 ms^{-1} and the other, downward at an angle of $45°$ to the horizontal at an initial velocity such that the bodies collide in flight. The horizontal distance between points A and B equals 8 m. Calculate the initial velocity v_0 of the body thrown at an angle $45°$, the co-ordinates $(x,\ y)$ of the point of collision, the time of flight t of the bodies before colliding and velocities v_A, v_B of the two bodies at the instant of collision. The trajectories lie in a single plane.

SOLUTION

The situation is shown here.



Let the two bodies collide after t second.

From figure, $x = 8 \times t$ and $x_1 = \left(\frac{v_0}{\sqrt{2}}\right) \times t$

$$\Rightarrow \quad x + x_1 = 8t + \left(\frac{v_0}{\sqrt{2}}\right) t$$

But given that $x + x_1 = 8$

$$\Rightarrow \quad 8 = 8t + \left(\frac{v_0}{\sqrt{2}}\right) t \qquad \ldots(1)$$

Again $y_1 = \frac{1}{2} g t^2$

and $y = 2 - y_1 = 2 - \frac{1}{2} g t^2 \qquad \ldots(2)$

Further, $y_2 = \left(\frac{v_0}{\sqrt{2}}\right) t + \frac{1}{2} g t^2$

$$\Rightarrow \quad y = 6 - \left(\frac{v_0}{\sqrt{2}}\right) t - \frac{1}{2} g t^2 \qquad \ldots(3)$$

From equations (2) and (3), we get

$$2 - \frac{1}{2} g t^2 = 6 - \left(\frac{v_0}{\sqrt{2}}\right) t - \frac{1}{2} g t^2$$

$$\Rightarrow \quad 4 = \left(\frac{v_0}{\sqrt{2}}\right) t \qquad \ldots(4)$$

Substituting this value of t in equation (1), we get

$$8 = 8t + 4$$

$$\Rightarrow \quad t = 0.5 \text{ s}$$

From equation (4),

$$4 = \left(\frac{v_0}{\sqrt{2}}\right) \times 0.5$$

$$\Rightarrow \quad v_0 = 11.28 \text{ ms}^{-1}$$

Now, $x = 8t = 8 \times 0.5 = 4 \text{ m}$

$$y = 2 - \frac{1}{2} g t^2 = 0.775 \text{ m}$$

$$\Rightarrow \quad (x,\ y) = (4,\ 0.775)$$

At P, we have $v_A = \sqrt{v_{Ax}^2 + v_{Ay}^2}$

$$\Rightarrow \quad v_A = \sqrt{(8)^2 + (4.9)^2} = 9.4 \text{ ms}^{-1}$$

and $v_B = \sqrt{v_{Bx}^2 + v_{By}^2}$

$$\Rightarrow \quad v_B = \sqrt{\left(\frac{11.28}{\sqrt{2}}\right)^2 + \left(\frac{11.28}{\sqrt{2}} + \frac{9.8}{2}\right)^2}$$

$$\Rightarrow \quad v_B = 15.2 \text{ ms}^{-1}$$

Test Your Concepts-III

Based on Oblique Projectile

(Solutions on page H.146)

1. Find the angle of projection of a projectile for which the horizontal range and maximum height are equal.

2. Prove that the maximum horizontal range is four times the maximum height attained by the projectile.

3. Two projectiles are launched with same initial velocity at different launch angles so as to have identical range. Show that the sum of the maximum heights attained by them is independent of the launch angles.

4. Show that there are two values of time for which a projectile is at the same height. Also prove that the sum of these two times is equal to the time of flight. Also find the time lapse and the horizontal separation between the two events.

5. Two shots are fired simultaneously from the top and bottom of a vertical tower AB at angles α and β with horizontal respectively. Both shots strike at the same point C on the ground at distance s from the foot of the tower at the same time. Show that the height of the tower is $S(\tan\beta - \tan\alpha)$.

6. A ball is shot from the ground into the air. At a height of 9.1 m, its velocity is observed to be $\vec{v} = (7.6\hat{i} + 6.1\hat{j})$ ms^{-1}.

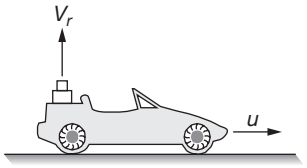
(a) To what maximum height does the ball rise?

(b) What total horizontal distance does the ball travel?

(c) Find the magnitude and direction of the ball's velocity just before it hits the ground?

7. The coach throws a baseball to a player with an initial speed of 20 ms^{-1} at an angle of 45° with the horizontal. At the moment the ball is thrown, the player is 50 m from the coach. At what speed and in what direction must the player run to catch the ball at the same height at which it was released?

8. A projectile is fired from the vertical tube mounted on the vehicle which is travelling at the constant speed $u = 30$ kmh^{-1}. The projectile leaves the tube with a velocity $v_r = 20$ ms^{-1} relative to the tube. If air resistance is neglected, show that the projectile will land on the vehicle at the tube location and calculate the distance s travelled by the vehicle during the flight of the projectile.



9. A stone is thrown from the top of a tower of height 50 m with a velocity of 30 ms^{-1} at an angle of 30° above the horizontal. Find the

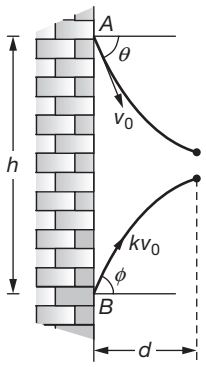
(a) time during which the stone will be in air,

(b) distance from the tower base to where the stone will hit the ground,

(c) speed with which the stone will hit the ground.

$\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$.

10. At the same instant two boys throw balls A and B from the window with a speed v_0 and kv_0, respectively, where k is a constant. Show that the balls with collide if $k = \dfrac{\cos\theta}{\cos\phi}$.

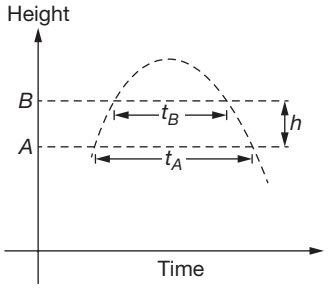


11. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a cm short of it when the elevation is α and goes b cm far when the elevation is β. Assume the speed of projection to be the same in all the cases, then show that the proper angle of projection is $\dfrac{1}{2}\sin^{-1}\left(\dfrac{a\sin(2\beta) + b\sin(2\alpha)}{a+b}\right)$.

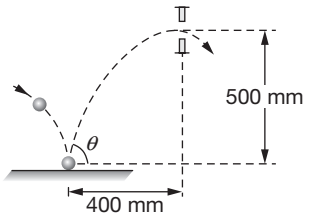
12. Two particles are simultaneously projected in the same vertical plane from the same point with velocities u and v at angles α and β with horizontal. Find the time that elapses when their velocities are parallel.

13. A projectile takes off with an initial velocity of $10\ \text{ms}^{-1}$ at an angle of elevation of 45°. It is just able to clear two hurdles of height 2 m each, separated from each other by a distance d. Calculate d. At what distance from the point of projection is the first hurdle placed? Take $g = 10\ \text{ms}^{-2}$.

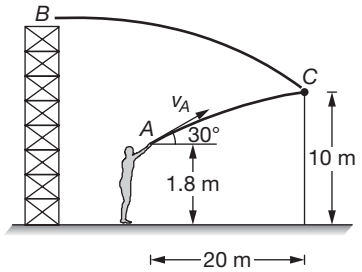
14. The acceleration of gravity can be measured by projecting a body upward and measuring the time it takes to pass two given points in both directions. Show that if the time the body takes to pass a horizontal line A in both directions is t_A and time to go by a second line B in both directions is t_B, then assuming that the acceleration is constant, its magnitude is $g = \dfrac{8h}{t_A^2 - t_B^2}$, where h is the height of the line B above line A.



15. To meet design criteria, small ball bearings must bounce through an opening of limited size at the top of their trajectory when rebounding from a heavy plate as shown. Calculate the angle θ made by the rebound velocity with the horizontal and the velocity v of the balls at the instant they pass through the opening.



16. A horizontal projectile is fired from a tower B. The shooter fires his gun from point A at an angle of 30°. Determine the muzzle speed of the bullet if it hits the projectile at C.



17. A particle is projected at an angle of elevation α and after t second it appears to have an elevation of β as seen from the point of projection. Find the initial velocity of projection.

18. A particle is projected with a velocity u at an angle α to the horizontal, in a vertical plane. At time t, it is moving in a direction making an angle β with the horizontal. Prove that $gt\cos\beta = u\sin(\alpha - \beta)$.

19. A ball is projected so as to just clear two walls, the first of height a at a distance b from the point of projection and the second of height b and at a distance a from point of projection. Show that the range on the horizontal plane is $R = \dfrac{a^2 + ab + b^2}{a + b}$.

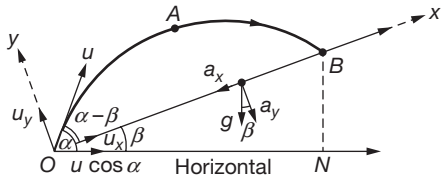
Also show that the angle of projection is given by $\tan^{-1}\left(\dfrac{a^2 + ab + b^2}{ab}\right)$.

20. The angular elevation of an enemy's position on a hill h m high is β. Show that, in order to shell it, the minimum initial velocity of the projectile must be $\sqrt{gh(1 + \text{cosec}\,\beta)}$.

21. A gun is fired from a moving platform and ranges of the shot are observed to be R_1 and R_2 when the platform is moving forward and backward respectively with velocity v. Find the elevation of the gun α in terms of the given quantities.

MOTION OF A PROJECTILE UP AN INCLINED PLANE

Consider a projectile to be launched with initial velocity u making an angle α with horizontal on an inclined plane which makes angle $\beta(<\alpha)$ with the horizontal. Then u and g can be resolved into two components each (one along the incline and the other perpendicular to the incline)



(a) $u_x = u\cos(\alpha-\beta)$ along the incline along $+x$ axis.

(b) $u_y = u\sin(\alpha-\beta)$ along the incline along $+y$ axis.

(c) $|a_x| = g\sin\beta$ along $-x$ axis, acting as retardation to motion along the incline i.e., $a_x = -g\sin\beta$.

(d) $|a_y| = g\cos\beta$ along $-y$ axis, acting as retardation to motion perpendicular to incline i.e., $a_y = -g\cos\beta$.

Time of Flight (T) (Up the Incline)

Time taken by the particle to go from O to A to B. When the particle reaches B in a time $t = T$, then $y = 0$.

Since, $y = u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad 0 = \{u\sin(\alpha-\beta)\}T + \frac{1}{2}(-g\cos\beta)T^2$$

$$\Rightarrow \quad T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta} \qquad \{\because\ T \neq 0\}$$

Range of Projectile (R) (Up the Incline)

Since the particle goes from O to N with velocity component $u\cos\alpha$, uniformly with no acceleration. So,

$$ON = (u\cos\alpha)T$$

$$\Rightarrow \quad ON = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos\beta}$$

But $OB = \frac{ON}{\cos\beta}$

$$\Rightarrow \quad \text{Range} = R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

We can also find R by the following method.

Since, $x = u_x t + \frac{1}{2}a_x t^2$ and at $t = T$, $x = R$

$$\Rightarrow \quad R = [u\cos(\alpha-\beta)]T + \frac{1}{2}(-g\sin\beta)T^2$$

Substituting $T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$, we get

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

Condition for Range to be Maximum (Up the Incline)

Since, $R = \frac{u^2}{g\cos^2\beta}[2\sin(\alpha-\beta)\cos\alpha]$

$$\Rightarrow \quad R = \frac{u^2}{g\cos^2\beta}[\sin(\alpha-\beta+\alpha)+\sin(\alpha-\beta-\alpha)]$$

$$\{\because\ 2\sin A\cos B = \sin(A+B)+\sin(A-B)\}$$

$$\Rightarrow \quad R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta)-\sin\beta]$$

For R to be MAXIMUM, $\sin(2\alpha-\beta)$ is MAXIMUM i.e., 1

$$\Rightarrow \quad \sin(2\alpha-\beta) = 1$$

$$\Rightarrow \quad 2\alpha-\beta = \frac{\pi}{2}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{4}+\frac{\beta}{2}$$

Maximum Range is given by

$$R_{\max} = \frac{u^2}{g\cos^2\beta}(1-\sin\beta)$$

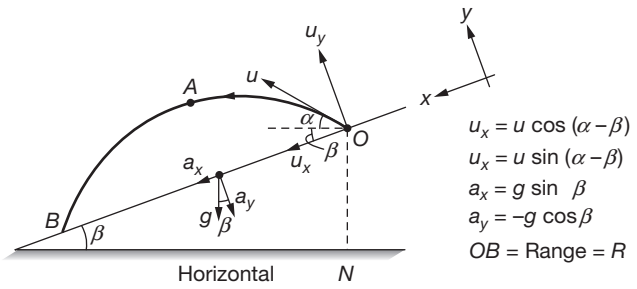
$$\Rightarrow \quad R_{\max} = \frac{u^2}{g(1-\sin^2\beta)}(1-\sin\beta)$$

$$\Rightarrow \quad R_{\max} = \frac{u^2}{g(1+\sin\beta)}$$

$$\Rightarrow \quad R_{\substack{\text{max. up}\\ \text{the incline}}} = \frac{R_{\text{max horizontal}}}{1+\sin\beta} \quad \left\{\because R_{\text{max horizontal}} = \frac{u^2}{g}\right\}$$

MOTION OF A PROJECTILE DOWN AN INCLINED PLANE

Consider a projectile to be launched with initial velocity u making a angle α with horizontal down an inclined plane which makes angle β with the horizontal. Then u and g can be resolved into two components each (one along the incline and the other perpendicular to the incline)



(a) $u_x = u\cos(\alpha+\beta)$ along the incline along $+x$ axis.

(b) $u_y = u\sin(\alpha+\beta)$ along the incline along $+y$ axis.

(c) $|a_x| = g\sin\beta$ along $-x$ axis, acting as acceleration to the motion along the incline.

(d) $|a_y| = g\cos\beta$ along $-y$ axis, acting as retardation to the motion perpendicular to incline i.e., $a_y = -g\cos\beta$.

Time of Flight (T) (Down the Incline)

Time taken by the particle to go from O to A to B. When the particle reaches B in a time $t = T$, then $y = 0$.

Since, $y = u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad 0 = \{u\sin(\alpha+\beta)\}T + \frac{1}{2}(-g\cos\beta)T^2$$

$$\Rightarrow \quad T = \frac{2u\sin(\alpha+\beta)}{g\cos\beta} \qquad \{\text{as } T \neq 0\}$$

Range of Projectile (R) (Down the Incline)

Since the particle goes from O to N with velocity component $u\cos\alpha$, uniformly with no acceleration. So,

$$ON = (u\cos\alpha)T$$

$$\Rightarrow \quad ON = \frac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos\beta}$$

But $OB = \frac{ON}{\cos\beta}$

$$\Rightarrow \quad \text{Range} = R = \frac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$$

We can also find R by the following method.

Since, $x = u_x t + \frac{1}{2}a_x t^2$ and

at $t = T$, $x = R$

$$\Rightarrow \quad R = \left[u\cos(\alpha+\beta)\right]T + \frac{1}{2}(g\sin\beta)T^2$$

Substituting $T = \frac{2u\sin(\alpha+\beta)}{g\cos\beta}$, we get

$$R = \frac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$$

Condition for Range to be Maximum (Down the Incline)

Since, $R = \frac{u^2}{g\cos^2\beta}\left[2\sin(\alpha+\beta)\cos\alpha\right]$

$$\Rightarrow \quad R = \frac{u^2}{g\cos^2\beta}[\sin(\alpha+\beta+\alpha)+\sin(\alpha+\beta-\alpha)]$$

$$\{\because\ 2\sin A\cos B = \sin(A+B)+\sin(A-B)\}$$

$$\Rightarrow \quad R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha+\beta)+\sin\beta\right]$$

For R to be MAXIMUM, $\sin(2\alpha+\beta)$ is MAXIMUM i.e. 1

$$\Rightarrow \quad \sin(2\alpha+\beta) = 1$$

$$\Rightarrow \quad 2\alpha+\beta = \frac{\pi}{2}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{4} - \frac{\beta}{2}$$

Maximum range is given by

$$R_{\max} = \frac{u^2}{g\cos^2\beta}(1+\sin\beta)$$

$$\Rightarrow \quad R_{\max} = \frac{u^2}{g(1-\sin^2\beta)}(1+\sin\beta)$$

$$\Rightarrow \quad R_{\max} = \frac{u^2}{g(1-\sin\beta)}$$

Since $R_{\max \text{ horizontal}} = \frac{u^2}{g}$

$$\Rightarrow \quad R_{\substack{\text{max. down} \\ \text{the incline}}} = \frac{R_{\max \text{ horizontal}}}{1-\sin\beta}$$

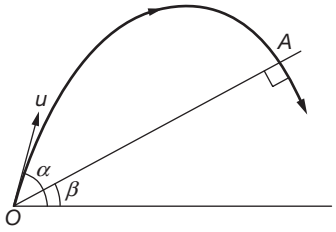
Condition for the Particle Launched Up the Incline to Strike it Normally

Let T be the time of flight from O to A, then

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta} \qquad \ldots(1)$$

Now we shall consider the motion of the particle along OA. Initial velocity along OA is

$$u_x = u\cos(\alpha-\beta)$$



Since the particle strikes the plane A at right angles, so the final velocity along OA is $v_x = 0$.

Acceleration due to gravity along OA is $a_x = -g\sin\beta$

Since $x = u_2 t + \frac{1}{2}a_2 t^2$

$$\Rightarrow \quad 0 = u\cos(\alpha-\beta) - (g\sin\beta)t \qquad \{\because\ v_x = u_x + a_x t\}$$

$$\Rightarrow \quad t = \frac{u\cos(\alpha-\beta)}{g\sin\beta} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$T = t$$

$$\Rightarrow \quad \frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{u\cos(\alpha-\beta)}{g\sin\beta}$$

$$\Rightarrow \quad 2\tan(\alpha-\beta) = \cot\beta$$

$$\Rightarrow \quad 2\left(\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}\right) = \cot\beta$$

$$\Rightarrow \quad 2\tan\alpha - 2\tan\beta = \cot\beta + \tan\alpha$$

$$\Rightarrow \quad \tan\alpha = \cot\beta + 2\tan\beta \qquad \ldots(3)$$

Condition for the Particle Launched Up the Incline to Strike it Horizontally

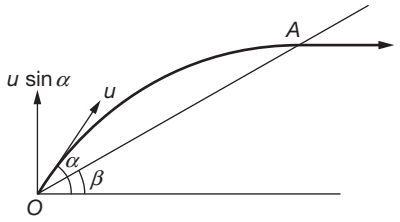
When the particle strikes the plane horizontally. Again in this case, time of flight is given by

$$t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

Since the particle strikes the plane horizontally, so vertical component of velocity at that instant is zero. Hence we get

$$0 = u\sin\alpha - gt$$

$$\Rightarrow \quad u\sin\alpha = g\left[\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\right]$$



$$\Rightarrow \quad \sin\alpha\cos\beta = 2[\sin\alpha\cos\beta - \cos\alpha\sin\beta]$$

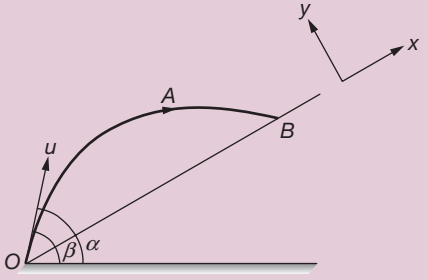
$$\Rightarrow \quad 2\cos\alpha\sin\beta = \sin\alpha\cos\beta$$

$$\Rightarrow \quad 2\tan\beta = \tan\alpha$$

Problem Solving Technique(s)

Following two points are important regarding the projectile motion in inclined planes:

(a) Time taken by the projectile to move from O to A is half the time of flight. Here A is a point where velocity of projectile is parallel to x-direction.



i.e., $t_{OA} = \frac{T}{2} = \frac{u\sin(\alpha-\beta)}{g\cos\beta}$

This can be proved as under,

At A, $v_y = 0 = u_y + a_y t_{OA}$

$$\Rightarrow \quad t_{OA} = -\frac{u_y}{a_y} = -\left[\frac{u\sin(\alpha-\beta)}{-g\cos\beta}\right] = \frac{u\sin(\alpha-\beta)}{g\cos\beta}$$

(b) At point B, where the projectile strikes the plane the y-component of its velocity (v_y) is just equal and opposite to the component with which it was projected from point O. i.e., $v_y = -u_y$ at B

This can be proved as:

$$v_y = u_y + a_y T$$

$$v_y = u\sin(\alpha-\beta) + (-g\cos\beta)\left[\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\right]$$

$$v_y = -u\sin(\alpha-\beta) = -u_y$$

ILLUSTRATION 31

A ball starts falling with zero initial velocity on a smooth inclined plane which forms an angle α with the horizontal. Having fallen through a height h, the ball rebounds elastically off the inclined plane. At what distance from the impact point will the ball rebound for the second time?

SOLUTION

Resolving the component(s) of acceleration w.r.t. the inclined plane, we have

(a) $g\sin\alpha$, acceleration along the inclined plane, and

(b) $g\cos\alpha$, acceleration perpendicular to the inclined plane.

After falling freely through a height h, velocity at the point of striking is

$$v_0 = \sqrt{2gh} \qquad \ldots(1)$$

Further since the collision is perfectly elastic, hence, angle of rebound is also α.

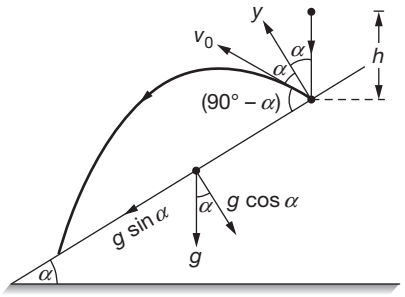
Motion Along the Inclined Plane

$$x = v\cos(90-\alpha)t + \frac{1}{2}(g\sin\alpha)t^2 \qquad \ldots(2)$$

Motion Perpendicular to the Inclined Plane

$$\Rightarrow \quad 0 = v_o\sin(90-\alpha)t + \frac{1}{2}(-g\cos\alpha)t^2$$

$$\Rightarrow \quad t = \frac{2v_o}{g} = \frac{2\sqrt{2gh}}{g}$$



Put value of t in (2) we get

$$x = v_o\sin\alpha\left(\frac{2v_o}{g}\right) + \frac{1}{2}g\sin\alpha\left(\frac{4v_o^2}{g^2}\right)$$

$$\Rightarrow \quad x = \frac{2v_o^2\sin\alpha}{g} + \frac{2v_o^2\sin\alpha}{g}$$

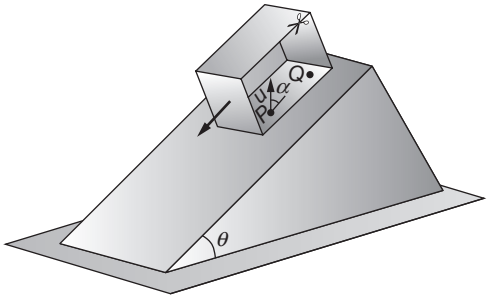
$$\Rightarrow \quad x = \frac{4v_o^2\sin\alpha}{g}$$

$$\Rightarrow \quad x = \frac{4\left(\sqrt{2gh}\right)^2\sin\alpha}{g} \qquad \{\text{using (1)}\}$$

$$\Rightarrow \quad x = 8h\sin\alpha$$

ILLUSTRATION 32

A large heavy box is sliding without friction down a smooth inclined plane of inclination θ. From a point P on bottom of box, a particle is projected inside the box. The initial speed of particle with respect to box is u and direction of projection makes and angle α with the bottom as shown in the figure.



(a) Find the distance along the bottom of box between point of projection P and point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).

(b) If the horizontal displacement of a particle as seen by an observer on ground is zero, find the speed of box with respect to ground at the instant the particle was projected.

SOLUTION

Lets consider reference frame x-y fixed with box with origin at one corner O and moving down with box. The acceleration of this frame down the incline is $g\sin\theta$.

Acceleration of the particle launched inside the box w.r.t. ground is g (vertically downwards) with components

(a) $g\sin\theta$; (along the negative x-axis) acting as retardation for particle's motion along the incline, and

(b) $g\cos\theta$; (along the negative y-axis) acting as retardation for particle's motion perpendicular to the incline.

Relative acceleration of the particle in reference frame along x-axis is

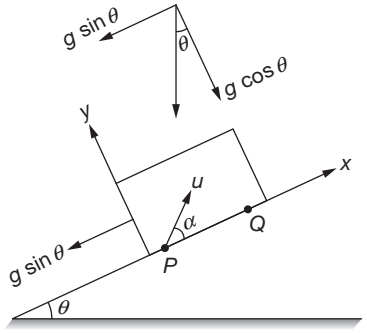
$$a_x = -g\sin\theta - (-g\sin\theta) = 0$$

Relative acceleration of the particle in reference frame along y-axis is

$$a_y = -g\cos\theta = 0$$

$$\Rightarrow \quad a_y = -g\cos\theta$$

Also the x component of particle's velocity in box $u_x = u\cos\alpha$ and the y component of the particle's velocity in box is $u_y = u\sin\alpha$.



When the particle (strikes) arrives at Q, net displacement perpendicular to the incline is zero.

$$\Rightarrow \quad 0 = (u\sin\alpha)\ t + \frac{1}{2}(-g\cos\theta)\ t^2$$

$$\Rightarrow \quad t\left[\left(\frac{1}{2}g\cos\theta\right)\ t - (u\sin\alpha)\right] = 0$$

Since $t \neq 0$

$$\Rightarrow \quad t = \frac{2u\sin\alpha}{g\cos\theta}$$

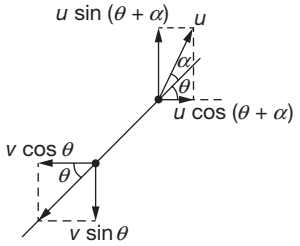
Since $a_x = 0$, so horizontal motion is non accelerated motion with uniform velocity, we have

$$PQ = (u\cos\alpha)t = \left(\frac{2u\sin\alpha}{g\cos\theta}\right)u\cos\alpha$$

$$\Rightarrow \quad PQ = \frac{u^2\sin 2\alpha}{g\cos\theta}$$

METHOD I

If the incline moves down with a velocity v and moves a distance QP, then it will appear to an observer on ground that horizontal displacement of particle is zero



For Box

$$PQ = vt + \frac{1}{2}(a_x)t^2$$

$$\Rightarrow \quad PQ = v\frac{2u\sin\alpha}{g\cos\theta} + \frac{1}{2}(g\sin\theta)\left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

$$\Rightarrow \quad \frac{2u\sin\alpha}{g\cos\theta}\left[v + \frac{u\sin\theta\sin\alpha}{\cos\theta}\right] = \frac{2u^2\sin\alpha\cos\alpha}{g\cos\theta}$$

{using part (a)}

$$\Rightarrow \quad v = u\left[\cos\alpha - \frac{\sin\theta\sin\alpha}{\cos\theta}\right]$$

$$\Rightarrow \quad v = u\frac{\cos(\theta+\alpha)}{\cos\theta}$$

METHOD II

For no horizontal displacement,

$$\Rightarrow \quad u\cos(\theta+\alpha) - v\cos\theta = 0$$

$$\Rightarrow \quad v = \frac{u\cos(\theta+\alpha)}{\cos\theta}$$

ILLUSTRATION 33

Two bodies are projected from the same point with equal speeds in such directions that they both strike the same point on a plane whose inclination is β. If α be the angle of projection of the first body with the horizontal show that the ratio of their times of flight is $\frac{\sin(\alpha-\beta)}{\cos\alpha}$.

SOLUTION

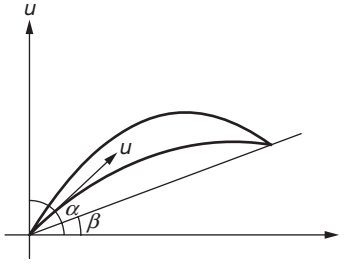
For the first body, we have

$$R = \frac{u^2}{g\cos^2\beta}\{\sin(2\alpha-\beta) - \sin\beta\}$$

Let α' be the angle of projection of the second body. Since, range of both the bodies is same. Therefore,

$$\sin(2\alpha-\beta) = \sin(2\alpha'-\beta)$$

$$\Rightarrow \quad 2\alpha' - \beta = \pi - (2\alpha-\beta)$$



$$\Rightarrow \quad \alpha' = \frac{\pi}{2} - (\alpha-\beta)$$

Now, $T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$ and $T' = \frac{2u\sin(\alpha'-\beta)}{g\cos\beta}$

$$\Rightarrow \quad \frac{T}{T'} = \frac{\sin(\alpha-\beta)}{\sin(\alpha'-\beta)} = \frac{\sin(\alpha-\beta)}{\sin\left(\frac{\pi}{2} - (\alpha-\beta) - \beta\right)}$$

$$\Rightarrow \quad \frac{T}{T'} = \frac{\sin(\alpha-\beta)}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{\sin(\alpha-\beta)}{\cos\alpha}$$

ILLUSTRATION 34

A particle projected with velocity u strikes at right angles a plane through the point of projection inclined at an angle β to the horizon. Show that the height of the point struck above the horizontal plane through the point of projection is $\frac{2u^2}{g}\left(\frac{\sin^2\beta}{1+3\sin^2\beta}\right)$ and that the time of flight up to that instant is, $t = \frac{2u}{g\sqrt{1+3\sin^2\beta}}$.

SOLUTION

$$\text{Time of flight } T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta} \qquad \ldots(1)$$

Also, we have, at B, at time t

$$v_x = 0$$

$$\Rightarrow \quad u_x + a_x t = 0$$

$$\Rightarrow \quad u\cos(\alpha-\beta) - g\sin\beta t = 0$$

$$\Rightarrow \quad t = \frac{u\cos(\alpha-\beta)}{g\sin\beta} \qquad \ldots(2)$$

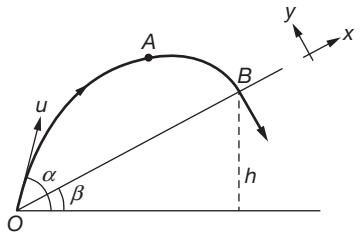
These two times being the same, can be equated. So, we get

$$T = t$$

$$\Rightarrow \quad \frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{u\cos(\alpha-\beta)}{g\sin\beta}$$

$$\Rightarrow \quad 2\tan(\alpha-\beta) = \cot\beta$$

$$\Rightarrow \quad 2\left(\frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}\right) = \cot\beta$$



$$\Rightarrow \quad 2\tan\alpha - 2\tan\beta = \cot\beta + \tan\alpha$$

$$\Rightarrow \quad \tan\alpha = 2\tan\beta + \cot\beta \qquad \ldots(3)$$

Further $OB = \text{Range} = R = \dfrac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$ $\ldots(4)$

and $h = R\sin\beta$

So, let us find R in terms of u and β only by eliminating α, after taking help from equation (3).

Since $\tan\alpha = 2\tan\beta + \cot\beta = 2\tan\beta + \dfrac{1}{\tan\beta}$

$$\Rightarrow \quad \tan\alpha = \frac{2\tan^2\beta + 1}{\tan\beta} = \frac{1+\sin^2\beta}{\sin\beta\cos\beta}$$

$$\Rightarrow \quad \cos\alpha = \frac{\sin\beta\cos\beta}{\sqrt{(1+\sin^2\beta)^2 + \sin^2\beta\cos^2\beta}}$$

$$\Rightarrow \quad \cos\alpha = \frac{\sin\beta\cos\beta}{\sqrt{1+3\sin^2\beta}}$$

$$\Rightarrow \quad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{\cos\beta}{\sqrt{1+3\sin^2\beta}}$$

Substituting these values in equation (4), we get

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

$$\Rightarrow \quad R = \frac{2u^2}{g\cos^2\beta}\left(\frac{\cos\beta}{\sqrt{1+3\sin^2\beta}}\right)\left(\frac{\sin\beta\cos\beta}{\sqrt{1+3\sin^2\beta}}\right)$$

$$\Rightarrow \quad R = \frac{2u^2\sin\beta}{g\left(1+3\sin^2\beta\right)}$$

So, $h = R\sin\beta = \dfrac{2u^2\sin^2\beta}{g\left(1+3\sin^2\beta\right)}$

Substituting value of $\sin(\alpha-\beta) = \dfrac{\cos\beta}{\sqrt{1+3\sin^2\beta}}$ in (1), we get

$$T = \frac{2u\cos\beta}{g\cos\beta\sqrt{1+3\sin^2\beta}} = \frac{2u}{g\sqrt{1+3\sin^2\beta}}$$

Test Your Concepts-IV

Based on Projectile on an Inclined Plane

(Solutions on page H.150)

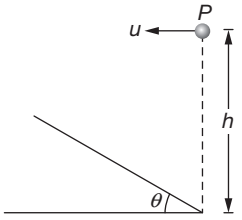
1. A heavy particle is projected from a point at the foot of a fixed plane, inclined at an angle 45° to the horizontal, in the vertical plane containing the line of greatest slope through the point. If $\phi(>45°)$ is the inclination to the horizontal of the initial direction of projection, for what value of $\tan\phi$ will the particle strike the plane
(a) horizontally
(b) at right angle

2. Two parallel straight lines are inclined to the horizontal at an angle α. A particle is projected from a point mid way between them so as to graze one of the lines and strike the other normally. Show that if θ is the angle between the direction of projection and either of lines, then

$$\tan\theta = (\sqrt{2}-1)\cot\alpha$$

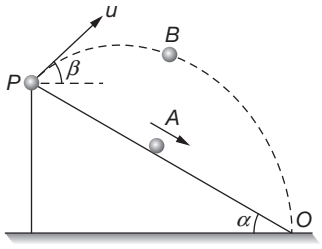
3. A projectile is fired with a velocity u at right angles to the slope, which is inclined at an angle θ with the horizontal. Derive an expression for the distance R to the point of impact.

4. Determine the horizontal velocity u with which a stone must be projected horizontally from a point P, so that it may hit the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and point P is at a height h above the foot of the incline, as shown in the figure.



5. A perfectly elastic ball is thrown from the foot of a plane whose inclination to horizontal is β. If after striking the plane at a distance R from the point of projection it rebounds and retraces its former path, find the velocity of projection.

6. Particle A is released from a point P on a smooth inclined plane inclined at an angle α with the horizontal. At the same instant another particle B is projected with initial velocity u making an angle β with the horizontal. Both the particles meet again on the inclined plane. Find the relation between α and β.



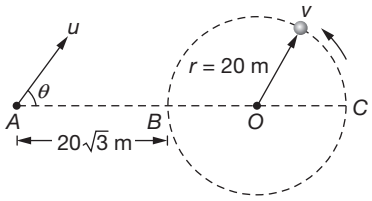
SOLVED PROBLEMS

PROBLEM 1

A particle is moving along a vertical circle of radius $r = 20$ m with a constant speed $v = 31.4$ ms^{-1} as shown in figure. Straight line ABC is horizontal and passes through the centre of the circle. A shell is fired from point A at the instant when the particle is at C. If distance AB is $20\sqrt{3}$ m and the shell collide with the particle at B, then prove

$$\tan\theta = \frac{(2n-1)^2}{\sqrt{3}}$$

where n is an integer. Further, show that smallest value of θ is 30°.



SOLUTION

At the time of firing of the shell, the particle was at C and the shell collides with it at B, therefore the number of revolutions completed by the particle is odd multiple of half, i.e., $\frac{(2n-1)}{2}$, where n is an integer.

Let t be the time period of revolution of the particle, then

$$t = \frac{2\pi r}{v} = \frac{2\times 3.14\times 20}{31.4} = 4 \text{ s}$$

If T be the time of flight of the shell, then this time T equals the time of $\left(\frac{2n-1}{2}\right)$ revolutions of the particle

$$\Rightarrow \quad T = \frac{(2n-1)}{2}\times 4 = 2(2n-1) \text{ s}$$

For a projectile, the time of flight is given by

$$T = \frac{2u\sin\theta}{g}$$

So, $T = t$

$$\Rightarrow \quad \frac{2u\sin\theta}{g} = 2(2n-1) \qquad \ldots(1)$$

The range of the projectile is given by

$$R = \frac{2u^2\sin\theta\cos\theta}{g} = 20\sqrt{3} \qquad \ldots(2)$$

From (1) and (2), we get

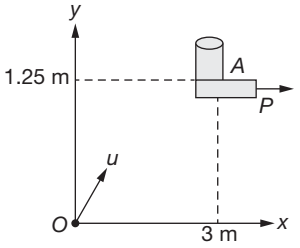
$$\tan\theta = \frac{(2n-1)^2}{\sqrt{3}}$$

For smallest θ, $n = 1$

$$\Rightarrow \quad \theta_{\min} = 30° = \frac{\pi}{3} \text{ radian}$$

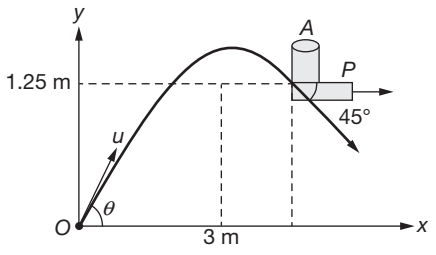
PROBLEM 2

An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$, the plank starts moving along the $+x$ direction with an acceleration 1.5 ms^{-2}. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the x-y plane. Find u and the time after which the stone hits the object. Take $g = 10$ ms^{-2}.



SOLUTION

For stone to hit the object, A, at time t



$$x = (u\cos\theta)t = x_o + \frac{1}{2}at^2$$

$$\Rightarrow \quad (u\cos\theta)t = 3 + \frac{1}{2}(1.5)t^2 \qquad \ldots(1)$$

$$\Rightarrow \quad y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad (u\sin\theta)t - \frac{1}{2}gt^2 = 1.25 \qquad \ldots(2)$$

Since stone hits A moving along $45°$ with horizontal during its downward motion, so

$$\tan(-45°) = \frac{u\sin\theta - gt}{u\cos\theta} = -1 \qquad \ldots(3)$$

$$\Rightarrow \quad u\sin\theta - gt = -u\cos\theta$$

$$\Rightarrow \quad (u\sin\theta)t - gt^2 = -(u\cos\theta)t \qquad \ldots(4)$$

Substituting values of $(u\sin\theta)t$ and $(u\cos\theta)t$ from (2) and (1) respectively and taking $g = 10\ \text{ms}^{-2}$ we get

$$1.25 + 5t^2 - 10t^2 = -3 - (0.75)t^2$$

$$\Rightarrow \quad (5 - 0.75)t^2 = (3 + 1.25)$$

$$\Rightarrow \quad t^2 = 1$$

$$\Rightarrow \quad t = 1\ \text{s}$$

So, from (1), $u\cos\theta = 3.75$ and $u\sin\theta = 6.25$

$$\Rightarrow \quad \vec{u} = u_x\hat{i} + u_y\hat{j}$$

$$\Rightarrow \quad \vec{u} = (3.75)\hat{i} + (6.25)\hat{j}$$

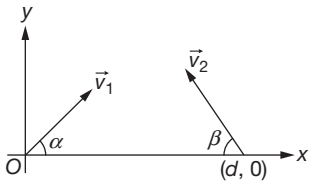
PROBLEM 3

Two particles A and B simultaneously leave 2 points O and A separated by a distance d, with velocities $\vec{v}_1$ and $\vec{v}_2$. The direction in which the particle travels forms an angle α and the second particle forms an angle β with the line OA. At what time, will the distance between the particles be minimum, also calculate this minimum distance between particles.

SOLUTION

METHOD I: FRAME FIXED w.r.t. EARTH

Let distance between the given particles be minimum at time t



For 1st particle, B

$$\Rightarrow \quad \vec{r}_1 = v_1\left(\cos\alpha\hat{i} + \sin\alpha\hat{j}\right)t$$

For 2nd particle, A

$$\Rightarrow \quad \vec{r}_2 = \left(d - v_2 t\cos\beta\right)\hat{i} + v_2 t\sin\beta\hat{j}$$

Separation between particles is $\vec{r} = \vec{r}_2 - \vec{r}_1$

$$|\vec{r}|^2 = r^2 = \left[d - (v_2\cos\beta + v_1\cos\alpha)t\right]^2 + \left[(v_2\sin\beta - v_1\sin\alpha)t\right]^2$$

For distance between particles to be minimum, $\frac{dr}{dt} = 0$

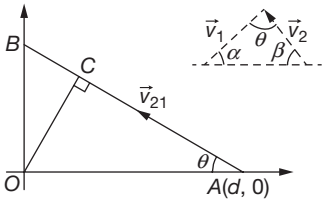
$$\Rightarrow \quad 2r\frac{dr}{dt} = 2\left[d - (v_2\cos\beta + v_1\cos\alpha)t\right]\times \left(-v\cos\beta - v_1\cos\alpha\right) + 2\left(v_2\sin\beta - v_1\sin\alpha\right)^2 t$$

$$\Rightarrow \quad t = \frac{d(v_2\cos\beta + v_1\cos\alpha)}{[v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha+\beta)]}$$

$$\Rightarrow \quad r_{\min} = \left[\frac{d(v_2\sin\beta - v_1\sin\alpha)}{\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha+\beta)}}\right]$$

METHOD II: FRAME FIXED w.r.t. PARTICLE

Since both projectiles are moving under the influence of gravity having a constant value of $g = 9.8\ \text{ms}^{-2}$ acting vertically downwards for both. Hence, we arrive at the following conclusion(s)



(a) Relative vertical acceleration of one projectile w.r.t. other is zero.

(b) Relative horizontal acceleration of one projectile w.r.t. other is zero.

(c) As a consequence of (a) and (b), both the projectiles must follow a straight line motion along AC with uniform relative velocity $\vec{v}_{21}$, where

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 \qquad \ldots(1)$$

Hence, minimum distance between the particles is OC i.e. perpendicular distance i.e. length of the perpendicular dropped from O on AC to meet at C.

Further, the angle between $\vec{v}_1$ and $\vec{v}_2$ is $180° - (\alpha + \beta)$.

Therefore,

$$v_{21}^2 = v_1^2 + v_2^2 - 2v_1v_2\cos\left[180° - (\alpha+\beta)\right]$$

$$\Rightarrow \quad v_{21}^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha+\beta) \qquad \ldots(2)$$

Let $\vec{v}_{21}$ make an angle θ with the horizontal axis (i.e. x-axis)

$$\Rightarrow \quad \vec{v}_{21} = v_{21}\left(-\cos\theta\hat{i} + \sin\theta\hat{j}\right) \qquad \ldots(3)$$

$$\Rightarrow \quad \vec{v}_2 = v_2\left(-\cos\beta\hat{i} + \sin\beta\hat{j}\right) \qquad \ldots(4)$$

$$\Rightarrow \quad \vec{v}_1 = v_1\left(\cos\alpha\hat{i} + \sin\alpha\hat{j}\right) \qquad \ldots(5)$$

Substituting (3), (4) & (5) in equation (1) and comparing we get

$$v_{21}\cos\theta = v_2\cos\beta + v_1\cos\alpha \qquad \ldots(6)$$

$$v_{21}\sin\theta = v_2\sin\beta - v_1\sin\alpha \qquad \ldots(7)$$

$$\Rightarrow \quad \cos\theta = \frac{v_2\cos\beta + v_1\cos\alpha}{v_{21}}$$

$$\Rightarrow \quad \cos\theta = \frac{v_2\cos\beta + v_1\cos\alpha}{\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha+\beta)}}$$

$$\Rightarrow \quad r_{\min} = OC = d\sin\theta = \frac{d\left(v_2\sin\beta - v_1\sin\alpha\right)}{\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha+\beta)}}$$

In right triangle OAC,

$$r_{\min} = OC = d\sin\theta = \frac{d\left(v_2\sin\beta - v_1\sin\alpha\right)}{\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha+\beta)}}$$

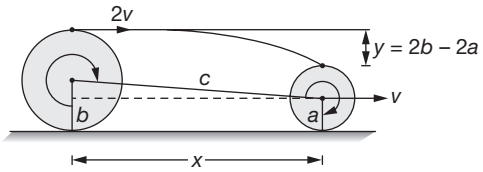
$$t = \frac{d\cos\theta}{v_{21}} = \frac{d\left(v_2\cos\beta + v_1\cos\alpha\right)}{\left[v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha+\beta)\right]}$$

PROBLEM 4

The radii of the front and rear wheels of a carriage are a, b and c is the distance between the axle centres. A particle of dust driven from the highest point of the rear wheel is observed to alight on the highest point of the front wheel. Show that the velocity of the carriage is $\dfrac{\sqrt{(c+b-a)(c-b+a)g}}{4(b-a)}$.

SOLUTION

Velocity of projection of the dust particle from the top of the wheel is $2v$ horizontally.



For horizontal projectile

$$y = \frac{gx^2}{2v^2}$$

$$2b - 2a = \frac{g\left[c^2 - (b-a)^2\right]}{2v^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{g(c+b-a)(c-b+a)}{4(b-a)}}$$

PROBLEM 5

A batsman hits a pitched cricket ball at a height of 1.2 m above the ground so that its angle of projection is $45°$ and its horizontal range is 110 m. The ball is lifted towards the left field line where a fence of 7.5 m is located 100 m from the position of the batsman. Will the ball clear the fence?

SOLUTION

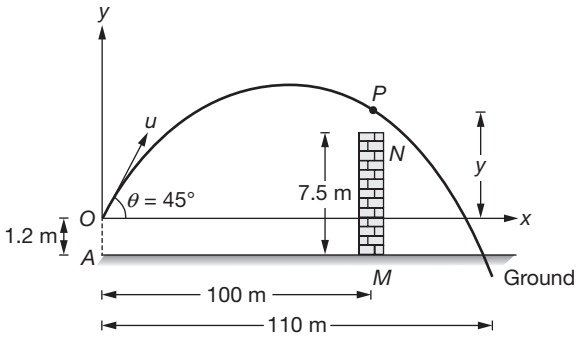
Lets firstly calculate initial velocity of launch of projectile. Considering origin at O we have

$$-1.2 = 110\tan(45°) - \frac{g(110)^2}{2u^2\cos^2(45°)}$$

$$\Rightarrow \quad u = 32.66 \text{ ms}^{-1}$$

Lets calculate the vertical distance of ball from x-axis at the position of fence. i.e. we are to find y for

$$x = 100 \text{ m}, \ \theta = 45°, \ u = 32.66 \text{ ms}^{-1}$$



$$y = 100\tan(45°) - \frac{9.8\times(100)^2}{2\times(32.66)^2\cos^2(45°)}$$

$$y = 100 - 91.87 = 8.13 \text{ m}$$

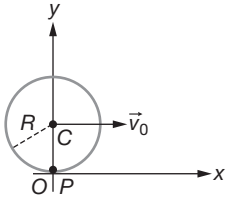
Total height of ball from the ground is

$$(1.2 + 8.13) = 9.33 \text{ m}$$

Since $9.33 \text{ m} > 7.5 \text{ m}$, therefore, the ball will clear the fence.

PROBLEM 6

A wheel of radius R rolls without slipping along the x-axis with constant speed v_0 (as shown in the figure). Investigate the motion of a point P on the rim of the wheel which starts from the origin O and find the total distance covered by the point between two successive moments at which it touches the surface.



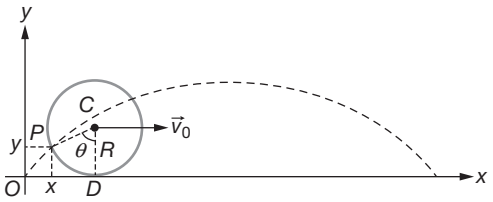
SOLUTION

After the time interval t, the center C of the wheel will have travelled a distance $v_0 t$ as shown, and since it rolls without slipping, the arc DP will also have the length $v_0 t$. Thus the angle DCP will be $\theta = \frac{v_o t}{R}$, Then from the geometry of the figure, we can express the coordinates x and y of the point P as follows:

$$x = v_o t - R\sin\left(\frac{v_o t}{R}\right)$$

$$y = R - R\cos\left(\frac{v_o t}{R}\right) \qquad \ldots(1)$$

With t as a parameter, these two equations define in rectangular coordinates the path of point P which is called a **CYCLOID**. In other words the two equations give the motion law for the point P on the rim of the wheel in the case of pure rolling with uniform motion.



Differentiating equation (1) with respect to time gives the velocity-time equations as follows:

$$v_x = v_o\left[1 - \cos\left(\frac{v_o t}{R}\right)\right]$$

$$v_y = v_o \sin\left(\frac{v_o t}{R}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = v_0\sqrt{2\left[1 - \cos\left(\frac{v_0 t}{R}\right)\right]}$$

$$\Rightarrow \quad v = 2v_0 \sin\left(\frac{v_0 t}{2R}\right) \qquad \ldots(3)$$

From this expression, we see that the maximum speed of point P is $2v_0$ when $t = \frac{\pi R}{v_o}$, that is, when the point P is at the top of its path.
Differentiating equation (2) again with respect to time, we obtain the acceleration-time equations

$$\Rightarrow \quad a_x = \frac{v_o^2}{R}\sin\left(\frac{v_o t}{R}\right)$$

$$a_y = \frac{v_o^2}{R}\cos\left(\frac{v_o t}{R}\right) \qquad \ldots(4)$$

Therefore, $a = \sqrt{a_x^2 + a_y^2} = \frac{v_o^2}{R}$ $\ldots(5)$

Thus the point P has acceleration of constant magnitude always directed towards the centre C of the rolling wheel.

The total distance traversed by the point P between two successive moments at which it touches the surface.

$$\Rightarrow \quad s = \int v dt = \int_0^{\frac{2\pi R}{v_o}} \left[2v_o \sin\left(\frac{v_o t}{2R}\right)\right] dt$$

$$\Rightarrow \quad s = 8R$$

PROBLEM 7

A canon fires successively two shells with velocity u, the first at an angle θ_1 and second at an angle θ_2 with horizontal. Neglecting air drag. Find the time interval between firings leading to collision of shells.

SOLUTION

METHOD I

Let the time of flight before collision be t, i.e., let first shell reach $P(x, y)$ in time t.

Let second shell be fired with a delay say Δt.

So it falls short of time by Δt and has net time $(t - \Delta t)$ to be at P.

For First shell

$$x = (u\cos\theta_1)t \qquad \ldots(1)$$

For Second shell

$$x = (u\cos\theta_2)(t - \Delta t) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{t - \Delta t}{t} = \frac{\cos\theta_1}{\cos\theta_2}$$

$$\Rightarrow \quad \frac{\Delta t}{t} = \frac{\cos\theta_2 - \cos\theta_1}{\cos\theta_2} \qquad \ldots(3)$$

From (1), $t = \dfrac{x}{u\cos\theta_1}$

$$\Rightarrow \quad \frac{\Delta t(u\cos\theta_1)}{x} = \frac{\cos\theta_2 - \cos\theta_1}{\cos\theta_2}$$

$$\Rightarrow \quad \frac{u\Delta t}{x} = \frac{\cos\theta_2 - \cos\theta_1}{\cos\theta_1 \times \cos\theta_2} \qquad \ldots(4)$$

Also, $y = x\tan\theta_1 - \dfrac{gx^2}{2u^2\cos^2\theta_1}$ $\qquad \ldots(5)$

$$\Rightarrow \quad y = x\tan\theta_2 - \frac{gx^2}{2u^2\cos^2\theta_2} \qquad \ldots(6)$$

$$\Rightarrow \quad \tan\theta_1 - \tan\theta_2 = \frac{gx}{2u^2}\left[\frac{1}{\cos^2\theta_1} - \frac{1}{\cos^2\theta_2}\right]$$

{using (5) & (6)}

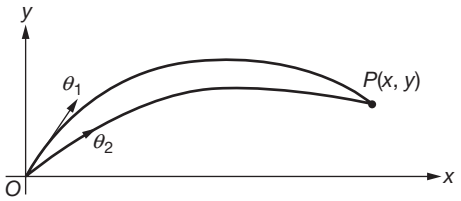
$$\Rightarrow \quad \frac{\sin\theta_1}{\cos\theta_1} - \frac{\sin\theta_2}{\cos\theta_2} = \frac{gx}{2u^2}\left[\frac{\cos^2\theta_2 - \cos^2\theta_1}{\cos^2\theta_2 \times \cos^2\theta_1}\right]$$

$$\Rightarrow \quad \frac{\sin(\theta_1 - \theta_2)}{\cos\theta_1 + \cos\theta_2} = \frac{gx}{2u^2}\left(\frac{u\Delta t}{x}\right)$$

$$\Rightarrow \quad \Delta t = \frac{2u}{g}\left[\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_1 + \cos\theta_2}\right]$$

METHOD II

Since no friction is there, so let us assume both shells are fired at same instant. Let they pass through common point $P(x, y)$ at times t_1 and t_2 from the start, i.e., they miss each other by time $(t_1 - t_2)$ and hence, if this is the time lapse between firing of shells then they will collide at $P(x, y)$



For First Shell

$$x = (u\cos\theta_1)t_1 \qquad \ldots(1)$$

$$y = \left(u\sin\theta_1 t_1 - \frac{1}{2}gt_1^2\right) \qquad \ldots(2)$$

Also

For Second Shell

$$x = (u\cos\theta_2)t_2 \qquad \ldots(3)$$

$$u(\sin\theta_2 t_2 - \sin\theta_1 t_1) = \frac{1}{2}g(t_1 + t_2)(t_1 - t_2) \qquad \ldots(4)$$

Using relations (1) and (3), we get

$$u\left(\sin\theta_2 t_2 - \sin\theta_1 \frac{\cos\theta_2}{\cos\theta_1} t_2\right) = \frac{1}{2}g\left(\frac{\cos\theta_1 + \cos\theta_2}{\cos\theta_1}\right)t_2\Delta t \qquad \ldots(5)$$

Also using relations (3) and (4), we get

$$u\sin\theta_2 t_2 - \frac{1}{2}gt_2^2 = u\sin\theta_1 t_1 - \frac{1}{2}gt_1^2$$

$$\Rightarrow \quad u(\sin\theta_2 t_2 - \sin\theta_1 t_1) = \frac{1}{2}g(t_1+t_2)(t_1-t_2)$$

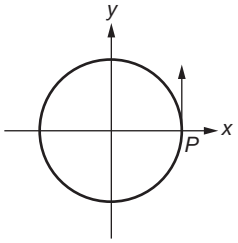
Using (5), we get

$$u\left(\sin\theta_2 t_2 - \sin\theta_1 \frac{\cos\theta_2}{\cos\theta_1} t_2\right) = \frac{1}{2}g\left(\frac{\cos\theta_1+\cos\theta_2}{\cos\theta_1}\right)t_2\Delta t$$

$$\Rightarrow \quad \Delta t = \frac{2u}{g}\left[\frac{\sin(\theta_1-\theta_2)}{\cos\theta_1+\cos\theta_2}\right]$$

PROBLEM 8

A train is moving with a constant speed of $10\ \text{ms}^{-1}$ in a circle of radius $\frac{16}{\pi}$ m. The plane of the circle lies in horizontal x-y plane. At time $t=0$ train is at point P and moving in counter-clockwise direction. At this instant a stone is thrown from the train with speed $10\ \text{ms}^{-1}$ relative to train towards negative x-axis at an angle of $37°$ with vertical z-axis. Find



(a) the velocity of particle relative to train at the highest point of its trajectory.

(b) the co-ordinates of points on the ground where it finally falls and that of the highest point of its trajectory.

Take $g = 10\ \text{ms}^{-2}$, $\sin(37°) = 0.6$

SOLUTION

At $t=0$, $\vec{v}_T = (10\hat{j})\ \text{ms}^{-1}$

$$\vec{v}_{ST} = 10\cos 37°\hat{k} - 10\sin 37°\hat{j} = (8\hat{k} - 6\hat{i})\ \text{ms}^{-1}$$

Since, $\vec{v}_{ST} = \vec{v}_S - \vec{v}_T$

$$\Rightarrow \quad \vec{v}_S = \vec{v}_{ST} + \vec{v}_T = (-6\hat{i} + 10\hat{j} + 8\hat{k})\ \text{ms}^{-1}$$

(a) At highest point, vertical component $(\hat{k})$ of $\vec{v}_S$ will become zero. Hence, velocity of particle at highest point will become $(-6\hat{i} + 10\hat{j})\ \text{ms}^{-1}$

(b) Time of flight,

$$T = \frac{2v_Z}{g} = \frac{2\times 8}{10} = 1.6\ \text{s}$$

$$\Rightarrow \quad y = (10)(1.6) = 16\ \text{m} \ \text{ and } \ z = 0$$

Therefore coordinates of particle where it finally lands on the ground are $(-4.5\ \text{m}, 16\ \text{m}, 0)$

At highest point, we have

$$t = \frac{T}{2} = 0.8\ \text{s}$$

$$\Rightarrow \quad x = \frac{16}{\pi} - (6)(0.8) = 0.3\ \text{m}$$

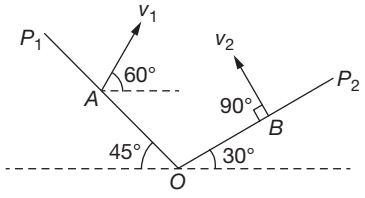
$$\Rightarrow \quad y = (10)(0.8) = 8\ \text{m}$$

and $z = \frac{v_Z^2}{2g} = \frac{(8)^2}{20} = 3.2\ \text{m}$

Therefore, coordinates at highest point are $(0.3\ \text{m}, 8\ \text{m}, 3.2\ \text{m})$.

PROBLEM 9

A particle is projected from an inclined plane OP_1 from A with velocity $v_1 = 8\ \text{ms}^{-1}$ at an angle $60°$ with horizontal. An another particle is projected at the same instant from B with velocity $16\ \text{ms}^{-1}$ and perpendicular to the plane OP_2 as shown in figure. After time $10\sqrt{3}$ s separation between them was minimum and found to be 70 m. Find the



(a) distance AB.

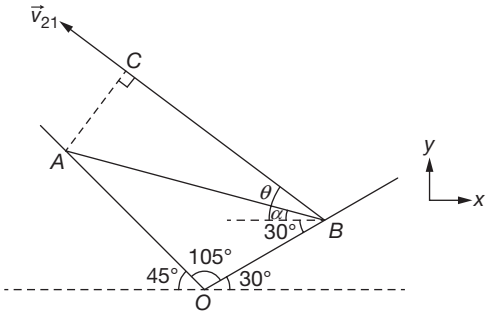
(b) height of A and B from O

SOLUTION

$$\left|(v_{21})_x\right| = (v_1+v_2)\cos 60° = 12\ \text{ms}^{-1}$$

$$\left|(v_{21})_y\right| = (v_1+v_2)\sin 60° = 4\sqrt{3}\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_{21} = \sqrt{(12)^2 + (4\sqrt{3})^2} = 13.9\ \text{ms}^{-1}$$



and $\theta = \tan^{-1}\left(\frac{4\sqrt{3}}{12}\right) = 30°$, $BC = (v_{21})t = 240$ m and $AC = 70$ m {given}

(a) Hence using Pythagora's Theorem, we have

$$AB = \sqrt{(240)^2 + (70)^2} = 250 \text{ m}$$

(b) $\angle\alpha = \angle\theta - \angle ABC = 30° - \sin^{-1}\left(\frac{AC}{AB}\right)$

$$\Rightarrow \angle\alpha = 30° - \sin^{-1}\left(\frac{70}{250}\right) = 13.7°$$

Applying sine law in ΔABO, we get

$$\frac{AB}{\sin(105°)} = \frac{AO}{\sin(\alpha + 30°)} = \frac{BO}{\sin(180° - (\alpha + 30° + 105°))}$$

Solving this we get,

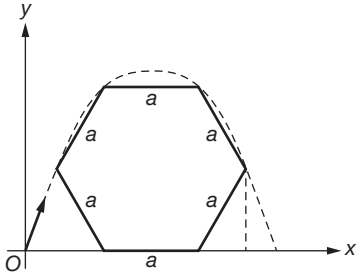
$AO = 181.2$ m and $BO = 134.6$ m

So, $h_A = AO \sin 45° = 128$ m and

$h_B = BO \sin 30° = 67.3$ m

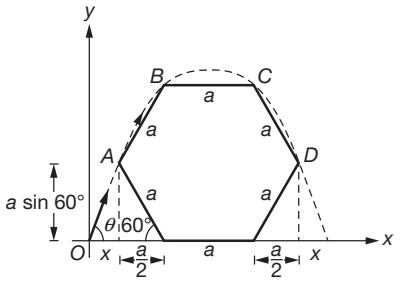
PROBLEM 10

A particle is launched such that it grazes the four vertices of a regular hexagon of side a as shown. Find the range of the particle.



SOLUTION

The coordinates of the points A, B, C and D are calculated here for convenience.



$$A\left(x, \frac{\sqrt{3}}{2}x\right)$$

$$B\left(x + \frac{a}{2}, \sqrt{3}a\right)$$

$$C\left(x + \frac{3a}{2}, \sqrt{3}a\right)$$

$$D\left(x + 2a, \frac{\sqrt{3}a}{2}\right)$$

Range, $R = x + \frac{a}{2} + a + \frac{a}{2} + x = 2a + 2x$

Also we know that

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

Since A lies on Trajectory, so

$$\frac{\sqrt{3}a}{2} = x\tan\theta\left(1 - \frac{x}{2a + 2x}\right) \quad \ldots(1)$$

Also, B lies on Trajectory, so

$$\sqrt{3}a = \left(x + \frac{a}{2}\right)\tan\theta\left(1 - \frac{\left(x + \frac{a}{2}\right)}{2a + 2x}\right) \quad \ldots(2)$$

Dividing (1) and (2), we get

$$\frac{\frac{\sqrt{3}}{2}a}{\sqrt{3}a} = \frac{x\cancel{\tan\theta}\left(1 - \frac{x}{2a + 2x}\right)}{\left(x + \frac{a}{2}\right)\cancel{\tan\theta}\left(1 - \frac{\left(x + \frac{a}{2}\right)}{2a + 2x}\right)}$$

$$\Rightarrow 2 = \frac{\left(x + \frac{a}{2}\right)}{x}\left(\frac{2a + 2x - x - \frac{a}{2}}{2a + 2x - x}\right)$$

$$\Rightarrow \quad 2=\left(\frac{2x+a}{2x}\right)\left(\frac{\frac{3a}{2}+x}{2a+x}\right)$$

$$\Rightarrow \quad 2=\frac{1}{4}\left(\frac{2x+a}{x}\right)\left(\frac{3a+2x}{2a+x}\right)$$

$$\Rightarrow \quad 8x(2a+x)=(2x+a)(3a+2x)$$

$$\Rightarrow \quad 16ax+8x^2=4x^2+6ax+2ax+3a^2$$

$$\Rightarrow \quad 4x^2+8ax-3a^2=0$$

$$\Rightarrow \quad x=-\frac{8a\pm\sqrt{64a^2+48a^2}}{8}$$

$$\Rightarrow \quad 8x=-8a\pm 4a\sqrt{4+3}$$

$$\Rightarrow \quad x=-a\pm\frac{a}{2}\sqrt{7}$$

$$\Rightarrow \quad x+a=\frac{a}{2}\sqrt{7}$$

$$\Rightarrow \quad 2x+2a=a\sqrt{7}$$

$$\Rightarrow \quad R=\sqrt{7}a$$

PROBLEM 11

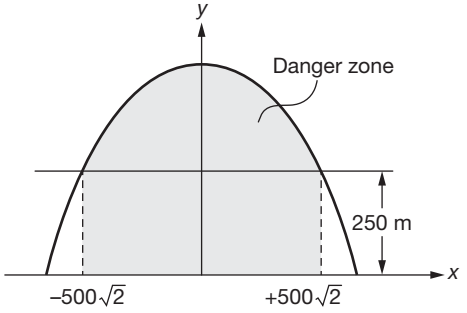
An enemy fighter jet is flying at a constant height of 250 m with a velocity of 500 ms^{-1}. The fighter jet passes over an anti-aircraft gun that can fire at any time and in any direction with a speed of 100 ms^{-1}. Determine the time interval during which the fighter jet is in danger of being hit by the gun bullets.

SOLUTION

The equation of trajectory of bullets is

$$y=x\tan\theta-\left(\frac{gx^2}{2u^2}\right)\sec^2\theta$$

$$\Rightarrow \quad y=x\tan\theta-\frac{gx^2}{2u^2}(1+\tan^2\theta) \qquad \ldots(1)$$



For a given value of x, maximum y can be determined from

$$\frac{dy}{d(\tan\theta)}=0$$

$$\Rightarrow \quad x-\frac{gx^2}{2u^2}(2\tan\theta)=0$$

$$\left\{\because\ \frac{d(\tan^2\theta)}{d(\tan\theta)}=2\tan\theta \text{ and } \frac{d(\tan\theta)}{d(\tan\theta)}=1\right\}$$

$$\Rightarrow \quad \tan\theta=\frac{u^2}{gx}$$

On substituting the expression for $\tan\theta$ in equation (1), we get

$$y_{MAX}=x\left(\frac{u^2}{gx}\right)-\frac{gx^2}{2u^2}\left(1+\frac{u^4}{g^2x^2}\right)$$

$$\Rightarrow \quad y_{MAX}=\frac{u^2}{2g}-\frac{gx^2}{2u^2}$$

The shell can hit an area defined by

$$y\le y_{MAX}$$

$$\Rightarrow \quad y\le\frac{u^2}{2g}-\frac{gx^2}{2u^2}$$

On substituting numerical values, $y=250$ m, $u=100$ ms^{-1}, $g=10$ ms^{-2}, we get

$$\frac{x^2}{2000}\le 250$$

$$\Rightarrow \quad x^2\le 500000$$

$$\Rightarrow \quad -500\sqrt{2}\le x\le 500\sqrt{2}$$

The fighter jet, can travel $1000\sqrt{2}$ m while it can be hit. So the plane is in danger for a period of

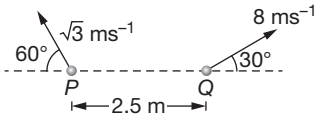
$$t=\frac{x}{u}=\frac{1000\sqrt{2}}{500}=2\sqrt{2}\text{ s.}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

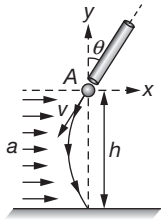
1. Two particles P and Q are moving as shown in the figure. At this moment of time the angular speed of P w.r.t. Q is



(A) $5\ \text{rads}^{-1}$ (B) $4\ \text{rads}^{-1}$

(C) $2\ \text{rads}^{-1}$ (D) $1\ \text{rads}^{-1}$

2. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y-axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x-direction. If the particle strikes the ground at a point directly under its released position and the downward y-acceleration is taken as g then



(A) $h=\dfrac{2v^2\sin\theta\cos\theta}{a}$

(B) $h=\dfrac{2v^2\sin\theta\cos\theta}{g}$

(C) $h=\dfrac{2v^2}{g}\sin\theta\left(\cos\theta+\dfrac{a}{g}\sin\theta\right)$

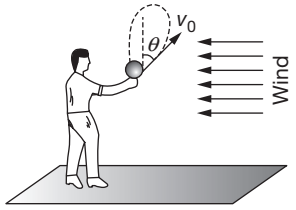
(D) $h=\dfrac{2v^2}{a}\sin\theta\left(\cos\theta+\dfrac{g}{a}\sin\theta\right)$

3. The speed of a projectile when it is at its greatest height is $\sqrt{\dfrac{2}{5}}$ times its speed at half the maximum height. The angle of projection is

(A) $30°$ (B) $\tan^{-1}\left(\dfrac{3}{4}\right)$

(C) $45°$ (D) $60°$

4. A boy throws a ball upwards with velocity $v_0=20\ \text{ms}^{-1}$. The wind imparts a horizontal acceleration of $4\ \text{ms}^{-2}$ to the left. The angle θ at which the ball must be thrown so that the ball returns to the boy's hand is $\left(g=10\ \text{ms}^{-2}\right)$



(A) $\tan^{-1}(1.2)$ (B) $\tan^{-1}(0.2)$

(C) $\cot^{-1}(2)$ (D) $\cot^{-1}(2.5)$

5. A particle is moving in a circle of radius R in such a way that at any instant the total acceleration makes an angle of $45°$ with radius. Initial speed of particle is v_0. The time taken to complete the first revolution is

(A) $\dfrac{R}{v_0}$ (B) $\dfrac{2R}{v_0}$

(C) $\dfrac{R}{v_0}e^{-2\pi}$ (D) $\dfrac{R}{v_0}\left(1-e^{-2\pi}\right)$

6. A large number of bullets are fired in all the directions with the same speed v. The maximum area on the ground on which these bullets will spread is

(A) $\dfrac{\pi v^2}{g}$ (B) $\dfrac{\pi v^4}{g^2}$

(C) $\dfrac{\pi^2 v^4}{g^2}$ (D) $\dfrac{\pi^2 v^2}{g^2}$

7. Starting from rest, a particle rotates in a circle of radius $R=\sqrt{2}\,\text{m}$ with an angular acceleration $\alpha=\dfrac{\pi}{4}\ \text{rads}^{-2}$. The magnitude of average velocity of the particle over the time it rotates quarter circle is

(A) $1\ \text{ms}^{-1}$ (B) $1.25\ \text{ms}^{-1}$

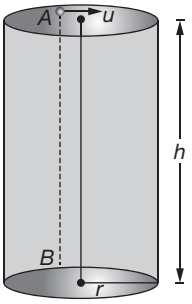
(C) $1.5\ \text{ms}^{-1}$ (D) $2\ \text{ms}^{-1}$

8. A projectile has a horizontal range R for two different angles. If h_1 and h_2 are the maximum heights reached, then

(A) $R = h_1 h_2$ (B) $R = \sqrt{h_1 h_2}$

(C) $R = 4\sqrt{h_1 h_2}$ (D) $R = 2\sqrt{h_1 h_2}$

9. A hollow vertical cylinder of radius r and height h has a smooth internal surface. A small particle is placed in contact with the inner side of the upper rim, at point A and given a horizontal speed u, tangential to the rim. It leaves the lower rim at point B, vertically below A. If n is an integer then



(A) $\dfrac{u}{2\pi r}\sqrt{\dfrac{2h}{g}} = n$ (B) $\dfrac{h}{2\pi r} = n$

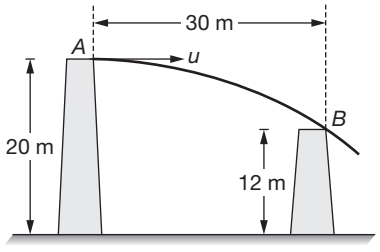
(C) $\dfrac{2\pi r}{h} = n$ (D) $\dfrac{u}{\sqrt{2gh}} = n$

10. A ball is projected so as to pass a wall at a distance a from the point of projection at an angle of $45°$ and falls at a distance b on the other side of the wall. If h is the height of the wall then

(A) $h = a\sqrt{2}$ (B) $h = b\sqrt{2}$

(C) $h = \dfrac{\sqrt{2}ab}{a+b}$ (D) $h = \dfrac{ab}{a+b}$

11. A boy wants to throw a ball from a point A so as to just clear the obstruction at B. The minimum horizontal velocity with which the boy should throw the ball is



(A) $23.5\ \text{ms}^{-1}$ (B) $235\ \text{ms}^{-1}$

(C) $47\ \text{ms}^{-1}$ (D) $32.5\ \text{ms}^{-1}$

12. A ball is thrown vertically upward with a speed v from a point h metre above the ground. The time taken for the ball to hit the ground is

(A) $\left(\dfrac{v}{g}\right)\left(1+\sqrt{1+\dfrac{2hg}{v^2}}\right)$ (B) $\sqrt{1+\left(\dfrac{2hg}{v^2}\right)}$

(C) $\left(\dfrac{v}{g}\right)\sqrt{1+\left(\dfrac{2hg}{v^2}\right)}$ (D) $\left(\dfrac{v}{g}\right)\sqrt{1-\left(\dfrac{2hg}{v^2}\right)}$

13. The muzzle velocity for a certain rifle is $600\ \text{ms}^{-1}$. If the rifle is pointed vertically upward and fired from an automobile moving horizontally at a speed of $72\ \text{kmh}^{-1}$, the radius of curvature of the path of the bullet at maximum altitude is (neglect friction of the air and take $g = 10\ \text{ms}^{-2}$)

(A) 400 m (B) 200 m

(C) 40 m (D) 20 m

14. A particle starts from the origin of co-ordinates at time $t = 0$ and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y = \beta x^2$. Its velocity component in the x-direction is

(A) $\dfrac{\alpha}{2\beta}$ (B) $\sqrt{\dfrac{\alpha}{2\beta}}$

(C) $\sqrt{\dfrac{2\alpha}{\beta}}$ (D) not constant

15. The speed of a projectile u reduces by 50% on reaching maximum height. The range on the horizontal plane is

(A) $\dfrac{2u^2}{3g}$ (B) $\dfrac{u^2}{g}$

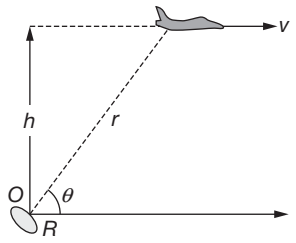
(C) $\dfrac{\sqrt{3}u^2}{2g}$ (D) $\dfrac{3u^2}{g}$

16. It is observed that a projectile is at the same height at 3 s and 5 s from the start. The time of flight of the projectile is equal to

(A) 1 s (B) 2 s

(C) 4 s (D) 8 s

17. A jet plane flying at a constant velocity v at a height $h = 8$ kilometre is being tracked by a radar R located at O directly below the line of flight. If the angle θ is decreasing at the rate of $0.025\ \text{rads}^{-1}$, the velocity of the plane when $\theta = 60°$ is



(A) $1440\ \text{kmh}^{-1}$ (B) $960\ \text{kmh}^{-1}$

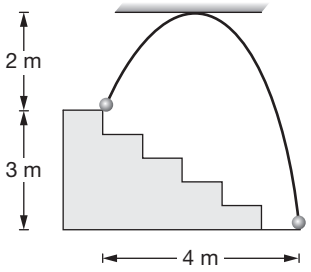
(C) $1920\ \text{kmh}^{-1}$ (D) $480\ \text{kmh}^{-1}$

18. A helicopter ascending at the rate of $12\ \text{ms}^{-1}$ drops a food packet from a height of $80\ \text{m}$ above the ground. The time the packet takes to reach the ground is

(A) 2.7 s (B) 5.5 s

(C) 11 s (D) 16.5 s

19. A ball is thrown from the top of a staircase which just touches the ceiling and finally hits the bottom of the steps. The initial speed of the ball is



(A) $6.3\ \text{ms}^{-1}$ (B) $7.6\ \text{ms}^{-1}$

(C) $63\ \text{ms}^{-1}$ (D) $76\ \text{ms}^{-1}$

20. A stone is projected with a velocity of $10\ \text{ms}^{-1}$ at $60°$ to the horizontal. At any instant, the angles of elevation of the stone from the two extremities of the range are α and β. Then $\tan\alpha + \tan\beta$ equals

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3}$

(C) 3 (D) $\sqrt{3}$

21. A particle is projected from the ground with a speed of $20\ \text{ms}^{-1}$ making an angle of $60°$ with the horizontal. The radius of curvature of the path of the particle, when its velocity makes an angle of $30°$ with horizontal is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 10.6 m (B) 12.8 m

(C) 15.4 m (D) 24.2 m

22. A particle is projected with a certain velocity at an angle α above the horizontal from the foot of an inclined plane of inclination $30°$. If the particle strikes the plane normally then α is equal to

(A) $30° + \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ (B) $30° + \tan^{-1}\left(2\sqrt{3}\right)$

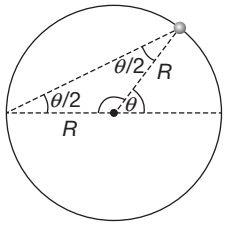
(C) $60°$ (D) $30° + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

23. A particle moves along a parabolic path $y = 9x^2$ in such a way that the x component of velocity remains constant and has a value $\frac{1}{3}\ \text{ms}^{-1}$. The acceleration of the particle is

(A) $\frac{1}{3}\hat{j}\ \text{ms}^{-2}$ (B) $3\hat{j}\ \text{ms}^{-2}$

(C) $\frac{2}{3}\hat{j}\ \text{ms}^{-2}$ (D) $2\hat{j}\ \text{ms}^{-2}$

24. A particle moves along a circle of radius $R = 2\ \text{m}$ so that its radius vector $\vec{r}$ relative to a point on its circumference rotates with the constant angular velocity $\omega = 2\ \text{rads}^{-1}$. The linear speed of the particle is



(A) $4\ \text{ms}^{-1}$ (B) $2\ \text{ms}^{-1}$

(C) $1\ \text{ms}^{-1}$ (D) $8\ \text{ms}^{-1}$

25. For two projectiles launched with same initial velocity, the maximum heights corresponding to equal ranges are $4\ \text{m}$ and $16\ \text{m}$. The range has a value

(A) 4 m (B) 8 m

(C) 16 m (D) 32 m

26. When a particle is moving along a circular path with uniform speed, it has

(A) radial velocity and radial acceleration

(B) radial velocity and transverse acceleration

(C) transverse velocity and radial acceleration

(D) transverse velocity and transverse acceleration

27. A particle moves in the xy plane with constant acceleration a directed along the negative y-axis. The equation of motion of the particle has the form $y = \alpha x - \beta x^2$,

where α and β are positive constants. The velocity of the particle at the origin of coordinates is

(A) $\sqrt{a\left(\frac{\alpha^2+1}{2\beta}\right)}$ (B) $\sqrt{a\left(\frac{\beta+1}{2\alpha^2}\right)}$

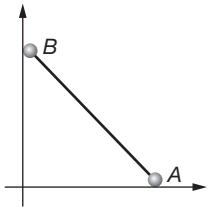
(C) $\sqrt{a\left(\frac{\alpha^2+1}{4\beta}\right)}$ (D) $\sqrt{a\left(\frac{\beta+1}{4\alpha^2}\right)}$

28. A body of mass m thrown horizontally with velocity v from the top of the tower of height h touches the ground at a distance of 250 m from the foot of the tower. A body of mass $2m$ thrown with a velocity of $\frac{v}{2}$ from the top of the tower of height $4h$ will touch the ground at a distance of

(A) 250 m (B) 500 m

(C) 1000 m (D) 125 m

29. Two particles A and B are connected by a rigid rod AB. The rod slides along perpendicular rails as shown here. The velocity of A to the left is $10\ \text{ms}^{-1}$. The speed of B when angle $\theta = 45°$ is



(A) $5\ \text{ms}^{-1}$ (B) $5\sqrt{2}\ \text{ms}^{-1}$

(C) $7.5\ \text{ms}^{-1}$ (D) $10\ \text{ms}^{-1}$

30. A stone is projected so as to pass two walls of heights a and b at distances b and a respectively from the point of projection. If α is the angle of projection then

(A) minimum value of $\tan\alpha$ is $\sqrt{3}$.

(B) minimum value of $\tan\alpha$ is 3.

(C) maximum value of $\tan\alpha$ is $\sqrt{3}$.

(D) maximum value of $\tan\alpha$ is 3.

31. Two stones are projected so as to reach the same distance from the point of projection on a horizontal surface. The maximum height reached by one exceeds the other by an amount equal to half the sum of the heights attained by them. Then the angles of projection for the stones are

(A) 45°, 135° (B) 0°, 90°

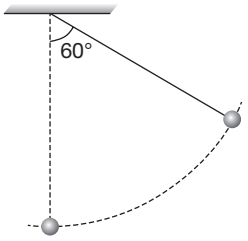
(C) 30°, 60° (D) 20°, 70°

32. Two particles are projected from a point at the same instant with velocities whose horizontal components and vertical components are (u_1, v_1) and (u_2, v_2), respectively. The time interval between their passing through the other common point of their path (other than origin) is

(A) $\frac{2}{g}\left(\frac{v_1u_1 - v_2u_2}{u_1+u_2}\right)$ (B) $\frac{2}{g}\left(\frac{v_1^2+v_2^2}{u_1+u_2}\right)$

(C) $\frac{2}{g}\left(\frac{u_1^2+u_2^2}{v_1+v_2}\right)$ (D) $\frac{2}{g}\left(\frac{v_1u_2 - v_2u_1}{u_1+u_2}\right)$

33. A pendulum of length $l = 1\ \text{m}$ is released from $\theta_0 = 60°$. The rate of change of speed of the bob at $\theta = 30°$ is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) $2.5\ \text{ms}^{-2}$ (B) $5\ \text{ms}^{-2}$

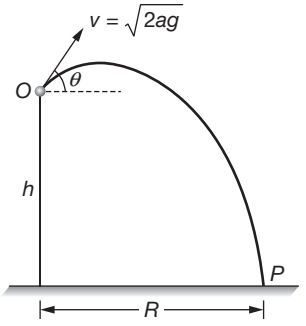
(C) $5\sqrt{3}\ \text{ms}^{-2}$ (D) $10\ \text{ms}^{-2}$

34. A projectile is thrown in a viscous medium offering resistance equal to one-tenth of acceleration due to gravity. The time of flight of projectile will

(A) increase by 1% (B) decrease by 1%

(C) increase by 2% (D) decrease by 2%

35. A particle is projected under gravity with velocity $\sqrt{2ag}$ from a point at a height h above the level plane. The maximum range R on the ground is



(A) $\sqrt{(a^2+1)h}$ (B) $\sqrt{a^2h}$

(C) $\sqrt{ah}$ (D) $2\sqrt{a(a+h)}$

36. The position vector of particle, moving in x-y plane, at any time t is $\vec{r} = \left[(2t)\hat{i} + (2t^2)\hat{j}\right]$ m. If θ be the angle which its velocity vector makes with positive x-axis) then the rate of change of θ at time $t = 0.5$ s.

(A) 6 rads^{-1} (B) 4 rads^{-1}

(C) 2 rads^{-1} (D) 1 rads^{-1}

37. Two particles are projected simultaneously in the same vertical plane, from the same point, but with different speeds and at different angles with the horizontal. The path followed by one, as seen by the other, is

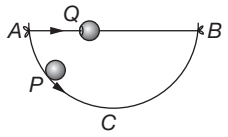
(A) a vertical straight line.

(B) a straight line making a constant angle $(\neq 90°)$ with the horizontal.

(C) a parabola.

(D) a hyperbola.

38. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of same mass as P is ejected from A at $t = 0$ along the horizontal string AB, with a speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B, then



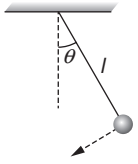
(A) $t_P < t_Q$

(B) $t_P = t_Q$

(C) $t_P > t_Q$

(D) $\dfrac{t_P}{t_Q} = \dfrac{\text{Length of arc } ACB}{\text{Length of chord } AB}$

39. For the simple pendulum shown, $l = 200$ mm, when $\theta = 30°$, $\dfrac{d\theta}{dt} = -9 \text{ rad s}^{-1}$. The magnitude of the total acceleration of the pendulum for this position is



(A) 9.81 ms^{-2} (B) 5 ms^{-2}

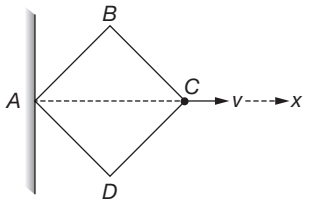
(C) 16.2 ms^{-2} (D) 17 ms^{-2}

40. A bomber flying with a horizontal velocity of 500 kmh^{-1} at a vertical height of 5 km above the ground wants to hit a train moving with a velocity of 100 kmh^{-1} in the same direction and in the same vertical plane. The angle θ between the line of sight of the target and the horizontal at the instant the bomb shell should be released is approximately

(A) 55° (B) 57°

(C) 72° (D) 27°

41. Four rods each of length l have been hinged to form a rhombus. Vertex A is fixed to rigid support, vertex C is being moved along the x-axis with a constant velocity v as shown in the figure. The rate at which vertex B is approaching the x-axis at the moment the rhombus is in the form of a square is



(A) $\dfrac{v}{\sqrt{2}}$ (B) $\dfrac{v}{2}$

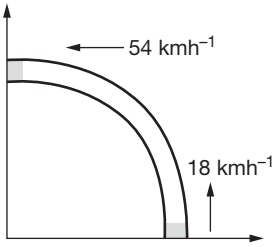
(C) $\dfrac{v}{3}$ (D) $\dfrac{v}{4}$

42. The distance r from the origin of a particle moving in x-y plane varies with time as, $r = 2t$ and the angle made by the radius vector with positive x-axis is $\theta = 4t$. Here, t is in second, r in metres and θ in radian. The speed of the particle at $t = 1$ s is

(A) 12 ms^{-1} (B) 10 ms^{-1}

(C) 8.25 ms^{-1} (D) 8 ms^{-1}

43. A car enters a curved road in the form of a quarter of a circle, the path length being 200 metre. Its speed at the entrance is 18 kmh^{-1} but when it leaves, it increases to 54 kmh^{-1}. If the car is travelling with constant acceleration along the curve, the acceleration when the car leaves the curved road is



(A) 0.9 ms^{-2} (B) 0.45 ms^{-2}

(C) 1.9 ms^{-2} (D) 3.68 ms^{-2}

44. The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as

(A) $2a\cos(\omega t)$ (B) $2a\sin(\omega t)$

(C) $2a\cos\left(\dfrac{\omega t}{2}\right)$ (D) $2a\sin\left(\dfrac{\omega t}{2}\right)$

45. Two bodies are thrown simultaneously from the same point. One thrown straight up and the other at an angle α with the horizontal. Both bodies have velocity equal to v_0. Neglecting the air drag, the separation between the particles at time t is

(A) $2v_0t\sqrt{1-\cos\alpha}$ (B) $v_0t\sqrt{1-\cos\alpha}$

(C) $2v_0t\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$ (D) $2v_0t\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

46. A boat is moving directly away from the gun on the shore with speed v_1. The gun fires a shell with speed v_2 at an angle α and hits the boat. The distance of the boat from the gun at the moment it is fired is

(A) $\frac{2v_2\sin\alpha(v_1\cos\alpha-v_2)}{g}$

(B) $\frac{2v_1\sin\alpha(v_1\cos\alpha-v_2)}{g}$

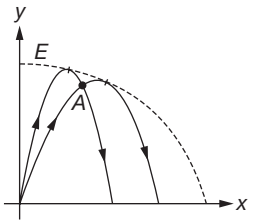
(C) $\frac{2v_2\sin\alpha(v_2\cos\alpha-v_1)}{g}$

(D) $\frac{2v_2\cos\alpha(v_1\sin\alpha-v_2)}{g}$

47. A projectile is thrown into space so as to have maximum possible range of 400 m. Taking the point of projection as the origin, the co-ordinate of the point where the velocity of the projectile is minimum is

(A) (400, 100) (B) (200, 100)
(C) (400, 200) (D) (200, 200)

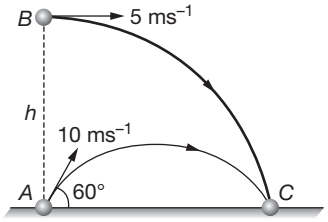
48. A number of projectiles each with a fixed muzzle velocity u are fired at different angles lying between $0°$ and $90°$ as shown. Neglecting air resistance and assuming g to be constant, then the equation of the envelope E of the parabolic trajectories (as shown in figure) is



(A) $y=\frac{u^2}{2g}-\frac{gx^2}{2u^2}$ (B) $y=\frac{gx^2}{2u^2}+\frac{u^2}{2g}$

(C) $y=\frac{2u^2}{gx^2}-\frac{2g}{u^2}$ (D) $y=\frac{2u^2}{gx^2}+\frac{2g}{u^2}$

49. A particle A is projected from the ground with an initial velocity of $10\ \text{ms}^{-1}$ at an angle of $60°$ with horizontal. At the same instant, another particle B is projected horizontally with velocity $5\ \text{ms}^{-1}$ from height h above A so that both the particles collide at point C on the ground. Taking $g=10\ \text{ms}^{-2}$, then



(A) $h=10$ m (B) $h=30$ m
(C) $h=15$ m (D) $h=25$ m

50. A body is projected with a velocity u at an angle α with the horizontal. It passes over a wall at a distance x from the point of projection. The maximum height of the wall corresponds to

(A) $\tan\alpha=\frac{u}{gx}$ (B) $\tan\alpha=\frac{u^2}{2gx}$

(C) $\tan\alpha=\frac{u^2}{gx}$ (D) $\tan\alpha=\frac{u^2}{2g}$

51. In PROBLEM 50, the maximum height of the wall is

(A) $h=\frac{u^2}{2g}$ (B) $h=\frac{gx^2}{2u^2}$

(C) $h=\frac{u^2}{2g}+\frac{gx^2}{2u^2}$ (D) $h=\frac{u^2}{2g}-\frac{gx^2}{2u^2}$

52. Two bodies are projected simultaneously in the same vertical plane, from the same point with different speeds and at different angles of projection with the horizon. The path followed by one projectile as seen from the other is

(A) a horizontal straight line.
(B) a parabola.
(C) a hyperbola.
(D) a straight line inclined at an acute angle with the horizontal.

53. Two particles are projected from the ground simultaneously with speeds $10\ \text{ms}^{-1}$ and $\frac{10}{\sqrt{3}}\ \text{ms}^{-1}$ at angles $30°$ and $60°$ with the horizontal in the same direction. The maximum distance between them till both of them strike the ground is approximately $\left(g=10\ \text{ms}^{-2}\right)$

(A) 10 m (B) 16 m
(C) 23 m (D) 30 m

54. Two particles are projected simultaneously in the same vertical plane from the same point, with different speeds u_1 and u_2, making angles θ_1 and θ_2 respectively with the vertical, such that $u_1 \sin\theta_1 = u_2 \sin\theta_2$. The path followed by one, as seen by the other (as long as both are in flight), is
(A) a straight line making an angle $|\theta_1 - \theta_2|$ with the horizontal.
(B) a parabola.
(C) a horizontal straight line.
(D) a vertical straight line.

55. A ball rolls of the top of a stair way with a horizontal velocity u ms^{-1}. If the steps are h m high and b m wide, the ball will hit the edge of the nth step, if

(A) $n = \dfrac{2hu}{gb^2}$ (B) $n = \dfrac{2hu^2}{gb^2}$

(C) $n = \dfrac{2hu^2}{gb}$ (D) $n = \dfrac{hu^2}{gb^2}$

56. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a, b are constants and x, y are respectively the horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

(A) $\dfrac{b^2}{2a}$, $\tan^{-1}(2a)$

(B) $\dfrac{a^2}{b}$, $\tan^{-1}(2a)$

(C) $\dfrac{a^2}{4b}$, $\tan^{-1}(a)$

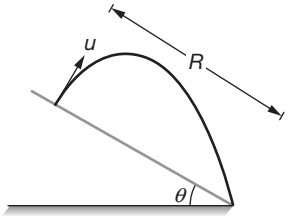
(D) $\dfrac{2a^2}{b}$, $\tan^{-1}(a)$

57. The velocity of a particle moving in the x-y plane is given by

$$\frac{dx}{dt} = 8\pi\sin(2\pi t) \text{ and } \frac{dy}{dt} = 8\pi\cos(2\pi t)$$

when $t = 0$, $x = 8$ and $y = 0$. The path of the particle is
(A) a straight line (B) a circle
(C) an ellipse (D) a parabola

58. A projectile is fired with a velocity u at right angle to the slope which is inclined at an angle θ with the horizontal. The expression for R is



(A) $\dfrac{2u^2}{g}\tan\theta$ (B) $\dfrac{2u^2}{g}\sec\theta$

(C) $\dfrac{u^2}{g}\tan^2\theta$ (D) $\dfrac{2u^2}{g}\tan\theta\sec\theta$

59. The horizontal range and maximum height attained by a projectile are R and H, respectively. If a constant horizontal acceleration $a = \dfrac{g}{2}$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

(A) $(R+H)$, $\dfrac{H}{2}$ (B) $\left(R+\dfrac{H}{2}\right)$, H

(C) $(R+2H)$, H (D) $(R+H)$, H

60. A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\dfrac{dx}{dt} = c = \dfrac{dy}{dt}$, where a, b and c are constants. The acceleration of the particle is

(A) $\left(2ax^2 + 6by^2\right)\hat{k}$ (B) $\left(6ac^2x + 2bc^2\right)\hat{k}$

(C) $\left(bc^2x + 2by\right)\hat{k}$ (D) $\left(4bc^2x + 3ac^2\right)\hat{k}$

61. A stone is thrown from a point at a distance a from a wall of height b. If it just clears the wall then the maximum height h reached by the stone for angle of projection α is

(A) $\dfrac{a^2\tan^2\alpha}{4(a\tan\alpha - b)}$ (B) $\dfrac{a^2\sec^2\alpha}{4(a\sec\alpha - b)}$

(C) $\dfrac{a^2\tan^2\alpha}{4b}$ (D) $\dfrac{a^2\tan^2\alpha}{4(a - b\cot\alpha)}$

62. An aeroplane flying at a constant speed releases a food-packet for flood victims. As the packet drops away from the aeroplane,
(A) it will always be vertically below the aeroplane only if the aeroplane was flying at an angle of 45° to the horizontal.
(B) it will always be vertically below the aeroplane only if the aeroplane was flying horizontally.
(C) it will gradually fall behind the aeroplane if the aeroplane was flying horizontally.
(D) it will always be vertically below the aeroplane.

63. Position vector of a particle moving in x-y plane at time t is $\vec{r} = a(1-\cos(\omega t))\hat{i} + a\sin(\omega t)\hat{j}$. The path of the particle is

(A) an ellipse

(B) a circle of radius a and centre at $(0, 0)$

(C) a circle of radius a and centre at $(a, 0)$

(D) neither a circle nor an ellipse

64. Ratio of minimum kinetic energies of two projectiles of same mass is $4:1$. The ratio of the maximum height attained by them is also $9:1$. The ratio of their ranges would be

(A) $16:1$ (B) $8:1$

(C) $6:1$ (D) $2:1$

65. A person at the point P aims his rifle at an angle of $20°$ with the horizontal so that the bullet fired is to hit an object at A but the bullet hits at point B, a vertical distance δ below A. If the initial velocity of the bullet is $600\ \text{ms}^{-1}$ and the point P is at a horizontal distance of 1 km from the point A then,

(A) $\delta = 1543$ m (B) $\delta = 154$ m

(C) $\delta = 154.3$ m (D) $\delta = 15.43$ m

66. A particle moves along the positive branch of the curve $y = \frac{x^2}{2}$ with x governed by $x = \frac{1}{2}t^2$, where x and y are measured in metre and t in second. At $t = 2$ s, the velocity of the particle is

(A) $2\hat{i} - 4\hat{j}\ \text{ms}^{-1}$ (B) $2\hat{i} + 4\hat{j}\ \text{ms}^{-1}$

(C) $4\hat{i} + 2\hat{j}\ \text{ms}^{-1}$ (D) $4\hat{i} - 2\hat{j}\ \text{ms}^{-1}$

67. A particle moves in the x-y plane according to the equations $x = 4t^2 + 4t + 5$ and $y = -t^3 + 12t + 3$. At $t = 1$ s, the acceleration of the particle is

(A) $5\ \text{ms}^{-2}$ (B) $10\ \text{ms}^{-2}$

(C) $15\ \text{ms}^{-2}$ (D) $20\ \text{ms}^{-2}$

68. A particle is moving in x-y plane. At certain instant of time, the components of its velocity and acceleration are $v_x = 3\ \text{ms}^{-1}$, $v_y = 4\ \text{ms}^{-1}$, $a_x = 2\ \text{ms}^{-2}$ and $a_y = 1\ \text{ms}^{-2}$. The rate of change of speed at this moment is

(A) $\sqrt{5}\ \text{ms}^{-2}$ (B) $\sqrt{10}\ \text{ms}^{-2}$

(C) $2\ \text{ms}^{-2}$ (D) $4\ \text{ms}^{-2}$

69. A particle is projected at an angle of $60°$ above the horizontal with a speed of $10\ \text{ms}^{-1}$. After some time the direction of its velocity makes an angle of $30°$ above the horizontal. The speed of the particle at this instant is

(A) $\frac{5}{\sqrt{3}}\ \text{ms}^{-1}$ (B) $5\sqrt{3}\ \text{ms}^{-1}$

(C) $5\ \text{ms}^{-1}$ (D) $\frac{10}{\sqrt{3}}\ \text{ms}^{-1}$

70. A particle is thrown with a speed u at an angle θ with the vertical. When the particle makes an angle ϕ with the vertical, its speed changes to v.

(A) $v = u\cos\theta$ (B) $v = u\sin\theta\cos\phi$

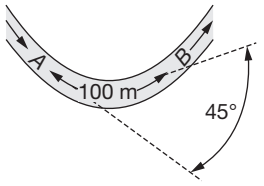
(C) $v = u\left(\frac{\sec\phi}{\sec\theta}\right)$ (D) $v = u\left(\frac{\text{cosec}\,\phi}{\text{cosec}\,\theta}\right)$

71. At a height 0.4 m from the ground, the velocity of a projectile in vector form is $\vec{v} = (6\hat{i} + 2\hat{j})\ \text{ms}^{-1}$. The angle of projection with the vertical is $(g = 10\ \text{ms}^{-2})$

(A) $45°$ (B) $60°$

(C) $30°$ (D) $\tan^{-1}\left(\frac{3}{4}\right)$

72. A car moves round a turn of constant curvature between A and B (curve $AB = 100$ metre) with a steady speed of $72\ \text{kmh}^{-1}$. If an accelerometer were mounted in the car, the magnitude of acceleration it would record between A and B is



(A) ZERO (B) $3.14\ \text{ms}^{-2}$

(C) $31.4\ \text{ms}^{-2}$ (D) $6.28\ \text{ms}^{-2}$

73. After one second the velocity of a projectile makes an angle of $45°$ with the horizontal. After another 3 s, it is travelling horizontally. The magnitude of its initial velocity and angle of projection are $(g = 10\ \text{ms}^{-2})$

(A) $50\ \text{ms}^{-1}$, $\tan^{-1}\left(\frac{3}{4}\right)$ (B) $25\ \text{ms}^{-1}$, $\tan^{-1}\left(\frac{4}{3}\right)$

(C) $50\ \text{ms}^{-1}$, $\tan^{-1}\left(\frac{4}{3}\right)$ (D) $25\ \text{ms}^{-1}$, $\tan^{-1}\left(\frac{3}{4}\right)$

74. The minimum speed with which a particle must be projected from the origin so that it just passes through the point $P(30\text{ m}, 40\text{ m})$, taking $g = 10\ \text{ms}^{-2}$, is

(A) $30\ \text{ms}^{-1}$ (B) $40\ \text{ms}^{-1}$

(C) $50\ \text{ms}^{-1}$ (D) $60\ \text{ms}^{-1}$

75. Two shots are projected from a gun at the top of a cliff with the same velocity u at angles of projection α and β with the horizon respectively. If the shots strike the horizontal ground through the foot of the cliff at the same point and if h is the height of the cliff, then

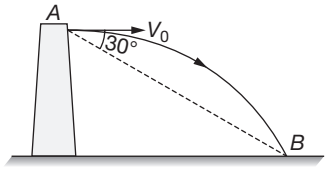
(A) $h=\dfrac{2u^2}{g}\left(\dfrac{1-\tan\alpha}{1-\tan\beta}\right)$

(B) $h=\dfrac{2u^2}{g}(\tan\alpha-\tan\beta)$

(C) $h=\dfrac{2u^2}{g}(\tan\alpha+\tan\beta)$

(D) $h=\dfrac{2u^2}{g}\cot(\alpha+\beta)$

76. An object is thrown horizontally from a tower at A and hits the ground 3 second later at B. The line of sight from A to B makes an angle of $30°$ with the horizontal. The initial velocity of the object, taking $g=10\ \text{ms}^{-2}$ is



(A) $10\ \text{ms}^{-1}$ (B) $13\ \text{ms}^{-1}$

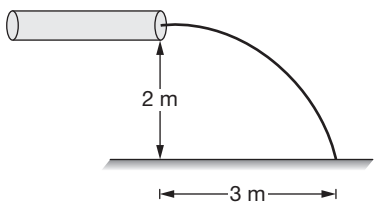
(C) $26\ \text{ms}^{-1}$ (D) $28.8\ \text{ms}^{-1}$

77. A point on the rim of a flywheel has a peripheral speed of $10\ \text{ms}^{-1}$ at an instant when it is decreasing at the rate of $60\ \text{ms}^{-2}$. If the magnitude of the total acceleration of the point at this instant is $100\ \text{ms}^{-2}$, the radius of the flywheel is

(A) 25 metre (B) 12.5 metre

(C) 2.5 metre (D) 1.25 metre

78. Water flows from a horizontal pipe which is fixed at a height of 2 m from the ground. If it falls at a distance of 3 m as shown in figure, the speed of water when it leaves the pipe is



(A) $4.7\ \text{ms}^{-1}$ (B) $7.4\ \text{ms}^{-1}$

(C) $14.7\ \text{ms}^{-1}$ (D) $47\ \text{ms}^{-1}$

79. A particle is projected from the ground with an initial speed of u at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

(A) $\dfrac{u}{2}\sqrt{1+2\cos^2\theta}$ (B) $\dfrac{u}{2}\sqrt{1+\cos^2\theta}$

(C) $\dfrac{u}{2}\sqrt{1+3\cos^2\theta}$ (D) $u\cos\theta$

80. Shots are fired at the same instant from the top and bottom of a vertical cliff at angle α and β and they strike an object simultaneously at the same point. If the horizontal distance of the object from the cliff is l and h be the height of the cliff then

(A) $h=l(\cot\beta-\cot\alpha)$ (B) $h=l(\cos\beta-\cos\alpha)$

(C) $h=l(\tan\beta-\tan\alpha)$ (D) $h=l(\sin\beta-\sin\alpha)$

81. A ball thrown upward at an angle of $30°$ to the horizontal lands on the top edge of a building 20 metre away. The top edge is 5 metre above the throwing point. The ball was thrown with a velocity of

(A) $20\ \text{ms}^{-1}$ (B) $30\ \text{ms}^{-1}$

(C) $40\ \text{ms}^{-1}$ (D) $60\ \text{ms}^{-1}$

82. A particle is projected vertically upwards from O with velocity v and a second particle is projected at the same instant from P (at a height h above O) with velocity v at an angle of projection θ. The time when the distance between them is minimum is

(A) $\dfrac{h}{2v\sin\theta}$ (B) $\dfrac{h}{2v\cos\theta}$

(C) $\dfrac{h}{v}$ (D) $\dfrac{h}{2v}$

83. A projectile is given an initial velocity of $\hat{i}+2\hat{j}$. The cartesian equation of its path is $\left(g=10\ \text{ms}^{-2}\right)$

(A) $y=2x-\dfrac{x^2}{5}$ (B) $y=2x-x^2$

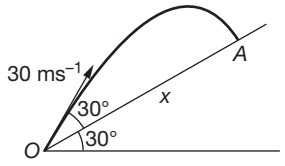
(C) $y=x-5x^2$ (D) $y=2x-5x^2$

84. A ball is launched with an initial velocity of $20\sqrt{2}\ \text{ms}^{-1}$ making at angle $45°$ with horizontal. The angular velocity of the particle at highest point of its journey about point of projection is

(A) $0.1\ \text{rads}^{-1}$ (B) $0.2\ \text{rads}^{-1}$

(C) $0.3\ \text{rads}^{-1}$ (D) $0.4\ \text{rads}^{-1}$

85. An object is projected up the incline at the angle shown in figure with an initial velocity of $30\ \text{ms}^{-1}$. The distance x up the incline at which the object lands is



(A) 600 m (B) 104 m
(C) 60 m (D) 208 m

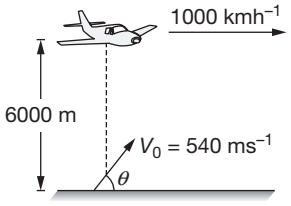
86. A very broad elevator is going up vertically with a constant acceleration of $2\ \text{ms}^{-2}$. At the instant when its velocity is $4\ \text{ms}^{-1}$ a ball is projected from the floor of the lift with a speed of $4\ \text{ms}^{-1}$ relative to the floor at an elevation of $30°$. The time taken by the ball to return the floor is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 1 s (B) $\frac{1}{2}$ s
(C) $\frac{1}{3}$ s (D) $\frac{1}{4}$ s

87. In PROBLEM 86, range of the ball over the floor of the lift is

(A) 2 m (B) $\frac{2}{\sqrt{3}}$ m
(C) $\sqrt{3}$ m (D) $2\sqrt{3}$ m

88. An aircraft moving with a speed of $1000\ \text{kmh}^{-1}$ is at a height of 6000 m, just overhead of an anti-aircraft gun. If the muzzle velocity is $540\ \text{ms}^{-1}$, the firing angle θ should be

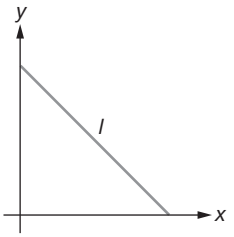


(A) 73° (B) 30°
(C) 59° (D) 45°

89. A particle moves in the x-y plane with velocity $v_x = 8t - 2$ and $v_y = 2$. If it passes through the point $(14,\ 4)$ m at $t = 2$ s, the equation of the path is

(A) $x = y^2 - y + 2$ (B) $x = y + 2$
(C) $x = y^2 + 2$ (D) $x = y^2 + y - 2$

90. A rod of length l leans by its upper end against a smooth vertical wall, while its other end leans against the floor. The end that leans against the wall moves uniformly downward with velocity v_0. Then



(A) the other end also moves uniformly
(B) the speed of other end goes on decreasing
(C) the speed of other end goes on increasing
(D) the speed of other end first decreases and then increases

91. A particle is projected from the ground with an initial velocity of $30\ \text{ms}^{-1}$ at an angle of $60°$ with horizontal. The magnitude of change in velocity in 2 s is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) $5\ \text{ms}^{-1}$ (B) $10\ \text{ms}^{-1}$
(C) $20\ \text{ms}^{-1}$ (D) $40\ \text{ms}^{-1}$

92. A boy throws a ball with a velocity u at an angle α with the vertical. At the same instant he starts running with uniform velocity to catch the ball before it hits the ground. To achieve this, he should run with a velocity of

(A) $u\cos\alpha$ (B) $u\sin\alpha$
(C) $u\tan\alpha$ (D) $\sqrt{u^2\tan\alpha}$

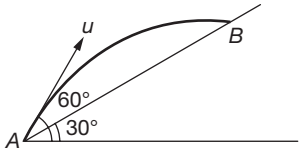
93. The rotor of a turbine rotates at the rate of 2000 rpm. If the diameter of the rotor is 5 m, the centripetal acceleration at the edge of the rotor is $(\text{take } \pi^2 = 10)$

(A) $11\times10^5\ \text{ms}^{-2}$ (B) $1.1\times10^5\ \text{ms}^{-2}$
(C) $2.2\times10^4\ \text{ms}^{-2}$ (D) $2.2\times10^5\ \text{ms}^{-2}$

94. A projectile is thrown with an initial velocity of $\left(a\hat{i} + b\hat{j}\right)\text{ms}^{-1}$. If the range of the projectile is twice the maximum height reached by it, then

(A) $a = 2b$ (B) $b = a$
(C) $b = 2a$ (D) $b = 4a$

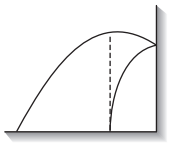
95. Time taken by the projectile to reach from A to B is t. Thet the distance AB is equal to



(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$

(C) $\sqrt{3}ut$ (D) $2ut$

96. A stone is projected from a horizontal plane. It attains maximum height H and strikes a stationary smooth wall and falls on the ground vertically below the maximum height. Assume the collision to be elastic, the height of the point on the wall where stone will strike is



(A) $\frac{H}{2}$ (B) $\frac{H}{4}$

(C) $\frac{3H}{4}$ (D) $\frac{H}{3}$

97. A circular disc of radius $r = 5$ m is rotating in horizontal plane about y-axis. Y-axis is vertical axis passing through the centre of disc and x-z is the horizontal plane at ground. The height of disc above ground is $h = 5$ m. Small particles are ejecting from disc in horizontal direction with speed $12\ \text{ms}^{-1}$ from the circumference of disc then the distance of these particles from origin when they hits the x-z plane is

(A) 12 m (B) 13 m

(C) 5 m (D) 6 m

98. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ radians should strike a given target. In actual practice it was found that a hill just prevented in the trajectory. At what angle of elevation should the gun be fired to hit the target.

(A) $\frac{5\pi}{36}$ radians (B) $\frac{11\pi}{36}$ radians

(C) $\frac{7\pi}{36}$ radians (D) $\frac{13\pi}{36}$ radians

99. A body is thrown with speed of $30\ \text{ms}^{-1}$ at angle $30°$ with horizontal from a perfectly inelastic horizontal floor. The time after which it is moving perpendicular to its initial direction of motion is

(A) 6 s (B) 3 s

(C) 1.5 s (D) never

100. A body is thrown from a point with speed $50\ \text{ms}^{-1}$ at an angle $37°$ with horizontal. When it has moved a horizontal distance of 80 m then its distance from point of projection is

(A) 40 m (B) $40\sqrt{2}$ m

(C) $40\sqrt{5}$ m (D) None of these

101. A ball is thrown from ground level so as to just clear a wall 4 meters high at a distance of 4 meters and falls at a distance of 14 meters from the wall, then the magnitude of the velocity of the ball is

(A) $\sqrt{281}\ \text{ms}^{-1}$ (B) $\sqrt{812}\ \text{ms}^{-1}$

(C) $\sqrt{182}\ \text{ms}^{-1}$ (D) $\sqrt{128}\ \text{ms}^{-1}$

102. An object is thrown at an angle α to the horizontal $(0° < \alpha < 90°)$ with a velocity. Then during ascent (ignoring air drag) the acceleration

(A) with which the object moves is $\vec{g}$, at all points.

(B) tangential to the path decreases.

(C) normal to the path increases, becoming equal to g at the highest point.

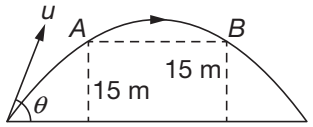
(D) All of these

103. A projectile is thrown with a velocity of $20\ \text{ms}^{-1}$, at an angle of $60°$ with the horizontal. After how much time the velocity vector will make an angle of $45°$ with the horizontal? $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

(A) $\sqrt{3}$ s (B) $\frac{1}{\sqrt{3}}$ s

(C) $\left(\sqrt{3}+1\right)$ s (D) $\left(\sqrt{3}-1\right)$ s

104. A golfer standing on level ground hits a ball with a velocity of $u = 52\ \text{ms}^{-1}$ at an angle θ above the horizontal. If $\tan\theta = \frac{5}{12}$, then the time for which the ball is at least 15 m above the ground (i.e. between A and B) will be $\left(\text{take } g = 10\ \text{ms}^{-2}\right)$



(A) 1 s (B) 2 s

(C) 3 s (D) 4 s

105. Which of the following ideas is helpful in understanding projectile motion?

(A) $v_x^2 + v_y^2 =$ constant

(B) Acceleration is $+g$ when the object is rising and $-g$ when falling

(C) In the absence of friction the trajectory will depend on the object's mass as well as its initial velocity and launch angle

(D) The horizontal motion is independent of the vertical motion

106. A ball is projected upwards from the top of the tower with a velocity $50\ \text{ms}^{-1}$ making an angle $30°$ with the horizontal. The height of tower is $70\ \text{m}$. After how many seconds the ball will strike the ground?

(A) 3 s (B) 5 s

(C) 7 s (D) 9 s

107. The equation of projectile is $y = \sqrt{3}x - \frac{g}{2}x^2$. The angle of projection and initial velocity is

(A) $30°$, $4\ \text{ms}^{-1}$ (B) $60°$, $2\ \text{ms}^{-1}$

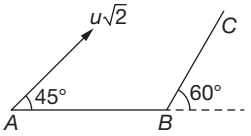
(C) $60°$, $4\ \text{ms}^{-1}$ (D) $90°$, $4\ \text{ms}^{-1}$

108. A projectile is fired at an angle of $30°$ to the horizontal such that the vertical component of its initial velocity is $80\ \text{ms}^{-1}$. Its time of flight is T. Its velocity at $t = \frac{T}{4}$ has a magnitude of nearly

(A) $200\ \text{ms}^{-1}$ (B) $300\ \text{ms}^{-1}$

(C) $140\ \text{ms}^{-1}$ (D) $100\ \text{ms}^{-1}$

109. A particle is projected from a point A with velocity $u\sqrt{2}$ at an angle of $45°$ with horizontal as shown in figure. It strikes the plane BC at right angles. The velocity of the particle at the time of collision is:



(A) $\frac{\sqrt{3}u}{2}$ (B) $\frac{u}{2}$

(C) $\frac{2u}{\sqrt{3}}$ (D) u

110. Consider a boy on a trolley who throws a ball with speed $20\ \text{ms}^{-1}$ at an angle $37°$ with respect to trolley in direction of motion of trolley which moves horizontally with speed $10\ \text{ms}^{-1}$ then what will be maximum distance travelled by ball parallel to road

(A) 20.2 m (B) 12 m

(C) 31.2 m (D) 62.4 m

111. A projectile is thrown at angle β with vertical. It reaches a maximum height H. The time taken to reach highest point of its path is

(A) $\sqrt{\frac{H}{g}}$ (B) $\sqrt{\frac{2H}{g}}$

(C) $\sqrt{\frac{H}{2g}}$ (D) $\sqrt{\frac{2H}{g\cos\beta}}$

112. A body is projected at angle $45°$ to horizontal with velocity $20\ \text{ms}^{-1}$ from ground. If there is an acceleration in horizontal direction of $2\ \text{ms}^{-2}$, then calculate horizontal range of this particle

(A) 40 m (B) 48 m

(C) 8 m (D) 20 m

113. A particle is projected with a velocity of $20\ \text{ms}^{-1}$ at an angle of $30°$ to an inclined plane of inclination $30°$ to the horizontal. The particle hits the inclined plane at an angle $30°$, during its journey. The time of flight is

(A) $\frac{20\sin(60°)}{g}$ (B) $\frac{20\sin(60°)}{g\cos(30°)}$

(C) $\frac{20\sin(30°)}{g\cos(60°)}$ (D) $\frac{20\sin(30°)}{g}$

114. A particle starts flying in the xy-plane with a speed of $2\hat{i} + 5x\hat{j}$. Initial position of the particle was the origin $(0,\ 0)$ of the plane. The trajectory of the particle is represented by the equation

(A) $y = 1.25x^2$ (B) $y = 5x^2$

(C) $y = 2.5x^2$ (D) $x = 5y^2$

115. The ceiling of a tunnel is $5\ \text{m}$ high. What is the maximum horizontal distance that a ball thrown with a speed of $20\ \text{ms}^{-1}$, can go without hitting the ceiling of the tunnel? $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

(A) 30 m (B) 40 m

(C) $30\sqrt{2}$ m (D) $20\sqrt{3}$ m

116. In projectile motion, the modulus of rate of change of speed

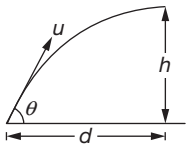
(A) is constant

(B) first increases then decreases

(C) first decreases then increases

(D) None of these

117. If a stone is to hit at a point which is at a horizontal distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u if the stone is launched at an angle θ?



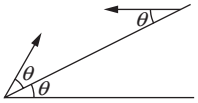
(A) $\frac{g}{\cos\theta}\sqrt{\frac{d}{2(d\tan\theta-h)}}$

(B) $\frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta-h)}}$

(C) $\sqrt{\frac{gd^2}{h\cos^2\theta}}$

(D) $\sqrt{\frac{gd^2}{d-h}}$

118. From an inclined plane two particles are projected with same speed at same angle θ, one up and other down the plane as shown in figure. Which of the following statement(s) is/are correct?



(A) The particles will collide the plane with same speed
(B) The times of flight of each particle are same
(C) Both particles strikes the plane perpendicularly
(D) None of these

119. A large box is moving on horizontal floor with constant acceleration $a = g$. A particle is projected inside box with velocity u and angle θ with horizontal with respect to box frame. For the given u, the value of θ for which horizontal range inside box will be maximum is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{8}$

(C) $\frac{3\pi}{8}$ (D) $\frac{\pi}{3}$

120. An object is projected with a velocity of $20\ \text{ms}^{-1}$ making an angle of $45°$ with horizontal. The equation for the trajectory is $h = Ax - Bx^2$ where h is height, x is horizontal distance. A and B are constants. The ratio $A:B$ is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 1 : 5 (B) 5 : 1
(C) 1 : 40 (D) 40 : 1

121. The x and y coordinates of a particle at any time t are given by $x = 2t + 4t^2$ and $y = 5t$, where x and y are in metre and t in second. The acceleration of the particle at $t = 5$ s is

(A) $40\ \text{ms}^{-2}$ (B) $20\ \text{ms}^{-2}$
(C) $8\ \text{ms}^{-2}$ (D) ZERO

122. A particle starts from the origin at $t = 0$. It moves in a plane with a velocity given by $\vec{v} = v_0\hat{i} + (a\omega\cos\omega t)\hat{j}$. The equation of trajectory of the particle is

(A) $y = a\sin(\omega t)$ (B) $y = a\cos(\omega t)$

(C) $y = a\sin\left(\frac{\omega x}{v_0}\right)$ (D) $y = a\cos\left(\frac{\omega x}{v_0}\right)$

123. Ratio of minimum kinetic energies of two projectiles of same mass is $4:1$. The ratio of the maximum height attained by them is also $4:1$. The ratio of their ranges would be

(A) 16 : 1 (B) 4 : 1
(C) 8 : 1 (D) 2 : 1

124. At a height 0.4 m from the ground, the velocity of a projectile is, $\vec{v} = \left(6\hat{i} + 2\hat{j}\right)\ \text{ms}^{-1}$. The angle of projection is $\left(g = 10\ \text{ms}^{-2}\right)$

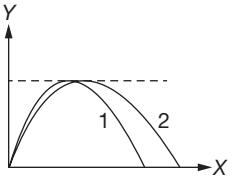
(A) $45°$ (B) $60°$

(C) $30°$ (D) $\tan^{-1}\left(\frac{3}{4}\right)$

125. A person standing on a truck moving with a uniform velocity $14.7\ \text{ms}^{-1}$ on a horizontal road throws a ball in such a way that it returns to him after 4 s. The speed and angle of projection as seen by a man on the road are

(A) $19.6\ \text{ms}^{-1}$, vertical
(B) $24.5\ \text{ms}^{-1}$, vertical
(C) $19.6\ \text{ms}^{-1}$, $53°$ with the road
(D) $24.5\ \text{ms}^{-1}$, $53°$ with the road

126. Trajectories of two projectiles are shown in the figure. Let T_1 and T_2 be the time periods and u_1 and u_2 be their speeds of projection. Then

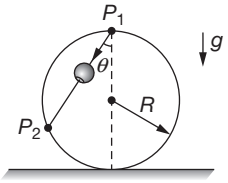


(A) $T_2 > T_1$ (B) $T_1 > T_2$
(C) $u_1 > u_2$ (D) $u_1 < u_2$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A bead is free to slide down a smooth wire tightly stretched between the points P_1 and P_2 on a vertical circle of radius R. If the bead starts from rest from P_1, the highest point on the circle and P_2 lies anywhere on the circumference of the circle. Then,



(A) time taken by bead to go from P_1 to P_2 is dependent on position of P_2 and equals $2\sqrt{\frac{R}{g}}\cos\theta$.

(B) time taken by bead to go from P_1 to P_2 is independent of position of P_2 and equals $2\sqrt{\frac{R}{g}}$.

(C) acceleration of bead along the wire is $g\cos\theta$.

(D) velocity of bead when it arrives at P_2 is $2\sqrt{gR}\cos\theta$.

2. For an oblique projectile, if T is the total time of flight, H the maximum height and R is the horizontal range, then x and y co-ordinates at any time t are related as (neglect air drag)

(A) $y = 4H\left(\frac{t}{T}\right)\left(1-\frac{t}{T}\right)$

(B) $y = 4H\left(\frac{T}{t}\right)\left(1-\frac{T}{t}\right)$

(C) $y = 4H\left(\frac{x}{R}\right)\left(1-\frac{x}{R}\right)$

(D) $y = 4H\left(\frac{R}{x}\right)\left(1-\frac{R}{x}\right)$

3. From a point P, a particle is projected with a velocity u at an angle θ with horizontal. At a certain point Q, the particle moves at right angles to its initial direction of motion. Then

(A) velocity of particle at Q is $u\sin\theta$.

(B) time of flight from P to Q is $\left(\frac{u}{g}\right)\sec\theta$

(C) speed of particle at Q is $u\cot\theta$.

(D) time of flight from P to Q is $\left(\frac{u}{g}\right)\text{cosec}\,\theta$

4. A particle is projected with a velocity $2\sqrt{hg}$ so that it just clears two walls of equal height h at horizontal separation $2h$ from each other. Then the

(A) angle of projection is $30°$ with vertical.

(B) angle of projection is $30°$ with horizon.

(C) time of passing between the walls is $\sqrt{\frac{2h}{g}}$.

(D) time of passing between the walls is $2\sqrt{\frac{h}{g}}$.

5. A particle is launched from the origin with an initial velocity $\vec{u} = \left(3\hat{i}\right)\text{ms}^{-1}$ under the influence of a constant acceleration $\vec{a} = -\left(\hat{i}+\frac{1}{2}\hat{j}\right)\text{ms}^{-2}$. Its velocity $\vec{v}$ and position vector $\vec{r}$ when it reaches its maximum x-co-ordinate are

(A) $\vec{v} = \left(-1.5\hat{j}\right)\text{ms}^{-1}$ (B) $\vec{v} = -2\hat{j}$

(C) $\vec{r} = \left(3\hat{i}-2\hat{j}\right)\text{m}$ (D) $\vec{r} = \left(4.5\hat{i}-2.25\hat{j}\right)\text{m}$

6. A projectile is projected from the ground making an angle α with the horizontal. Air exerts a drag which is proportional to the velocity of the projectile

(A) the time of ascent will be equal to the time of descent

(B) the time of ascent will be greater than time of descent

(C) the time of descent will be greater than time of ascent

(D) at highest point velocity will be horizontal

7. Two particles projected from the same point with same speed u at angles of projection α and β strike the horizontal ground at the same point. If h_1 and h_2 are the maximum heights attained by projectiles, R be the range for both and t_1 and t_2 be their time of flights respectively then

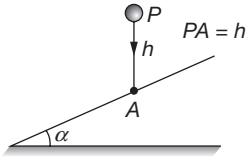
(A) $\alpha+\beta=\frac{\pi}{2}$ (B) $R = 4\sqrt{h_1h_2}$

(C) $\frac{t_1}{t_2}=\tan\alpha$ (D) $\tan\alpha=\sqrt{\frac{h_1}{h_2}}$

8. The co-ordinates of a particle moving in a plane are given by $x = a\cos(pt)$ and $y = b\sin(pt)$, where a, $b(<a)$ and p are positive constants of appropriate dimensions. Then

(A) the path of the particle is an ellipse.
(B) the velocity and acceleration of the particle are normal to each other at $t = \frac{\pi}{2p}$.
(C) the acceleration of the particle is always directed towards the focus.
(D) the distance travelled by the particle in time interval $t = 0$ to $t = \frac{\pi}{2p}$ is a.

9. A ball starts falling freely from a height h from a point on the inclined plane forming an angle α with the horizontal as shown. After collision with the incline it rebounds elastically off the inclined plane. Then



(A) it again strikes the incline at $t = \sqrt{\frac{8h}{g}}$ after it strikes the incline at A.
(B) it again strikes the incline at $t = \sqrt{\frac{2h}{g}}$ after it strikes the incline at A.
(C) it again strikes the incline at a distance $4h\sin\alpha$ from A along the incline.
(D) it again strikes the incline at a distance $8h\sin\alpha$ from A along the incline.

10. A cart is moving along $+x$ direction with a velocity of $4\ \text{ms}^{-1}$. A person on the cart throws a stone with a velocity of $6\ \text{ms}^{-1}$ with respect to himself. In the frame of reference of the cart the stone is thrown in the y-z plane making an angle of $30°$ with the vertical z-axis. Then with respect to an observer on the ground
(A) the initial velocity of the stone is $10\ \text{ms}^{-1}$.
(B) the initial velocity of the stone is $2\sqrt{13}\ \text{ms}^{-1}$.
(C) the velocity at the highest point of its motion is zero.
(D) the velocity at the highest point of its motion is $5\ \text{ms}^{-1}$.

11. Two shells are fired from a cannon successively with speed u each at angles of projection α and β, respectively. If the time interval between the firing of shells is t and they collide in mid air after a time T from the firing of the first shell. Then
(A) $T\cos\alpha = (T-t)\cos\beta$
(B) $\alpha > \beta$
(C) $(T-t)\cos\alpha = t\cos\beta$
(D) $(u\sin\alpha)T - \frac{1}{2}gT^2 = (u\sin\beta)(T-t) - \frac{1}{2}g(T-t)^2$

12. Choose the correct alternative (s)
(A) A ball is thrown vertically up and another ball is thrown at an angle θ with the vertical such that both of them remain in air for the same period of time, then the ratio of heights attained by the two balls is $1:1$
(B) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R\tan\theta$, where θ is the angle of projection
(C) For a projectile whose range R is n times its maximum height H the angle of projection is $\tan^{-1}\left(\frac{4}{n}\right)$
(D) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is $2h$

13. A radar observer on the ground is watching an approaching projectile. At a certain instant he has the following information.
(i) The projectile has reached the maximum altitude and is moving with a horizontal velocity v;
(ii) The straight line distance of the observer to the projectile is l;
(iii) The line of sight to the projectile is at an angle θ above the horizontal.
Assuming earth to be flat and the observer lying in the plane of the projectile's trajectory then,
(A) the distance between the observer and the point of impact of the projectile is $D = \frac{v^2\sin\theta}{g} - l\cos\theta$.
(B) the distance between the observer and the point of impact of the projectile is $D = v\sqrt{\frac{2l\sin\theta}{g}} - l\cos\theta$.
(C) the projectile will pass over the observer's head for $l < \frac{2v^2\tan\theta\sec\theta}{g}$.
(D) the projectile will pass over the observer's head for $l > \frac{2v^2\tan\theta\sec\theta}{g}$.

14. A projectile has the same range R for two angles of projections. If T_1 and T_2 be the times of flight in the two cases and θ be the angle of projection corresponding to the time T_1, then

(A) $T_1T_2 \propto R^2$ (B) $\frac{T_1}{T_2} = \cot\theta$

(C) $\frac{T_1}{T_2} = \tan\theta$ (D) $T_1T_2 \propto R$

15. Two guns situated at the top of a hill of height 10 m, fire one shot each with the same speed of $5\sqrt{3}$ ms^{-1} at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in mid air at the point P. Taking the origin of the coordinate system at the foot of the hill right below the muzzle, trajectories in x-y plane and $g = 10\ ms^{-2}$, then

(A) the first shell reaches the point P at $t_1 = 1$ s from the start.

(B) the second shell reaches the point P at $t_2 = 2$ s from the start.

(C) the first shell is fired 1 s after the firing of the second shell.

(D) they collide at P whose coordinates are given by $\left(5\sqrt{3}, 5\right)$ m.

16. A projectile is thrown with an initial velocity u, at an angle of projection θ first from the equator and then from the pole. The fractional decrement in the range of projectile is

(A) $\frac{dR}{R} = -\frac{1}{291}$ (B) $\frac{dR}{R} = \frac{1}{291}$

(C) $\frac{dR}{R} = g\left(\frac{1}{g_p} - \frac{1}{g_e}\right)$ (D) $\frac{dR}{R} = -g\left(\frac{1}{g_p} - \frac{1}{g_e}\right)$

17. A boat is moving directly away from a cannon on the shore with a speed v_1. The cannon fires a shell with a speed v_2 at an angle α and the shell hits the boat. Then,

(A) the shell hits the boat when the time equal to $\frac{2v_2\sin\alpha}{g}$ is lapsed.

(B) the boat travels a distance $\frac{2v_1v_2\sin\alpha}{g}$ from its original position.

(C) the distance of the boat from the cannon at the instant the shell is fired is $\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha - v_1)$.

(D) the distance of the boat from the cannon when the shell hits the boat is $\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha)$.

18. A particle is projected from the ground with a velocity $40\sqrt{2}$ ms^{-1} which makes an angle of 45° with the horizontal. At time $t = 2$ s

(A) displacement of particle is 100 m

(B) vertical component of velocity is 20 ms^{-1}

(C) velocity makes an angle of $\tan^{-1}(2)$ with the horizontal

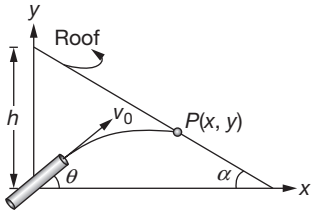
(D) particle is at height of 60 m from ground

19. A particle P lying on smooth horizontal x-y plane starts from $\left(6\hat{i} + 8\hat{j}\right)$ m with velocity $\left(2\hat{i}\right)$ ms^{-1}. Another particle Q is projected (horizontally from origin with velocity $\left(a\hat{i} + b\hat{j}\right)$ so that is strikes P after 2 s. Then

(A) $a = 2$ (B) $a = 5$

(C) $b = 4$ (D) $b = 8$

20. A projectile is fired upward with velocity v_0 at an angle θ and strikes a point $P(x, y)$ on the roof of the building (as shown). Then,



(A) the projectile hits the roof in minimum time if $\theta + \alpha = \frac{\pi}{2}$.

(B) the projectile hits the roof in minimum time if $\theta + \alpha = \frac{\pi}{4}$.

(C) the minimum time taken by the projectile to hit the roof is $t_{min} = \frac{v_0 - \sqrt{v_0^2 - 2gh\cos^2\alpha}}{g\cos\alpha}$.

(D) the projectile never reaches the roof for $v_0 < \sqrt{2gh}\cos\alpha$.

21. A particle is fired from a point on the ground with speed u making an angle θ with the horizontal. Then

(A) the radius of curvature of the projectile at the highest point is $\frac{u^2\cos^2\theta}{g}$

(B) the radius of curvature of the projectile at the highest point of launch is $\frac{u^2}{g\cos\theta}$

(C) at the point of projection tangential acceleration is $g\sin\theta$

(D) at the highest point tangential acceleration is zero

22. A shot is fired with a velocity u at an angle $(\alpha+\theta)$ with the horizon from the foot of an incline plane of angle α through the point of projection. If it hits the plane horizontally then

(A) $\tan\theta=\dfrac{\tan\alpha}{1+2\tan^2\alpha}$ (B) $\tan\theta=2\tan\alpha$

(C) $\tan\theta=\dfrac{2\tan\alpha}{1+2\tan^2\alpha}$ (D) $\tan\theta=\dfrac{\sin\alpha\cos\alpha}{1+\sin^2\alpha}$

23. Two second after projection, a projectile is travelling in a direction inclined at $30°$ to the horizon. After one more second it is travelling horizontally. Then

(A) the velocity of projection is $20\ \text{ms}^{-1}$.

(B) the velocity of projection is $20\sqrt{3}\ \text{ms}^{-1}$.

(C) the angle of projection is $30°$ with vertical.

(D) the angle of projection is $30°$ with horizon.

24. A particle is projected from point A with speed u and angle of projection is $60°$. At some instant, magnitude of velocity of particle is v and it makes an angle θ with horizontal. If radius of curvature of path of particle at the given instant is $\dfrac{8}{3\sqrt{3}}$ times minimum radius of curvature during the whole flight, then

(A) $\theta=37°$ (B) $\theta=30°$

(C) $v=\dfrac{u}{\sqrt{3}}$ (D) $v=\dfrac{u}{2\sqrt{3}}$

25. Path of a particle moving in x-y plane is $y=3x+4$. At some instant suppose x-component of velocity is $1\ \text{ms}^{-1}$ and it is increasing at a rate of $1\ \text{ms}^{-2}$. Then at this instant

(A) speed of particle is $\sqrt{10}\ \text{ms}^{-1}$

(B) acceleration of particle is $\sqrt{10}\ \text{ms}^{-2}$

(C) velocity-time graph is a straight line

(D) acceleration-time graph is a straight line

26. The co-ordinate of the particle in x-y plane are given as $x=2+2t+4t^2$ and $y=4t+8t^2$. The motion of the particle is

(A) along a straight line

(B) uniformly accelerated

(C) along a parabolic path

(D) non-uniformly accelerated

27. Velocity of a particle moving in a curvilinear path varies with time as $\vec{v}=\left(2t\hat{i}+t^2\hat{j}\right)\ \text{ms}^{-1}$, where, t is in second. At $t=1\ \text{s}$, the

(A) acceleration of particle is $8\ \text{ms}^{-2}$.

(B) tangential acceleration of particle is $\dfrac{6}{\sqrt{5}}\ \text{ms}^{-2}$.

(C) radial acceleration of particle is $\dfrac{2}{\sqrt{5}}\ \text{ms}^{-2}$.

(D) None of these

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** For an oblique projectile launched from the ground at an angle θ with the horizontal, $R=H$ at $\theta=\tan^{-1}(4)$.

Statement-2: Maximum range of projectile is proportional to square of initial velocity and inversely proportional to g.

2. **Statement-1:** The net acceleration of a particle in circular motion is always directed radially inwards.
Statement-2: Whenever a particle moves in a circular path, an acceleration exists which is directed towards the centre.

3. **Statement-1:** In uniform circular motion, acceleration is constant.
Statement-2: In uniform circular motion, speed is constant.

4. **Statement-1:** When a particle is thrown obliquely from the surface of the Earth, it always moves in a parabolic path, provided the air resistance is negligible.
Statement-2: A projectile motion is a two-dimensional motion.

5. **Statement-1:** In uniform circular motion acceleration is constant.

Statement-2: In uniform circular motion magnitude of acceleration is $\frac{v^2}{r}$ and direction is always towards the centre.

6. **Statement-1:** A particle is moving on a horizontal surface. Its path will be straight line if initial velocity and acceleration are collinear and/or either of them is zero.
Statement-2: Angle between $\vec{u}$ and $\vec{a}$ determine path therefore, it may be accelerated or retarded curvilinear.

7. **Statement-1:** When a particle moves in a circle with a uniform speed, its velocity and acceleration both changes.

Statement-2: The centripetal acceleration in circular motion is dependent on angular velocity of the body.

8. **Statement-1:** A coin is allowed to fall in a train moving with constant velocity. Its trajectory is parabola as seen by an observer sitting in the train.
Statement-2: An observer on ground will see the path of coin as a parabola.

9. **Statement-1:** A particle is projected with a velocity u making at an angle $\theta < 90°$ with the horizontal. When particle strikes the ground its speed is again u.

Statement-2: Velocity along horizontal direction remains same but velocity along vertical direction is changed. When particle strikes the ground, magnitude of final vertical velocity is equal to magnitude of initial vertical velocity.

10. **Statement-1:** Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particles will be same.
Statement-2: The maximum height of projectile is independent of the mass of the particle.

11. **Statement-1:** When speed of projection of a body is made n-times, its time of flight becomes n times
Statement-2: This is because range of projectile become n times.

12. **Statement-1:** Horizontal velocity of a particle moving under the influence of gravity remains constant.
Statement-2: Acceleration due to gravity acts vertically downwards.

13. **Statement-1:** An oblique projectile is launched from the ground to attain a maximum range. The maximum height attained by the projectile is 25% of range.

Statement-2: $R = \frac{u^2 \sin(2\theta)}{g}$ and $H = \frac{u^2 \sin^2\theta}{2g}$

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A point moves in the plane x, y according to the law $x = kt$, $y = kt(1 - \alpha t)$, where k and α are positive constants and t is the time. Based on the above facts, answer the following questions.

1. The trajectory $y(x)$ followed by the particle is a/an
(A) straight line (B) ellipse
(C) parabola (D) cycloid

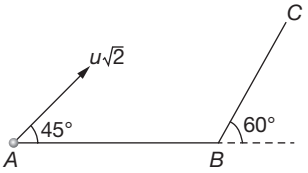
2. The velocity v of the point is minimum at time
(A) $t = \frac{1}{\alpha}$ (B) $t = \frac{1}{2\alpha}$
(C) $t = \frac{2}{\alpha}$ (D) $t = \frac{1}{3\alpha}$

3. The acceleration of the particle at time when the velocity has a minimum value is
(A) $(2k\alpha)\hat{j}$ (B) $(k\alpha)\hat{j}$
(C) $\left(\frac{k\alpha}{2}\right)\hat{j}$ (D) $-2(k\alpha)\hat{j}$

4. The moment t_0 at which the velocity vector forms an angle $\frac{\pi}{4}$ with the acceleration vector is
(A) $\frac{1}{\alpha}$ (B) $\frac{1}{2\alpha}$
(C) $\frac{2}{\alpha}$ (D) $\frac{1}{3\alpha}$

Comprehension 2

A particle is projected from a point A with velocity $u\sqrt{2}$ at an angle of $45°$ with horizontal as shown in figure. It strikes the plane BC at right angles. Based on the above facts, answer the following questions.



5. The velocity of the particle at the time of collision is

(A) u (B) $\frac{u}{2}$

(C) $\frac{\sqrt{3}u}{2}$ (D) $\frac{2u}{\sqrt{3}}$

6. The time after which collision takes place is

(A) $\frac{u}{g}$ (B) $\frac{\sqrt{3}u}{g}$

(C) $\frac{u}{g}\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ (D) $\left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)\frac{u}{g}$

Comprehension 3

An oblique projectile is launched from a point O with an initial velocity u that makes an angle θ with the horizontal. It is observed that the projectile attains a maximum height H at the point $\left(\frac{R}{2}, H\right)$, where R is the maximum distance from O at which the projectile strikes the ground. Now, if the velocity of the projectile is $\vec{v} = \left(20\hat{i} + 10\hat{j}\right)$ ms^{-1} when it is at a height of 15 m above the ground, then based on this information and taking $g = 10$ ms^{-2}, answer the following questions.

7. The launch speed in ms^{-1} is

(A) 20 ms^{-1} (B) $20\sqrt{2}$ ms^{-1}

(C) 40 ms^{-1} (D) $40\sqrt{2}$ ms^{-1}

8. The value of θ (in radian) is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

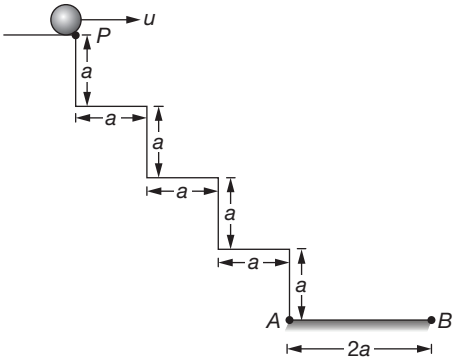
9. The coordinate, where the maximum height is attained is

(A) $(80, 20)$ m (B) $(20, 40)$ m

(C) $(20, 80)$ m (D) $(40, 20)$ m

Comprehension 4

Consider the situation shown in the figure. A particle has to be projected from point P in horizontal direction. It is required for the particle to strike the plane AB (see figure).



10. The minimum value of horizontal velocity of particle at point P so that particle may strike the plane AB is

(A) $\sqrt{\frac{9}{8}ag}$ (B) $\sqrt{\frac{25}{8}ag}$

(C) $\sqrt{\frac{1}{2}ag}$ (D) $\sqrt{3ag}$

11. The time taken by particle in going from point P to plane AB is

(A) $4\sqrt{\frac{a}{g}}$ (B) $\sqrt{\frac{4a}{g}}$

(C) $\sqrt{\frac{2a}{g}}$ (D) $\sqrt{\frac{8a}{g}}$

12. The maximum value of horizontal velocity of particle at point P so that it may strike the plane AB is

(A) $\sqrt{\frac{9}{8}ag}$ (B) $\sqrt{\frac{25}{8}ag}$

(C) $\sqrt{\frac{1}{2}ag}$ (D) $\sqrt{3ag}$

Comprehension 5

A ball is projected horizontally from a height of 100 m from the ground with a speed of 20 ms^{-1}. Taking $g = 10$ ms^{-2} and based on the information provided, answer the following questions.

13. The time taken to reach the ground is

(A) $\sqrt{5}$ s (B) $2\sqrt{5}$ s

(C) $3\sqrt{5}$ s (D) $4\sqrt{5}$ s

14. The horizontal distance it covers before striking the ground is

(A) $20\sqrt{5}$ m (B) $40\sqrt{5}$ m

(C) $60\sqrt{5}$ m (D) $80\sqrt{5}$ m

15. The angle with the vertical which it strikes the ground is

(A) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(C) $\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$ (D) $\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$

Comprehension 6

For a particle moving in the x-y plane the x, y coordinates as a function of time are given by $x = 6t$ and $y = 8t - 5t^2$, where x and y are in metre and t is in second. Assume no air drag, answer the following questions. Based on the above facts, answer the following questions.

16. Select the correct statement from the following

(A) The particle will follow a parabolic path to have a projectile motion with an initial velocity of $5\ \text{ms}^{-1}$ and an acceleration of $3\ \text{ms}^{-2}$ acting along $-y$ axis

(B) The particle will follow a parabolic path to have a projectile motion with an initial velocity of $6\ \text{ms}^{-1}$ and a constant acceleration of $5\ \text{ms}^{-2}$ acting along $-y$ axis

(C) The particle will follow a parabolic path to have a projectile motion with an initial horizontal velocity of $6\ \text{ms}^{-1}$ and a constant acceleration of $10\ \text{ms}^{-2}$ acting along $-y$ axis

(D) None of the above statement seems to be correct

17. The velocity, of the projectile, along x-axis after 0.2 second is

(A) $> 6\ \text{ms}^{-1}$ (B) $< 6\ \text{ms}^{-1}$

(C) $6\ \text{ms}^{-1}$ (D) zero

18. The initial vertical velocity is

(A) $6\ \text{ms}^{-1}$ (B) $7\ \text{ms}^{-1}$

(C) $8\ \text{ms}^{-1}$ (D) $10\ \text{ms}^{-1}$

19. The velocity of projection of the projectile is

(A) $6\ \text{ms}^{-1}$ (B) $7\ \text{ms}^{-1}$

(C) $8\ \text{ms}^{-1}$ (D) $10\ \text{ms}^{-1}$

20. The time of ascent of the projectile is

(A) 0.2 s (B) 0.4 s

(C) 0.6 s (D) 0.8 s

21. The maximum height attained by the projectile is

(A) 0.8 m (B) 1.6 m

(C) 2.9 m (D) 3.2 m

22. The horizontal range of the projectile is

(A) 3.2 m (B) 4.9 m

(C) 8.7 m (D) 9.6 m

Comprehension 7

The maximum height attained by an oblique projectile is 8 m and the horizontal range is 24 m. Based on the above facts, answer the following questions.

23. The vertical component of the velocity of projection is

(A) $\sqrt{g}$ (B) $2\sqrt{g}$

(C) $3\sqrt{g}$ (D) $4\sqrt{g}$

24. The horizontal component of the velocity of projection is

(A) $2\sqrt{g}$ (B) $3\sqrt{g}$

(C) $4\sqrt{g}$ (D) $5\sqrt{g}$

25. The velocity of projection is

(A) $2\sqrt{g}$ (B) $3\sqrt{g}$

(C) $4\sqrt{g}$ (D) $5\sqrt{g}$

26. The angle of projection is

(A) $\cos^{-1}(0.8)$ (B) $\sin^{-1}(0.8)$

(C) $\tan^{-1}(0.6)$ (D) $\cot^{-1}(0.8)$

Comprehension 8

A particle initially at rest and starting from the origin is moving under the influence of acceleration given by $\vec{a} = \left(6t\hat{i} + 8t\hat{j}\right)\text{ms}^{-2}$. Based on the above facts, answer the following questions.

27. Velocity of particle at $t = 3$ s

(A) $45\ \text{ms}^{-1}$ (B) $40\ \text{ms}^{-1}$

(C) $35\ \text{ms}^{-1}$ (D) $22\ \text{ms}^{-1}$

28. Displacement of particle at $t = 3$ s is

(A) 28 m (B) 30 m

(C) 35 m (D) 45 m

29. Path of particle will be

(A) Straight line (B) Parabola

(C) Circle (D) None of these

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

1. A particle is launched with an initial velocity of $20\sqrt{2}$ ms^{-1} making an angle of $45°$ with the horizontal. Based on this information and $g = 10$ ms^{-2} match the contents of **COLUMN-I** with their counterparts in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Magnitude of average velocity, in ms^{-1}, at $t = 1$ s.	(p) $25\sqrt{5}$
(B) Magnitude of average acceleration, in ms^{-2}, at $t = 2$ s.	(q) $80\sqrt{2}$
(C) Radius of curvature, in m, at $t = 0$ s.	(r) 25
(D) Radius of curvature, in m, at $t = 1$ s.	(s) 40
(E) Radius of curvature, in m, at $t = 2$ s.	(t) 10

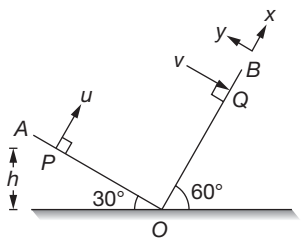
2. A particle is moving in a circle such that its speed varies with time t as $v = (2t)$ ms^{-1}. The quantities in **COLUMN-I** are at $t = 2$ s against their values mentioned in **COLUMN-II**. Match them correctly.

COLUMN-I	COLUMN-II
(A) Distance travelled	(p) 2
(B) Displacement	(q) $\sin(2)$
(C) Average speed	(r) 4
(D) Average velocity	(s) $2\sin(2)$
	(t) None of these

3. A particle is moving in a curvilinear path such that its velocity $\vec{v}$, in ms^{-1}, at any instant of time t, in second, is given by $\vec{v} = 2t\hat{i} + t^2\hat{j}$. Match the quantities in **COLUMN-I** calculated at $t = 1$ s, with the respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Tangential acceleration, in ms^{-2}	(p) $2\sqrt{2}$
(B) Radial acceleration, in ms^{-2}	(q) $\frac{5\sqrt{5}}{2}$
(C) Acceleration, in ms^{-2}	(r) $\frac{6}{\sqrt{5}}$
(D) Radius of curvature, in m	(s) $\frac{2}{\sqrt{5}}$
	(t) None of these

4. Two inclined planes OA and OB having inclinations $30°$ and $60°$ with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}$ ms^{-1} along a direction perpendicular to plane OA. If the particle strikes plane OB normally at Q. Based on the information provided and taking $g = 10$ ms^{-2}, match the quantities in **COLUMN-I** with the respective values in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Time of flight, in s from P to Q.	(p) 5
(B) Velocity, in ms^{-1}, with which the particle strikes the plane OB.	(q) 2
(C) Vertical height, in m, of the point P above O	(r) 20
(D) Separation PQ, in m	(s) 10
	(t) None of these

5. A ball launched with some initial velocity from the origin moves in x-y plane such that x and y vary with time t as $x = \alpha t$ and $y = \alpha t(1 - \beta t)$ where α and β are positive constants. Based on this information match the quantities in **COLUMN-I** with the respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) The maximum horizontal distance travelled, as a multiple of $\frac{\alpha}{2\beta}$	(p) 1
(B) The maximum vertical displacement attained, as a multiple of $\frac{\alpha}{16\beta}$	(q) 2
(C) The time taken by the ball to hit the x-axis again, as a multiple of $\frac{1}{8\beta}$	(r) 4
(D) The acceleration of the ball, in magnitude, as a multiple of $\frac{\alpha\beta}{8}$	(s) 8
(E) The velocity of the ball at half the value of time calculated in (C), as a multiple of α	(t) 16

6. Match the following

COLUMN-I	COLUMN-II
(A) Uniform motion	(p) Projectile motion
(B) Uniform accelerated motion	(q) Uniform circular motion
(C) Non uniform accelerated motion	(r) Motion along a straight line
(D) Uniform velocity	(s) Motion along ellipse

7. Two projectiles are launched with same initial speed from ground at angles $30°$ and $60°$. If R_1 is range of first and R_2 is range of second, similarly H_1 and H_2 are their maximum heights and T_1 and T_2 are time of flights, then match the ratios in **COLUMN-I** to the values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $\frac{R_1}{R_2}$	(p) $\frac{1}{3}$
(B) $\frac{H_1}{H_2}$	(q) 1
(C) $\frac{T_2}{T_1}$	(r) $\sqrt{3}$
(D) $\frac{T_1H_1R_1}{T_2H_2R_2}$	(s) $\frac{1}{3\sqrt{3}}$

8. A body is projected with speed $20\sqrt{2}$ ms^{-1} at an angle $45°$ with horizontal. After 1 s of it motion match the following columns. $(g = 10\ ms^{-2})$.

COLUMN-I	COLUMN-II
(A) Average velocity (in magnitude)	(p) $10\sqrt{5}$ ms^{-1}
(B) Change in velocity (in magnitude)	(q) 25 ms^{-1}

(*Continued*)

COLUMN-I	COLUMN-II
(C) Instantaneous speed	(r) $10\ \text{ms}^{-1}$
(D) Change in speed (nearly) (in magnitude)	(s) $6\ \text{ms}^{-1}$

9. Trajectory of particle launched obliquely from the ground is given as $y = x - \frac{x^2}{80}$, where, x and y are in metre. For this projectile motion match the following if $g = 10\ \text{ms}^{-2}$.

COLUMN-I	COLUMN-II
(A) Angle of projection	(p) 20 m
(B) Angle of velocity with horizontal after 4 s	(q) 80 m
(C) Maximum height	(r) 45°
(D) Horizontal range	(s) $\tan^{-1}\left(\frac{1}{2}\right)$

10. Match the following

COLUMN-I	COLUMN-II
(A) Particle moving in circle	(p) $\vec{a}$ may be perpendicular to $\vec{v}$
(B) Particle moving in straight line	(q) $\vec{a}$ may be in the direction of $\vec{v}$
(C) Particle undergoing projectile motion	(r) $\vec{a}$ may make same acute angle with $\vec{v}$
(D) Particle moving into space	(s) $\vec{a}$ may be opposite velocity

11. A body is projected from the ground with velocity v at an angle of projection θ. Then match the following.

COLUMN-I	COLUMN-II
(A) Change in momentum	(p) Remains unchanged
(B) Angle at the highest point	(q) Independent of projected velocity
(C) Kinetic energy of body	(r) At highest point is zero
(D) Horizontal component of velocity	(s) Minimum at highest point

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $30°$ and the angle of the barrel to the horizontal $60°$. The initial velocity of the shell is $21\ \text{ms}^{-1}$. Find the distance, in metre, from the gun to the point at which the shell falls.

2. A particle is projected with velocity $2\sqrt{gh}$ so that it just clears two walls of equal height h which are at a distance $2h$ from each other. The time of passing between the walls is $*\sqrt{\frac{h}{g}}$, where $*$ is not readable. Find $*$.

3. On a cricket field, the batsman is at the origin of co-ordinates and a fielder stands in position $\left(46\hat{i} + 28\hat{j}\right)$ m. The batsman hits the ball so that it rolls along the ground with constant velocity $\left(7.5\hat{i} + 10\hat{j}\right)\ \text{ms}^{-1}$. The fielder can run with a speed of $5\ \text{ms}^{-1}$. If he starts to run immediately the ball is hit what is the shortest time, in seconds, in which he could intercept the ball?

4. A particle is projected from a point at the foot of a fixed plane, inclined at an angle of $45°$ to the horizontal, in the vertical plane containing the line of greatest slope through the point. If the particle strikes the plane

horizontally and $\phi(>45°)$ is the angle of launch measured to the horizontal, then find the value of $\tan\phi$.

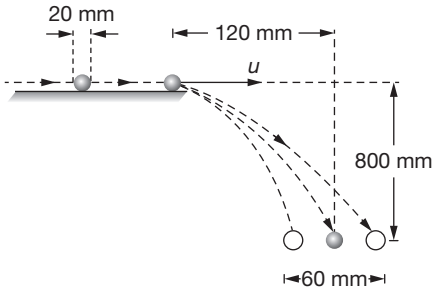
5. A particle is moving in a circle of radius $\frac{1}{4}$ m with a linear speed of $2\ \text{ms}^{-1}$. Calculate the angular speed of a particle, in rads^{-1}.

6. A boy whirls a stone in a horizontal circle of radius 0.5 m and at height 20 m above level ground. The string breaks, and the stone flies off horizontally to strike the ground after travelling a horizontal distance of 10 m. Find the magnitude of the centripetal acceleration, in ms^{-2}, of the stone while in circular motion. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

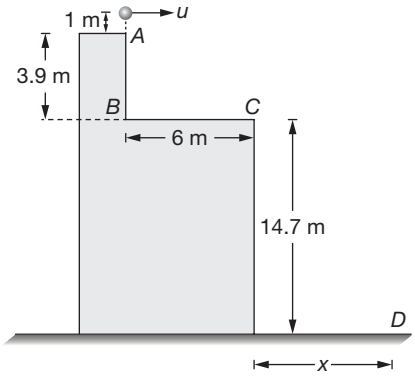
7. A highway curve is designed such that the cars travelling at a constant speed of $25\ \text{ms}^{-1}$ must not have an acceleration that exceeds $3\ \text{ms}^{-2}$. Determine the minimum radius of curvature, in metre, of the curve to the nearest integer.

8. At a given instant, a car travels along a circular curved road with a speed of $20\ \text{ms}^{-1}$ while decreasing its speed at the rate of $3\ \text{ms}^{-2}$. If the magnitude of the car's acceleration is $5\ \text{ms}^{-2}$, determine the radius of curvature of the road, in metre.

9. Ball bearings of diameter 20 mm leave the horizontal with a velocity of magnitude u and fall through the 60 mm diameter hole at a depth of 800 mm as shown. Calculate the permissible range of u, in cms^{-1} which will enable the ball bearings to enter the hole. Take the dotted positions to represent the limiting conditions. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$



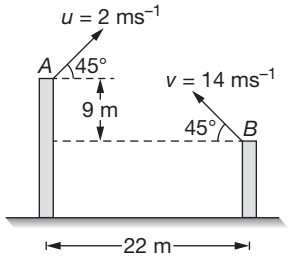
10. A ball is launched by a man standing on the top of a building who holds the ball at a distance 1 m above the edge A. Calculate the minimum velocity u, in ms^{-1}, along the horizontal such that the ball just clears the edge C. Also find x, in metre, where the ball strikes the ground. Take $g = 9.8\ \text{ms}^{-2}$.



11. A particle is projected from a point at the foot of a fixed plane, inclined at an angle of $45°$ to the horizontal, in the vertical plane containing the line of greatest slope through the point. If the particle strikes the plane at right angles and $\phi(>45°)$ is the angle of launch measured to the horizontal, then find the value of $\tan\phi$.

12. The minimum speed in ms^{-1} with which a projectile must be thrown from origin at ground so that it is able to pass through a point $P(30\ \text{m}, 40\ \text{m})$ is $\left(g = 10\ \text{ms}^{-2}\right)$

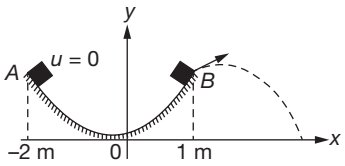
13. Two particles are simultaneously thrown from top of two towers as shown. Their velocities are $2\ \text{ms}^{-1}$ and $14\ \text{ms}^{-1}$. Horizontal and vertical separation between these particles are 22 m and 9 m, respectively. Then the minimum separation between the particles in process of their motion in meters is $\left(g = 10\ \text{ms}^{-2}\right)$.



14. A projectile is fixed at an angle $60°$ with horizontal. Ratio of initial and final kinetic energy velocity vector of projectile makes an angle $15°$ with velocity of projection is

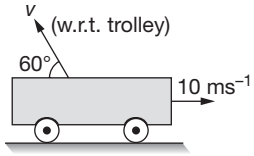
15. A particle is projected from the bottom of an inclined plane of inclination $30°$. At what angle α (from the horizontal), in degree, should the particle be projected to get the maximum range on the inclined plane?

16. A small body is released from point A of smooth parabolic path $y = x^2$, where y is vertical axis and x is horizontal axis at ground as shown. The body leaves the surface from point B. If $g = 10\ \text{ms}^{-2}$, then total horizontal distance in meters travelled by body before it hits ground is _____



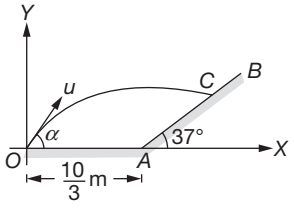
17. A particle is projected towards north with speed $20\ \text{ms}^{-1}$ at an angle $45°$ with horizontal. Ball get horizontal acceleration of $7.5\ \text{ms}^{-2}$ towards east due to wind. Range of ball (in meter) minus 42 m will be

18. For an observer on trolley, direction of projection of particle is shown in figure, while for observer on ground ball rises vertically. Maximum height (in meter) reached by ball minus 10 m is _____

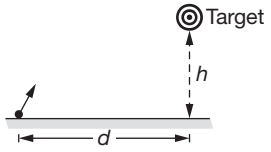


19. Two seconds after projection, a projectile is travelling in a direction inclined at $30°$ with horizontal. After one more second, it is travelling horizontally. One tenth of the angle of projection (in degree) with horizontal is _____

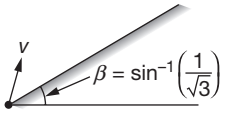
20. A particle is projected from O on the ground with velocity $u = 5\sqrt{5}\ \text{ms}^{-1}$ at angle $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$. It strikes at a point C on a fixed plane AB having inclination of $37°$ with horizontal as shown, then the x-coordinate of point C in meters is $\left(g = 10\ \text{ms}^{-2}\right)$



21. A particle is projected from ground with minimum speed required to hit a target at a height $h = 10$ m at a horizontal distance $d = \sqrt{300}$ m as shown. Then find the time taken by particle (in seconds) to hit the target. $\left(g = 10\ \text{ms}^{-2}\right)$



22. A particle is projected with initial velocity $v = 10\sqrt{2}\ \text{ms}^{-1}$ as shown. After elastic collision with the inclined plane, the particle rebounds normally with the plane and retraces its path to come back at its point of projection. Then find the time in seconds in which particle returns to the point of projection. $\left(g = 10\ \text{ms}^{-2}\right)$

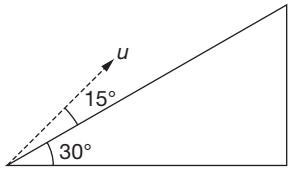


23. A football is thrown with a velocity of $10\ \text{ms}^{-1}$ at an angle of $30°$ above the horizontal. What will the time of flight, in seconds? $\left(g = 10\ \text{ms}^{-2}\right)$

ARCHIVE: JEE MAIN

1. **[Online April 2019]**

A plane is inclined at an angle $30°$ with respect to the horizontal. A particle is projected with a speed $2\ \text{ms}^{-1}$, from the base of the plane, making an angle $15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to (Take $g = 10\ \text{ms}^{-2}$)



(A) 18 cm (B) 20 cm

(C) 14 cm (D) 26 cm

2. **[Online April 2019]**
A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is

(A) $\frac{R}{2g}$ (B) $\frac{2R}{g}$

(C) $\frac{R}{g}$ (D) $\frac{R}{4g}$

3. **[Online April 2019]**
The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then $\left(g = 10\ \text{ms}^{-2}\right)$

(A) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\ \text{ms}^{-1}$

(B) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ \text{ms}^{-1}$

(C) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\ \text{ms}^{-1}$

(D) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ \text{ms}^{-1}$

4. **[Online April 2019]**
Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct?

(A) $R^2 = 4h_1h_2$ (B) $R^2 = 16h_1h_2$

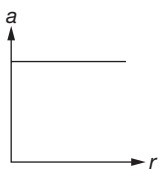
(C) $R^2 = 2h_1h_2$ (D) $R^2 = h_1h_2$

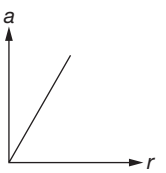
5. **[Online January 2019]**
Two guns A and B can fire bullets at speeds $1\ \text{kms}^{-1}$ and $2\ \text{kms}^{-1}$ respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is

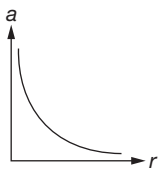
(A) 1 : 4 (B) 1 : 8

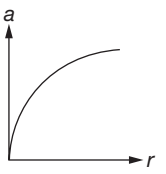
(C) 1 : 2 (D) 1 : 16

6. **[Online 2015]**
If a body moving in a circular path maintains constant speed of $10\ \text{ms}^{-1}$, then which of the following correctly describes relation between acceleration and radius?

(A) 

(B) 

(C) 

(D) 

7. **[2013]**
A projectile is given an initial velocity of $\left(\hat{i} + 2\hat{j}\right)\ \text{ms}^{-1}$, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ \text{ms}^{-2}$, the equation of its trajectory is

(A) $4y = 2x - 25x^2$ (B) $y = x - 5x^2$

(C) $y = 2x - 5x^2$ (D) $4y = 2x - 5x^2$

8. **[2012]**
A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

(A) 10 m (B) $10\sqrt{2}$ m

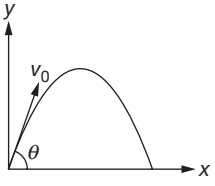
(C) 20 m (D) $20\sqrt{2}$ m

9. **[2011]**
A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(A) $\pi\frac{v^2}{g}$ (B) $\pi\frac{v^4}{g^2}$

(C) $\frac{\pi}{2}\frac{v^4}{g^2}$ (D) $\pi\frac{v^2}{g^2}$

10. **[2010]**
A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(A) $\frac{1}{2}mgv_0t^2\cos\theta\hat{i}$ (B) $-mgv_0t^2\cos\theta\hat{j}$

(C) $mgv_0t\cos\theta\hat{k}$ (D) $-\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

11. [2010]

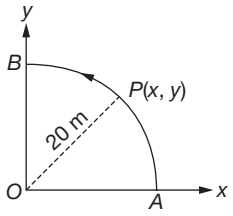
For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R,\theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

(A) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$ (B) $-\frac{v^2}{R}\cos\theta\hat{i}+\frac{v^2}{R}\sin\theta\hat{j}$

(C) $-\frac{v^2}{R}\sin\theta\hat{i}+\frac{v^2}{R}\cos\theta\hat{j}$ (D) $-\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$

12. [2010]

A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s=t^3+5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t=2$ s is nearly



(A) 14 ms^{-2} (B) 13 ms^{-2}

(C) 12 ms^{-2} (D) 7.2 ms^{-2}

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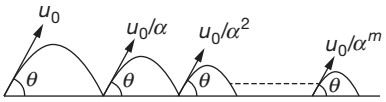
Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2019]

A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of $\frac{u_0}{\alpha}$. Its motion continues for a long time as shown in Figure.



If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is ……..

2. [JEE (Advanced) 2018]

A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is _____.

3. [JEE (Advanced) 2011]

A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 ms^{-1}, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in ms^{-2}, is

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Curvilinear Motion)

3. 8 rads^{-1}

4. 2 rads^{-2}

5. 9.5 ms^{-2}

6. 50 ms^{-2}

7. 20 s, 0.32 ms^{-2}

8. 1.8 ms^{-1}, 1.2 ms^{-2}

9. 38.7 ms^{-1}

10. 63 ms^{-1}

11. 0.488 ms^{-2}

12. 208 m

13. 100 m

14. 3.2 m

Test Your Concepts-II (Based on Horizontal Projectile)

1. 44.1 m, 29.4 ms^{-1}

2. 20 ms^{-1}

3. 0.7 s, 7 m

4. 14 ms^{-1}, 2.4 m

5. 34 ms^{-1}

6. 6 ms^{-1}, $(1+3.9+14.7)=\frac{1}{2}(9.8)t^2$, 6 m

7. $u_{\text{MIN}}=25\ \text{cms}^{-1}$ and $u_{\text{MAX}}=35\ \text{cms}^{-1}$

Test Your Concepts-III (Based on Oblique Projectile)

1. 76°

6. (a) 11 m

(b) 23 m

(c) 17 ms^{-1}, 63° with horizontal

7. 3.16 ms^{-1}

9. (a) 5 s

(b) 130 m

(c) 43.6 ms^{-1}

12. $t=\dfrac{uv\sin(\alpha-\beta)}{(v\cos\beta-u\cos\alpha)}$

13. 4.5 m, 2.75 m

15. 68.2°, 1.253 ms^{-1}

16. 28 ms^{-1}

17. $\dfrac{gt\cos\beta}{2\sin(\alpha-\beta)}$

21. $\tan\alpha=\dfrac{g(R_1-R_2)^2}{4v^2(R_1+R_2)}$

Test Your Concepts-IV (Based on Projectile on an Inclined Plane)

1. (a) 2

(b) 3

3. $\dfrac{2u^2\tan\theta\sec\theta}{g}$

4. $\sqrt{\dfrac{2gh}{2+\cot^2\theta}}$

5. $\sqrt{\dfrac{gR(1+3\sin^2\beta)}{2\sin\beta}}$

6. $\alpha+\beta=\dfrac{\pi}{2}$

Single Correct Choice Type Questions

1. D	**2.** D	**3.** D	**4.** D	**5.** D	**6.** B	**7.** A	**8.** C	**9.** A	**10.** D
11. A	**12.** A	**13.** C	**14.** B	**15.** C	**16.** D	**17.** B	**18.** B	**19.** A	**20.** D
21. C	**22.** D	**23.** D	**24.** D	**25.** D	**26.** C	**27.** A	**28.** A	**29.** D	**30.** B
31. C	**32.** D	**33.** B	**34.** A	**35.** D	**36.** D	**37.** B	**38.** A	**39.** D	**40.** A

41. B	**42.** C	**43.** C	**44.** D	**45.** C	**46.** C	**47.** B	**48.** A	**49.** C	**50.** C
51. D	**52.** D	**53.** C	**54.** D	**55.** B	**56.** C	**57.** B	**58.** D	**59.** B	**60.** B
61. A	**62.** D	**63.** C	**64.** C	**65.** D	**66.** C	**67.** B	**68.** C	**69.** D	**70.** D
71. B	**72.** B	**73.** C	**74.** A	**75.** D	**76.** C	**77.** D	**78.** A	**79.** C	**80.** C
81. A	**82.** D	**83.** D	**84.** B	**85.** C	**86.** C	**87.** B	**88.** C	**89.** A	**90.** B
91. C	**92.** B	**93.** B	**94.** C	**95.** A	**96.** C	**97.** B	**98.** D	**99.** D	**100.** C
101. C	**102.** A	**103.** D	**104.** B	**105.** D	**106.** C	**107.** B	**108.** C	**109.** C	**110.** D
111. B	**112.** B	**113.** A	**114.** A	**115.** D	**116.** C	**117.** B	**118.** B	**119.** B	**120.** D
121. C	**122.** C	**123.** B	**124.** C	**125.** D	**126.** D				

Multiple Correct Choice Type Questions

1. B, C, D	**2.** A, C	**3.** C, D	**4.** A, D	**5.** A, D
6. C, D	**7.** A, B, C, D	**8.** A, B, C	**9.** A, D	**10.** B, D
11. A, B, D	**12.** A, B, C, D	**13.** B, C	**14.** C, D	**15.** A, B, C, D
16. B, D	**17.** A, B, C, D	**18.** A, B, D	**19.** B, C	**20.** A, C, D
21. A, B, C, D	**22.** A, D	**23.** B, C	**24.** B, C	**25.** A, B
26. A, B	**27.** B, C			

Reasoning Based Questions

1. B	**2.** D	**3.** D	**4.** D	**5.** D	**6.** A	**7.** B	**8.** D	**9.** A	**10.** A
11. C	**12.** A	**13.** A							

Linked Comprehension Type Questions

1. C	**2.** B	**3.** D	**4.** A	**5.** D	**6.** C	**7.** B	**8.** B	**9.** D	**10.** A
11. D	**12.** B	**13.** B	**14.** B	**15.** C	**16.** C	**17.** C	**18.** C	**19.** D	**20.** D
21. D	**22.** D	**23.** D	**24.** B	**25.** D	**26.** B				

Matrix Match/Column Match Type Questions

1. A → (r)	B → (t)	C → (q)	D → (p)	E → (s)
2. A → (r)	B → (s)	C → (p)	D → (q)	
3. A → (r)	B → (s)	C → (p)	D → (q)	
4. A → (q)	B → (s)	C → (p)	D → (r)	
5. A → (q)	B → (r)	C → (s)	D → (t)	E → (p)
6. A → (q, r)	B → (p, r)	C → (q, r, s)	D → (r)	
7. A → (q)	B → (p)	C → (r)	D → (s)	
8. A → (q)	B → (r)	C → (p)	D → (s)	
9. A → (r)	B → (r)	C → (p)	D → (q)	
10. A → (p, r)	B → (q, s)	C → (p, q, r, s)	D → (p, q, r, s)	
11. A → (q)	B → (r)	C → (s)	D → (p)	

Integer/Numerical Answer Type Questions

1. 30	**2.** 2	**3.** 4	**4.** 2	**5.** 8
6. 50	**7.** 208	**8.** 100	**9.** Min. 25 & Max. 35	**10.** 6, 6
11. 3	**12.** 30	**13.** 6	**14.** 2	**15.** 60
16. 8	**17.** 8	**18.** 5	**19.** 6	**20.** 5
21. 2	**22.** 6	**23.** 1		

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1. B	**2.** B	**3.** C	**4.** B	**5.** D	**6.** C	**7.** C	**8.** C	**9.** B	**10.** D
11. D	**12.** A								

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Integer/Numerical Answer Type Questions

1. 4.00	**2.** 30	**3.** 5

CHAPTER

6 Newton's Laws of Motion

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Newton's Laws of Motion
(b) Pseudo Force
(c) Friction
(d) Dynamics of Circular Motion

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

DYNAMICS: AN INTRODUCTION

In kinematics, we studied the motion of a particle, with emphasis on motion along a straight line and motion in a plane and simply described it in terms of vectors $\vec{r}$, $\vec{v}$ and $\vec{a}$. Now comes the time when we shall be discussing about the cause producing motion. This treatment, happens to be an aspect of mechanics, known as **dynamics**. In this chapter, we shall be primarily discussing the forces along with their respective nature and properties that account for the motion of a body. As done before, the bodies will be treated as if they were single particles. However, in the later chapters, we shall be extending the properties of this chapter to discuss the motion of a group of particles and extended bodies as well. So, this branch of Physics dealing with motion along with the cause producing motion is called **Dynamics**.

FORCE

Force is a pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity. Its SI unit is newton (N) and cgs unit is dyne

$$1\ \text{N} = 10^5\ \text{dyne}$$

A force acting on a body

(a) may change only speed.
(b) may change only direction of motion.
(c) may change both the speed and direction of motion.
(d) may change size and shape of a body.

Unit of Force

SI unit of force is newton (N) and $1\ \text{N} = 1\ \text{kgms}^{-2}$
cgs unit of force is dyne (dyn) and $1\ \text{dyne} = 1\ \text{gcms}^{-2}$
Also, $1\ \text{newton} = 10^5$ dyne .

The dimensional formula of force is $[MLT^{-2}]$

Another commonly used unit of force is **kilogram force (kgf).** It is the force with which a body of mass 1 kg is attracted towards the centre of the

earth. So 1 kgf can also be thought as the weight corresponding to a body of mass 1 kg is the weight

$$1 \text{ kgf} = 9.8 \text{ N} \text{ and } 1 \text{ gf} = 980 \text{ dyne}$$

Conceptual Note(s)

A force is a vector quantity. Since we know that a vector quantity is a quantity which has both magnitude and direction. To fully describe the force acting upon an object, you must describe both the magnitude (size or numerical value) and the direction. Thus, 10 N, is not a full description of the force acting upon an object. In contrast, 10 N, downwards is a complete description of the force acting upon an object because, both the magnitude (10 N) and the direction (downwards) are given.

NEWTON'S LAWS OF MOTION

Newton gave three laws of motion, based on which motion associated with a particle can be explained easily.

(a) The First Law or The Law of Inertia

(b) The Second Law $(\vec{F} = m\vec{a})$

(c) The Third Law or The Action - Reaction Law

NEWTON'S FIRST LAW OR LAW OF INERTIA

A body continues to maintain its state of rest or of uniform motion or of direction unless and until some external unbalanced force acts on it. If there is an interaction between the body and objects present in the environment, the effect may be to change the "natural" state of the body's motion.

EXAMPLES:

(a) When a bus suddenly starts/stops, the passengers tends to move backward/forward.

(b) A carpet is beaten with a stick to remove its dust.

(c) A coin kept on a cardboard on a glass tumbler, on striking hard the cardboard, the coin falls into the glass tumbler.

(d) Athlete runs some distance before making a jump.

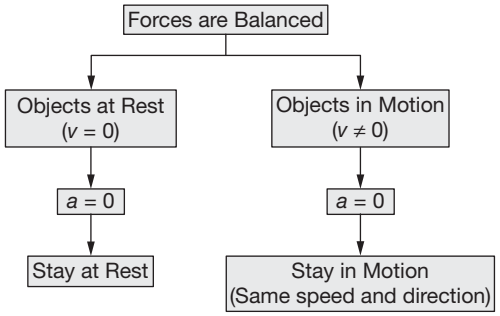
In other words, an object at rest tend to stay at rest and an object in motion tend to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

The behaviour of all objects can be described by saying that objects tend to keep on doing what they are doing (unless acted upon by an unbalanced force). There are two parts of the statement to Newton's First Law.

(a) one which predicts the **behaviour of stationary objects** and

(b) the other which predicts the **behaviour of moving objects**.

These two parts are summarized in the following diagram.



Conceptual Note(s)

It has been a common observation that every object **resists** any effort to change its velocity (both in magnitude and direction). This property expressing the degree of insusceptibility of a body to any change in its velocity is called the **property of inertia**. Different bodies reveal this property in different degrees. A measure of inertia is provided by the quantity called **mass**. A body possessing a greater mass has more inertia i.e., if equal forces are applied on two different bodies, the body with the greater mass possesses smaller acceleration.

MOMENTUM ($\vec{p}$)

It is observed that more the mass of the body, the more is the momentum possessed by it. Similarly, the more the velocity possessed by a body, the more is its momentum. In simpler words, actually the momentum is a measure of the amount of motion possessed by a body. It is defined as the **product of mass and velocity** or **momentum** is just equal to the **mass times velocity**. The momentum is always

directed along the velocity of the body whose momentum is being measured. Mathematically,

$$\vec{p} = m\vec{v}$$

It is a vector quantity with SI unit $kgms^{-1}$ and dimensional formula MLT^{-1}.

NEWTON'S SECOND LAW OF MOTION

The external force applied on a body is equal to the rate of change of momentum of a body.

$$\vec{F}_{ext} = \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

For constant mass system(s)

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

For variable mass system(s)

$$\vec{F} = \vec{v}\frac{dm}{dt} + m\frac{d\vec{v}}{dt}$$

Special Case

If $\vec{v} = \text{constant}$, then $\frac{d\vec{v}}{dt} = \vec{0}$

$$\Rightarrow \quad |\vec{F}| = v\frac{dm}{dt}$$

$$\text{Thrust} = |\vec{F}| = \left|v\frac{dm}{dt}\right|$$

e.g., Rocket propulsion

ILLUSTRATION 1

The velocity of a particle of mass 2 kg is given by $\vec{v} = at\hat{i} + bt^2\hat{j}$. Find the force acting on the particle.

SOLUTION

From Second Law of Motion

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$\Rightarrow \quad F = 2\frac{d}{dt}\left(at\hat{i} + bt^2\hat{j}\right)$$

$$\Rightarrow \quad \vec{F} = 2a\hat{i} + 4bt\hat{j}$$

Conceptual Note(s)

(a) The Second Law is obviously consistent with the First Law as $F = 0$ implies $a = 0$.

(b) The Second Law of motion is a vector law. It is actually a combination of three equations, one for each component of the vectors.

$$F_x = \frac{dp_x}{dt} = ma_x$$

$$\Rightarrow \quad F_y = \frac{dp_y}{dt} = ma_y$$

$$\Rightarrow \quad F_z = \frac{dp_z}{dt} = ma_z$$

This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

(c) The Second Law of motion given above is strictly applicable to a single point mass. The force F in the law stand for the net external force on the particle and a stands for the acceleration of the particle. Any internal forces in the system are not to be included in F.

NEWTON'S THIRD LAW OF MOTION

To every action there is equal and opposite reaction **and both must act on two different bodies**. In other words, all interaction forces exist in pair, a pair which has two forces of equal magnitude, opposite direction and acting on different bodies along a straight line connecting them. Also the two forces comprising the pair have the same nature.

i.e., $\vec{F}_{AB} = -\vec{F}_{BA}$ …(1)

$\vec{F}_{AB}$ = Force on body A due to B

$\vec{F}_{BA}$ = Force on body B due to A

From (1), we get

$$|\vec{F}_{AB}| = |\vec{F}_{BA}|$$

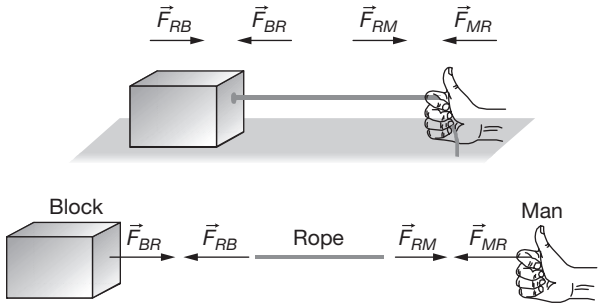
$$\Rightarrow \quad m_A a_A = m_B a_B$$

$$\Rightarrow \quad a_B = \frac{m_A a_A}{m_B}$$

If $m_A \ll m_B$, then $a_B \to 0$ (Think of apple-earth example)

EXAMPLE:

A block is being pulled by means of a rope. I have characterised some action-reaction pairs and have shown them in figures. Please see these figures carefully.



We also have, $\vec{F}_{BR} = -\vec{F}_{RB}$ and $\vec{F}_{RM} = -\vec{F}_{MR}$

Conceptual Note(s)

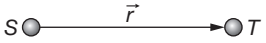
Action reaction forces always act on different bodies and their line of action is the same.

FUNDAMENTAL FORCES IN NATURE

All the forces observed in nature such as muscular force, tension, reaction, friction, elastic, weight, electric, magnetic, nuclear etc., can be explained in terms of only following four basic interactions.

Gravitational Force and Properties

The force of interaction which exists between two particles due to their masses is called gravitational force. The gravitational force between two masses m_1 (called the source mass S) and m_2 (called the test mass T) separated by a distance r is given by



$$\vec{F} = -G\frac{m_1 m_2}{r^3}\vec{r}$$

where, $\vec{r}$ is position vector of test particle T with respect to source particle S and G is universal gravitational constant whose value is given by $G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.

The negative sign tells about the attractive nature of the gravitational force as the test mass is always attracted towards the source mass (i.e. opposite to $\vec{r}$).

(i) It is the weakest force and is always attractive.

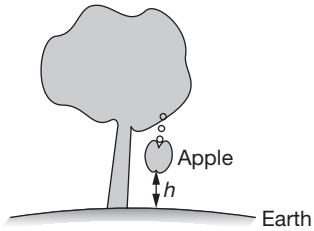
(ii) It is a long range force as it acts between any two particles situated at any distance in the universe.

(iii) Gravitational force between two bodies is independent of the presence of other bodies and it is also independent of the nature of intervening medium (i.e. medium present between the bodies).

(iv) It is a central force i.e. it acts along the line joining the centres of the two bodies.

(v) It is negligible for lighter bodies, however it dominates in case of planetary bodies and their motion.

(vi) Gravitons are exchange particles between two bodies and are responsible for the gravitational interaction between them.



Consider an apple to be falling freely as shown in figure. When it is at a height h, force between earth and apple is given by

$$F = \frac{GmM_e}{r^2} = \frac{GmM_e}{(R_e + h)^2}$$

where M_e is the mass of earth, R_e is the radius of earth. It acts towards earth's centre. Now rearranging above result, we get

$$F = m\left(\frac{GM_e}{R_e^2}\right)\left(\frac{R_e}{R_e + h}\right)^2$$

$$\Rightarrow \quad F = mg\left(\frac{R_e}{R_e + h}\right)^2 \qquad \left\{\because g = \frac{GM_e}{R_e^2}\right\}$$

For $h \ll R_e$, we have $\frac{R_e}{R_e + h} \simeq 1$

$\Rightarrow \quad F = mg$

This is the force exerted by earth on any particle of mass m near the earth surface.

The value of $g = 9.81\ \text{ms}^{-2} \simeq 10\ \text{ms}^{-2} \simeq \pi^2\ \text{ms}^{-2} \simeq 32\ \text{fts}^{-2}$. It is also called acceleration due to gravity near the surface of earth.

Electromagnetic Force

It includes the electrical and the magnetic forces which may be attractive or repulsive. Photons are exchange particles for electromagnetic interactions. So, force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.
Following are the main characteristics of electromagnetic force

(a) These can be attractive or repulsive.
(b) These are long range forces.
(c) These depend on the nature of medium between the charged particles.
(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between the atoms or molecules.

Nuclear Force

An attractive force, it is the force between two nucleons. It is the strongest force that binds all the nucleons in a tiny volume (corresponding to radius of order of 1 fermi) inspite of large electric repulsion between protons. Mesons are the exchange particles responsible for nuclear interactions. It acts within the nucleus that too upto a very small distance. Radioactivity, fission and fusion etc. result because of unbalancing of nuclear forces.

Weak Force

It exists between elementary particles and is responsible for the change in nucleus . Under its action a neutron can change into proton during negative beta decay by emitting an electron and a particle called antineutrino. It is not the same force as gravitational, electromagnetic or nuclear force. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.
It has been found that for two protons at a distance of 1 fermi

$$F_N : F_{EM} : F_W : F_G :: 1 : 10^{-2} : 10^{-7} : 10^{-38}$$

CLASSIFICATION OF FORCES ON THE BASIS OF CONTACT

On the basis of contact, forces can be classified as

Field Force

Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called **Field Force or Action at a Distance Force**.

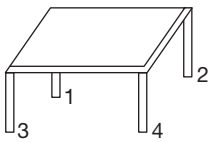
EXAMPLES:
(a) Gravitation force
(b) Electromagnetic force

Contact Force

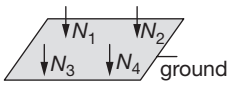
Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

EXAMPLES:
Normal Force, Mechanical Force, Force of Friction
Normal force (*N*) or Normal Reaction (*N*)

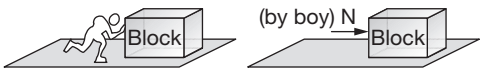
It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force.
A table is placed on Earth as shown in figure.



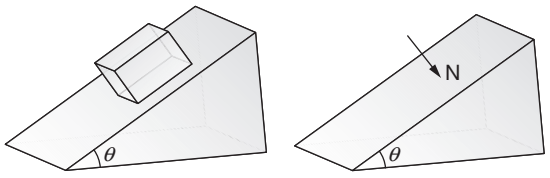
Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



Now a boy pushes a block kept on a frictionless surface. Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block. Normal force exerted by block on the surface of inclined plane is shown in figure. Force acts perpendicular to the surface.



Frictional Force (*f*)

A contact force, it arises due to a contact between the surfaces of the two bodies and comes into being when there is a relative motion between the surfaces in contact. Actually, it is the component of the reaction force tangential to the surface on which the body is kept.

SYSTEM

Two or more than two objects which interact with each other form a system. The classification of forces on the basis of boundary of system are given.

Internal Forces

Forces acting each with in a system among its constituents.

External Forces

Forces exerted on the constituents of a system by the outside surroundings are called as external forces.

Real Force

Force which acts on an object due to other object is called as real force.

An isolated object (far away from all objects) does not experience any real force.

CONCEPT OF IMPULSE AND IMPULSE AS AREA UNDER $\vec{F}$-t GRAPH

Whenever a large force $(\vec{F})$ acts on a body for an extremely small time (say dt), then we introduce the concept of Impulse $(\vec{I})$. So, impulse is just defined as the product of the large force with the small time.

$$\Rightarrow \quad d\vec{I} = \vec{F}dt$$

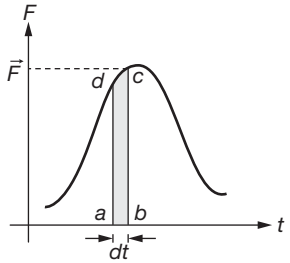
Impulse is also defined as the integral of force with respect to time.

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}dt$$

Since force is a vector and time is a scalar, the result of the integral in above equation is a vector. If the force is constant (both in magnitude and direction), it may be removed from the integral so that the integral is reduced to

$$I = F\int_{t_i}^{t_f} dt = F\left(t_f - t_i\right) = F\Delta t$$

Graphically, the impulse is the area between the force curve and the $F = 0$ axis, as shown in figure.



The SI unit of impulse is Ns.

If more than one force are acting on a particle, then the net impulse is given by the time integral of the net force.

$$\vec{I}_{\text{net}} = \int_{t_i}^{t_f} \vec{F}_{\text{net}} dt$$

For example: A cricketer hitting a ball to score a six. The bat hits the ball with the large force for a very short time.

If $\vec{F}$ be the large force acting on a body for a small time dt, then

$$d\vec{I} = \vec{F}dt$$

$$\Rightarrow \quad \vec{I} = \int d\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$$

Since $|d\vec{I}|$ = Area $abcd = \vec{F}dt$

$$\Rightarrow \quad |\vec{I}| = \int \vec{F}dt = \text{Area under a curve in a } \vec{F}\text{-}t \text{ graph.}$$

ILLUSTRATION 2

Find the impulse due to the force $\vec{F} = a\hat{i} + bt\hat{j}$, where $a = 2$ N and $b = 4$ Ns^{-1}, if this force acts from $t_i = 0$ to $t_f = 0.3$ s.

SOLUTION

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}dt = \int_0^{0.3} \left(a\hat{i} + bt\hat{j}\right)dt$$

$$\Rightarrow \quad \vec{I} = a\hat{i}\int_0^{0.3} dt + b\hat{j}\int_0^{0.3} t\,dt$$

$$\Rightarrow \quad \vec{I} = at\hat{i}\Big|_0^{0.3} + \frac{bt^2}{2}\hat{j}\Big|_0^{0.3}$$

$$\Rightarrow \quad \vec{I} = (2)(0.3)\hat{i} + \frac{(4)(0.3)^2}{2}\hat{j}$$

$$\Rightarrow \quad \vec{I} = \left(0.6\hat{i} + 0.18\hat{j}\right) \text{ Ns}$$

ILLUSTRATION 3

A ball falling with velocity $\vec{v}_i = \left(-0.65\hat{i} - 0.35\hat{j}\right)$ ms^{-1} is subjected to a net impulse $\vec{I} = \left(0.6\hat{i} + 0.18\hat{j}\right)$ Ns^{-1}. If the ball has a mass of 275 g, calculate its velocity immediately following the impulse.

SOLUTION

Using Impulse - Momentum Theorem

$$m\vec{v}_f - m\vec{v}_i = \vec{I}$$

$$\Rightarrow \quad \vec{v}_f = \vec{v}_i + \frac{\vec{I}}{m}$$

Thus, $\vec{v}_f = -0.65\hat{i} - 0.35\hat{j} + \dfrac{0.6\hat{i} + 0.18\hat{j}}{0.275}$

$$\Rightarrow \quad \vec{v}_f = \left(-0.65\hat{i} - 0.35\hat{j}\right) + \left(2.18\hat{i} + 0.655\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_f = \left(1.53\hat{i} + 0.305\hat{j}\right) \text{ ms}^{-1}$$

ILLUSTRATION 4

An 150 g ball is thrown at 30 ms^{-1}. It is struck by a bat, which gives it a velocity of 40 ms^{-1} in the opposite direction. If the time of contact is 10^{-2} s, what is the average force on the ball?

SOLUTION

If we choose the original direction as $+x$-axis, then

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = m\left(-40\hat{i} - 30\hat{i}\right)$$

The average force is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{-0.15 \text{ kg} \times 70\hat{i}}{10^{-2}} = -1050\hat{i} \text{ N}$$

Notice that this is much larger than the weight (1.5 N) of the ball.

IMPULSE – MOMENTUM THEOREM

According to Newton's Second Law, we have

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \quad d\vec{p} = \vec{F}dt$$

$$\Rightarrow \quad \int_i^f d\vec{p} = \int_i^f F dt = \vec{I}$$

$$\Rightarrow \quad \vec{I} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}}$$

$$\Rightarrow \quad \text{Impulse} = \text{Change in momentum}$$

{called **Impulse Momentum Theorem**}

$$\Rightarrow \quad \vec{I} = m(\vec{v} - \vec{u})$$

Conceptual Note(s)

In the following cases shown, the change in momentum is calculated.

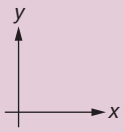
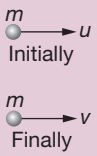
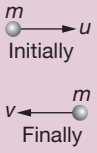
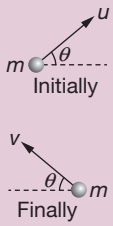
		Change in Momentum $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$
CASE-1		$\Delta p = m(v-u)$, along $+x$ direction
CASE-2		$\Delta p = -m(v+u)$ $\Rightarrow \lvert\Delta p\rvert = m(v+u)$, along $-x$ direction
CASE-3		$\Delta p_x = m(v_x - u_x)$ $\Rightarrow \Delta p_x = -m(v\cos\phi + u\cos\theta)$, along $-x$ direction $(\Delta p)_y = m(v_y - u_y)$ $\Rightarrow (\Delta p)_y = m(v\sin\phi - u\sin\theta)$, along $+y$ direction. So, $\Delta\vec{p} = (\Delta p_x)\hat{i} + (\Delta p_y)\hat{j}$

ILLUSTRATION 5

A box is put on a scale which is adjusted to read zero, when the box is empty. A stream of pebbles is then poured into the box from a height h above its bottom at the rate of n pebbles per second. Each pebble has a mass m. If the pebbles collide with the box such that they immediately come to rest after the collision. Find the reading at time t after which the pebbles begin to fill the box.

SOLUTION

Velocity (v) of each pebble just before striking the box is $v = \sqrt{2gh}$

$$\begin{pmatrix}\text{Reading of the scale at time } t\end{pmatrix} = \begin{pmatrix}\text{Sum of weights of pebbles collected in time } t\end{pmatrix} + \begin{pmatrix}\text{The additional force exerted by the pebbles due to a change in momentum}\end{pmatrix}$$

Total weight of pebbles in time t is $(mg)(nt)$

Change in momentum of 1 pebble is $m\left(\sqrt{2gh}\right)$

Change in momentum (Δp) of nt pebbles is

$$\Delta p = \left(m\sqrt{2gh}\right)(nt)$$

$$\Rightarrow \text{Additional force} = \frac{\Delta p}{t} = mn\sqrt{2gh}$$

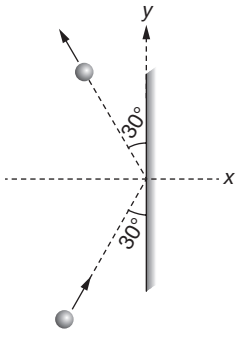
$$\Rightarrow \text{Total force} = mn\left(gt + \sqrt{2gh}\right)$$

Test Your Concepts-I

Based on Impulse Momentum

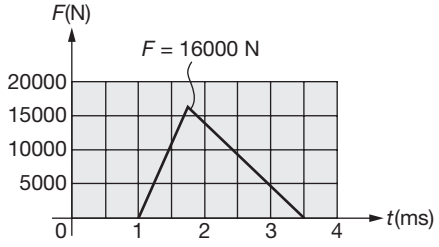
(Solutions on page H.193)

1. A steel ball of mass 3 kg strikes a wall with a speed of 10 ms^{-1} at an angle of 30° with the surface. It bounces off with the same speed and angle as shown. If the ball is in contact with the wall for 200 ms, calculate the average force exerted by the wall on the ball.



2. A ball of mass 150 g is dropped from a height of 20 m. It rebounds from the floor to reach a height of 5 m. What impulse was given to the ball by the floor? Take $g = 10\ \text{ms}^{-2}$.

3. An estimated force-time curve for a ball struck by a bat is shown in figure. From the curve, determine.
(a) the impulse delivered to the ball
(b) the average force exerted on the ball and
(c) the peak force exerted on the ball.



4. A garden hose is originally full of motionless water. What additional force is necessary to hold the nozzle stationary after the water flow is turned on, if the discharge rate is $0.6\ \text{kgs}^{-1}$ with a speed of $25\ \text{ms}^{-1}$?

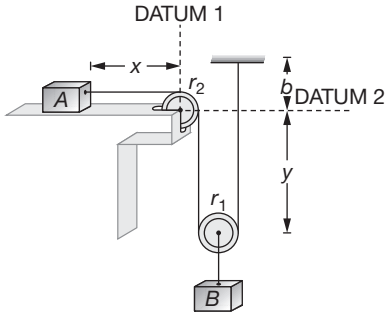
5. A professional driver of mass 60 kg performs a dive from a platform 10 m above the water surface. Find the magnitude of the average impact force experienced by him if the impact time is 1 s on collision with the water surface. Assume that the velocity of the diver just after entering the water surface is $4\ \text{ms}^{-1}$. Take $g = 9.8\ \text{ms}^{-2}$.

CONSTRAINED MOTION OF CONNECTED PARTICLES

Sometimes it is observed that the motions of particles are interrelated because of the constraints imposed by interconnecting members. In such cases it becomes necessary for us to account for these constraints in order to determine the respective motions of the particles.

ONE DEGREE OF FREEDOM

Let us consider a simple system of two interconnected particles A and B shown in figure. It is quite evident by inspection that the horizontal motion of A is twice the vertical motion of B. However, we will use this example to illustrate the method of analysis which applies to more complex situations where the results cannot be easily obtained by inspection.



The motion of B is clearly the same as that of the center of its pulley, so we establish position coordinates y and x measured from a convenient fixed **datum** (a horizontal or a vertical fixed axis). The total length of the rope is

$$x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b = L \qquad \ldots(1)$$

Since L, r_2, r_1, and b are all constant, so the first and second time derivatives of the equation (1) give

$$\dot{x} + 2\dot{y} = 0$$

$$\Rightarrow \quad v_A + 2v_B = 0 \qquad \ldots(2)$$

$$\Rightarrow \quad \dot{v}_A + 2\dot{v}_B = 0 \ \text{ or } \ \ddot{x} + 2\ddot{y} = 0$$

$$\Rightarrow \quad a_A + 2a_B = 0 \qquad \ldots(3)$$

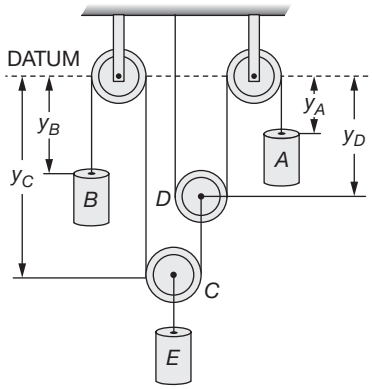
The velocity and acceleration constraint equations indicate that, for the coordinates selected, the velocity of A must have a sign which is opposite to that of the velocity of B, and similarly for the accelerations. The constraint equations are valid for the motion of the system in either direction. We emphasize that $v_A = \dot{x}$ is positive to the left and that $v_B = \dot{y}$ is positive down. The system shown is said to have one degree of freedom since only one variable, either x or y, is needed to specify the positions of all parts of the system.

TWO DEGREES OF FREEDOM

Consider a system, shown, having two degrees of freedom. Here the positions of the lower cylinder E and pulley C depend on the separate specifications of the two coordinates y_A and y_B. The lengths of the cables (measured from DATUM), attached to cylinders A and B can be written, respectively, as

$$y_A + 2y_D + \text{constant} = L_A \quad \ldots(1)$$

$$y_B + y_C + (y_C - y_D) + \text{constant} = L_B \quad \ldots(2)$$



and their time derivatives are

$$\dot{y}_A + 2\dot{y}_D = 0 \text{ and } \dot{y}_B + 2\dot{y}_C - \dot{y}_D = 0$$

$$\ddot{y}_A + 2\ddot{y}_D = 0 \text{ and } \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D = 0$$

Eliminating the terms in $\dot{y}_D$ and $\ddot{y}_D$ gives

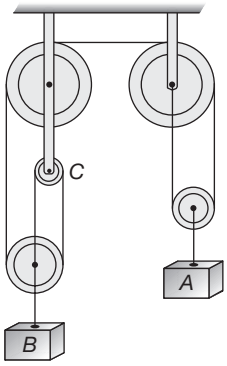
$$\dot{y}_A + 2\dot{y}_B + 4\dot{y}_C = 0 \text{ or } v_A + 2v_B + 4v_C = 0$$

$$\ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C = 0 \text{ or } a_A + 2a_B + 4a_C = 0$$

It is clearly impossible for the signs of all three terms to be positive simultaneously. So, for example, if both A and B have downward (positive) velocities, then C will have an upward (negative) velocity.

ILLUSTRATION 6

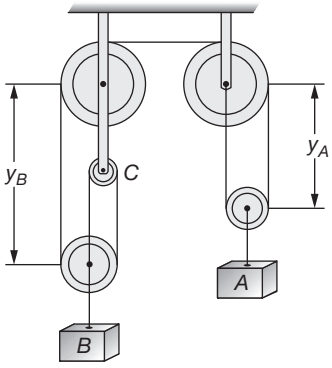
In the pulley configuration shown, cylinder A has a downward velocity of $0.3\ \text{ms}^{-1}$. Determine the velocity of B.



SOLUTION

The centers of the pulleys at A and B are located by the coordinates y_A and y_B measured from fixed positions. The total constant length of cable in the pulley system is

$$L = 3y_B + 2y_A + \text{constants}$$



where the constants account for the fixed lengths of cable in contact with the circumferences of the pulleys, and the constant vertical separation between the two upper left-hand pulleys. Differentiation with time gives

$$0 = 3\dot{y}_B + 2\dot{y}_A$$

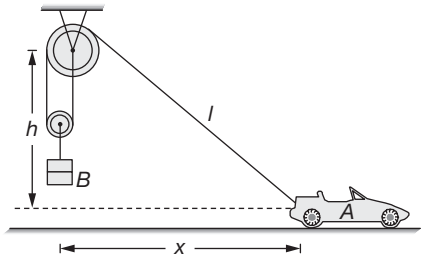
Substitution of $v_A = \dot{y}_A = 0.3\ \text{ms}^{-1}$ and $v_B = \dot{y}_B$ gives

$$0 = 3(v_B) + 2(0.3)$$

$$\Rightarrow \quad v_B = -0.2\ \text{ms}^{-1}$$

ILLUSTRATION 7

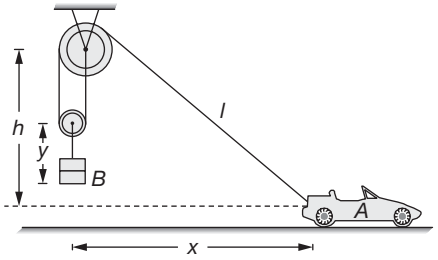
The car A is used to pull a load B with the pulley arrangement shown. If A has a forward velocity v_A, determine an expression for the upward velocity v_B of the load in terms of x.



SOLUTION

We designate the position of the car by the coordinate x and the position of the load by the coordinate y, both measured from a fixed reference. The total constant length of the cable is

$$L = 2(h-y)+l = 2(h-y)+\sqrt{h^2+x^2}$$



Differentiation with time yields

$$0 = -2\dot{y} + \frac{x\dot{x}}{\sqrt{h^2+x^2}}$$

Substituting $v_A = \dot{x}$ and $v_B = \dot{y}$ gives

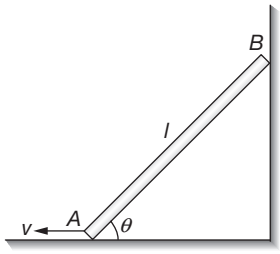
$$v_B = \frac{1}{2}\frac{xv_A}{\sqrt{h^2+x^2}}$$

SIMPLE CONSTRAINT MOTION OF BODIES AND PARTICLES IN TWO DIMENSIONS

Similar to projectile motion, there can be several two dimensional motions, in which the laws of motion can be separately applied to x and y directions and later on the developed relations can be linked for getting the required parameters. Sometime x and y directional motion or any two directions of the motion are related by some specific rule, we call such rules as constraint rules. These rules relate one direction of motion of an object with some other direction of the same object or some other object also.

ILLUSTRATION 8

Figure shows a rod of length l resting on a wall and the floor. Its lower end A is pulled towards left with a constant velocity u. Find the velocity of the other end B downward when the rod makes an angle θ with the horizontal.



SOLUTION

METHOD I

Here if the distance from the corner to the point A is x and that up to B is y. The velocity of point A can then be given by

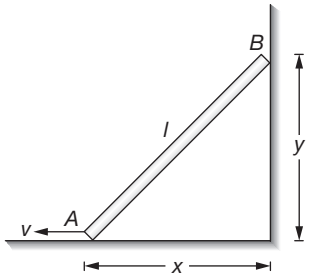
$$v = \frac{dx}{dt}$$

and that of B can be given as

$$v_B = \frac{dy}{dt}$$

Since, we have,

$$x^2 + y^2 = l^2$$



Differentiating with respect to t, we get

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$xv+yv_B=0$$

$$\Rightarrow \quad v_B=-\left(\frac{x}{y}\right)v$$

$$\Rightarrow \quad v_B=-v\cot\theta$$

{negative sign indicates, y decreasing with time}

METHOD II

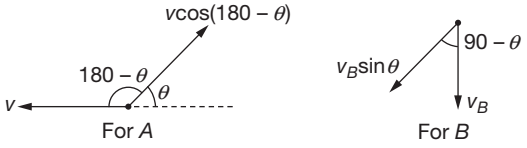
In cases when the relation between two points of a rigid body is required, we can make use of the fact that in a rigid body the distance between two points always remains same. Thus the relative velocity of one point of an object with respect to any other point of the same object in the direction of line joining them will always remain zero, as their separation always remains constant.

Here in above example the distance between the points A and B of the rod always remains constant, thus, the two points must have same velocity components in the direction of their line joining i.e., along the length of the rod.

If point B is moving down with velocity v_B, its component along the length of the rod is $v_B\sin\theta$. Similarly the velocity component of point A along the length of rod is $v\cos\theta$. Thus we have.

$$v_B\sin\theta=v\cos\theta$$

$$v_B=v\cot\theta$$

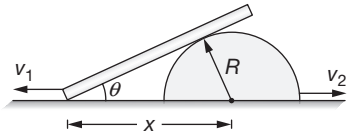


Conceptual Note(s)

In such type of problems, when velocity of one part of a body is given and that of other is required or in cases, when relation in two velocities is required, we first find the relation between the two displacements then differentiate with respect to time.

ILLUSTRATION 9

Figure shows a hemisphere and a supported rod. Hemisphere is moving in right direction with a uniform velocity v_2 and the end of rod which is in contact with ground is moving in left direction with a velocity v_1. Find the rate at which the angle θ is changing in terms of v_1, v_2, R and θ.



SOLUTION

Here x is the separation between centre of hemisphere and the end of rod. Rate of change of x is actually the relative velocity of end of rod and centre of hemisphere i.e., (v_1+v_2). We are required to find the rate of change of θ, i.e., $\frac{d\theta}{dt}$, knowing that $\frac{dx}{dt}=v_1+v_2$

Since, $x=R\,\text{cosec}\,\theta$

Differentiating with respect to time we get

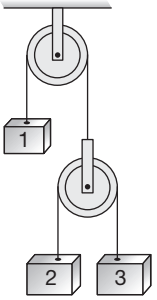
$$\frac{dx}{dt}=-R\,\text{cosec}\,\theta\cot\theta\frac{d\theta}{dt}$$

$$\left\{\because \frac{d}{dt}(\text{cosec}\,\theta)=-\text{cosec}\,\theta\cot\theta\right\}$$

$$\Rightarrow \quad \frac{d\theta}{dt}=\frac{(v_1+v_2)\sin^2\theta}{R\cos\theta}$$

ILLUSTRATION 10

In the system shown, if a_1, a_2 and a_3 be the respective accelerations of 1, 2 and 3, then find a_1 in terms of a_2 and a_3.



SOLUTION

Let the datum pass from the centre of the fixed pulley. Since the points 1, 2, 3 and 4 are movable, so let their displacements at any instant from the datum be x_1, x_2, x_3 and x_4. We observe that

$$x_1 + x_4 = l_1 \qquad \ldots(1)$$

(length of first string between 1 and 4 is constant)

and $(x_2 - x_4) + (x_3 - x_4) = l_2$

(length of second string between 2 and 3 is constant)

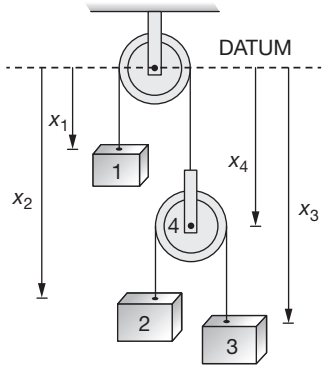
$$\Rightarrow \quad x_2 + x_3 - 2x_4 = l_2 \qquad \ldots(2)$$

Differentiating twice with respect to time, we get

$$\ddot{x}_1 + \ddot{x}_4 = 0 \text{ and } \ddot{x}_2 + \ddot{x}_3 - 2\ddot{x}_4 = 0$$

$$\Rightarrow \quad a_1 + a_4 = 0 \qquad \ldots(3)$$

and $a_2 + a_3 - 2a_4 = 0 \qquad \ldots(4)$



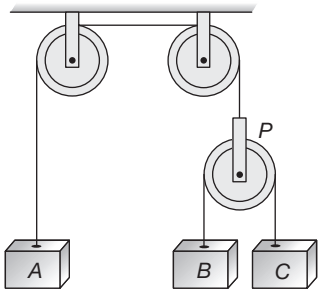
Now, since $a_4 = -a_1$, so we get {from equation (3)}

$$a_2 + a_3 + 2a_1 = 0$$

$$\Rightarrow \quad a_1 = \frac{a_2 + a_3}{2} \quad \text{(in magnitude)}$$

ILLUSTRATION 11

In the arrangement shown, the three blocks move with constant velocities. Knowing that the relative velocity of A with respect to C is 300 mms^{-1} upwards and that the relative velocity of B with respect to A is 200 mms^{-1} downwards, find the velocity of each block.



SOLUTION

Let v be the velocity of block A (upwards) and v_r be the velocity of block B with respect to moving pulley P (upwards).

Taking upward direction as positive, we have

$$v_A = +v, \; v_B = v_r - v \text{ and } v_C = -(v_r + v)$$

Now it is given that,

$$v_{AC} = +300 \text{ mms}^{-1}$$

$$\Rightarrow \quad v_A + v_C = 300 \text{ mms}^{-1} \qquad \ldots(1)$$

$$\Rightarrow \quad 2v + v_r = 300 \text{ mms}^{-1}$$

Similarly $v_{BA} = -200 \text{ mms}^{-1}$

$$\Rightarrow \quad v_B - v_A = -200 \text{ mms}^{-1}$$

$$\Rightarrow \quad v_r - 2v = -200 \text{ mms}^{-1} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_r = 50 \text{ mms}^{-1} \text{ and } v = 125 \text{ mms}^{-1}$$

So, $v_A = +v = 125 \text{ mms}^{-1}$ {upwards}

$$v_B = v_r - v = -75 \text{ mms}^{-1}$$

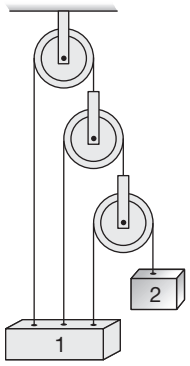
$$\Rightarrow \quad v_B = 75 \text{ mms}^{-1}, \text{ downwards}$$

and $v_C = -v_r - v = -175 \text{ mms}^{-1}$

$$\Rightarrow \quad v_C = 175 \text{ mms}^{-1}, \text{ downwards.}$$

ILLUSTRATION 12

In the system shown, if a_0 be the acceleration of block 1, then find the acceleration of block 2.



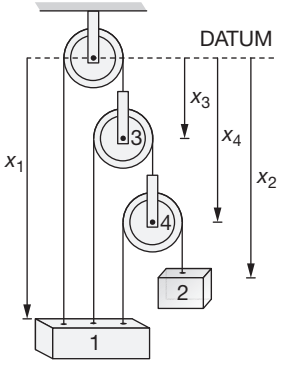
SOLUTION

Let the datum pass through the centre of the fixed pulley. Since the points 1, 2, 3 and 4 are movable, so let their displacements from the datum be x_1, x_2, x_3 and x_4. We observe that the length of the strings between 1 and 3, 1 and 4, 1 and 2 is constant. So,

$$x_1 + x_3 = l_1$$

$$(x_1 - x_3) + (x_4 - x_3) = l_2$$

$$(x_1 - x_4) + (x_2 - x_4) = l_3$$



On double differentiating with respect to time, we get

$$a_1 + a_3 = 0 \quad \ldots(1)$$

$$a_1 + a_4 - 2a_3 = 0 \quad \ldots(2)$$

$$a_1 + a_2 - 2a_4 = 0 \quad \ldots(3)$$

Solving equations (1), (2) and (3), we get

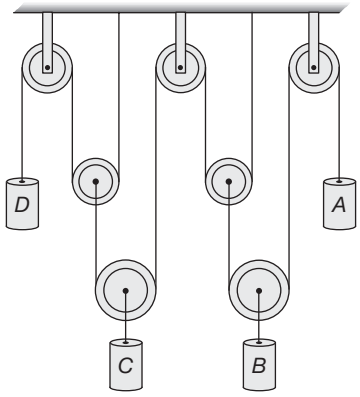
$$a_2 = -7a_1 = -7a_0$$

$$\Rightarrow \quad |a_2| = 7a_0$$

When 1 moves up by x, 3 goes down by x, 4 by $2x$ and 2 by $4x$, so that total distance moved down by 2 becomes $(x + 2x + 4x) = 7x$.

ILLUSTRATION 13

Determine the relationship which governs the velocities of the four cylinders. Express all velocities as positive down. How many degrees of freedom are there?



SOLUTION

$$L_1 = y_B + y_A + (y_B - y_1) + C_1$$

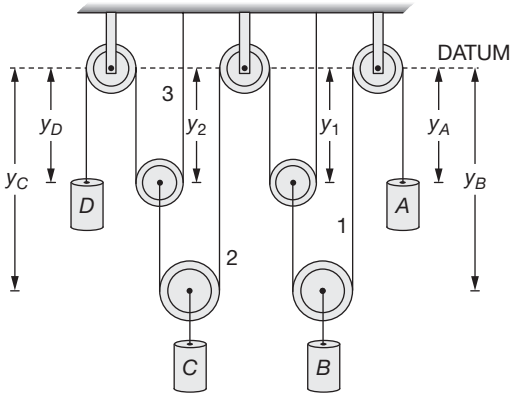
$$\Rightarrow \quad 2\dot{y}_B + \dot{y}_A - \dot{y}_1 = 0$$

$$L_2 = y_C + 2y_1 + (y_C - y_2) + C_2$$

$$\Rightarrow \quad 2\dot{y}_C + 2\dot{y}_1 - \dot{y}_2 = 0$$

$$L_3 = 2y_2 + y_D + C_3$$

$$\Rightarrow \quad 2\dot{y}_2 + \dot{y}_D = 0$$



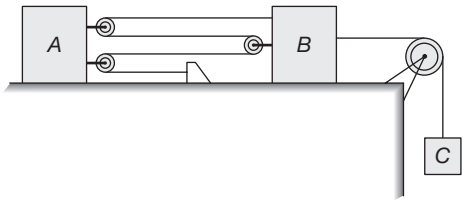
3 degrees of freedom

Eliminate $\dot{y}_1$ and $\dot{y}_2$, and get

$$4v_A + 8v_B + 4v_C + v_D = 0$$

ILLUSTRATION 14

Block C shown in figure is going down at acceleration $2\ \text{ms}^{-2}$. Find the acceleration of blocks A and B.



SOLUTION

The analysis is shown in figure. As block B and C are connected by a string there accelerations must be same hence we can directly state

$$a_B = 2\ \text{ms}^{-2}$$

Block A is also constrained to move with block B, with pulleys X, Y and Z. As shown in figure, we assume if block B and C moves by a distance y, A will move by x and due to this the parts (like length ab) of strings 1, 2, 3 and 4 which are passing over pulleys X and Y are slackened by a length $4x$. This will be tightened by the displacement of pulley Z along with the block B and the string ℓ which is attached to B at point d, by a distance y and this will pull the same string by $3y$ (like the length cd). Thus we have $4x = 3y$ and similarly we have the constrained relation for blocks A and B as.

$$a_A = \frac{3}{4}a_B = 1.5\ \text{ms}^{-2}$$

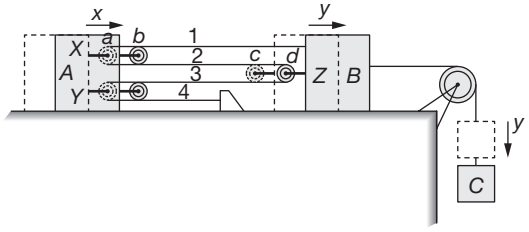
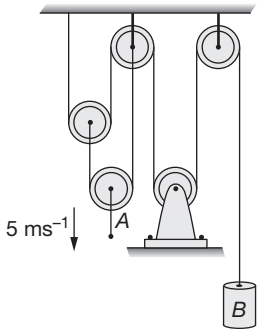


ILLUSTRATION 15

In the arrangement shown, if the end A of the rope moves downward with a speed of $5\ \text{ms}^{-1}$, determine the speed of cylinder B.



SOLUTION

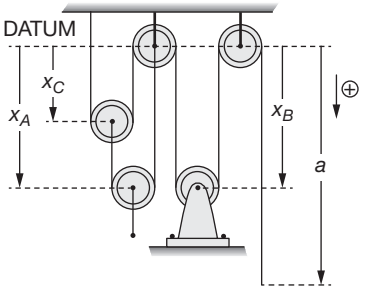
Let the datum be passing through both the fixed pulleys as shown. The length of the two ropes in terms of the position coordinates x_A, x_B and x_C are

$$x_B + 2a + 2x_C = l_1$$

$$\Rightarrow \quad x_B + 2x_C = l_1 - 2a \qquad \ldots(1)$$

and $\quad x_A + (x_A - x_C) = l_2$

$$\Rightarrow \quad 2x_A - x_C = l_2 \qquad \ldots(2)$$



Eliminating x_C from equations (1) and (2), we get

$$x_B + 4x_A = l_1 - 2a + 2l_2 \qquad \ldots(3)$$

Taking the time derivative of equation (3), we get

$$v_B + 4v_A = 0$$

Taking downward direction as positive, we get

$$v_A = 5\ \text{ms}^{-1}$$

$\Rightarrow \quad v_B + 4(5) = 0$

$\Rightarrow \quad v_B = -20 \text{ ms}^{-1} = 20 \text{ ms}^{-1}$, upwards

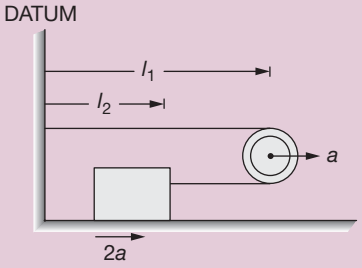
Problem Solving Technique(s)

For pulleys interconnected with strings, we can have the following methods to analyse constraint equations.

(a) Branch wise Analysis.

(b) Law of Conservation of Length of strings connecting the pulleys.

(c) If one end of a string passing over a moving pulley is fixed, then the acceleration of the other end (free or connected to something) is twice the acceleration of the moving pulley. Following situations show this to be made as a good rule for applications.

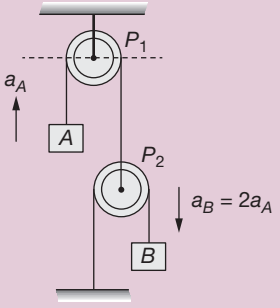


$l_1 + (l_1 - l_2) = \text{constant}$

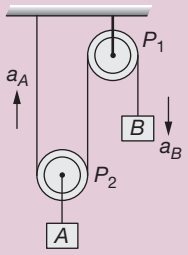
$\Rightarrow \quad 2l_1 - l_2 = \text{constant}$

$\Rightarrow \quad 2\dot{l}_1 - \dot{l}_2 = 0 \quad \Rightarrow \quad \dot{l}_2 = 2\dot{l}_1$

$\Rightarrow \quad \ddot{l}_2 = 2\ddot{l}_1 \quad \Rightarrow \quad a_2 = 2a_1$



$a_B = 2(\text{Acc. of Pulley } P_2) = 2a_A$



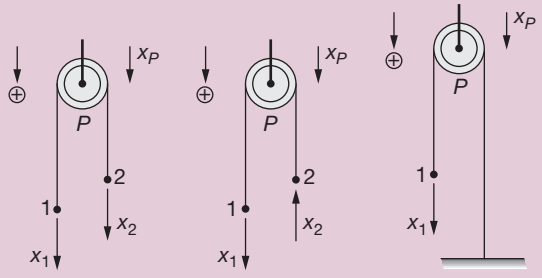
$a_B = 2(\text{Acc. of Pulley } P_2)$

$\Rightarrow \quad a_B = 2a_A$

(d) For a moving pulley, the displacement of the pulley equals the average of the displacements on the left and the right of the pulley.

$$x_P = \frac{x_1 + x_2}{2} \qquad x_P = \frac{x_1 - x_2}{2} \qquad x_P = \frac{x_1 + 0}{2}$$

$\Rightarrow \quad x_1 = 2x_P$ (As discussed in 1)

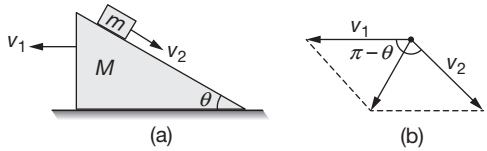


WEDGE CONSTRAINTS

Till now we have discussed the motion of blocks connected by strings governed by pulleys connected in several ways possible. Here we will discuss the relation between the motion of two or more bodies which are in contact and responsible for motion of bodies.

EXAMPLE 1:

First we consider a very simple case shown in Figure (a).



Here a triangular block of mass M is free to move on ground and m is free to move on inclined surface of M. Here M is constrained to move only along horizontal ground and m is also constrained to move only along the inclined surface of M relative to it. Here if M is going toward left with speed v_1 (say), and if on its inclined surface m is going down with speed v_2, then we can state that the net speed of M is v_1 but m is also moving to the left along with M, thus, its net speed is given by vector sum of the two v_1 and v_2 as shown in Figure (b).

$$v_m = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\pi - \theta)}$$

$$v_m = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$

Same can also be evaluated by using the velocity components in horizontal and vertical directions of small mass m. Here it is going along the incline with a velocity v_2 relative to M and it is also moving with M toward left with velocity v_1, thus we have
Horizontal velocity of m relative to ground is

$$v_x = v_2\cos\theta - v_1$$

Vertical velocity of m relative to ground is

$$v_y = v_2\sin\theta$$

Net velocity of m is

$$v_m = \sqrt{v_1^2 + v_y^2} = \sqrt{v_1^2 + v_y^2 - 2v_1v_2\cos\theta}$$

EXAMPLE 2:

Now consider the situation shown in figure. Here block A and B are constrained to move on their contact surface as well as horizontal ground and vertical wall. Here as B goes down, A will move to the left. If B goes down by a distance x, A will move toward left by a distance $x\cot\theta$. Thus if velocity and acceleration of B are v and a downward, velocity and acceleration A will be $v\cot\theta$ and $a\cot\theta$ toward left.

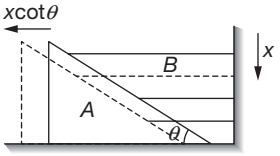
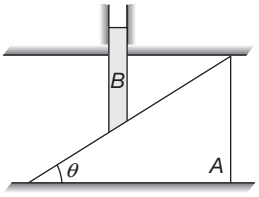


ILLUSTRATION 16

Find the relation among accelerations of wedge A and the rod B supported on wedge A. Rod B is restricted to move vertically by two fixed wall corners shown in figure.



SOLUTION

Here we can observe that the rod is restricted to move only in vertical direction and wedge can move along horizontal plane only. Here if wedge moves toward right by a distance x, figure shows that the rod moves vertically down by a distance $x\tan\theta$. Thus if wedge is moving toward right with an acceleration a_1, rod will go down with acceleration a_2, given as

$$a_2 = a_1\tan\theta$$

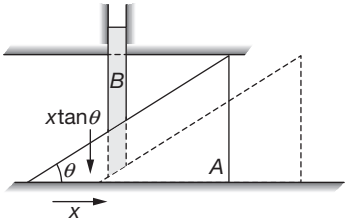
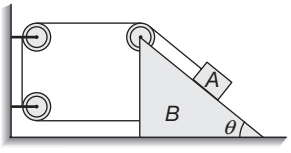


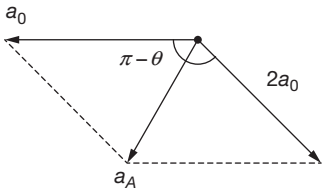
ILLUSTRATION 17

Figure shown a block A constrained to slide along the incline plane of the wedge B shown. Block A is attached with a string which passes through three ideal pulleys and connected to the wedge B. If wedge moves towards left with an acceleration a_0, find the acceleration of the block with respect to the wedge and the ground.



SOLUTION

When the wedge moves to the left towards the wall by x, then a portion $2x$ of the string is loosened which makes the block A to go down by $2x$. So, acceleration of the block with respect to the wedge is $2a_0$. However, with respect to the ground the situation is shown for



$$a_A = \sqrt{a_0^2 + 4a_0^2 - 4a_0^2\cos\theta}$$

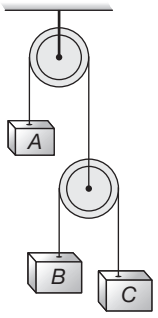
$$\Rightarrow \quad a_A = a_0\sqrt{5-4\cos\theta}$$

Test Your Concepts-II

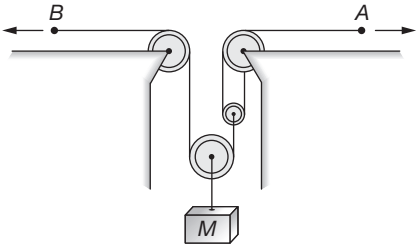
Based on Constraints

(Solutions on page H.193)

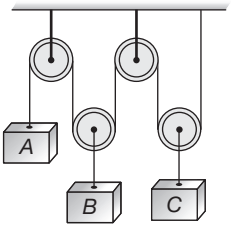
1. At certain moment of time, velocities of A and B both are 1 ms^{-1} upwards. Find the velocity of C at that moment.



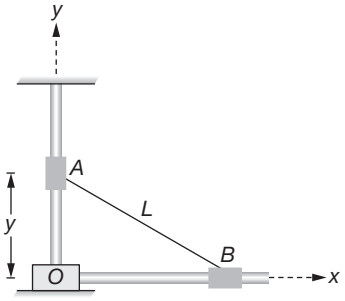
2. For the pulley system each of the cables at A and B is given a velocity of 2 ms^{-1} in the direction shown. Determine the upward velocity v of the load M.



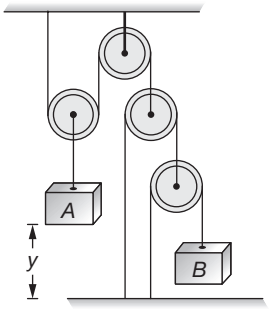
3. In the arrangement shown, if the block A of the pulley system is moving downward with a speed of 2 ms^{-1} while block C is moving up at 1 ms^{-1}, then determine the speed of block B.



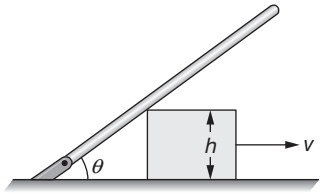
4. Collars A and B slide along the fixed right angled rods and are connected by a cord of length L. Determine the acceleration a_x of collar B as a function of y, if collar A is given a constant upward velocity v_A.



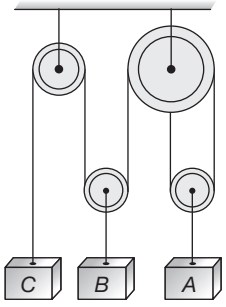
5. The vertical displacement of block A in meter is given by $y = \frac{t^2}{4}$, where t is in second. Calculate the downward acceleration a_B of block B.



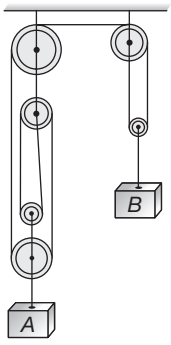
6. In the arrangement shown, the rod is freely pivoted at *O* and is in the contact with the block which moves on the horizontal frictionless ground. As the block is given a speed *v* forward, the rod rotates about *O*. Find the angular velocity of rod as a function of θ.



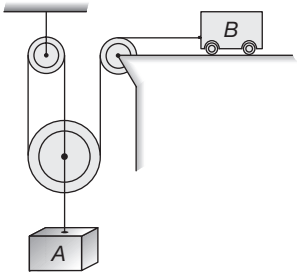
7. Find the relation which governs the accelerations of *A*, *B* and *C*, all measured positive down. Identify the number of degrees of freedom.



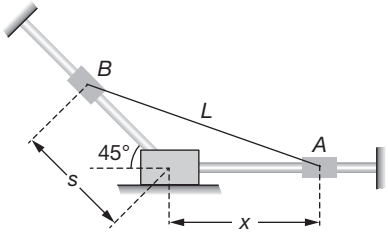
8. Cylinder *B* has a downward velocity in metre per second given by $v_B = \frac{t^2}{2} + \frac{t^3}{6}$, where *t* is in seconds. Calculate the acceleration of *A* when $t = 2$ s.



9. If block *B* has a leftward velocity of 1.2 ms^{-1}, determine the velocity of cylinder *A*.

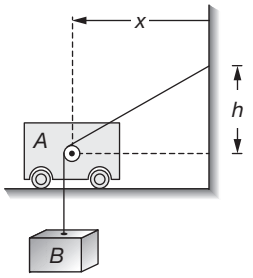


10. Collars *A* and *B* slide along the fixed rods and are connected by a cord of length *L*. If collar *A* has a velocity $v_A = \frac{dx}{dt}$ to the right, express the velocity $v_B = -\frac{ds}{dt}$ of *B* in terms of *x*, v_A, and *s*.

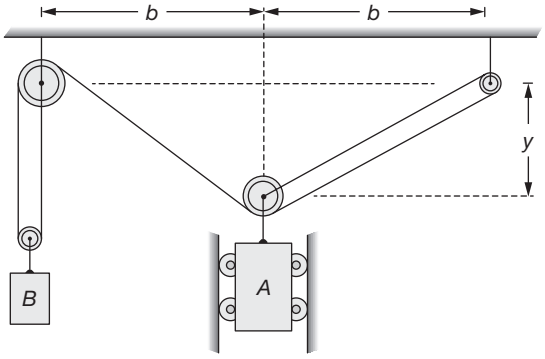


11. Neglect the diameter of the small pulley attached to body *A* and determine the magnitude of the total velocity of *B* in terms of the velocity v_A which body *A* has to the right. Assume that the cable between

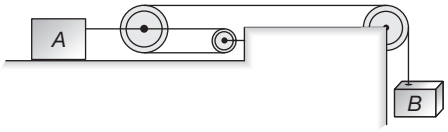
B and the pulley remains vertical and solve for a given value of *x*



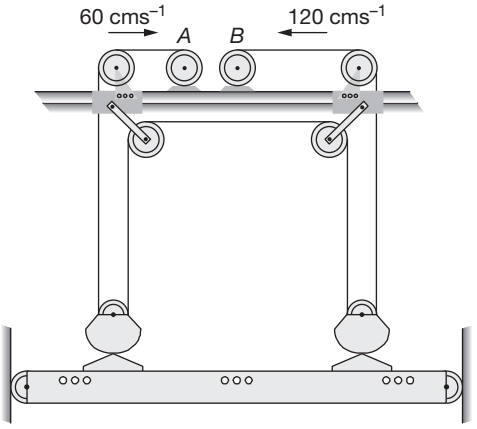
12. Establish the relationship between the velocity of *A* and the velocity of *B* for a given value of *y*. Neglect the diameters of the small pulley.



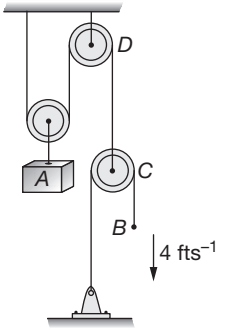
13. If block *A* has a velocity of 0.6 ms^{-1} to the right, determine the velocity of cylinder *B*.



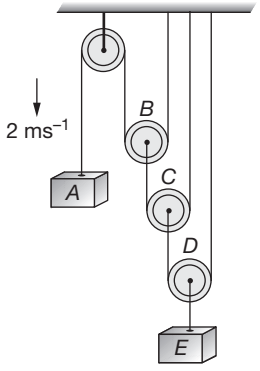
14. The crane is used to hoist the load. If the motors at *A* and *B* are drawing in the cable at a speed of 60 cms^{-1} and 120 cms^{-1}, respectively, determine the speed of the load.



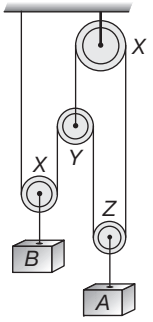
15. The cable at *B* is pulled downwards at 4 ms^{-1} and the speed is decreasing at 2 ms^{-2}. Determine the velocity and acceleration of block *A* at this instant.



16. If the end of the cable at *A* is pulled down with a speed of 2 ms^{-1}, determine the speed at which block *E* rises.



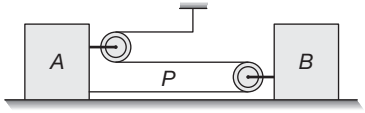
17. Figure shows a system of four pulleys with two masses *A* and *B*. Find the



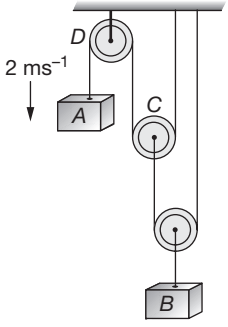
(a) speed of block *A* when the block *B* is going up at $1\,ms^{-1}$ and pulley *Y* is going up at $2\,ms^{-1}$.

(b) acceleration of block *A* if block *B* is going up at $3\,ms^{-2}$ and pulley *Y* is going down at $4\,ms^{-2}$.

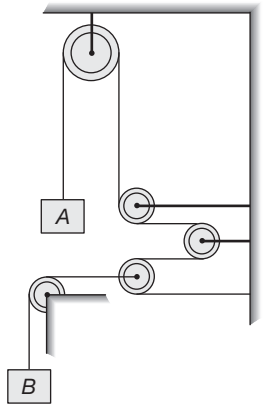
18. Block *A* shown in figure move by a distance 3 m toward left. Find the distance and direction in which the point *P* on string shown in figure is displaced.



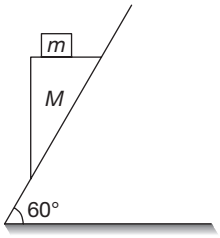
19. In the arrangement shown, if the end of the cable at *A* is pulled down with a speed of $2\,ms^{-1}$, determine the speed at which block *B* rises.



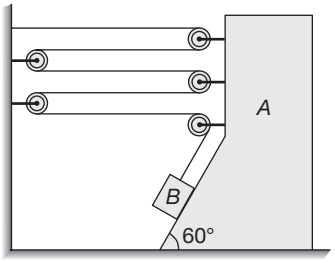
20. Find the velocity of the block *A* shown in figure, if *B* moves up with a velocity v_0.



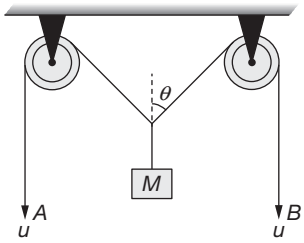
21. In the situation shown in figure, if mass *M* is going down along the incline at an acceleration of $5\,ms^{-2}$ and *m* is moving toward right relative to *M* horizontally with $3\,ms^{-2}$. Find the net acceleration of *m*.



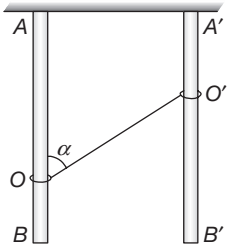
22. Find the acceleration of block *B* relative to the block *A* and relative to the ground, if the block *A* moves to the left with an acceleration a_0.



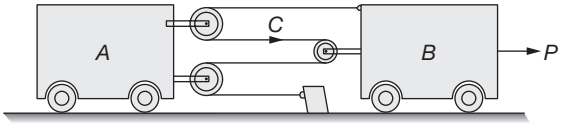
23. In the arrangement shown in figure, the ends *A* and *B* of an inextensible string move downwards with uniform speed *u*. Pulleys *A* and *B* are fixed. Find the speed with which the mass *M* moves upwards.



24. Two rings O and O' are put on two vertical stationary rods AB and $A'B'$, respectively as shown in figure. An inextensible string is fixed at point A' and on ring O and is passed through O'. Assuming that ring O' moves downwards at a constant speed v, find the velocity of the ring O in terms of α.



25. Under the action of force P, the constant acceleration of block B is $3\ ms^{-2}$ to the right. At the instant when the velocity of B is $2\ ms^{-1}$ to the right, determine the velocity of B relative to A, the acceleration of B relative to A and the absolute velocity of point C of the cable.



FREE BODY DIAGRAM (FBD)

A Free Body Diagram (FBD) consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

Steps for Drawing FBD

STEP-1: Identify the object or system and isolate it from other objects clearly specify its boundary.
STEP-2: First draw non-contact external force in the diagram. Generally it is weight.
STEP-3: Draw contact forces which act at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.
STEP-4: In F.B.D., draw all the forces acting on the isolated body. Do not draw the forces which the body is exerting on others.

WEIGHT OF A BODY (W)

The weight $\vec{W}$ of a body is a force that pulls the body directly towards the earth. The force is due to gravitational attraction between two bodies. An object of mass m located at a point where the acceleration due to gravity has magnitude g will have a weight W whose magnitude is given by

$$W = mg$$

And it is always directed vertically downward (towards the centre of the earth). **Normally we assume that weight is measured in an inertial frame. If it is measured in a non-inertial frame, it is called apparent weight.**

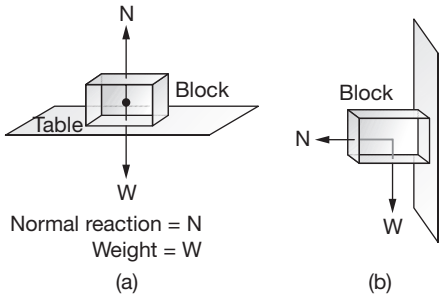
WEIGHT OF A BODY IS A NON-CONTACT FORCE. It acts on the body whether the body is in contact with a surface or not.

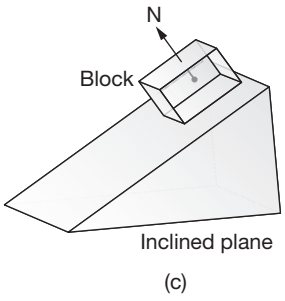
Weight is the force with which the earth attracts other bodies. It is also called the **force of gravity** or the **gravitational force**. It acts upon all the bodies near the earth. If they do not fall to the earth, then their motion is restricted by certain other bodies : a support, string, spring, etc. Bodies that restrict the motion of other bodies are called **constraints**. These bodies restrict the motion of the given bodies and hence oppose free motion.

EXAMPLE:
The surface of the table is the constraint for all objects lying on it, the floor serves as the constraint for the table, etc.

NORMAL REACTION/ NORMAL CONTACT FORCE

Normal Reaction (N) acts on a body when a body is lying on a surface or leaning against (supporting against) a surface. Normal reaction always acts normal to surface on which the body is kept or against which the body is leaning.





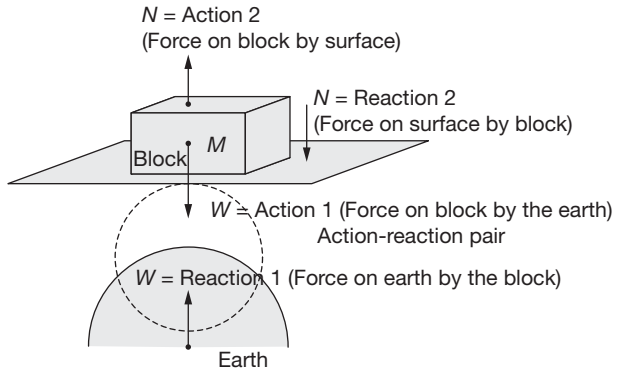
Conceptual Note(s)

Generally students think that the weight of the body, *W*, acts on the surface on which the body is kept. This is not so because WEIGHT IS A NON-CONTACT FORCE AND WILL ACT ON THE BODY IRRESPECTIVE OF THE FACT WHETHER THE BODY IS LYING ON THE SURFACE OR NOT.

However, when the body is kept on the surface, then the body will make its presence felt to the surface by exerting a force called normal reaction *N* on the surface.

CASE-1: Consider a block of mass M, weight W, place on a rigid horizontal surface. Then the forces are drawn as shown.

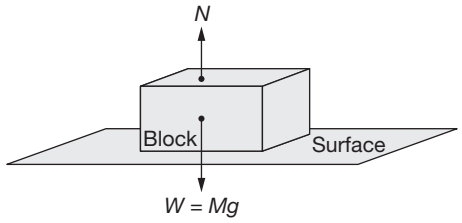
Out of these two action-reaction force pairs, the Reaction 1 (force on earth by the block) and Reaction 2 (force on surface by the block) do not contribute anything significant to the motion as the earth is massive and the surface is rigid and not moving, so our free body diagram can be redrawn as shown.


N = Action 2
(Force on block by surface)
N = Reaction 2
(Force on surface by block)
Block
M
W = Action 1 (Force on block by the earth)
Action-reaction pair
W = Reaction 1 (Force on earth by the block)
Earth

So, for the block to stay in equilibrium on the rigid surface, we have from Newton's Second Law,

$$N - Mg = 0$$

$$\Rightarrow \quad N = Mg$$



Word of Advice

You may come across situations, when the forces are equal in magnitude and opposite in directions. Please **do not treat them as Action-Reaction pair**, because action and reaction must act on different bodies. So, $N = mg$ is not a consequence of Newton's Third Law. N and mg are not to be taken as action-reaction pair. Hence the equation $N = mg$ actually follows from Newton's Second Law and not from Newton's Third Law.

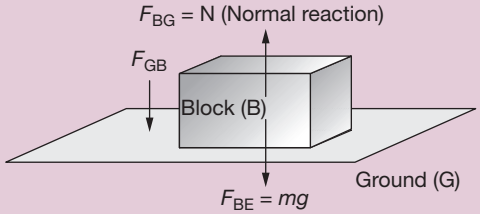
According to *NSL*,

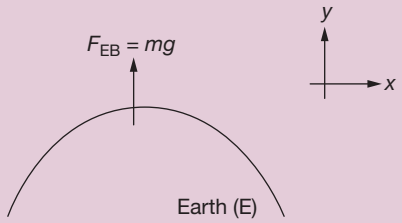
$$\sum \vec{F} = m\vec{a}$$

$$\Rightarrow \quad \sum F_x = ma_x$$

$$\Rightarrow \quad \sum F_y = ma_y$$

$$\Rightarrow \quad \sum F_z = ma_z$$





Since there is no motion along y-axis,

$$N(\text{up}) + mg(\text{down}) = m(0)$$

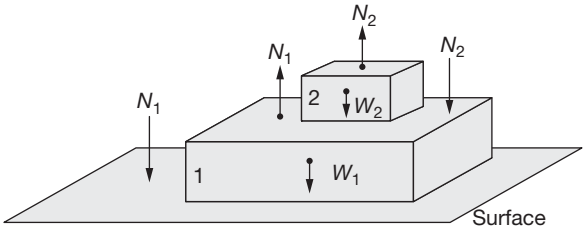
$$N - mg = 0$$

$$N = mg$$

So, this equation is a consequence of Newton's Second Law.

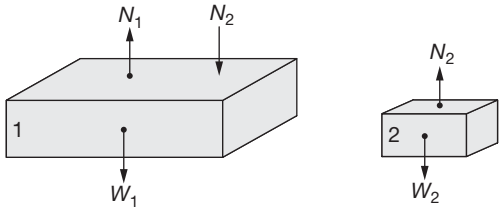
CASE-2: Now let us give CASE-1 an extension, where we place another the block 2 on block 1 (as in CASE-1), where we are designating the block placed on ground/surface as 1 and the block placed on 1 as 2 as shown. **Here, now we have not shown the earth in the diagram as the forces acting on it are negligible to produce any significant effect on it.**
To understand the forces drawn, read the following stepwise arguments.

(a) First of all, both the blocks will be attracted towards the centre of the earth. This force of attraction, called the weight is shown as W_1 and W_2 in the figure.



(b) Now the block 1 is placed on the rigid surface, so the surface exerts a normal force N_1 on the block 1 upwards. Correspondingly the block 1 exerts a normal force N_1 (due to Newton's Third Law) downwards on the surface as shown. Actually till this point we have just rediscussed CASE-1.

(c) However, now the block 2 is placed on block 1, so the block 1 exerts a normal force N_2, upwards on block 2. Correspondingly the block 2 also exerts a normal force of equal magnitude but opposite direction on block 1 as shown.



Again using Newton's Second Law, we get

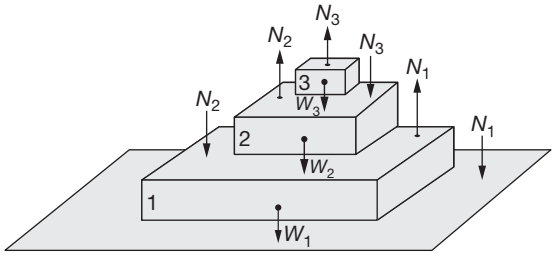
$$N_1 = N_2 + W_1 \qquad \ldots(1)$$

$$N_2 = W_2 \qquad \ldots(2)$$

So, from (1) and (2), we get

$$N_1 = W_1 + W_2$$

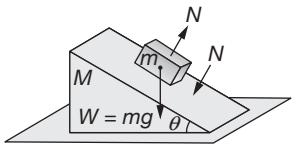
Similarly, for three blocks we can proceed in the same manner, to get $N_1 = (W_1 + W_2 + W_3)$.



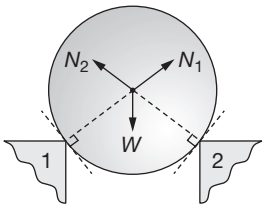
NORMAL REACTION FOR VARIOUS SITUATIONS

Normal reactions in various situations is shown in Figures given below.

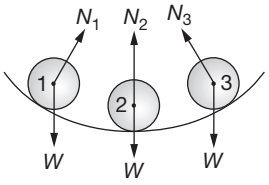
(a)



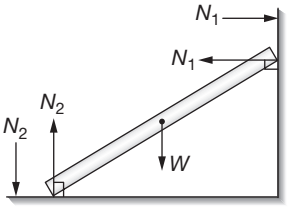
(b)



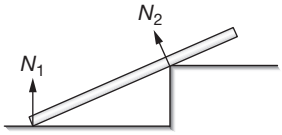
(c)



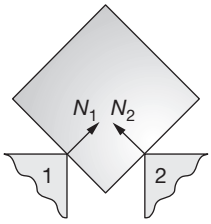
(d)



(e)



(f)

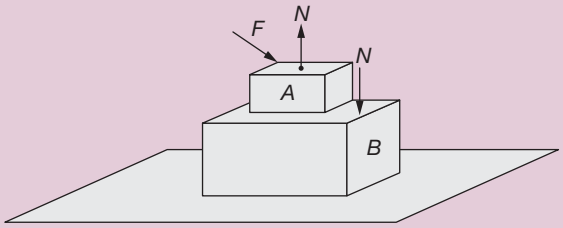


Conceptual Note(s)

Whenever a force acts on a body, it changes the motion of the body. As the motion of a body or bodies is concerned there can be two type of forces.

(a) External Forces. External forces are those which act from outside of the system, only action acts on the system, reaction of these forces are not utilized by the system.

(b) Internal Forces. Internal forces are those which are developed within the system bodies, hence, both action and reaction of these forces are in the system. If we consider a situation, shown in figure, box *A* is placed over box *B*, and a force *F* is applied on box *A*. Here system includes two blocks, *A* and *B*, and the force *F* which is acting from outside of the system is an external force and the normal contact force between the two blocks is the internal force of this system. Now will discuss properties and applications for some important forces.



TENSION IN A LIGHT STRING

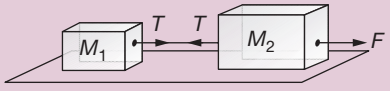
The force exerted at any point in the light rope/string/wire/rod is called the **tension** at that point. We may measure the tension at any point in the light rope by cutting a suitable length from it and inserting a spring scale, then the tension is the reading of the scale.

The tension is same at all points in the rope only if the rope is unaccelerated and assumed to be massless.

Whenever a thread or a rope or a wire is exposed to some kind of force, a tension (T) develops in it.

Conceptual Note(s)

(a) Tension in any branch of string must be shown as a pair.
(The pair is shown with two arrows facing each other).
When two blocks (M_1 and M_2 say) connected by a string are pulled, then tension on M_1 acts towards M_2 and that on M_2 acts towards M_1 as shown in the figure.



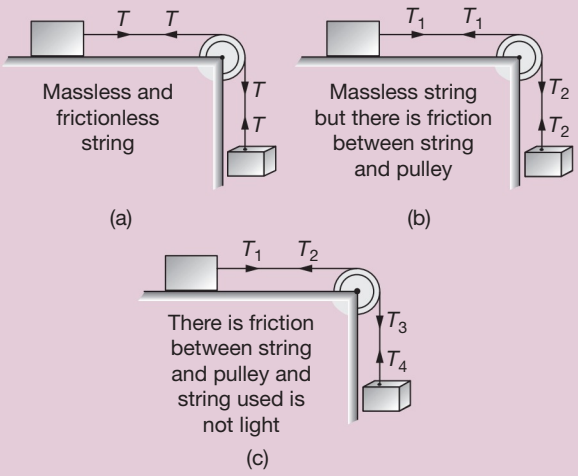
(b) If a body *A* attached to an ideal light inextensible string, is pulled with a force *F* and it is placed on a horizontal frictionless surface, then

Tension in the string = Pull (F)

(c) Force of tension act on a body in the direction away from the point of contact or tied ends of the string. For example consider figure. A man pulls a box with a string. The tension in string acts on the box towards right or in the direction away from the tied point and on the man it is again away from it. The way of showing the direction of tension is shown in figure.

(d) If string is massless and frictionless, tension throughout the string remains constant as shown in figure (a). But if the string is massless and not frictionless, at every contact in the length of the string tension changes and if it is not light, tension at each point will be different depending on the acceleration of the string.

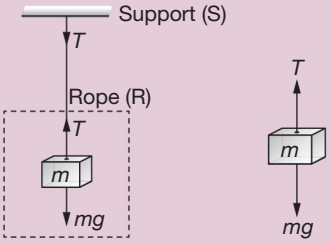
EXAMPLE 1: Consider the situation shown in figure (a). A box is tied to another mass with a string going over a pulley. If string is massless and there is no friction between the contact of string and pulley surface, tension throughout the string remains same as T and as there is no friction between pulley and string, string will not be able to rotate the pulley and it will slide on the surface of pulley. But if there is friction between surface of pulley and the string, due to friction, pulley will rotate on its axis as the string slides on it. In this case, due to friction between pulley and the string, tensions in string on two sides of the pulley will be different as shown in figure (b). If string has a mass, it will accelerate and tension at each point will be different on the string as shown in figure (c). How this tension can be obtained, we will explain in further sections.



EXAMPLE 2:

$$T - mg = 0$$

$$\Rightarrow \quad T = mg$$



EXAMPLE 3:

$$T_1 = m_1 g + T_2 \qquad \ldots(1)$$

$$T_2 = m_2 g \qquad \ldots(2)$$

From (1) and (2), we get

$$T_1 = (m_1 + m_2)g$$

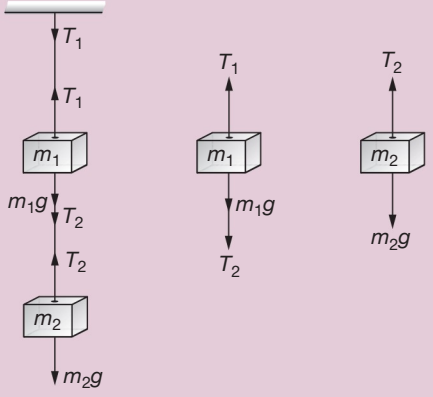
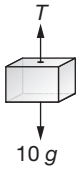


ILLUSTRATION 18

A block of mass 10 kg is suspended with string as shown in figure. Find tension in the string. $\left(g = 10\ \text{ms}^{-2}\right)$.

SOLUTION

F.B.D. of block



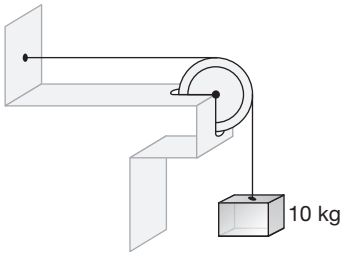
$$\Sigma F_y = 0$$

$$T - 10g = 0$$

$$\Rightarrow \quad T = 100\ \text{N}$$

ILLUSTRATION 19

Find magnitude of force exerted by string on pulley.

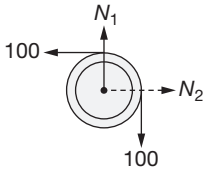


SOLUTION

F.B.D. of 10 kg **block**

$$T = 10g = 100\ \text{N}$$

F.B.D. of pulley



Since string is massless, so tension in both sides of string is same. Force exerted by string is

$$F = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ \text{N}$$

Conceptual Note(s)

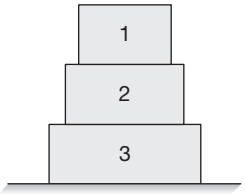
Since pulley is in equilibrium position, so net forces on it is zero. However force exerted by hinge on the pulley is $100\sqrt{2}$ N.

Test Your Concepts-III

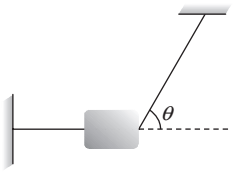
Based on FBD

(Solutions on page H.198)

1. Three blocks 1, 2 and 3 are placed one over the other as shown in figure. Draw free body diagrams of all the three blocks.

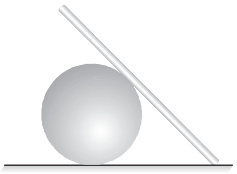


2. A block of mass *m* is attached with two strings as shown in figure. Draw the free body diagram of the block.



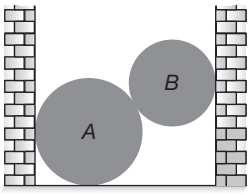
3. A spherical ball of radius R, weight W is resting on a V-groove as shown in figure. Draw its free body diagram.

4. The diagram shows a rough plank resting on a cylinder (shown cross-sectionally) with one end of the plank on rough ground. Neglect friction between plank and cylinder. Draw diagrams to show the



(a) forces acting on the cylinder.
(b) forces acting on the plank.

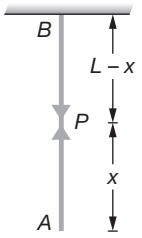
5. Two spheres A and B are placed between two vertical walls as shown in figure. Friction is absent everywhere. Draw the free body diagrams of both the spheres.



TENSION IN A ROPE HAVING UNIFORM MASS DISTRIBUTION

Now, if the rope/thread has a uniform mass, then the tension at a point P at distance x from the free end is equal to the weight of rope that hangs below P. So,

$$\begin{pmatrix}\text{Tension at}\\ \text{point } P\end{pmatrix} = \begin{pmatrix}\text{Weight of rope that}\\ \text{hangs below the point } P\end{pmatrix}$$



$$\Rightarrow \quad T_P = (\lambda x)g$$

where $\lambda = \dfrac{M}{L}$

M = total mass of rope
L = total length of rope and
λ = linear mass density

Tension at A

$x = 0$

$\Rightarrow \quad T_A = 0$

(we can think that no part of rope hangs below A, so $T_A = 0$)

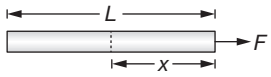
Tension at B

$x = L$

$\Rightarrow \quad T_A = (\lambda L)g = Mg$ = Total weight of rope.

ILLUSTRATION 20

A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance x from the end where force is applied.

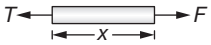


SOLUTION

Considering rod as a system, the acceleration of rod is

$$a = \frac{F}{m}$$

Now draw F.B.D. of rod having length x as shown in figure



According to Newton's Second Law, we have

$$\Sigma F = ma$$

$$\Rightarrow \quad F - T = \left[\frac{M}{L}\right] xa$$

$$\Rightarrow \quad T = F - \left(\frac{M}{L}x\right)\left(\frac{F}{M}\right)$$

$$\Rightarrow \quad T = F\left(1 - \frac{x}{L}\right)$$

ILLUSTRATION 21

For the situation given in figure, find the acceleration of each body for the following two cases.

(a) if the thread has a mass m.
(b) if the thread is massless.

It is given that masses of A and B are M_1 and M_2 respectively and the horizontal surface is smooth.

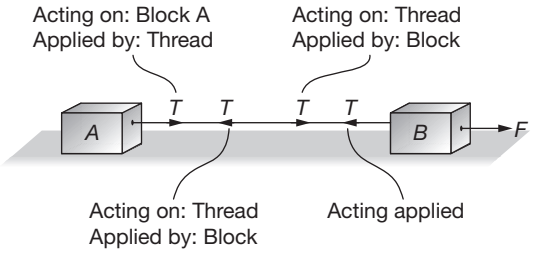
SOLUTION

(a) Suppose the two bodies and the thread are moving along the force F with an acceleration of a, as shown in figure. Then, we have,

$$T_1 = M_1 a \qquad \ldots(1)$$

$$T_2 - T_1 = ma \qquad \ldots(2)$$

$$F - T_2 = M_2 a \qquad \ldots(3)$$



Adding (1), (2) and (3), we get

$$F = (M_1 + M_2 + m)a$$

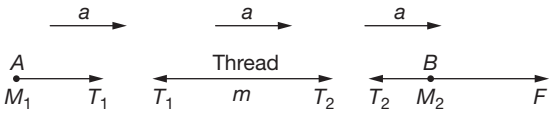
$$\Rightarrow \quad a = \frac{F}{M_1 + M_2 + m}$$

$$\Rightarrow \quad T_1 = M_1 a$$

$$T_1 = \frac{M_1 F}{M_1 + M_2 + m}$$

and $T_2 = F - M_2 a$

$$\Rightarrow \quad T_2 = \frac{(M_1 + m)F}{M_1 + M_2 + m}$$



(b) If thread is massless, then putting $m = 0$ (for any a) in equation (2), we get

$$T_1 = T_2$$

Again, after solving (1) and (3), we get

$$a = \frac{F}{M_1 + M_2}$$

and $T_1 = T_2 = \dfrac{M_1 F}{M_1 + M_2}$

Conceptual Note(s)

(a) Tensions at different points in a massless string is the same.

(b) Acceleration of each body could be calculated very easily if we would have assumed M_1, M_2 and thread as parts of a single system. In this way, action and reaction forces acting on different parts of the same system must get cancelled out and we would have to take care of only external forces. In this way, we can write.

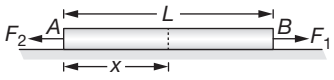
$$F_{ext} = M_{sys} a$$

$$\Rightarrow \quad a = \frac{F_{ext}}{M_{sys}} = \frac{F}{M_1 + M_2 + m}$$

(Put $m = 0$, if the thread is massless)

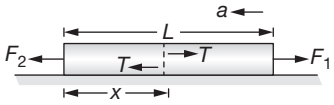
ILLUSTRATION 22

Two forces F_1 and $F_2\,(>F_1)$ are applied at the free ends of uniform rod kept on a horizontal frictionless surface. Find tension in rod at a distance x from end A.

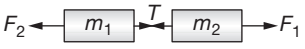


SOLUTION

$$a=\frac{F_2-F_1}{m}$$



Since, $T-F_1=m_2a$, where $m_2=\frac{m}{L}(L-x)$



$$\Rightarrow \quad T-F_1=\frac{m}{L}(L-x)\frac{F_2-F_1}{m}$$

$$\Rightarrow \quad T=F_1+\left(1-\frac{x}{L}\right)(F_2-F_1)$$

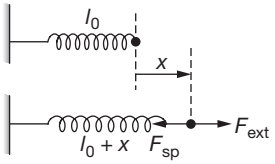
$$\Rightarrow \quad T=F+F_2-F_1-\frac{x}{L}(F_2-F_1)$$

$$\Rightarrow \quad T=F_2-(F_2-F_1)\frac{x}{L}$$

SPRING FORCE

Every spring opposes the attempts to change its length i.e. **every spring opposes the phenomenon of compression or extension.** This opposing or the resistive force called as the Restoring Force or the spring force F increases with change in length of the spring i.e., Extension or Compression (x).

When spring is in its natural length l_0, spring force (F_{sp}) is zero.



$$F_{sp}=0$$

$$F_{sp}=F=-kx$$

Spring force is given by $F_{sp}=-kx$, where x is the change in length of the spring (also called as compression or extension) and k is the Spring Constant or Force constant having SI unit $\mathrm{Nm^{-1}}$ and dimensional formula MT^{-2}. The spring constant (k) is also known as **force constant/spring factor/force factor.**

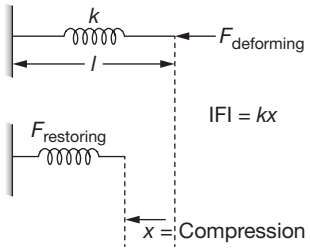
The spring constant k depends on geometry of the spring and on the material property. For us, it is important to know that the spring constant is inversely proportional to its natural length l, other things remaining the same i.e.,

$$k\propto\frac{1}{l} \qquad \{\text{everything else constant}\}$$

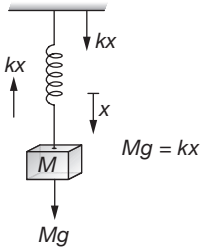
$$\Rightarrow \quad kl=\text{constant}$$

Therefore if you cut a spring into two parts whose length are in ratio $1:2$, their spring constants will be in ratio of $2:1$.

The negative sign in the above relation signifies that the spring force is always directed opposite to the compression or extension in the spring.



CASE-1: Horizontal Placement



CASE-2: Vertical Placement

As in case of rope, we will usually deal with a massless spring for which the force at each point is the same. Such springs are normally referred to as ideal.

Conceptual Note(s)

(a) For a spring of natural length ℓ, spring constant k, we have,

$$k\ell=\text{constant}$$

(b) No need to take the negative sign as the direction has already been taken.

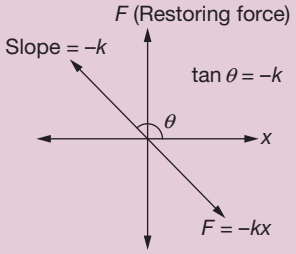
(c) The more the value of k, the more the elastic a spring is.

(d) Soft springs have small k in comparison to hard spring.

$$k_{soft} \ll k_{hard}$$

e.g., Spring used in a ball pen (soft spring). Spring used in the shocker of the car (hard spring).

(e) The graph showing variation of restoring force F with extension/compression x is shown here.



(f) If a spring with spring constant k is divided into n equal parts, the spring constant of each part is nk.

(g) If springs of spring constants k_1, k_2, k_3 ... are connected in series, then effective force constant

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots$$

(h) If springs of spring constants k_1, k_2, k_3 ... are connected in parallel, then effective spring constant.

$$k_{eff} = k_1 + k_2 + k_3 + \ldots$$

(i) SI unit of k is Nm^{-1} and $[k] = MT^{-2}$.

(j) When a spring is cut, then to calculate the force constant of each divided part we have

$$k\ell = k_1\ell_1 = k_2\ell_2 = \ldots = k_n\ell_n$$

ILLUSTRATION 23

A spring of force constant k and natural length l is cut into two parts of lengths $\frac{l}{3}$ and $\frac{2l}{3}$. Find the new force constant of the divided parts.

SOLUTION

Since, kl = constant

$$\Rightarrow \quad kl = k_1 l_1 = k_2 l_2 = \ldots\ldots$$

So, $kl = k_1 l_1$

$$\Rightarrow \quad kl = k_1\left(\frac{2l}{3}\right)$$

$$\Rightarrow \quad k_1 = \frac{3k}{2}$$

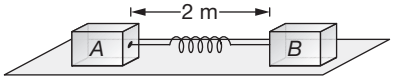
So, $kl = k_2 l_2$

$$\Rightarrow \quad kl = k_2\left(\frac{l}{3}\right)$$

$$\Rightarrow \quad k_2 = 3k$$

ILLUSTRATION 24

Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 Nm^{-1}. Find spring force in following situations.



(a) If block A and B both are displaced by 0.5 m in same direction.

(b) If block A and B both are displaced by 0.5 m in opposite direction.

SOLUTION

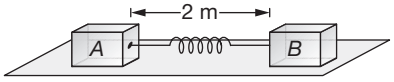
(a) Since both blocks are displaced by 0.5 m in same direction, so change in length of spring is zero. Hence, spring force is zero.

(b) In this case, change in length of spring is 1 m. So spring force is

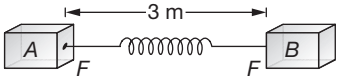
$$F = -Kx = -(200)(1)$$

$$\Rightarrow \quad F = -200 \text{ N}$$

Natural length



When spring is extended



When spring is compressed

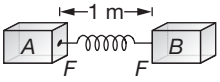
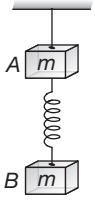


ILLUSTRATION 25

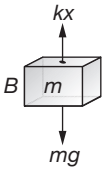
Two blocks A and B of same mass m attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block A and B just after the string is cut.



SOLUTION

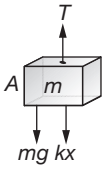
When block A and B are in equilibrium position, then

F.B.D. of B



$$kx = mg \qquad \ldots(1)$$

F.B.D. of A

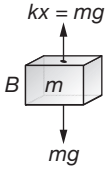


$$T = mg + kx \qquad \ldots(2)$$

$$\Rightarrow \quad T = 2mg$$

Now, when string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of blocks A and B as shown in figure.

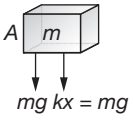
F.B.D. of B



$$kx - mg = ma_B$$

$$\Rightarrow \quad a_B = 0$$

F.B.D. of A



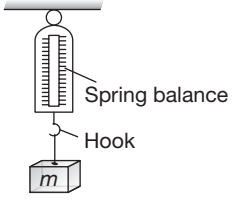
$$mg + kx = ma_A$$

$$\Rightarrow \quad 2mg = ma_A$$

$$\Rightarrow \quad a_A = 2g \quad \text{(downwards)}$$

SPRING BALANCE

A spring balance does not measure the weight. It measures the force exerted by the object at the hook. Symbolically, it is represented as shown in figure. A block of mass m is suspended at hook.



When spring balance is in equilibrium, we draw the F.B.D. of mass m for calculating the reading of balance.

Unless and until mentioned, a spring balance is assumed to be light or having negligible mass.

F.B.D. of m

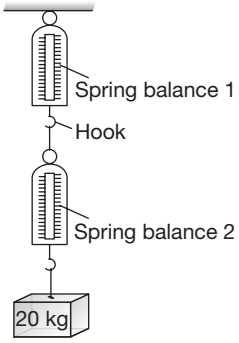
$$mg - T = 0$$

$$\Rightarrow \quad T = mg$$

Magnitude of T gives the reading of spring balance.

ILLUSTRATION 26

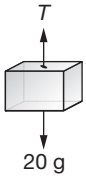
A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the



(a) reading of spring balance 1.
(b) reading of spring balance 2.

SOLUTION

For calculating the reading, first we draw F.B.D. of 20 kg block.



$$mg - T = 0$$

$$\Rightarrow \quad T = 20g = 200 \text{ N}$$

Since both balances are light so, both the scales will read 20 kg.

NEWTON'S SECOND LAW: REVISITED

We know from the first law, what happens when there is no unbalanced force on an object: its velocity remains constant. Now let us see What happens when there is an unbalanced force on an object? The Newton's second Law gives answer to this question, that is, net force acting on a body will produce an acceleration.

When there is a constant unbalanced force on an object, the object moves with constant acceleration. Furthermore, if the force varies, the acceleration varies in direct proportion with larger force producing larger acceleration. Twice the force produces twice the acceleration in the same mass.

The magnitude of the acceleration produced depends on the quantity of matter being pushed. The quantity of matter is referred to as the inertial mass.

Newton's Second Law states the relation between the net force and the inertial mass.

$$\sum \vec{F} = m\vec{a}$$

Note that the direction of acceleration is in the direction of the net force.

In terms of components

$$\sum F_x = ma_x,$$

$$\sum F_y = ma_y \text{ and}$$

$$\sum F_z = ma_z$$

Problem Solving Technique(s)

HOW TO MAKE FREE BODY DIAGRAMS (FBD): REVISITED

With the help of free body diagrams, it becomes easy for us to consider the motion of the object after taking the account the forces acting on it.

Rest is the case of motion where the net force acting on the body is zero. This is also the case of Equilibrium.

(a) Select a convenient co-ordinate axes.
(b) Isolate the object of interest.

(c) Draw all the forces that are acting on the isolated object keeping in mind that every force must be a part of action-reaction pair acting on two different bodies.
When drawing the FBD of the isolated object, **never take into account the forces that the isolated object is acting on others. Just draw the forces that are acting directly on the isolated object.**

(d) Apply Newton's Second Law such that we have

$$\sum \vec{F} = m\vec{a}$$

$$\Rightarrow \quad \sum F_x = ma_x, \sum F_y = ma_y, \sum F_z = ma_z$$

and obtain the equations required.

(e) If the body is at Rest or moving with a constant velocity, then the acceleration of the body is ZERO and this is also the CASE OF EQUILIBRIUM.

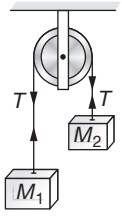
ATWOOD'S MACHINE

Masses M_1 and M_2 are tied to a string, which goes over a frictionless light pulley.
The string is light and inextensible.

(a) If $M_1 > M_2$ and they move with acceleration a, then

$$M_1 g - T = M_1 a$$

$$T - M_2 g = M_2 a$$



where T is the tension in the string. It gives

$$a = \left(\frac{M_1 - M_2}{M_1 + M_2}\right) g \text{ and } T = \left(\frac{2M_1 M_2}{M_1 + M_2}\right) g$$

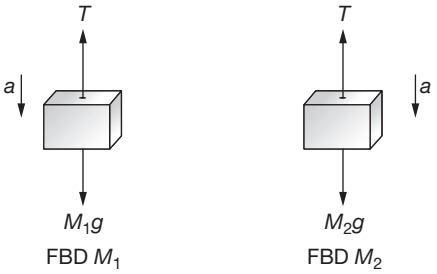
Thrust is given by $2T$. So,

$$\text{Thrust} = \left(\frac{4m_1 m_2}{m_1 + m_2}\right) g$$

(b) If the pulley begins to move with acceleration $\vec{a}_0$, then

$$a = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)(\vec{g} - \vec{a}_0)$$

and $$T = \left(\frac{2M_1 M_2}{M_1 + M_2}\right)(\vec{g} - \vec{a}_0)$$



So, if pulley accelerates up, then

$$a = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)(g + a_0)$$

and $$T = \left(\frac{2M_1 M_2}{M_1 + M_2}\right)(g + a_0)$$

and when it accelerates down, then

$$a = \left(\frac{M_1 - M}{M_1 + M_2}\right)(g - a_0)$$

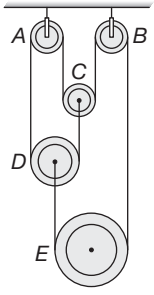
and $$T = \left(\frac{2M_1 M_2}{M_1 + M_2}\right)(g - a_0)$$

and thrust in the pulley is

$$\vec{T} = \frac{4M_1 M_2}{M_1 + M_2}(\vec{g} - \vec{a}_0)$$

ILLUSTRATION 27

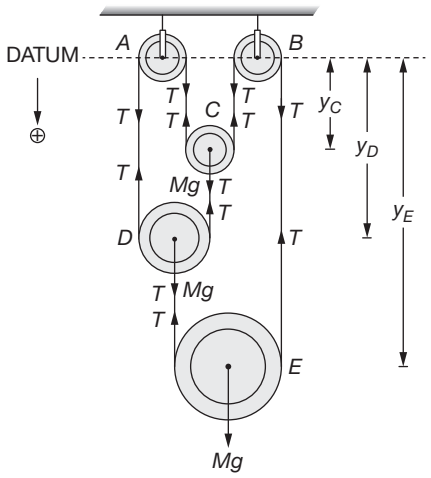
Consider identical pulleys A, B, C, D and E as shown in figure.



A and B are fixed while C, D and E are free to move. The pulleys are connected by light inextensible string. All the pulleys have identical masses say M each. At a particular instant, it is observed that C and D move down with velocity, $V_C = 2\ \text{ms}^{-1}$ and $V_D = 2\ \text{ms}^{-1}$. Calculate the velocity of pulley E at that instant. Also find the acceleration in the pulleys and the tension in the string.

SOLUTION

Let us first draw the fixed axis (called Datum) passing through the centre of pulleys A and B as shown in figure.



At any instant, the distance of the pulley C, D and E from the Datum be y_C, y_D and y_E respectively. Start from centre of C and go to centre, we observe a single thread

$\therefore$ Thread length = constant

$$\left(y_D - \cancel{y_C}\right) + \cancel{y_D} + \cancel{y_C} + y_C + y_E + \left(y_E - \cancel{y_D}\right) = \text{constant}$$

$$\Rightarrow \quad y_D + y_C + 2y_E = \text{constant} \qquad \ldots(1)$$

Take derivative w.r.t. time

$$\Rightarrow \quad \dot{y}_D + \dot{y}_C + 2\dot{y}_E = 0$$

$$\Rightarrow \quad V_D + V_C + 2V_E = 0 \qquad \ldots(2)$$

Now, take downwards as positive

$$V_C = +2\ \text{ms}^{-1}$$

$$V_D = +2\ \text{ms}^{-1}$$

$$\Rightarrow \quad 2 + 2 + 2V_E = 0$$

$$\Rightarrow \quad V_E = -2\ \text{ms}^{-1}$$

Since the pulleys are identical and the string is light and inextensible and connects from C to D as a single curve, therefore tension throughout the string will remain the same.

From (2) we get,

$$\dot{V}_D + \dot{V}_C + 2\dot{V}_E = 0$$

$$\Rightarrow \quad a_D + a_C + 2a_E = 0 \qquad \ldots(3)$$

FBD C

$$\left(Mg + T\right) - 2T = Ma_C \qquad \ldots(4)$$

$$Mg - T = Ma_C$$

FBD D

$$Mg + T - 2T = Ma_D$$

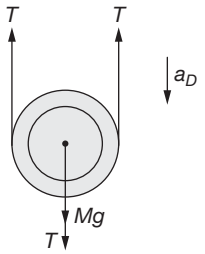
$$Mg - T = Ma_D \qquad \ldots(5)$$

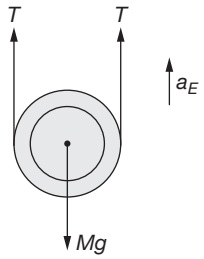
From (4) and (5), we get

$$a_C = a_D \qquad \ldots(6)$$

From (6) and (3), we get

$$a_C = a_D = -a_E \qquad \ldots(7)$$





FBD E

$$2T - Mg = Ma_E \qquad \ldots(8)$$

From (4), we get

$$Mg - T = Ma \qquad \ldots(9)$$

From (8), we get

$$2T - Mg = Ma \qquad \ldots(10)$$

From (9) and (10)

$$\left(2 \times 9 + 10\right)$$

$$2Mg - Mg = 2Ma + Ma$$

$$\Rightarrow \quad a = \frac{g}{3}$$

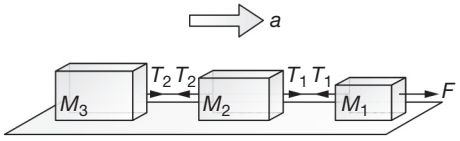
Putting a in (9),

$$Mg - T = \frac{Mg}{3}$$

$$\Rightarrow \quad 3Mg - 3T = Mg$$
$$\Rightarrow \quad 3T = 2Mg$$
$$\Rightarrow \quad T = \frac{2Mg}{3}$$

MASSES CONNECTED WITH STRINGS

Three masses M_1, M_2, M_3 are connected with strings as shown in figure and lie on a frictionless surface.



They are pulled with a force F attached to M_1.

$$F - T_1 = M_1 a$$
$$T_1 - T_2 = M_2 a$$
$$T_2 = M_3 a$$

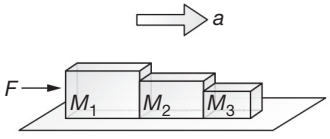
Such that the acceleration of system is

$$a = \frac{F}{M_1 + M_2 + M_3} \text{ and } T_1 = \left(\frac{M_2 + M_3}{M_1 + M_2 + M_3}\right)F$$
$$T_2 = \left(\frac{M_3}{M_1 + M_2 + M_3}\right)F$$

MASSES ON A SMOOTH SURFACE IN CONTACT WITH EACH OTHER

Three masses M_1, M_2, M_3 are placed on a smooth surface in contact with each other as shown in the figure. A force F pushes them and the three masses move with acceleration a, which is given by

$$a = \frac{F}{M_1 + M_2 + M_3}$$
$$F - F_2 = M_1 a$$
$$F_2 - F_3 = M_2 a$$
$$F_3 = M_3 a$$

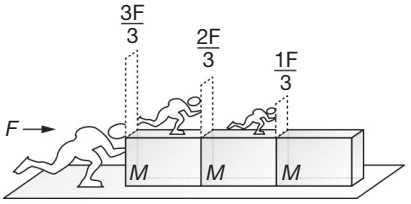


The forces acting are as follows

On M_1, the force is $F_1 = F$

On M_2, the force is $F_2 = \left(\frac{M_2 + M_3}{M_1 + M_2 + M_3}\right)F$

On M_3, the force is $F_3 = \left(\frac{M_3}{M_1 + M_2 + M_3}\right)F$



F_2 and F_3 are the forces of contact between M_1, M_2 and M_2, M_3 respectively.

If $M_1 = M_2 = M_3 = M(\text{say})$, then $a = \frac{F}{3M}$.

The forces acting are as shown in above figure.

Similarly we have seven identical masses then

$$a = \frac{F}{7M}$$

The forces acting are shown in above figure.

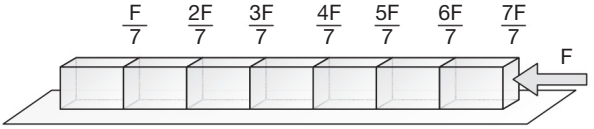
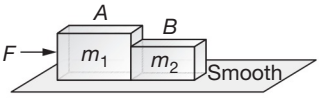


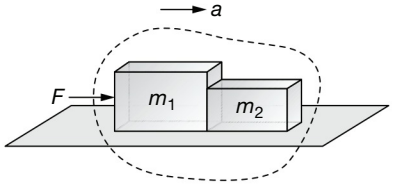
ILLUSTRATION 28

A force F is applied horizontally on mass m_1 as shown in figure. Find the contact force between m_1 and m_2.



SOLUTION

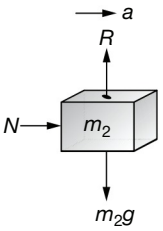
Considering both blocks as a system to find the common acceleration.



Common acceleration

$$a = \frac{F}{(m_1 + m_2)} \qquad \ldots(1)$$

To find the contact force between A and B we draw F.B.D. of mass m_2.



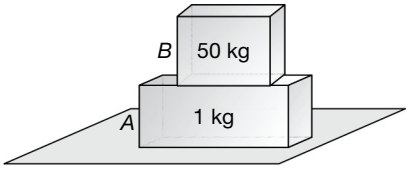
$$\Sigma F_x = ma_x$$

$$N = m_2 \cdot a$$

$$N = \frac{m_2 F}{(m_1 + m_2)}$$

ILLUSTRATION 29

A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 1 kg block. Assuming all surfaces to be smooth and taking $g = 10\ \text{ms}^{-2}$. Calculate the

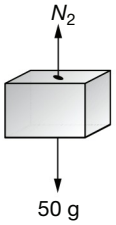


(a) acceleration of block A and B.

(b) force exerted by B on A.

SOLUTION

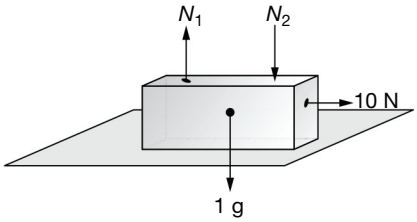
(a) F.B.D. of 50 kg block



$$N_2 = 50g = 500\ \text{N}$$

Along horizontal direction, there is no force, so $a_B = 0$.

(b) F.B.D. of 1 kg block



Along horizontal direction

$$10 = 1a_A$$

$$\Rightarrow \quad a_A = 10\ \text{ms}^{-2}$$

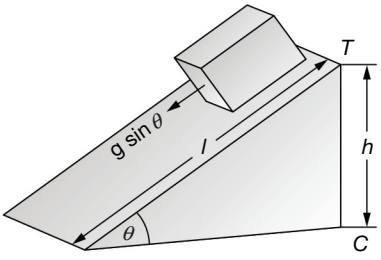
Along vertical direction

$$\Rightarrow \quad N_1 = N_2 + 1g$$

$$\Rightarrow \quad N_1 = 500 + 10 = 510\ \text{N}$$

BODY ON A SMOOTH INCLINED PLANE

Consider a body starting from rest moves along a smooth inclined plane of length l, height h and having angle of inclination θ.



(a) Its acceleration down the plane is $g\sin\theta$.

(b) Its velocity at the bottom of the inclined plane will be

$$\sqrt{2gh} = \sqrt{2gl\sin\theta}$$

(c) Time taken to reach the bottom will be

$$t = \sqrt{\frac{2l}{g\sin\theta}} = \sqrt{\frac{2h}{g\sin^2\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$

(d) If angles of inclination are θ_1 and θ_2 keeping the height constant then $\frac{t_1}{t_2}=\frac{\sin\theta_2}{\sin\theta_1}$.

(e) If the angle of inclination is changed from θ_1 to θ_2 keeping the length constant then $\frac{t_1}{t_2}=\sqrt{\frac{\sin\theta_2}{\sin\theta_1}}$.

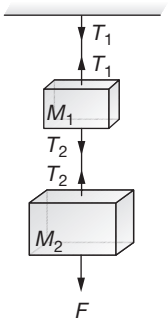
(f) The ratio of times to reach the bottom in the two cases are related to each other as

$$\left[\frac{t_1}{t_2}\right]_{\text{constant height}}=\left[\frac{t_1}{t_2}\right]^2_{\text{constant length}}$$

WHEN MASSES ARE SUSPENDED VERTICALLY FROM A RIGID SUPPORT

Two blocks of masses M_1 and M_2 are suspended vertically from a rigid support with the help of strings as shown in the figure. The mass M_2 is pulled down with a force F. The tension between the masses M_1 and M_2 will be

$$T_2=F+M_2g$$



Tension between the support and the mass M_1 will be

$$T_1=F+(M_1+M_2)g$$

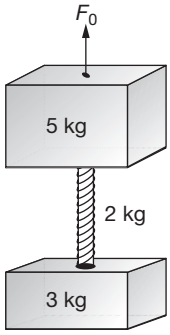
Problem Solving Technique(s)

Whenever a light string is pulled then tension in the light string at all the points is equal to the pull.

ILLUSTRATION 30

A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at $2\ \text{ms}^{-2}$ by an external force F_0.

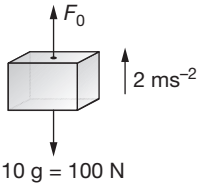
(a) What is F_0?
(b) What is the net force on rope?
(c) What is the tension at middle point of the rope? $(g=10\ \text{ms}^{-2})$



SOLUTION

For calculating the value of F_0, consider two blocks with the rope as a single system.

(a) F.B.D. of whole system



$$F_0-100=10\times2$$

$$\Rightarrow\quad F_0=120\ \text{N} \qquad \ldots(1)$$

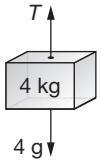
(b) According to Newton's Second Law, net force on rope

$$F=ma \quad \text{where } m=m_{\text{rope}}=2\ \text{kg}$$

$$\Rightarrow\quad F=(2)(2)$$

$$\Rightarrow\quad F=4\ \text{N} \qquad \ldots(2)$$

(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown. So, total mass becomes 4 kg.



$$T-4g=4(2)$$

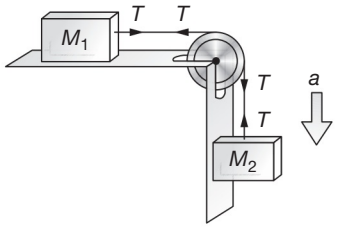
$$\Rightarrow\quad T=48\ \text{N}$$

WHEN TWO MASSES ARE ATTACHED TO A STRING WHICH PASSES OVER A PULLEY ATTACHED TO THE EDGE OF A HORIZONTAL TABLE

Let the mass M_1 lies on the frictionless surface of the table (as shown in the figure). Let the tension in the string be T and the acceleration of the system be a. Then from Cause-Effect equations, we get

$M_2 g - T = M_2 a$ and $T = M_1 a$, such that

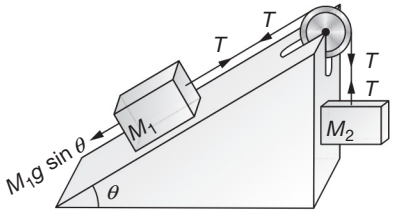
$$a = \left(\frac{M_2}{M_1 + M_2}\right) g \text{ and } T = \left(\frac{M_1 M_2}{M_1 + M_2}\right) g$$



The system is accelerated for all values of M_1 and M_2 (as long as friction is assumed to be absent)
Thrust on pulley is $\sqrt{2}T$.

WHEN TWO MASSES ARE ATTACHED TO A STRING WHICH PASSES OVER A PULLEY ATTACHED TO THE EDGE OF AN INCLINED PLANE

Two masses M_1 and M_2 are attached to the ends of a string, which passes over a frictionless pulley at the top of the inclined plane of inclination θ. Let the tension in the string be T.



Two cases arise.

(a) When the mass M_1 moves upwards with acceleration a, then

$$a = \left(\frac{M_2 - M_1 \sin\theta}{M_1 + M_2}\right) g$$

$$T = \left(\frac{M_1 M_2 (1 + \sin\theta)}{M_1 + M_2}\right) g$$

(b) When the mass M_1 moves downwards with acceleration a, then

$$a = \left(\frac{M_1 \sin\theta - M_2}{M_1 + M_2}\right) g$$

$$T = \left(\frac{M_1 M_2 (1 + \sin\theta)}{M_1 + M_2}\right) g$$

Thrust F on the pulley in both the cases is the resultant of T and T inclined at an angle of $(90 - \theta)$. So,

$$F = \sqrt{T^2 + T^2 + 2T^2 \cos(90 - \theta)}$$

$$F = \sqrt{2}T\sqrt{1 + \sin\theta}$$

Since $\sqrt{1 + \sin\theta} = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)$

$$F = \sqrt{2}T\left[\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right]$$

From the above relations we can derive the following conclusions.

(i) System does not accelerate $(a = 0)$ when $M_2 = M_1 \sin\theta$.

(ii) If the positions of the masses are reversed, the tension in the string remains unchanged. Also, the acceleration of the system remains unchanged.

(iii) If $M_1 = M_2 = M$ (say), then

$$a = \frac{1}{2}(1 - \sin\theta) g$$

$$\Rightarrow \quad a = \frac{1}{2}\left\{\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right\}^2 g$$

$$\Rightarrow \quad a = \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2 \left(\frac{g}{2}\right) \text{ and}$$

$$T = \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2 \left(\frac{Mg}{2}\right)$$

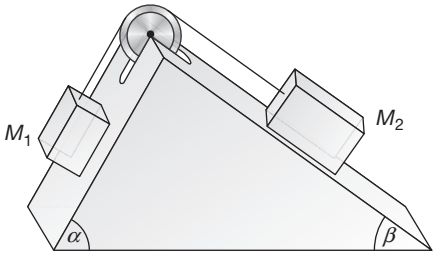
WHEN TWO MASSES ARE ATTACHED TO A STRING WHICH PASSES OVER A PULLEY ATTACHED TO THE TOP OF A DOUBLE INCLINED PLANE

Two masses M_1 and M_2 are attached to the ends of a string over a pulley fixed to the top of a double inclined plane of angles of inclination α and β.
Let M_2 move downwards with acceleration a and the tension in the string be T. Then

$$a = \left(\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right) g$$

$$T = \frac{M_1 M_2 g}{(M_1 + M_2)}[\sin\beta + \sin\alpha]$$

For system not to accelerate we must have $M_2 \sin\beta = M_1 \sin\alpha$.



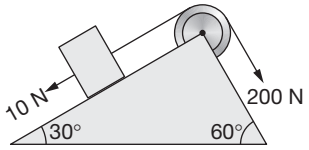
Thrust F on the pulley is the resultant of T and T inclined at an angle of $180-(\alpha+\beta)$. So,

$$\text{Thrust} = \sqrt{T^2 + T^2 + 2T^2\cos\{180-(\alpha+\beta)\}}$$

$$\Rightarrow \quad F = \sqrt{2}T\sqrt{1-\cos(\alpha+\beta)} = 2T\sin\left(\frac{\alpha+\beta}{2}\right)$$

ILLUSTRATION 31

A 10 kg block kept on an inclined plane is pulled by a string applying 200 N force. A 10 N force is also applied on 10 kg block as shown in figure. Calculate the

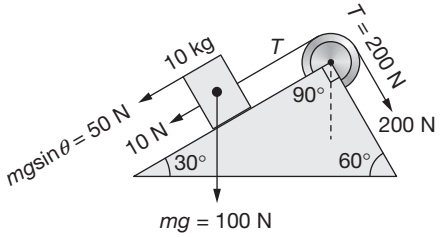


(a) tension in the string.
(b) acceleration of 10 kg block.
(c) net force on pulley exerted by string.

SOLUTION

(a) $T = 200$ N

(b) $T - 10 - mg\sin\theta = ma$

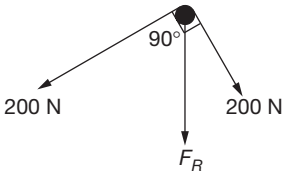


$$\Rightarrow \quad T - 10 - 50 = 10a$$

$$\Rightarrow \quad 200 - 60 = 10a$$

$$\Rightarrow \quad a = \frac{140}{10} = 14 \text{ ms}^{-2}$$

(c)



$$\Rightarrow \quad F_R = \sqrt{(200)^2 + (200)^2}$$

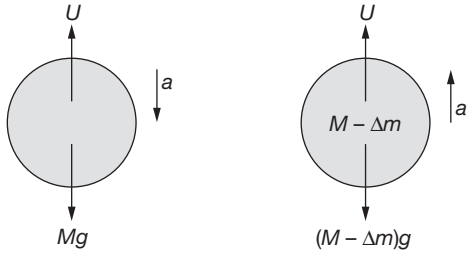
$$\Rightarrow \quad F_R = 200\sqrt{2} \text{ N}$$

ILLUSTRATION 32

An aerostat of mass m starts coming down with a constant acceleration a. Determine the ballast mass to be dumped from the aerostat to reach the upward acceleration of the same magnitude. The air drag is to be neglected.

SOLUTION

$$Mg - U = Ma \qquad \ldots(1)$$



Let the mass Δm be dumped

$$U - (M-\Delta m)g = (M-\Delta m)a \qquad \ldots(2)$$

Adding (1) and (2),

$$Mg - (M-\Delta m)g = Ma + (M-\Delta m)a$$

$$\Delta mg = a(2m - \Delta M)$$

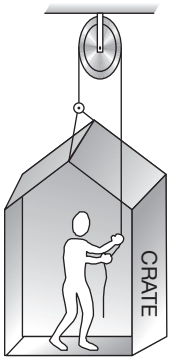
$$\Delta Mg = 2ma - \Delta Ma$$

$$\Delta M(g+a) = 2ma$$

$$\Delta M = \frac{2Ma}{g+a}$$

ILLUSTRATION 33

A painter of mass 100 kg is raising himself and the crate (such an arrangement is called Bosun's chair) on which he stands as shown. When he pulls the rope the force exerted by him on the crate's floor is 450 N. If the crate weighs 25 kg then, find the acceleration of the system and the tension in the rope.



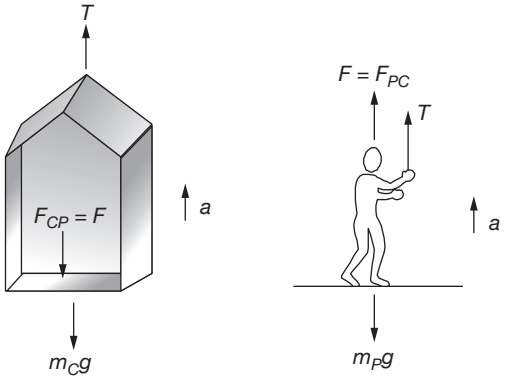
SOLUTION

$$T + F - m_P g = m_P a \quad \text{(For Painter)} \qquad \ldots(1)$$

$$T - F - m_C g = m_C a \quad \text{(For crate)} \qquad \ldots(2)$$

Subtract (2) from (1), we get

$$2F + (m_C - m_P)g = (m_P - m_C)a$$



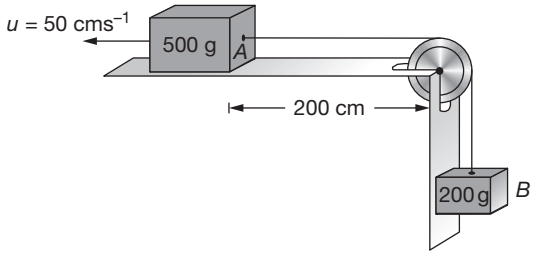
$$\Rightarrow \quad 900 + (25 - 100)(10) = 75a$$

$$\Rightarrow \quad 900 - 750 = 75a$$

$$\Rightarrow \quad a = 2 \text{ ms}^{-2},\ T = 750 \text{ N}$$

ILLUSTRATION 34

A block A of mass 500 g is placed on a smooth horizontal table with a light string attached to it. The string passes over a smooth pulley P to the right of A at the end of the table and connected to another block B of mass 200 g. The portion of the string over the table is parallel to the table surface and the portion beyond the pulley is vertical. Initially the block A is at a distance of 200 cm from pulley and is moving with a speed of 50 cms^{-1} to the left. Find the position and velocity of A at $t = 1$ s.



SOLUTION

$$T = M_1 g$$

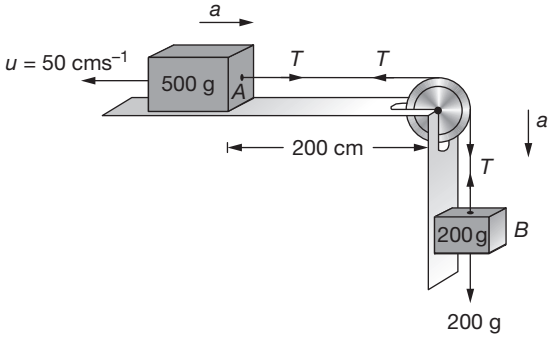
$$T = 500 \text{ g}$$

$$M_B g - T = M_B a \qquad \ldots(1)$$

$$T = M_A a \qquad \ldots(2)$$

$$a = \frac{M_B g}{M_A + M_B}$$

$$a = \left(\frac{200}{700}\right)(9.8)$$



$$a = 280 \text{ cms}^{-2}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = (-50)(1) + \frac{1}{2}(280)(1)^2$$

$$s = 90 \text{ cm}$$

Distance covered $= 110$ cm

Using, $v = u + at$

$$v = -50 + (280)(1)$$

$$v = 280 - 50$$

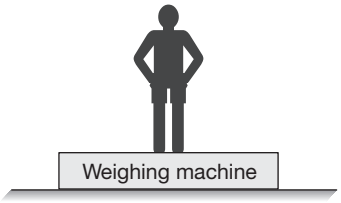
$$v = 230 \text{ cms}^{-1}$$

WEIGHING MACHINE

A weighing machine does not measure the weight but measures the force exerted by object at the upper surface of the machine (i.e. the surface on which the person is standing).

ILLUSTRATION 35

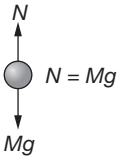
A man of mass 60 kg is standing on a weighing machine placed on ground. Calculate the reading of machine $\left(g = 10 \text{ ms}^{-2}\right)$.

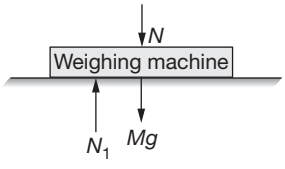


SOLUTION

For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately. F.B.D. of man, F.B.D. of weighing machine.
Here force exerted by object on upper surface is N, so the reading of weighing machine is

$$N = Mg$$



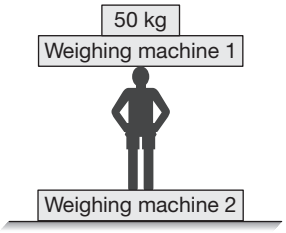


$$\Rightarrow \quad N = 60 \times 10$$

$$\Rightarrow \quad N = 600 \text{ N}$$

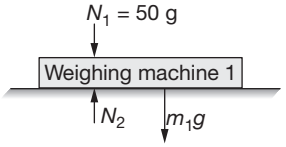
ILLUSTRATION 36

A man of mass 60 kg is standing on a weighing machine (2) of mass 5 kg placed on ground. Another same weighing machine is placed over man's head. A block of mass 50 kg is put on the weighing machine (1). Calculate the readings of weighing machines (1) and (2).



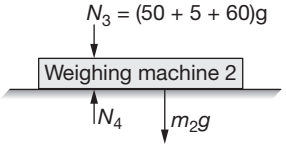
SOLUTION

Since a weighing machine measures the force applied at its upper surface, so we have the reading of weighing machine 1 given by



$$R_1 = N_1 = 50g = 500 \text{ N}$$

Similarly, we have the reading of weighing machine 2 given by



$$R_2 = N_3$$

$$\Rightarrow \quad R_2 = (50 + 5 + 60)\, g$$

$$\Rightarrow \quad R_2 = 115 \times 10$$

$$\Rightarrow \quad R_2 = 1150 \text{ N}$$

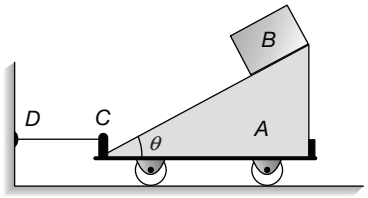
Test Your Concepts-IV

Based on Newton's Laws of Motion: Acceleration Systems

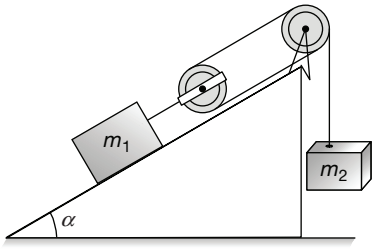
(Solutions on page H.199)

1. A weight of 200 kg hangs freely from the free end of a rope. The weight is hauled up vertically from rest by winding up the rope. The pull starts at 250 kgwt and diminishes uniformly at the rate of 1 kgwt per metre wound up. Find the velocity after 20 metre have been wound up. Neglect the weight of the rope $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$.

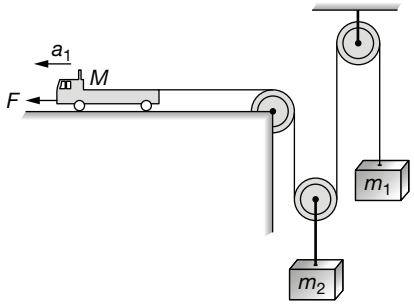
2. Block B has a mass m and is released from rest when it is on top of wedge A, which has a mass 3 m. Determine the tension in cord CD required to hold the wedge from moving while B is sliding down A. Neglect friction.



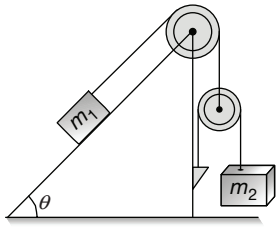
3. Find the acceleration of masses m_1 and m_2. The string and the pulley are massless and frictionless.



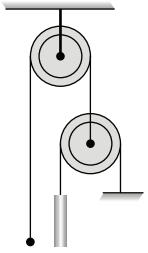
4. A toy truck of mass $M = 2$ kg is moving towards left with an acceleration a_1 due to a force $F = 20$ N as shown in figure. It is connected to a mass $m_1 = 1$ kg with a light and frictionless string, passing over a movable massless pulley, to which another mass $m_2 = 0.5$ kg is connected. Find the force acting on the truck towards right and the accelerations of mass m_1 and m_2. Take $g = 10 \text{ ms}^{-2}$.



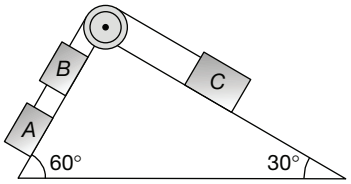
5. Find the acceleration of the body of mass m_2 in the arrangement shown in figure. If the mass m_2 is η times great as the mass m_1 and the angle that the inclined plane forms with the horizontal is equal to θ. The masses of the pulleys and threads, as well as the friction, are assumed to be negligible.



6. In the arrangement shown in figure the mass of the ball is η times as great as that of the rod. The length of the rod is l, the masses of the pulleys and the threads, as well as the friction, are negligible. The ball is set on the same level as the lower end of the rod and then released. How soon will the ball be opposite the upper end of the rod?

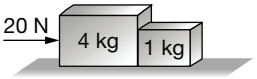


7. In the arrangement shown, mass of A, B and C are 1 kg, 3 kg and 2 kg, respectively, and friction is absent. Find the

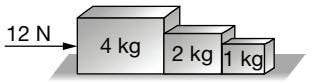


(a) acceleration of the system and
(b) tensions in the string.
Take $g = 10\ \text{ms}^{-2}$

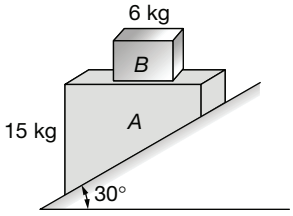
8. Two blocks of mass 4 kg and 1 kg are placed side by side on a smooth horizontal surface as shown in the figure. A horizontal force of 20 N is applied on 4 kg block. Find
(a) the acceleration of each block
(b) the normal reaction between two blocks.



9. Three blocks having mass 4 kg, 2 kg and 1 kg are placed side by side on a smooth surface as shown in figure. A horizontal force of 14 N is applied on 4 kg block. Find the net force on 2 kg block.



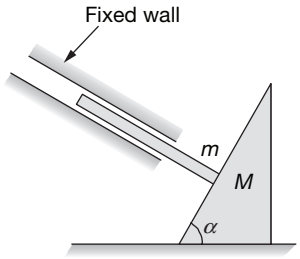
10. A 6 kg block B rests as shown on the upper surface of a 15 kg wedge A. Neglecting friction, determine the acceleration of A, the acceleration of B relative to A, immediately after the system is released from rest $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$.



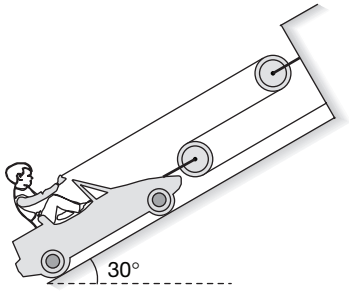
11. A heavy uniform chain of length $2l$, hangs over a small smooth fixed pulley, the length $l + c$ being at one side and $l - c$ at the other. If the end of the shorter portion be held and then let go, show that the chain will slip off the pulley in time given by

$$\sqrt{\frac{l}{g}}\log\left(\frac{l+\sqrt{l^2-c^2}}{c}\right)$$

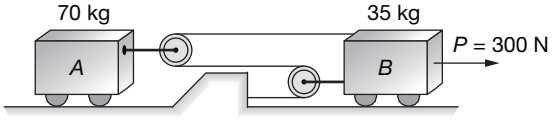
12. In the arrangement shown in the figure, the rod of mass m held by two smooth walls, remains always perpendicular to the surface of the wedge of mass M. Assuming all the surfaces are frictionless, find the acceleration of the rod and that of the wedge.



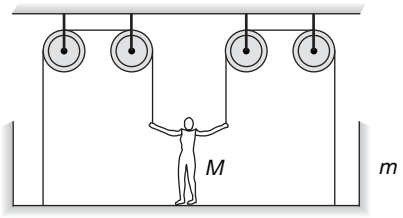
13. A man pulls himself up the 30° incline by the method shown. If the combined mass of the man and cart is 100 kg, determine the acceleration of the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys and wheels.



14. Determine the accelerations of bodies A and B and the tension in the cable due to the application of the 300 N force. Neglect all friction and the masses of the pulleys.



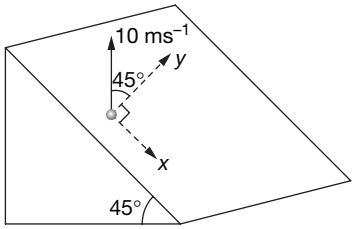
15. A painter of mass M stand on a crate of mass m and pulls himself up by two ropes which hang over pulley as shown. He pulls each rope with force F and moves upward with a uniform acceleration a. Find a, neglecting the fact that no one could do this for long time.



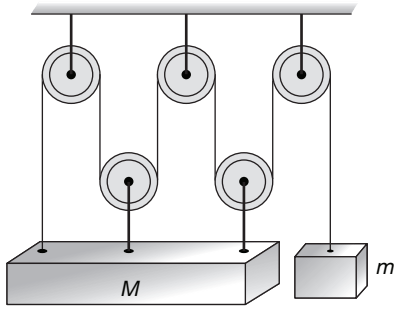
16. Two men A and B having masses M and $M+m$, start simultaneously from the ground and climb with uniform vertical accelerations up the free ends of a light inextensible rope which passes over a smooth pulley at a height h from the ground. If the lighter of the two men reaches the pulley in time t second, show that the heavier cannot get nearer to it than $\frac{m}{M+m}\left(\frac{gt^2}{2}+h\right)$.

17. A block is kept on the floor of an elevator at rest. The elevator starts descending with an acceleration of $a_0 = 12\ \text{ms}^{-2}$. Find the displacement of the block during the first 0.2 s after the start. Take $g = 10\ \text{ms}^{-2}$.

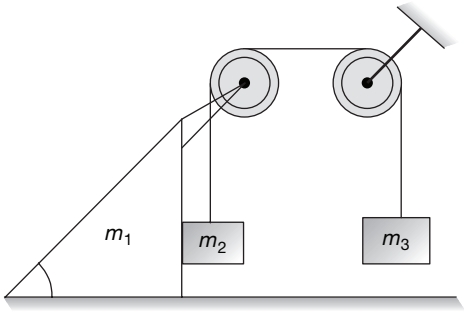
18. The small marble is projected with a velocity of $10\ \text{ms}^{-1}$ in a direction 45° from the horizontal y-direction on the smooth inclined plane. Calculate the magnitude v of its velocity after 2 second. Take $g = 10\ \text{ms}^{-2}$.



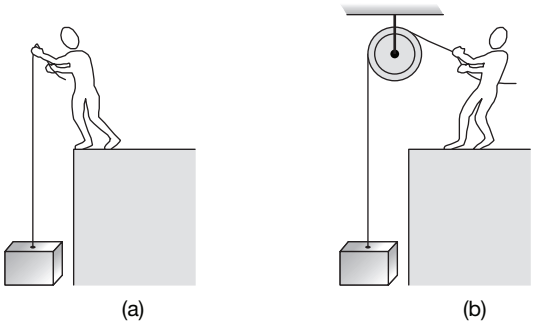
19. In the arrangement shown, the pulleys are light and frictionless and strings are light and inextensible. Find the acceleration of the two masses as shown in figure.



20. In the shown arrangement both pulleys and the string are massless and all the surfaces are frictionless. Find the acceleration of the wedge.



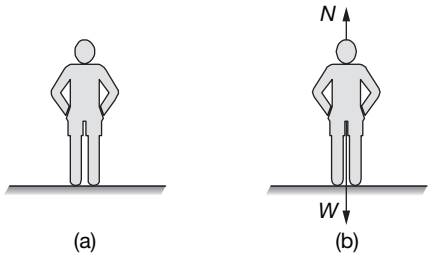
21. A man of 50 kg raises a block of mass 25 kg in two different ways as shown in figure. Calculate the action on the floor by the man in the two cases. If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding? Take $g = 9.8\ \text{ms}^{-2}$.



EQUILIBRIUM OF COPLANAR FORCES

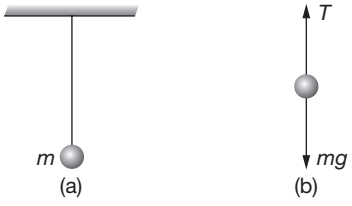
A body or a system is said to be in equilibrium if it does not tend to undergo any further change of its own. Any further change must be produced by external means (e.g., force).

The simplest kind of equilibrium situation is one where two forces act on a body. When you stand motionless, you experience the downward gravitational pull of the earth, your weight $\vec{W}$. The weight is balanced by an upward force exerted on you by the floor. This force is perpendicular to the floor and it is called the **normal force** $\vec{N}$. Note that although $\vec{N}$ and $\vec{W}$ are equal and opposite, they do not constitute an action-reaction pair.

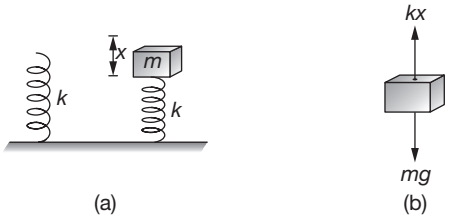


(a) A man standing on floor is in equilibrium.
(b) The free body diagram of the man gives $W = N$.

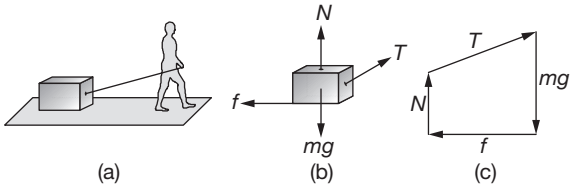
Some other examples of static equilibrium are shown in the following figures.



(a) A ball of mass m suspended from the ceiling with an inextensible string in equilibrium.
(b) The free body diagram of the ball gives $T = mg$.



(a) A block of mass m supported by a spring is in equilibrium.
(b) The free body diagram of the block gives $kx = mg$.



(a) A block of mass m is being pulled with a constant velocity on a horizontal surface.
(b) The free body diagram of the block.
(c) The block is in dynamic equilibrium because the vector sum of the forces is zero.

There is no net force $(\vec{F})$ and no net torque $(\vec{\tau})$ acting on the body, i.e., net force and torque acting on the body have to be zero.

Equilibrium	
Translational Equilibrium	**Rotational Equilibrium**
• $\sum \vec{F} = \vec{0}$	• $\sum \vec{\tau} = \vec{0}$
• $\sum F_x = 0$	• $\sum \tau_x = 0$
• $\sum F_y = 0$	• $\sum \tau_y = 0$
• $\sum F_z = 0$	• $\sum \tau_z = 0$

Conceptual Note(s)

(a) In the arrangement shown, for translational equilibrium to exist, we have

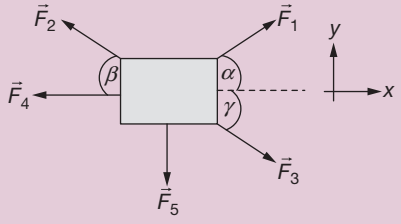
$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

$$\sum F_x = 0 \text{ gives}$$

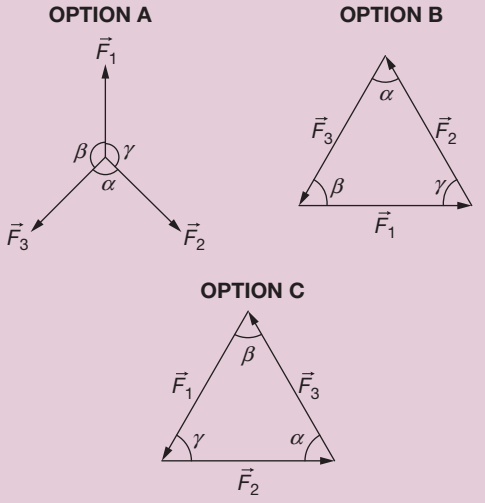
$$F_1 \cos\alpha + F_3 \cos\gamma - F_4 - F_2 \cos\beta = 0 \quad ...(1)$$

$$\sum F_y = 0 \text{ gives}$$

$$F_1 \sin\alpha + F_2 \sin\beta - F_5 - F_3 \sin\gamma = 0 \quad ...(2)$$



(b) If three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ happen to be in equilibrium, as shown (non-collinear lying in same plane)



$$\Rightarrow \quad \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

or $$\frac{\sin\alpha}{F_1} = \frac{\sin\beta}{F_2} = \frac{\sin\gamma}{F_3}$$

ROTATIONAL EQUILIBRIUM (LAW OF CONSERVATION OF MOMENTS OF FORCE)

$$\sum \vec{\tau} = \vec{0}$$

In an easy to use scalar form we must remember that

$$\sum F_i (r_i)_\perp = 0$$

where $(r_i)_\perp$ is the $\perp$ distance of the ith force F_i, from the point about which moments are to be taken.

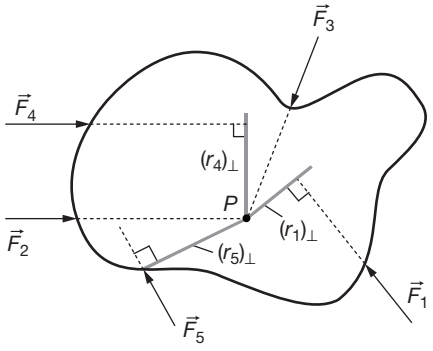
$$\Rightarrow \quad \sum_{CW} F_i (r_i)_\perp = \sum_{CCW} F_i (r_i)_\perp$$

Law of conservation of moments

$$\left(\begin{array}{c}\text{Total clockwise}\\ \text{moments of}\\ \text{force}\end{array}\right) = \left(\begin{array}{c}\text{Total counter}\\ \text{clockwise moments}\\ \text{of force}\end{array}\right)$$

EXAMPLE:

Consider a body on which forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, $\vec{F}_4$ and $\vec{F}_5$ are acting as shown. To check for the rotational equilibrium of the body, we shall be following the steps given.



STEP-1: Assume clockwise (*CW*) sense as positive.

STEP-2: Extend all the forces to check out which of them passes through the point about which moments have to be calculated (in this case *P*).

STEP-3: For the forces that pass through point *P* (like $\vec{F}_2$ and $\vec{F}_3$), $(r_2)_\perp = (r_3)_\perp = 0$.

So, the forces that pass through the point about which moments are to calculated will have no contribution to the moments of force.

STEP-4: For the left out forces (like $\vec{F}_1$, $\vec{F}_4$ and $\vec{F}_5$), let us calculate the moments taking each one by one as if the remaining others are absent.

For this we drop perpendiculars from the point *P* on the extended force lines. So that we get $((r_1)_\perp, (r_4)_\perp$ and $(r_5)_\perp)$.

STEP-5: Now for equilibrium, net moment of force is zero.

$$\Rightarrow \quad \underbrace{F_1(r_1)_\perp}_{CCW} + \underbrace{F_4(r_4)_\perp}_{CW} + \underbrace{F_5(r_5)_\perp}_{CW} = 0$$

Since *CW* sense is taken as positive, so we have

$$-F_1(r_1)_\perp + F_4(r_4)_\perp + F_5(r_5)_\perp = 0$$

$$\Rightarrow \quad F_1(r_1)_\perp = F_4(r_4)_\perp + F_5(r_5)_\perp$$

Conceptual Note(s)

(a) If all forces give *CW* moments, net moments cannot be zero.

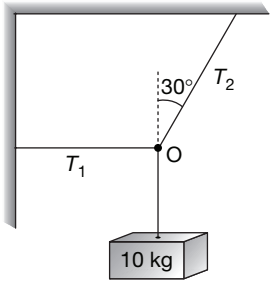
(b) If the net moment of force is non-zero, then net moment is given by

$$\tau = F_4(r_4)_\perp + F_5(r_5)_\perp - F_1(r_1)_\perp$$

If τ is $\oplus$, net moment is clockwise (CW)
If τ is $\ominus$, net moment is counter clockwise (CCW).

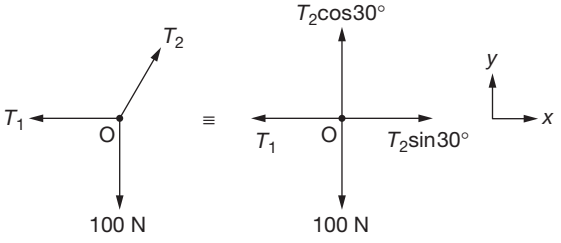
ILLUSTRATION 37

A block of mass 10 kg is suspended with two strings, as shown in the figure. Find the tensions T_1 and T_2 in the two strings.



SOLUTION

The free body diagram of the joint O is drawn as shown in the following figure.



Applying equations for equilibrium.

$$\sum F_x = 0$$

$$T_2 \sin 30° - T_1 = 0 \qquad \ldots(1)$$

$$\sum F_y = 0$$

$$T_2 \cos 30° - 100 = 0 \qquad \ldots(2)$$

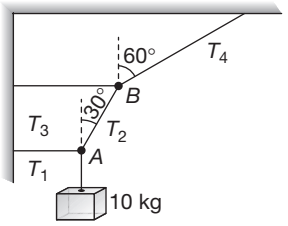
Thus, $T_2 = \dfrac{100}{\cos 30°} = \dfrac{200}{\sqrt{3}}$ N

Substituting the value of T_2 in equation (1), we get

$$T_1 = T_2 \sin 30° = \frac{100}{\sqrt{3}} \text{ N}$$

ILLUSTRATION 38

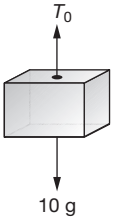
The system shown in figure is in equilibrium.



Find the magnitude of tension in each string T_1, T_2, T_3 and T_4. (Take $g = 10\ \text{ms}^{-2}$)

SOLUTION

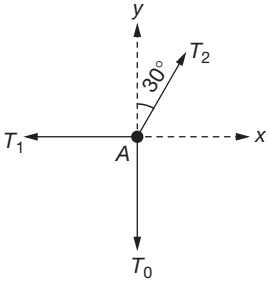
F.B.D. of 10 kg block



$$T_0 = 10g$$

$$\Rightarrow \quad T_0 = 100 \text{ N}$$

F.B.D. of point A



$$\Sigma F_y = 0$$

$$\Rightarrow \quad T_2 \cos(30°) = T_0 = 100 \text{ N}$$

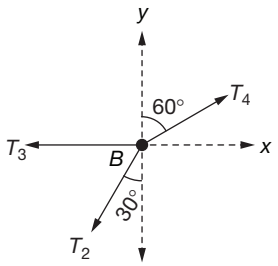
$$\Rightarrow \quad T_2 = \frac{200}{\sqrt{3}} \text{ N}$$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad T_1 = T_2 \sin(30°)$$

$$\Rightarrow \quad T_1 = \left(\frac{200}{\sqrt{3}}\right)\left(\frac{1}{2}\right) = \frac{100}{\sqrt{3}} \text{ N}$$

F.B.D. of point B



$$\Sigma F_y = 0$$

$$\Rightarrow \quad T_4 \cos 60° = T_2 \cos 30°$$

and $\Sigma F_x = 0$

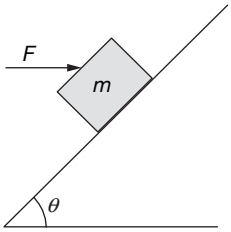
$$\Rightarrow \quad T_3 + T_2 \sin 30° = T_4 \sin 60°$$

$$\Rightarrow \quad T_3 = \frac{200}{\sqrt{3}} \text{ N and}$$

$$T_4 = 200 \text{ N}$$

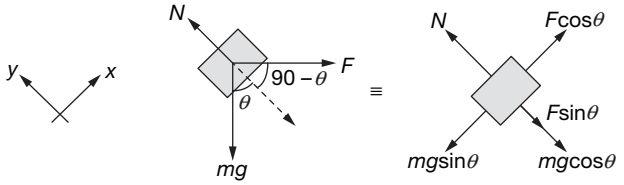
ILLUSTRATION 39

Find the magnitude of the horizontal force F required to keep the block of mass m stationary on the smooth inclined plane as shown in the figure.



SOLUTION

The forces acting on the block are shown in the free body diagram.



Applying equations for equilibrium,

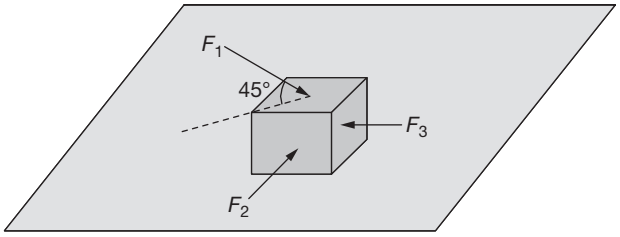
$$\sum F_x = 0$$

$$F\cos\theta - mg\sin\theta = 0$$

$$\Rightarrow \quad F = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta$$

ILLUSTRATION 40

A cubical block is experiencing three forces as shown in figure. Find the friction force acting on the block if it is at rest. Given that $F_1 = 30$ N; $F_2 = 50$ N and $F_3 = 42$ N.



SOLUTION

As shown in figure force F_1 is having three components, one along vertical $\left(=\frac{F_1}{\sqrt{2}}\right)$, other along force $F_2\left(=\frac{F}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)$ and one against $F_3\left(=\frac{F}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)$. Here the net horizontal force acting on the block is given by

$$F_{\text{net}} = \sqrt{\left(F_2 + \frac{F}{\sqrt{4}}\right)^2 + \left(F_3 - \frac{F}{\sqrt{4}}\right)^2}$$

$$\Rightarrow \quad F_{\text{net}} = \sqrt{(50+15)^2 + (42-15)^2} = 70.384 \text{ N}$$

As it is given that block is in static equilibrium, thus sum of all horizontal forces acting on it is must be zero. Sum of given external horizontal forces is 70.384 N and is in a direction $\theta = \tan^{-1}\left(\frac{65}{67}\right)$ from the direction of force F_3. We can state that friction force acting on block must be exactly opposing this force so as to keep the block in static equilibrium.

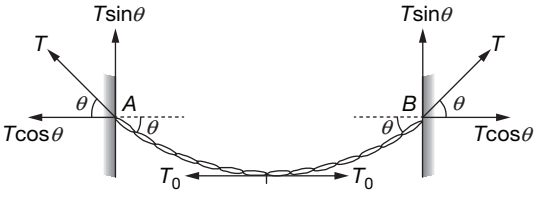
ILLUSTRATION 41

A chain of mass m is attached at two points A and B of two fixed walls as shown in figure. Due to its weight a sag is there in the chain such that at point A and B it makes an angle θ with the normal to the wall. Find the tension in the chain at : (Assume tension is always along the length of chain).

(a) point A and B.
(b) mid point of the chain

SOLUTION

(a) Let the tension in the chain at point A and B is T, so at these points chain will pull the wall hinges with the same force and wall hinges will also exert same force on chain in tangential direction as shown in figure.



Now for vertical equilibrium of chain we have

$$2T\sin\theta = mg$$

$$\Rightarrow \quad T = \frac{1}{2}mg\,\text{cosec}\,\theta$$

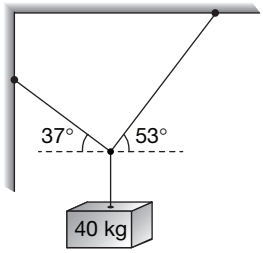
(b) Let tension at the mid point of chain be T_0. It must be along horizontal direction as at midpoint slope is zero. For horizontal equilibrium of chain we can state that at every point horizontal component of the tension in the chain must be equal as no other external force is acting on it in horizontal direction. Thus we have

$$T_0 = T\cos\theta$$

$$\Rightarrow \quad T_0 = \frac{1}{2}mg\cot\theta$$

ILLUSTRATION 42

The object in figure weighs 40 kg and hangs at rest. Find the tensions in the three cords that hold it.



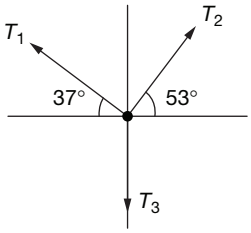
SOLUTION

Because the object is at equilibrium, the vector sum of the forces acting directly on it must be zero. There are only two such forces, the tension in the lower cord and the pull of gravity, 400 N. Therefore, the tension in the lower cord must be 400 N. It is the tension that supports the object. Figure shows the junction where the three cords meet. As the system is in equilibrium, net sum of all the forces at the junction must be zero. For this we resolve the tensions in horizontal and perpendicular direction as
In horizontal direction

$$0.6T_2 - 0.8T_1 = 0 \qquad \ldots(1)$$

In vertical direction

$$0.6T_1 + 0.8T_2 - 400 = 0 \qquad \ldots(2)$$



On solving the above equations we get

$$T_1 = 240 \text{ N} \quad \text{and} \quad T_2 = 320 \text{ N}$$

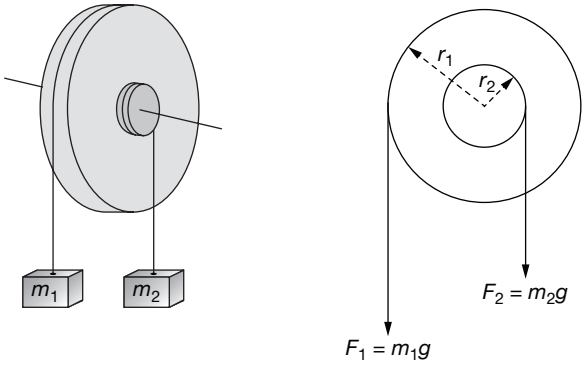
and the tension in third cord we already have

$$T_2 = 400 \text{ N}$$

EXAMPLES AND SITUATIONS FOR ROTATIONAL EQUILIBRIUM

To learn how forces and rotations are related, we can perform the experiment shown in figure. We see there a wheel that consists of two disks cemented together. It is free to rotate on a stationary axle that we call the axis, or pivot, of rotation. By hanging objects from the two cords, we can determine the turning effect of a force. The force F_2 tries to turn the wheel clockwise, while F_1 tries to turn the wheel counterclockwise. By experimenting with different radii r_1 and r_2 for the two disks, we find that the two turning effects balance whenever

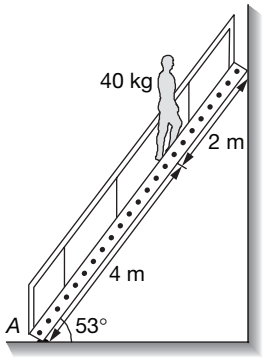
$$F_1 r_1 = F_2 r_2$$



The above relation, product of force and the radii, is known as torque. Torque is the physical quantity which measures the turning effect of a force on a body. Its magnitude is given by the product of the force and the perpendicular distance from the axis of rotation or pivot. In above case it is simply the product of force and the radii.

ILLUSTRATION 43

The uniform 20 kg ladder hinged at the bottom in figure, leans against a smooth wall. If a 40 kg person stands on the ladder shown, how large are the forces at the wall the ground.



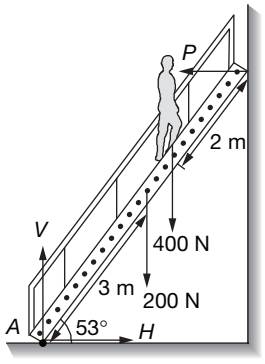
SOLUTION

Consider figure, all the forces acting on the ladder are shown. For horizontal and vertical equilibrium, we use

$$H - P = 0 \qquad \{\text{along } x\text{-direction}\}$$
$$V - 200 - 400 = 0 \qquad \{\text{along } y\text{-direction}\}$$

On solving we get

$$V = 600 \text{ N}$$



For rotational equilibrium about point A, we have

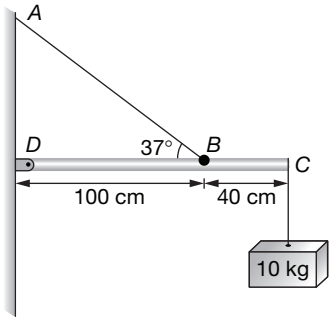
$$\sum \tau_A = 0$$

$$\Rightarrow \quad P(6)(0.8) - 200(3)(0.6) - 400(4)(0.6) = 0$$

$$\Rightarrow \quad P = H = 275 \text{ N}$$

ILLUSTRATION 44

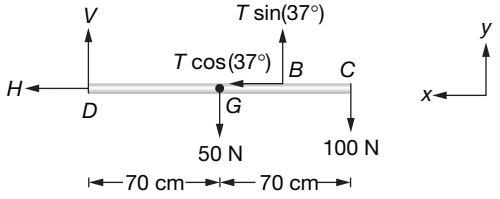
For the uniform 5 kg beam shown in figure, how large is the tension in the supporting cable AB and what are the components of the force exerted by the hinge on the beam at point D? Take $g = 10 \text{ ms}^{-2}$.



SOLUTION

The forces acting on the beam at difference locations are shown in figure. Note that the weight of the beam, 50 N, is taken as acting at the beam's centre of mass. Further, we have replaced the tension in the cable by its components. For equilibrium of the beam, we have

$$\Sigma\tau_D = 0,\ \Sigma F_x = 0 \text{ and } \sum F_y = 0$$



$$\Rightarrow \quad T\sin(37°)\times 1 - 50\times 0.7 - 100\times 1.4 = 0 \qquad \ldots(1)$$

$$\Rightarrow \quad H + T\cos(37°) = 0 \qquad \ldots(2)$$

$$\Rightarrow \quad V + T\sin(37°) - 50 - 100 = 0 \qquad \ldots(3)$$

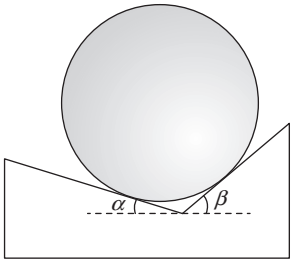
From (1), (2) and (3), we get

$$T = 292 \text{ N},\ H = -234 \text{ N} \text{ and } V = -25.2 \text{ N}$$

Here, V and H come with a negative sign. Which simply implies that the direction of vertical force from hinge on beam shown in figure is downwards and of horizontal force is towards the right.

ILLUSTRATION 45

A solid sphere of radius R and mass M is placed in a trough as shown in figure. The inner surfaces of the trough are frictionless. Determine the forces exerted by the trough on the sphere at the two contact points.



SOLUTION

Let the normal forces be denoted by F_1 and F_2. They make angles α and β with the vertical

$$\Sigma F_x = 0$$

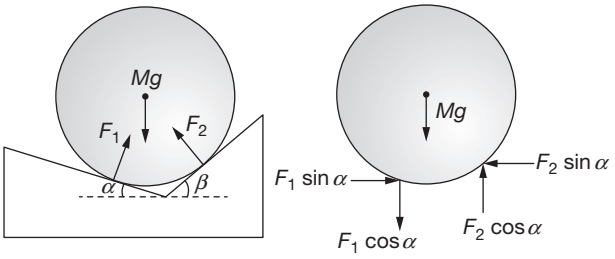
$$\Rightarrow \quad F_1\sin\alpha - F_2\sin\beta = 0$$

$$\Rightarrow \quad \Sigma F_y = 0$$

$$\Rightarrow \quad F_1\cos\alpha - Mg + F_2\cos\beta = 0$$

Substitute $F_2 = \dfrac{F\sin\alpha}{\sin\beta}$

$$F_1\cos\alpha + F_1\cos\beta\left(\frac{\sin\alpha}{\sin\beta}\right) = Mg$$

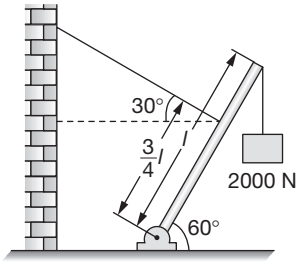


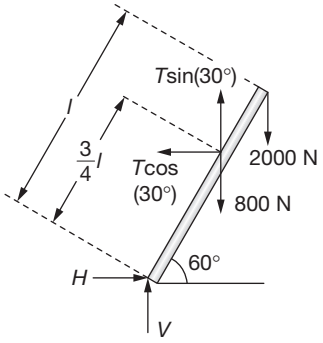
$$\Rightarrow \quad F_1(\cos\alpha\sin\beta + \sin\alpha\cos\beta) = Mg\sin\beta$$

$$\Rightarrow \quad F_1 = Mg\frac{\sin\beta}{\sin(\alpha+\beta)} \text{ and } F_2 = Mg\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

ILLUSTRATION 46

A uniform rod of 800 N weight pivoted at the bottom, is supported by a cable as in figure. A 2000 N object hangs from its top. Find the tension in the cable and the components of the reaction force exerted by the floor on the rod. Take $\sqrt{3} = 1.7$.





SOLUTION

$$\sum \tau_{\text{pivot}} = 0 \text{ gives}$$

$$(T\cos 30°)\left(\frac{3l}{3}\sin 60°\right)+(T\sin 30°)\left(\frac{3l}{4}\cos 60°\right)=$$

$$(2000)\left(l\cos 60°\right)+(800)\left(\frac{l}{2}\cos 60°\right)$$

$$\Rightarrow \quad T\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3l}{4}\right)\left(\frac{\sqrt{3}}{2}\right)+T\left(\frac{1}{2}\right)\left(\frac{3l}{4}\right)\left(\frac{1}{2}\right)=$$

$$(2000)(l)\left(\frac{1}{2}\right)+(800)\left(\frac{l}{2}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \frac{9}{16}T+\frac{3T}{16}=1000+200$$

$$\Rightarrow \quad T=1600 \text{ N}$$

Since, $\sum F_x = 0$, so we get

$$H = T\cos(30°) = 1360 \text{ N (towards right)}$$

Also, $\sum F_y = 0$, gives

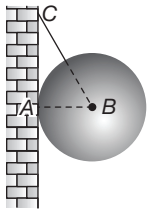
$$V=(2000+800)-T\sin(30°)=2000 \text{ N (upwards)}$$

Test Your Concepts-V

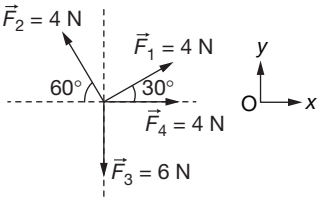
Based on Equilibrium

(Solutions on page H.204)

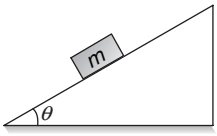
1. A point *A* on a sphere of mass *M*, radius *a* rests in contact with a smooth vertical wall and is supported by a string of length 2*a* which joins a point *B* on the sphere to a point *C* on the wall. Find tension in the string and the force exerted by the wall on the sphere.



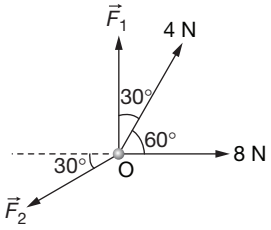
2. Write down the components of four forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ and $\vec{F}_4$ along *x* and directions as shown in figure. Also find the resultant of these forces.



3. A block of mass *m* is at rest on a rough wedge as shown in figure. What is the force exerted by the wedge on the block?

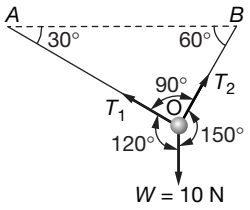


4. An object is in equilibrium under four concurrent forces in the directions shown in figure. Find the magnitude of $\vec{F}_1$ and $\vec{F}_2$.

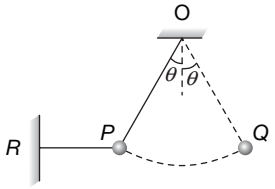


5. A force $\vec{F} = \vec{v} \times \vec{A}$ is exerted on a particle in addition to the force of gravity where $\vec{v}$ is the velocity of the particle and $\vec{A}$ is a constant vector in the horizontal direction. Find the minimum speed of projection of a particle of mass *m* so that it continues to move undeflected with the same constant velocity?

6. A ball of mass 1 kg hangs in equilibrium from two strings *OA* and *OB* as shown in figure. What are the tensions in strings *OA* and *OB*? Take $g = 10\ \text{ms}^{-2}$.



7. A ball of mass 1 kg is at rest in position *P* by means of two light strings *OP* and *RP*. The string *RP* is now cut and the ball swings to position *Q*. If $\theta = 45°$. Find the tensions in the strings in positions *OP* (when *RP* was not cut) and *OQ* (when *RP* was cut). Take $g = 10\ \text{ms}^{-2}$.



PSEUDO FORCE

FRAMES OF REFERENCE

Whenever any physical phenomena has to be observed, a suitable platform has to be taken by the observer. This platform from where a physical phenomena is being observed is called **the frame of reference.** Frames of reference are of two types

(a) Inertial: Non-Accelerated
(b) Non-Inertial: Accelerated

Inertial Frame

All non-accelerated frames (frames either at rest or moving with uniform velocity) are inertial frames.

Conceptual Note(s)

A frame of reference moving with constant velocity w.r.t. an inertial frame is also an Inertial Frame.

OR

A frame of reference at rest w.r.t. an inertial frame is also an Inertial Frame.

The Newton's Laws which we have already studied have been fabricated from an inertial frame of reference.

Non-inertial Frame

All accelerated frames are non-inertial frames.

Most of the students have a misconception that the Newton's Laws are not obeyed in the non-inertial frames. However this is not the case, because we have to introduce the concept of the **pseudo force** to keep the laws validated and acquire identical force equations irrespective of the frame from where the physical phenomenon is being observed.

Pseudo force is to be applied on a body whenever the body is lying in a non-inertial frame or the body is being observed from a non inertial frame.

Pseudo force is always directed opposite to the acceleration of the non inertial frame and has a value given by

$$F_{\text{pseudo}} = \begin{pmatrix}\text{Mass of}\\ \text{the block}\end{pmatrix}\begin{pmatrix}\text{Acceleration of}\\ \text{the frame}\end{pmatrix}$$

Pseudo force is a fictitious force because it has no physical origin, i.e., it is not caused by any of the basic interactions in the nature. Note that it has existence only in a non-inertial frame.

These forces are, then, simply a technique that permits us to apply $\vec{F} = m\vec{a}_0$ in the normal way to events if we insist on viewing the events from an accelerating reference frame.

In mechanics problems, then, we have two choices

(a) select an inertial frame as a reference frame and consider only "real" forces i.e., forces that we can associate with definite bodies (e.g. string, the earth, table etc.)
(b) select a non-inertial frame and consider ***both the real forces and pseudo forces****.*

Although we usually select the first alternative, but sometimes have to go for the second alternative, but both are completely equivalent and the choice is purely a matter of convenience.

Conceptual Note(s)

(a) All physical phenomenon should give identical results whether seen from an inertial frame or a non-inertial frame.
(b) All accelerated frames are non-inertial frames.
(c) Let us consider a block of mass M placed in an inertial frame. Let a force $\vec{F}$ be applied on the block. If $\vec{a}$ be the acceleration of the block, then

$$\vec{F} = M\vec{a} \quad \text{...(1)}$$

Suppose that the same block is now placed in a non-inertial frame and the same force $\vec{F}$ is applied on it. If $\vec{a}'$ be the acceleration of the block in non-inertial frame, then

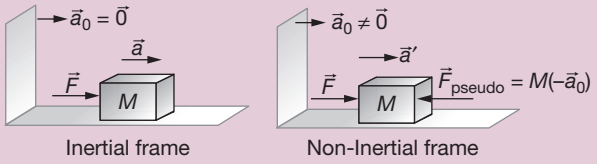
$$M\vec{a}' = \vec{F} - \vec{F}_{\text{pseudo}}$$

$$\Rightarrow \quad \vec{F} = M\vec{a}' + \vec{F}_{\text{pseudo}} \quad \text{...(2)}$$

So, we get from (1) and (2), that

$$M\vec{a} = M\vec{a}' + \vec{F}_{pseudo}$$

$$\text{i.e., } \begin{pmatrix}\text{Force in}\\ \text{an inertial}\\ \text{frame}\end{pmatrix} = \begin{pmatrix}\text{Force in a}\\ \text{Non-inertial}\\ \text{frame}\end{pmatrix} + \begin{pmatrix}\text{Pseudo}\\ \text{Force}\end{pmatrix} \quad ...(3)$$

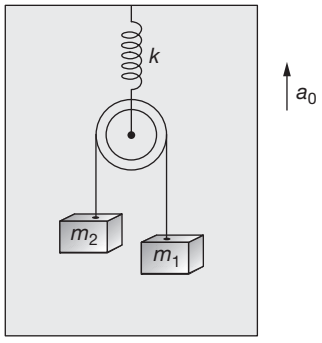


Clearly we observe that equation (3) establishes a relation between the phenomenon observed in an inertial frame and a non-inertial frame and the equivalency of both phenomenon with the help of pseudo force.

So donot develop a misconception that observers in different frames will get different result for an identical physical phenomenon. Just keep in mind that for the observer in an inertial frame Newton's Laws are valid as studied but for an observer in a non-inertial frame the Newton's Laws are being made suitably valid using the concept of Pseudo Force.

ILLUSTRATION 47

A pulley with two blocks system is attached to the ceiling of a lift with the help of a spring a force constant k. The lift is moving upward with an acceleration a_0. Find the deformation in the spring as observed by the inertial and non-inertial reference frame observer.



SOLUTION

We shall be discussing this problem taking into account two observers, one in an inertial frame and the other in the non-inertial frame. The observer in the inertial frame will taken into account all the real forces acting on the system, where as the observer in the non-inertial frame will take into account all the real + fictitious forces into account and both will arrive at the same results.

Observer in an Inertial Frame

Let us draw the free body diagram of the masses as seen by an observer standing on the ground or an inertial frame.

Applying Newton's Second Law

$$m_1 g - T = m_1 a_1 \quad ...(1)$$

$$-m_2 g + T = m_2 a_2 \quad ...(2)$$

Constraint relation

$$(x_0 - x_2) + (x_0 - x_1) = \text{constant}$$

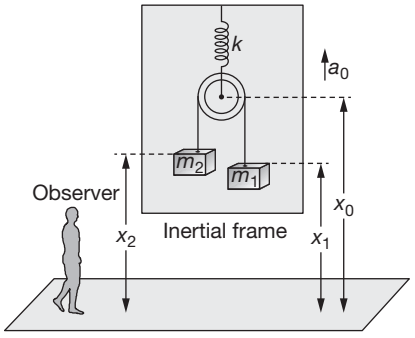
$$\Rightarrow \quad 2x_0 - x_1 - x_2 = \text{constant}$$

Differentiating twice w.r.t. time, we get

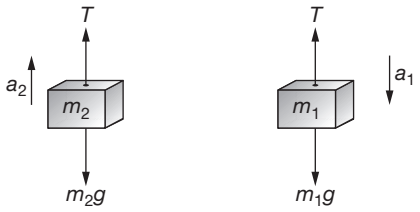
$$2\frac{d^2x_0}{dt^2} - \frac{d^2x_1}{dt^2} - \frac{d^2x_2}{dt^2} = 0$$

$$\Rightarrow \quad 2a_0 - (-a_1) - a_2 = 0$$

$$\Rightarrow \quad a_2 = 2a_0 + a_1 \quad ...(3)$$



Observer in inertial frame



FBD of masses from ground/inertial frame

Solving equations (1), (2) and (3), we get

$$T = \left[\frac{2m_1m_2}{m_1+m_2}\right](g+a_0)$$

The stretching force on the spring is

$$F = 2T$$

Using Hooke's Law

$$F = kx$$

where x is the deformation in the spring. Thus,

$$x = \frac{F}{k} = \left[\frac{4m_1m_2}{m_1+m_2}\right]\frac{(g+a_0)}{k}$$

Observer in a Non-inertial Frame

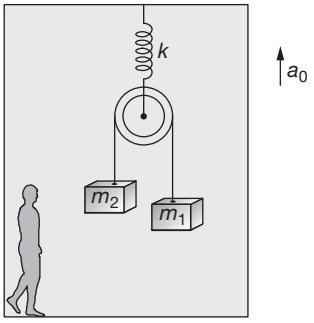
Let us draw the free body diagram of the masses as seen by an observer standing inside the cabin of the lift which happens to be a non-inertial frame due to the upward acceleration a_0 of the lift.

Relative to the centre of the pulley m_1 accelerates downward with a_{rel} and m_2 accelerates upward with a_{rel}.

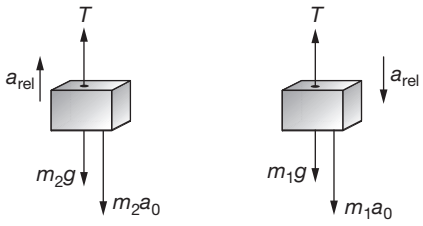
Applying Newton's Second Law

$$m_1g + m_1a_0 - T = m_1a_{rel} \quad \ldots(4)$$

$$-m_2g - m_2a_0 + T = m_2a_{rel} \quad \ldots(5)$$



Observer in non-inertial frame



FBD of masses from non-inertial frame

On adding equations

$$a_{rel} = \left(\frac{m_1-m_2}{m_2+m_2}\right)(g+a_0)$$

Substituting a_{rel} in equation (4)

$$T = \left[\frac{2m_1m_2}{m_1+m_2}\right](g+a_0)$$

$$F = 2T$$

$$x = \frac{F}{k} = \frac{2T}{k}$$

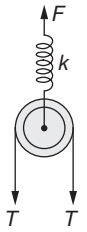
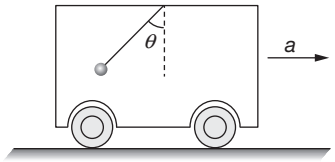


ILLUSTRATION 48

A bob of mass m is suspended from the ceiling of a train moving with an acceleration a as shown in figure. Find the angle θ in equilibrium position.

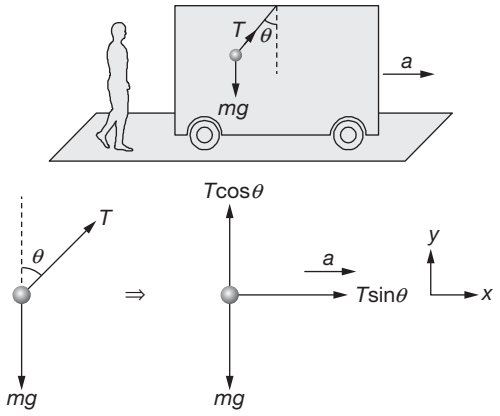


SOLUTION

This problem can also be solved by both the methods.

Inertial frame of reference (Ground)

F.B.D. of bob w.r.t. ground (only real forces):



FBD of bob w.r.t. ground

with respect to ground, bob is also moving with an acceleration a.

$\Rightarrow \quad \Sigma F_y = 0$

$\Rightarrow \quad T\cos\theta = mg \qquad \ldots(1)$

and $\quad \Sigma F_x = ma$

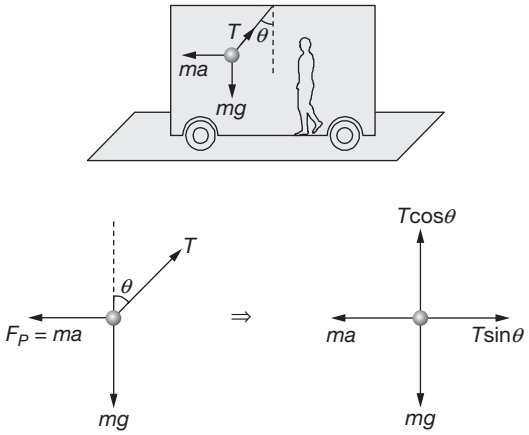
$\Rightarrow \quad T\sin\theta = ma \qquad \ldots(2)$

From equation (1) and (2), we get

$$\tan\theta = \frac{a}{g} \text{ or } \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Non-inertial frame of reference (Train)

F.B.D. of bob w.r.t. train. (real forces + pseudo force):



FBD of bob w.r.t. train

with respect to train, bob is in equilibrium

$\Rightarrow \quad \Sigma F_y = 0$

$\Rightarrow \quad T\cos\theta = mg \qquad \ldots(3)$

and $\quad \Sigma F_x = 0$

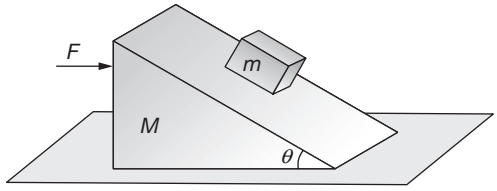
$\Rightarrow \quad T\sin\theta = ma \qquad \ldots(4)$

From equation (3) and (4), we get the same result, i.e.,

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

ILLUSTRATION 49

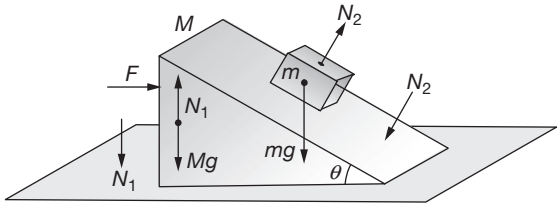
Figure shows a box of mass m is placed on a wedge of mass M on a smooth surface. How much force F is required to be applied on M so that during motion m remains at rest on its surface.



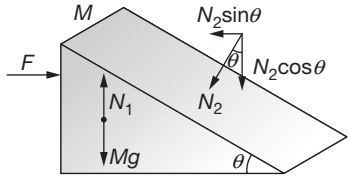
SOLUTION

The force acting on the bodies m and M are shown in figure along with free body diagrams of mass m and M. As the two bodies move together, we can find the acceleration of system towards right directly as

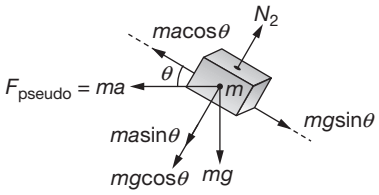
$$a = \frac{F}{m+M}$$



Forces acting on the system



FBD of M



FBD of m

Here the condition is, the small block of mass m should remain at rest on the incline surface of the wedge block. Look at the FBD of m in figure, the force acting on it towards left ma is the pseudo force on it as its reference frame is the wedge block. As wedge block is moving with an acceleration, we consider m relative to it. Now with respect to wedge

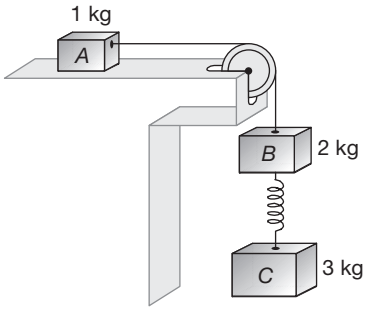
block m is at rest or in equilibrium, we can balance all the forces along the tendency of motion of body (i.e., inclined plane) and perpendicular to it shown in *FBD* of it.
For m to be at rest, from *FBD* of m, along the plane

$$mg\sin\theta = ma\cos\theta$$
$$\Rightarrow \quad a = g\tan\theta$$
$$\Rightarrow \quad \frac{F}{m+M} = g\tan\theta$$
$$\Rightarrow \quad F = (m+M)g\tan\theta$$

ILLUSTRATION 50

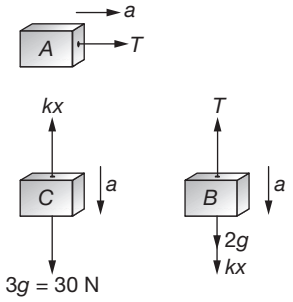
In the system shown in figure all surfaces are smooth, string is massless and inextensible. Find:

(a) acceleration of the system
(b) tension in the string and
(c) extension in the spring if force constant of spring is $k = 50\ \text{Nm}^{-1}$ $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$.



SOLUTION

The free body diagrams for the masses is shown here



$$T = (1)a \qquad \ldots(1)$$
$$2g + kx - T = 2a \qquad \ldots(2)$$
$$3g - kx = 3a \qquad \ldots(3)$$

(a) Adding (1), (2) and (3), we get

$$a = \frac{50}{6}\ \text{ms}^{-2}$$

(b) $T = ma = (1)\left(\frac{50}{6}\right) = \frac{50}{6}\ \text{N}$

(c) Substituting $a = \frac{50}{6}\ \text{ms}^{-2}$ in (3), we get

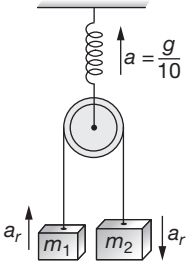
$$30 - kx = 3\left(\frac{50}{6}\right)$$
$$\Rightarrow \quad kx = 5$$
$$\Rightarrow \quad x = \frac{5}{50} = 0.1\ \text{m}$$
$$\Rightarrow \quad x = 10\ \text{cm}$$

ILLUSTRATION 51

An Atwood's machine having two blocks of mass $m_1 = 1\ \text{kg}$ and $m_2 = 2\ \text{kg}$ is suspended from a spring balance attached to ceiling of a lift moving upwards with an acceleration of $\frac{g}{10}$. Determine the reading on the spring balance. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

The problem is represented diagrammatically in the figure. Let a_r be the relative acceleration of blocks with respect to pulley.
In the figure,



absolute acceleration of m_1 is $a_r + a = a_r + \frac{g}{10}$, upwards and absolute acceleration of m_2 is $a_r - a = a_r - \frac{g}{10}$, downwards

From Newton's Second Law, we have

$$T - m_1 g = m_1\left(a_r + \frac{g}{10}\right) \quad \ldots(1)$$

$$m_2 g - T = m_2\left(a_r - \frac{g}{10}\right) \quad \ldots(2)$$

Solving equations (1) and (2), with $m_1 = 1\text{ kg}$ and $m_2 = 2\text{ kg}$, we get

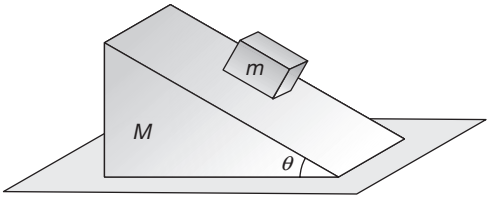
$$T = \frac{44}{3}\text{ N}$$

The reading of the spring balance is therefore

$$F = 2T = \frac{88}{3}\text{ N}$$

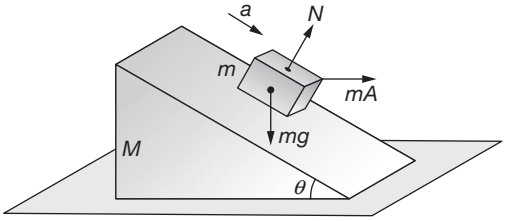
ILLUSTRATION 52

All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.

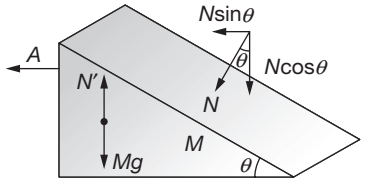


SOLUTION

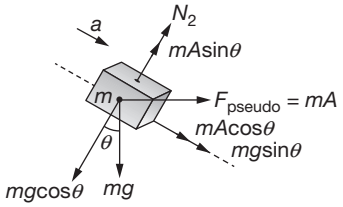
Suppose the acceleration of block with respect to prism is a down the plane and let the acceleration of the prism be A in the backward direction. Consider the motion of the smaller block from the frame of the prism. The forces acting on the system, free body diagram of M and m are shown here.



Forces acting on the system



FBD of M



FBD of m

The following three forces will act on the block in the directions shown in figure.

(a) Normal reaction N.
(b) Weight mg
(c) Pseudo force mA

The block slides down the plane. Component of the forces parallel to the incline gives

$$mA\cos\theta + mg\sin\theta = ma$$

$$\Rightarrow \quad a = A\cos\theta + g\sin\theta \quad \ldots(1)$$

Component of the forces perpendicular to the incline gives

$$N + mA\sin\theta = mg\cos\theta \quad \ldots(2)$$

Now, consider the motion of the prism from the ground frame. No pseudo force is needed as the frame used in inertial. The forces are:

(a) Mg downward
(b) N normal to the incline (by the block)
(c) N' upward (by the horizontal surface)

Horizontal components give,

$$N\sin\theta = MA$$

$$\Rightarrow \quad N = \frac{MA}{\sin\theta} \quad \ldots(3)$$

Putting in equation (2)

$$\frac{MA}{\sin\theta} + mA\sin\theta = mg\cos\theta$$

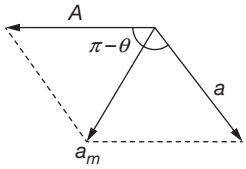
$$\Rightarrow \quad A = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}$$

From equation (1),

$$a = \frac{mg\sin\theta\cos^2\theta}{M + m\sin^2\theta} + g\sin\theta = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta}$$

The net acceleration of m, is the vector addition of A and a, as m is also moving with A toward left along with M.

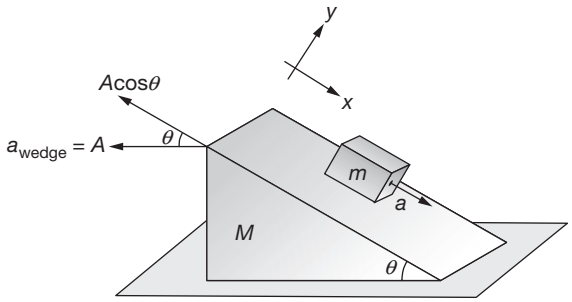
Net acceleration of m can be obtained by the vector sum shown in figure.



$$a_m = \sqrt{A^2 + a^2 - 2aA\cos\theta}$$

Inertial Method: Acceleration of wedge along x-axis is $A\cos\theta$, so net acceleration of block downwards along $+x$-axis is $(a - A\cos\theta)$.

Since block lies on frictionless incline, so it must accelerate down the incline with an acceleration $g\sin\theta$.



Observed from ground frame

Hence, $a - A\cos\theta = g\sin\theta$ …(1)

From (1),

$$N\sin\theta = MA$$

$$N = \frac{MA}{\sin\theta}$$

Now in (4),

$$N + mA\sin\theta = mg\cos\theta$$

$$\frac{MA}{\sin\theta} + mA\sin\theta = mg\cos\theta$$

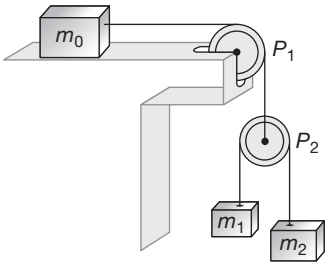
$$MA + mA\sin^2\theta = mg\cos\theta\sin\theta$$

$$A\left(M + m\sin^2\theta\right) = mg\cos\theta\sin\theta$$

$$A = \frac{mg\cos\theta\sin\theta}{M + m\sin^2\theta}$$

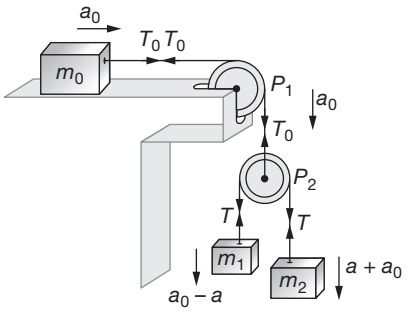
ILLUSTRATION 53

Three blocks of mass m_0, m_1 and m_2 are connected as shown in the figure. All the surfaces are frictionless and the string and the pulleys are light. Find the acceleration of m_0.



SOLUTION

Suppose the acceleration of m_0 is a_0 towards right. The acceleration of pulley P_2 will also be a_0 downwards, because the string connecting m_0 and P_2 is constant in length. Also the string connecting m_1 and m_2 has a constant length. This implies that the decrease in the separation between m_1 and P_2 equals the increase in the separation between m_2 and P_2. So, the upward acceleration of m_1 with respect to P_2 equals the downward acceleration of m_2 with respect to P_2. Let this acceleration be a.



As seen from ground frame

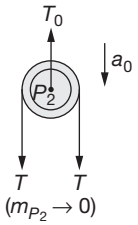
The acceleration of m_1 with respect to the ground is $a_0 - a$ (downward) and the acceleration of m_2 with respect to the ground is $a_0 + a$ (downward).

Let the tension be T_0 in the upper string and T in the lower string.

Consider the motion of the pulley P_2. The forces on this light pulley are

(a) T_0 upwards by the upper string and

(b) $2T$ downwards by the lower string



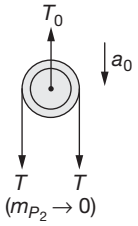
As the mass of the pulley is negligible,

$$2T - T_0 = 0 \text{ giving } T = \frac{T_0}{2} \qquad \ldots(1)$$

Motion of m_0:

In the horizontal direction, the equation is

$$T_0 = m_0 a_0 \qquad \ldots(2)$$



Motion of m_1:

$$m_1 g - \frac{T_0}{2} = m_1 (a_0 - a) \qquad \ldots(3)$$

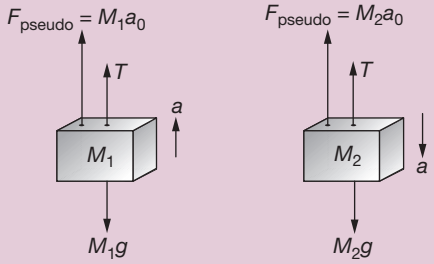
Motion of m_2:

$$m_2 g - \frac{T_0}{2} = m_2 (a_0 + a) \qquad \ldots(4)$$

Solving these four equations, we get

$$a_0 = \frac{g}{1 + \frac{m_0}{4}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$$

We can also do the problem w.r.t. an observer on pulley P_2, a non-inertial frame and get results identical to the ones obtain when the observer was in an inertial frame. Since P_2 acceleration down with a_0, so it is a non-inertial frame accelerating downwards.



For mass m_1

$$M_1 a_0 + T - M_1 g = M_1 a$$

$$\Rightarrow \quad T - M_1 g = M_1 (a - a_0) \qquad \ldots(5)$$

Equation (5) is identical as equation (3)

For mass m_2

$$M_2 g - (T + M_2 a_0) = M_2 a$$

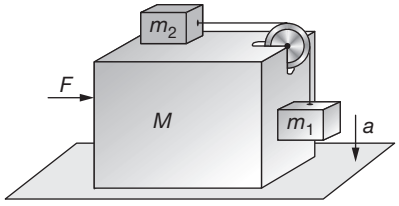
$$\Rightarrow \quad M_2 g - T = M_2 (a + a_0) \qquad \ldots(6)$$

Equation (6) is identical as equation (4)

Please be careful while solving the problem in earth frame you have to consider the acceleration of mass relative to its surface (accelerating) but write the equations in horizontal and vertical directions relative to earth (ground).

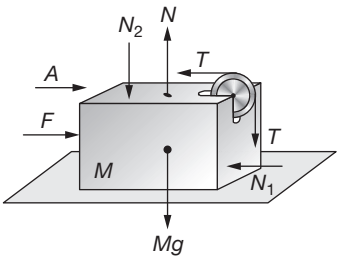
ILLUSTRATION 54

Figure shows a large block of mass M, supporting two small masses m_1 and m_2, connected by a light, frictionless thread. A force F is acting on M, such that the block m_1 is sliding down, with an acceleration a relative to M. Find the force F applied on M and also the acceleration of M. Assuming all surfaces are frictionless.

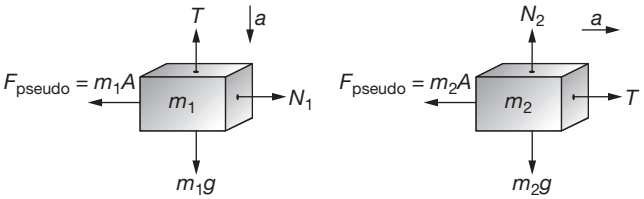


SOLUTION

Let T be the tension in the string, N_1 be the normal reaction between M and m_1, N_2 be the normal reaction between M and m_2, N be the normal reaction between M and ground, A be the acceleration of M, then drawing the free body diagrams of different blocks and writing the equations of motion



FBD M



FBD m_1 FBD m_2

for mass M, we get

$$F - N_1 - T = MA \quad \ldots(1)$$

$$N - N_2 - T - Mg = 0 \quad \ldots(2)$$

for mass m_1, we get

$$m_1 g - T = m_1 a \quad \ldots(3)$$

$$N_1 - m_1 A = 0 \quad \ldots(4)$$

for mass m_2, we get

$$T - m_2 A = m_2 a \quad \ldots(5)$$

and $N_2 - m_2 g = 0 \quad \ldots(6)$

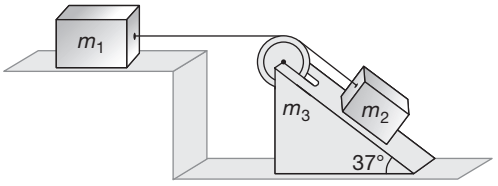
Solving these six equations, we get

$$A = \frac{m_1}{m_2} g - \left(1 + \frac{m_1}{m_2}\right) a$$

and $F = m_1 g + \dfrac{(M + m_1) m_1}{m_2} (g - a) - (M + 2m_1) a$

ILLUSTRATION 55

In the arrangement shown in figure, a wedge of mass $m_3 = 3.45$ kg is placed on a smooth horizontal surface. A small and light pulley is connected on its top edge. A light flexible thread passes over the pulley. Two blocks having mass $m_1 = 1.3$ kg and $m_2 = 1.5$ kg are connected at the ends of the thread, m_1 is on smooth horizontal surface and m_2 rests on inclined surface of the wedge.



If the whole system is released from rest, calculate

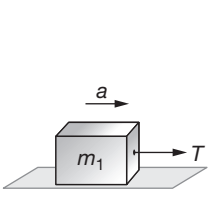
(a) the tension in the string and
(b) acceleration of m_1 and m_3.

All surfaces are smooth. Take $g = 10\ \text{ms}^{-2}$ and $\tan(37°) = \dfrac{3}{4}$

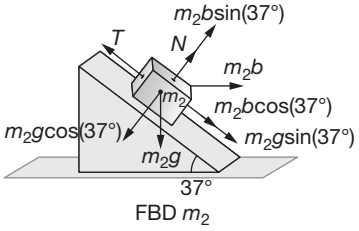
SOLUTION

Let, a be the acceleration of mass m_1 towards right and b be the acceleration of wedge towards left.
So, the acceleration of m_2 relative to m_3 is $(a + b)$ down the plane.

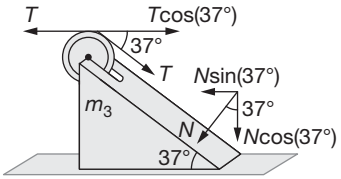
Let T be the tension in the thread and N, the normal reaction between m_2 and m_3. Drawing free body diagrams for m_1, m_2 and m_3 (see figure) and writing equations of motion,



FBD m_1



FBD m_2



FBD m_3

for mass m_1, we have

$$T = m_1 a \qquad \ldots(1)$$

for mass m_2, we have

along the plane

$$m_2 b\cos 37^\circ + m_2 g \sin 37^\circ - T = m_2 (a+b) \qquad \ldots(2)$$

perpendicular to the plane

$$N + m_2 b \sin 37^\circ - m_2 g \cos 37^\circ = 0 \qquad \ldots(3)$$

for mass m_3, we have

$$T + N\sin 37^\circ - T\cos 37^\circ = m_3 b \qquad \ldots(4)$$

We have four unknowns T, N, a and b

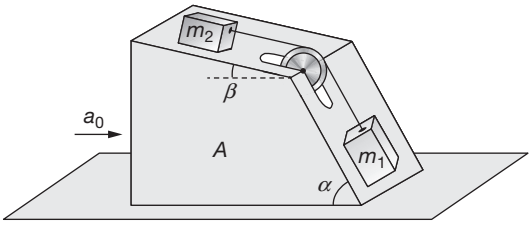
Since $\tan(37^\circ)=\frac{3}{4}$, so $\sin(37^\circ)=\frac{3}{5}$ and $\cos(37^\circ)=\frac{4}{5}$

Substituting $m_1 = 1.3$ kg, $m_2 = 1.5$ kg, $m_3 = 3.45$ kg, $g = 10$ ms^{-2}, we get

$$T = 3.9 \text{ N},\ a = 3 \text{ ms}^{-2} \text{ and } b = 2 \text{ ms}^{-2}$$

ILLUSTRATION 56

Two cubes of masses m_1 and m_2 lie on frictionless slopes of a block A which rests on a horizontal table. The cubes are connected by a string which passes over a pulley as shown in figure. If a_0 be the horizontal acceleration to which the whole system (block + masses) is subjected so that m_1 and m_2 do not move and T be the tension in the string in that situation then, find a_0 and T.

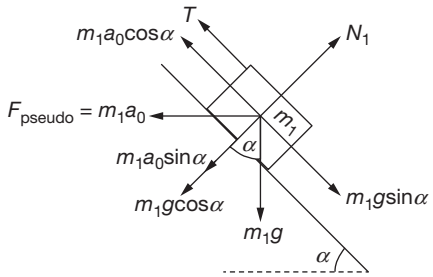


SOLUTION

For m_1

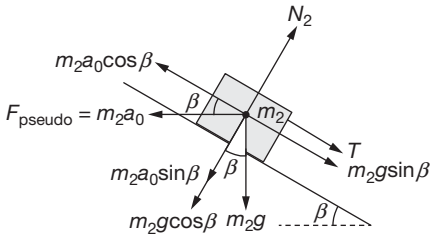
$$T + m_1 a_0 \cos\alpha = m_1 g \sin\alpha$$

$$\Rightarrow \quad T = m_1 g \sin\alpha - m_1 a_0 \cos\alpha \qquad \ldots(1)$$



For m_2

$$T + m_2 g \sin\beta = m_2 a_0 \cos\beta$$



$$\Rightarrow \quad T = m_2 a_0 \cos\beta - m_2 g \sin\beta \qquad \ldots(2)$$

Equating (1) and (2), we get

$$m_1 g \sin\alpha - m_1 a_0 \cos\alpha = m_2 a_0 \cos\beta - m_2 g \sin\beta$$

$$\Rightarrow \quad (m_1 \sin\alpha + m_2 \sin\beta) g = (m_1 \cos\alpha + m_2 \cos\beta) a_0$$

$$\Rightarrow \quad a_0 = \left(\frac{m_1 \sin\alpha + m_2 \sin\beta}{m_1 \cos\alpha + m_2 \cos\beta}\right) g \qquad \ldots(3)$$

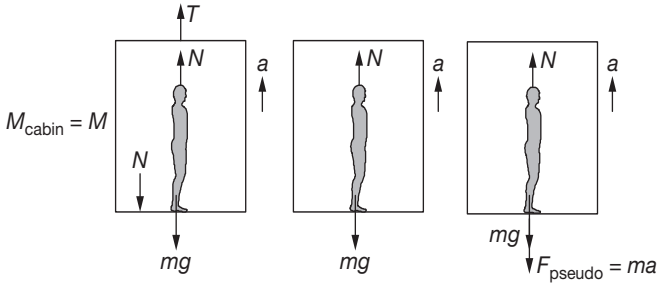
Substituting (3) either in (2) or in (1), we get

$$T = \left[\frac{m_1 m_2}{m_1 \cos\alpha + m_2 \cos\beta}\right] g \sin(\alpha - \beta) \qquad \ldots(4)$$

MAN IN A LIFT

Lift Accelerating Up/Retarding Down

Consider a man of mass m standing in a lift, accelerating upwards with an acceleration a, whose cabin has mass M. The force on man by the lift's floor is N, upwards. Then, the force on the lift by the man is also N, downwards. Let T be the tension in the cable supporting the cabin (lift + man), then the following two cases have been discussed both from the inertial and the non-inertial observer's view point. Force on man by the lift is N, upwards, so force on lift by man is also N, downwards.



Forces acting on man and lift | FBD man, inertial frame | FBD man, non-inertial frame

FBD Man (Inertial Frame):

According to the observer in the inertial frame, the man accelerates up, so

$$N - mg = ma$$

$$\Rightarrow \quad N = m(g + a)$$

FBD Man (Non inertial Frame attached to lift):

According to the non inertial frame, the man is in equilibrium, so

$$N = mg + ma$$

$$\Rightarrow \quad N = m(g + a)$$

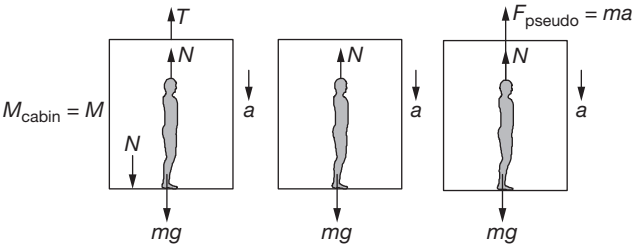
Effective weight or the apparent weight of the man in the lift accelerating up is greater than the actual weight mg. The tension in the cable supporting the lift is found by writing the combined equation for the cabin + man system. So, we have

$$T - (M + m)g = (M + m)a$$

$$\Rightarrow \quad T = (M + m)(g + a)$$

Lift Accelerating Down/Retarding Up

Consider a man of mass m standing in a lift, accelerating upwards with an acceleration a, whose cabin has mass M. The force on man by the lift's floor is N, upwards. Then, the force on the lift by the man is also N, downwards. Let T be the tension in the cable supporting the cabin (lift + man), then the following two cases have been discussed both from the inertial and the non-inertial observer's view point.



Forces acting on man and lift | FBD man, inertial frame | FBD man, non-inertial frame

FBD (Inertial):

According to the observer in the inertial frame, the man accelerates down, so

$$mg - N = ma$$

$$\Rightarrow \quad N = m(g - a)$$

FBD (Non-inertial):

According to the non inertial frame, the man is in equilibrium, so

$$mg - N - ma = 0$$

$$\Rightarrow \quad N = m(g - a)$$

Effective weight or the apparent weight of the man in the lift accelerating down is less than the actual weight mg. The tension in the cable supporting the lift is found by writing the combined equation for the cabin + man system. So, we have

$$(M + m)g - T = (M + m)a$$

$$\Rightarrow \quad T = (M + m)(g - a)$$

Problem Solving Technique(s)

(a) If M is the mass of the cabin including the person(s) in it, then tension in the cable of the lift when lift accelerates up is $T = M(g + a)$ and when it accelerates down is $T = M(g - a)$.

(b) A person of mass M climbs up a rope with acceleration a. The tension in the rope will be $T = M(g + a)$. For rope not to break, $T <$ Breaking Tension of Rope.

(c) If the person climbs down along the rope with acceleration a, the tension in the rope will be $T = M(g - a)$. For rope not to break, $T <$ Breaking Tension of Rope or $a < g$.

(d) When the person climbs up or down with uniform speed, tension in the string will be Mg.

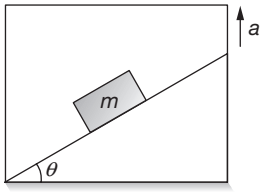
(e) When a man jumps with load on his head, the apparent weight of the load and the man is zero.

Test Your Concepts-VI

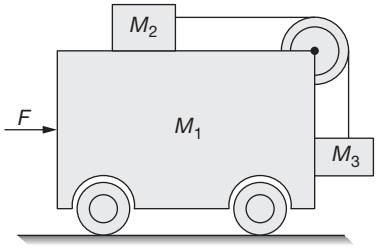
Based on Non-inertial Frames: Pseudo Force

(Solutions on page H.206)

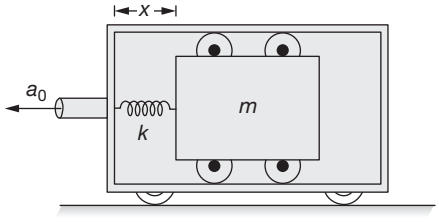
1. In the figure, a wedge is fixed to an elevator moving upwards with an acceleration a. A block of mass m is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.



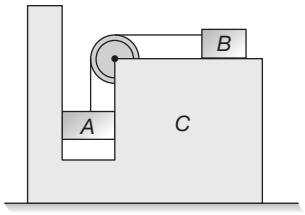
2. In the figure all surfaces are frictionless. What force F must by applied to M_1 to keep M_3 free from rising or falling?



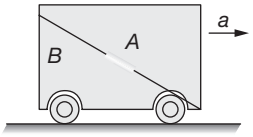
3. The block of mass m is attached to the frame by the spring of stiffness k and moves horizontally with negligible friction within the frame. The frame and block are initially at rest with $x = x_0$, the uncompressed length of the spring. If the frame is given a constant acceleration a_0, determine the maximum velocity $v_{max} = (v_{rel})_{max}$ of the block relative to the frame.



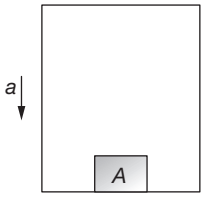
4. In the system shown in figure $m_A = 4\,\text{m}$, $m_B = 3\,\text{m}$ and $m_C = 8\,\text{m}$. Friction is absent everywhere. String is light and inextensible. If the system is released from rest find the acceleration of each block.



5. The collar A is free to slide along the smooth shaft B mounted in the frame. The plane of the frame is vertical. Determine the horizontal acceleration a of the frame necessary to maintain the collar in a fixed position on the shaft.



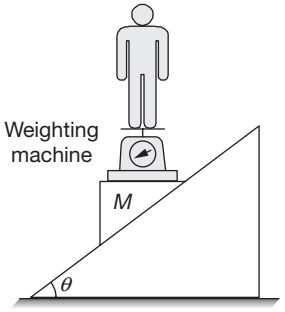
6. Calculate the acceleration a of the lift cabin shown in figure so that the block A of mass m exerts a force $\frac{mg}{4}$ on the floor of the cabin.



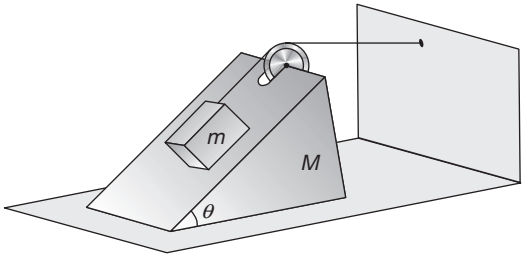
7. A plumb bob of mass 2 kg is hung from the ceiling of a car. The car moves on an inclined plane with constant velocity. If the angle of incline is 30°. Find the angle made by the string with the normal to the ceiling. Also find the tension in the string. $\left(g = 10\ \text{ms}^{-2}\right)$

8. Repeat both parts of the above question if the car moves with an acceleration $a_0 = \frac{g}{2}$ up the plane.

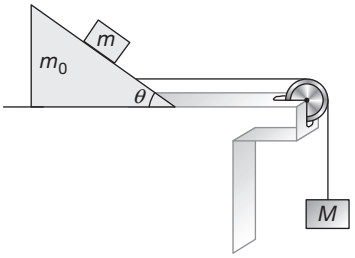
9. Find the weight shown by the weighing machine on which a man of mass m is standing at rest relative to it as shown in figure. Assume that the wedge of mass M is in free fall.



10. In the figure a bar of mass m is on the smooth inclined face of the wedge of mass M, the inclination to the horizontal being θ. The wedge is resting on a smooth horizontal plane. Assuming the pulley to be smooth and the string is light and inextensible. Find the acceleration of M, when M and m are always in contact.



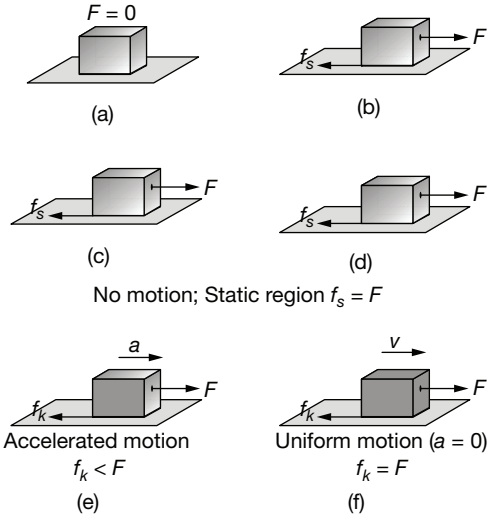
11. Find the mass M of the hanging block shown in figure, which will prevent the smaller block from slipping over the triangular block. All the surfaces are frictionless and the strings and the pulleys are light.



FRICTION

INTRODUCTION

Actually, whenever the surface of one body slides or has a tendency to slide on the surface of another then each body exerts a frictional force on the other, parallel to the surfaces. The frictional force on each body is in a direction opposite to its motion relative to the other body. Frictional forces automatically oppose the relative motion and never aid it. Even when there is no relative motion, frictional forces may exist between surfaces (but there must be a tendency of relative motion).



Consider a block at rest on a horizontal table as shown in figure. We find that the block will not move even though we apply a small force. We say that our applied force is balanced by an opposite frictional force exerted on the block by the table, acting along the surface of contact. As we increase the applied force we find that there is some definite value of the applied force at which the block just begins to move. Once motion has started, this same force (without increasing any further) produces accelerated motion. By reducing the force once motion has started, we find that it is possible to keep the block in uniform motion without acceleration; this force may be small, but it is never zero. Now, analyze the diagrams given in figure once again.

In Figure (a), no force is applied on the block due to which it does not have tendency to slide over the surface of the table and hence, the surface does not apply any opposing (i.e., frictional) force on the block.

In Figure (b), we have applied a small force on the block and its magnitude is being continuously increased. It is clear from the second, third and fourth figures, that as we are increasing external force, frictional force acting upon the block also increases and prevents the block from sliding over the surface of the table.

But at a certain instant friction reaches its **maximum value**, referred to as **limiting friction**, and now if we increase the external force, even slightly, the block will start sliding in the direction of the external force. We can also say that now the external force has sufficient magnitude to break molecular bonds between the surfaces.

Conclusion

Therefore, if the applied force is less than limiting friction, the block will not move and the frictional force will have the same magnitude as that of the external applied force. Since the body is at rest with respect to the surface on which it is placed so, this frictional force is called **static frictional force** and static friction being self adjusting, so static frictional force can have a magnitude lying between zero and limiting value of frictional force (which is the maximum value of static friction), or we can write

$0 \le f_{\text{static}} \le f_{\text{limiting}}$, where

$f_{\text{limiting}} = (f_s)_{\max}$ is the maximum value of static friction.

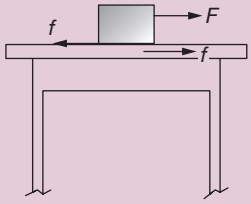
In Figure (d), frictional force has already reached its maximum value (i.e., its limiting value) and is just balancing the horizontal force F applied on the block. If F is increased further by even a negligible amount, the block starts sliding. Once the body has started sliding it is observed that the magnitude of friction (or we can say the magnitude of kinetic friction) is constant and is smaller than limiting friction (i.e., maximum value of static friction). We can write

$f_{\text{kinetic}} < f_{\text{limiting}}$

The frictional forces acting between surfaces at rest with respect to each other are called forces of **static friction**. The maximum force of static friction will be the same as the smallest force necessary to start motion. Once motion has started, the frictional forces acting between the surfaces usually decrease so that a smaller force is necessary to maintain uniform motion. The forces acting between surfaces in relative motion are called forces of **kinetic friction**.

Conceptual Note(s)

Note that in figure the block, of course exerts an equal and opposite frictional force on the table (as shown in figure), tending to drag it in the direction of the horizontal force we exert. This frictional force is due to the bonding of the molecules of the block and the table at the places where the surfaces are in very close contact. If we focus on the table only, we see that friction tends to move the table in the same direction in which force F tends to move the block. This observation leads us to two very important conclusions.



(a) friction does not always oppose motion (because it is friction only which is trying to move the table)

(b) friction always opposes relative motion between surfaces in contact (because it is trying to move the table in the same direction in which the block has a tendency to move).

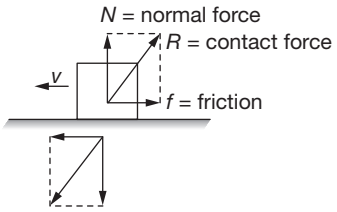
REASONS FOR FRICTION

(a) Frictional forces arise due to molecular interactions due to which a bonding between the molecules of the two surfaces or objects in contact comes into being due to which it becomes difficult to move one surface on the other.

(b) Inter locking of extended parts of one object into the extended parts of the other object.

CONTACT FORCE AND FRICTION

When two bodies are kept in contact then each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal in magnitude and opposite in direction. So the contact forces obey Newton's Third Law.



The contact force acting on a particular body is not necessarily perpendicular to the contact surface. So, contact force can be resolved into two components.

(a) Perpendicular to the contact surface and

(b) Parallel to contact surface.

The perpendicular component is called the Normal Contact Force or Normal Reaction (generally written as N) and the parallel component is called Friction (generally written as f).

Therefore if R is contact force then

$$R = \sqrt{f^2 + N^2}$$

STATIC AND KINETIC FRICTION

The frictional force between two surfaces before the relative motion actually starts is called **static frictional force** or **static friction**, while the frictional force between two surfaces in contact and in relative motion is called **kinetic frictional force** or **kinetic friction**.

Static friction is a self-adjusting force and it adjusts both in magnitude and direction automatically. Its magnitude is always equal to external effective applied force, tending to cause the relative motion and its direction is always opposite to that of external applied force.

So, when a body is not in motion or is in equilibrium, then **force of static friction is equal to the component of the applied External force(s) tangential to the surface.**

$$\begin{pmatrix}\text{Force of}\\ \text{Static Friction}\end{pmatrix} = \begin{pmatrix}\text{Applied External}\\ \text{Force Tangential to the}\\ \text{surfaces in Contact}\end{pmatrix}$$

Conceptual Note(s)

Static Friction exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

For Example

Consider a bed inside a room. When we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the direction opposite to the applied force.

Kinetic friction exists between two contact surfaces only when there is relative motion between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

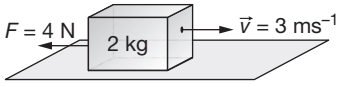
The direction of kinetic friction on an object is opposite to the relative velocity of the object with respect to the other object in contact considered.

Conceptual Note(s)

Note that, the direction of kinetic friction is not opposite to the force applied. It is opposite to the relative motion of the body considered which is in contact with the other surface.

ILLUSTRATION 57

Find the direction of kinetic force of friction.



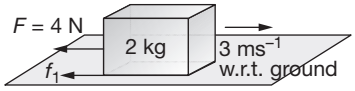
(a) f_1 on the block, exerted by the ground.

(b) f_2 on the ground, exerted by the block.

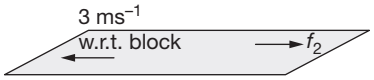
Also find the relation between f_1 and f_2

SOLUTION

(a) Since the block is moving to the right at the instant shown, so the kinetic friction on the block due to the surface is towards the left.



(b) Since friction force obeys Newton's Third Law, so the friction on the surface due to the block is to the right.

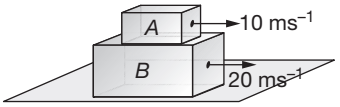


Since f_1 and f_2 obey Newton's Third Law, so they are equal in magnitude and opposite in direction. So $f_1 = f_2$.

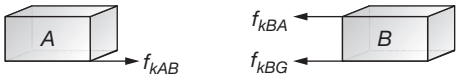
Also note that the direction of kinetic friction has nothing to do with applied force F.

ILLUSTRATION 58

All surfaces as shown in the figure are rough. Draw the friction force on A and B.



SOLUTION



Kinetic friction acts in such a way so as to reduce relative motion.

ILLUSTRATION 59

Calculate the acceleration of the block for different ranges of the applied force F.

SOLUTION

$$0 \le f \le \mu_S N$$

$$\Rightarrow \quad 0 \le f \le \mu_S (mg)$$

If the applied force $F \le \mu_S mg$, then

$$a = 0$$

and if the applied force $F > \mu mg$, then

$$a = \frac{F - \mu mg}{m}$$

LAWS OF FRICTION

In Static Region

The **maximum force of static friction** between any pair of dry unlubricated surfaces follows these two empirical laws.

(a) It is approximately independent of the area of contact, and
(b) it is proportional to the normal force.

$$f_s \propto N$$

The ratio of the magnitude of the maximum force of static friction to the magnitude of the normal force is called the **coefficient of static friction** (μ_s) for the surfaces involved.

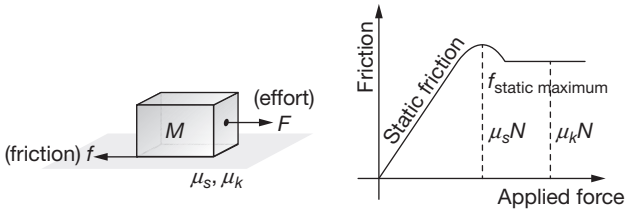
The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. This means it is a variable and self-adjusting force. However it has a maximum value called limiting friction. i.e.

$$(f_s)_{\max} = f_{\text{limiting}} = f_l, \quad \text{where } f_{\max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body.

The magnitude of frictional force is equal to that required to keep the body at relative rest. So, we have $0 \le f_s \le f_l$.

The plot of force of friction with the applied force is shown here.



It is observed that

1. initially, when there is no applied force, then force of friction is zero.
2. as the applied force is increased and the body is still at rest, then the force of static friction is equal to the applied force.
3. for a particular maximum value of the applied force, the body is on the verge of motion or is just about to move. This maximum value of force of static friction is called the force of limiting friction.
4. when the applied force is increased beyond this maximum value, the body starts moving with respect to the surface on which it is kept and now the friction between the body and the surface is kinetic in nature.

Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and Teflon	0.04
Glass and glass	1.00	Wood and metal	0.40
Wood and wood	0.35	Rubber and rubber	1.16
Rubber tyre on dry concrete road	1.0	Rubber tyre on wet concrete road	0.7

In Kinetic Region

The force of kinetic friction, f_k, between dry unlubricated surfaces follows the same two laws as those of static friction.

(a) It is approximately independent of the area of contact and
(b) It is proportional to the normal force.

$$f_k \propto N$$

The force of kinetic friction is also reasonably independent of the relative speed with which the surfaces move over each other (provided it is neither too small, nor too large).

The ratio of the magnitude of the force of the kinetic friction to the magnitude of the normal force is called the **coefficient of kinetic friction** (μ_k). If f_k represents the magnitude of the force of kinetic friction, N the normal reaction and μ_k the coefficient of kinetic friction, then

$$f_k = \mu_k N$$

Conceptual Note(s)

(a) Note that the equation $f = \mu N$ is the relation between the magnitudes of the normal and frictional forces. These forces (frictional force and normal contact force) are always directed perpendicularly to each other. **So it would absolutely be incorrect to write** $\vec{f} = \mu\vec{N}$.

(b) Both μ_s and μ_k are dimensionless constants, each being the ratio of the magnitudes of the two forces (frictional force and normal contact force). Usually, for a given pair of surfaces $\mu_s > \mu_k$. The actual values of μ_s and μ_k depend on the nature of both the surfaces in contact. Both μ_s and μ_k can exceed unity, although commonly they are less than one.

(c) Coefficient of limiting friction is the ratio of the force of limiting friction $f_\ell \left(= (f_s)_{\max}\right)$ to the normal reaction.

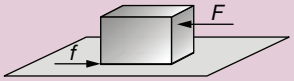
(d) If not mentioned then $\mu_s = \mu_k$ can be taken.

(e) Value of μ can be from 0 to ∞.

(f) **Static friction comes into play** whenever the surfaces in contact donot move with respect to each other
OR whenever a surface has a tendency to slip on another surface
OR whenever force applied on a body kept on a surface tends to move the body
and in all these cases the force of friction is opposite to the tendency of motion of the body due to the applied force.

(g) **Kinetic friction comes into play** whenever there is a relative motion between the two surfaces in contact and then the force of kinetic friction is always directed opposite to the direction of instantaneous relative velocity of the body with respect to the surface on which the body is kept.

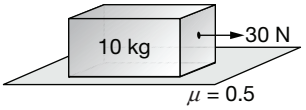
(h) Since force of friction is a contact force and hence it is shown tangentially to the surfaces in contact.



(i) For a given pair of surfaces, we have $\mu_s > \mu_k$, where μ_s and μ_k are proportionality constants called coefficients of static and kinetic friction. They are dimensionless quantities independent of shape and area of contact. Coefficient of friction μ is a property of the two surfaces in contact.

ILLUSTRATION 60

Calculate the acceleration of the block as shown in the figure.



SOLUTION

Since $F_{\text{app}} = 30\ \text{N}$ and $f_{\text{limiting}} = (0.5)(100) = 50\ \text{N}$

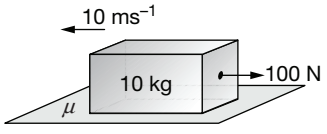
Since $F_{\text{app}} < f_{\text{limiting}}$

$\Rightarrow \quad a = 0$

and $\quad f_s = F_{\text{app}} = 30\ \text{N}$

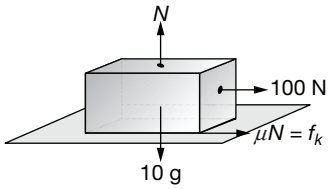
ILLUSTRATION 61

A block of mass $10\ \text{kg}$ is given a velocity of $10\ \text{ms}^{-1}$ and a force of $100\ \text{N}$ in addition to friction force is also acting on the block. Calculate the retardation of the block?



SOLUTION

Since there is relative motion, so the kinetic friction will act to reduce this relative motion.



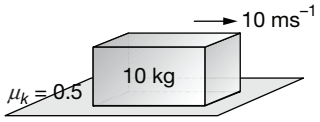
$$f_k = \mu N = (0.1)(10)(10) = 10 \text{ N}$$

$$\Rightarrow \quad 100 + 10 = 10a$$

$$\Rightarrow \quad a = \frac{110}{10} = 11 \text{ ms}^{-2}$$

ILLUSTRATION 62

Calculate the distance travelled by the block shown in the figure before it stops.



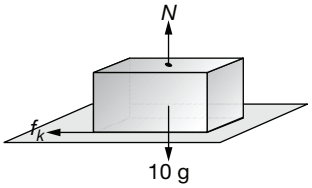
SOLUTION

Since, $f_k = \mu_k N$

When not mentioned, we have

$$\mu = \mu_s = \mu_k = 0.5$$

$$\Rightarrow \quad f_k = (0.5)(100) = 50 \text{ N}$$



Since, $N - 10g = 0$

$$\Rightarrow \quad N = 100 \text{ N}$$

Now, $F = ma$

$$\Rightarrow \quad 50 = 10a$$

$$\Rightarrow \quad a = 5$$

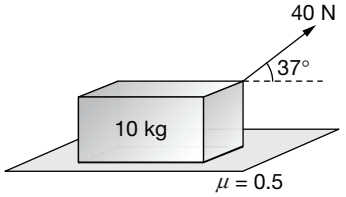
Since, $v^2 = u^2 + 2as$

$$\Rightarrow \quad 0^2 = 10^2 + 2(-5)(5)$$

$$\Rightarrow \quad s = 10 \text{ m}$$

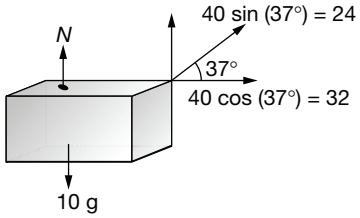
ILLUSTRATION 63

Calculate the acceleration of the block. Assume that initially the block is at rest.



SOLUTION

$N + 24 - 100 = 0$, for vertical direction



$$\Rightarrow \quad N = 76 \text{ N}$$

Now $0 \le f_s \le f_l$

$$\Rightarrow \quad 0 \le f_s \le 76 \times 0.5$$

$$0 \le f \le 38 \text{ N}$$

Since applied force is less than limiting force

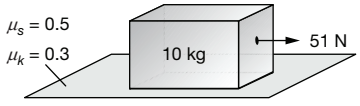
$$32 < 38$$

Hence $f = 32$

$\Rightarrow$ Acceleration of block is zero.

ILLUSTRATION 64

In the arrangement shown, calculate the acceleration of the block. Assume that initially the block is at rest.



SOLUTION

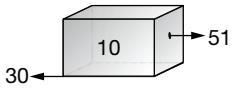
Since, the limiting friction is given by

$$f_l = \mu N = 0.5(100)$$

$$\Rightarrow \quad f_l = 50 \text{ N} < F_{\text{app}}$$

$$\Rightarrow \quad 0 \le f_s \le \mu_s N$$

$$\Rightarrow \quad 0 \le f_s \le 50$$



So, block will move and hence kinetic friction will come into play. So

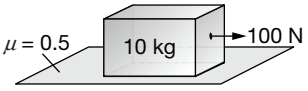
$$f_k = \mu_k N = 0.3 \times 100 = 30 \text{ N}$$

$$\Rightarrow \quad 51 - 30 = 10a$$

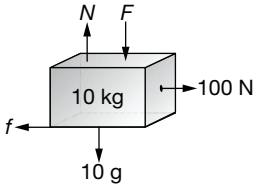
$$\Rightarrow \quad a = 2.1 \text{ ms}^{-2}$$

ILLUSTRATION 65

Calculate the minimum force that must be applied on the block vertically downwards so that the block does not move.



SOLUTION



Since, $100 - f_s = 0$

$$\Rightarrow \quad f_s = 100 \qquad \ldots(1)$$

$$F + 10g = N$$

$$\Rightarrow \quad N = 100 + F \qquad \ldots(2)$$

Now $0 \le f_S \le \mu N$

$$\Rightarrow \quad 100 \le 0.5N$$

$$\Rightarrow \quad 100 \le 0.5(100 + F)$$

$$\Rightarrow \quad 200 \le 100 + F$$

$$\Rightarrow \quad F \ge 100 \text{ N}$$

$$\Rightarrow \quad \text{Minimum force is } F_{\min} = 100 \text{ N}$$

Conceptual Note(s)

Direction of Static Friction Force

The static friction force on an object is opposite to its impending motion relative to the surface or tendency of motion of body w.r.t. the surface. The steps to be followed in determining the direction of static friction force on an object are

STEP-1: Draw the free body diagram with respect to the other object or the surface on which it is kept.

STEP-2: Also include pseudo force if contact surface is accelerating.

STEP-3: Find the resultant force.

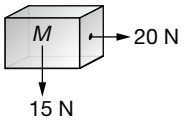
STEP-4: Calculate the component of this force tangential to the surfaces in contact i.e. parallel to the contact surfaces.

STEP-5: The direction of static friction is opposite to the tangential component of resultant contact force.

Here once again, we must note that the static friction is involved when there is no relative motion between two surfaces.

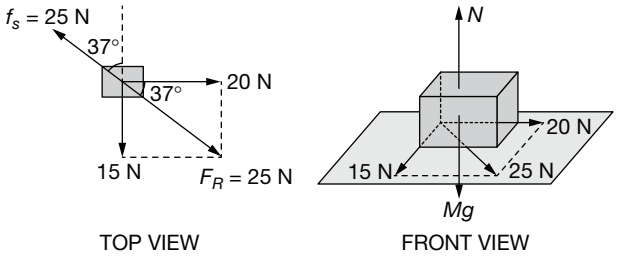
ILLUSTRATION 66

In the figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.



SOLUTION

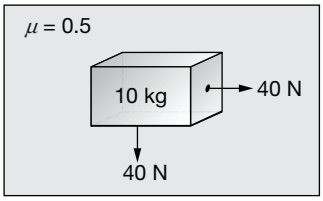
In the above Illustration we first draw the free body diagram of find the resultant force.



As the object does not move so this is not a case of limiting friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure. Its magnitude is equal to 25 N.

ILLUSTRATION 67

A block of mass 10 kg is placed on a horizontal rough table surface. The top view of the block on the table is shown. Calculate the acceleration of the block, taking $\mu = 0.5$ and $g = 10\ \text{ms}^{-2}$.



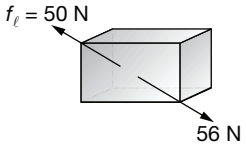
SOLUTION

The resultant force acting on the block is

$$F_R = \sqrt{40^2 + 40^2} = 40\sqrt{2}\ \text{N}$$

$$\Rightarrow \quad F_R = 56\ \text{N}$$

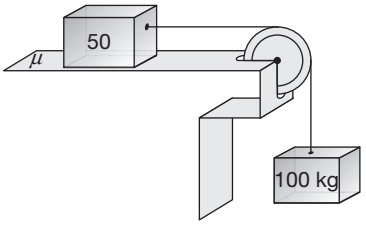
Since, $f_l = (0.5)(100) = 50\ \text{N} < F_R$



So, the block will move in the direction of resultant force with an acceleration $a = \dfrac{56-50}{10} = 0.6\ \text{ms}^{-2}$

ILLUSTRATION 68

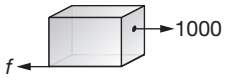
Find minimum μ so that the blocks remain stationary.



SOLUTION

$$T = 100g = 1000\ \text{N}$$

$$\Rightarrow \quad f = 1000 \text{ to keep the block stationary}$$



Now $f_{\max} = 1000$

$$\mu N = 1000$$

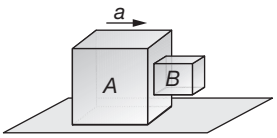
$$\mu = 2$$

Can μ be greater than 1?

Yes $0 < \mu \leq \infty$

ILLUSTRATION 69

In the arrangement shown, the coefficient of friction between A and B is 0.5. Calculate the minimum acceleration of block A so that the block B of mass 10 kg doesn't fall.



SOLUTION

Applying Newton's Law in horizontal direction, we get

$$N = 10a \qquad \ldots(1)$$

For block not to slip, we have

$$10g \leq f_l$$

$$\Rightarrow \quad mg \leq \mu N$$

$$\Rightarrow \quad mg \leq \mu ma$$

$$\Rightarrow \quad a \geq \frac{g}{\mu}$$

$$\Rightarrow \quad a_{\min} = \frac{g}{\mu} = 20\ \text{ms}^{-2}$$

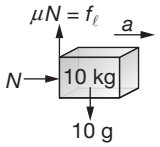
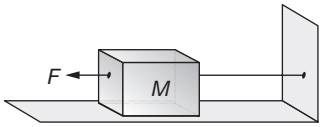


ILLUSTRATION 70

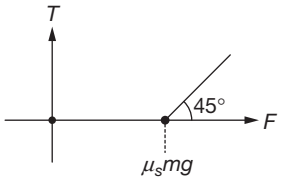
In the figure shown, the force F is gradually increased from zero.



Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ.

SOLUTION

As the external force F is gradually increased from zero it is compensated by the static friction and the string bears no tension. So, till $F_{app} < f_{limiting}$, we have $T = 0$. So for $F_{app} > f_l$



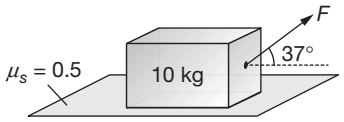
$$\Rightarrow \quad F_{app} > \mu mg$$

We have, $F = T + \mu mg$

$$\Rightarrow \quad T = F - \mu mg$$

ILLUSTRATION 71

In the arrangement shown, the force F is gradually increased from zero. Determine whether the block will first slide or lift up.

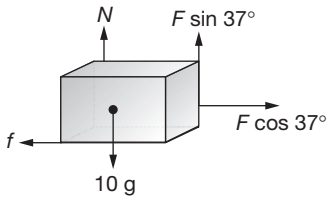


SOLUTION

We have to calculate the minimum magnitude of forces required both in horizontal and vertical direction either to slide or lift up the block. The block will first slide or lift up depends upon which minimum magnitude of force is lesser.

CASE-1: For block to start lifting up in vertical direction, we have

$$F\sin(37°) + N - Mg \geq 0$$



N becomes zero just before lifting, so

$$F_{lift} \geq \frac{10g}{3/5}$$

$$\Rightarrow \quad F_{lift} \geq \frac{500}{3}\ \text{N}$$

CASE-2: For block to start sliding in horizontal direction, we have

$$F\cos(37°) \geq \mu_s N$$

Since $N = 10g - F\sin(37°)$

$$\Rightarrow \quad F\cos(37°) \geq 0.5\left(10g - F\sin(37°)\right)$$

Hence $F_{slide} \geq \dfrac{50}{\cos(37°) + 0.5\sin(37°)}$

$$\Rightarrow \quad F_{slide} \geq \frac{500}{11}\ \text{N}$$

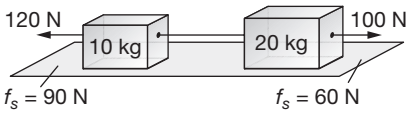
Since $F_{lift} \geq \dfrac{500}{3}\ \text{N}$

$$\Rightarrow \quad F_{slide} < F_{lift}$$

Hence the block will begin to slide horizontally before it begins to lift up.

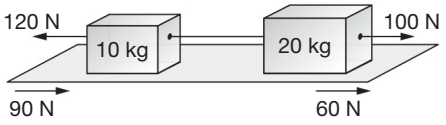
ILLUSTRATION 72

Calculate the tension in the string in situation as shown in the figure below. Forces 120 N and 100 N start acting when the system is at rest and the maximum value of static friction for 10 kg is 90 N and that for 20 kg is 60 N?



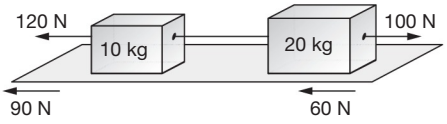
SOLUTION

(a) Let us assume that system moves towards left then as it is clear from *FBD*, net force in horizontal direction is towards right. Therefore the assumption is not valid.



Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

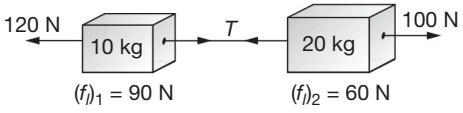
(b) Similarly let us assume that system moves towards right.

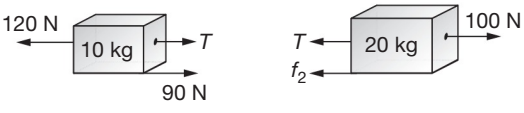


Above assumption is also not possible as net force on the system is towards left in this situation. Hence assumption is again not valid.

Therefore we conclude that the system remains stationary.

Assuming that the 10 kg block reaches limiting friction first then using FBD's





For 10 kg block to be in equilibrium, we have

$$120 = T + 90$$

$$\Rightarrow \quad T = 30 \text{ N}$$

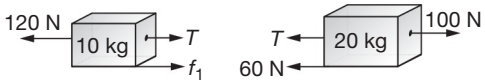
Also $T + f_2 = 100$

$$\Rightarrow \quad 30 + f_2 = 100$$

$$\Rightarrow \quad f_2 = 70 \text{ N}$$

Which is not possible as the limiting value is 60 N for this surface of block.

So, our assumption is wrong and hence now let us take the 20 kg block to be in limiting situation, hence



$$T + 60 = 100 \text{ N}$$

$$\Rightarrow \quad T = 40 \text{ N}$$

Also $f_1 + T = 120$ N

$$\Rightarrow \quad f_1 = 80 \text{ N} < (f_l)_1$$

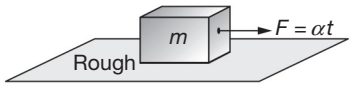
This is acceptable as static friction at this surface should be less than 90 N.

Hence the tension in the string is

$$T = 40 \text{ N}$$

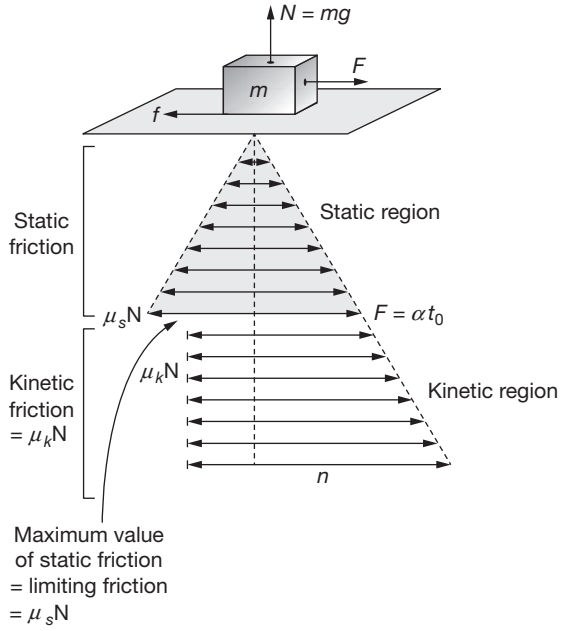
ILLUSTRATION 73

A block of mass m is placed on a rough horizontal surface, as shown in figure. At $t = 0$, a horizontal force $F = \alpha t$, where α is a positive constant, is applied on the block. If μ_s and μ_k be coefficients of static and kinetic frictions respectively, find the frictional force acting on the block as a function of time. Also plot the frictional force against the applied horizontal force F.



SOLUTION

The detailed analysis of the given situation is shown in figure. At $t = 0$, when $F = 0$, frictional force is also zero but as F starts increasing from zero, frictional force also increases and balances it or we can say that friction prevents the block from sliding over the horizontal surface. As time passes, F increases and f also increases.



In fact friction and F have the same magnitude until the block starts sliding. At some time t_0, static friction reaches its maximum value $\mu_s N$. After $t = t_0$, F will keep increasing but friction will not increase and hence the block can not remain in equilibrium and eventually it acquires motion along the direction of F. But as soon as the block starts sliding, the nature of the friction becomes kinetic and the magnitude of the frictional force comes down to $\mu_k N$ and thereafter remains constant as long as the block is in motion. If the block starts sliding immediately after $t = t_0$, then at $t = t_0$.

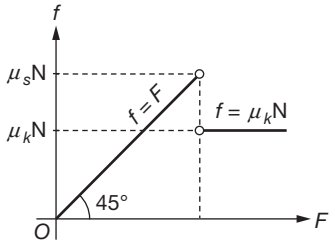
$$F = f$$

$$\Rightarrow \quad \alpha t_0 = \mu_s N$$

$$\Rightarrow \quad \alpha t_0 = \mu_s mg$$

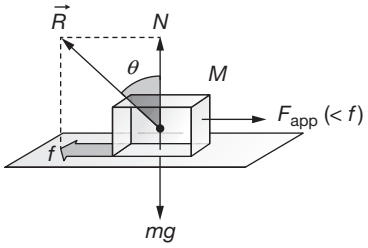
$$\Rightarrow \quad t_0 = \frac{\mu_s mg}{\alpha}$$

Therefore, for the time interval $(0,\ t_0)$: $f = F(= \alpha t)$
and for the time interval $(t_0,\ \infty)$: $f = \mu_k N = \mu_k mg$
The plot of f against F is shown in figure.



COEFFICIENT OF FRICTION, LIMITING FRICTION AND ANGLE OF FRICTION

Consider a block resting on a rough horizontal surface. The forces acting on the block are its weight mg downwards and normal reaction N acting upward, such that $N = mg$.



Now suppose a force F_{app} is applied on the block to the right, then there will arise a frictional force f directed to the left (opposite to direction of applied force), which prevents the motion of the block. Let the resultant of $\vec{N}$ and $\vec{f}$ be $\vec{R}$ which makes an angle θ with normal reaction $\vec{N}$. Resolving $\vec{R}$, we get

$$R\cos\theta = N \text{ and } R\sin\theta = f \qquad \ldots(1)$$

For equilibrium $N = mg$ and $f = F_{app}$ $\qquad \ldots(2)$

If we increase the pull F_{app} continuously, the force of friction increases and a stage comes when the body is just on the state of moving. This state is called **limiting equilibrium**. Under this condition the frictional force is maximum and is equal to applied force.

THE COEFFICIENT OF FRICTION (μ)

It is defined as the ratio of limiting friction f to the normal reaction N between two surfaces in contact,

i.e., $\mu = \frac{f}{N}$

Limiting Friction

The maximum value of static frictional force exerted between two surfaces in contact parallel to surfaces for a given normal force between them, when the body is on the verge of motion, is called **limiting friction**.

Angle of Friction

Angle of friction (θ) is the angle which the resultant of force of static friction (f) and normal reaction (N) makes with the normal reaction.
From (1), we get

$$\tan\theta = \frac{f}{N}$$

Since $\mu = \frac{f}{N}$

$\Rightarrow \quad \tan\theta = \mu$

Hence coefficient of static friction is equal to tangent of the angle of friction.

RESULTANT FORCE EXERTED BY A SURFACE ON THE BLOCK

The Reaction Force is the resultant of force of friction and the normal reaction, so from the above figure, we have

$$R = \sqrt{f^2 + N^2}$$

$$\Rightarrow \quad R = \sqrt{(\mu mg)^2 + (mg)^2}$$

$$\Rightarrow \quad R = mg\sqrt{\mu^2 + 1}$$

when there is no friction $(\mu = 0)$, the reaction force R will be minimum

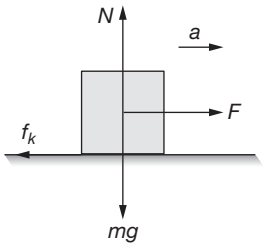
$$\Rightarrow \quad R = mg$$

Hence the range of Reaction Force R is

$$\Rightarrow \quad mg \le R \le mg\sqrt{\mu^2 + 1}$$

ACCELERATION OF BLOCK ON ROUGH HORIZONTAL SURFACE

When body is moving under application of force $F > f_l$, then kinetic friction f_k opposes its motion.



Let a be the acceleration of the body, then from the figure, we observe that

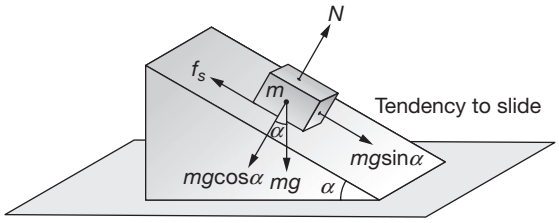
$$F - f_k = ma$$

So, $a = \frac{F - f_k}{m}$, for $F > f_l = \mu_s N = \mu_s (mg)$ and

$a = 0$, for $F \le f_l = \mu_s N = \mu_s (mg)$

ANGLE OF REPOSE (α)

This is concerned with an inclined plane on which a block rests, exerting its weight on the plane.



The angle of repose α is the angle which an inclined plane makes with the horizontal such that a body placed on it is on the verge of motion (is just about to loose the state of rest).
Under this condition the forces acting on the block are:

(a) its weight mg, downward,

(b) normal reaction N, normal to plane,

(c) a force of friction f_s, parallel and tangential to plane upward.

Taking α as angle of inclination of the plane with the horizontal and resolving mg, parallel and

perpendicular to inclined plane, then for equilibrium, we get

$$N = mg\cos\alpha \quad \text{and} \quad f_s = mg\sin\alpha$$

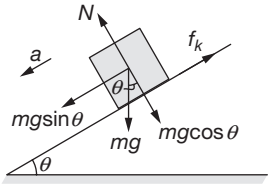
$$\Rightarrow \quad \tan\alpha = \frac{f_s}{N} = \mu_s \qquad \left\{\because\ \mu_s = \frac{f_s}{N}\right\}$$

Reaction force i.e. contact force is

$$R = \sqrt{f_s^2 + N^2} = \sqrt{(mg\sin\theta)^2 + (mg\cos\theta)^2} = mg$$

ACCELERATION OF BLOCK DOWN A ROUGH INCLINE

When angle of inclined plane (θ) is more than angle of repose (α), then the body placed on the inclined plane starts sliding down with an acceleration a. From the figure



$$mg\sin\theta - f_k = ma$$

$$\Rightarrow \quad ma = mg\sin\theta - \mu_k N$$

$$\Rightarrow \quad ma = mg\sin\theta - \mu_k mg\cos\theta$$

$$\Rightarrow \quad \text{Acceleration, } a = g(\sin\theta - \mu_k\cos\theta)$$

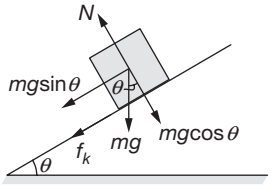
Conceptual Note(s)

For frictionless inclined plane $\mu = 0$, so we get $a = g\sin\theta$.

RETARDATION OF BLOCK MOVING UP A ROUGH INCLINE

When the angle of inclined plane (θ) is less than the angle of repose (α), then for the upward motion of the block, retardation is given by

$$\text{Retardation} = \frac{\text{Net retarding force}}{\text{Total mass}}$$



$$\Rightarrow \quad a = \frac{mg\sin\theta + f_k}{m}$$

$$\Rightarrow \quad a = \frac{mg\sin\theta + \mu mg\cos\theta}{m}$$

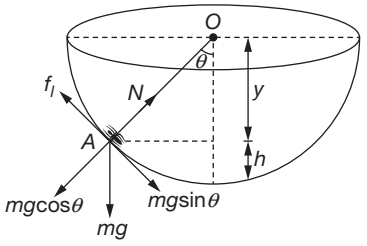
$$\Rightarrow \quad \text{Retardation, } a = g(\sin\theta + \mu\cos\theta)$$

Conceptual Note(s)

For frictionless inclined plane $\mu = 0$, so the retardation is given by $a = g\sin\theta$.

MAXIMUM HEIGHT (H) TO WHICH AN INSECT CAN CRAWL UP A ROUGH HEMISPHERICAL BOWL

The insect can crawl up the bowl, up to a certain height h only till the component of its weight along the bowl is balanced by limiting frictional force.



Let m be the mass of the insect, r be the radius of the bowl and μ be the coefficient of friction, then at the point A, we have

$$N = mg\cos\theta \qquad \ldots(1)$$

$$f_l = mg\sin\theta \qquad \ldots(2)$$

Dividing (2) by (1), we get

$$\tan\theta = \frac{f_l}{N} = \mu \qquad \left\{\because\ \mu = \frac{f_l}{N}\right\}$$

$$\Rightarrow \quad \frac{\sqrt{r^2-y^2}}{y}=\mu$$

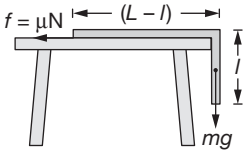
$$\Rightarrow \quad y=\frac{r}{\sqrt{1+\mu^2}}$$

Since, $h=r-y=r\left[1-\frac{1}{\sqrt{1+\mu^2}}\right]$

$$\Rightarrow \quad h=r\left[1-\frac{1}{\sqrt{1+\mu^2}}\right]$$

MAXIMUM LENGTH OF CHAIN THAT CAN HANG FROM THE TABLE WITHOUT FALLING FROM IT

Let a uniform chain of length L be placed on the table such that its length l is hanging over the edge of table without sliding. So, the weight of the hanging part of the chain must be balanced by the force of friction that is acting on the part of chain lying on the table



$$\Rightarrow \quad \mu N=mg$$

$$\Rightarrow \quad \mu\left[\lambda(L-l)g\right]=\lambda l g$$

where λ is mass per unit length of the chain

$$\Rightarrow \quad \mu(L-l)=l$$

$$\Rightarrow \quad l=\frac{\mu L}{\mu+1}$$

So percentage of length of chain hanging is $\frac{l}{L}\times 100\%$.

MINIMUM FORCE FOR MOTION ALONG HORIZONTAL SURFACE AND ITS DIRECTION

Consider a block of mass m on which a force F is applied such that it makes an angle θ with the horizontal. Then for the motion of the block to just begin (or for the block to be on the verge of motion), by resolving F in horizontal and vertical direction (as shown in figure), we get

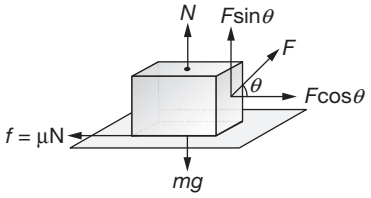
$$F\cos\theta \geq \mu N \text{ and} \qquad \ldots(1)$$

$$N+F\sin\theta=mg$$

$$\Rightarrow \quad N=mg-F\sin\theta \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$F\cos\theta \geq \mu mg-\mu F\sin\theta$$



$$\Rightarrow \quad F\geq \frac{\mu mg}{\cos\theta+\mu\sin\theta} \qquad \ldots(3)$$

Let $Z=\text{Denominator}=\cos\theta+\mu\sin\theta$

For F to be MINIMUM, Z must be MAXIMUM

$$\Rightarrow \quad \frac{dZ}{d\theta}=0$$

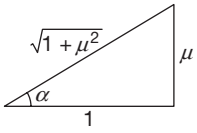
$$\Rightarrow \quad -\sin\theta+\mu\cos\theta=0$$

$$\Rightarrow \quad \tan\theta=\mu$$

$$\Rightarrow \quad \theta=\tan^{-1}\mu$$

So F will be minimum when its angle with the horizontal is equal to the angle of friction i.e. $\theta=\tan^{-1}\mu$.

Since $\tan\theta=\mu$, so from the figure, we get



$$\sin\theta=\frac{\mu}{\sqrt{1+\mu^2}} \text{ and } \cos\theta=\frac{1}{\sqrt{1+\mu^2}}$$

By substituting these value in equation (3), we get

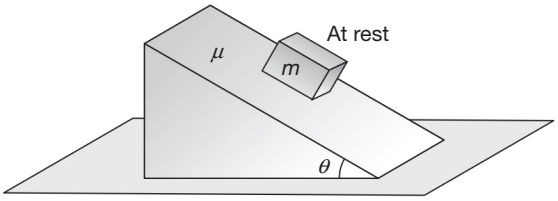
$$F\geq \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}}+\frac{\mu^2}{\sqrt{1+\mu^2}}}$$

$$\Rightarrow \quad F \geq \frac{\mu mg\sqrt{1+\mu^2}}{1+\mu^2} \quad \Rightarrow \quad F \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\Rightarrow \quad F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

ILLUSTRATION 74

A block of mass m, as shown in figure, is resting on a rough inclined plane with angle of inclination θ. If μ be the coefficient of friction between the block and the inclined surface, find:



(a) frictional force acting on the block
(b) the maximum angle of inclination, θ_0, for which the block stays in equilibrium
(c) if $\theta > \theta_0$, find the acceleration of the block.

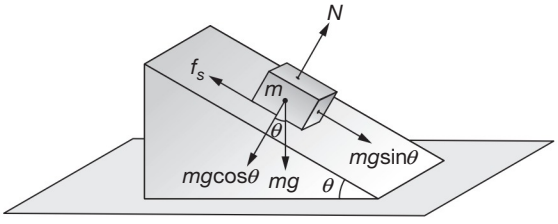
SOLUTION

The forces acting on the block and FBD of the block are shown in figure (a). The weight of the block, mg, has been resolved along the surface and the direction perpendicular to the inclined surface, as shown in figure (a). From figure (a) it is obvious that the component of the weight of the block parallel to the surface, $mg\sin\theta$, tries to slide the block down the inclined surface and therefore, the surface applies a frictional force opposite to the direction of $mg\sin\theta$.

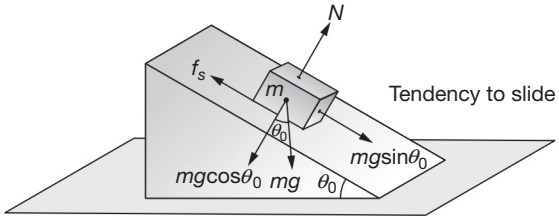
(a) Since, the block is in equilibrium, we have,

$$f = mg\sin\theta \qquad \ldots(1)$$

and $N = mg\cos\theta$ $\qquad \ldots(2)$



(a)



(b)

(b) As the frictional force acting on the block is static in nature (therefore the block is at rest w.r.t. the inclined surface), it must not exceed its limiting value which is equal to μN. Therefore, from figure (b), we have

$$f \leq \mu N$$

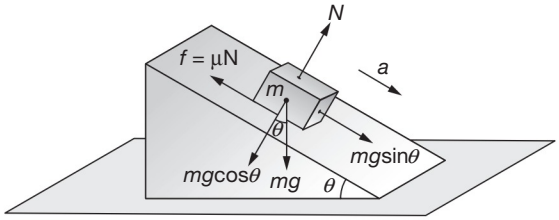
$$\Rightarrow \quad mg\sin\theta_0 \leq \mu mg\cos\theta_0$$

$$\Rightarrow \quad \tan\theta_0 \leq \mu$$

$$\Rightarrow \quad \theta_0 \leq \tan^{-1}\mu \qquad \ldots(3)$$

Therefore, the maximum angle of inclination θ_0 for which the block remains in equilibrium is $\tan^{-1}\mu$.

(c) If $\theta > \tan^{-1}\mu$, then obviously the block can not remain in equilibrium or we can also say that for this angle the limiting friction can not balance the component of the gravitational pull parallel to the surface. Therefore, the block will accelerate down the incline, as shown in the figure (c).



(c)

In this case the frictional force is kinetic in nature and hence its magnitude is constant and is equal to μN. Applying Newton's Second Law along the inclined surface, from figure (c), we get,

$$mg\sin\theta - f = ma$$

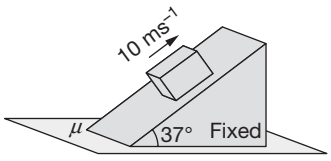
$$\Rightarrow \quad mg\sin\theta - \mu N = ma$$

$$\Rightarrow \quad mg\sin\theta - \mu mg\cos\theta = ma$$

$$\Rightarrow \quad a = g(\sin\theta - \mu\cos\theta) \qquad \ldots(4)$$

ILLUSTRATION 75

Find out the distance travelled by the block on incline before it stops. Initial velocity of the block is $10\ \text{ms}^{-1}$ and coefficient of friction between the block and incline is $\mu = 0.5$.



SOLUTION

$$N = mg\cos(37°)$$

Since $mg\sin(37°) + \mu N = ma$

$$\Rightarrow \quad a = g\left[\sin(37°) + \mu\cos(37°)\right]$$

$$\Rightarrow \quad a = 10\left[\frac{3}{5} + 0.5\left(\frac{4}{5}\right)\right] = 10\ \text{ms}^{-2}$$

$$\Rightarrow \quad a = 10\ \text{ms}^{-2} \text{ down the incline}$$

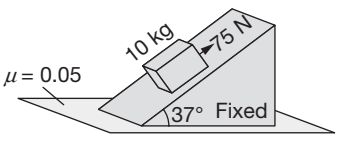
Since, $v^2 = u^2 + 2as$

$$\Rightarrow \quad 0 = 10^2 + 2(-10)\ s$$

$$\Rightarrow \quad s = 5\ \text{m}$$

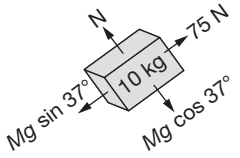
ILLUSTRATION 76

In the arrangement shown, calculate the acceleration of the block. Assume that the block is initially at rest. Calculate the force of friction acting between the block and incline. Also calculate the additional force (excess of 75 N) for which the block starts to move up the incline. Find the minimum force that can replace 75 N so that the block does not move.



SOLUTION

Let us first draw F.B.D. of the block excluding the friction.

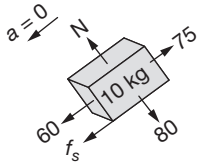


$$N = 10g\cos(37°) = 80\ \text{N}$$

Since the applied force on the block is

$$Mg\sin(37°) = (100)\left(\frac{3}{5}\right) = 60\ \text{N} < 75\ \text{N}$$

So, net force has a tendency to move the block up the incline due to which friction acts down the incline as shown. Also, $f_l = \mu N = 40\ \text{N}$



But $\left(F_{\text{applied}}\right)_{\text{net}} = 75 - 60 = 15\ \text{N} < f_l$

So, block will not move, i.e., $a = 0$ and hence static friction acts between the block and the incline. Now the block is at rest or in equilibrium with respect to the incline, so we have

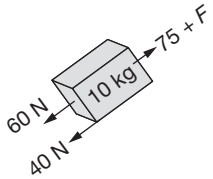
$$f_s + 60 = 75$$

$$\Rightarrow \quad f_s = 15\ \text{N}$$

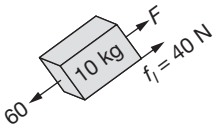
Let F be the additional force for which the block starts moving up the incline (i.e. still the block is in equilibrium or it does not move with respect to the incline), then we have

$$75 + F = 60 + 40$$

$$\Rightarrow \quad F = 25\ \text{N}$$



In this case the block has a tendency to move downwards. So the friction acts upwards.



$\Rightarrow \quad F + 40 = 60$

$\Rightarrow \quad F = 20$ N

Problem Solving Technique(s)

Block Over Block Problems

Method of Solving

STEP-1: Make free body diagram.

STEP-2: Calculate $f_\ell = \mu N$, i.e., the limiting value of friction between the required surface(s).

STEP-3: Denote static friction force by f_s (or f) because value of friction is not known. Then $f_s \le f_\ell$.

STEP-4: Calculate the static friction separately for the following two cases.

CASE-1: When the Blocks Move Together

STEP-5: Calculate acceleration, for the blocks moving together $a = \dfrac{F_{\text{applied}}}{\text{Total Mass}} = \dfrac{F}{m_1 + m_2}$.

STEP-6: Calculate f_s (or f) for above calculated value of a.

STEP-7(a): If $f_s < f_\ell$, which is true, then it means that the blocks will move together with acceleration $a = \dfrac{F}{m_1 + m_2}$.

CASE-2: When the Blocks Move Separately

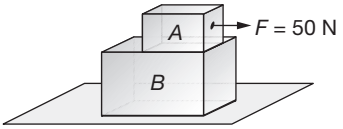
STEP-7(b): If $f_s > f_\ell$, which is impossible, so the blocks will move separately and kinetic friction $f_k = \mu_k N$ is involved.

STEP-8: Calculate acceleration of each block separately for this case.

STEP-9: Also keep in mind that for block A kept on block B (which is kept on a rough surface), the friction on B due to A, i.e. (f_{BA}) will be the part of external force applied on B and if this external force is less than friction between B and ground, then B will not move and friction between B and ground will be the applied force on block B, i.e., f_{BA}.

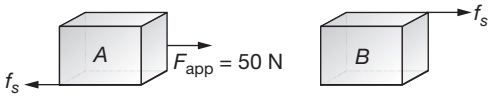
ILLUSTRATION 77

In the arrangement shown, calculate the acceleration of the two blocks each of mass 10 kg. The system is initially at rest and the friction coefficient between A and B is 0.5 whereas no friction exists between B and ground. Also calculate the maximum force applied on A for which the blocks move together.



SOLUTION

$$f_{\lim} = \mu N = (0.5)(10)(10) = 50 \text{ N}$$



Now let us assume that both blocks move together, then

$$a = a_A = a_B = \frac{50}{10+10} = 2.5 \text{ ms}^{-2}$$

Let us now calculate the friction between A and B for $a_A = a_B = 2.5 \text{ ms}^{-2}$. Since

$$f_s = m_B a_B = (10)(2.5) = 25 \text{ N} < f_{\text{limiting}}$$

So, the blocks A and B will move together with an acceleration of 2.5 ms^{-2} and friction between A and B is $25 \text{ N} < f_l (= 50 \text{ N})$.

Let $F_{\max} = F$ be the maximum force for which the blocks move together. Then the friction between the blocks will be limiting i.e.

$$f_l = \mu (m_A g) = (0.5)\ (10)\ (10) = 50 \text{ N}$$

So, we have

For A, $\quad F_{\max} - 50 = 10a \qquad \ldots(1)$

For B, $\quad 50 = 10a \qquad \ldots(2)$

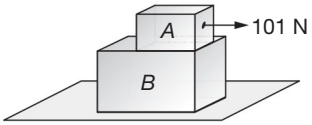
From (2), we get $a = 5 \text{ ms}^{-2}$

Substituting in (1), we get

$$F_{\max} = 100 \text{ N}$$

ILLUSTRATION 78

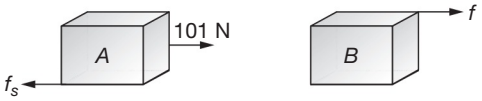
Find the acceleration of the two blocks A and B each of mass 10 kg . The system is initially at rest and the friction coefficient between A and B is 0.5 whereas the ground is smooth.



SOLUTION

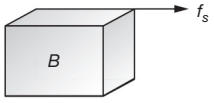
$f_{\lim} = 50$ N

Also, $f_s \le f_l = 50$ N



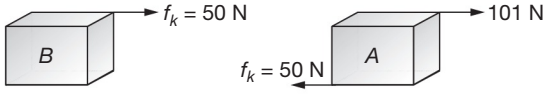
STEP-1: Assuming that blocks A and B move together, then $a = \dfrac{101}{20} = 5.05 \text{ ms}^{-2}$

STEP-2: Calculating static friction on B due to A



$f_s = 10 \times 5.05 = 50.5$

Since, $f_s > f_{\lim}$ which is not possible, so both blocks A and B move separately and so kinetic friction is involved.



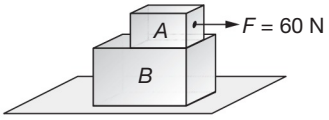
So, for A, $a_A = \dfrac{101-50}{10} = 5.1 \text{ ms}^{-2}$ and

for B, $a_B = \dfrac{50}{10} = 5 \text{ ms}^{-2}$

We observe that $a_A > a_B$ as force is applied on A.

ILLUSTRATION 79

Find the acceleration of the two blocks A (of mass 10 kg) and B (of mass 20 kg). The system is initially at rest and the friction coefficient between A and B is 0.5 whereas the ground is smooth. Also calculate the maximum force F applied horizontally on A so that blocks A and B move together.

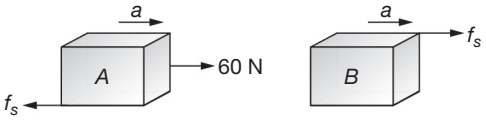


SOLUTION

$f_{\lim} = \mu m_A g = 50$ N

Assuming that the blocks A and B move together, then

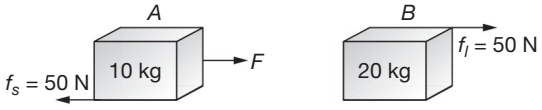
$$a_A = a_B = a = \frac{60}{10+20} = 2 \text{ ms}^{-2}$$



Let f_s be static friction between A and B, then $f_s = m_B a = (20)(2) = 40 \text{ N} < f_{\lim}$

So the blocks move together with an acceleration of 2 ms^{-2} and friction between the blocks is static and 40 N.

The maximum force for which A and B move together corresponds to the situation when friction between both A and B is limiting i.e., $f_l = 50$ N.



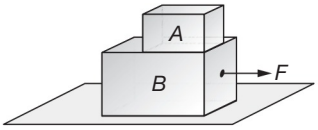
For A, $F - f_{\max} = 10a$ …(1)

For B, $f_{\max} = 20a$ …(2)

$\Rightarrow \quad F = 75$ N

ILLUSTRATION 80

In the arrangement shown, the blocks A (of mass 10 kg) and B (of mass 20 kg) are initially at rest. Calculate the minimum value of F for which sliding starts between the two blocks. Assuming the friction coefficient between A and B to be 0.5 whereas the ground is smooth.

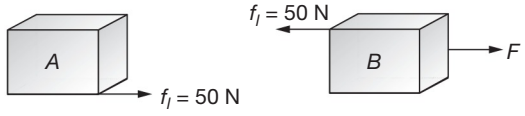


SOLUTION

When the sliding between A and B starts, then limiting friction is acting between A and B.

Since $f_l = \mu m_A g$

$\Rightarrow \quad f_l = 50$ N



For A, $f_l = 10a$

$\Rightarrow \quad a = 5 \text{ ms}^{-2}$

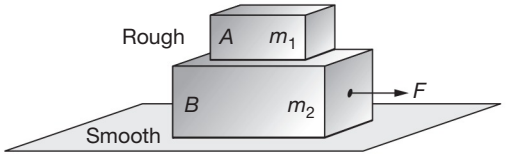
For B, $F - 50 = 20a$

$\Rightarrow \quad F = 50 + 20(5) = 150$ N

$\Rightarrow \quad F_{\min} = 150$ N

ILLUSTRATION 81

Block A is placed over block B which is placed over some smooth horizontal surface, as shown in figure. It is given that there is no friction between the horizontal surface and the lower block and appreciable friction exists between the two blocks. If block B is pulled by constant horizontal force, as shown in figure, then



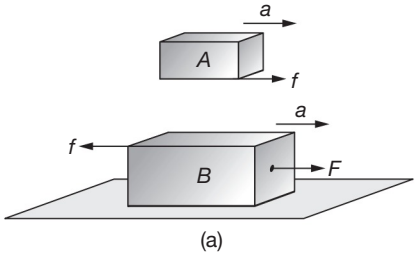
(a) find the acceleration of each block and the frictional force acting on each block, if the two blocks are moving together.

(b) if coefficient of friction between m_1 and m_2 be μ, what is the maximum value of F for which the two blocks will move together?

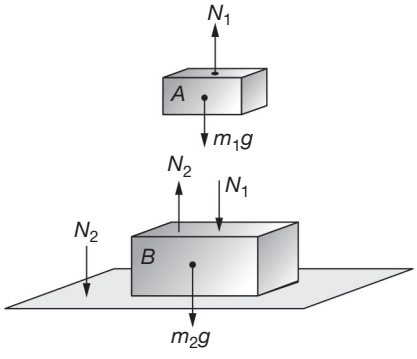
(c) if the force F is given by $F = \alpha t$ (α is a positive constant), find how the accelerations of the block A and of the block B depend on t, if the coefficient of friction between the blocks is equal to μ. Draw the approximate plot of these dependencies.

SOLUTION

(a) There is no friction between m_2 and ground. Also m_1 moves backwards relative to m_2. So, friction on m_1 is forward and the reaction pair of this friction will act on m_2 in the backward direction as shown.



(a)



(b)

For m_1: $\quad f = m_1 a \quad \ldots(1)$

For m_2: $\quad F - f = m_2 a \quad \ldots(2)$

Solving equations (1) and (2), we get

$$f = \frac{m_1 F}{m_1 + m_2} \text{ and } a = \frac{F}{m_1 + m_2}$$

(b) Since frictional force acting on the block A is static in nature, we can write,

$$f \le \mu N_1$$

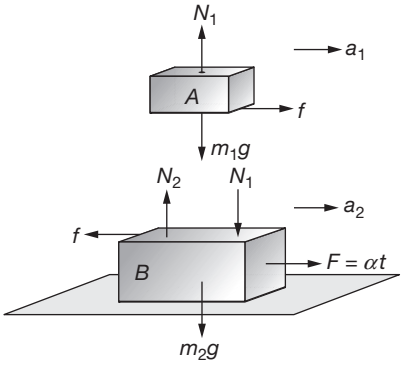
$$\Rightarrow \quad \frac{m_1 F}{m_1 + m_2} \le \mu m_1 g$$

$$\Rightarrow \quad F \le \mu (m_1 + m_2) g$$

$$\Rightarrow \quad F_{\max} = \mu (m_1 + m_2) g$$

Therefore, the two bodies will move together only if the applied force F is not greater than $\mu (m_1 + m_2) g$.

(c) It is quite obvious from the previous discussion that from $t = 0$ to some instant, lets say t_0, the blocks will move together, that is, there will be no slipping between the blocks and for this duration of time friction will be static in nature. After t_0, there will be relative motion between the blocks and the friction would be kinetic in nature. The forces acting on the two blocks and their assumed accelerations are shown in figure.



For $t < t_0$:

$$a_1 = a_2 \qquad (= a, \text{ say})$$

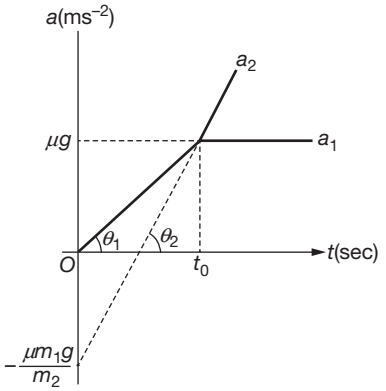
For m_1: $f = m_1 a$

For m_2: $F - f = m_2 a$

$$\Rightarrow \quad a = \frac{m_1 F}{m_1 + m_2}$$

$$\Rightarrow \quad a = \frac{\alpha m_2 t}{m_1 + m_2} \text{ and } f = m_1 a$$

$$\Rightarrow \quad f = \frac{\alpha m_1 m_2}{m_1 + m_2} t$$



At $t = t_0$:

$$f = \mu N_1$$

$$\Rightarrow \quad f = \frac{\alpha m_1 m_2}{m_1 + m_2} t_0 = \mu m_1 g$$

$$\Rightarrow \quad t_0 = \frac{\mu (m_1 + m_2) g}{\alpha m_2}$$

For $t > t_0$:

$$f = \mu N_1 = \mu m_1 g$$

$$\Rightarrow \quad a_1 = \frac{f}{m_1} = \frac{\mu m_1 g}{m_1} = \mu g$$

and $$a_2 = \frac{F - f}{m_2} = \frac{\alpha t - \mu m_1 g}{m_2} = \frac{\alpha t}{m_2} - \frac{\mu m_1 g}{m_2}$$

It is clear from the obtained expressions for the accelerations of the blocks that till t_0, the bodies are moving together but at t_0, friction reaches its limiting value and thereafter acceleration of the upper block becomes constant and that of the lower block keeps on increasing due to increase in magnitude of F. Dependencies of a_1 and a_2 on time are shown in figure.

$$\tan\theta_1 = \frac{\alpha m_1}{m_1 + m_2} \text{ and } \tan\theta_2 = \frac{\alpha}{m_2}$$

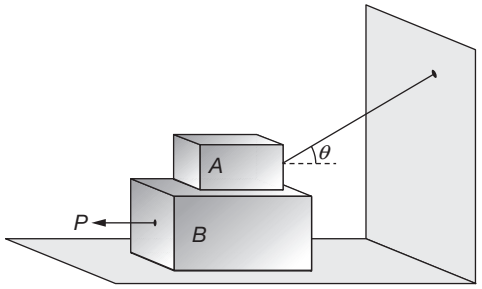
Conceptual Note(s)

Now, in order to understand this result intuitively read the following. In part (a) we got $f = \frac{m_1 F}{m_1 + m_2}$ (for the case when the two bodies are moving together). Here you can see that $f \propto F$. It is quite obvious why

we got $f \propto F$. When we increase F, acceleration of the block B increases and now to prevent relative motion between the block A and the block B, the acceleration of the block A also must increase. Here friction comes to the rescue. Friction increases its value to increase the acceleration of the block A and to decrease the acceleration of the block B and therefore, accelerations of the blocks are still equal. But if we keep on increasing F, then friction can increase only if its limiting value is not achieved. If we increase F when friction has already achieved its limiting value, then the acceleration of the lower block will increase but that of the upper block cannot increase because friction cannot increase anymore and hence acceleration of the lower block would be greater than that of the upper block. Now, there will be relative motion between the two blocks and the nature of the friction would be kinetic.

ILLUSTRATION 82

In the arrangement shown block A has weight $W_1 = 100$ N, block B has weight $W_2 = 200$ N and $\mu = 0.25$ for all surfaces in contact. A pull P is applied on W_2 such that it just slides under W_1. Find the tension in the string and the pull P, if $\theta = 45°$.

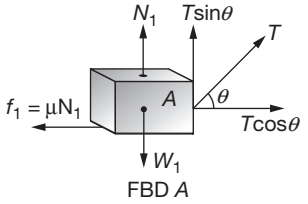


For all surfaces in contact, $\mu = 0.25$

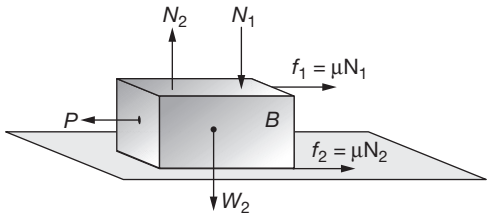
SOLUTION

For block *A*

Other than the routine forces shown in FBD, our main concern is to find the direction of friction acting on W_1. Since, W_1 is lying on W_2 (tendency to move along P) therefore, W_1 has the tendency to move opposite to P and hence force of friction of W_1 must be acting in the direction of P. The reaction pair of which will be acting of W_2 opposite to P.



FBD *A*



FBD *B*

For block *B*

Since W_2 is placed on ground, and hence will have a tendency to move along P therefore friction on W_2 due to ground will act opposite to P.

Apart from the other routine forces, f_1 is also acting on W_2 opposite to P (reaction pair of f_1 action on W_1). Now for W_1 to just move on W_2, i.e., condition for equilibrium to be just achieved, we have

For W_1

$$N_1 + T\sin\theta = W_1 \quad \ldots(1)$$

$$T\cos\theta = f_1 = \mu N_1 \quad \ldots(2)$$

For W_2

$$N_2 = N_1 + W_2 \quad \ldots(3)$$

$$P = f_1 + f_2 = \mu(N_1 + N_2) \quad \ldots(4)$$

Now, it is given that $W_1 = 100$ N, $W_2 = 200$ N, $\mu = 0.25$ and $\theta = 45°$. So from (1), (2), (3) and (4) we get

$$N_1 + \frac{T}{\sqrt{2}} = 100 \quad \ldots(5)$$

$$\frac{T}{\sqrt{2}} = (0.25)N_1 \quad \ldots(6)$$

$$N_2 = N_1 + 200 \quad \ldots(7)$$

$$P = 0.25(N_1 + N_2) \quad \ldots(8)$$

Substituting (6) in (5), we get

$$N_1 + 0.25N_1 = 100$$

$$\Rightarrow \quad 1.25N_1 = 100$$

$$\Rightarrow \quad N_1 = 80 \text{ N}$$

So, from (7), we get

$N_2 = 280$ N

Substituting $N_1 = 80$ N in (6), we get

$T = (0.25)\sqrt{2}(80)$

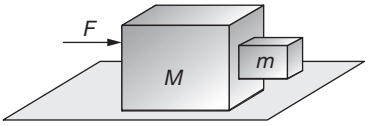
$\Rightarrow \quad T = 20\sqrt{2}$ N

From (8), we get

$P = 0.25(80 + 280) = 90$ N

ILLUSTRATION 83

In the figure, the coefficient of friction between box (of mass M) and block (of mass m) is μ. Find the magnitude of horizontal force F required to keep the block stationary with respect to wedge.



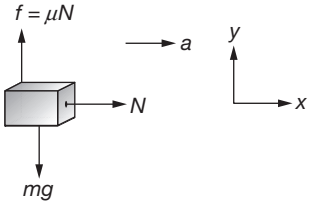
SOLUTION

Such problems can be solved with or without using the concept of pseudo force. Let us, solve the problem by both the methods.

a = Acceleration of (box + block) in horizontal direction

$$a = \frac{F}{M+m}$$

Inertial frame of reference (Ground): F.B.D. of block with respect to ground (only real forces have to applied) with respect to ground block is moving with an acceleration a. Therefore, $\Sigma F_y = 0$ and $\Sigma F_x = ma$.



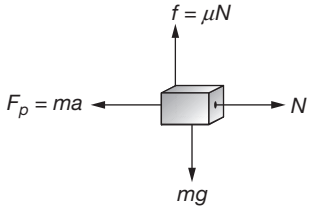
$mg = \mu N$ and $N = ma$

$\Rightarrow \quad a = \dfrac{g}{\mu}$

$\Rightarrow \quad F = (M+m)a$

$\Rightarrow \quad F = (M+m)\dfrac{g}{\mu}$

Non-inertial frame of reference (Box): F.B.D. of m with respect to box (real + one pseudo force)



With respect to wedge block is stationary.

$\Rightarrow \quad \Sigma F_x = 0 = \Sigma F_y$

$\Rightarrow \quad mg = \mu N$ and $N = ma$

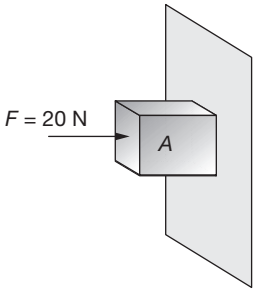
$\Rightarrow \quad a = \dfrac{g}{\mu}$ and $F = (M+m)a$

$\Rightarrow \quad F = (M+m)\dfrac{g}{\mu}$

From the above discussion, we can see that from both the methods results are same.

ILLUSTRATION 84

A block of mass 1 kg is pushed against a rough vertical wall with a force of 20 N, coefficient of static friction being $\frac{1}{4}$. Another horizontal force of 10 N is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction? If no, find the frictional force exerted by the wall on the block. $\left(g = 10 \text{ ms}^{-2}\right)$.



SOLUTION

Normal reaction on the block from the wall is

$$N = F = 20 \text{ N}$$

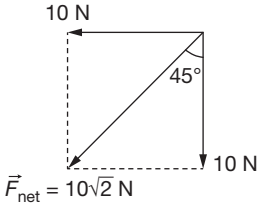
Therefore, limiting friction is given by

$$f_L = \mu N$$

$$\Rightarrow \quad f_L = \left(\frac{1}{4}\right)(20) = 5 \text{ N}$$

Weight of the block is

$$W = mg = (1)(10) = 10 \text{ N}$$



Since a horizontal force of 10 N is applied to the block so, the resultant of these two forces will be $10\sqrt{2}$ N in the direction shown in figure. Now, this resultant is greater than the limiting friction and so the block will move in the direction of $\vec{F}_{net}$ with an acceleration a given by

$$a = \frac{F_{net} - f_L}{m} = \frac{10\sqrt{2} - 5}{1} = 9.14 \text{ ms}^{-2}$$

ILLUSTRATION 85

A small bar starts sliding down an inclined plane forming an angle α with the horizontal. The friction coefficient depends on the distance x covered as $\mu = kx$, where k is a constant. Find the distance covered by the bar till it stops and its maximum velocity over this distance.

SOLUTION

Equation of motion down the incline is given by

$$mg \sin\alpha - kxmg \cos\alpha = ma$$

$$\Rightarrow \quad a = g \sin\alpha - kxg \cos\alpha \qquad \ldots(1)$$

where a is the acceleration of bar

Since $a = v\dfrac{dv}{dx}$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^x (g \sin\alpha - kxg \cos\alpha)\,dx$$

$$\Rightarrow \quad v^2 = 2g \sin\alpha - gkx^2 \cos\alpha \qquad \ldots(2)$$

$$\Rightarrow \quad v = \sqrt{(2x \sin\alpha - kx^2 \cos\alpha)g} \qquad \ldots(3)$$

It can be seen that the velocity again becomes zero after covering a distance $x = \dfrac{2}{k}\tan\alpha$.

Therefore, the distance covered by the bar till it stops is $\dfrac{2}{k}\tan\alpha$.

Further, the maximum velocity of the bar will be when

$$\frac{dv}{dx} = 0$$

$$\Rightarrow \quad a = 0 \qquad \left\{\because\ a = v\frac{dv}{dx}\right\}$$

$$\Rightarrow \quad x = \frac{\tan\alpha}{k} \qquad \{\text{from equation (1)}\}$$

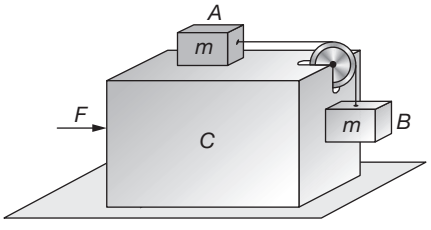
Substituting this in equation (3), we get the maximum velocity as

$$v_{max} = \sqrt{\left(\frac{2\sin\alpha\tan\alpha}{k} - \frac{\tan^2\alpha\cos\alpha}{k}\right)g}$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{g}{k}\tan\alpha\sin\alpha}$$

ILLUSTRATION 86

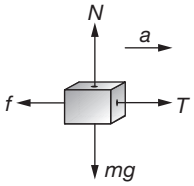
Consider the situation shown in figure. The horizontal surface below the bigger block is smooth. The coefficient of friction between the blocks is μ. Find the minimum and the maximum force F that can be applied in order to keep the smaller blocks at rest with respect to the bigger block.



SOLUTION

Suppose that the minimum force needed to prevent slipping between the blocks is F. Considering $A+B+C$ as the system, the acceleration of the system is

$$a = \frac{F}{M+2m} \qquad \ldots(1)$$

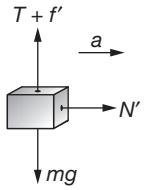


FBD of A

Now, consider the F.B.D. of A. The forces on A shown in figure are

(a) tension T by the string towards right,
(b) friction f by the block C towards left,
(c) weight mg downward and
(d) normal force N upwards

For vertical equilibrium $N = mg$. As the minimum force needed to prevent slipping is applied, the friction is limiting. Thus, $f = \mu N = \mu mg$



FBD of B

As the block moves towards right with an acceleration a,

$$T - f = ma$$

$$\Rightarrow \quad T - \mu mg = ma \qquad \ldots(2)$$

Now, consider the F.B.D. of B. The forces on B shown in figure are

(a) tension T upwards,
(b) weight mg downward,
(c) normal force N' towards right and
(d) friction f' upwards.

As the block moves towards right with an acceleration a, so we have $N' = ma$. Since the friction is limiting, hence

$$f' = \mu N' = \mu(ma) \qquad \ldots(3)$$

For vertical equilibrium

$$T + f' = mg \qquad \ldots(4)$$

Solving these equations, we get

$$a_{\min} = \left(\frac{1-\mu}{1+\mu}\right)g$$

When a large force is applied the block A slips on C towards left and the block B slips on C in the upward direction. The friction on A is towards right and that on B is downwards. Solving as above, the acceleration in this case is $a_{\max} = \dfrac{1+\mu}{1-\mu}g$.

Thus, a lies between $\left(\dfrac{1-\mu}{1+\mu}\right)g$ and $\left(\dfrac{1+\mu}{1-\mu}\right)g$, i.e.,

$$\left(\frac{1-\mu}{1+\mu}\right)g \le a \le \left(\frac{1+\mu}{1-\mu}\right)g.$$

From equation (1) the force F lies between

$$\left(\frac{1-\mu}{1+\mu}\right)(M+2m)g \text{ and } \left(\frac{1+\mu}{1-\mu}\right)(M+2m)g$$

i.e., $$\left(\frac{1-\mu}{1+\mu}\right)(M+2m)g \le F \le \left(\frac{1+\mu}{1-\mu}\right)(M+2m)g.$$

Test Your Concepts-VII

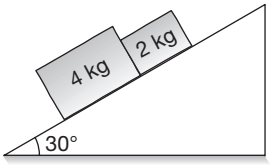
Based on Friction

(Solutions on page H.209)

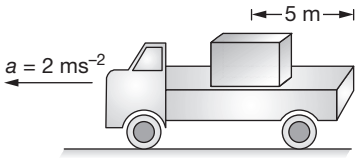
1. Calculate the height upto which an insect can crawl up a fixed bowl in the form of a hemisphere of radius *r*. Given, coefficient of friction is $\frac{1}{\sqrt{3}}$.

2. A particle of mass 2 kg rests on rough plane inclined at 30° to the horizontal and is just about to slip. Find the coefficient of friction between the plane and the particle.

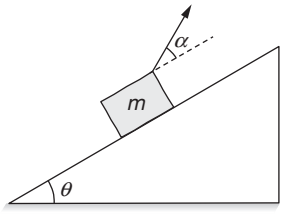
3. Figure shows two blocks in contact sliding down an inclined surface of inclination 30°. The friction coefficient between the block of mass 2 kg and the incline is $\mu_1 = 0.2$ and that between the block of mass 4 kg and the incline is $\mu_2 = 0.3$. Find the acceleration of 2 kg block. Take $g = 10\ \text{ms}^{-2}$.



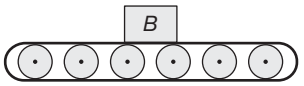
4. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in figure. The coefficient of friction between the box and the surface below it is $\mu = 0.1$. On a straight road, the truck starts from rest and accelerates with $2\ \text{ms}^{-2}$, find the time when box falls off the truck. Take $g = 10\ \text{ms}^{-2}$.



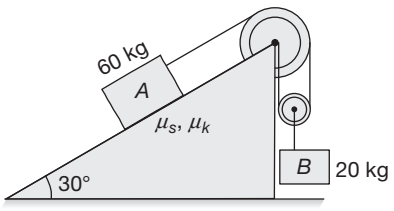
5. A block of mass *m* is pulled by means of a force *F* up an inclined plane forming an angle θ with the horizontal as shown in figure. The coefficient of friction is μ. Find the minimum force required for the block to move up and the angle α at that instant.



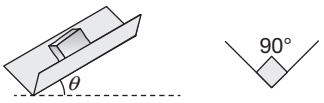
6. The conveyor belt is designed to transport packages of various weights. Each 10 kg package has a coefficient of kinetic friction $\mu_k = 0.15$. If the speed of the conveyor is 5 ms^{-1} and then it suddenly stops, determine the distance the package will slide on the belt before it comes to rest.



7. The system is released from rest with cable taut. Calculate the acceleration of each body and the tension *T* in the cable attached with *A* for $\mu_s = 0.25$ and $\mu_k = 0.2$. Neglect the small mass and friction of the pulleys. Take $g = 10\ \text{ms}^{-2}$.

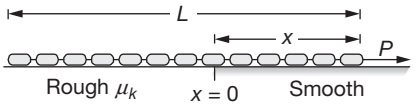


8. In figure, a crate slides down an inclined right-angled trough. The coefficient of kinetic friction between the crate and the trough is μ_k. What is the acceleration of the crate in terms of μ_k, θ and *g*?

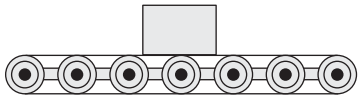


9. A heavy chain with a mass per unit length ρ is pulled by the constant force *F* along a horizontal surface consisting of a smooth section and a rough

section. The chain is initially at rest on the rough surface with $x = 0$. If the coefficient of kinetic friction between the chain and the rough surface is μ_k, determine the velocity v of the chain when $x = L$. The force F is greater than $\mu_k \rho g L$ in order to initiate motion.

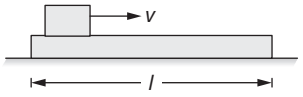


10. A package is at rest on a conveyor belt which is initially at rest. The belt is started and moves to the right for 1.3 s with a constant acceleration of $2\ \text{ms}^{-2}$. The belt then moves with a constant deceleration a_2 and comes to a stop after a total displacement of 2.2 m. Knowing that the coefficients of friction between the package and the belt are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine



(a) the deceleration a_2 of the belt,
(b) the displacement of the package relative to the belt as the belt comes to a stop.

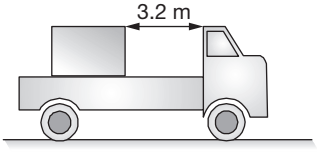
11. A small block of mass m is projected on a larger block of mass 10 m and length ℓ with a velocity v as shown in the figure. The coefficient of friction between the two blocks is μ_2 while that between the lower block and the ground is μ_1. Given that $\mu_2 > 11\mu_1$.



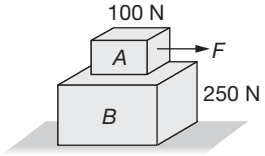
(a) Find the minimum value of v such that the mass m falls off the block of mass 10 m.
(b) If v has this minimum value, find the time taken by block m to do so.

12. A uniform rod is made to lean between a rough vertical wall and the rough ground. Show that the least angle at which the rod can be leaned without slipping is given by $\theta = \tan^{-1}\left(\frac{7}{4}\right)$, where coefficient of friction between the rod and the wall is $\mu_1 = \frac{1}{2}$ and that between the rod and the ground is $\mu_2 = \frac{1}{4}$.

13. Coefficients of friction between the flat bed of the truck and crate are $\mu_s = 0.8$ and $\mu_k = 0.7$. The coefficient of kinetic friction between the truck tires and the road surface is 0.9. If the truck stops from an initial speed of $15\ \text{ms}^{-1}$ with maximum braking (wheels skidding), determine where on the bed the crate finally comes to rest or the velocity v_{rel} relative to the truck with which the crate strikes the wall at the forward edge of the bed. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$.

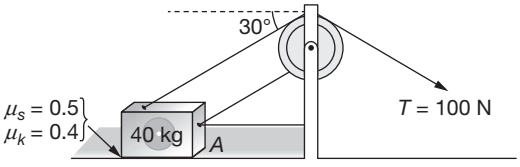


14. Block B rests on a smooth surface. If the coefficient of static friction between A and B is $\mu = 0.4$, determine the acceleration of each if

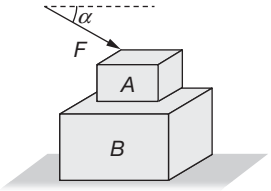


(a) $F = 30$ N and
(b) $F = 250$ N
Take $g = 10\ \text{ms}^{-2}$

15. In the arrangement shown, find the acceleration of block A for the instant depicted. Neglect the mass of the pulley. Take $g = 10\ \text{ms}^{-2}$.



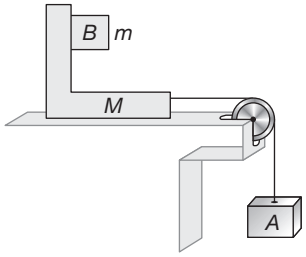
16. Two blocks *A* and *B* of mass 1 kg and 2 kg respectively are placed over a smooth horizontal surface as shown in figure. The coefficient of friction between blocks *A* and *B* is $\mu = \frac{1}{2}$. An external force of magnitude *F* is applied to the top block at an angle $\alpha = 30°$ below the horizontal.



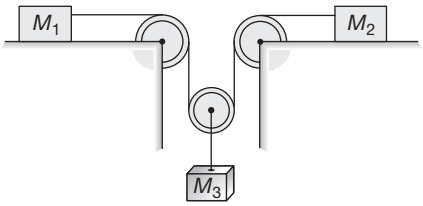
(a) If the two blocks move together, find their acceleration in terms of *F*.

(b) Find the maximum values of *F* so that both the blocks move together with same acceleration. $\left(g = 10\ \text{ms}^{-2}\right)$.

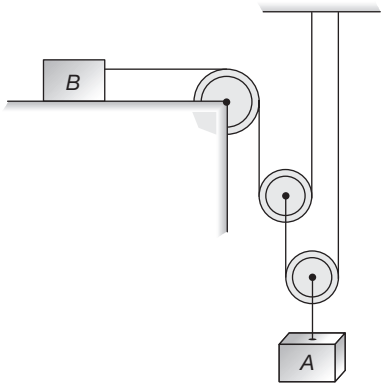
17. The figure shows an *L* shaped body of mass *M* placed on smooth horizontal surface. The block *A* is connected to the body by means of an inextensible string, which is passing over a smooth pulley of negligible mass. Another block *B* of mass *m* is placed against a vertical wall of the body. Find the minimum value of the mass of block *A* so that block *B* remains stationary relative to the wall. Coefficient of friction between the block *B* and the vertical wall is μ.



18. The system shown in figure uses massless pulley and rope. The coefficient of friction between the masses and horizontal surfaces is μ. Assume that M_1 and M_2 are sliding. Find the tension *T* in the rope.

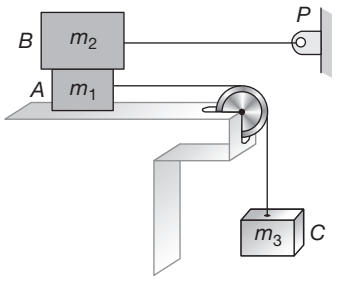


19. Determine the acceleration of the 5 kg block *A*. Neglect the mass of the pulley and cords. The block *B* has a mass of 10 kg. The coefficient of kinetic friction between block *B* and the surface is $\mu_k = 0.1$. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$.



20. A worker wishes to pile a cone of sand onto a circular area in his yard. The radius of the circle is *r* and no sand is to spill onto the surrounding area. If μ is the static coefficient of friction between each layer of sand along the slope and the sand, show that the greatest volume of sand that can be stored in this manner is $\frac{1}{3}\pi\mu r^3$.

21. A block *A* of mass m_1 rests on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. A uniform plank *B* of mass m_2 rests on *A*. *B* is prevented from moving by connecting it to a light rod and hinged at point *P*. The coefficient of friction between *A* and *B* is μ. Find the acceleration of blocks *A* and *C*.



22. A 4 m long ladder weighing 25 kg rests with its upper end against a smooth wall and lower end on rough ground. What should be the minimum coefficient of friction between the ground and the ladder for it to be inclined at 60° with the horizontal without slipping? Take $g = 10\ \text{ms}^{-2}$.

DYNAMICS OF CIRCULAR MOTION

CIRCULAR MOTION: AN INTRODUCTION

For a particle which moves in a plane such that its distance remains constant from a fixed or a moving point, then its motion is known as circular motion with respect to that fixed point or moving point. This point is called the centre of the circle and the distance of the particle from this point is called the radius.

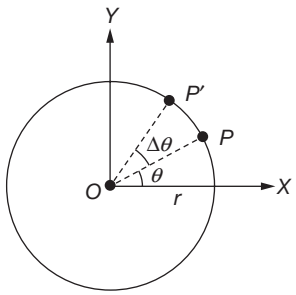
VARIABLES OF CIRCULAR MOTION

Angular Position (θ)

The angular position of a point in space is decided by specifying the

(i) origin and
(ii) reference line.

The angle made by the position vector w.r.t. origin, with the reference line is called angular position. Clearly, angular position depends on the choice of the origin as well as the reference line.



Circular motion is a two dimensional motion or motion in a plane.

Suppose a particle P is moving in a circle of radius r centred at O, then the angular position of the particle P at a given instant may be described by the angle θ between OP and OX. This angle θ is called the **angular position** of the particle.

Angular Displacement ($\Delta\theta$)

Definition: Angle through which the position vector of the moving particle rotates in a given time interval is called its angular displacement. Angular displacement depends on origin, but it does not depends on the reference line. As the particle moves on above circle its angular position θ changes. Suppose that the point rotates through an angle $\Delta\theta$ in time Δt, then $\Delta\theta$ is angular displacement.

Average Angular Velocity

$$\omega_{av} = \frac{\text{Angular displacement}}{\text{Total time taken}}$$

$$\Rightarrow \quad \omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2. Since angular displacement is a scalar, average angular velocity is also a scalar.

Instantaneous Angular Velocity

It is the limit of average angular velocity as Δt approaches zero. i.e.,

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Since infinitesimally small angular displacement $d\theta$ is a vector quantity, so instantaneous angular velocity ω is also a vector, whose direction is given by Right Hand Thumb Rule.

ILLUSTRATION 87

If angular displacement of a particle is given by $\theta = a - bt + ct^2$, then find its angular velocity.

SOLUTION

$$\omega = \frac{d\theta}{dt} = -b + 2ct$$

ILLUSTRATION 88

Is the angular velocity of rotation of hour hand of a watch greater or smaller than the angular velocity of Earth's rotation about its own axis?

SOLUTION

Hour hand completes one rotation in 12 hours while Earth completes one rotation in 24 hours. So, angular velocity of hour hand is double the angular velocity of Earth, because

$$\omega = \frac{2\pi}{T}$$

Conceptual Note(s)

(a) Angular displacement is a dimensionless quantity. Its SI unit is radian, some other units are degree and revolution.

$$2\pi \text{ rad} = 360° = 1 \text{ rev}$$

(b) Infinitesimally small angular displacement is a vector quantity, but finite angular displacement is a scalar, because while the addition of the infinitesimally small angular displacements is commutative, addition of finite angular displacement is not.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \text{ but } \theta_1 + \theta_2 \neq \theta_2 + \theta_1$$

(c) Direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point then thumb will represents the direction of angular displacement.

(d) Angular velocity has dimension formula $M^0L^0T^{-1}$ and SI unit rad s^{-1}.

(e) If a body makes n rotations in t seconds then average angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t}$$

(f) If T is the period and f the frequency of uniform circular motion, then

$$\omega_{av} = \frac{2\pi}{T} = 2\pi f$$

(g) For a rigid body, as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g. angular velocity of all points of earth about earth's axis is

$$\frac{2\pi}{24} \text{ radhr}^{-1} = \frac{2\pi}{86400} \text{ rads}^{-1}$$

(h) Average Angular Acceleration: Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

(i) Instantaneous Angular Acceleration: It is the limit of average angular acceleration as Δt approaches zero, i.e.,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Since, $\omega = \frac{d\theta}{dt}$

$$\Rightarrow \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Also, $\alpha = \omega\frac{d\omega}{d\theta}$

(j) Both average and instantaneous angular acceleration are axial vectors with dimension $[T^{-2}]$ and unit rads^{-2}.

(k) If $\alpha = 0$, circular motion is said to be uniform.

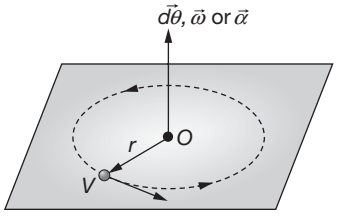
KINEMATICS OF CIRCULAR MOTION

Motion with Constant Angular Velocity

If a particle is moving in a circle with uniform angular velocity, then $\alpha = 0$ and hence $\theta = \omega t$.

Motion with Constant Angular Acceleration

Circular motion with constant angular acceleration is analogous to one dimensional translational motion with constant acceleration. Hence even here equation of motion have same form.



$$\omega = \omega_0 + \alpha t$$

$$\Rightarrow \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \quad \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\Rightarrow \quad \theta = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$\Rightarrow \quad \theta_{n\text{th}} = \omega_0 + \frac{\alpha}{2}(\theta_n - \theta_{n-1})$$

where ω_0 is the initial angular velocity,
ω is the final angular velocity,
α is the constant angular acceleration,
θ is the angle traversed from $t = 0$ to t and
$\theta_{n\text{th}}$ is the angle traversed in the nth second of motion.

ILLUSTRATION 89

A particle is moving with constant speed in a circular path. Find the ratio of average velocity to its instantaneous velocity when the particle describes an angle $\theta = \frac{\pi}{2}$.

SOLUTION

Time taken to describe angle θ

$$t = \frac{\theta}{\omega} = \frac{\theta R}{v} = \frac{\pi R}{2v}$$

$$v_{\text{av}} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{\sqrt{2}R}{\pi R/2v} = \frac{2\sqrt{2}}{\pi}v$$

Instantaneous velocity $= v$
The ratio of average velocity to its instantaneous velocity is

$$\frac{v_{av}}{v_{ins}} = \frac{2\sqrt{2}}{\pi}$$

ILLUSTRATION 90

A fan is rotating with angular velocity 100 revs^{-1}. Then it is switched off. It takes 5 minutes to stop.

(a) Find the total number of revolution made before it stops. (Assume uniform angular retardation).
(b) Find the value of angular retardation.
(c) Find the average angular velocity during this interval.

SOLUTION

(a) $\theta = \left(\frac{\omega + \omega_0}{2}\right)t = \left(\frac{100 + 0}{2}\right) \times 5 \times 60$

$\Rightarrow \quad \theta = 15000$ revolution.

(b) $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad 0 = 100 - \alpha(5 \times 60)$

$\Rightarrow \quad \alpha = \frac{1}{3}$ revs^{-2}

(c) $\omega_{av} = \frac{\text{Total angle of rotation}}{\text{Total time taken}}$

$\Rightarrow \quad \omega_{av} = \frac{15000}{50 \times 60}$

$\Rightarrow \quad \omega_{av} = 50$ revs^{-1}

ILLUSTRATION 91

A fan rotating with $\omega = 100$ rads^{-1}, is switched off. After $2n$ rotation its angular velocity becomes 50 rads^{-1}. Find the angular velocity of the fan after n rotations.

SOLUTION

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\Rightarrow \quad 50^2 = (100)^2 + 2\alpha(2\pi 2n) \qquad \ldots(1)$$

If angular velocity after n rotation is ω_n, then

$$\omega_n^2 = (100)^2 + 2\alpha(2\pi n) \qquad \ldots(2)$$

From equations (1) and (2), we get

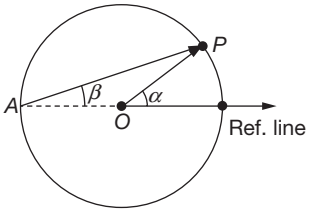
$$\frac{50^2 - 100^2}{\omega_n^2 - 100^2} = \frac{2\alpha(2\pi 2n)}{2\alpha 2\pi n} = 2$$

$$\Rightarrow \quad \omega_n^2 = \frac{50^2 + 100^2}{2}$$

$$\Rightarrow \quad \omega = 25\sqrt{10} \text{ rads}^{-1}$$

RELATIVE ANGULAR VELOCITY

Just as velocities are always relative, similarly angular velocity is also always relative. There is no such thing as absolute angular velocity. Angular velocity is defined with respect to origin, the point from which the position vector of the moving particle is drawn.



Consider a particle P moving along a circular path shown in the figure.

Here angular velocity of the particle P w.r.t. O (i.e. P is revolving with O as the centre) and the angular velocity of P w.r.t. A (i.e. P is revolving with A as the centre) will be different.

Angular velocity of a particle P w.r.t. O is

$$\omega_{PO} = \frac{d\alpha}{dt}$$

Angular velocity of a particle P w.r.t. A is

$$\omega_{PA} = \frac{d\beta}{dt}$$

Definition of Angular Velocity

Angular velocity of a particle A with respect to the other moving particle B is the rate at which position vector of A with respect to B rotates at that instant. (or it is simply, angular velocity of A with origin fixed at B). Angular velocity of A w.r.t. B, ω_{AB} is mathematically define as

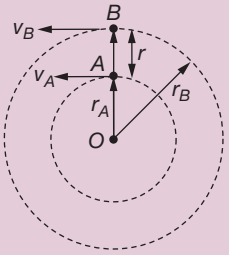
$$\omega_{AB} = \frac{\left(\begin{array}{c}\text{Component of relative}\\ \text{velocityof A w.r.t. B,}\\ \text{perpendicular to line}\end{array}\right)}{\text{Separation between } A \text{ and } B}$$

$$\Rightarrow \quad \omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}}$$

Conceptual Note(s)

(a) If two particles are moving on two different concentric circles with different velocities then angular velocity of B as observed by A will depend on their positions and velocities. Consider the case when A and B are closest to each other moving in same direction as shown in figure. In this situation

$$(v_{AB})_{\perp} = v_B - v_A$$



Separation between A and B is

$$r_{BA} = r_B - r_A$$

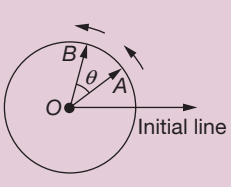
So, $\omega_{AB} = \dfrac{(v_{AB})_{\perp}}{r_{AB}} = \dfrac{v_B - v_A}{r_B - r_A}$

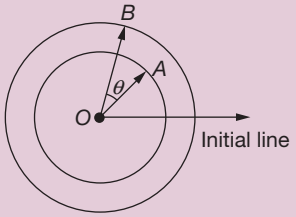
(b) If two particles are moving on the same circle or different coplanar concentric circles in same direction with different uniform angular speed ω_A and ω_B respectively, the rate of change of angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is

$$\frac{d\theta}{dt} = \omega_B - \omega_A$$

So, the time taken by one to complete one revolution around O w.r.t. the other

$$T = \frac{2\pi}{\omega_{\text{rel}}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2}$$

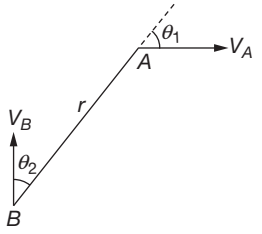




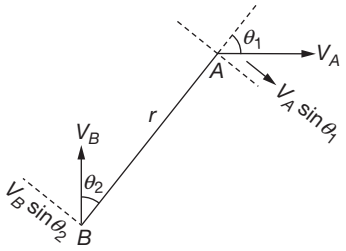
(c) $\omega_B - \omega_A$ is rate of change of angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. This is not the angular velocity of B w.r.t. A. (which is rate at which line AB rotates).

ILLUSTRATION 92

Find the angular velocity of A with respect to B in the figure given below:



SOLUTION



Angular velocity of A with respect to B

$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$$

From the figure drawn, we observe that

$$(V_{AB})_\perp = V_A \sin\theta_1 + V_B \sin\theta_2$$

and $r_{AB} = r$

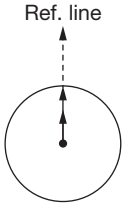
$$\Rightarrow \quad \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

ILLUSTRATION 93

Find the time period of meeting of minute hand and second hand of a wall clock.

SOLUTION

For second and minute hand to meet again, we must have



$$\theta_{sec} - \theta_{min} = 2\pi$$

$$\Rightarrow \quad (\omega_{sec} - \omega_{min})t = 2\pi$$

Since, $\omega_{min} = \frac{2\pi}{60}$ radmin^{-1} and $\omega_{sec} = \frac{2\pi}{1}$ radmin^{-1}, so we have

$$2\pi\left(1 - \frac{1}{60}\right)t = 2\pi$$

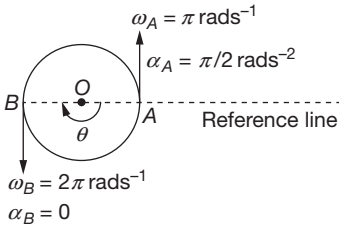
$$\Rightarrow \quad t = \frac{60}{59} \text{ min}$$

ILLUSTRATION 94

Two particles A and B move on a circle. Initially particle A and B are diagonally opposite to each other. Particle A move with angular velocity π rads^{-1}, angular acceleration $\frac{\pi}{2}$ rads^{-2} and particle B moves with constant angular velocity 2π rads^{-1}. Find the time after which both the particles A and B will collide.

SOLUTION

Suppose angle between OA and OB, be θ, then, rate of change of θ is called angular velocity.



With respect to the point A, we have

$$\omega_{BA} = \omega = \omega_B - \omega_A = 2\pi - \pi = \pi \text{ rads}^{-1}$$

$$\alpha_{BA} = \alpha = \alpha_B - \alpha_A = -\frac{\pi}{2} \text{ rads}^{-2}$$

If angular displacement is $\Delta\theta$, then

$$\Delta\theta = \omega t + \frac{1}{2}\alpha t^2$$

For A and B to collide, angular displacement is given by

$$\Delta\theta = \pi$$

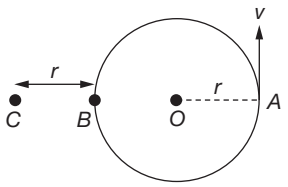
$$\Rightarrow \quad \pi = \pi t + \frac{1}{2}\left(\frac{-\pi}{2}\right)t^2$$

$$\Rightarrow \quad t^2 - 4t + 4 = 0$$

$$\Rightarrow \quad t = 2 \text{ sec}$$

ILLUSTRATION 95

A particle is moving with constant speed in a circle as shown, find the angular velocity of the particle A with respect to fixed point B and C if angular velocity with respect to O is ω.



SOLUTION

Angular velocity of A with respect to O (i.e. A is revolving with O as the centre) is

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$$

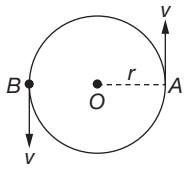
Similarly, we have

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{v}{2r} = \frac{\omega}{2}$$

and $\omega_{AC} = \dfrac{(v_{AC})_\perp}{r_{AC}} = \dfrac{v}{3r} = \dfrac{\omega}{3}$

ILLUSTRATION 96

Particles A and B move with constant and equal speeds in a circle as shown, find the angular velocity of the particle A with respect to B, if angular velocity of particle A w.r.t. O is ω.



SOLUTION

Angular velocity of A with respect to O is

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$$

Now, $\omega_{AB} = \dfrac{(v_{AB})_\perp}{r_{AB}}$

$$\Rightarrow \quad v_{AB} = 2v$$

Since v_{AB} is perpendicular to r_{AB}

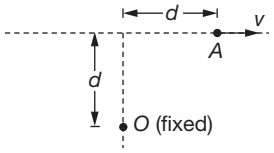
$$\Rightarrow \quad (v_{AB})_\perp = v_{AB} = 2v$$

$$r_{AB} = 2r$$

$$\Rightarrow \quad \omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{2v}{2r} = \omega$$

ILLUSTRATION 97

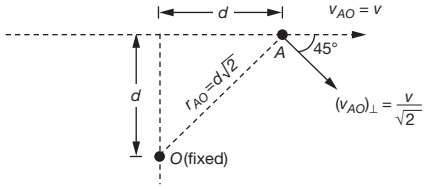
Find angular velocity of A with respect to O at the instant shown in the figure.



SOLUTION

Angular velocity of A with respect to O is

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}}$$



Since, $v_{AO} = v$

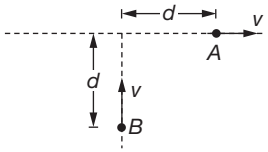
$$\Rightarrow \quad (v_{AO})_{\perp} = \frac{v}{\sqrt{2}}$$

and $r_{AO} = d\sqrt{2}$

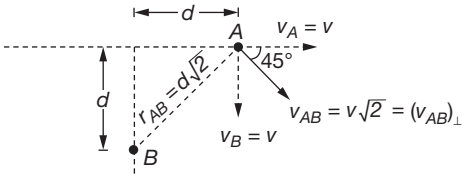
$$\Rightarrow \quad \omega_{AO} = \frac{(v_{AO})_{\perp}}{r_{AO}} = \frac{v/\sqrt{2}}{d\sqrt{2}} = \frac{v}{2d}$$

ILLUSTRATION 98

Find angular velocity of A with respect to B at the instant shown in the figure.



SOLUTION



Angular velocity of A with respect to B is

$$\omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}}$$

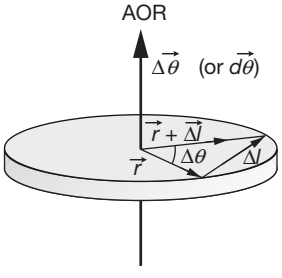
$$\Rightarrow \quad v_{AB} = \sqrt{2}v = (v_{AB})_{\perp}$$

$$\Rightarrow \quad r_{AB} = \sqrt{2}d$$

$$\Rightarrow \quad \omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}} = \frac{v\sqrt{2}}{d\sqrt{2}} = \frac{v}{d}$$

ANGULAR DISPLACEMENT ($d\vec{\theta}$): REVISITED

Finite angular displacement θ is NEVER a vector.



Infinitesimal (extremely small) angular displacement $d\vec{\theta}$ is a vector and

$$\Delta\vec{l} = \Delta\vec{\theta} \times \vec{r} \qquad \{\text{In magnitude } \Delta l = r\Delta\theta\}$$

For infinitesimal displacement

$$d\vec{l} = d\vec{\theta} \times \vec{r}$$

ANGULAR VELOCITY ($\vec{\omega}$): REVISITED

Rate of change of angular displacement is angular velocity.

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

Since $d\vec{l} = d\vec{\theta} \times \vec{r}$

Divide both sides by dt

$$\Rightarrow \quad \frac{d\vec{l}}{dt} = \frac{d\vec{\theta}}{dt} \times \vec{r}$$

$$\Rightarrow \quad \vec{v} = \vec{\omega} \times \vec{r}$$

In magnitude

$$\Rightarrow \quad v = r\omega$$

Conceptual Note(s)

(a) Commonly used units for angular velocity ω are rpm, rph, rps, 1 degrees^{-1}.

1 revolution ≡ 2π radian

$$\text{rps} \xrightarrow{\times 2\pi} \text{rads}^{-1}$$

$$\text{rph} \xrightarrow{\times \frac{2\pi}{3600}} \text{rads}^{-1}$$

(b) Direction of ω

Direction of ω is given by Right Hand Thumb Rule (RHTR) according to which curl the fingers of right hand in the direction of rotation then thumb gives the direction of ω.

e.g.: The angular velocity for the second's hand of watch is inwards, perpendicular to the plane of watch.

ANGULAR ACCELERATION ($\vec{\alpha}$): REVISITED

Rate of change of angular velocity is angular acceleration

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

Since $\vec{v} = \vec{\omega} \times \vec{r}$

Differentiating both sides with respect to t, we get

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

Using formula

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{A} \times \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \times \vec{B}$$

$$\Rightarrow \quad \vec{a} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r}$$

$$\Rightarrow \quad \vec{a} = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$$

$$\Rightarrow \quad \vec{a} = \vec{a}_C + \vec{a}_T$$

where $\vec{a}_C$ is the centripetal acceleration also called as Radial acceleration and $\vec{a}_T$ is the tangential acceleration.
Mathematically, we have

$$\vec{a}_C = \vec{\omega} \times \vec{v}$$

$$\vec{a}_T = \vec{\alpha} \times \vec{r}$$

In magnitude, $a_C = v\omega = r\omega^2 = \frac{v^2}{r}$ and $a_T = r\alpha$

Also, $\vec{a}_C \perp \vec{a}_T$ and $\vec{a}_C = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

RADIAL AND TANGENTIAL ACCELERATION

There are two types of acceleration in circular motion; Tangential acceleration and centripetal acceleration.

Tangential Acceleration (a_T)

Component of acceleration directed along tangent of circle is called tangential acceleration. It is responsible for changing the speed of the particle. It is defined as

$$a_T = \frac{dv}{dt} = \frac{d|\vec{v}|}{dt} = \text{Rate of change of speed}$$

Also, $a_T = \alpha r$

Conceptual Note(s)

(a) In vector form $\vec{a}_T = \vec{\alpha} \times \vec{r}$

(b) Tangential acceleration is defined as the rate of change of speed (not velocity). So

$$a_T = \frac{d|\vec{v}|}{dt} \text{ or simply } a_T = \frac{dv}{dt}.$$

(c) If tangential acceleration is directed in direction of velocity then the speed of the particle increases.

(d) If tangential acceleration is directed opposite to velocity then the speed of the particle decreases.

Centripetal Acceleration (a_C)

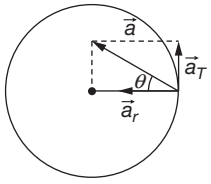
It is responsible for change in direction of velocity. In circular motion, there is always a centripetal acceleration.

Centripetal acceleration is always variable because it changes in direction. Centripetal acceleration is also called radial acceleration (a_r) or normal acceleration $(a_\perp)$. For a particle moving with speed v in a curved track of radius r, centripetal acceleration is given by

$$a_C = \frac{v^2}{r}$$

Total Acceleration

Total acceleration is vector sum of centripetal acceleration and tangential acceleration.



$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_C + \vec{a}_T$$

$$\Rightarrow \quad a = \sqrt{a_T^2 + a_C^2}$$

$$\Rightarrow \quad \tan\theta = \frac{a_C}{a_T}$$

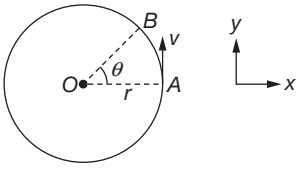
Conceptual Note(s)

(a) Differentiation of velocity $(\vec{v})$ gives total acceleration.

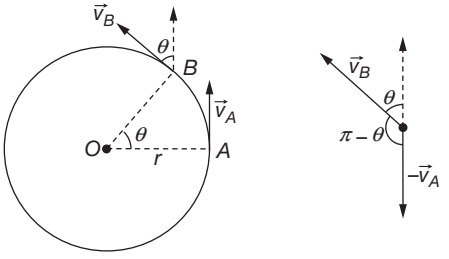
(b) $\left|\frac{d\vec{v}}{dt}\right|$ and $\frac{d|\vec{v}|}{dt}$ are not same physical quantities, $\left|\frac{d\vec{v}}{dt}\right|$ is the magnitude of rate of change of velocity, i.e., magnitude of total acceleration and $\frac{d|\vec{v}|}{dt}$ is a rate of change of speed, i.e., tangential acceleration.

CALCULATION OF CENTRIPETAL ACCELERATION

Consider a particle which moves in a circle with constant speed v as shown in Figure.



Change in velocity between the point A and B is



$$\Delta\vec{v} = \vec{v}_B - \vec{v}_A$$

Magnitude of change in velocity

$$|\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{v_B^2 + v_A^2 + 2v_A v_B \cos(\pi - \theta)}$$

$$(v_A = v_B = v, \text{ since speed is same})$$

$$\Rightarrow \quad |\Delta\vec{v}| = 2v\sin\left(\frac{\theta}{2}\right)$$

Distance travelled by particle between A and $B = r\theta$

Hence time taken, $\Delta t = \frac{r\theta}{v}$

Net acceleration, $|\vec{a}_{\text{net}}| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{2v\sin\left(\frac{\theta}{2}\right)}{r\theta/v}$

$$\Rightarrow \quad |\vec{a}_{\text{net}}| = \frac{v^2}{r}\left[\frac{2\sin\left(\frac{\theta}{2}\right)}{\theta}\right]$$

If $\Delta t \to 0$, then θ is small, $\sin\left(\frac{\theta}{2}\right) = \frac{\theta}{2}$

$$\lim_{\Delta t \to 0}\left|\frac{\Delta\vec{v}}{\Delta t}\right| = \left|\frac{d\vec{v}}{dt}\right| = \frac{v^2}{r}$$

i.e., net acceleration is $\frac{v^2}{r}$ but speed is constant so that tangential acceleration

$$a_t = \frac{dv}{dt} = 0$$

$$\Rightarrow \quad a_{\text{net}} = a_r = \frac{v^2}{r}$$

Through we have derived the formula of centripetal acceleration under condition of constant speed, the same formula is applicable even when speed is variable.

Conceptual Note(s)

In vector form

$$\vec{a}_c = \vec{\omega} \times \vec{v}$$

ILLUSTRATION 99

A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5 ms^{-1} to 6 ms^{-1} in 2 s, find the angular acceleration

SOLUTION

Since speed increases uniformly, average tangential acceleration is equal to instantaneous tangential acceleration

The instantaneous tangential acceleration is given by

$$a_T = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\Rightarrow \quad a_T = \frac{6-5}{2}\ \text{ms}^{-2} = 0.5\ \text{ms}^{-2}$$

The angular acceleration is $\alpha = \frac{a_T}{r}$.

$$\Rightarrow \quad \alpha = \frac{0.5\ \text{ms}^{-2}}{20\ \text{cm}} = 2.5\ \text{rads}^{-2}$$

ILLUSTRATION 100

Find the magnitude of the acceleration of a particle moving in a circle of radius 10 cm with uniform speed completing the circle in $4s$.

SOLUTION

The distance covered in completing the circle is

$$2\pi r = 2\pi \times 10\ \text{cm}.$$

The linear speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 10\ \text{cm}}{4s} = 5\pi\ \text{cms}^{-1}$$

The acceleration is

$$a_C = \frac{v^2}{r} = \frac{(5\pi\ \text{cms}^{-1})^2}{10\ \text{cm}} = 2.5\pi^2\ \text{cms}^{-2}$$

ILLUSTRATION 101

A particle moves in a circle of radius 2 cm at a speed given by $v = 4t$, where v is in cms^{-1} and t is in seconds.

(a) Find the tangential acceleration at $t = 1$ s.
(b) Find total acceleration at $t = 1$ s.

SOLUTION

(a) Tangential acceleration

$$a_T = \frac{dv}{dt}$$

$$\Rightarrow \quad a_T = \frac{d}{dt}(4t) = 4\ \text{cms}^{-2}$$

$$\Rightarrow \quad a_C = \frac{v^2}{R} = \frac{(4)^2}{2} = 8\ \text{cms}^{-2}$$

$$\Rightarrow \quad a = \sqrt{a_T^2 + a_C^2} = \sqrt{(4)^2 + (8)^2}$$

$$\Rightarrow \quad a = 4\sqrt{5}\ \text{cms}^{-2}$$

ILLUSTRATION 102

A particle is moving with a constant angular acceleration of 4 rads^{-2} in a circular path. At time $t = 0$ particle was at rest. Find the time at which the magnitudes of centripetal acceleration and tangential acceleration are equal.

SOLUTION

$$a_t = \alpha R$$

$$\Rightarrow \quad v = 0 + \alpha R t$$

$$a_c = \frac{v^2}{R} = \frac{\alpha^2 R^2 t^2}{R}$$

$$\Rightarrow \quad |a_t| = |a_c|$$

$$\Rightarrow \quad \alpha R = \frac{\alpha^2 R^2 t^2}{R}$$

$$\Rightarrow \quad t^2 = \frac{1}{\alpha} = \frac{1}{4}$$

$$\Rightarrow \quad t = \frac{1}{2}\ \text{s}$$

DYNAMICS OF CIRCULAR MOTION

If there is no force acting on a body it will move in a straight line (with constant speed). Hence if a body is moving in a circular path or any curved path, there must be some force acting on the body.

If speed of body is constant, the net force acting on the body is along the inside normal to the path of the body and it is called centripetal force.

$$\text{Centripetal force } (F_C) = ma_C = \frac{mv^2}{r} = mr\omega^2$$

However if speed of the body varies then, in addition to above centripetal force which acts along inside normal, there is also a force acting along the tangent of the path of the body which is called tangential force.

$$\text{Tangential force } (F_T) = Ma_T = M\frac{dv}{dt} = Mr\alpha,$$

where α is the angular acceleration.

Conceptual Note(s)

Remember $\frac{mv^2}{r}$ is not a force itself. It is just the value of the net force acting along the inside normal (towards the centre) which is responsible for circular motion.

This force may be friction, normal, tension, spring force, gravitational force or a combination of them.

So to solve any problem in uniform circular motion we identify all the forces acting along the normal (towards centre), calculate their resultant and equate it to $\frac{mv^2}{r}$.

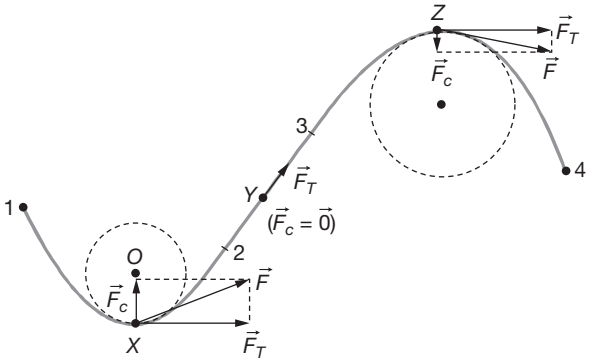
If circular motion is non-uniform then in addition to above step we also identify all the forces acting along the tangent to the circular path, calculate their resultant and equate it to $m\frac{dv}{dt}$ or $m\frac{d|\vec{v}|}{dt} = a_T$.

Some Common Examples of Centripetal Force are

SITUATION	CENTRIPETAL FORCE
A particle tied to a string and whirled in a horizontal circle.	Tension in the string.
Earth in orbit around the sun.	Gravitational force exerted by the sun.
An electron revolving around the nucleus in an atom.	Coulomb attraction exerted by the protons in the nucleus.
A charged particle describing a circular path in a magnetic field.	Magnetic force exerted by the agent that sets up the magnetic field.
Vehicle taking a turn on a level road.	Frictional force exerted by the road on the tyres.

MOTION OF A PARTICLE IN A CURVED TRACK AND RADIUS OF CURVATURE

Consider a curved track, 1234, having portions 12, 23 and 34 on which points X, Y and Z are taken respectively.



We must understand that when a single force acts on any particle it will just accelerate in the direction of force. So, for a particle to move in a curved track it must be experiencing two forces, as shown already,

(a) Centripetal force $\vec{F}_C$, acting radially inwards.

(b) Tangential force $\vec{F}_T$, acting tangentially.

So, at the point X, the curved track is due to the two forces $\vec{F}_C$ and $\vec{F}_T$ at that point.

At the point Y, $r \to \infty$, so $\vec{F}_C \to 0$, hence from 2 to 3, the particle just follows a straight track 23 under the influence of a single force $\vec{F}_T$.

At Z, story similar to the point X is repeated. So, we can say that for a particle moving in a curved track, net force and hence net acceleration are given by

$$F = \sqrt{F_C^2 + F_T^2} \qquad \{\because \vec{F}_C \perp \vec{F}_T\}$$

$$\Rightarrow \quad F = m\sqrt{a_C^2 + a_T^2}$$

$$\Rightarrow \quad F = m\sqrt{(r\omega^2)^2 + (r\alpha)^2}$$

$$\Rightarrow \quad F = mr\sqrt{\omega^4 + \alpha^2} \quad \text{and} \quad a = \frac{F}{m} = r\sqrt{\omega^4 + \alpha^2}$$

Radius of Curvature

As observed, any curved track/path can be assumed to be made of a large number of circular arcs of variable radii. The radius of curvature at a point is the radius of the circular arc that suitably fits on the curve at that point.

Since $a_C = \frac{v^2}{r}$, where v is the tangential velocity and can be denoted by v_T.

$$\Rightarrow \quad a_C = \frac{v_T^2}{r}$$

$$\Rightarrow \quad r = \frac{v_T^2}{a_C} = \frac{v_\perp^2}{a_\parallel}$$

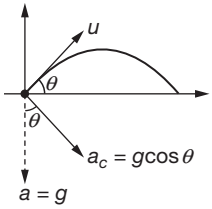
where $v_T = v_\perp$, i.e., tangential velocity equals the velocity of the particle **perpendicular to the radius** of the curve and hence v_T can also be denoted by $v_\perp$ (read as v perpendicular) and $a_C = a_\parallel$, i.e., centripetal acceleration equals the acceleration of the particle **parallel to the radius** of the curve and hence a_C can also be denoted by $a_\parallel$.

ILLUSTRATION 103

A particle of mass m is projected with speed u at an angle θ with the horizontal. Find the radius of curvature of the path traced out by the particle at the point of projection and also at the highest point of trajectory.

SOLUTION

At point of projection, we have



$$R = \frac{v_T^2}{a_C} = \frac{u^2}{g\cos\theta}$$

$$\Rightarrow \quad R = \frac{u^2}{g\cos\theta}$$

At highest point, we have

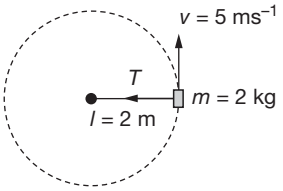
$$a_C = g\,,\ v = u\cos\theta$$

$$\Rightarrow \quad R = \frac{v_T^2}{a_C} = \frac{u^2\cos^2\theta}{g}$$

ILLUSTRATION 104

A block of mass 2 kg is tied to a string of length 2 m, the other end of which is fixed. The block is moved on a smooth horizontal table with constant speed $5\ \text{ms}^{-1}$. Find the tension in the string.

SOLUTION

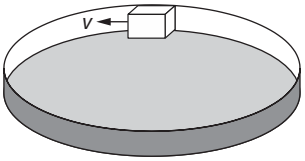


In this case, the centripetal force is provided by tension, so,

$$T = \frac{mv^2}{r} = \frac{2\times 5^2}{2} = 25\ \text{N}$$

ILLUSTRATION 105

A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.



Find the

(a) normal reaction of the floor on the block.
(b) normal reaction of the vertical wall on the block.

SOLUTION

Here centripetal force is provided by normal reaction of vertical wall

(a) normal reaction due to the floor

$$N_F = mg$$

(b) normal reaction due to the vertical wall

$$N_W = \frac{mv^2}{r}$$

ILLUSTRATION 106

A block of mass m is kept on the edge of a horizontal turn table of radius R, which is rotating with constant angular velocity ω (along with the block) about its axis. If coefficient of friction is μ, find the friction force between block and table if the block is at rest with respect to table.

SOLUTION

The centripetal force, in this case is provided by friction force

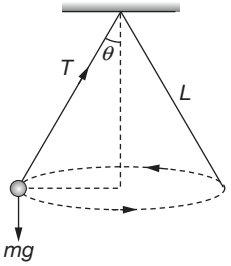
$$\text{Friction force} = \text{centripetal force} = mR\omega^2$$

ILLUSTRATION 107

A string breaks under a load of $50\ \text{kg}$. A mass of $1\ \text{kg}$ is attached to one end of the string $10\ \text{m}$ long and is rotated in horizontal circle. Calculate the greatest number of revolutions that the mass can make in one second without breaking the string.

SOLUTION

The situation is shown in the figure. Since we have



$$\omega = 2\pi n$$

$$T_{\max} = 500\ \text{N}$$

$$r = L\sin\theta$$

$$T\sin\theta = m\omega^2 r$$

$$\Rightarrow \quad T = m\omega^2 L$$

$$\Rightarrow \quad T_{\max} = m\omega_{\max}^2 L$$

$$\Rightarrow \quad T_{\max} = m\left(2\pi n_{\max}\right)^2 L$$

$$\Rightarrow \quad n_{\max} = \frac{1}{2\pi}\sqrt{\frac{T_{\max}}{mL}}$$

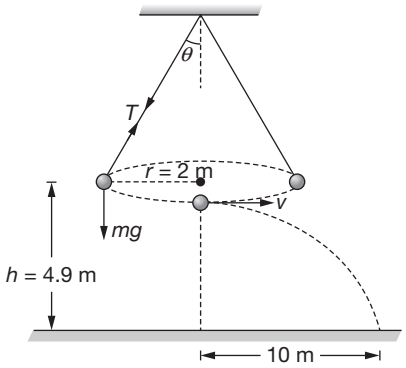
$$\Rightarrow \quad n_{\max} = \frac{1}{2\pi}\sqrt{\frac{500}{1\times 10}}$$

$$\Rightarrow \quad n_{\max} = \frac{\sqrt{50}}{2\pi}\ \text{rps}$$

ILLUSTRATION 108

A boy whirls a stone in a horizontal circle of radius $2\ \text{m}$ and at height $4.9\ \text{m}$ above level ground. The string breaks and the stone files off horizontally and strikes the ground at a point which is $10\ \text{m}$ away from the point on the ground directly below the point where the string had broken. What is the magnitude of the centripetal acceleration of the stone while in circular motion? $\left(g = 9.8\ \text{ms}^{-2}\right)$.

SOLUTION



$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 4.9}{9.8}} = 1\ \text{s}$$

$$\Rightarrow \quad v = \frac{10}{t} = 10\ \text{ms}^{-1}$$

$$\Rightarrow \quad a = \frac{v^2}{R} = 50\ \text{ms}^{-2}$$

ILLUSTRATION 109

A particle begins to move with a tangential acceleration of constant magnitude $0.6\ \text{ms}^{-2}$ in a circular path. If it slips when its total acceleration becomes $1\ \text{ms}^{-2}$. Find the angle through which it would have turned before it starts to slip.

SOLUTION

$$a_{\text{Net}} = \sqrt{a_T^2 + a_C^2} \qquad \ldots(1)$$

Since, $\omega^2 = \omega_0^2 + 2\alpha\theta$ and $\omega_0 = 0$

$$\Rightarrow \quad \omega^2 = 2\alpha\theta$$

$$\Rightarrow \quad \omega^2 R = 2(\alpha R\theta)$$

$$\Rightarrow \quad a_C = \omega^2 R = 2a_T\theta$$

From (1), we get

$$\sqrt{0.36+(1.2\times\theta)^2}=1$$

$$\Rightarrow \quad 1-0.36=(1.2\theta)^2$$

$$\Rightarrow \quad \frac{0.8}{1.2}=\theta$$

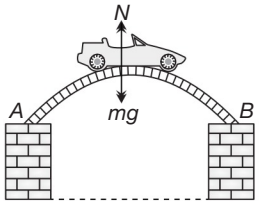
$$\Rightarrow \quad \theta=\frac{2}{3} \text{ radian}$$

ILLUSTRATION 110

Prove that a motor car moving over a convex bridge is lighter than the same car resting on the same bridge.

SOLUTION

The motion of the motor car over a convex bridge AB is the motion along the segment of a circle AB as shown in the figure.



The centripetal force is provided by the difference of weight mg of the car and the normal reaction N of the bridge.

$$\Rightarrow \quad mg-N=\frac{mv^2}{r}$$

$$\Rightarrow \quad N=mg-\frac{mv^2}{r}$$

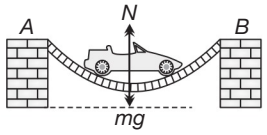
Since $N<mg$, so, the weight of the moving car is less than the weight of the stationary car.

ILLUSTRATION 111

Prove that a motor car moving over a concave bridge is heavier than the same car resting on the same bridge.

SOLUTION

The motion of the motor car over a concave bridge AB is the motion along the segment of a circle AB as shown in the figure.



The centripetal force is provided by the difference of normal reaction N of the bridge and weight mg of the car.

$$\Rightarrow \quad N-mg=\frac{mv^2}{r}$$

$$\Rightarrow \quad N=mg+\frac{mv^2}{r}$$

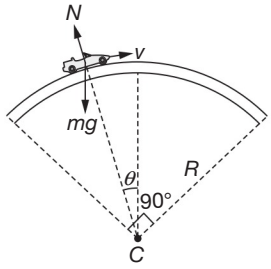
Since $N>mg$, so, the weight of the moving car is greater than the weight of the stationary car.

ILLUSTRATION 112

A car is moving with uniform speed over a circular bridge of radius R which subtends an angle of $90°$ at its centre. Find the minimum possible speed so that the car can cross the bridge without losing the contact anywhere.

SOLUTION

Let the car loses the contact at angle θ with the vertical, then $mg\cos\theta-N=\frac{mv^2}{R}$.



$$\Rightarrow \quad N=mg\cos\theta-\frac{mv^2}{R} \qquad \ldots(1)$$

For losing the contact $N=0$

$$\Rightarrow \quad v=\sqrt{Rg\cos\theta} \qquad \{\text{from (1)}\}$$

For minimum speed, cos θ should be minimum so that θ should be maximum

$$\theta_{max}=45°$$

$$\Rightarrow \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad v_{\min} = \sqrt{\frac{Rg}{\sqrt{2}}}$$

So, that if car cannot lose the contact at initial or final point, car cannot be lose the contact anywhere.

ILLUSTRATION 113

A simple pendulum is constructed by attaching a bob of mass m to a string of length L fixed at its upper end. The bob oscillates in a vertical circle. If is found that the speed of the bob is v when the string makes an angle θ with the vertical. Find the tension in the string and the magnitude of net force on the bob at the instant.

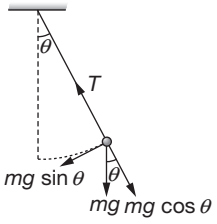
SOLUTION

(a) The forces acting on the bob are

(i) the tension T

(ii) the weight mg

As the bob moves in a circle of radius L with centre at O, A centripetal force of magnitude $\frac{mv^2}{L}$ is required towards O.



This force will be provided by the resultant of T and $mg\cos\theta$. Thus,

$$T - mg\cos\theta = \frac{mv^2}{L}$$

$$\Rightarrow \quad T = m\left(g\cos\theta + \frac{v^2}{L}\right)$$

(b) Since a_C is provided by $(T - mg\cos\theta)$ acting radially inwards, so $a_C = \frac{T - mg\cos\theta}{m} = \frac{v^2}{l}$.

$$a_{\text{net}} = \sqrt{a_T^2 + a_C^2} = \sqrt{(g\sin\alpha)^2 + \left(\frac{v^2}{l}\right)^2}$$

$$|\vec{F}_{\text{net}}| = ma_{\text{net}} = m\sqrt{g^2\sin^2\alpha + \frac{v^4}{L^2}}$$

CENTRIFUGAL FORCE

When a body is rotating in a circular path and if the centripetal force vanishes, then the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release.

To another observer B, who is sharing the motion along the circular path (i.e., the observer B is also rotating with the body which is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force called centrifugal force).

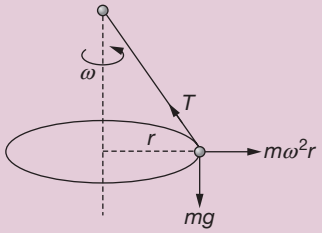
Its magnitude is equal to that of the centripetal force i.e.,

$$|F_{\text{centrifugal}}| = \frac{mv^2}{r} = mr\omega^2$$

Conceptual Note(s)

(a) Direction of centrifugal force, it is always directed radially outward.

(b) Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion in that frame. FBD of ball w.r.t. non inertial frame rotating with the ball is shown here.



(c) Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyse

the dynamics of a particle of mass m kept at a distance r from the axis of rotation, we have to assume that a force $mr\omega^2$ react radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

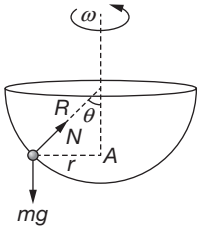
(d) While drawing FBD from rotating frame(s), we must draw the centrifugal force whereas while drawing FBD from ground or inertial frame we must not show the centripetal force. Instead we draw all forces and then the resultant of these forces acting towards the centre will provide the necessary centripetal force.

ILLUSTRATION 114

A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. It the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is θ. Find the angular speed at which the bowl is rotating.

SOLUTION

Let ω be the angular speed of rotation of the bowl. Two forces are acting on the ball.



(a) Normal reaction (N)
(b) Weight (mg)

The ball is rotating in a circle of radius $r(=R\sin\theta)$ with centre at A at an angular speed ω. Thus,

$$N\sin\theta = mr\omega^2 = mR\omega^2\sin\theta$$

$$\Rightarrow \quad N = mR\omega^2 \qquad \ldots(1)$$

and $$N\cos\theta = mg \qquad \ldots(2)$$

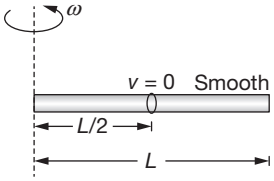
Dividing (1) by (2), we get

$$\frac{1}{\cos\theta} = \frac{\omega^2 R}{g}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R\cos\theta}}$$

ILLUSTRATION 115

A ring which can slide along the rod are kept at mid-point of a smooth rod of length L. The rod is rotated with constant angular velocity ω about vertical axis passing through its one end. Ring is released from mid-point. Find the velocity of the ring when it just leave the rod.



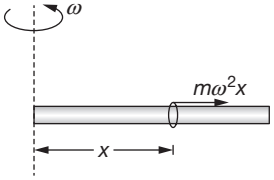
SOLUTION

Centrifugal force at distance x is

$$m\omega^2 x = ma$$

$$\Rightarrow \quad \omega^2 x = \frac{vdv}{dx}$$

Rearranging and integrating, we get



$$\int_{L/2}^{L} \omega^2 x\,dx = \int_0^v v\,dv$$

$$\Rightarrow \quad \omega^2\left(\frac{x^2}{2}\right)_{L/2}^{L} = \left(\frac{v^2}{2}\right)_0^v$$

$$\Rightarrow \quad \omega^2\left(\frac{L^2}{2} - \frac{L^2}{8}\right) = \frac{v^2}{2}$$

$$\Rightarrow \quad v = \frac{\sqrt{3}}{2}\omega L$$

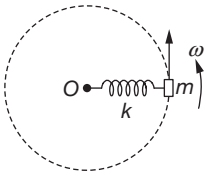
Velocity at time of leaving the rod is the resultant of tangential speed and the radial speed of the particle. So,

$$v' = \sqrt{(\omega L)^2 + \left(\frac{\sqrt{3}}{2}\omega L\right)^2}$$

$$\Rightarrow \quad v' = \frac{\sqrt{7}}{2}\omega L$$

ILLUSTRATION 116

A block of mass m is tied to a spring of spring constant k, natural length l and the other end of spring is fixed at O. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity ω, find tension in the spring.



SOLUTION

Assume extension in the spring is x.
Here centripetal force is provided by spring force
Centripetal force

$$kx = m\omega^2(l + x)$$

$$\Rightarrow \quad x = \frac{m\omega^2 l}{k - m\omega^2}$$

$$\Rightarrow \quad \text{Tension} = kx = \frac{km\omega^2 l}{k - m\omega^2}$$

ILLUSTRATION 117

A block of mass m is kept on rough horizontal turn table at a distance r from centre of table. Coefficient of friction between turn table and block is μ. Now turn table starts rotating with uniform angular acceleration α.

(a) Find the time after which slipping occurs between block and turn table.
(b) Find angle made by friction force with velocity at the point of slipping.

SOLUTION

(a) $a_t = \alpha r$

Speed after t time

$$\frac{dv}{dt} = \alpha r$$

$$\Rightarrow \quad v = 0 + \alpha r t$$

Centripetal acceleration

$$a_c = \frac{v^2}{r} = \alpha^2 r t^2$$

Net acceleration $a_{\text{net}} = \sqrt{a_t^2 + a_c^2}$

$$\Rightarrow \quad a_{\text{net}} = \sqrt{\alpha^2 r^2 + \alpha^4 r^2 t^4}$$

Block just start slipping

$$\mu mg = ma_{\text{net}} = m\sqrt{\alpha^2 r^2 + \alpha^4 r^2 t^4}$$

$$\Rightarrow \quad t = \left(\frac{\mu^2 g^2 - \alpha^2 r^2}{\alpha^4 r^2}\right)^{1/4}$$

$$\Rightarrow \quad t = \left[\left(\frac{\mu g}{\alpha^2 r}\right)^2 - \left(\frac{1}{\alpha}\right)^2\right]^{1/4}$$

(b) $\tan\theta = \frac{a_c}{a_t}$

$$\Rightarrow \quad \tan\theta = \frac{\alpha^2 r t^2}{\alpha r}$$

$$\Rightarrow \quad \theta = \tan^{-1}(\alpha t^2)$$

ROTOR OR DEATH WELL

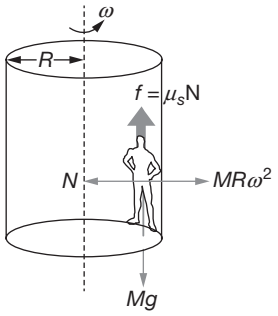
It is a hollow cylindrical room which is capable of rotating about a central vertical axis of cylinder. A person stands up against the wall. For some time the rotational speed of rotor increases and at a certain speed when the floor of the rotor room is removed the person in it does not fall but remains 'pinned up' against the wall of the rotor. The relation between rotational speed ω and the coefficient of friction between person and wall may be found as follows.

Let M be the mass of person and R the radius of cylindrical room. The forces acting on the person are

(a) Weight Mg (vertically downward)
(b) Normal reaction N (radially inwards)
(c) Frictional force $f = \mu_s N$ (upward)
(d) Centrifugal force $= mR\omega^2$ (radially outwards)

For vertical equilibrium

$$Mg = f \text{ or } Mg = \mu_s N \qquad \ldots(1)$$



The person will remain in contact with the wall only if

$$MR\omega^2 \geq N$$

$$\Rightarrow \quad MR\omega^2 \geq \frac{Mg}{\mu_s} \quad \text{or} \quad \omega^2 \geq \frac{g}{\mu_s R}$$

$$\Rightarrow \quad \omega \geq \sqrt{\frac{g}{\mu_s R}}$$

Minimum speed of rotor, $\omega_{\min} = \sqrt{\frac{g}{\mu_s R}}$

If v is the linear speed of rotor, then

$$v = R\omega = \sqrt{\frac{gR}{\mu_s}}$$

ILLUSTRATION 118

Two blocks of mass $m_1 = 10$ kg and $m_2 = 5$ kg connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of the turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rads^{-1} about a vertical axis passing through its centre O. The masses are placed along the diameter of the table on either side of the centre O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table. $\left(g = 9.8 \text{ ms}^{-2}\right)$.

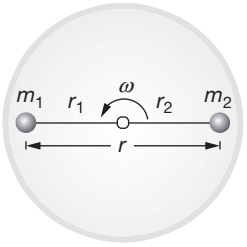
(a) Calculate the frictional force on m_1.
(b) What should be the minimum angular speed of the turn table so that the masses will slip from this position?
(c) How should the masses be placed with the string remaining taut so that there is no frictional force acting on the mass m_1?

SOLUTION

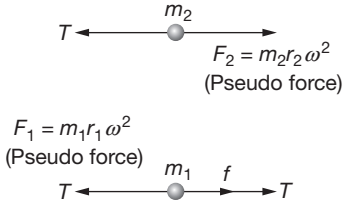
Given $m_1 = 10$ kg, $m_2 = 5$ kg, $\omega = 10$ rads^{-1}

$r = 0.3$ m, $r_1 = 0.124$ m

$\Rightarrow \quad r_2 = r - r_1 = 0.176$ m



(a) Masses m_1 and m_2 are at rest with respect to rotating table. Let f be the friction between mass m_1 and table. Free body diagram of m_1 and m_2 with respect to table (non inertial frame of reference are shown in figure).



Equilibrium of m_2 gives

$$T = m_2 r_2 \omega^2 \qquad \ldots(1)$$

Since, $m_2 r_2 \omega^2 < m_1 r_1 \omega^2$ $\quad \{m_2 r_2 < m_1 r_1\}$

Therefore, $m_1 r_1 \omega^2 > T$ and friction on m_1 will be inwards (towards centre)

Equilibrium of m_1 gives $f + T = m_1 r_1 \omega^2$ …(2)

From equations (1) and (2), we get

$$f = m_1 r_1 \omega^2 - m_2 r_2 \omega^2$$

$$\Rightarrow \quad f = (m_1 r_1 - m_2 r_2)\omega^2 \quad \text{…(3)}$$

$$\Rightarrow \quad f = (10 \times 0.124 - 5 \times 0.176)(100) \text{ Newton}$$

$$\Rightarrow \quad f = 36 \text{ N}$$

Therefore, frictional force on m_1 is 36 N (inwards).

(b) Since, from equation (3), we have

$$f = (m_1 r_1 - m_2 r_2)\omega^2$$

So, the masses will start slipping when this force is greater than f_{max}, i.e.,

$$(m_1 r_1 - m_2 r_2)\omega^2 > f_{max} > \mu m_1 g$$

Hence, the minimum value of ω is

$$\omega_{min} = \sqrt{\frac{\mu m_1 g}{m_1 r_1 - m_2 r_2}} = \sqrt{\frac{0.5 \times 10 \times 9.8}{10 \times 0.124 - 5 \times 0.176}}$$

$$\Rightarrow \quad \omega_{min} = 11.67 \text{ rads}^{-1}$$

(c) From equation (3), frictional force $f = 0$ when $m_1 r_1 = m_2 r_2$

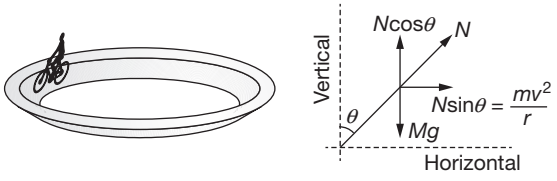
$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{5}{10} = \frac{1}{2} \text{ and } r = r_1 + r_2 = 0.3 \text{ m}$$

$$\Rightarrow \quad r_1 = 0.1 \text{ m and } r_2 = 0.2 \text{ m}$$

i.e., mass m_2 should be placed at 0.2 m and m_1 at 0.1 m from the centre O.

MOTION OF A CYCLIST

Let a cyclist moving on a circular path of radius r bend away from the vertical by an angle θ.



If R is the normal reaction of the ground, then R may be resolved into two components horizontal and vertical. The vertical component $R\cos\theta$ balances the weight mg of the cyclist and the horizontal component $R\sin\theta$ provides the necessary centripetal force to the cyclist to move in circular path i.e.,

$$R\sin\theta = \frac{mv^2}{r} \quad \text{…(1)}$$

and $R\cos\theta = mg$ …(2)

Dividing (1) by (2), we get

$$\tan\theta = \frac{v^2}{rg} \quad \text{…(3)}$$

For less bending of cyclist, his speed v should be smaller and radius r of circular path should be greater. If μ is coefficient of friction, then for no skidding of cycle (or overturning of cyclist)

$$\mu \geq \tan\theta = \frac{v^2}{rg} \quad \text{…(4)}$$

CIRCULAR TURNING ON ROADS

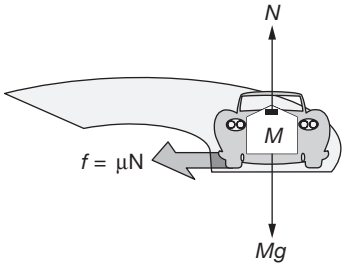
When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

(a) By friction only.
(b) By banking of roads only.
(c) By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

BY FRICTION ONLY: VEHICLE ON A LEVEL ROAD

When a vehicle goes around a curve, it has a tendency of skidding sideways i.e. away from the centre of the curve. Due to this tendency, the static friction f_s acts towards the centre and provides the necessary centripetal force for motion along the curve.



The forces acting on the vehicle are

(a) Weight Mg acting vertically downward
(b) Normal reaction N
(c) Static frictional force f_s

The static friction is self adjusting and if μ_s is coefficient of static friction, then $f_s \le \mu_s N$. So,

For vertical equilibrium, $N = Mg$

As frictional force provides the necessary centripetal force $f_s = \dfrac{Mv^2}{r}$

As $f_s \le \mu_s N$

$$\Rightarrow \quad \frac{Mv^2}{r} \le \mu_s N$$

$$\Rightarrow \quad \frac{Mv^2}{r} \le \mu_s Mg$$

$$\Rightarrow \quad v \le \sqrt{\mu_s rg}$$

So, maximum speed for no skidding is

$$v_{max} = \sqrt{\mu_s rg}$$

ILLUSTRATION 119

A bend in a level road has a radius of 100 m. Calculate the maximum speed which a car turning this bend may have without skidding. Given: $\mu = 0.8$.

SOLUTION

$$v_{max} = \sqrt{\mu rg} = \sqrt{0.8 \times 100 \times 10}$$

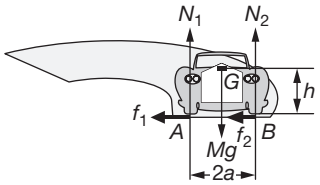
$$\Rightarrow \quad v_{max} = \sqrt{0.8 \times 100 \times 10}$$

$$\Rightarrow \quad v_{max} = \sqrt{800}$$

$$\Rightarrow \quad v_{max} = 28 \text{ ms}^{-1}$$

MAXIMUM VELOCITY FOR SKIDDING AND OVERTURNING

Let A and B be inner and outer wheels of a vehicle moving on a circular track. The forces acting on the vehicle are



(a) normal reactions N_1 and N_2 vertically upward
(b) frictional forces $f_1 = \mu N_1$ and $f_2 = \mu N_2$
(c) Weight of vehicle Mg vertically downward
(d) centripetal force F (horizontally towards centre of turn)

For translational equilibrium

$$N_1 + N_2 = Mg \qquad \ldots(1)$$

and frictional force provides the necessary centripetal force

$$F = f_1 + f_2 = \mu N_1 + \mu N_2 \ge \frac{mv^2}{r}$$

$$\Rightarrow \quad \mu(N_1 + N_2) \ge \frac{mv^2}{r} \qquad \ldots(2)$$

where r is the radius of path.
Using (1), equation (2) gives

$$\Rightarrow \quad \mu Mg \ge \frac{mv^2}{r} \quad \text{(for no skidding)}$$

$$\Rightarrow \quad v \le \sqrt{\mu rg}$$

Thus maximum speed for no skidding is

$$v_{max} = \sqrt{\mu rg} \qquad \ldots(3)$$

For No Overturning

If the wheels A and B are a distance $2a$ apart, then taking moments about G, we get $N_2 a = N_1 a + Fh$

where $F = \dfrac{mv^2}{r}$ given by (2)

The car tends to overturn when reaction N_1 the inner wheel is zero i.e. when inner wheel leaves contact with the ground $N_2 \cdot a \geq Fh$

If $N_1 = 0$, then from (1) $N_2 = Mg$

$$\Rightarrow \quad Mga \geq \frac{mv^2}{r}h$$

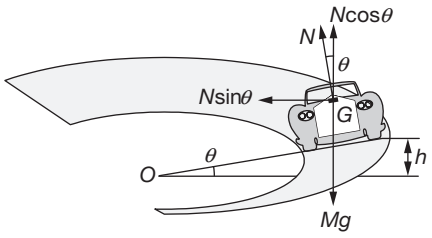
$$\Rightarrow \quad v \leq \sqrt{\frac{gra}{h}}$$

That is maximum speed for no overturning is

$$v_{max} = \sqrt{\frac{gra}{h}}$$

BY BANKING OF ROADS/TRACKS

When a vehicle moves round a curve on the road with sufficient speed, then there is a tendency of overturning for the vehicle. To avoid this the road is given a slope rising outwards. The phenomenon is known as **banking**. Consider a vehicle on a road having a slope θ. N is the normal reaction of the ground. This may be resolved into two components : A vertical component $N\cos\theta$ which balances the weight of vehicle and a horizontal component $N\sin\theta$ which provides the necessary centripetal force i.e.,



$$N\sin\theta = \frac{mv^2}{r} \qquad \ldots(1)$$

and $N\cos\theta = Mg$ $\qquad \ldots(2)$

Dividing equation (1) by (2), we get

$$\tan\theta = \frac{v^2}{rg}$$

This equation gives the angle of banking required.

Let $l(=OB)$ be the width of track and $h(=AB)$ be its height.

Assuming θ (the angle of banking to be small) then

$$\tan\theta \simeq \sin\theta = \frac{h}{l}$$

$$\Rightarrow \quad \frac{h}{l} = \frac{v^2}{rg}$$

$$\Rightarrow \quad h = \frac{v^2 l}{rg}$$

is the height through which outer part of the track has to be raised.

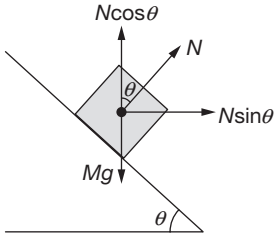
ILLUSTRATION 120

A circular track of radius 600 m is to be designed for cars at an average speed of 180 kmh^{-1}. What should be the angle of banking of the track?

SOLUTION

Let the angle of banking be θ. The forces on the car are (figure)

(a) weight of the car Mg downward and
(b) normal force N.



For proper banking, static frictional force is not needed. For vertical direction the acceleration is zero. So,

$$N\cos\theta = Mg \qquad \ldots(1)$$

For horizontal direction, the acceleration is $\frac{v^2}{r}$ towards the centre, so that

$$N\sin\theta = \frac{Mv^2}{r} \qquad \ldots(2)$$

From (1) and (2)

$$\tan\theta = \frac{v^2}{rg}$$

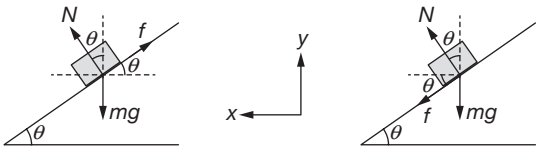
Substituting the values

$$\tan\theta = \frac{180 \text{ kmh}^{-2}}{600 \text{ m} \times 10 \text{ ms}^{-2}} = 0.4167$$

$$\Rightarrow \quad \theta = 22.6°$$

BY FRICTION AND BANKING OF ROAD BOTH

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.



The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road)

The direction of the third force i.e., friction f can be either inwards or outwards and its magnitude can be varied upto a maximum limit $(f_L = \mu N)$.

So the magnitude of normal reaction N, direction of friction and magnitude of friction f are so adjusted so that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.

Since, m and r are also constant so, magnitude of normal reaction N, direction of friction and magnitude of friction f mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem.

Also, we observe that

(a) friction f will be outwards if the vehicle is at rest $v = 0$. Because in that case the component of weight $mg\sin\theta$ is balanced by f.

(b) friction f will be inwards if

$$v > \sqrt{rg\tan\theta}$$

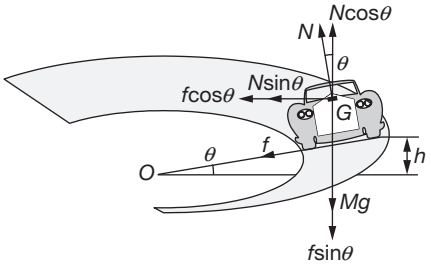
(c) friction f will be outwards if

$$v < \sqrt{rg\tan\theta} \text{ and}$$

(d) friction f will be zero if

$$v = \sqrt{rg\tan\theta}$$

(e) for maximum safe speed (shown in figure (ii)) and redrawn below



$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \quad \ldots(1)$$

$$N\cos\theta - f\sin\theta = mg \quad \ldots(2)$$

As maximum value of friction

$$f = \mu N$$

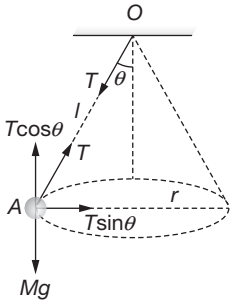
$$\Rightarrow \quad \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}$$

Similarly, $v_{\min} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$

CONICAL PENDULUM

It consists of a string OA, whose upper end O is fixed and a bob is tied at the other free end. The bob is given a horizontal push a little through angular displacement θ and arranged such that the bob describes a horizontal circle with uniform angular velocity ω in such a way that the string always makes an angle θ with the vertical. As the string traces the surface of the cone, the arrangement is called **a conical pendulum**.



Let T be the tension in the string of length l and r the radius of circular path. The vertical component of tension T balances the weight of the bob and horizontal component provides the necessary centripetal force.

Thus

$$T\cos\theta = Mg \qquad \ldots(1)$$

and $T\sin\theta = Mr\omega^2$ …(2)

Dividing (2) by (1), we get

$$\tan\theta = \frac{r\omega^2}{g} \text{ i.e., } \omega = \sqrt{\frac{g\tan\theta}{r}} \qquad \ldots(3)$$

But $r = l\sin\theta$ and $\omega = \frac{2\pi}{t}$, t being period of completing one revolution

$$\Rightarrow \quad \frac{2\pi}{t} = \sqrt{\frac{g\tan\theta}{l\sin\theta}}$$

This gives

$$t = 2\pi\sqrt{\frac{l\sin\theta}{g\left(\frac{\sin\theta}{\cos\theta}\right)}}$$

$$\Rightarrow \quad t = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

ILLUSTRATION 121

A turn of radius 20 m is banked for the vehicle of mass 200 kg going at a speed of 10 ms^{-1}. Find the direction and magnitude of frictional force acting on a vehicle if it moves with a speed (a) 5 ms^{-1} (b) 15 ms^{-1}.
Take $g = 10\ ms^{-2}$ and assume that friction is sufficient to prevent slipping.

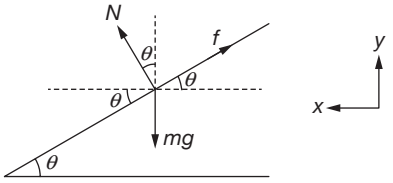
SOLUTION

(a) The turn is banked for speed $v = 10\ ms^{-1}$. If θ is the angle of banking, then

$$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{(20)(10)} = \frac{1}{2}$$

Now, as the speed is decreased, force of friction f acts outwards.
Using the equations

$$\Sigma F_x = \frac{mv^2}{r} \text{ and } \Sigma F_y = 0, \text{ we get}$$



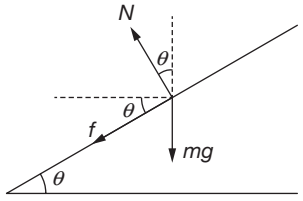
$$N\sin\theta - f\cos\theta = \frac{mv^2}{r} \qquad \ldots(1)$$

$$N\cos\theta + f\sin\theta = mg \qquad \ldots(2)$$

Substituting, $\theta = \tan^{-1}\left(\frac{1}{2}\right)$, $v = 5\ ms^{-1}$, $m = 200$ kg and $r = 20$ m, in equations (1) and (2), we get

$$f = 300\sqrt{5}\ \text{N} \quad \text{(outwards)}$$

(b) In the second case force of friction f will now act inwards.



Using $\Sigma F_x = \frac{mv^2}{r}$ and $\Sigma F_y = 0$, we get

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \qquad \ldots(3)$$

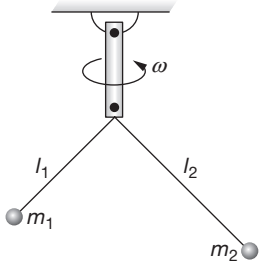
$$N\cos\theta - f\sin\theta = mg \qquad \ldots(4)$$

Substituting $\theta = \tan^{-1}\left(\frac{1}{2}\right)$, $v = 15\ ms^{-1}$, $m = 200$ kg and $r = 20$ m in equations (3) and (4), we get

$$f = 500\sqrt{5}\ \text{N} \quad \text{(inwards)}$$

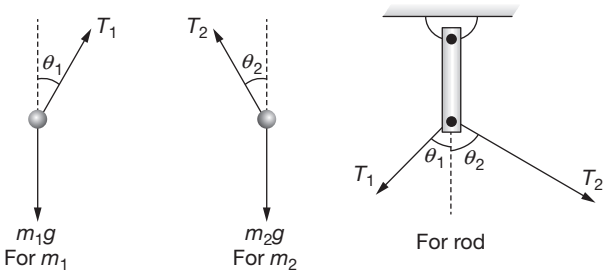
ILLUSTRATION 122

Two balls of mass m_1 and m_2 are suspended by two threads of lengths l_1 and l_2 at the end of a freely hanging rod. Determine the angular velocity ω at which the rod must be rotated about the vertical axis so that it remains vertical.



SOLUTION

Free body diagrams of two masses and the rod are as shown in figure.



Equations of motion are,

$$T_1 \sin\theta_1 = m_1\omega^2 l_1 \sin\theta_1 \quad \ldots(1)$$

$$T_1 \cos\theta_1 = m_1 g \quad \ldots(2)$$

$$T_2 \sin\theta_2 = m_2\omega^2 l_2 \sin\theta_2 \quad \ldots(3)$$

$$T_2 \cos\theta_2 = m_2 g \quad \ldots(4)$$

For the rod to remain in vertical position,

$$T_1 \sin\theta_1 = T_2 \sin\theta_2 \quad \ldots(5)$$

Solving the above equations, we get

$$\omega^4 = \left(\frac{m_1^2 g^2 - m_2^2 g^2}{m_1^2 l_1^2 - m_2^2 l_2^2}\right)$$

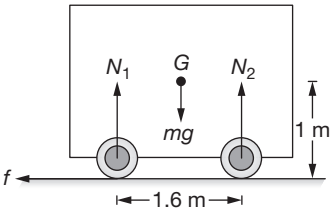
ILLUSTRATION 123

A vehicle whose wheel track is 1.6 m wide and whose centre of gravity is 1 m above the road, centred between the wheels, takes a curve whose radius is 50 m, on a level road. Taking $g = 10\ \text{ms}^{-2}$, find the speed at which the inner wheel would leave the road.

SOLUTION

The situation is shown in figure. Let N_1 and N_2 be the reactions at inner wheels and outer wheels respectively. f is the frictional force of the tracks. G represents the centre of gravity and mg, the weight of the vehicle acting downwards at centre of gravity. For vertical equilibrium,

$$N_1 + N_2 = mg \quad \ldots(1)$$



For circular motion

$$f = \frac{mv^2}{r} \quad \ldots(2)$$

For rotational equilibrium, net moment of all the forces about G should be zero. Hence,

$$f(1) + N_1\left(\frac{1.6}{2}\right) = N_2\left(\frac{1.6}{2}\right)$$

$$\Rightarrow \quad 0.8N_1 + f = 0.8N_2 \quad \ldots(3)$$

When the inner wheel leaves the road, then $N_1 = 0$. Therefore from (3), we get

$$f = 0.8N_2 \quad \ldots(4)$$

and from equation (1) we get

$$N_2 = mg \quad \ldots(5)$$

Solving equations (2), (4) and (5), we get

$$v = \sqrt{(0.8)\times(50)\times(10)} = 20\ \text{ms}^{-1}$$

Test Your Concepts-VIII

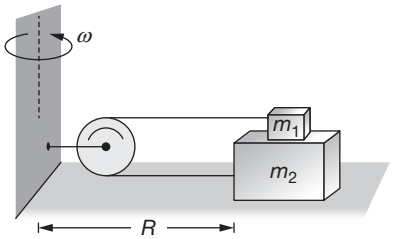
Based on Circular Motion

(Solutions on page H.214)

1. A circular table with smooth horizontal surface is rotating at an angular speed ω about its axis. A groove is made on the surface along a radius and a smooth ball of diameter equal to the width of the groove is gently placed inside the groove at a distance a from the centre. Find the speed of the particle with respect to the table when its distance from the centre becomes L.

2. A block of mass m moves on a horizontal circle against the wall of a cylindrical room of radius R. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ. The block is given an initial speed v_0. Find the
- **(a)** normal force by the wall on the block
- **(b)** frictional force by the wall
- **(c)** tangential acceleration of the block.
- **(d)** speed of block after one revolution.

3. If the system shown in the figure is rotated in a horizontal circle about the dotted axis, with angular velocity ω, find the minimum value of ω to start relative motion between the two blocks. Also calculate the tension in the string connecting m_1 and m_2 when slipping just starts between the blocks. The coefficient of friction between the two masses is 0.5, there is no friction between m_2 and ground, $R = 0.5$ m, $m_1 = 2$ kg, $m_2 = 1$ kg and $g = 10$ ms^{-2}. The dimensions of the blocks can be neglected.



4. The coefficient of friction between a block of mass m placed on a horizontal ruler is μ. The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end.
- **(a)** What can the maximum angular speed be for which the block does not slip?
- **(b)** If the angular speed of the ruler is uniformly increased from zero at an angular acceleration α, at what angular speed will the block slip?

5. A bicyclist travels in a circle of radius 25 m at a constant speed of 9 ms^{-1}. The bicycle-rider mass is 85 kg. Calculate the magnitude of
- **(a)** the force of friction on the bicycle from the road and
- **(b)** the net force on the bicycle from the road.

6. A small bead can slide without friction on a circular hoop that is in a vertical plane and has a radius of 0.1 m. The hoop rotates at a constant rate of 3 revs^{-1} about a vertical diameter.
- **(a)** Find the angle β at which the bead is in vertical equilibrium. (of course, it has a radial acceleration towards the axis).
- **(b)** Is it possible for the bead to ride at the same elevation as the centre of the hoop?

Take $g = 10$ ms^{-2} and $\pi^2 = 10$.

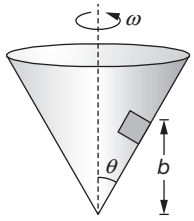
7. A string has one end attached to the corner of a square board fixed on a smooth horizontal table and is wound round the square carrying a particle at its other end. The particle is projected with velocity u at right angles to the side of a square of size $a \times a$. If the length of the string is $4a$. Find the time that the string takes to unwrap itself form the squares, assuming that the speed of the particle remains the same throughout the motion.

8. A wheel of radius R rolls along the ground with velocity v. A pebble is carefully released on top of the wheel so that it is instantaneously at rest on the wheel.
- **(a)** Show that the pebble will immediately fly off the wheel if $v > \sqrt{Rg}$.

(b) Show that in the case when $v < \sqrt{Rg}$ and the coefficient of friction is $\mu = 1$, the pebble starts to slide when it has rotated through an angle given by $\theta = \cos^{-1}\left(\frac{v^2}{\sqrt{2}Rg}\right) - \frac{\pi}{4}$.

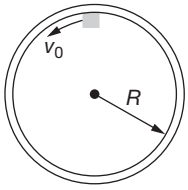
9. A turn of radius 20 m is banked for the vehicles going at a speed of 10 ms^{-1}. If the coefficient of static friction between the road and the tyre is 0.4, what are the possible speeds of a vehicle so that it neither slips down nor skids up? $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$

10. A small block of mass m is placed inside a hollow cone rotating about a vertical axis with angular velocity ω as shown in figure. The semi-vertex angle of the cone is θ and the coefficient of friction between the cone and the block is μ. If the block is to remain at a constant height b above the apex of the cone, as shown, what are the maximum and minimum values of ω?

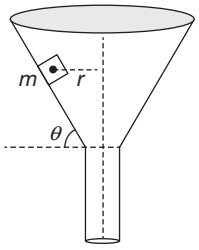


11. The width of the track of a car is 1.15 m and the centre of gravity is 0.6 m above the road and 0.075 m to the right of the centre line of the car. The road bends first to the right and then to the left, the radius of each curve being 40 m. If the coefficient of friction between the tyres and the road is 0.9 show that if the car were driven too fast, it would skid at the first bend and overturn at the second. Take $g = 9.8$ ms^{-2}.

12. A block of mass m slides on a frictionless table. It is constrained to move inside a ring of radius R. At time $t = 0$, block is moving along the inside of the ring (i.e., in the tangential direction) with velocity v_0. The coefficient of friction between the block and the ring is μ. Find the speed of the block at time t.



13. A very small cube of mass m is placed on the inside of a funnel rotating about a vertical axis at a constant rate of n revolutions per sec. The wall of the funnel makes an angle θ with the horizontal. If the coefficient of static friction between the cube and funnel is μ and the centre of the cube is at a distance r from the axis of rotation, what are the largest and smallest values of n for which the block will not move with respect to the funnel?



14. A bicyclist travels in a circle of radius 25 m at a constant speed of 9 ms^{-1}. The bicycle rider mass is 85 kg. Calculate the magnitude of (a) the force of friction on the bicycle from the road and (b) the net force on the bicycle from the road.

SOLVED PROBLEMS

PROBLEM 1

Two small beads of weight W_1 and W_2 each capable of sliding freely on a smooth circular ring fixed in the vertical plane are connected by a light string. Show that in the position of equilibrium in which the string be straight and inclined at an angle θ to the horizontal we have

$$(W_1+W_2)\tan\theta=(W_1-W_2)\tan\frac{\alpha}{2}$$

Here, α is the angle subtended by the string at the centre.

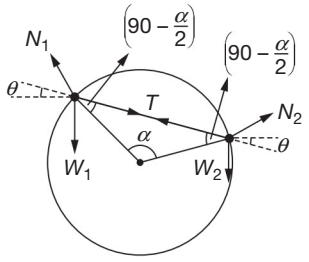
SOLUTION

Beads are in equilibrium under three concurrent forces. So applying Lami's theorem,

For Bead 1:

$$\frac{W_1}{\sin\left\{180-\left(90-\frac{\alpha}{2}\right)\right\}}=\frac{T}{\sin\left\{90+\theta+90-\frac{\alpha}{2}\right\}}$$

$$\Rightarrow \quad T=\frac{W_1\sin\left(\frac{\alpha}{2}-\theta\right)}{\cos\frac{\alpha}{2}} \qquad \ldots(1)$$



For Bead 2:

$$\frac{W_2}{\sin\left\{180-\left(90-\frac{\alpha}{2}\right)\right\}}=\frac{T}{\sin\left\{90+90-\frac{\alpha}{2}-\theta\right\}}$$

$$\Rightarrow \quad T=\frac{W_2\sin\left(\frac{\alpha}{2}+\theta\right)}{\cos\frac{\alpha}{2}} \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$W_1\sin\left(\frac{\alpha}{2}-\theta\right)=W_2\sin\left(\frac{\alpha}{2}+\theta\right)$$

Since $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$

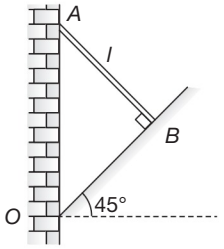
$$\Rightarrow \quad W_1\sin\left(\frac{\alpha}{2}\right)\cos\theta-W_1\sin\theta\cos\left(\frac{\alpha}{2}\right)=W_2\sin\left(\frac{\alpha}{2}\right)\cos\theta+W_2\sin\theta\cos\left(\frac{\alpha}{2}\right)$$

Dividing the equation by $\cos\theta\cos\left(\frac{\alpha}{2}\right)$, we get

$$(W_1+W_2)\tan\theta=(W_1-W_2)\tan\left(\frac{\alpha}{2}\right)$$

PROBLEM 2

At the bottom edge of a smooth vertical wall, an inclined plane is kept at an angle of $45°$. A uniform ladder of length l and mass M rests on the inclined plane against the wall such that it is perpendicular to the incline.



(a) If the plane is also smooth, which way will the ladder slide.

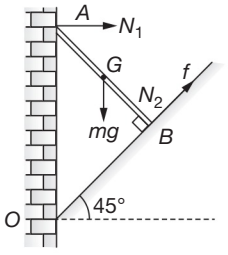
(b) What is the minimum coefficient of friction necessary so that the ladder does not slip on the incline.

SOLUTION

(a) N_2 and mg pass through G. N_1 has clockwise moment about G, so the ladder has a tendency to slip by rotating clockwise and the force of friction (f) at B is then up the plane.

(b) $\Sigma\tau_A=0$

$$\Rightarrow \quad fl=mg\left(\frac{l}{2}\sin(45°)\right) \qquad \ldots(1)$$



Also, for equilibrium $\Sigma F_V = 0$,

$$\Rightarrow \quad mg = N_2\cos(45°) + f\sin(45°) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$N_2 = \frac{3mg}{2\sqrt{2}} \text{ and } f = \frac{mg}{2\sqrt{2}}$$

Since, we have, by definition

$$\mu_{MIN} = \frac{f}{N_2}$$

$$\Rightarrow \quad \mu_{MIN} = \frac{f}{N_3} = \frac{\left(\frac{mg}{2\sqrt{2}}\right)}{\left(\frac{3mg}{2\sqrt{2}}\right)} = \frac{1}{3}$$

PROBLEM 3

A uniform ladder of length L and mass m_1 rests against a frictionless wall. The ladder makes an angle θ with the horizontal.

(a) Find the horizontal and vertical forces the ground exerts on the base of the ladder when a fire-fighter of mass m_2 is a distance x from the bottom.

(b) If the ladder is just on the verge of slipping when the fire-fighter is a distance d from the bottom, what is the coefficient of static friction between ladder and ground?

SOLUTION

(a) $\Sigma F_x = f - N_w = 0 \qquad \ldots(1)$

$\Sigma F_y = N_g - m_1 g - m_2 g = 0 \qquad \ldots(2)$

$$\Sigma \tau_A = 0$$

$$-m_1 g\left(\frac{L}{2}\right)\cos\theta - m_2 g x\cos\theta + N_w L\sin\theta = 0$$

From the torque equation,

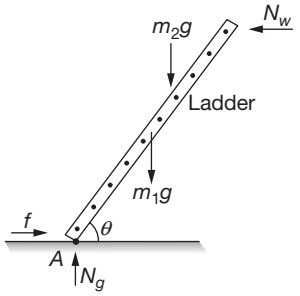
$$N_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$

Then, from equation (1), we get

$$f = N_w = \left(\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right)\cot\theta$$

and from equation (2), we get

$$N_g = (m_1 + m_2)g$$



(b) If the ladder is on the verge of slipping when $x = d$, then

$$\mu = \frac{f|_{x=d}}{N_g} = \frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L}\right)\cot\theta}{m_1 + m_2}$$

PROBLEM 4

A ladder of uniform density and mass m rests against a frictionless vertical wall, making an angle of $60°$ with the horizontal. The lower end rests on a flat surface where the coefficient of static friction is $\mu_s = \frac{1}{\sqrt{3}}$. A window cleaner with mass $M = 2m$ attempts to climb the ladder. What fraction of the length L of the ladder will the worker have reached when the ladder begins to slip?

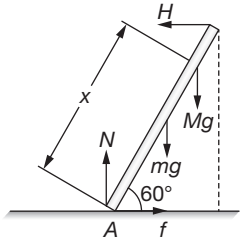
SOLUTION

$$N = (M+m)g \text{ and } H = f$$

So, $H_{max} = f_{max} = \mu_s(m+M)g$

$$\Sigma \tau_A = 0$$

$$\frac{mgL}{2}\cos 60° + Mgx\cos 60° - HL\sin 60° = 0$$



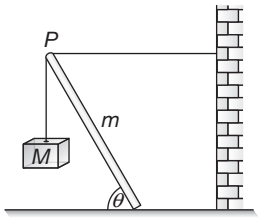
$$\Rightarrow \quad \frac{x}{L} = \frac{H\tan(60°)}{Mg} - \frac{m}{2M}$$

$$\Rightarrow \quad \frac{x}{L} = \frac{\mu_s(m+M)\tan(60°)}{M} - \frac{m}{2M}$$

$$\Rightarrow \quad \frac{x}{L} = \frac{3}{2}\mu_s\tan(60°) - \frac{1}{4} = 0.25$$

PROBLEM 5

A uniform beam of mass m is inclined at an angle θ to the horizontal. Its upper end produces a ninety degree bend in a very rough rope tied to a wall and its lower end rests on a rough floor.



(a) If the coefficient of static friction between beam and floor is μ_s, determine an expression for the maximum mass M that can be suspended from the top before the beam slips.

(b) Determine the magnitude of the reaction force at the floor and the magnitude of the force exerted by the beam on the rope at P in terms of m, M and μ_s.

SOLUTION

(a) For equilibrium, $\sum F_x = \sum F_y = 0$ and $\sum \tau = 0$ with pivot point at the contact on the floor.

Then $\sum F_x = T - \mu_s N = 0$ …(1)

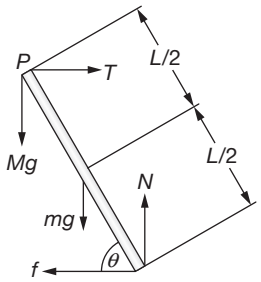
$\Sigma F_y = N - Mg - mg = 0$ …(2)

$\Sigma \tau = 0$

$$Mg(L\cos\theta) + mg\left(\frac{L}{2}\cos\theta\right) - T(L\sin\theta) = 0 \quad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$M = \frac{m}{2}\left(\frac{2\mu_s \sin\theta - \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$$



This answer is the maximum value for M if $\mu_s < \cot\theta$.

If $\mu_s \geq \cot\theta$, the mass M can increase without limit. It has no maximum value, and part (b) cannot be answered as stated either. In the case $\mu_s < \cot\theta$, we proceed.

(b) At the floor, we have the normal force in the y-direction and frictional force in the x-direction. The reaction force then is

$$R = \sqrt{N^2 + (\mu_s N)^2} = (M+m)g\sqrt{1+\mu_s^2}$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = g\sqrt{M^2 + \mu_s^2 (M+m)^2}$$

PROBLEM 6

At the moment $t = 0$, the force $F = kt$ is applied to a small body of mass m resting on a smooth horizontal plane (k is a constant). The direction of this force is always constant. Find the

(a) velocity of the body at the moment of its breaking off the plane.

(b) distance traversed by the body up to this moment.

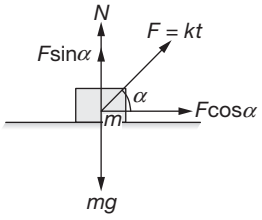
SOLUTION

(a) Along horizontal:

$$F\cos\alpha = kt\cos\alpha = m\frac{dv}{dt} \quad \ldots(1)$$

Along vertical:

$$N + kt\sin\alpha = mg$$



At break-off, we have

$$N = 0$$

$$\Rightarrow \quad t = t_0 = \frac{mg}{(k\sin\alpha)}$$

From (1), we get

$$m\,dv = kt\cos\alpha\,dt$$

Integrating both sides,

$$m\int_0^v dv = \int_0^t k\cos\alpha\, t\, dt$$

$$\Rightarrow \quad v(t) = \frac{kt^2\cos\alpha}{2m} \qquad \ldots(2)$$

At break-off, $t_0 = \dfrac{mg}{k\sin\alpha}$

$$\Rightarrow \quad v(t_0) = \frac{k\cos\alpha}{2m}\left(\frac{m^2g^2}{k^2\sin^2\alpha}\right)$$

$$\Rightarrow \quad v(t_0) = \left(\frac{mg^2\cos\alpha}{2k\sin^2\alpha}\right)$$

(b) From (2), we get

$$v(t) = \frac{dx}{dt} = \frac{kt^2\cos\alpha}{2m}$$

$$\Rightarrow \quad \int_0^x dx = \int_0^t \frac{k\cos\alpha}{2m} t^2 dt$$

$$\Rightarrow \quad x(t_0) = \frac{k\cos\alpha}{6m}\frac{m^3g^3}{k^3\sin^3\alpha}$$

$$\Rightarrow \quad x(t_0) = \frac{m^2g^3\cos\alpha}{6k^2\sin^3\alpha}$$

PROBLEM 7

A small disc A is placed on an inclined plane forming an angle α with the horizontal and is imparted an initial velocity v_0. Find how the velocity of the disc depends on the angle ϕ if the friction coefficient $\mu = \tan\alpha$ and at the initial moment $\phi_0 = \dfrac{\pi}{2}$.

SOLUTION

Let a be the acceleration of disc in the direction of its velocity (tangential acceleration). Then

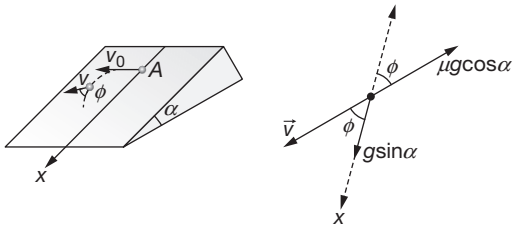
$$a = g\sin\alpha\cos\phi - \mu g\cos\alpha$$

$$\Rightarrow \quad a = g\sin\alpha(\cos\phi - 1) \qquad (\because\ \mu = \tan\alpha) \qquad \ldots(1)$$

Similarly, if a_x be the acceleration of disc along x-direction, then

$$a_x = g\sin\alpha - (\mu g\cos\alpha)\cos\phi$$

$$\Rightarrow \quad a_x = g\sin\alpha(1-\cos\phi) \qquad \ldots(2)$$



Adding equations (1) and (2), we get

$$a + a_x = 0$$

Integrating, we get

$$v + v_x = c$$

Here, c is a constant and $v_x = v\cos\phi$

$$\Rightarrow \quad v + v\cos\phi = c \qquad \ldots(3)$$

Since, at $\phi_0 = \dfrac{\pi}{2}$, we have $v = v_0$

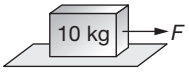
$$\Rightarrow \quad c = v_0$$

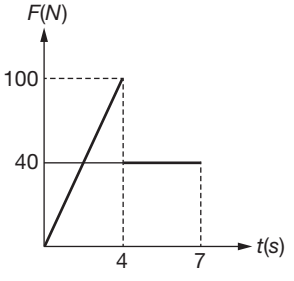
Substituting in equation (3), we get

$$v = \frac{v_0}{1+\cos\phi}$$

PROBLEM 8

The 10 kg block is resting on the horizontal surface when the force F is applied to it for 7 s. The variation of F with time is shown. Calculate the maximum velocity reached by the block and the total time t during which the block is in motion. The coefficients of static and kinetic friction are both 0.5. Take $g = 10\ \text{ms}^{-2}$.





SOLUTION

For $0 < t < 4$ s, the force applied on the block increases as a function of time and is given by

$$F_{app} = 25t \qquad \ldots(1)$$

The block will start moving when applied force equals the limiting value of friction. So, we get, at time t, (when the block starts moving)

$$F = \mu mg$$

$$\Rightarrow \quad 25t_1 = (0.5)(10)(10) = 50 \text{ N}$$

$$\Rightarrow \quad t_1 = 2 \text{ s}$$

Velocity is maximum at the end of 4 s

$$\Rightarrow \quad \frac{dv}{dt} = \frac{F - \mu mg}{m} = \frac{25t - 50}{10} = \frac{5}{2}t - 5$$

$$\Rightarrow \quad \int_0^{v_{\max}} dv = \int_2^4 \left(\frac{5}{2}t - 5\right)dt$$

$$\Rightarrow \quad v_{\max} = \left(\frac{5}{4}t^2 - 5t\right)\Big|_2^4 = \frac{5}{4}(4^2 - 2^2) - 5(4-2)$$

$$\Rightarrow \quad v_{\max} = 15 - 10 = 5 \text{ ms}^{-2}$$

For $4s < t < 7s$

Net retardation is

$$a_1 = \frac{50 - 40}{10} = 1 \text{ ms}^{-2}$$

$$\Rightarrow \quad v = v_{\max} - a_1 t_1 = 5 - (1)(3) = 2 \text{ ms}^{-1}$$

For $t > 7s$

Retardation $a_2 = \frac{50}{10} = 5 \text{ ms}^{-2}$

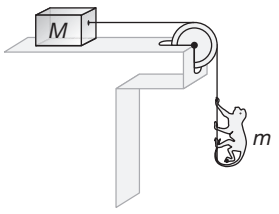
$$\Rightarrow \quad t = \frac{v}{a_2} = \frac{2}{5} = 0.4 \text{ s}$$

So, total time $T = (4-2) + (7-4) + (0.4)$

$$\Rightarrow \quad T = 5.4 \text{ s}$$

PROBLEM 9

A monkey of mass m clings to a rope slung over a fixed pulley. The opposite end of the rope is tied to a weight of mass M lying on a horizontal plate. The coefficient of friction between the weight and the plate is μ. Find the acceleration of weight and the tension of the rope for three cases.



(a) the monkey does not move with respect to the rope

(b) the monkey moves upwards with respect to the rope with an acceleration b.

(c) the monkey moves downwards with respect to the rope with an acceleration b.

SOLUTION

(a) Since monkey does not move w.r.t. rope so, the acceleration of block (or rope) and of monkey are equal. Hence,

$$T - \mu Mg = Ma \qquad \ldots(1)$$

$$mg - T = ma \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$a = \left(\frac{m - \mu M}{m + M}\right)g \text{ and } T = \frac{mMg(1+\mu)}{M+m}$$

(b) Monkey moves upwards w.r.t. rope with an acceleration b. Thus, its absolute acceleration downwards is $(a-b)$, where a is the acceleration of block (or rope). The equations of motion are,

$$T - \mu Mg = Ma \qquad \ldots(3)$$

$$mg - T = m(a-b) \qquad \ldots(4)$$

Solving equations (3) and (4), we get

$$a = \frac{m(g+b) - \mu Mg}{M+m}$$

and $$T = \frac{mM[g(1+\mu)+b]}{M+m}$$

(c) The absolute acceleration is $(a+b)$, so in this case equations of motion are

$$T - \mu Mg = Ma \qquad \ldots(5)$$

$$mg - T = m(a+b) \qquad \ldots(6)$$

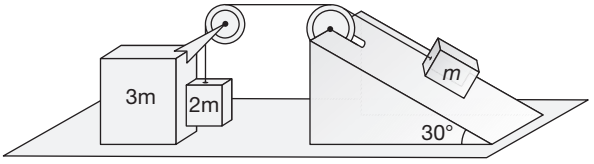
Solving equations (5) and (6), we get

$$a = \frac{m(g-b) - \mu Mg}{m+M}$$

and $$T = \frac{mM[g(1+\mu)-b]}{m+M}$$

PROBLEM 10

Find accelerations of m, $2m$ and $3m$ as shown in the figure. The wedge is fixed and friction is absent for all the contact surfaces.



SOLUTION

Drawing free body diagrams for $3m$, $2m$ and m and writing equations of motion, we get

$$T - N = 3ma_1 \quad \ldots(1)$$

$$N = 2ma_1 \quad \ldots(2)$$

$$2mg - T = 2ma_2 \quad \ldots(3)$$

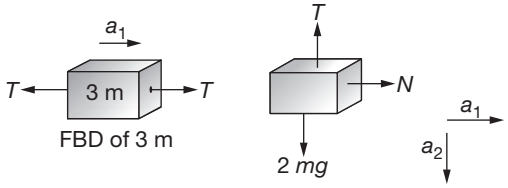
$$T - \frac{mg}{2} = ma_3 \quad \ldots(4)$$

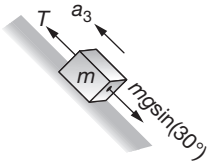
Also from constraint equation, we have

$$a_2 = a_1 + a_3 \quad \ldots(5)$$

Solving the above five equations we get

$$a_1 = \frac{3}{17}g,\ a_2 = \frac{19}{34}g \text{ and } a_3 = \frac{13}{34}g$$





FBD of m

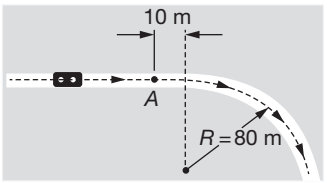
Acceleration of m is $a_3 = \frac{13}{34}g$

Acceleration of $2m$ is $\sqrt{a_1^2 + a_2^2} = \frac{\sqrt{397}}{34}g$ and

Acceleration of $3m$ is $a_1 = \frac{3}{17}g$

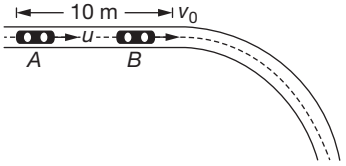
PROBLEM 11

Each tyre on the 1350 kg car can support a maximum friction force parallel to the road surface of 2500 N. This force limit is nearly constant over all possible rectilinear and curvilinear car motions and is attainable only if the car does not skid. Under this maximum braking, determine the total stopping distance s if the brakes are first applied at point A when the car speed is 25 ms^{-1} and if the car follows the center line of the road.



SOLUTION

Total force of friction available on four tyres is $f = 10{,}000$ N



So, $a_{\max} = \frac{10{,}000}{1350} = 7.4 \text{ ms}^{-2}$

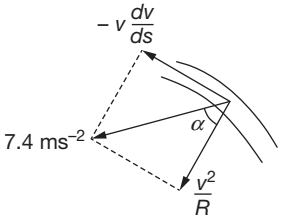
For straight track from A to B, we have

$$v_0^2 = (25)^2 - (2 \times 7.4 \times 10)$$

$$\Rightarrow \quad v_0 = 21.84 \text{ ms}^{-1}$$

For circular track, we have

$$a_{\max} \cos\alpha = \frac{v^2}{R}$$



$$\Rightarrow \quad \frac{v^2}{80} = 7.4\cos\alpha$$

$$\Rightarrow \quad \cos\alpha = \frac{v^2}{592}$$

Also, for circular path,

$$a_T = -a_{\max}\sin\alpha$$

$$\Rightarrow \quad v\frac{dv}{ds} = -7.4\sin\alpha$$

$$\Rightarrow \quad v\frac{dv}{ds} = -7.4\sqrt{1-\frac{v^4}{(592)^2}} \quad \left\{\because \sin\alpha = \sqrt{1-\cos^2\alpha}\right\}$$

$$\Rightarrow \quad -\int_{v_0}^{0} \frac{vdv}{7.4\sqrt{1-\frac{v^4}{(592)^2}}} = \int_0^s ds\,,$$

where $v_0 = 21.84\ \text{ms}^{-1}$

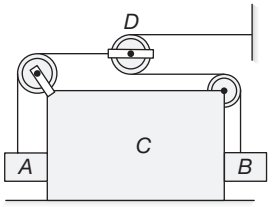
Solving this equation, we get

$s = 37.4$ m

Hence, total stopping distance is $10 + 37.4 = 47.4$ m

PROBLEM 12

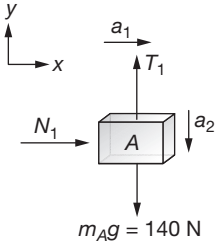
In the arrangement shown in figure, pulleys are light and smooth. Strings are light and inextensible. The masses of blocks A, B and C are 14 kg, 11 kg and 52 kg respectively. The block A can slide freely along a vertical rail fixed to left vertical face of block C. Assuming all the surfaces to be smooth, calculate magnitude of resultant acceleration of each of the blocks A, B and C. $\left(g = 10\ \text{ms}^{-2}\right)$.



SOLUTION

Let the block C have an acceleration a_1 in horizontal direction towards right. Then horizontal acceleration of A and B will be a_1.

Further, let the acceleration of block A, vertically downwards, be a_2 and the acceleration of block B, vertically upwards, be a_3.



Let, T_1 = tension in the string attached to A

T_2 = tension in the string attached to B

N_1 = force between A and C

and N_2 = normal reaction between B and C

Drawing free body diagram of A and then using Newtons Second Law, we get

$$\Sigma F_x = ma_x \text{ and } \Sigma F_y = ma_y$$

$$\Rightarrow \quad N_1 = 14a_1 \qquad \ldots(1)$$

$$\Rightarrow \quad 140 - T_1 = 14a_2 \qquad \ldots(2)$$

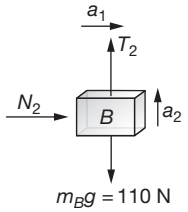
Drawing free body diagram of B and again applying Newton's Second Law, i.e., $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$, we get

$$N_2 = 11a_1 \qquad \ldots(3)$$

$$T_2 - 110 = 11a_3 \qquad \ldots(4)$$

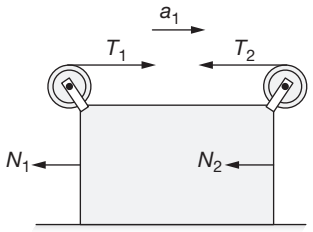
Since the pulley is light, so we get

$$2T_2 = T_1 \qquad \ldots(5)$$



Drawing the free body diagram of block C (showing only horizontal forces) and using $\Sigma F_x = ma_x$, we get

$$T_1 - T_2 - N_1 - N_2 = 52a_1 \qquad \ldots(6)$$



Further, by constraint relations, we can show that

$$a_3 = 2a_2 \qquad \ldots(7)$$

Now, we have seven unknowns, T_1, T_2, N_1, N_2, a_1, a_2 and a_3 to solve from seven equations. On solving, we get

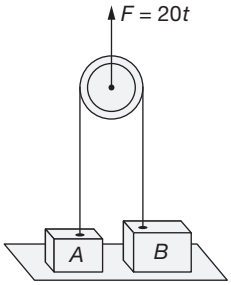
$$a_1 = 1.03\ \text{ms}^{-2},\ a_2 = -1.38\ \text{ms}^{-2},\ a_3 = -2.76\ \text{ms}^{-2}$$

Thus, acceleration of different blocks A, B and C will be as follows:

Block	Horizontal acceleration	Vertical acceleration	Resultant acceleration
A	$a_1 = 1.03\ \text{ms}^{-2}$	$1.38\ \text{ms}^{-2}\uparrow$	$1.72\ \text{ms}^{-2}$
B	$a_1 = 1.03\ \text{ms}^{-2}$	$2.76\ \text{ms}^{-2}\downarrow$	$2.95\ \text{ms}^{-2}$
C	$a_1 = 1.03\ \text{ms}^{-2}$	0	$1.03\ \text{ms}^{-2}$

PROBLEM 13

Two blocks A and B of mass 1 kg and 2 kg respectively are connected by a string, passing over a light frictionless pulley. Both the blocks are resting on a horizontal floor and the pulley is held such that string remains just taut. At the moment $t = 0$, a force $F = 20t$ newton starts acting on the pulley along vertically upward direction as shown in figure. Calculate



(a) velocity of A when B loses contact with the floor.

(b) height raised by the pulley upto that instant.

Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

(a) If T be the tension in the string, then

$$F = 2T = 20t$$

$$\Rightarrow \quad T = 10t \text{ newton}$$

Let the block A lose its contact with the floor at time $t = t_1$ (say). This happens when the tension in string becomes equal to the weight of block A. So,

$$T = mg$$

$$\Rightarrow \quad 10t_1 = 1 \times 10$$

$$\Rightarrow \quad t_1 = 1\ \text{s} \qquad \ldots(1)$$

Similarly, for block B, we have

$$10t_2 = 2 \times 10$$

$$\Rightarrow \quad t_2 = 2\ \text{s} \qquad \ldots(2)$$

i.e., the block B loses contact with the floor after $t_2 = 2$ s.

For block A, at time t such that $t \geq t_1$ let a be its acceleration in upward direction. Then

$$10t - (1)(10) = (1)(a) = \left(\frac{dv}{dt}\right)$$

$$\Rightarrow \quad dv = 10(t-1)dt \qquad \ldots(3)$$

Integrating, we get

$$\int_0^v dv = 10\int_1^t (t-1)dt$$

$$\Rightarrow \quad v = 5t^2 - 10t + 5 \qquad \ldots(4)$$

Substituting $t = t_2 = 2$ s, we get

$$v = 20 - 20 + 5 = 5\ \text{ms}^{-1} \qquad \ldots(5)$$

(b) If y is the vertical displacement of block A at time $t(\geq t_1)$, then from equation (4), we have

$$dy = (5t^2 - 10t + 5)dt \qquad \ldots(6)$$

Integrating, we get

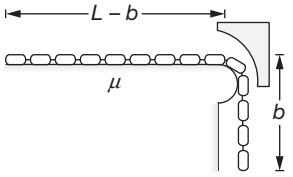
$$\int_{y=0}^{y=h} dy = \int_{t=1}^{t=2} (5t^2 - 10t + 5)dt$$

$$\Rightarrow \quad h = 5\left(\frac{t^3}{3}\right)\Big|_1^2 - 10\left(\frac{t^2}{2}\right)\Big|_1^2 + 5(t)\Big|_1^2 = \frac{5}{3}\ \text{m}$$

So, the height raised by pulley upto that instant is $\frac{h}{2} = \frac{5}{6}$ m

PROBLEM 14

The chain is released from rest with the length b of overhanging links just sufficient to initiate motion. The coefficients of static and kinetic friction between the links and the horizontal surface have essentially the same value μ. Determine the velocity v of the chain when the last link leaves the edge neglect any friction at the corner.



SOLUTION

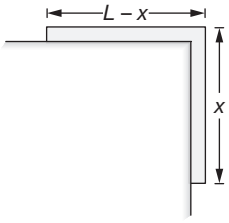
For equilibrium, we have

$$\begin{pmatrix}\text{Weight of the}\\ \text{hanging part}\end{pmatrix} = \begin{pmatrix}\text{Force of friction on the}\\ \text{part lying on the table}\end{pmatrix}$$

So, $m\left(\frac{b}{L}\right)g = \mu m\left(\frac{L-b}{L}\right)g$

$$\Rightarrow \quad \mu = \frac{b}{L-b} \qquad \ldots(1)$$

$$\Rightarrow \quad b = \left(\frac{\mu L}{1+\mu}\right)$$



Let the length of the chain hanging at any instant be x and if a is the acceleration of the chain, then we have

$$a = \frac{\left(m\frac{x}{L} - \mu m\frac{(L-x)}{L}\right)g}{m}$$

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{g}{L}\left(x - \mu(L-x)\right) = \frac{g}{L}\left((1+\mu)x - \mu L\right)$$

$$\Rightarrow \quad \int_0^v v\,dv = \frac{g}{L}\int_b^L \left((1+\mu)x - \mu L\right)dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{g}{L}\left((1+\mu)\frac{x^2}{2} - \mu Lx\right)\Bigg|_b^L$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{g}{L}\left((1+\mu)\frac{L^2}{2} - \mu L^2 - (1+\mu)\frac{b^2}{2} - \mu Lb\right)$$

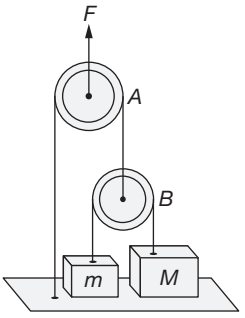
$$\Rightarrow \quad v^2 = \frac{g}{L}\left(L^2 + \mu L^2 - 2\mu L^2 - b^2 - \mu b^2 - 2\mu Lb\right)$$

Substituting the value of b from equation (1), we get

$$v = \sqrt{\frac{gL}{1+\mu}}$$

PROBLEM 15

Two blocks of mass $m = 5$ kg and $M = 10$ kg are connected by a string passing over a pulley B as shown. Another string connects the centre of pulley B to the floor and passes over another pulley A as shown. An upward force F is applied at the centre of pulley A. Both the pulleys are massless.



Find the acceleration of blocks m and M if F is:

(a) 100 N

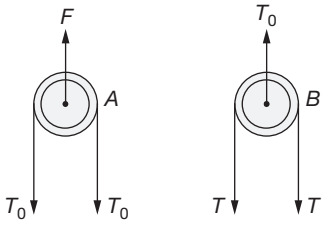
(b) 300 N

(c) 500 N $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$

SOLUTION

Let T_0 = tension in the string passing over A and

T = tension in the string passing over B



From free body diagram of pulleys A and B, we get

$$2T_0 = F \text{ and } 2T = T_0$$

$$\Rightarrow \quad T = \frac{F}{4}$$

(a) $T = \frac{F}{4} = 25$ N

The weights of blocks are

$$mg = 50 \text{ N} \text{ and } Mg = 100 \text{ N}$$

As T happens to be less than mg and Mg both so, the blocks will remain stationary on the floor.

(b) $T = \frac{F}{4} = 75$ N

As $T < Mg$ and $T > mg$, M will remain stationary on the floor, whereas m will move.

Acceleration of m is given by

$$a = \frac{T - mg}{m} = \frac{75 - 50}{5} = 5 \text{ ms}^{-2}$$

(c) $T = \frac{F}{4} = 125$ N

As T now happens to be greater than mg and Mg so, both the blocks will accelerate upwards.

Acceleration of m is given by

$$a_1 = \frac{T - mg}{m} = \frac{125 - 50}{5} = 15 \text{ ms}^{-2}$$

Acceleration of M is given by

$$a_2 = \frac{T - Mg}{M} = \frac{125 - 100}{10} = 2.5 \text{ ms}^{-2}$$

PROBLEM 16

A body of mass m is thrown straight up with velocity v_0. Find the velocity v' with which the body comes down if the air drag equals kv^2, where k is a constant and v is the velocity of the body.

SOLUTION

When the body is projected upwards

Net acceleration $a_1 = -\left(g + \frac{kv^2}{m}\right)$

$$\Rightarrow \quad v\frac{dv}{ds} = -\left(g + \frac{kv^2}{m}\right)$$

$$\Rightarrow \quad \left(\frac{vdv}{g + \frac{kv^2}{m}}\right) = -ds$$

Integrating this expression, we get

$$\int_{v_0}^{0} \frac{vdv}{\left(g + \frac{kv^2}{m}\right)} = -\int_0^h ds \qquad \ldots(1)$$

where h is the maximum height attained by the body.

Integrating, we get

$$\Rightarrow \quad h = \frac{m}{2k}\ln\left(1 + \frac{kv_0^2}{mg}\right) \qquad \ldots(2)$$

When the body falls downwards

In this case, net acceleration $a_2 = g - \frac{kv^2}{m}$

$$\Rightarrow \quad \int_0^{v'} \frac{vdv}{\left(g - \frac{kv^2}{m}\right)} = \int_0^h ds$$

Integrating, we get

$$h = -\frac{m}{2k}\ln\left(1 - \frac{kv'^2}{mg}\right) \qquad \ldots(3)$$

Equating equations (2) and (3), we get

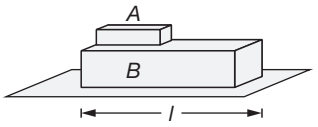
$$v' = \frac{v_0}{\sqrt{1 + \left(\frac{kv_0^2}{mg}\right)}}$$

PROBLEM 17

Figure shows a small block A of mass m kept at the left end of a plank B of mass $M = 2m$ and length l. The system can slide on a horizontal road. The system is started towards right with the initial velocity v. The friction coefficients between the road and the plank is $\frac{1}{2}$ and that between the plank and the block is $\frac{1}{4}$. Find

(a) the time elapsed before the block separates from the plank.

(b) displacement of block and plank relative to ground till that moment.



SOLUTION

There will be relative motion between block and plank and plank and road. So at each surface limiting friction will act. The direction of friction forces at different surfaces are as shown in figure.

Here, $f_1 = \left(\frac{1}{4}\right)(mg)$

and $f_2 = \left(\frac{1}{2}\right)(m+2m)g = \left(\frac{3}{2}\right)mg$

Retardation of A is

$$a_1 = \frac{f_1}{m} = \frac{g}{4}$$

and retardation of B is

$$a_2 = \frac{f_2 - f_1}{2m} = \frac{5}{8}g$$

Since, $a_2 > a_1$

Relative acceleration of A with respect to B is

$$a_r = a_2 - a_1 = \frac{3}{8}g$$

Initial velocity of both A and B is v. So there is no relative initial velocity. Hence,

(a) Applying $s = \frac{1}{2}at^2$

$$\Rightarrow \quad l = \frac{1}{2}a_r t^2 = \frac{3}{16}gt^2$$

$$\Rightarrow \quad t = 4\sqrt{\frac{l}{3g}}$$

(b) Displacement of block is given by

$$s_A = u_A t - \frac{1}{2}a_A t^2 = 4v\sqrt{\frac{l}{3g}} - \frac{1}{2}\left(\frac{g}{4}\right)\left(\frac{16l}{3g}\right) \qquad \left\{a_A = a_1 = \frac{g}{4}\right\}$$

$$\Rightarrow \quad s_A = 4v\sqrt{\frac{l}{3g}} - \frac{2}{3}l$$

Displacement of plank is given by

$$s_B = u_B t - \frac{1}{2}a_B t^2 = 4v\sqrt{\frac{l}{3g}} - \frac{1}{2}\left(\frac{5}{8}g\right)\left(\frac{16l}{3g}\right) \qquad \left\{a_B = a_2 = \frac{5}{8}g\right\}$$

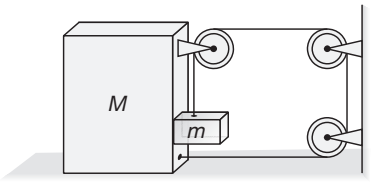
$$\Rightarrow \quad s_B = 4v\sqrt{\frac{l}{3g}} - \frac{5}{3}l$$

Here we observe that $s_A - s_B = l$.

This is a quite obvious result because the block A has moved a distance l relative to plank.

PROBLEM 18

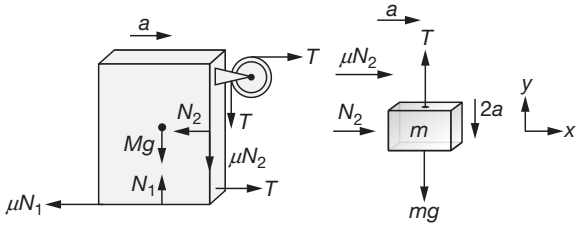
In the arrangement shown, $M = 2m$ and the coefficient of friction between all surfaces in contact is μ. Assuming the string to be light and inextensible and all the pulleys to be smooth find the acceleration of the bigger block M.



SOLUTION

From our knowledge of constraint relations we get that acceleration of block of mass m in vertical direction is two times the acceleration of block of mass M and in the horizontal direction, the acceleration of both blocks is equal.

So let a be the acceleration of block M in horizontal direction (towards right). Then acceleration of block m will be a in horizontal direction and $2a$ in vertical direction. Free body diagrams of both the blocks are shown in figure:



Let T be the tension in the string, N_1 be the normal reaction between bigger block and ground and N_2 be the normal reaction between both the blocks.

Using $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$,

for block of mass $M(=2m)$, we get

$$N_1 - \mu N_2 - 2mg - T = 0$$

$$\Rightarrow \quad N_1 = T + 2mg + \mu N_2 \qquad \ldots(1)$$

and $\quad T + T - N_2 - \mu N_1 = 2ma$

$$\Rightarrow \quad 2T - N_2 - \mu N_1 = 2ma \qquad \ldots(2)$$

for block of mass m, we get

$$N_2 = ma \quad \ldots(3)$$

and $$mg - T - \mu N_2 = 2ma \quad \ldots(4)$$

Solving these four equations for the four unknowns i.e., T, N_1, N_2 and a, we get

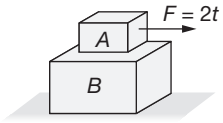
$$a = \left(\frac{2-3\mu}{7}\right)g$$

Hence, acceleration of the bigger block of mass M is

$$a = \left(\frac{2-3\mu}{7}\right)g.$$

PROBLEM 19

Two blocks A and B of mass 2 kg and 4 kg are placed one over the other as shown in figure. A time varying horizontal force $F = 2t$ is applied on the upper block as shown in figure. Here t is in second and F is in newton. Draw a graph showing accelerations of A and B on y-axis and time on x-axis. Coefficient of friction between A and B is $\mu = \frac{1}{2}$ and the horizontal surface over which B is placed is smooth. $\left(g = 10 \text{ ms}^{-2}\right)$.



SOLUTION

Limiting friction between A and B is

$$f_L = \mu m_A g = \left(\frac{1}{2}\right)(2)(10) = 10 \text{ N}$$

Block B moves due to friction only. Therefore, maximum acceleration of B can be

$$a_{\max} = \frac{f_L}{m_B} = \frac{10}{4} = 2.5 \text{ ms}^{-2}$$

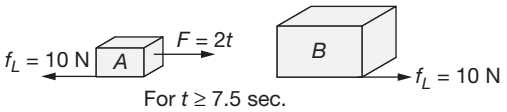
Thus, both the blocks move together with same acceleration till the common acceleration becomes 2.5 ms^{-2}, after that acceleration of B will become constant while that of A will go on increasing. To find the time when the acceleration of both the blocks becomes 2.5 ms^{-2} (or when slipping will start between A and B) we will write:

$$2.5 = \frac{F}{(m_A + m_B)} = \frac{2t}{6}$$

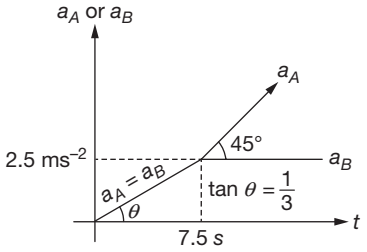
$$\Rightarrow \quad t = 7.5 \text{ s}$$

Hence, for $t \le 7.5$ s

$$a_A = a_B = \frac{F}{m_A + m_B} = \frac{2t}{6} = \frac{t}{3}$$



Thus, a_A versus t or a_B versus t graph is a straight line passing through origin of slope $\frac{1}{3}$.



For, $t \ge 7.5$ s

$$a_B = 2.5 \text{ ms}^{-2} = \text{constant}$$

and $$a_A = \frac{F - f_L}{m_A}$$

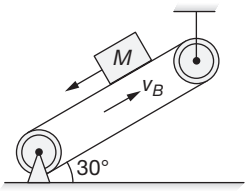
$$\Rightarrow \quad a_A = \frac{2t - 10}{2}$$

$$\Rightarrow \quad a_A = t - 5$$

Thus, a_A versus t graph is a straight line of slope 1 and intercept -5. While a_B versus t graph is a straight line parallel to t axis. The corresponding graph is as shown in figure.

PROBLEM 20

A conveyor belt is inclined at $30°$ with the horizontal. A block of mass $M = 1$ kg is kept on the belt as shown in figure. The frictional force, in newton, between the block and belt depends upon the relative speed of the body with respect to belt as $f = 0.4v_{\text{rel}}$. The belt moves up at a constant speed vB while initially the block has a speed $v_M = 2 \text{ ms}^{-1}$ relative to ground in a direction down the belt.



(a) Find the speed of the belt in order for the block to come permanently at rest relative to ground.

(b) Using the speed of belt calculated in part (a) find the time when block M attains a speed $1\ \text{ms}^{-1}$ relative to ground $\left(g = 10\ \text{ms}^{-1}\right)$.

SOLUTION

(a) In equilibrium (in this case, permanently at rest), the net force on block should be zero.

Hence, $Mg\sin\theta = f$

$$\Rightarrow \quad (1)(10)\left(\frac{1}{2}\right) = 0.4\left(v_B + 0\right)$$

$$\Rightarrow \quad v_B = 12.5\ \text{ms}^{-1}$$

(b) Let at time t velocity of block be $v_M\ \text{ms}^{-1}$

$$\Rightarrow \quad M\frac{dv_M}{dt} = Mg\sin\theta - 0.4\left(v_B + v_M\right)$$

$$\Rightarrow \quad \frac{dv_M}{dt} = 5 - 0.4\left(v_M + 12.5\right) = -0.4\,v_M$$

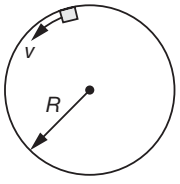
$$\Rightarrow \quad \int_2^1 \frac{dv_M}{v_M} = -0.4\int_0^t dt$$

$$\Rightarrow \quad 0.4t = \ln(2)$$

$$\Rightarrow \quad t = 2.5\ln(2) = 1.73\ \text{s}$$

PROBLEM 21

The small collar of mass m is given an initial velocity of magnitude v_0 on the horizontal circular track fabricated from a slender rod. If the coefficient of kinetic friction is μ_k, determine the distance travelled before the collar comes to rest.



SOLUTION

Net normal force acting on the collar is

$$N = m\sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$$

Since the frictional force depends upon the net normal force, so we have

$$f = \mu_K N = m\mu_K\sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$$

Now, $a_t = -\frac{f}{m} = -\mu_K\sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$

$$\Rightarrow \quad v\frac{dv}{ds} = -\mu_K\sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$$

$$\Rightarrow \quad -\int_{v_0}^{0}\frac{vdv}{\sqrt{v^4 + r^2g^2}} = \frac{\mu_k}{r}\int_0^s ds \qquad \ldots(1)$$

Substitute $v^2 = y$

$$\Rightarrow \quad 2vdv = dy$$

$$\Rightarrow \quad vdv = \frac{dy}{2}$$

$$\Rightarrow \quad \text{Integral } I = \frac{1}{2}\int\frac{dy}{\sqrt{y^2 + r^2g^2}}$$

Since $\int\frac{dx}{\sqrt{a^2 + x^2}} = \log_e\left(x + \sqrt{x^2 + a^2}\right)$

$$\Rightarrow \quad I = \frac{1}{2}\log_e\left(y + \sqrt{y^2 + r^2g^2}\right)$$

$$\Rightarrow \quad I = \frac{1}{2}\log_e\left(v^2 + \sqrt{v^4 + r^2g^2}\right) \qquad \left\{\because\ y = v^2\right\}$$

Substituting in (1), we get

$$-\frac{1}{2}\log\left(v^2 + \sqrt{v^4 + r^2g^2}\right)\Big|_{v_0}^{0} = \frac{\mu_k s}{r}$$

$$\Rightarrow \quad -\frac{1}{2}\left[\log_e(rg) - \log_e\left(v_0^2 + \sqrt{v_0^4 + r^2g^2}\right)\right] = \frac{\mu_k s}{r}$$

$$\Rightarrow \quad s = \frac{r}{2\mu_k}\log_e\left(\frac{v_0^2 + \sqrt{v_0^4 + r^2g^2}}{rg}\right)$$

PRACTICE EXERCISES

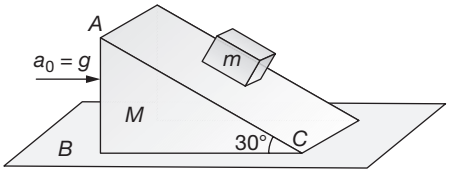
SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A body of mass 2 kg is moving towards east with a uniform speed of $2\ \text{ms}^{-1}$. A force of 3 N is applied to it towards north. The magnitude of the displacement of the body 2 s after the application of force is

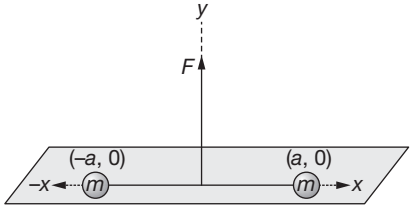
(A) 4 m (B) 5 m
(C) 6 m (D) 7 m

2. A block of mass m, is placed on an inclined plane of mass M moving towards right horizontally with an acceleration $a_0 = g$. The length of the plane $AC = 1$ m. Friction is absent everywhere. The time taken by the block to reach C starting from A is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 0.42 s (B) 0.74 s
(C) 1.2 s (D) 2.56 s

3. Two masses each equal to m are lying on x-axis at $(-a, 0)$ and $(+a, 0)$, respectively, as shown in figure. They are connected by a light string. A force F is applied at the origin along vertical direction. As a result the masses move towards each other without loosing contact with ground. The acceleration of each mass at the instant when the masses are at $(-x, 0)$ and $(x, 0)$ respectively is



(A) $\dfrac{2F}{m}\dfrac{\sqrt{a^2-x^2}}{x}$ (B) $\dfrac{2F}{m}\dfrac{x}{\sqrt{a^2-x^2}}$
(C) $\dfrac{F}{2m}\dfrac{x}{\sqrt{a^2-x^2}}$ (D) $\dfrac{F}{m}\dfrac{x}{\sqrt{a^2-x^2}}$

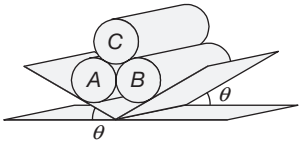
4. A particle of mass M is attached to the lower end of a vertical rope of mass m. An upward force P acts on the upper end of the rope. The system is free to move. The force exerted by the rope on the block is $\dfrac{PM}{M+m}$

(A) only if the rope is uniform
(B) in gravity-free space only
(C) only if $P > (M+m)g$
(D) in all cases

5. A log of weight W is pulled at a constant velocity and with a force F by means of a rope of length L. The distance between the free end of the rope and the ground is h. Neglecting the thickness of the log the coefficient of friction between the log and the ground is

(A) $\mu = \dfrac{F\sqrt{L^2-h^2}}{WL-Fh}$ (B) $\mu = \dfrac{\sqrt{L^2-h^2}}{WL-Fh}$
(C) $\mu = \dfrac{WL-Fh}{F\sqrt{L^2-h^2}}$ (D) $\mu = \dfrac{WL-Fh}{\sqrt{L^2-h^2}}$

6. Three identical rigid circular cylinders A, B and C are arranged on smooth inclined surfaces as shown in figure. The least value of θ that prevents the arrangement from collapsing is

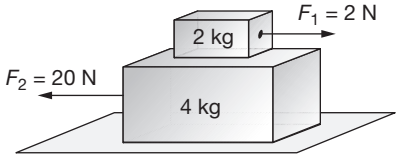


(A) $\tan^{-1}\left(\dfrac{1}{2}\right)$ (B) $\tan^{-1}\left(\dfrac{1}{2\sqrt{3}}\right)$
(C) $\tan^{-1}\left(\dfrac{1}{3\sqrt{3}}\right)$ (D) $\tan^{-1}\left(\dfrac{1}{4\sqrt{3}}\right)$

7. The greatest height h of the sand pile that can be erected without spilling the sand onto the surrounding circular area of radius R (if μ is the coefficient of friction between sand particles) is

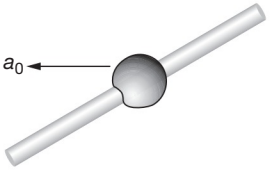
(A) R (B) $\dfrac{R}{\mu}$
(C) μR (D) $\mu^2 R$

8. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = \dfrac{1}{2}$. The force of friction acting between the two blocks is



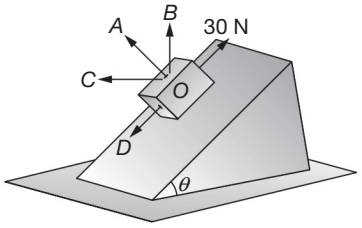
(A) 4 N (B) 6 N
(C) 8 N (D) 10 N

9. A bead of mass m is fitted on a rod and can move on it without friction. Initially the bead is at the middle of the rod and the rod moves translationally in a horizontal plane with an acceleration a_0 in a direction forming an α with the rod. The acceleration of bead with respect to rod is



(A) $g\sin\alpha$ (B) $(g+a_0)\sin\alpha$
(C) $g\sin\alpha + a_0\cos\alpha$ (D) $g\sin\alpha - a_0\cos\alpha$

10. A block of mass 10 kg lies on a rough inclined plane of inclination $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ with the horizontal when a force of 30 N is applied on the block parallel to and upward the plane, the total force exerted by the plane on the block is nearly along $\left(\text{coefficient of friction is } \mu = \frac{3}{4}\right)$ $\left(g = 10\ \text{ms}^{-2}\right)$



(A) OA (B) OB
(C) OC (D) OD

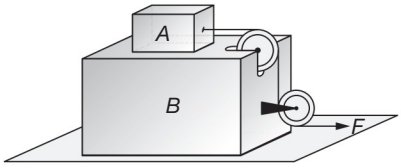
11. In **PROBLEM 10**, the acceleration of the block is

(A) 3 ms^{-2} up the plane
(B) 3 ms^{-2} down the plane
(C) 6 ms^{-2} up the plane
(D) zero

12. A ball of mass m is thrown upward with a velocity v. If air exerts an average resisting force F, the velocity with which the ball returns to the thrower is

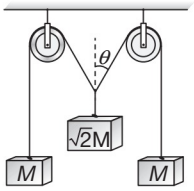
(A) $v\sqrt{\frac{mg}{mg+F}}$ (B) $v\sqrt{\frac{F}{mg+F}}$
(C) $v\sqrt{\frac{mg-F}{mg+F}}$ (D) $v\sqrt{\frac{mg+F}{mg-F}}$

13. In the arrangement shown in figure $m_A = m_B = 2$ kg. String is massless and pulley is frictionless. Block B is resting on a smooth horizontal surface, while friction coefficient between blocks A and B is $\mu = 0.5$. The maximum horizontal force F that can be applied so that block A does not slip over the block B is $\left(\text{take } g = 10\ \text{ms}^{-2}\right)$.



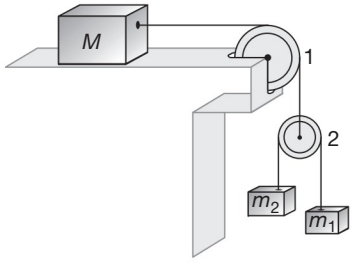
(A) 25 N (B) 40 N
(C) 30 N (D) 20 N

14. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be



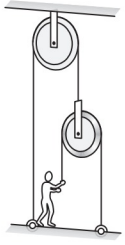
(A) 0° (B) 30°
(C) 45° (D) 60°

15. In the arrangement shown, $m_1 = 1$ kg, $m_2 = 2$ kg, the pulleys and strings are ideal and all surfaces in contact are frictionless. The value of M for which the mass m_1 moves with constant velocity is



(A) 6 kg (B) 4 kg
(C) 8 kg (D) 10 kg

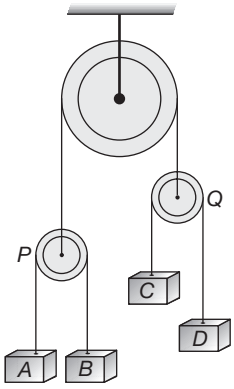
16. In the figure shown the force with which the man should pull the rope to hold the plank in position is F. If weight of the man is 60 kgf, the plank and pulleys have negligible masses, then



(A) $F = 150$ N (B) $F = 300$ N

(C) $F = 600$ N (D) $F = 1200$ N

17. In the arrangement shown, acceleration of A is 1 ms^{-2} upwards, acceleration of B is 7 ms^{-2} upwards and acceleration of C is 2 ms^{-2} upwards. Then acceleration of D has to be



(A) 7 ms^{-2}, downwards (B) 2 ms^{-2}, downwards

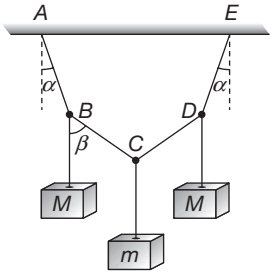
(C) 10 ms^{-2}, downwards (D) 8 ms^{-2}, downwards

18. A particle of mass m is joined to a very heavy mass M by a light string passing over a light and frictionless pulley. Both the bodies are then set free. The total thrust on the pulley is

(A) mg (B) $2mg$

(C) $4mg$ (D) much greater than mg

19. The figure represents a light inextensible string $ABCDE$ in which $AB = BC = CD = DE$ and to which are attached masses M, m and M at the points B, C and D, respectively. The system hangs freely in equilibrium with ends A and E of the string fixed in the same horizontal line shown in figure. It is given that $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{12}{5}$. Then the tension in the string BC is

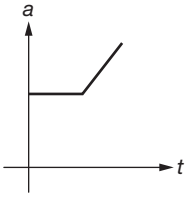


(A) $2mg$ (B) $\frac{13}{10}mg$

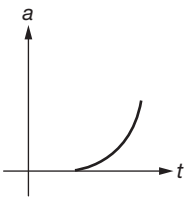
(C) $\frac{3}{10}mg$ (D) $\frac{20}{11}mg$

20. A block is placed on a rough horizontal plane. A time dependent horizontal force $F = Kt$ acts on the block, where K is a positive constant. The acceleration-time graph of the block is

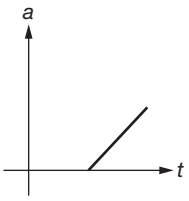
(A)



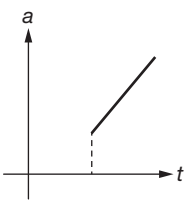
(B)



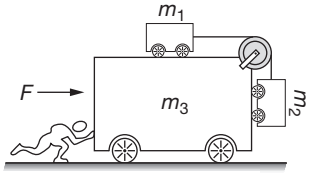
(C)



(D)



21. A mass m_1, placed on top of a trolley of mass m_3, is connected to another mass m_2 by means of a string passing over a smooth pulley as shown in figure. The friction between surfaces is negligible. For m_1 and m_2 not to move with respect to trolley, the horizontal force F to be applied on trolley is



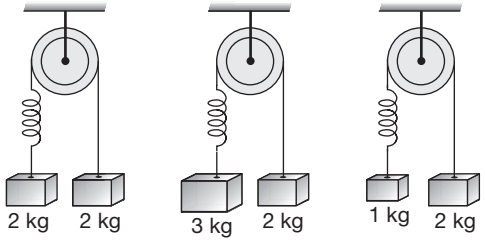
(A) $F = m_3 g$

(B) $F = (m_1 + m_2)g$

(C) $F = (m_1 + m_2 + m_3)\frac{m_2 g}{m_1}$

(D) $F = m_1 g$

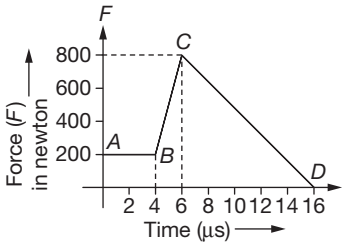
22. Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown in figure. If x_1, x_2 and x_3 be the extensions in the spring in these three cases then



(A) $x_1 = 0$, $x_3 > x_2$ (B) $x_2 > x_1 > x_3$

(C) $x_3 > x_1 > x_2$ (D) $x_1 > x_2 > x_3$

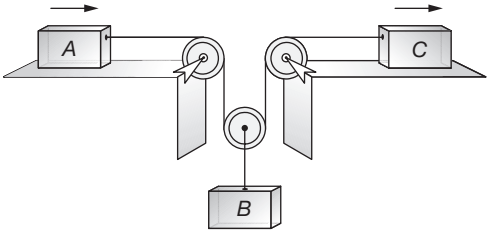
23. The magnitude of the force (in N) acting on a body varies with time t (in μs) as shown in figure. AB, BC and CD are straight line segments. The magnitude of the total impulse of the force on the body from $t = 4\ \mu s$ to $t = 16\ \mu s$ is



(A) 5×10^{-3} Ns (B) 5×10^{-4} Ns

(C) 5×10^{-2} Ns (D) 5×10^{-1} Ns

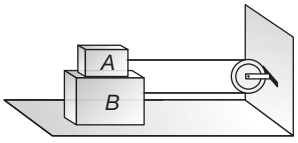
24. Blocks A and C start from rest and move to the right with acceleration $a_A = 12t\ \text{ms}^{-2}$ and $a_C = 3\ \text{ms}^{-2}$. Here t is in second. The time when block B again comes to rest is



(A) 2 s (B) 1 s

(C) $\frac{3}{2}$ s (D) $\frac{1}{2}$ s

25. Two blocks A and B of masses m and M respectively are placed on each other and their combination rests on a fixed horizontal surface C. A light string passing over the smooth light pulley is used to connect A and B as shown. The coefficient of sliding friction between all surfaces in contact is μ. If A is dragged with a force F then for both A and B to move with a uniform speed we have



(A) $F = \mu(M+m)g$ (B) $F = \mu mg$

(C) $F = \mu(3M+m)g$ (D) $F = \mu(3m+M)g$

26. In a horizontal circle of radius R a particle P is rotating with a constant speed v. Another particle Q is moving along the diameter of the circle with a constant acceleration $a = \frac{v^2}{R}$. Net force on P as observed by Q is

(A) constant both in magnitude and direction
(B) constant in magnitude only
(C) zero at least once during one rotation
(D) None of these

27. Sixteen balls of equal masses are connected like beads on a string. Some balls are placed on a smooth inclined plane of inclination $\sin^{-1}\left(\frac{1}{3}\right)$ and the remaining balls hang over the top of the plane. The number of balls hanging so as to produce an acceleration of $\frac{g}{2}$ is

(A) 8 (B) 9

(C) 10 (D) 11

28. A block is kept on a smooth inclined plane of angle of inclination 30° that moves with a constant acceleration so that the block does not slide relative to the inclined plane. Let F be the contact force between the block and the plane. Now the inclined plane stops and let f be the contact force between the two in this case. Then $\frac{F}{f}$ is

(A) 1 (B) $\frac{4}{3}$

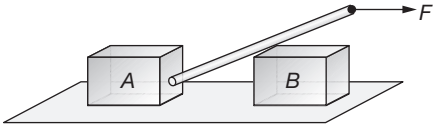
(C) 2 (D) $\frac{3}{2}$

29. A box is put on a scale which is adjusted to read zero when the box is empty. A stream of pebbles is then poured into the box from a height h above its bottom at a rate of n pebbles per second. Each pebble has a mass m. If the pebbles collide with the box such that they immediately come to rest after collision. Then the scale reading at time t after the pebbles begin to fill the box is

(A) $mngt$ (B) $mn\sqrt{2gh}$

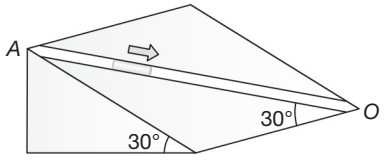
(C) mg (D) $mn\left(\sqrt{2gh}+gt\right)$

30. A horizontal force F is applied on a very light rod inserted between the two identical blocks A and B placed over a rough surface as shown in figure. If the force F is gradually increased, then which of the blocks will move first?



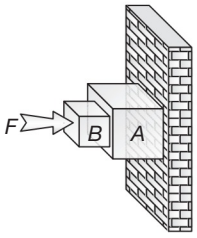
(A) A
(B) B
(C) both move simultaneously
(D) This depends on the friction coefficient between the blocks and the ground

31. An inclined plane makes an angle 30° with the horizontal. A groove $OA = 5$ m cut in the plane makes an angle 30° with OX. A short smooth body is free to slide down the influence of gravity. The time taken by the body to reach from A to O is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 4 s (B) 2 s
(C) $2\sqrt{2}$ s (D) 1 s

32. In the arrangement shown, the wall is smooth, but the surfaces of blocks A and B in contact are rough. In the state of equilibrium, the force of friction on B due to A is



(A) zero
(B) upwards
(C) downwards
(D) such that the system cannot stay in equilibrium

33. An elevator accelerates upward at a constant rate. A uniform string of length L and mass m supports a small block of mass M that hangs from the ceiling of an elevator which accelerates upwards at a constant rate. The tension at distance l from the ceiling is T. The acceleration of the elevator is

(A) $\dfrac{T}{\left(2M + m - \dfrac{ml}{L}\right)} + g$ (B) $\dfrac{T}{\left(2M - m + \dfrac{ml}{L}\right)} - g$

(C) $\dfrac{T}{\left(M + m - \dfrac{ml}{L}\right)} - g$ (D) $\dfrac{T}{\left(M + \dfrac{ml}{L}\right)} - g$

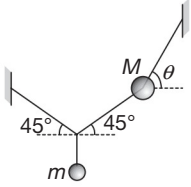
34. A plumb bob is hung from the ceiling of a train compartment. The train moves on an inclined track of inclination 30° with horizontal. Acceleration of train up the plane is $a_0 = \dfrac{g}{2}$. The angle which the string supporting the bob makes with normal to the ceiling in equilibrium is

(A) 30° (B) $\tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

(C) $\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ (D) $\tan^{-1}(2)$

35. A solid sphere will be in stable equilibrium when its centre of gravity lies
(A) vertically above its centre
(B) vertically below its centre
(C) on the horizontal line through its centre
(D) none of the above

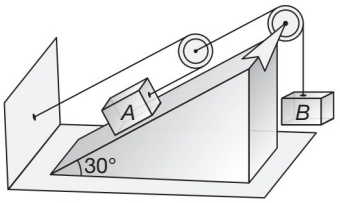
36. Two masses m and M are attached with strings as shown. For the system to be in equilibrium we have



(A) $\tan\theta = 1 + \dfrac{2M}{m}$ (B) $\tan\theta = 1 + \dfrac{2m}{M}$

(C) $\tan\theta = 1 + \dfrac{M}{2m}$ (D) $\tan\theta = 1 + \dfrac{m}{2M}$

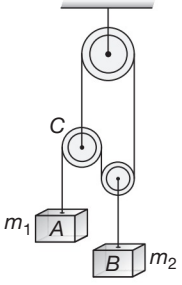
37. In the system shown in figure $m_B = 4$ kg, and $m_A = 2$ kg. The pulleys are massless and friction is absent everywhere. The acceleration of block A is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) $\dfrac{10}{3}\ \text{ms}^{-2}$ (B) $\dfrac{20}{3}\ \text{ms}^{-2}$

(C) $2\ \text{ms}^{-2}$ (D) $4\ \text{ms}^{-2}$

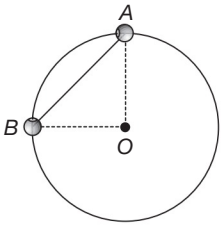
38. In the arrangement shown the pulleys and the strings are ideal. The respective accelerations of the blocks A and B are



(A) $g, \dfrac{g}{2}$ (B) $\dfrac{g}{2}, g$

(C) $\dfrac{3g}{2}, \dfrac{3g}{4}$ (D) g, g

39. Beads A and B, each of mass m, are connected by a light inextensible cord. They are constrained (restricted) to move on a frictionless ring in a vertical plane as shown. The beads are released from rest at the positions shown. The tension in the cord just after the release is



(A) $\sqrt{2}mg$ (B) mg

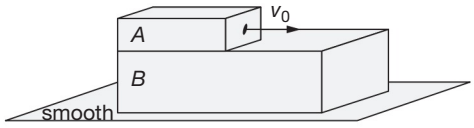
(C) $\frac{mg}{\sqrt{2}}$ (D) $2mg$

40. Two unequal masses are connected by a light inextensible string which passes over a frictionless light pulley. After the system has moved from rest for 3 s, the string is cut. If the lighter mass ascends $\frac{49}{90}$ m before it begins to descend and the heavier mass is 5 kg then the lighter mass is

(A) 4 kg (B) 3 kg

(C) 10 kg (D) 12 kg

41. A block A of mass m is placed over a plank B of mass $2m$. Plank B is placed over a smooth horizontal surface. The coefficient of friction between A and B is $\frac{1}{2}$. Block A is given a velocity v_0 towards right. Acceleration of B relative to A is



(A) $\frac{g}{2}$ (B) g

(C) $\frac{3g}{4}$ (D) zero

42. An automobile enters a turn whose radius is R. The road is banked at angle θ. Friction is negligible between wheels of the automobile and road. Mass of the automobile is m and speed is v. Select the correct alternative

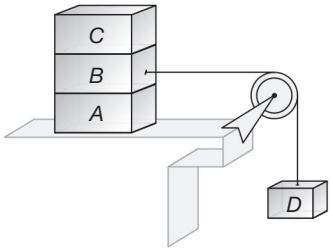
(A) net force on the automobile is zero

(B) normal reaction on the automobile is $mg\cos\theta$

(C) normal reaction on the automobile is $mg\sec\theta$

(D) net force on the automobile is $\sqrt{(mg)^2+\left(\frac{mv^2}{R}\right)^2}$

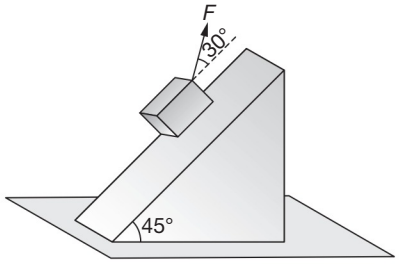
43. Three blocks A, B and C of equal mass m are placed one over the other on a smooth horizontal ground as shown in figure. Coefficient of friction between any two blocks of A, B and C is $\frac{1}{2}$ and that between C and ground is zero. The maximum value of mass of block D so that the blocks A, B and C move without slipping over each other is



(A) 3 m (B) 4 m

(C) 5 m (D) 6 m

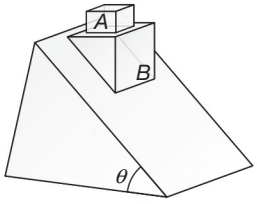
44. A block of mass $m=4$ kg is placed over a rough inclined plane having coefficient of friction $\mu=0.6$ as shown in figure. A force $F=10$ N is applied on the block at an angle of $30°$. The contact force between the block and the plane is



(A) 10.65 N (B) 16.32 N

(C) 27.15 N (D) 32.16 N

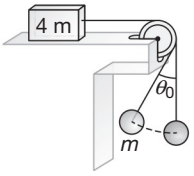
45. On a fixed wedge, two blocks A and B are placed as shown. The coefficient of friction between the block A of mass m and block B of mass $2m$ is μ. There is no friction between block B and the inclined plane. If the system of blocks A and B is released from rest, then for no slipping between A and B, we have



(A) $\theta\le\cos^{-1}(2\mu)$ (B) $2\theta\le\tan^{-1}\left(\frac{\mu}{2}\right)$

(C) $2\theta\le\tan^{-1}(2\mu)$ (D) $\theta\le\tan^{-1}(\mu)$

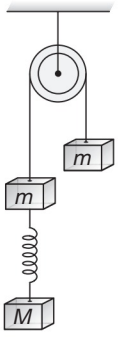
46. In the arrangement shown, two bodies of mass m and $4m$ are attached with string. The body of mass m hanging from the string of length l is executing oscillations of angular amplitude θ_0 while the other body is at rest. The minimum coefficient of friction between the mass $4m$ and the horizontal surface is



(A) $\mu_{MIN} = \left(\frac{2-\cos\theta_0}{3}\right)$ (B) $\mu_{MIN} = 2\cos^2\left(\frac{\theta_0}{2}\right)$

(C) $\mu_{MIN} = \left(\frac{3-\cos\theta_0}{4}\right)$ (D) $\mu_{MIN} = \left(\frac{3-2\cos\theta_0}{4}\right)$

47. The system shown in figure is released from rest. The spring gets elongated

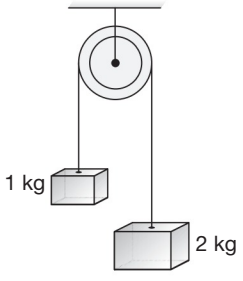


(A) if $M > m$ (B) if $M > 2m$

(C) if $M > \frac{m}{2}$ (D) for any value of M

(Neglect friction and masses of pulley, string and spring)

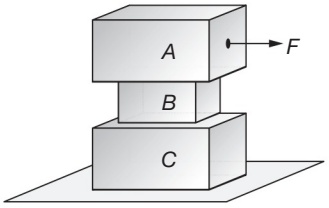
48. Two unequal masses are connected on two sides of a light string passing over a light and smooth pulley as shown in figure. The system is released from rest. The larger mass is stopped for a moment, 1 second after the system is set into motion. The time elapsed before the string is tight again is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) $\frac{1}{4}$ s (B) $\frac{1}{2}$ s

(C) $\frac{2}{3}$ s (D) $\frac{1}{3}$ s

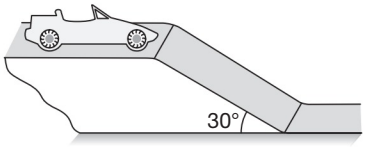
49. Given m_A = 30 kg, m_B = 10 kg, m_C = 20 kg. Between A and B $\mu_1 = 0.3$, between B and C $\mu_2 = 0.2$ and between C and ground $\mu_3 = 0.1$. The least horizontal force F to start motion of any part of the system of three blocks resting upon one another as shown in figure is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 60 N (B) 90 N

(C) 80 N (D) 150 N

50. A car going at a speed of 6 ms^{-1} encounters an incline (inclination angle $30°$) of length 15 m. The friction coefficient between the road and tyres is 0.5. The minimum speed of car with which it can reach the bottom is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 10 ms^{-1} (B) 7.5 ms^{-1}

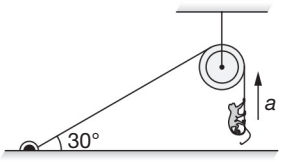
(C) 5 ms^{-1} (D) 2.5 ms^{-1}

51. In order to raise a block of mass 100 kg a man of mass 60 kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with an acceleration $\frac{5g}{4}$ relative to rope. The tension in the rope is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 1432 N (B) 928 N

(C) 1218 N (D) 642 N

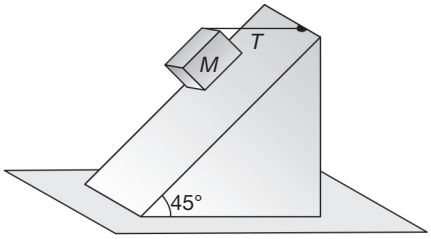
52. A light string fixed at one end to a clamp on ground passes over a fixed pulley and hangs at the other side. It makes an angle of $30°$ with the ground. A monkey of mass 5 kg climbs up the rope. The clamp can tolerate a vertical force of 40 N only. The maximum acceleration in upward direction with which the monkey can climb safely is (Neglect friction and take $g = 10\ \text{ms}^{-2}$)



(A) $2\ ms^{-2}$ (B) $4\ ms^{-2}$

(C) $6\ ms^{-2}$ (D) $8\ ms^{-2}$

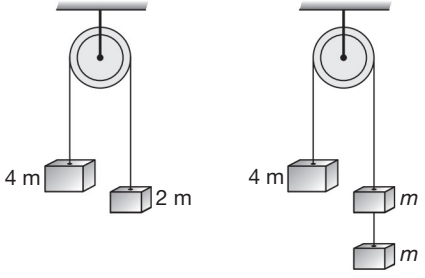
53. A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact is ($g = 10\ ms^{-2}$)



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{3}{4}$ (D) $\frac{1}{4}$

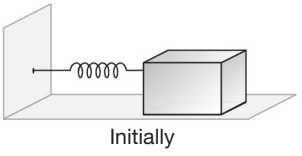
54. In the arrangement shown in figure, pulley is smooth and massless and all the strings are light. Let F_1 be the force exerted on the pulley in CASE-1 and F_2 the force in CASE-2. Then

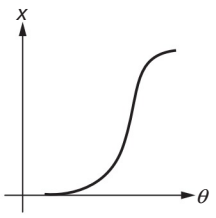


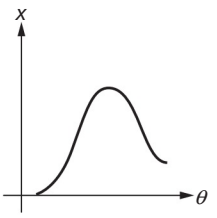
(A) $F_1 > F_2$ (B) $F_1 < F_2$

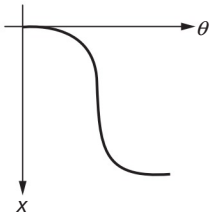
(C) $F_1 = F_2$ (D) $F_1 = 2F_2$

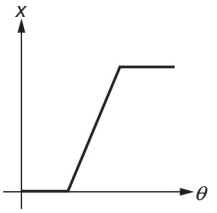
55. A block of mass m is placed on a rough horizontal plane attached with an elastic spring of spring constant k as shown in figure. Initially spring is unstretched. If the plane is now gradually lifted from $\theta = 0°$ to $\theta = 90°$, then the graph showing extension in the spring (x) versus angle (θ) is



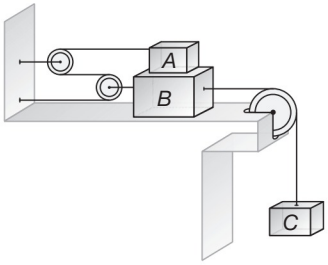
(A) 

(B) 

(C) 

(D) 

56. The maximum value of mass of block C so that neither A nor B moves is (Given that mass of A is 100 kg and that of B is 140 kg. Pulleys are smooth and friction coefficient between A and B and between B and horizontal surface is $\mu = 0.3$) $g = 10\ ms^{-2}$



(A) 210 kg (B) 190 kg

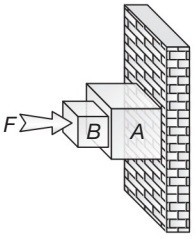
(C) 185 kg (D) 162 kg

57. A homogeneous chain of length L lies on a table. The coefficient of friction between the chain and the table is μ. The maximum length which can hang over the table in equilibrium is (The vertical portion of table is smooth)

(A) $\left(\frac{\mu}{\mu+1}\right)L$ (B) $\left(\frac{1-\mu}{\mu}\right)L$

(C) $\left(\frac{1-\mu}{1+\mu}\right)L$ (D) $\left(\frac{2\mu}{2\mu+1}\right)L$

58. Consider the arrangement shown. Suppose all the surfaces are rough, the direction of friction on B due to A, on A due to wall and on A due to B is

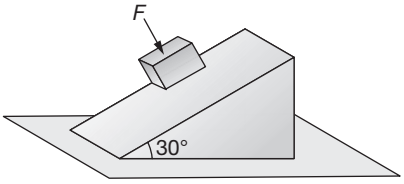


(A) up, down, up
(B) up, up, down
(C) down, up, up
(D) depends on the masses of A and B and friction coefficients at different surfaces

59. A block of mass m slides down an inclined plane of inclination θ with uniform speed. The coefficient of friction between the block and the plane is μ. The contact force between the block and the plane is

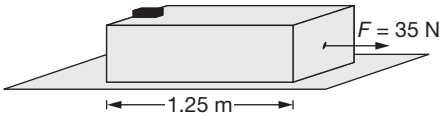
(A) $mg\sin\theta\sqrt{1+\mu^2}$
(B) $\sqrt{(mg\sin\theta)^2+(\mu mg\cos\theta)^2}$
(C) $mg\sin\theta$
(D) mg

60. A block of mass $m = 2$ kg is at rest on a rough inclined plane of inclination 30° as shown. The coefficient of friction between the block and the plane is $\mu = 0.5$. The minimum force F that must be applied on the block perpendicular to the plane, so that block does not slip on the plane is ($g = 10$ ms^{-2})



(A) zero (B) 6.24 N
(C) 2.68 N (D) 4.34 N

61. On a plank of mass 2 kg, length 1.25 m lying on a smooth horizontal ground, a block of mass 1 kg is placed. The coefficient of friction between the block and the plank is 0.5. A constant force $F = 35$ N is applied on the plank in horizontal direction. The time after which the block will separate from the plank is ($g = 10$ ms^{-2})

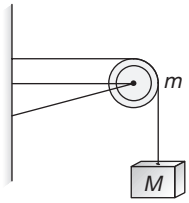


(A) 0.25 s (B) 0.5 s
(C) 1 s (D) 2 s

62. A body slides down an inclined plane of inclination θ. The coefficient of friction down the plane varies in direct proportion to the distance moved down the plane as $\mu = kx$, where k is a constant. The body will move down the plane with a

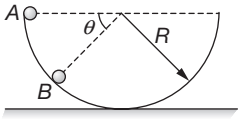
(A) constant acceleration $g\sin\theta$.
(B) constant acceleration $g(\sin\theta-\mu\cos\theta)$.
(C) variable acceleration $g(\mu\sin\theta-\cos\theta)$.
(D) variable acceleration first decreasing from $g\sin\theta$ to zero and then becoming negative.

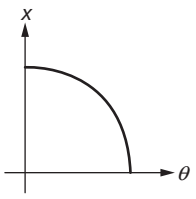
63. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by

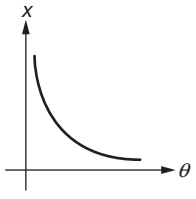


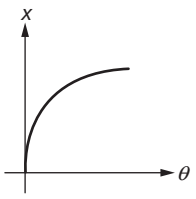
(A) $\sqrt{2}Mg$
(B) $\sqrt{2}mg$
(C) $\left[\sqrt{(M+m)^2+m^2}\right]g$
(D) $\left[\sqrt{(M+m)^2+M^2}\right]g$

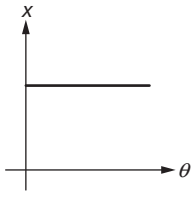
64. A small particle of mass m is released from rest from point A inside a smooth hemispherical bowl as shown in figure. The ratio (x) of magnitude of centripetal force and normal reaction on the particle at any point B varies with θ as



(A) 

(B) 

(C) 

(D) 

65. Three freight cars of mass M are pulled with force F by a locomotive. Friction is negligible. The ratio of the forces on each car from the end where F is applied is

(A) 3 : 2 : 1 (B) 1 : 2 : 3
(C) 1 : 1 : 1 (D) 3 : 1 : 2

66. A particle is moving in a circular path. The acceleration and momentum of the particle at a certain moment are $\vec{a} = \left(4\hat{i}+3\hat{j}\right)$ ms^{-2} and $\vec{p} = \left(8\hat{i}-6\hat{j}\right)$ kgms^{-1}. The motion of the particle is

(A) accelerated circular motion
(B) de-accelerated circular motion
(C) uniform circular motion
(D) data insufficient to say anything with $\vec{a}$ and $\vec{p}$ only

67. Assuming the gravity to be in negative z-direction, a force $\vec{F} = \vec{v} \times \vec{A}$ is exerted on a particle in addition to the force of gravity where $\vec{v}$ is the velocity of the particle and $\vec{A}$ is a constant vector in positive x-direction. The minimum speed $\vec{v}_{\min}$ with which a particle of mass m must be projected so that it continues to move undeflected with constant velocity is

(A) $\vec{v}_{\min} = -\frac{A}{mg}\hat{j}$ (B) $\vec{v}_{\min} = \frac{A}{mg}\hat{j}$

(C) $\vec{v}_{\min} = \frac{mg}{A}\hat{j}$ (D) $\vec{v}_{\min} = -\frac{mg}{A}\hat{j}$

68. Two masses m and M are connected by a light string passing over a smooth pulley. When set free m moves up by 1.4 m in 2 s. The ratio $\frac{m}{M}$ is

(A) $\frac{13}{15}$ (B) $\frac{15}{13}$

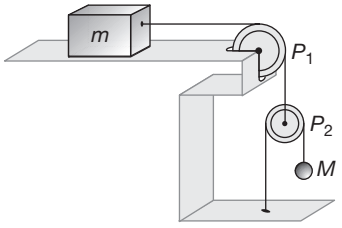
(C) $\frac{9}{7}$ (D) $\frac{7}{9}$

69. An airplane requires for take-off a speed of 80 kmh^{-1}, the run on the ground being 100 m. The mass of the airplane is 10,000 kg and the coefficient of friction between the airplane and the ground is 0.2. Assuming that the plane accelerates uniformly during take-off, the minimum force required by the engine for take-off is

(A) 2×10^4 N (B) 2.43×10^4 N

(C) 4.43×10^4 N (D) 8.86×10^4 N

70. In the pulley arrangement shown, the pulley P_2 is movable. Assuming coefficient of friction between m and surface to be μ, the minimum value of M for which m is at rest is



(A) $M = \frac{\mu m}{2}$ (B) $m = \frac{\mu M}{2}$

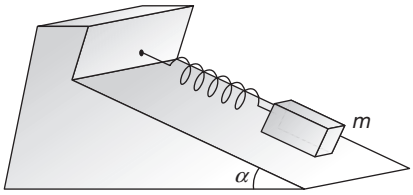
(C) $M = \frac{m}{2\mu}$ (D) $m = \frac{M}{2\mu}$

71. A block is gently dropped on a conveyor belt moving at 3 ms^{-1}. If the coefficient of friction between the belt and the block is 0.5. The displacement of block relative to the conveyor belt before the slipping between the two is stopped ($g = 10\ \text{ms}^{-2}$)

(A) 0.9 m (B) 1.2 m

(C) 1.8 m (D) 2.7 m

72. A block of mass m is held on a rough inclined plane by a spring parallel to the plane as shown. The natural length of the spring is l_0 and its force constant is λ. The frictional force is in magnitude proportional to the speed of the block. If l be the equilibrium length, then



(A) $l = l_0\left(1 + \frac{2mg\sin\alpha}{\lambda}\right)$ (B) $l = l_0\left(1 + \frac{mg\sin\alpha}{\lambda}\right)$

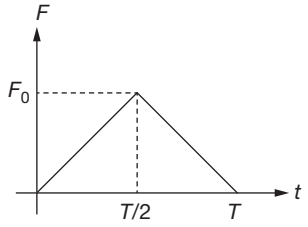
(C) $l = l_0\left(1 - \frac{2mg\sin\alpha}{\lambda}\right)$ (D) $l = l_0\left(1 - \frac{mg\sin\alpha}{\lambda}\right)$

73. A car starts from rest to cover a distance of 500 m on a road whose coefficient of friction with the tyres of the car is $\frac{1}{2}$. The minimum time in which the car can cover this distance is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 5 s (B) 10 s

(C) 20 s (D) 40 s

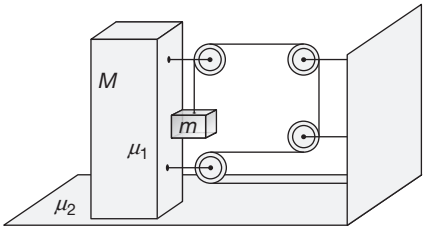
74. A particle of mass m moving with velocity u makes an elastic one dimensional collision with a stationary particle of mass m. They are in contact for a very brief time T. Their force of interaction increases from zero to F_0 linearly in time $\frac{T}{2}$, and decreases linearly to zero in a further time $\frac{T}{2}$. The magnitude of F_0 is



(A) $\frac{mu}{2T}$ (B) $\frac{mu}{T}$

(C) $\frac{2mu}{T}$ (D) None of these

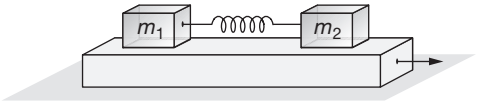
75. Two blocks M and m are arranged as shown in the figure. The coefficient of friction between the blocks M and m is $\mu_1 = 0.25$ and between the ground and M is $\mu_2 = \frac{1}{3}$. If $M = 8$ kg, then the value of m for which the system will remain at rest is



(A) $\frac{4}{3}$ kg (B) $\frac{8}{9}$ kg

(C) 1 kg (D) $\frac{8}{5}$ kg

76. Two blocks of masses m_1 and m_2 are connected with a light spring and placed over a plank moving with an acceleration a as shown in figure. The coefficient of friction between the blocks and plank is μ. The spring will



(A) neither be compressed nor be stretched for $a \le \mu g$

(B) be compressed if $a \le \mu g$

(C) be stretched if $a > \mu g$

(D) be in its natural length under all conditions

77. A block is about to slide down an inclined plane when its angle to the horizontal is θ. If now an another block of same mass is put on the block

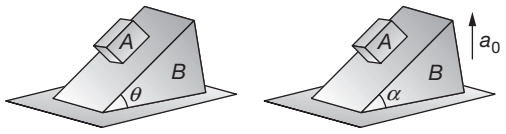
(A) it will not slide down the plane unless the inclination is increased

(B) it can remain in equilibrium even if the inclination is slightly decreased

(C) it will about to slide when the inclination is greater than $\frac{\theta}{2}$

(D) it is still about to slide down the plane

78. In CASE-1, the wedge B is stationary and block A starts sliding when the angle of plane is greater than θ, while in CASE-2, the wedge B is moving upwards with an acceleration a_0 and the block A starts sliding when the angle of plane is greater than α. The coefficient of friction between A and B in both the cases is same. Then

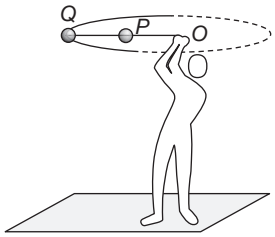


CASE-1 CASE-2

(A) $\frac{\tan\theta}{\tan\alpha} = \frac{g}{a_0}$ (B) $\theta < \alpha$

(C) $\theta > \alpha$ (D) $\theta = \alpha$

79. Two spheres of equal masses are attached to a string of length 2 m as shown. The string and the spheres are then whirled in a horizontal circle about O at a constant rate. If T_1 is the tension in the string between P & Q and T_2 is the tension in the string between P & O then $\frac{T_1}{T_2}$ equals

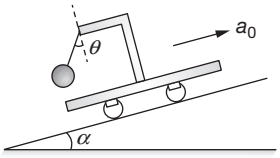


$OP = PQ = 1$ m

(A) 1 (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

80. A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is



(A) $\theta = \tan^{-1}\alpha$

(B) $\theta = \tan^{-1}\left(\frac{a_0}{g}\right)$

(C) $\theta = \tan^{-1}\left(\frac{g}{a_0}\right)$

(D) $\theta = \tan^{-1}\left(\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right)$

81. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 ms^{-1}. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1 m. The angle made by the rod with the track is

(A) 0° (B) 30°

(C) 45° (D) 60°

82. A block is placed on a rough inclined plane having inclination angle θ. Now, the angle θ is increased from 0° to 90°, then the contact force between the block and the plane

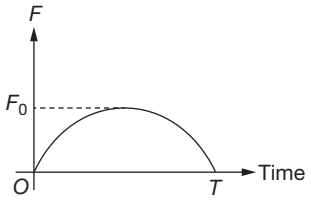
(A) remains constant

(B) first increases then decreases

(C) first decreases then increases

(D) first remains constant then decreases

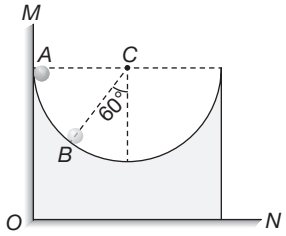
83. A particle of mass m, initially at rest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The curve is an ellipse



(A) $u=\dfrac{\pi F_0^2}{2m}$ (B) $u=\dfrac{\pi T^2}{8m}$

(C) $u=\dfrac{\pi F_0 T}{4m}$ (D) $u=\dfrac{F_0 T}{2m}$

84. A ball of mass 1 kg is released from position A inside a wedge which has a hemispherical cut of radius 0.5 m as shown. Assuming all surfaces to be smooth, the force exerted by the vertical wall OM on wedge, when the ball is in position B is $\left(g=10\ \text{ms}^{-2}\right)$



(A) $5\sqrt{3}$ N (B) 10 N

(C) $\dfrac{15\sqrt{3}}{2}$ N (D) 15 N

85. A bar of mass m resting on a smooth horizontal plane starts moving due to a force $F=\dfrac{mg}{3}$ of constant magnitude. In the process of its rectilinear motion, the angle θ between the direction of this force and the horizontal varies as $\theta=ks$, where k is a constant and s is the displacement of the particle. The velocity of bar as a function of θ is

(A) $v=\sqrt{\dfrac{2g}{3}\sin\theta}$ (B) $v=\sqrt{\dfrac{2g}{3k}\cos\theta}$

(C) $v=\sqrt{\dfrac{2g}{3k}\sin\theta}$ (D) $v=\sqrt{\dfrac{2g}{3}\cos\theta}$

86. Two small balls each having a radius R, having masses m_1 and $m_2\left(m_1>m_2\right)$, tied by a thin light thread are dropped from a certain height. Taking upward buoyancy force F into account, the tension T in the thread during the flight when the motion of the balls becomes uniform is

(A) $\left(m_1-m_2\right)g$ (B) $\left(m_1-m_2\right)\dfrac{g}{2}$

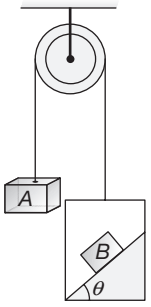
(C) $\left(m_1+m_2\right)g$ (D) $\left(m_1+m_2\right)\dfrac{g}{2}$

87. On an inclined plane of inclination angle $30°$, a block is placed. It is observed that the force to drag the block along the plane upwards is smaller than the force required to lift it. The maximum value of coefficient of friction is

(A) $\dfrac{\sqrt{3}}{2}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{1}{\sqrt{3}}$ (D) $\dfrac{2}{3}$

88. In the arrangement shown, the string is light, pulley is smooth, the total mass on left hand side of the pulley is m_1, on right hand side is m_2, the coefficient of friction between block B and the wedge is $\mu=\dfrac{1}{2}$ and the inclination angle is $\theta=30°$. Select the incorrect option.



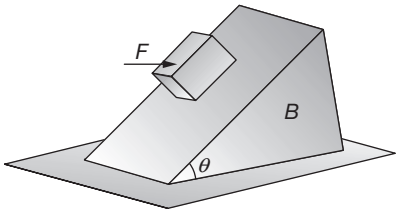
(A) Block B will slide down if $m_1=m_2$

(B) Block B may remain stationary with respect to wedge for suitable values of m_1 and m_2 with $m_2>m_1$

(C) Block B will slide down if $m_1>m_2$

(D) Block B cannot remain stationary with respect to wedge in any case

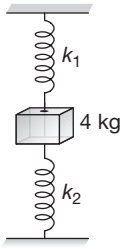
89. A horizontal force F is applied to a block of mass m on a smooth inclined plane of inclination θ to the horizontal as shown in figure. The resultant force on the block is



(A) $F\sin\theta-mg\cos\theta$ (B) $F+mg\tan\theta$

(C) $F\sin\theta+mg\cos\theta$ (D) $F\cos\theta-mg\sin\theta$

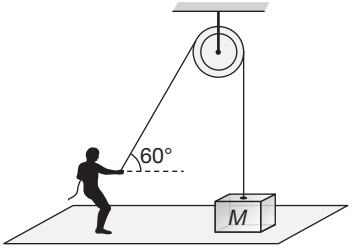
90. A 4 kg block is connected with two springs of force constants $k_1 = 100\ \text{Nm}^{-1}$ and $k_2 = 300\ \text{Nm}^{-1}$ as shown in figure. The block is released from rest with the springs unstretched. The acceleration of the block in its lowest position is ($g = 10\ \text{ms}^{-2}$)



(A) zero (B) $5\ \text{ms}^{-2}$ upwards

(C) $10\ \text{ms}^{-2}$ downwards (D) $10\ \text{ms}^{-2}$ upwards

91. A man of mass 60 kg is pulling a block of mass M by an inextensible light rope passing through a smooth and massless pulley as shown in figure. The coefficient of friction between the man and the ground is $\mu = \frac{1}{2}$. The maximum value of M that can be pulled by the man without slipping on the ground is approximately



(A) 26 kg (B) 32 kg

(C) 46 kg (D) 51 kg

92. A wooden block of mass M resting on a rough horizontal floor is pulled with a force F at an angle ϕ with the horizontal. If μ is the coefficient of kinetic friction between the block and the surface, then acceleration of the block is

(A) $\frac{F}{M}\sin\phi$

(B) $\mu F\cos\phi$

(C) $\frac{F}{M}(\cos\phi - \mu\sin\phi) - \mu g$

(D) $\frac{F}{M}(\cos\phi + \mu\sin\phi) - \mu g$

93. An electric fan is placed on a stationary boat and air is blown with it on the sail of the boat. Select the correct statement(s) from the following.

(A) The boat will start moving with a uniform speed.

(B) The boat will be uniformly accelerated in the direction of flow of the air.

(C) The boat will be uniformly accelerated opposite to the direction of the flow of air.

(D) The boat will remain stationary as before.

94. In a tug-of-war contest, two teams A and B pull on a horizontal rope from the opposite sides. The winner team will be the one who

(A) exerts greater force on the rope.

(B) exerts greater force on the ground.

(C) exerts a force on the rope greater than the tension existing in the rope.

(D) data insufficient to arrive at a result.

95. A bowl of soup rests on a table in the dining car of a train. If the acceleration of the train is a_0 in the forward direction then the angle which the surface of the soup makes with the horizontal is

(A) $\arctan\left(\frac{a_0}{g}\right)$ (B) $\arctan\left(\frac{g}{a_0}\right)$

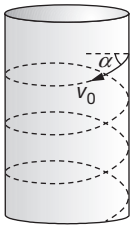
(C) $\arctan\left(\frac{a_0}{\sqrt{a_0^2 + g^2}}\right)$ (D) $\arctan\left(\frac{g}{\sqrt{a_0^2 + g^2}}\right)$

96. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet of water at a rate of $1\ \text{kgs}^{-1}$ at a speed of $5\ \text{ms}^{-1}$. The initial acceleration of the block is

(A) $\frac{2}{5}\ \text{ms}^{-2}$ (B) $\frac{5}{2}\ \text{ms}^{-2}$

(C) $5\ \text{ms}^{-2}$ (D) $\frac{1}{5}\ \text{ms}^{-2}$

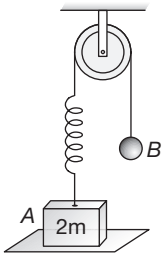
97. A particle of mass m moves along the internal smooth surface of a vertical cylinder of radius R as shown. The force which acts on the wall of the cylinder if initially the velocity v_0 of the particle makes an angle α with the horizontal (Assume that particle does not leave contact with the curved surface of the cylinder) is



(A) $\frac{mv_0^2}{R}$ (B) $\frac{mv_0^2\cos^2\alpha}{R}$

(C) $\frac{mv_0^2\sin^2\alpha}{R}$ (D) mg

98. In the figure, the ball B is released from rest when the spring is at its natural length. For the block A of mass $2m$ to leave contact with the ground at some instant, the minimum mass of B must be



(A) $2m$ (B) m

(C) $\frac{m}{2}$ (D) $\frac{m}{4}$

99. A blimp of mass m is descending with an acceleration a_0. A buoyant force is acting on the blimp which is equal to the weight of the air displaced by the blimp. Assuming the buoyant force to be constant the ballast that must be jettisioned from the blimp so that it rises with same acceleration a_0 is

(A) $\left(\frac{2g}{g+a_0}\right)m$ (B) $\left(\frac{2a_0}{g}\right)m$

(C) $\left(\frac{2a_0}{g+a_0}\right)m$ (D) $\left(\frac{2a_0}{g-a_0}\right)m$

100. A car with closed windows takes a left turn. A helium filled balloon in the car will be pushed to the

(A) right (B) left

(C) front (D) back

101. According to the basic postulates of the "Theory of Relativity", the laws of Physics are same in

(A) all frames of reference.

(B) Only those frames of reference which move at constant velocity with respect to an inertial frame.

(C) only those frames of reference which move at constant acceleration with respect to an inertial frame.

(D) only those frames which rotate with constant angular velocity with respect to an inertial frame.

102. A string with constant tension T is deflected through an angle $2\theta_0$ by a smooth fixed pulley. The force on the pulley is

(A) $2T\cos\theta_0$ (B) $T\cos\theta_0$

(C) $2T\sin\theta_0$ (D) $T\sin\theta_0$

103. A man running along a straight road leans a little in the forward direction. The angle between the vertical line and the line joining the man's centre of gravity with the point of support is θ_0. For the man not to slip, the coefficient of friction satisfies the condition given by

(A) $\mu \le \tan\theta_0$ (B) $\mu \ge \tan\theta_0$

(C) $\mu \ge \tan^2\theta_0$ (D) $\mu \le \tan^2\theta_0$

104. Vertically hanging curtains between the compartments in an airplane are seen to bend forward. This indicates that the airplane is

(A) taking off

(B) moving steadily at constant altitude

(C) slowing down for landing

(D) taking a bend

105. A particle of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. For the block to just move

(A) A minimum force $F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$ has to be applied at an angle $\theta = \tan^{-1}\left(\frac{1}{\mu}\right)$ with the horizontal.

(B) A minimum force $F_{\min} = \mu mg$ has to be applied at an angle $\theta = \tan^{-1}(\mu)$ with the horizontal.

(C) A minimum force $F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$ has to be applied at an angle $\theta = \tan^{-1}(\mu)$ with the horizontal.

(D) A minimum force $F_{\min} = \mu mg$ has to be applied at an angle $\theta = \tan^{-1}\left(\frac{1}{\mu}\right)$ with the horizontal.

106. A prism of mass M is placed on a horizontal surface. A block of mass m slides on it, which in turn slides on the horizontal surface. Assume all surfaces to be frictionless the acceleration of the block with respect to the prism is

(A) $g\sin\theta$ (B) $g\cos\theta$

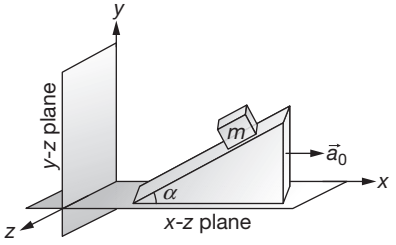
(C) $\left(\frac{M+m}{M+m\sin^2\theta}\right)g\sin\theta$ (D) $\left(\frac{M+m}{M+m\sin^2\theta}\right)g\cos\theta$

107. A body slides over an inclined plane forming an angle of $45°$ with the horizontal. The distance x travelled by the body in time t is described by the equation $x = kt^2$, where $k = 1.732$. The coefficient of friction between the body and the plane has a value

(A) $\mu = 0.5$ (B) $\mu = 1$

(C) $\mu = 0.25$ (D) $\mu = 0.75$

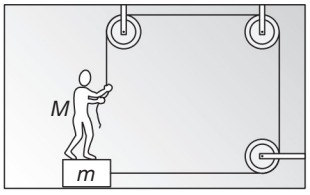
108. The inclined plane shown in figure has an acceleration $\vec{a}_0$ to the right. If $\mu_s = \tan\theta$ is the coefficient of static friction for all surfaces in contact. The block will slide on the plane for



(A) $a_0 > g\tan\theta$ (B) $a_0 < g\tan\theta$

(C) $a_0 > g\tan(\theta-\alpha)$ (D) $a_0 < g\tan(\theta-\alpha)$

109. A board of mass m is placed on the floor and a man of mass M is standing on the board as shown. The coefficient of friction between the board and the floor is μ. The maximum force that the man can exert on the rope so that the board does not slip on the floor is



(A) $F = \mu(M+m)g$ (B) $F = \mu mg$

(C) $F = \dfrac{\mu Mg}{\mu+1}$ (D) $F = \dfrac{\mu(M+m)g}{\mu+1}$

110. A body rests on a rough horizontal plane. A force is applied to the body directed towards the plane at an angle ϕ with the vertical. If θ is the angle of friction then for the body to move along the plane

(A) $\phi > \theta$ (B) $\phi < \theta$

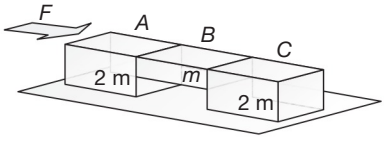
(C) $\phi = \theta$ (D) ϕ can take up any value

111. The force required to just move a body up an inclined plane is double the force required to just prevent it from sliding down. If ϕ is angle of friction and θ is the angle which incline makes with the horizontal then,

(A) $\tan\theta = \tan\phi$ (B) $\tan\theta = 2\tan\phi$

(C) $\tan\theta = 3\tan\phi$ (D) $\tan\phi = 3\tan\theta$

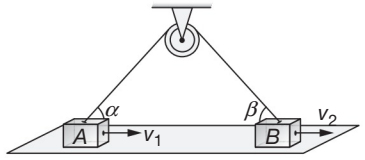
112. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from downward slipping is



(A) $\left(\dfrac{3}{2\mu}\right)mg$ (B) $\left(\dfrac{5}{2\mu}\right)mg$

(C) $\left(\dfrac{5}{2}\right)\mu mg$ (D) $\left(\dfrac{3}{2}\right)\mu mg$

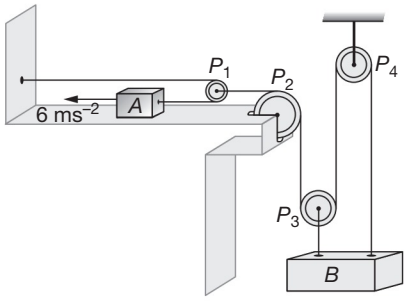
113. In the arrangement shown, blocks A and B connected with an inextensible string move with velocities v_1 and v_2 along horizontal direction. The ratio of $\dfrac{v_2}{v_1}$ is



(A) $\dfrac{\sin\alpha}{\sin\beta}$ (B) $\dfrac{\sin\beta}{\sin\alpha}$

(C) $\dfrac{\cos\beta}{\cos\alpha}$ (D) $\dfrac{\cos\alpha}{\cos\beta}$

114. In the arrangement shown, the block A moves left with an acceleration of $6\ \text{ms}^{-2}$, such that at all the instants, the block B always remains horizontal. The acceleration of B is



(A) $6\ \text{ms}^{-2}$ (B) $4\ \text{ms}^{-2}$

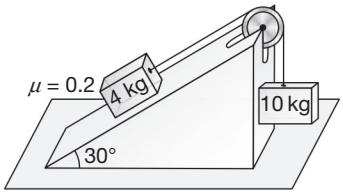
(C) $2\ \text{ms}^{-2}$ (D) $1\ \text{ms}^{-2}$

115. Springs of spring constant K, $3K$, $9K$, $27K$, …, ∞ are connected in series. The equivalent spring constant of the combination is

(A) $\dfrac{3K}{2}$ (B) $\dfrac{K}{2}$

(C) $\dfrac{2K}{3}$ (D) ∞

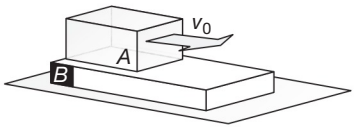
116. Two blocks of masses 4 kg and 10 kg are connected by flexible inextensible string as shown. The pulley is assumed to be light and frictionless. The coefficient of friction between the 4 kg block and the surface is 0.2 and angle of inclination of the plane is 30°. Assume the system to be released from rest, the velocity of 10 kg block when it has moved 4 m down is

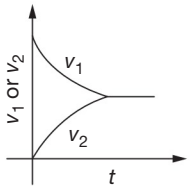


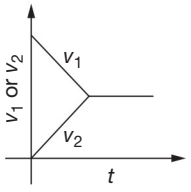
(A) $6.5\ \text{ms}^{-1}$ (B) $5.6\ \text{ms}^{-1}$

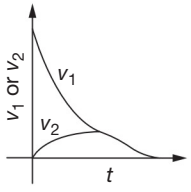
(C) $41.8\ \text{ms}^{-1}$ (D) $13\ \text{ms}^{-1}$

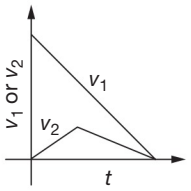
117. A block A is placed over a long rough plank B of same mass as shown in figure. The plank is placed over a smooth horizontal surface. At time $t = 0$, block A is given a velocity v_0 in horizontal direction. Let v_1 and v_2 be the velocities of A and B at time t. Then the correct graph between v_1 or v_2 and t is



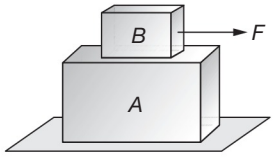
(A) 

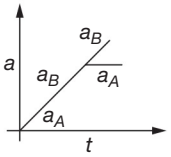
(B) 

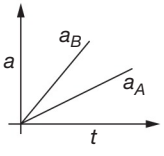
(C) 

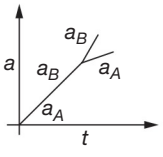
(D) 

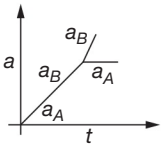
118. A block B is placed on block A. The mass of block B is less than the mass of block A. Friction exists between the blocks, whereas the ground on which the block A is placed is taken to be smooth. A horizontal force F, increasing linearly with time begins to act on B. The acceleration a_A and a_B of blocks A and B respectively are plotted against t. The correctly plotted graph is



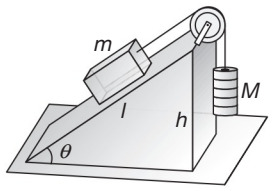
(A) 

(B) 

(C) 

(D) 

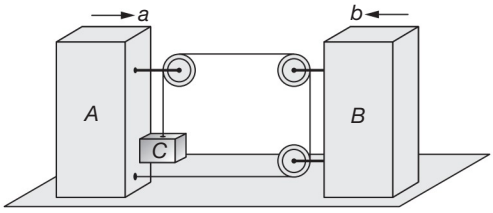
119. A mass m is drawn up a smooth inclined plane of height h and length l by means of a string passing over a pulley fixed at the vertex of the plane as shown. A mass M hangs from the other end. For m to just reach the top of the plane, M must be detached when m has moved through a distance



(A) $\frac{mh}{M}$ (B) $\frac{ml}{M}$

(C) $\frac{M}{M+m}\left(\frac{hl}{h+l}\right)$ (D) $\frac{M+m}{M}\left(\frac{hl}{h+l}\right)$

120. In the arrangement shown, if the blocks A and B move towards each other with acceleration values a and b, respectively, then acceleration of the block C is



(A) $2a + 2b$ (B) $\sqrt{4a^2 + 5b^2 + 8ab}$

(C) a (D) $\sqrt{5a^2 + 4b^2 + 8ab}$

121. A car starts from rest to cover a distance s. The coefficient of friction between the road and the tyres is μ. The minimum time in which the car can cover the distance s is proportional to

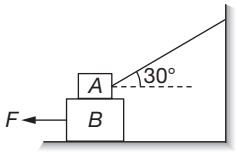
(A) $\frac{1}{\mu}$ (B) $\frac{1}{\sqrt{\mu}}$

(C) μ (D) $\sqrt{\mu}$

122. In the shown arrangement if f_1, f_2 and T be the frictional forces on 2 kg block, 3 kg block and tension in the string respectively, then their values are

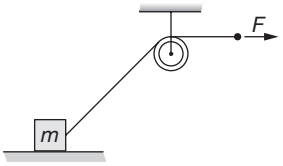
(A) 2 N, 6 N, 3.2 N
(B) 2 N, 6 N, 0 N
(C) 1 N, 6 N, 2 N
(D) Data insufficient to calculate the required values

123. In the figure given below, if all surface are assumed to be smooth and the force $F = 100$ N. If acceleration of block B of mass 20 kg is a and tension in string connecting block A of mass 20 kg is T then just after when the force F is applied



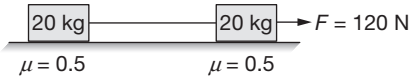
(A) $T = 0$ and $a = 5\ \text{ms}^{-2}$
(B) $T = 100\ \text{N}$ and $a = 0$
(C) $T = 200\ \text{N}$ and $a = 5\ \text{ms}^{-2}$
(D) None of these

124. The block of mass m is placed on a rough horizontal floor and it is pulled by an ideal string as shown by a constant force F. As the block moves towards right the friction force on block

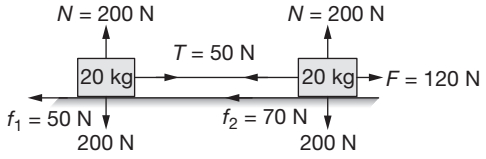


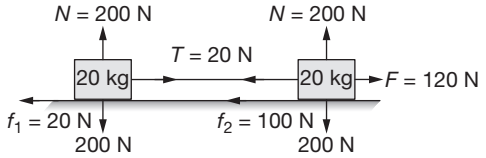
(A) remains constant (B) increases
(C) decreases (D) can be said

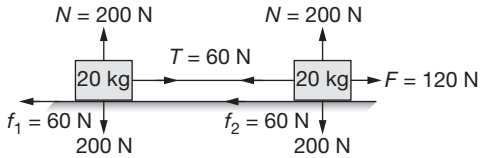
125. Two blocks each of mass 20 kg are connected by an ideal string and this system is kept on rough horizontal surface as shown. Initially the string is just tight then a horizontal force $F = 120\ \text{N}$ is applied on one block as shown.



If friction coefficient at every contact is $\mu = 0.5$ then which of the following represents the correct free body diagram.

(A)


(B)


(C)


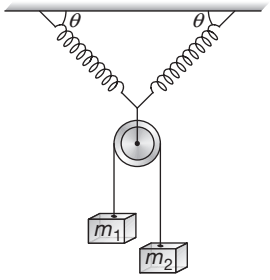
(D) All of the above

126. A particle is moving along a circular path with a constant speed. The acceleration of the particle is constant in
(A) magnitude
(B) direction
(C) both magnitude and direction
(D) neither magnitude nor direction

127. A particle moves in x-y plane. The position vector of particle at any time t is $\vec{r} = \left[(2t)\hat{i} + (2t^2)\hat{j}\right]$ m. The rate of change of θ at time $t = 2$ s. (where θ is the angle which its velocity vector makes with positive x-axis) is

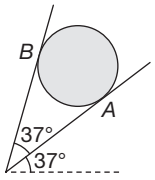
(A) $\frac{2}{17}\ \text{rads}^{-1}$ (B) $\frac{1}{14}\ \text{rads}^{-1}$
(C) $\frac{4}{7}\ \text{rads}^{-1}$ (D) $\frac{6}{5}\ \text{rads}^{-1}$

128. Two identical ideal springs of spring constant $1000\ \text{Nm}^{-1}$ are connected by a pulley as shown in figure and this arrangement is established in the vertical plane. The pulley is ideal and string passing over pulley is massless. At equilibrium of pulley θ is $60°$ and the masses m_1 and m_2 are 2 kg and 3 kg, respectively. Then the elongation in each spring when $\theta = 60°$ is



(A) $1.6\sqrt{3}$ cm (B) 1.6 cm
(C) 4.8 cm (D) None of these

129. A sphere of mass m is held between two smooth inclined walls A and B. For $\sin(37°) = \frac{3}{5}$, the normal reaction of the wall (2) is equal to



(A) mg (B) $mg\sin(74°)$
(C) $mg\cos(74°)$ (D) None of these

130. A block A of mass 2 kg rests on another block B of mass 8 kg which rests on a horizontal floor. The coefficient of friction between A and B is 0.2 while between B and

floor is 0.5. When a horizontal force 25 N is applied on B, the frictional force acting between A and B is

(A) 0 N (B) 3.9 N
(C) 5 N (D) 49 N

131. A block starts moving up a fixed inclined plane of inclination $60°$ with a velocity of 20 ms^{-1} and stops after 2 s. The approximate value of coefficient of friction is $(g = 10\ ms^{-2})$

(A) 3 (B) 3.3
(C) 0.27 (D) 0.33

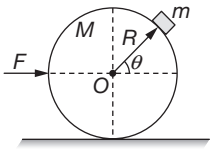
132. A point on the periphery of rotating disc has its acceleration vector making on angle $30°$ with velocity vector then the ratio of magnitude of centripetal acceleration to tangential acceleration is

(A) $\sin(30°)$ (B) $\cos(30°)$
(C) $\tan(30°)$ (D) None of these

133. Find the only correct statement from the following statements

(A) A body has a constant velocity and still it can have a varying speed
(B) A body has a constant speed but it can have a varying velocity
(C) A body having constant speed cannot have any acceleration
(D) A body in motion under a force acting upon it must always have work done upon it

134. A smooth sphere of radius R and mass M is placed on the smooth horizontal floor. Another smooth particle of mass m is placed on the sphere and a horizontal force F is applied on the sphere as shown. If the particle does not slip on the sphere then the value of force F is

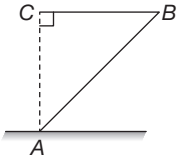


(A) $F = mg\cot\theta$ (B) $F = Mg\cot\theta$
(C) $F = (m+M)g\cot\theta$ (D) $F = (m+M)g\tan\theta$

135. The contact force exerted by a body on another body is perpendicular to the surfaces in contact. Then,

(A) the surfaces in contact must be frictionless
(B) the force of friction between the bodies may not the zero
(C) the contact force equals the normal reaction
(D) the contact force equals the friction

136. A rod AB of mass 10 kg tied with a string at C such that $AC = BC$ and rod remains in equilibrium then friction force by horizontal surface will be



(A) 50 N towards right (B) 50 N towards left
(C) 100 N (D) None of these

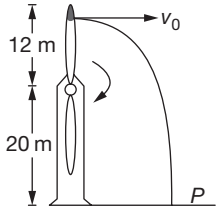
137. A block of mass m is moving on a rough horizontal surface where coefficient of friction between block and surface is $\mu = 0.5$. At $t = 0$, block is moving with a velocity $\vec{v}$. Then a horizontal force $F = 2t$ is applied on it in the direction of velocity, where t represents time, then the speed of block after the application of force

(A) decreases
(B) increases
(C) first decreases then increases
(D) first increases then decreases

138. A circular disc of radius $r = 5$ m is rotating in horizontal plane about y-axis. y-axis a vertical axis passing through the centre of disc and x-z is the horizontal plane at ground. The height of disc above ground is $h = 5$ m. Small particles are ejecting from disc in horizontal direction with speed 12 ms^{-1} from the circumference of disc then the distance of these particles from origin when they hits x-z plane is

(A) 5 m (B) 12 m
(C) 13 m (D) None of these

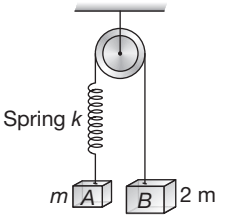
139. A wind farm generator uses a two-bladed propeller mounted on a pylon at a height of 20 m. The length of each propeller blade is 12 m. A tip of the propeller breaks off when the propeller is vertical. At that instant, the period of the motion of the propeller is 1.2 second. The fragment files off horizontally, falls and strikes the ground at point P.



In figure, the distance from the base of the pylon to the point where the fragment strikes the ground is closest to

(A) 120 m (B) 130 m
(C) 140 m (D) 160 m

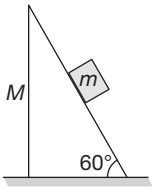
140. Two blocks A and B of masses m and $2m$, respectively are held at rest such that the string is in natural length. Find out the accelerations of both the blocks just after release.



(A) $g\downarrow, g\downarrow$ (B) $\frac{g}{3}\downarrow, \frac{g}{3}\uparrow$

(C) 0, 0 (D) $g\downarrow, 0$

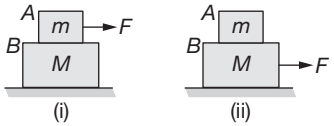
141. In the arrangement shown, the wedge of mass M moves towards left with an acceleration a. All surfaces are smooth. The acceleration of mass m relative to wedge is



(A) $\frac{a}{2}$ (B) $\frac{2Ma}{m}$

(C) $\frac{2(M+m)a}{m}$ (D) $\frac{(M+m)a}{m}$

142. A body A rests on B and friction coefficient between A and B is μ. Block M is placed on a frictionless surface. Then



(A) the maximum possible value of F so that both bodies move together in case (i) is $\mu(m+M)g$

(B) the maximum possible value of F so that both bodies move together in case (i) is μmg

(C) the maximum possible value of F so that both bodies move together in case (ii) is $\mu(M+m)g$

(D) the maximum possible value of F so that both bodies move together in case (ii) is μMg

143. A bicycle is in motion. When it is not pedalled, the force of friction exerted by the ground on two wheels is such that it acts

(A) in the backward direction on the front wheel and in the forward direction on the rear wheel

(B) in the forward direction on the front wheel and in the backward direction on rear wheel

(C) in the backward direction on both the front and the rear wheels

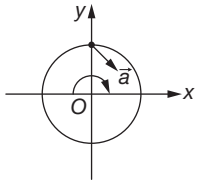
(D) in the forward direction on both the front and the rear wheels

144. A particle is moving in a circular path and its acceleration vector is making an angle of $30°$ with the velocity vector, then the ratio of centripetal acceleration to its tangential acceleration is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

145. A body is moving is x-y plane as shown in a circular path of radius 2 m. At a certain instant when the body is crossing the positive y-axis its acceleration is $\left(6\hat{i}-8\hat{j}\right)$ ms^{-2}. Then its angular acceleration and angular velocity at this instant will be



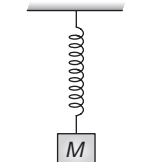
(A) $-3\hat{k}$ rads^{-2} and $-2\hat{k}$ rads^{-1} respectively

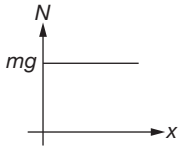
(B) $+3\hat{k}$ rads^{-2} and $+2\hat{k}$ rads^{-1} respectively

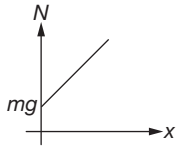
(C) $-4\hat{k}$ rads^{-2} and $-\sqrt{3}\hat{k}$ rads^{-1} respectively

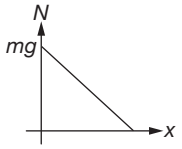
(D) $+4\hat{k}$ rads^{-2} and $+\sqrt{3}\hat{k}$ rads^{-1} respectively

146. A block of mass m is attached to an ideal spring and system lies in vertical plane as shown. Initially the supporting plane is placed so that spring remains in its natural length then the plane is moved very slowly downwards. The graph showing variation of normal reaction applied by mass on supporting plane with distance travelled by block is



(A)


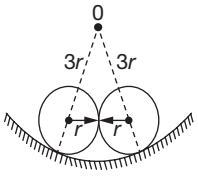
(B)


(C)


(D) None of these

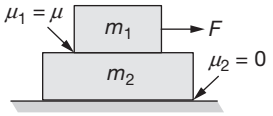
147. Two identical heavy spheres of equal mass are placed on smooth cup of radius $3r$, where r is radius of each sphere as shown. Then the ratio of reaction

force between cup and any sphere to reaction force between two sphere is



(A) 1 (B) 2
(C) 3 (D) None of these

148. In the given figure, the coefficient of friction between m_1 and m_2 is μ and m_2 and horizontal surface is zero



(A) If $F > \mu m_1 g$, then relative acceleration is found between m_1 and m_2
(B) If $F < \mu m_1 g$, then no relative acceleration is found between m_1 and m_2
(C) If $F > \mu m_1 g$, then both bodies move together
(D) (A) and (B) are correct

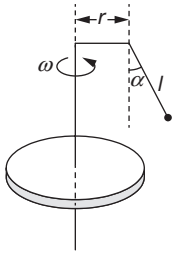
149. The force required to just move a body up the inclined plane is double the force required to just prevent the body from sliding down the plane. The coefficient of friction is μ. The inclination θ of the plane is

(A) $\tan^{-1}(\mu)$ (B) $\tan^{-1}\left(\frac{\mu}{2}\right)$
(C) $\tan^{-1}(2\mu)$ (D) $\tan^{-1}(3\mu)$

150. A circular tube of radius R and cross-sectional radius $r (r \ll R)$ is filled completely with iron balls of radius ρ. Iron balls are just fitting into the tubes. The tension in the tube when it is rotated about its axis perpendicular to its plane with angular velocity ω is

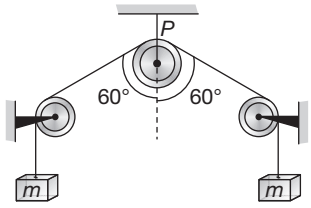
(A) $\frac{4}{3}\pi\rho\omega^2 r^3 R$ (B) $\frac{4}{3}\pi\rho\omega^2 r^2 R^2$
(C) $\frac{2}{3}\pi\rho\omega^2 r^3 R$ (D) $\frac{2}{3}\pi\rho\omega^2 r^2 R^2$

151. A plumb-line is set up on a rotating disk and makes an angle of α with the vertical, as in figure. The distance r from the point of suspension to the axis of rotation is known, and so is the length l of the thread. The angular velocity of rotation is



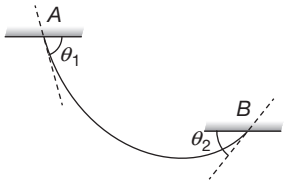
(A) $\omega = \sqrt{\frac{g\tan\alpha}{r + l\sin\alpha}}$ (B) $\omega = \sqrt{\frac{g\sin\alpha}{r + l\tan\alpha}}$
(C) $\omega = \sqrt{\frac{g}{r + l\sin\alpha}}$ (D) $\omega = \sqrt{\frac{g\cos\alpha}{r + l\sin\alpha}}$

152. The force exerted by the ideal string on the ideal pulley P is



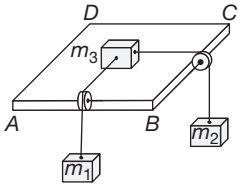
(A) mg (B) $2mg$
(C) $\sqrt{2}mg$ (D) $4mg$

153. A rope of mass M is hanged from two supports A and B as shown in figure. Maximum and minimum tension in the rope respectively are



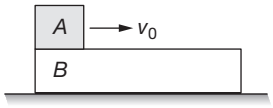
(A) $\frac{Mg\cos\theta_2}{\sin(\theta_1+\theta_2)}, \frac{Mg\cos\theta_1}{\sin(\theta_1+\theta_2)}$
(B) Mg, $Mg\cos\theta_1$
(C) $\frac{Mg\cos\theta_2}{\sin(\theta_1+\theta_2)}, \frac{Mg\cos\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)}$
(D) $\frac{Mg\cos\theta_2}{\cos(\theta_2-\theta_1)}, \frac{Mg\cos\theta_1}{\sin(\theta_1+\theta_2)}$

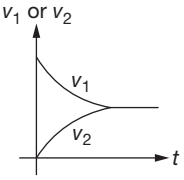
154. Three blocks are arranged as shown in which $ABCD$ is a horizontal plane. Strings are massless and both pulley stands vertical while the strings connecting blocks m_1 and m_2 are also vertical and are perpendicular to faces AB and BC which are mutually perpendicular to each other. If m_1 and m_2 are 3 kg and 4 kg respectively. Coefficient of friction between the block $m_3 = 10$ kg and surface is $\mu = 0.6$, then, frictional force on m_3 is

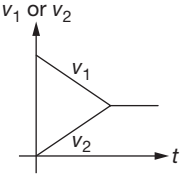


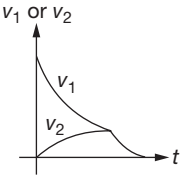
(A) 30 N (B) 40 N
(C) 50 N (D) 60 N

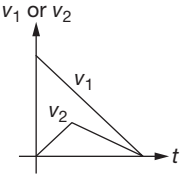
155. A block A is placed over a long rough plank B of same mass as shown in figure. The plank is placed over a smooth horizontal surface. At time $t = 0$, block A is given a velocity v_0 in horizontal direction. Let v_1 and v_2 be the velocities of A and B at time t. Then choose the correct graph between v_1 or v_2 and t



(A) 

(B) 

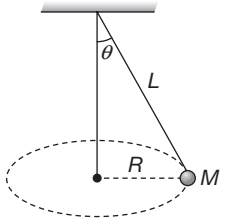
(C) 

(D) 

156. Which of the following correctly describes the centripetal acceleration vector for a particle moving in a circular path?

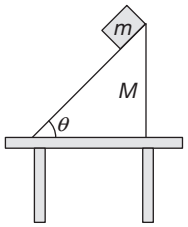
(A) Constant and always perpendicular to the velocity vector for the particle
(B) Constant and always parallel to the velocity vector for the particle
(C) Of constant magnitude and always perpendicular to the velocity for the particle
(D) Of constant magnitude and always parallel to the velocity vector for the particle

157. A string of length L is fixed at one end and carries a mass M at the other end. The string makes $\frac{2}{\pi}$ revolutions per second around the vertical axis through the fixed end as shown in the figure, then tension in the string is



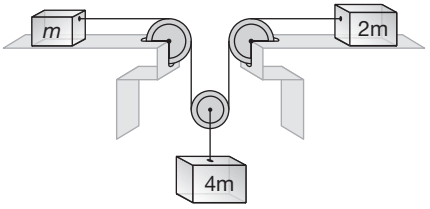
(A) ML (B) $2ML$
(C) $4ML$ (D) $16ML$

158. A block of mass m is slipping down a rough inclined wedge kept at rest on a horizontal table, with constant velocity. Force on wedge by the table is



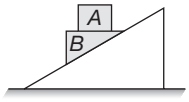
(A) less than $(m+M)g$
(B) equal to $(m+M)g$
(C) equal to $(M+m\cos^2\theta)g$
(D) will make some non-zero angle with vertical direction

159. All surface are frictionless and pulley and strings are light. Acceleration of block of mass m is



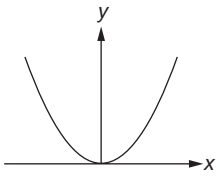
(A) $\frac{2}{5}g$ (B) $\frac{4g}{5}$
(C) $\frac{4g}{7}$ (D) $\frac{2g}{7}$

160. Block A is placed over the block B as shown in figure. Wedge is smooth and fixed. Force of friction on block A is



(A) towards right
(B) towards left
(C) zero
(D) always kinetic

161. A parabolic bowl with its bottom at origin has the shape $y = \frac{x^2}{20}$. Here x and y are in metre. The maximum height at which a small mass m can be placed on the bowl without slipping (coefficient of static friction is 0.5) is



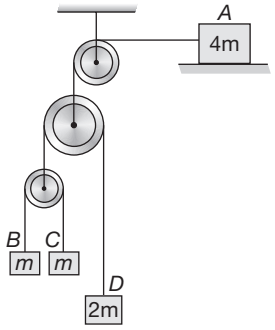
(A) 2.5 m (B) 1.25 m
(C) 1 m (D) 4 m

162. A particle describes a horizontal circle in a conical funnel whose inner surface is smooth with speed of $0.5\ \text{ms}^{-1}$. The height of the plane of circle from vertex of the funnel is
(A) 0.25 cm (B) 2 cm
(C) 4 cm (D) 2.5 cm

163. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is
(A) 60 (B) 30
(C) 15 (D) 25

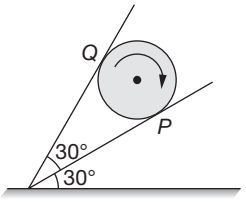
164. A bunch of bananas hangs from the end of a rope that passes over a light, frictionless pulley. A monkey of mass equal to the mass of the bananas hangs from the other end of the rope. The monkey and the bananas are initially balanced and at rest. Now the monkey starts to climb up the rope, moving away from the ground with speed v. What happens to the bananas?
(A) They move downward at speed v
(B) They remain stationary
(C) They move up at speed $\frac{1}{2}v$
(D) They move up at speed v

165. In the given arrangement, strings and pulleys are light and all surface are frictionless. Assuming at $t = 0$, system is released from rest, the speed of block A at $t = 2$ sec is



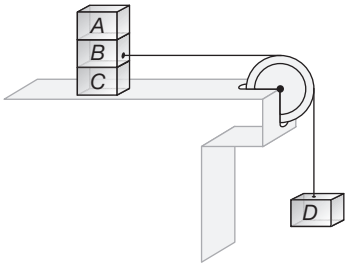
(A) $8\ \text{ms}^{-1}$ (B) $9\ \text{ms}^{-1}$
(C) $10\ \text{ms}^{-1}$ (D) None of these

166. A sphere is rotating between two rough inclined walls as shown in Figure.



The coefficient of friction between each wall and the sphere is $\frac{1}{3}$. If f_1 and f_2 be the friction forces at P and Q. Then $\frac{f_1}{f_2}$ is
(A) $\frac{4}{\sqrt{3}}+1$ (B) $\frac{1}{\sqrt{3}}+2$
(C) $\frac{1}{2}+\sqrt{3}$ (D) $1+2\sqrt{3}$

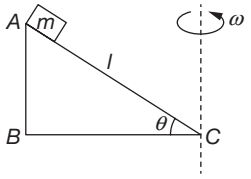
167. Three blocks A, B and C of equal mass m are placed one over the other on a smooth horizontal ground as shown in figure. Coefficient of friction between any two blocks of A, B and C is $\frac{1}{2}$. The maximum value of mass of block D so that the blocks A, B and C move without slipping over each other is



(A) $6m$ (B) $5m$
(C) $3m$ (D) $4m$

168. A small particle of mass 0.36 g rests on a horizontal turntable at a distance 25 cm from the axis of spindle. The turntable is accelerated at a rate of $\frac{1}{3}\ \text{rads}^{-2}$. The frictional force that the table exerts on the particle 2 s after the start up is
(A) 40 μN (B) 30 μN
(C) 50 μN (D) 60 μN

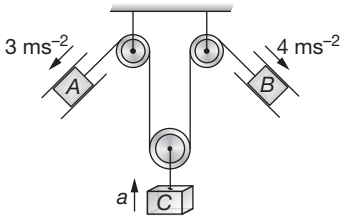
169. A block of mass m is placed at the top of a smooth wedge ABC. The wedge is rotated about an axis passing through C as shown in the figure. The minimum value of angular speed ω such that the block does not slip on the wedge is



(A) $\left(\sqrt{\frac{g\sin\theta}{l}}\right)\sec\theta$ (B) $\left(\sqrt{\frac{g}{l}}\right)\cos\theta$

(C) $\left(\sqrt{\frac{g}{l\cos\theta}}\right)\cos\theta$ (D) $\sqrt{\frac{g\sin\theta}{l}}$

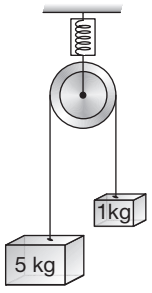
170. Assuming all the surfaces to be frictionless, the acceleration of block C shown in the figure is



(A) $5\ \text{ms}^{-2}$ (B) $7\ \text{ms}^{-2}$

(C) $3.5\ \text{ms}^{-2}$ (D) $4\ \text{ms}^{-2}$

171. In the figure a smooth pulley of negligible weight is suspended by a spring balance. Weights of 1 kg and 5 kg are attached to the opposite ends of a string passing over the pulley and move with acceleration because of gravity. During the motion, the spring balance reads a weight of



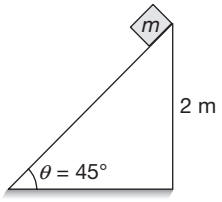
(A) 6 kg

(B) less than 6 kg

(C) more than 6 kg

(D) may be more or less than 6 kg

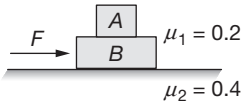
172. A wedge of mass $2m$ and a cube of mass m are shown in figure. Between cube and wedge, there is no friction. The minimum coefficient of friction between wedge and ground so that wedge does not move is



(A) 0.10 (B) 0.20

(C) 0.25 (D) 0.50

173. In the figure, $m_A = 2$ kg and $m_B = 4$ kg. The minimum value of F for which A starts slipping over B is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 24 N (B) 36 N

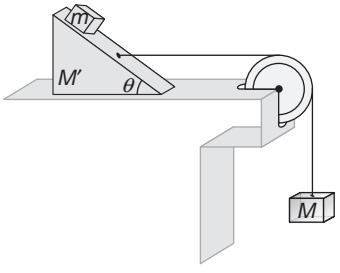
(C) 12 N (D) 20 N

174. Two particles starting from a point on a circle of radius 4 m in horizontal plane move along the circle with constant speeds of $4\ \text{ms}^{-1}$ and $6\ \text{ms}^{-1}$ respectively in opposite directions. The particles will collide with each other after a time of

(A) 3 s (B) 2.5 s

(C) 2 s (D) 1.5 s

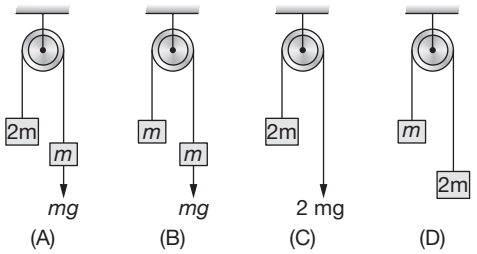
175. The mass M of the hanging block in figure which will prevent the smaller block from slipping over the triangular block (assuming all the surfaces to be frictionless and the strings and the pulleys to be light) is



(A) $\frac{M'+m}{\tan\theta}$ (B) $(M'+m)\tan\theta$

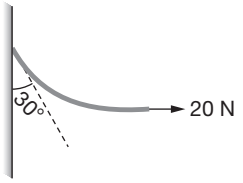
(C) $\frac{M'+m}{1-\tan\theta}$ (D) $\frac{M'+m}{\cot\theta-1}$

176. In the figures, shown which of the system is at rest.



(A) A only (B) A and C only
(C) A, B, and C only (D) C only

177. One end of the rope is fixed to vertical wall and other end is pulled by horizontal force of 20 N. The shape of flexible rope is shown in figure. The mass of rope is



(A) 2 kg (B) 3 kg
(C) 3.5 kg (D) 4.5 kg

178. A body of mass M is kept on a rough horizontal surface (friction coefficient $= \mu$). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on A is F, where

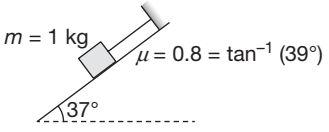
(A) $F = Mg$

(B) $F = \mu Mg$

(C) $Mg \le F \le Mg\sqrt{1+\mu^2}$

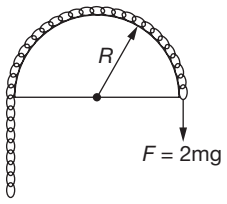
(D) $Mg \ge F \ge Mg\sqrt{1-\mu^2}$

179. For the arrangement shown in figure, the tension in the string to prevent it from sliding down is



(A) 6 N (B) 6.4 N
(C) 0.4 N (D) None of these

180. A chain of length $\frac{3}{2}\pi R$ and mass m is put on a mounted half cylinder as shown in figure. Chain is pulled by vertically downward force $2mg$. Assuming the surface to be frictionless, acceleration of chain is

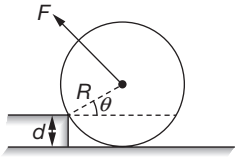


(A) $2g$ (B) $\frac{2g}{3}$
(C) $\frac{g}{2}$ (D) $\frac{5g}{3}$

181. Which of the following quantities are independent of reference frame Chosen. (Consider classical mechanics only)

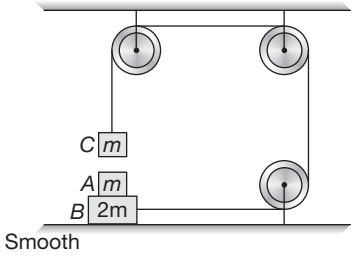
(A) Force
(B) Acceleration
(C) Total energy of an isolated system
(D) None of these

182. The wheel has weight W. The minimum value of F required to pull the wheel up the step is



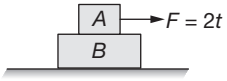
(A) $W\cos\theta$ (B) $W\tan\theta$
(C) $W\sin\theta$ (D) $W\cot\theta$

183. In the arrangement shown in figure. There is a friction force between the block of masses m and $2m$. Block of mass $2m$ is kept on a smooth horizontal plane. The mass of suspended block is m. Block A is stationary with respect to block of mass $2m$. The minimum value of coefficient of friction between m and $2m$ is



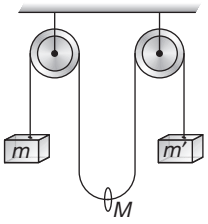
(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{4}$ (D) $\frac{1}{3}$

184. Two blocks A and B of mass 2 kg and 4 kg are placed one over the other. A horizontal force $F = 2t$, which varies with time is applied on the upper block. If coefficient of friction between blocks A and B is 0.5 and horizontal surface is smooth. Then assuming t is in seconds, the total time upto which both blocks will move together without slipping over each other is



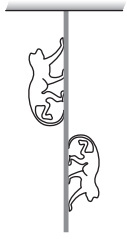
(A) 5 sec (B) 7.5 sec
(C) 10 sec (D) 12.5 sec

185. A smooth ring of mass M is threaded on a string as shown in the figure. Various portions of strings are vertical. If the ring alone is to remain at rest, then



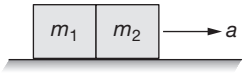
(A) $\frac{4}{M} = \frac{1}{m} + \frac{1}{m'}$ (B) $\frac{2}{M} = \frac{1}{m} + \frac{1}{m'}$
(C) $\frac{1}{M} = \frac{1}{m} + \frac{1}{m'}$ (D) $\frac{3}{M} = \frac{1}{m} + \frac{1}{m'}$

186. Two monkeys of masses 5 kg and 4 kg are moving along a vertical rope, the former climbing up with an acceleration of $2\ \text{ms}^{-2}$ while the later coming down with a uniform velocity of $2\ \text{ms}^{-1}$. Tension in the rope at the fixed support is



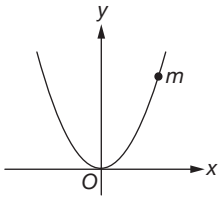
(A) 80 N (B) 100 N
(C) 98 N (D) 108 N

187. Two blocks of masses m_1 and m_2 are placed in contact with each other on a horizontal platform. The coefficient of friction between the platform and the two blocks is the same. The platform moves with an acceleration. The force of interaction between the blocks is



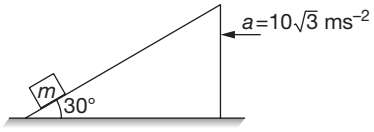
(A) zero in all cases
(B) zero only if $m_1 = m_2$
(C) nonzero only if $m_1 > m_2$
(D) nonzero only if $m_1 < m_2$

188. A bead of mass m is located on a parabolic wire with its axis vertical and vertex directed towards downward as in figure and whose equation is $x^2 = ay$. If the coefficient of friction is μ, the highest distance above the x-axis at which the particle will be in equilibrium is



(A) μa (B) $\mu^2 a$
(C) $\frac{1}{4}\mu^2 a$ (D) $\frac{1}{2}\mu a$

189. In the figure, the wedge is accelerated to the left with an acceleration of $10\sqrt{3}\ \text{ms}^{-2}$. It is seen that the block starts climbing upon the smooth inclined face of wedge. The time taken by the block to reach the top if length of the incline is 1.25 m is

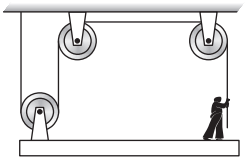


(A) 2 s (B) 1 s
(C) $\frac{1}{\sqrt{2}}$ s (D) $\frac{1}{2}$ s

190. A body is thrown up in a lift with a velocity $5\ \text{ms}^{-1}$ relative to the lift and the time of flight is found to be 0.8 sec. The acceleration with which the lift is moving up will be $\left(g = 10\ \text{ms}^{-2}\right)$

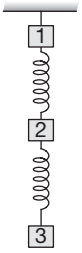
(A) $1.5\ \text{ms}^{-2}$ (B) $2\ \text{ms}^{-2}$
(C) $2.5\ \text{ms}^{-2}$ (D) $3\ \text{ms}^{-2}$

191. A person of 50 kg stands on a 25 kg platform. He pulls on the rope which is attached to the platform via the frictionless pulleys as shown in the figure. The platform moves upwards at a constant velocity if the force with which the person pulls the rope is



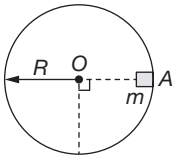
(A) 500 N (B) 250 N
(C) 25 N (D) 50 N

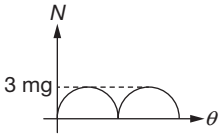
192. Three identical blocks are suspended on two identical springs one below the other as shown in figure. If thread is cut that supports block 1, then initially (choose one alternative only)

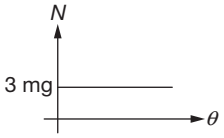


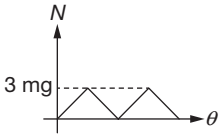
(A) the second block falls with zero acceleration
(B) the first block falls with maximum acceleration
(C) both (A) and (B) are wrong
(D) both (A) and (B) are correct

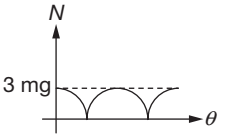
193. A particle of mass m is released from point A on smooth fixed circular track as shown. If the particle is released from rest at $t = 0$, then variation of normal reaction N with (θ) angular displacement from initial position is



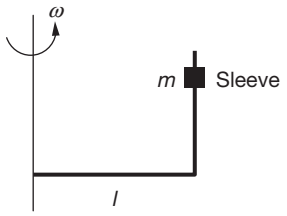
(A) 

(B) 

(C) 

(D) 

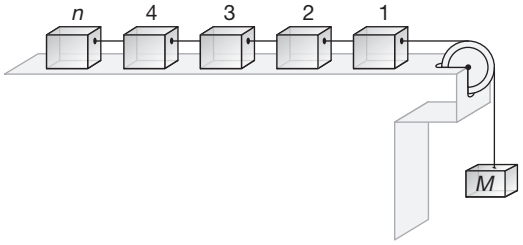
194. A L shaped rod whose one rod is horizontal and other is vertical is rotating about a vertical axis as shown with angular speed ω. The sleeve shown in figure has mass m and friction coefficient between rod and sleeve is μ. The minimum angular speed ω for which sleeve cannot sleep on rod is



(A) $\omega = \sqrt{\dfrac{g}{\mu l}}$ (B) $\omega = \sqrt{\dfrac{\mu g}{l}}$
(C) $\omega = \sqrt{\dfrac{l}{\mu g}}$ (D) None of these

195. A bullet moving with a velocity of 100 ms^{-1} can just penetrate two planks of equal thickness. The number of such planks penetrated by the same bullet, when the velocity is doubled, will be
(A) 4 (B) 6
(C) 8 (D) 10

196. In the given arrangement, n number of equal masses are connected by strings of negligible masses. The tension in the string connected to nth mass is

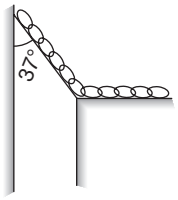


(A) $\dfrac{mMg}{nm + M}$ (B) $\dfrac{mMg}{nmM}$
(C) mg (D) mng

197. A chain consisting of 5 links each of mass 0.1 kg is lifted vertically with a constant acceleration of 2.5 ms^{-2}. The force of interaction between the top link and the link immediately below it, will be
(A) 6.15 N (B) 4.92 N
(C) 3.69 N (D) 2046 N

198. A ball of mass 400 g is dropped from a height of 5 m. A boy on the ground hits the ball vertically upwards with a bat with an average force of 100 N so that it attains a vertical height of 20 m. The time for which the ball remains in contact with the bat is $(g = 10\ ms^{-2})$
(A) 0.12 sec (B) 0.08 sec
(C) 0.04 sec (D) 12 sec

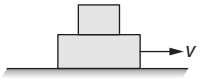
199. A uniform chain of length L has one of its ends attached to the wall at point A, while $\frac{3L}{4}$ of the length of chain is lying on table as shown in figure. The minimum coefficient of friction between table and chain, so that chain remains in equilibrium is



(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{3}{4}$ (D) $\frac{1}{5}$

200. A block is resting on a horizontal plate in the XY plane and co-efficient of friction between block and the plate is μ. The plate begins to move with velocity $v = bt^2$ in X-direction. At what time will the block starts sliding from plate



(A) $\frac{\mu b}{g}$ (B) $\frac{\mu b g}{2}$

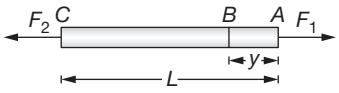
(C) $\frac{\mu g}{b}$ (D) $\frac{\mu g}{2b}$

201. 80 railway wagons all of same mass 5×10^3 kg are pulled by an engine with a force of 4×10^5 N. The tension in the coupling between 30^{th} and 31^{st} wagon from the engine is

(A) 25×10^4 N (B) 20×10^4 N

(C) 32×10^4 N (D) 40×10^4 N

202. A rod of length L and mass M is acted on by two unequal forces F_1 and $F_2 (< F_1)$ as shown in the following figure. The tension in the rod at a distance y from the end A is given by



(A) $F_1\left(1-\frac{y}{L}\right)+F_2\left(\frac{y}{L}\right)$ (B) $F_2\left(1-\frac{y}{L}\right)+F_1\left(\frac{y}{L}\right)$

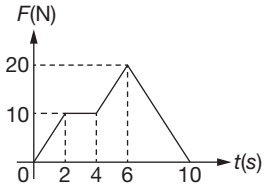
(C) $(F_1-F_2)\frac{y}{L}$ (D) $(F_1-F_2)\frac{y}{2L}$

203. A disk of mass 10 g is kept floating horizontally in the air by firing bullets each of mass 5 g with the same velocity. If 10 bullets are fired per second and the bullets rebound with the same speed, then the velocity of each bullet is

(A) 196 cms^{-1} (B) 98 cms^{-1}

(C) 49 cms^{-1} (D) None of these

204. A particle of mass 2 kg is initially at rest. A force acts on it whose magnitude changes with time. The force time graph is shown below. The velocity of the particle after 10 s is



(A) 20 ms^{-1} (B) 10 ms^{-1}

(C) 75 ms^{-1} (D) 50 ms^{-1}

205. A body is moving down a long inclined plane of slope $37°$. The coefficient of friction between the body and plane varies as $\mu = 0.3x$, where x is the distance travelled down the plane. The body will have maximum speed at

(Given that $\sin(37°)=\frac{3}{5}$ and $g = 10\ ms^{-2}$)

(A) $x = 1.16$ m (B) $x = 2$ m

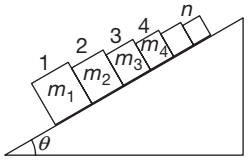
(C) the bottom of plane (D) $x = 2.5$ m

206. A 60 kg body is pushed with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficient of static friction and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is

(A) 6 ms^{-2} (B) 4.9 ms^{-2}

(C) 3.92 ms^{-2} (D) 1 ms^{-2}

207. n-blocks of different masses are placed on the frictionless inclined plane in contact. They are released at the same time. The force of interaction between $(n-1)$th and nth block is



(A) $(m_{n-1} - m_n)g\sin\theta$
(B) zero
(C) $mng\cos\theta$
(D) None of these

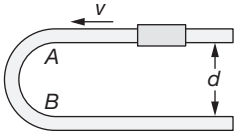
208. A rocket of mass 120 kg is fired in the gravity free space. It ejects gases with velocity 600 ms^{-1} at the rate of 1 kgs^{-1}. What will be the initial acceleration of the rocket?

(A) 1 ms^{-2} (B) 5 ms^{-2}
(C) 10 ms^{-2} (D) 15 ms^{-2}

209. In order to raise a mass of 100 kg, a man of mass 60 kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with an acceleration $\frac{5g}{4}$ relative to the rope. The tension in the rope is $(g = 10\ ms^{-2})$

(A) 1432 N (B) 928 N
(C) 1218 N (D) 642 N

210. A U-shaped smooth wire has a semicircular bending between A and B as shown in the figure. A bead of mass m moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B. The average force, exerted by the bead on the part AB of the wire is



(A) 0 (B) $\frac{4mv^2}{\pi d}$
(C) $\frac{2mv^2}{\pi d}$ (D) None of these

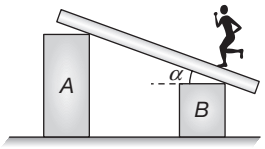
211. A suitcase is gently dropped on a conveyor belt moving at 3 ms^{-1}. If the coefficient of friction between the belt and the suitcase is 0.5, find the displacement of the suitcase relative to conveyor belt before the slipping between the two is stopped $(g = 10\ ms^{-2})$

(A) 2.7 m (B) 1.8 m
(C) 0.9 m (D) 1.2 m

212. A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance R from the centre of the record. The static coefficient of friction is μ. The coin will revolve with the record if

(A) $R > \frac{\mu g}{\omega^2}$ (B) $R = \frac{\mu g}{\omega^2}$ only
(C) $R < \frac{\mu g}{\omega^2}$ (D) $R \le \frac{\mu g}{\omega^2}$

213. A plank is held at an angle α to the horizontal (figure) on two fixed supports A and B. The plank can slide against the supports (without friction) because of it's weight Mg. Acceleration and direction in which a man of mass m should move so that the plank does not move

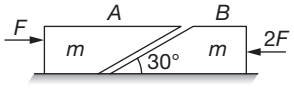


(A) $g\sin\alpha\left[1+\frac{m}{M}\right]$ down the incline
(B) $g\sin\alpha\left[1+\frac{M}{m}\right]$ down the incline
(C) $g\sin\alpha\left[1+\frac{m}{M}\right]$ up the incline
(D) $g\sin\alpha\left[1+\frac{M}{m}\right]$ up the incline

214. Given that $m_A = 20$ kg, $m_B = 10$ kg, $m_C = 20$ kg, the coefficient of friction between B and C is $\mu_2 = 0.3$ and that between C and ground $\mu_3 = 0.1$. The least horizontal force F to start the motion of any part of the system of three blocks resting upon one another as shown in figure is $(g = 10\ ms^{-2})$

(A) 60 N (B) 90 N
(C) 80 N (D) 50 N

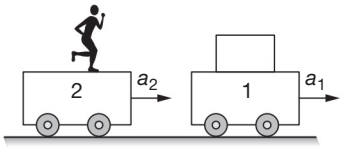
215. Two blocks A and B each of mass m are placed on a smooth horizontal surface. Two horizontal force F and $2F$ are applied on the 2 blocks A and B respectively as shown in figure. The block A does not slide on block B. Then the normal reaction acting between the two blocks is



(A) F (B) $\frac{F}{2}$

(C) $\frac{F}{\sqrt{3}}$ (D) $3F$

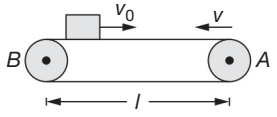
216. Two trolleys 1 and 2 are moving with acceleration a_1 and a_2 respectively in the same direction. A block of mass m on trolleys 1 is in equilibrium from the frame of observer stationary w.r.t. trolleys 2. The magnitude of friction on block due to trolley is



(A) $m(a_1 - a_2)$ (B) ma_2

(C) ma_1 (D) None of these

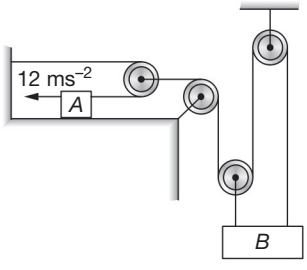
217. A conveyer belt of length l is moving with velocity v. A block of mass m is pushed against the motion of conveyer belt with velocity v_0 from end B with respect to conveyer belt. Co-efficient of friction between block and belt is μ. The value of v_0 so that the amount of heat liberated as a result of retardation of the block by conveyer belt is maximum is



(A) $\sqrt{\mu gl}$ (B) $\sqrt{2\mu gl}$

(C) $2\sqrt{\mu gl}$ (D) $\sqrt{3\mu gl}$

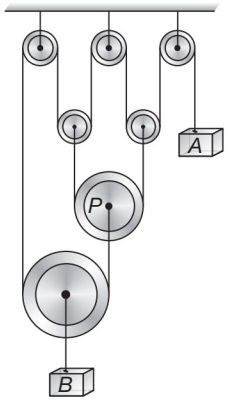
218. Assuming that the is block always remains horizontal, hence the acceleration of B is



(A) 6 ms^{-2} (B) 2 ms^{-2}

(C) 4 ms^{-2} (D) None of these

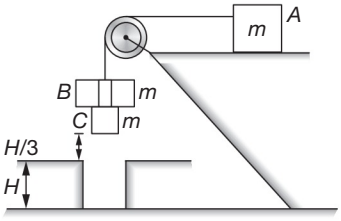
219. All pulley shown in figure are light and there is no friction between pulley and string. Acceleration of pulley P in figure is given by



(A) g upward (B) g downward

(C) zero (D) None of these

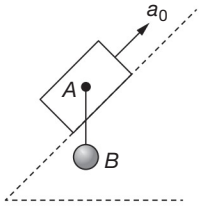
220. The system shown in figure is released from rest. After blocks have moved through a distance $\frac{H}{3}$, collar B stops and blocks A and C continue to move. The speed of C before it hits ground is



(A) $\frac{4}{3}\sqrt{gH}$ (B) $2\sqrt{\frac{gH}{3}}$

(C) $\frac{\sqrt{13gH}}{3}$ (D) $2\sqrt{2gH}$

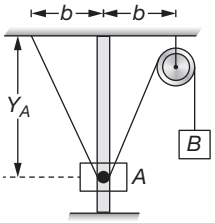
221. A block A is made to move over an inclined plane of inclination θ with constant acceleration a_0 as shown in figure. Initially bob B hanging from block A is held vertical. Magnitude of acceleration of block A relative to bob immediately after bob is released is



(A) a_0 (B) $a_0 \sin\theta$

(C) $a_0 \cos\theta$ (D) $(a_0 - g\sin\theta)$

222. The massless string connecting slider A and mass B passes over two small massless smooth pulley, one which is attached to A. The end of strings are attached to mass B and roof as shown. Mass B has constant velocity V_0 directed downwards. At the instant when $Y_A = b$, the magnitude of acceleration of A is



(A) $\frac{V_0^2}{b}$ (B) $\frac{V_0^2}{2b}$

(C) $\frac{V_0^2}{4b}$ (D) None of these

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels (front and rear) is F_1 and F_2 respectively. Then,

(A) F_1 is in backward direction and F_2 is in the forward direction.

(B) F_1 is in the forward direction and F_2 is in the backward direction.

(C) both F_1 and F_2 are in forward direction.

(D) both F_1 and F_2 are in backward direction.

2. A reference frame attached to the earth

(A) is an inertial frame by definition.

(B) cannot be an inertial frame because of revolution of earth round the sun.

(C) is an inertial frame because Newton's laws are applicable in this frame.

(D) cannot be an inertial frame because of rotation of earth about its own axis.

3. A small mirror of area A and mass m is suspended in a vertical plane by a light string. A beam of light of intensity I falls normally on the mirror and the string is deflected from the vertical by an angle θ. Assuming the mirror to be perfectly reflecting we have

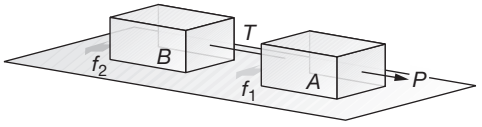
(A) radiation pressure equal to $\frac{2I}{c}$

(B) radiation pressure equal to $\frac{I}{2c}$

(C) $\tan\theta = \frac{2IA}{mgc}$

(D) $\tan\theta = \frac{IA}{2mgc}$

4. Two blocks A and B of the same mass are joined by a light inextensible string and placed on a horizontal surface. An external horizontal force P acts on A. The tension in the string is T. The forces of friction acting on A and B are f_1 and f_2 respectively. The limiting value of f_1 and f_2 is f_0. When P is gradually increased, then



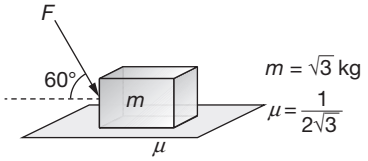
(A) for $P < f_0$, $T = 0$

(B) for $f_0 < P < 2f_0$, $T = P - f_0$

(C) for $P > 2f_0$, $T = \frac{P}{2}$

(D) None of the above

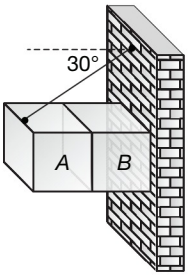
5. The maximum value of the force F such that the block shown in the arrangement, does not move is



(A) 20 N (B) 10 N

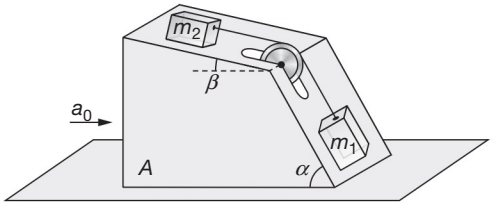
(C) 12 N (D) 15 N

6. Two blocks A and B of mass 10 kg and 20 kg, respectively are placed as shown in figure. Coefficient of friction between all the surfaces is 0.2 $(g = 10\ \text{ms}^{-2})$. Then



(A) tension in the string is 306 N
(B) tension in the string is 132 N
(C) force exerted by the wall on B is 265 N
(D) acceleration of block B is 4.7 ms^{-2}

7. Two cubes of masses m_1 and m_2 lie on frictionless slopes of a block A which rests on a horizontal table. The cubes are connected by a string which passes over a pulley as shown in figure. If a_0 be the horizontal acceleration to which the whole system (block + masses) is subjected so that m_1 and m_2 do not move and T be the tension in the string in that situation then find a_0 and T



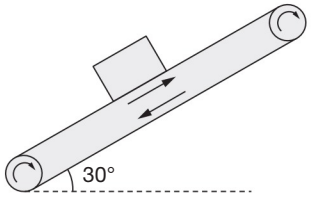
(A) $a_0 = \left(\dfrac{m_1 \sin\alpha + m_2 \sin\beta}{m_1 + m_2}\right) g$

(B) $a_0 = \left(\dfrac{m_1 \sin\alpha + m_2 \sin\beta}{m_1 \cos\alpha + m_2 \cos\beta}\right) g$

(C) $T = \dfrac{m_1 m_2}{m_1 + m_2} g \sin(\alpha + \beta)$

(D) $T = \left(\dfrac{m_1 m_2}{m_1 \cos\alpha + m_2 \cos\beta}\right) g \sin(\alpha - \beta)$

8. A block of mass 1 kg is stationary with respect to a conveyor belt that is accelerating with 1 ms^{-2} upwards at an angle of 30° as shown in figure. Which of the following statement(s) is/are correct. $(g = 10\ ms^{-2})$



(A) force of friction on the block is 1 N upwards
(B) force of friction on the block is 6 N upwards
(C) contact force between the block and the belt is 10.5 N
(D) contact force between the block and the belt is $5\sqrt{3}$ N

9. A body is kept on a smooth inclined plane having an inclination of 1 in x. Then,

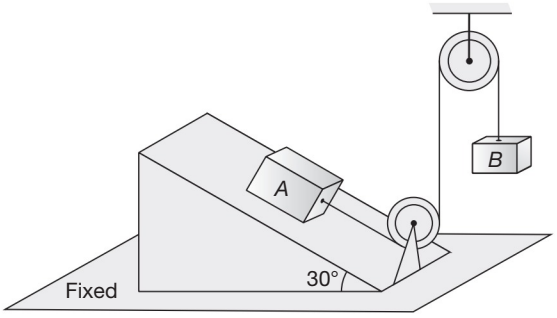
(A) slope of inclined plane is $\dfrac{1}{x}$.

(B) slope of inclined plane is $\dfrac{1}{\sqrt{x^2 - 1}}$.

(C) for the body of mass m to remain stationary relative to the incline, the incline must offer a normal reaction of $mg\dfrac{x}{\sqrt{x^2 - 1}}$.

(D) for the body to remain stationary relative to the incline, the incline must be given a horizontal acceleration of $\dfrac{g}{\sqrt{x^2 - 1}}$.

10. Two blocks A and B of equal mass m are connected through a light inextensible string and arranged as shown in figure. Friction is absent everywhere. When the system is released from rest



(A) tension in string is $\dfrac{mg}{2}$.

(B) tension in string is $\dfrac{mg}{4}$.

(C) acceleration of A is $\dfrac{g}{2}$.

(D) acceleration of A is $\dfrac{3}{4}g$.

11. A block of weight W is suspended from a spring balance. The lower surface of the block rests on a weighing machine. The spring balance reads W_1 and the weighing machine reads W_2. Assuming all W, W_1, W_2 to have the same units, which of the following is/are correct?

(A) $W = W_1 + W_2$, when the system is at rest.

(B) $W > W_1 + W_2$, when the system moves down with some acceleration.

(C) $W_1 > W_2$, when the system moves up with some acceleration.

(D) Data is insufficient to find the relation between W_1 and W_2.

12. A uniform chain of length L lies on a smooth horizontal table with its length perpendicular to the edge of the table and a small portion of the chain is hanging over the edge. The chain starts sliding due to the weight of the hanging part.

(A) The acceleration of the chain is $\frac{gx}{L}$; where x is the length of the hanging part of chain.

(B) The acceleration of the chain is $\frac{g}{L}(L-x)$; where x is the length of the hanging part of chain.

(C) The velocity of the chain is $x\sqrt{\frac{g}{L}}$, where x is the length of the hanging part of chain.

(D) The velocity of the chain is $(L-x)\sqrt{\frac{g}{L}}$; where x is the length of the hanging part of chain.

13. An amusement park ride called "The Spinning Terror" is a large vertical drum which spins so fast that everyone inside stays pinned against the wall when the floor drops away

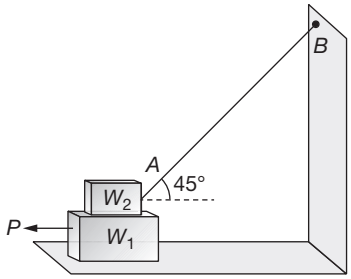
(A) The minimum angular velocity is $\omega_{\min} = \sqrt{\frac{g}{\mu R}}$ for everyone to stay inside.

(B) The minimum linear velocity is $v_{\min} = \sqrt{\frac{g}{\mu R}}$ for everyone to stay inside.

(C) The minimum angular velocity is $\omega_{\min} = \sqrt{\frac{gR}{\mu}}$ for everyone to stay inside.

(D) The minimum linear velocity is $v_{\min} = \sqrt{\frac{gR}{\mu}}$ for everyone to stay inside.

14. In the arrangement shown W_1 = 200 N, W_2 = 100 N, $\mu = 0.25$ for all surfaces in contact. The block W_1 just slides under the block W_2.



(A) A pull of 50 N is to be applied on W_1.

(B) A pull of 90 N is to be applied on W_1.

(C) Tension in the string AB is $10\sqrt{2}$ N.

(D) Tension in the string AB is $20\sqrt{2}$ N.

15. A weight W can be just supported on a rough inclined plane by a force P either acting along the plane or horizontally. The angle of friction is ϕ and θ is the angle which incline makes with the horizontal.

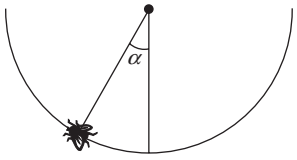
(A) The incline makes an angle with the horizontal twice the angle of friction i.e., $\theta = 2\phi$.

(B) The incline make an angle with the horizontal equal to the angle of friction i.e., $\theta = \phi$.

(C) The ratio of the force to the weight is $\frac{P}{W} = \cot\phi$.

(D) The ratio of the force to the weight is $\frac{P}{W} = \tan\phi$

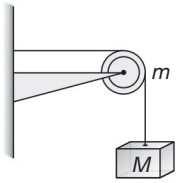
16. An insect crawls up a hemispherical surface very slowly as shown in figure. The coefficient of friction between the surface and the insect is $\frac{1}{3}$. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(A) $\cot\alpha = 3$ (B) $\tan\alpha = 3$

(C) $\sec\alpha = 3$ (D) $\operatorname{cosec}\alpha = 3$

17. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by



(A) $\sqrt{2}Mg$

(B) $\sqrt{2}mg$

(C) $\sqrt{(M+m)^2+m^2}g$

(D) $\sqrt{(M+m)^2+M^2}g$

18. A boy of mass m kg slides down a light rope attached to a fixed spring balance, with an acceleration a. The reading of the spring balance is M kg.

(A) The tension in the rope is $T = Mg$ N

(B) $m = M\left(1+\frac{a}{g}\right)$

(C) $m = \frac{Mg}{g-a}$

(D) The force of friction exerted by the rope on the boy is $f = m(g-a)$ N

19. A block of weight 9.8 N is placed on a table. The table surface exerts an upward force of 10 N on the block. Assume $g = 9.8\ \text{ms}^{-2}$.

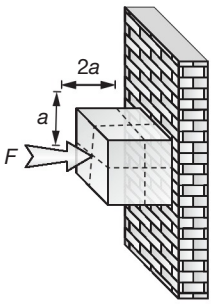
(A) The block exerts a force of 19.8 N on the table.

(B) The block exerts a force of 10 N on the table.

(C) The block has an upward acceleration.

(D) The block exerts a force of 9.8 N on the table.

20. A solid uniform cube of mass m is pressed against a rough vertical wall by applying a force F as shown in the figure. The cube is stationary. Choose the incorrect option



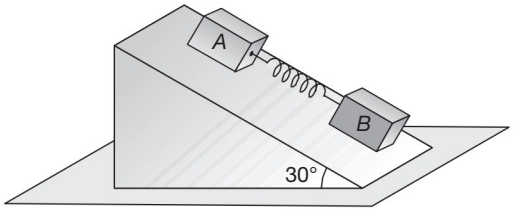
(A) normal force by the wall on the cube is F

(B) friction force on the cube is mg

(C) the torque of force F about the centre of mass of the cube is zero

(D) the torque of normal force about the centre of mass of the cube is zero

21. Two blocks A and B of mass 4 kg and 2 kg respectively connected by a spring of force constant $k = 100\ \text{Nm}^{-1}$ are placed on an inclined plane of inclination 30° as shown in figure. If the system is released from rest, which one of the following statement(s) is/are correct $\left(g = 10\ \text{ms}^{-2}\right)$



(A) there will be no compression/elongation in the spring if all surfaces are smooth

(B) maximum compression of the spring is 10 cm if all surfaces are smooth

(C) maximum elongation in the spring is 60 cm if all the surfaces are smooth

(D) there will be elongation in the spring if A is smooth and B is rough

22. Newton's laws are valid in

(A) any reference frame.

(B) a reference frame moving with constant velocity with respect to an inertial frame.

(C) all reference frames which are at rest with respect to an inertial frame.

(D) the reference frame attached to the earth.

23. A light vertical chain is used to haul up an object of mass M attached to its lower end. The vertical pull applied has a magnitude F at $t = 0$ and it decreases at a uniform rate of $f\ \text{Nm}^{-1}$ over a distance s through which the object is raised.

(A) The acceleration of the object is $\left(\frac{F-fy-Mg}{M}\right)$ when the object is raised through a distance $y(<s)$.

(B) The acceleration of the object is constant.

(C) The object has a velocity $\sqrt{\frac{2s}{M}\left(F-Mg-\frac{fs}{2}\right)}$ when it has been raised through a distance s.

(D) The object has a velocity $\sqrt{\frac{s}{g}}$ when it has been raised through a distance s.

24. A lift is moving downwards. A body of mass m kept on the floor of the lift is pulled horizontally. If μ is the coefficient of friction between the surfaces in contact then,

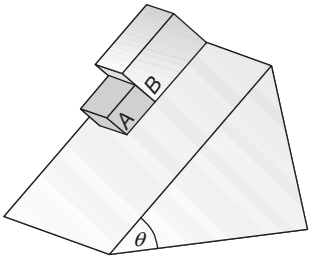
(A) frictional resistance offered by the floor is μmg when lift moves up with a uniform velocity of $5\ \text{ms}^{-1}$.

(B) frictional resistance offered by the floor is μmg when lift moves up with a uniform velocity of $3\ \text{ms}^{-1}$.

(C) frictional resistance offered by the floor is $5\ \text{m}\mu$ when lift accelerates down with an acceleration of $4.8\ \text{ms}^{-2}$.

(D) frictional resistance (f) offered by the floor must lie in the range $0 \le f < \infty$.

25. The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane and A and B are μ_1 and μ_2 respectively.



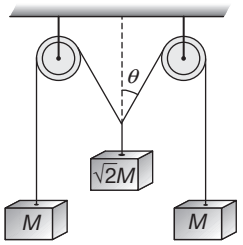
(A) If $\mu_1 > \mu_2$, the blocks will always remain in contact.

(B) If $\mu_1 < \mu_2$, the blocks will slide down with different accelerations.

(C) If $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1 + \mu_2)g\sin\theta$.

(D) If $\mu_1 < \mu_2$, the blocks will have a common acceleration $\frac{\mu_1\mu_2 g}{\mu_1 + \mu_2}\sin\theta$.

26. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be



(A) $0°$ (B) $30°$

(C) $45°$ (D) $60°$

27. A simple pendulum with a bob of mass m is suspended from the roof of a car moving with a horizontal acceleration a.

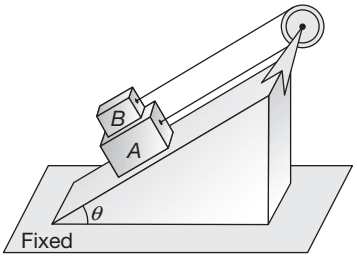
(A) The string makes an angle of $\tan^{-1}\left(\frac{g}{a}\right)$ with the vertical.

(B) The string makes an angle of $\tan^{-1}\left(\frac{a}{g}\right)$ with the vertical.

(C) The tension in the string is $m\sqrt{g^2 - a^2}$.

(D) The tension in the string is $m\sqrt{a^2 + g^2}$.

28. In the arrangement shown in figure pulley is smooth and massless and string is light. Friction coefficient between A and B is μ. Friction is absent between A and plane. Select the correct alternative(s)



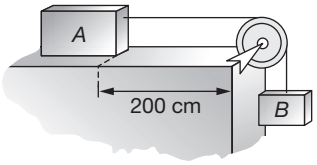
(A) acceleration of the system is zero if $\mu \ge \left(\frac{m_B - m_A}{2m_B}\right)\tan\theta$ and $m_B > m_A$

(B) force of friction between A and B is zero if $m_A = m_B$

(C) B moves upwards if $m_B < m_A$

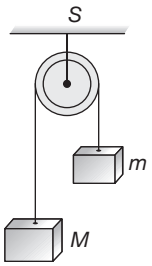
(D) tension in the string is $mg\sin\theta$ if $m_A = m_B = m$

29. A block A of mass 500 g is placed on a smooth horizontal table with a light string attached to it. The string passes over a smooth pulley P at the end of the table (as shown) and is connected to other block B of mass 200 g. Initially the block A is at a distance of 200 cm from the pulley and is moving with a speed of $50\ \text{cms}^{-1}$ to the left. At $t = 1$ s



(A) the block is at a distance of 90 cm from the pulley P.
(B) the block is at a distance of 110 cm from the pulley P.
(C) the block has a velocity of 230 cms^{-1} towards left.
(D) the block has a velocity of 230 cms^{-1} towards right.

30. Two masses m and $M(m<M)$ are joined by a light string passing over a smooth and light pulley (as shown).



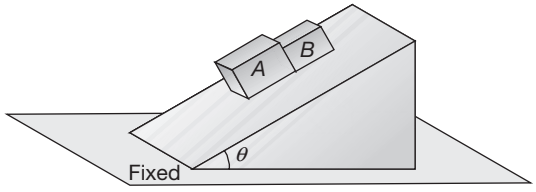
(A) The acceleration of each mass is $\left(\frac{M-m}{M+m}\right)g$.

(B) The tension in the string connecting masses is $\left(\frac{2Mm}{M+m}\right)g$.

(C) The thrust acting on the pulley is $\left(\frac{4Mm}{M+m}\right)g$.

(D) The centre of mass of the system (i.e. M and m) moves down with an acceleration of $\left(\frac{M-m}{M+m}\right)^2 g$.

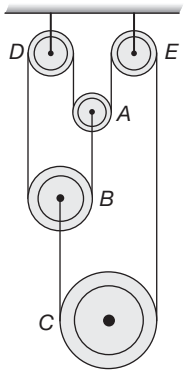
31. In the arrangement shown in figure all surfaces are smooth. Select the correct alternative(s)



(A) contact force between the two blocks is zero if $\frac{m_A}{m_B}=\tan\theta$
(B) for any value of θ acceleration of A and B are equal
(C) normal reactions exerted by the wedge on the blocks are equal
(D) contact force between the two is zero for any value of m_A or m_B

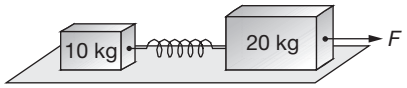
32. A cricket ball of mass 150 g is moving with a velocity of 12 ms^{-1} and is hit by a bat so that it turns back with a velocity of 20 ms^{-1}. The force of blow acts for a time of 0.01 s. Then,
(A) the change in momentum of ball is 4.8 $kgms^{-1}$.
(B) the average force exerted by the bat on the ball is 480 N.
(C) change in momentum of the ball is 1.2 $kgms^{-1}$.
(D) the average force exerted by the bat on the ball is 120 N.

33. In the pulley system shown the movable pulleys A, B and C have mass m each, D and E are fixed pulleys. The strings are vertical, light and inextensible. Then,



(A) the tension throughout the string is the same and equals $T=\frac{2mg}{3}$.
(B) pulleys A and B have acceleration $\frac{g}{3}$ each in downward direction and pulley C has acceleration $\frac{g}{3}$ in upward direction.
(C) pulleys A, B and C all have acceleration $\frac{g}{3}$ in downward direction.
(D) pulley A has acceleration $\frac{g}{3}$ in downward direction and pulleys B and C have acceleration $\frac{g}{3}$ each in upward direction.

34. Two masses of 10 kg and 20 kg are connected by a light spring as shown. A force of 200 N acts on a 20 kg mass as shown. At a certain instant the acceleration of 10 kg mass is 12 ms^{-2}.



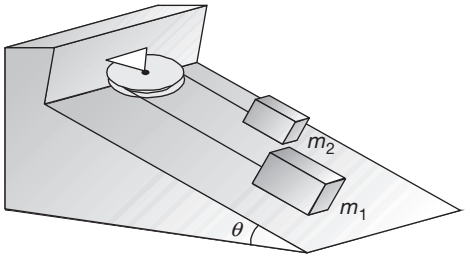
F = 200 N

(A) At that instant the 20 kg mass has an acceleration of $12\ \text{ms}^{-2}$.
(B) At that instant the 20 kg mass has an acceleration of $4\ \text{ms}^{-2}$.
(C) The stretching force in the spring is 120 N.
(D) The collective system moves with a common acceleration of $30\ \text{ms}^{-2}$ when the extension in the connecting spring is the maximum.

35. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
(A) its velocity is constant.
(B) its acceleration is constant.
(C) its kinetic energy is constant.
(D) it moves in a circular path.

36. A particle of mass M is moving with acceleration a_0 as measured by an observer 1 standing in a frame of reference moving with a uniform velocity. Another observer 2 is standing in a frame of reference moving with acceleration a.
(A) Observer 1 measures the force acting on the body as Ma_0.
(B) Observer 2 measures the force acting on the body as Ma.
(C) Observer 2 measures the force acting on the body as $Ma_0 - Ma$.
(D) Observer 1 is standing in an inertial frame of reference and observer 2 is standing in a non-inertial frame of reference.

37. Two blocks on a rough incline are connected by a light string that passes over a frictionless light pulley as shown. Assuming $m_1 > m_2$ and taking the coefficient of kinetic friction for each block to be μ we get acceleration of the blocks as



(A) $a = \dfrac{\left[(m_1 - m_2)\sin\theta - \mu(m_1 + m_2)\cos\theta\right]g}{(m_1 + m_2)}$ for m_1

(B) $a = \dfrac{\left[\mu(m_1 + m_2)\cos\theta - (m_1 - m_2)\sin\theta\right]g}{(m_1 + m_2)}$ for m_2

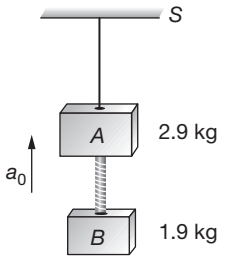
(C) $a = g(\sin\theta - \mu\cos\theta)$

(D) $a = g(\cos\theta - \mu\sin\theta)$

38. Two masses m_1 and m_2 are connected by a light string which passes over the top of an inclined plane making an angle of $30°$ with the horizontal such that one mass rests on the plane and other hangs vertically. It is found that m_1 hanging vertically can draw m_2 up the full length of the incline in half the time in which m_2 hanging vertically draws m_1 up the full length of the incline. Assuming the surfaces in contact to be frictionless and pulley to be light and smooth, we have
(A) the ratio of acceleration in the two cases respectively as 4.
(B) the ratio of acceleration in the two cases respectively as $\frac{1}{4}$.
(C) the ratio of masses m_1 and m_2 as $\frac{3}{2}$.
(D) the ratio of masses m_1 and m_2 as $\frac{2}{3}$.

39. Which of the following statement(s) is (are) correct about friction ?
(A) The coefficient of friction between two bodies is largely independent of area of contact.
(B) The frictional force can never exceed the reaction force on a body from the supporting surface.
(C) Rolling friction is generally smaller than sliding friction.
(D) Friction is due to irregularities of the surfaces in contact.

40. Two blocks of masses $2.9\ \text{kg}$ and $1.9\ \text{kg}$ are suspended from a rigid support S by two inextensible wires each of length $1\ \text{m}$ as shown in figure. The upper wire has negligible mass and lower wire has a uniform mass of $0.2\ \text{kgm}^{-1}$. The whole system of blocks, wires and support have an upward acceleration of $a_0 = 0.2\ \text{ms}^{-2}$. Taking $g = 9.8\ \text{ms}^{-2}$, we have

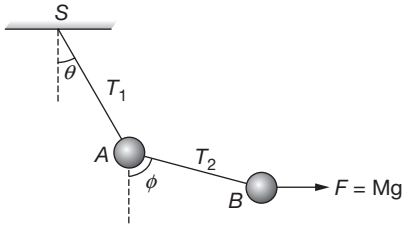


(A) tension at the midpoint of upper rope is 50 N.
(B) tension at the midpoint of upper rope is 40 N.

(C) tension at the midpoint of lower rope is 19.6 N.
(D) tension at the midpoint of lower rope is 20 N.

41. An engine of mass 5×10^4 kg pulls a coach of mass 4×10^4 kg. A resistance of 1 N per 100 kg is acting on both the coach and the engine and if the driving force of the engine is 4500 N then
(A) the acceleration of the system (coach + engine) is $0.04\ \text{ms}^{-2}$.
(B) the acceleration of the system (coach + engine) is $0.05\ \text{ms}^{-2}$.
(C) the tension in the coupling is 2000 N.
(D) the tension in the coupling is 2400 N.

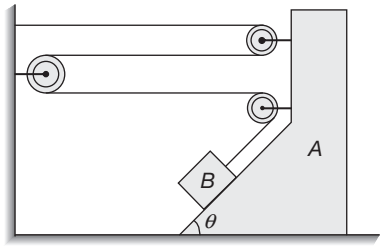
42. The spheres A and B as shown have mass M each. The strings SA and AB are light and inextensible with tensions T_1 and T_2 respectively. A constant horizontal force $F = Mg$ is acting on B. For the system to be in equilibrium we have



(A) $\tan\phi = 1$ (B) $\tan\theta = 0.5$
(C) $T_2 = \sqrt{2}\,Mg$ (D) $T_1 = \sqrt{5}\,Mg$

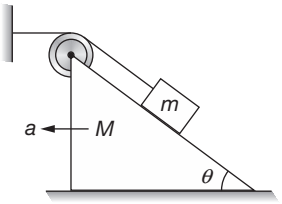
43. A person is sitting on a moving train and is facing the engine. He tosses up a coin which falls behind him.
(A) The train is moving forward and gaining speed.
(B) The train is moving forward and losing speed.
(C) The train is moving backward and losing speed.
(D) The train is moving backward and gaining speed.

44. In the diagram shown, the acceleration of the block B as shown in figure relative to the block A and relative to ground is a_{BA} and a_{BG}, respectively. If the block A is moving toward left with an acceleration a_0, then



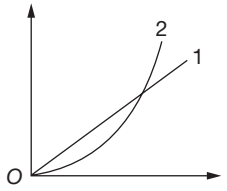
(A) $a_{BA} = 2a_0$ (B) $a_{BG} = 3a_0$
(C) $a_{BG} = 3a_0$ (D) $a_{BG} = a_0\sqrt{10+6\cos\theta}$

45. In the adjoining figure if acceleration of M with respect to ground is a, then



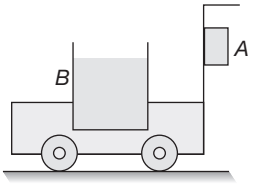
(A) acceleration of m with respect to M is a.
(B) acceleration of m with respect to ground is $2a\sin\left(\frac{\alpha}{2}\right)$.
(C) acceleration of m with respect to ground is a.
(D) acceleration of m with respect to ground is $a\tan\alpha$.

46. A particle is resting over a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time as $F = kt$, where k is a constant. The two curves are drawn for this particle as shown.



(A) Curve-1 shows acceleration versus time
(B) Curve-2 shows velocity versus time
(C) Curve-2 shows velocity versus acceleration
(D) None of these

47. A trolley of mass m_1 is to be moved such as to keep block A of mass m_2 at rest with respect to it. A bucket of mass m_3 (with water) in it is placed on trolley. Coefficient of friction between the block A and trolley is μ. The trolley is moved with acceleration so that block does not slip

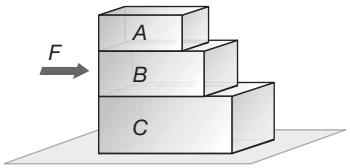


(A) The minimum coefficient of friction between bucket and trolley is $\frac{\mu}{2}$.
(B) The acceleration of trolley is $\frac{g}{\mu}$.

(C) The inclination of water surface in the bucket with horizontal in absence of any slipping is $\tan^{-1}\left(\frac{1}{\mu}\right)$.

(D) Force on trolley is $(m_1+m_2+m_3)\frac{g}{\mu}$.

48. Force F is applied on block B and the system is at rest. All surfaces are rough. Which of the following statements is/are correct?

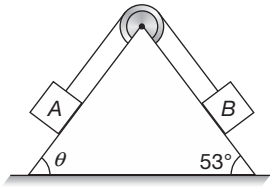


(A) Friction force on C due to B is zero.
(B) Friction on C due to ground is equal to friction force on B.
(C) Friction force on A must be zero.
(D) Friction force on A depends upon force applied F.

49. Which of the following statements are true for a moving body?
(A) if its speed changes, its velocity must change and it must have some acceleration
(B) if its velocity changes, its speed must change and it must have some acceleration
(C) if its velocity changes, its speed may or may not change, and it must have some acceleration
(D) if its speed changes, but direction of motion does not change, its velocity may remain constant

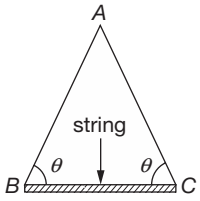
50. Speed of a body moving in a circular path changes with time as $v=2t$, then
(A) magnitude of acceleration remains constant
(B) magnitude of acceleration increases
(C) angle between velocity and acceleration remains constant
(D) angle between velocity and acceleration increases

51. Two blocks A and B of mass 1 kg each are connected by an ideal string that passes over a smooth pulley that is fixed on a smooth fixed wedge as shown. If the ratio of normal reaction on block A and on block B is $\frac{4}{3}$ then



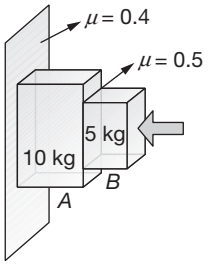
(A) $\cos\theta=\frac{4}{5}$
(B) $\sin\theta=\frac{4}{5}$
(C) acceleration of blocks is $\frac{g}{10}$ ms^{-2}
(D) acceleration of blocks is $\frac{g}{5}$ ms^{-2}

52. Two uniform identical ladders AB and AC each of mass m leans against each other and a string is tied between them which is horizontal. The system stands on a smooth horizontal surface as shown, then, string is connected from ends B and C.



(A) the force exerted by one rod on other at A is equal to the tension T in the string
(B) tension in string $T=\left(\frac{mg}{2}\right)\cot\theta$
(C) tension in string $T=\left(\frac{mg}{2}\right)\tan\theta$
(D) None of these

53. Two blocks are pushed against a rough wall and the system is at rest as shown in figure. Select the correct statement(s).



(A) Friction force on block A due to B is in upward direction
(B) Friction force on block A due to wall is 150 N
(C) Minimum force required to keep system at rest is 375 N
(D) Friction of block B due to A increases with increase in force applied F

54. A block of mass m is moving on a rough horizontal surface and its velocity at time $t=0$ is found to be $\vec{v}$ then a time varying horizontal force $F=2t$ is applied

on the body in the direction of its velocity then, after the application of force
(A) speed of block increases
(B) first decreases then increases
(C) work done by friction is negative
(D) work done by applied force is positive

55. A body moves in a circular path of radius R with deceleration so that at any moment of time its tangential and normal accelerations are equal in magnitude. At the initial moment $t = 0$, the velocity of body is v_0 then the velocity of body will be

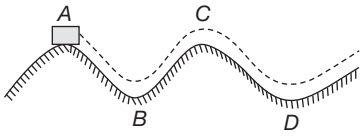
(A) $v = \dfrac{v_0}{1+\left(\dfrac{v_0 t}{R}\right)}$ at time t.

(B) $v = v_0 e^{-\frac{S}{R}}$ after it has moved S meter.

(C) $v = v_0 e^{-SR}$ after it has moved S meter.

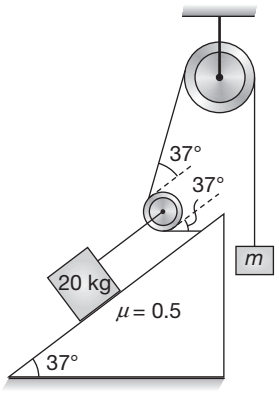
(D) None of these

56. A body is moving with constant velocity on the path shown.



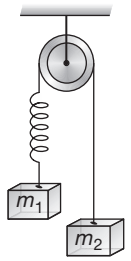
(A) Normal reaction exerted on body by path is greatest at N_B.
(B) Normal reaction exerted on body by path is least at N_C.
(C) Normal force is greater than weight at B.
(D) Normal force is lesser than weight at A, D.

57. In the figure shown the pulley is ideal and strings are massless. If mass m of hanging block is the minimum mass to keep the system in equilibrium, then $\left(g = 10\ \text{ms}^{-2}\right)$



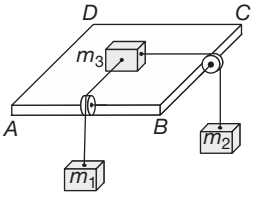
(A) $m = 2.5$ kg
(B) $m = 5$ kg
(C) force applied by 20 kg block on inclined plane is 179 N
(D) force applied by 20 kg block on inclined plane is 223 N

58. The blocks $\left(m_2 > m_1\right)$ are held in such a manner that spring is unstretched initially. Consider the situation at the time of releasing the blocks $(t = 0)$ and short time after releasing the blocks $(t > 0)$. Acceleration of block of mass m_1 is a_1 and m_2 is a_2, then



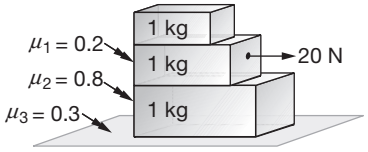
(A) $t = 0$, $a_1 = a_2$ (B) $t = 0$, $a_1 < a_2$
(C) $t > 0$, $a_1 = a_2$ (D) $t > 0$, $a_1 < a_2$

59. Three blocks are arranged as shown. $ABCD$ is the horizontal plane. Strings are massless and both pulley stands vertical while the strings connecting blocks m_1 and m_2 are also vertical and perpendicular to each other. The masses m_1 and m_2 are 3 kg and 4 kg respectively. Coefficient of friction between the block $m_3 = 10$ kg and surface is $\mu = 0.6$, then select the correct statement(s).



(A) The system of blocks can be in equilibrium
(B) At equilibrium frictional force on block m_3 is 60 N
(C) At equilibrium frictional force on block m_3 is 50 N
(D) Total force applied by surface on block m_3 is $50\sqrt{5}$ N

60. Three large box are kept stationary over one other as shown in figure. A horizontal force 20 N acts on middle block as shown in figure. Then select the correct statement(s).

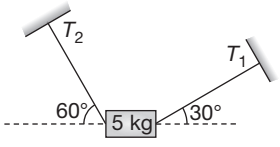


(A) Acceleration of lowest block is $4.5\ \text{ms}^{-2}$.
(B) Acceleration of upper most block is $2\ \text{ms}^{-2}$.
(C) Friction between middle and lower block is 12 N.
(D) Friction between middle and upper block is 2 N.

61. A car of mass M is moving horizontally on a circular path of radius r. At an instant its speed is v and is increasing at a rate a.
(A) The acceleration of the car is towards the centre of the path
(B) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$
(C) The friction coefficient between the ground and the car is not less than $\frac{a}{g}$
(D) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\left(\frac{v^2}{rg}\right)$

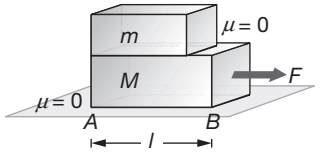
62. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen (in the plane of motion) from an inertial frame of reference.
(A) This is not possible
(B) There are other forces on the particle
(C) The resultant of the other forces is $\frac{mv^2}{r}$ towards the centre
(D) The resultant of the other forces varies in magnitude as well as in direction

63. A body of mass 5 kg is suspended by the strings making angles $60°$ and $30°$ with the horizontal as shown in figure $\left(g = 10\ \text{ms}^{-2}\right)$.



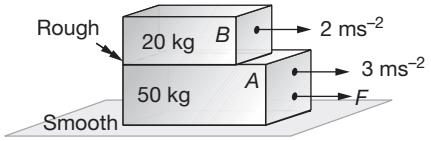
(A) $T_1 = 25$ N (B) $T_2 = 25$ N
(C) $T_1 = 25\sqrt{3}$ N (D) $T_2 = 25\sqrt{3}$ N

64. In the figure shown, a small block of mass m is kept on M, placed on a horizontal surface. All surfaces are smooth and a force F is applied on M.



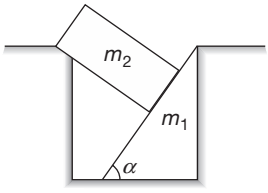
(A) The acceleration of m w.r.t. ground is $\frac{F}{m}$
(B) The acceleration of m w.r.t. ground is zero
(C) The time taken by m to separate from M is $\sqrt{\frac{2lm}{F}}$
(D) The time taken by m to separate from M is $\sqrt{\frac{2lM}{F}}$

65. A 20 kg block is placed on top of 50 kg block as shown in figure. A horizontal force F acting on A produces an acceleration of $3\ \text{ms}^{-2}$ in A and $2\ \text{ms}^{-2}$ in B as shown in figure. For this situation mark out the correct statement(s).



(A) The friction force between A and B is 40 N.
(B) The net force acting on A is 150 N.
(C) The value of F is 190 N.
(D) The value of F is 150 N.

66. A wedge of mass m_1 and a block of mass m_2 are in equilibrium as shown. Inclined surface of the wedge has an inclination α with the horizontal. Each surface is frictionless. The normal reaction on the wedge may be



(A) $m_2 g\cos\alpha$
(B) $m_2 g\sin\alpha\cos\alpha$
(C) $m_1 g + m_2 g\cos^2\alpha$
(D) $m_1 g + m_2 g\sin\alpha\cos\alpha$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** Rate of change of linear momentum is equal to external force.
Statement-2: There is equal and opposite reaction to every action.

2. **Statement-1:** A block of mass m is kept at rest on an inclined plane, the rest force applied by the surface to the block will be mg.
Statement-2: Normal contact force is the resultant of normal contact force and friction force.

3. **Statement-1:** Friction force always opposes the motion.
Statement-2: Friction force can support the motion.

4. **Statement-1:** A table cloth can be pulled from a table without disturbing a glass kept on it.
Statement-2: Every body opposes the change in its state.

5. **Statement-1:** The driver of a moving car sees a wall in front of him. To avoid collision, he should apply brakes rather than taking a turn away from the wall.
Statement-2: Friction force is needed to stop the car or taking a turn on a horizontal road.

6. **Statement-1:** Two teams having a tug of war always pull equally hard on one another.
Statement-2: The team that pushes harder against the ground, in a tug of war, wins.

7. **Statement-1:** A bird alights on a stretched wire depressing it slightly. The increase in tension of the wire is more than the weight of the bird
Statement-2: The tension must be more than the weight as it is required to balance weight.

8. **Statement-1:** Newton's First Law is merely a special case $(a = 0)$ of the Second Law.
Statement-2: Newton's First Law defines the frame from where Newton's Second Law; $\vec{F} = m\vec{a}$, $\vec{F}$ representing the net real force acting on a body; is applicable.

9. **Statement-1:** Two smooth blocks are kept on a smooth inclined plane such that one block is kept over other when a force is applied on upper block acceleration of lower block is unaffected.
Statement-2: Acceleration of a block on smooth inclined plane is $g\sin\theta$.

10. **Statement-1:** A uniform rope of mass m hangs freely from a ceiling. A monkey of mass M climbs up the rope with an acceleration a. The force exerted by the rope on the ceiling is $M(a+g)+mg$.
Statement-2: Action and reaction force are acting on two different bodies

11. **Statement-1:** According to Newton's Second Law of motion action and reaction forces are equal and opposite
Statement-2: Action and reaction forces never cancel out each other because they are acting on different objects.

12. **Statement-1:** Two balls are projected with different velocities at angles 30° and 45°. Horizontal range must be maximum for the ball which is projected at 45°.
Statement-2: For a given velocity $R = \dfrac{u^2 \sin 2\theta}{g}$.

13. **Statement-1:** A block of mass m is placed on a smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has a magnitude $mg\cos\theta$.
Statement-2: Normal reaction always acts perpendicular to the contact surface.

14. **Statement-1:** A particle is found to be at rest when seen from a frame S_1 and moving with a constant velocity when seen from another frame S_2. We can say both the frames are inertial.
Statement-2: All frames moving uniformly with respect to an inertial frame are themselves inertial.

15. **Statement-1:** Coefficient of friction can be greater than unity.
Statement-2: Force of friction is dependent on normal reaction and ratio of force of friction and normal reaction cannot exceed unity.

16. **Statement-1:** In high jump, it hurts less when an athlete lands on a heap of sand.
Statement-2: Because of greater distance and hence greater time over which the motion of an athlete is stopped, the athlete experience less force when lands on heap of sand.

17. **Statement-1:** For a boy it is difficult to run at high speed on a rainy day.
Statement-2: Coefficient of friction μ is decreased due to rain.

18. **Statement-1:** A body in equilibrium has to be at rest only.
Statement-2: A body in equilibrium may be moving with a constant speed along a straight line path.

19. **Statement-1:** Inertia is the property by virtue of which the body is unable to change by itself the state of rest.
Statement-2: The bodies do not change their state unless acted upon by an un-balanced external force.

20. **Statement-1:** Pulling (refer figure 1) is easier than pushing (refer figure 2) on a rough surface.

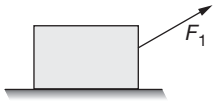


Figure 1

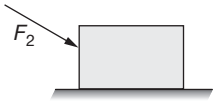
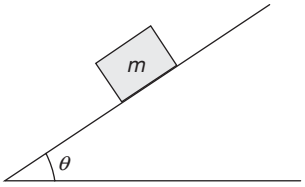


Figure 2

Statement-2: Normal reaction is less in pulling than is pushing.

21. **Statement-1:** A block is lying stationary as on inclined plane and coefficient of friction is μ. Friction on block is $\mu mg\cos\theta$.

Statement-2: Contact force on block is mg.



22. **Statement-1:** Static frictional force is always greater than kinetic frictional force.

Statement-2: (Coefficient of static friction) $\mu_s > \mu_k$ (coefficient of kinetic friction).

23. **Statement-1:** Two particles are moving towards each other due to mutual gravitational attraction. The momentum of each particle will increase.

Statement-2: Rate of change of momentum depends upon F_{ext}.

24. **Statement-1:** A concept of pseudo forces is valid both for inertial as well as non-inertial frame of reference.

Statement-2: A frame accelerated with respect to an inertial frame is a non-inertial frame.

25. **Statement-1:** For all bodies, momentum always remains same.

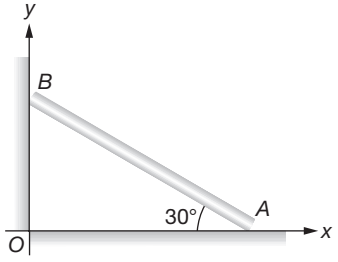
Statement-2: If two bodies of different masses have same momentum the lighter body possesses greater velocity.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct (For the sake of competitiveness there may be a few questions that may have more than one correct options).

Comprehension 1

A rod AB rests with the end A on rough horizontal ground and the end B against a smooth vertical wall. The rod is uniform and of weight W. If the rod is in equilibrium in the position shown in figure. Based on the above facts, answer the following questions.



1. The frictional force at A

(A) $\sqrt{3}W$ (B) $\frac{\sqrt{3}W}{2}$

(C) $\frac{\sqrt{3}W}{4}$ (D) $\frac{\sqrt{3}W}{8}$

2. The normal reaction at A

(A) W (B) $\sqrt{3}W$

(C) $\frac{\sqrt{3}W}{2}$ (D) $\frac{\sqrt{3}W}{4}$

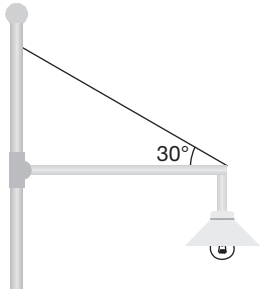
3. The normal reaction at B

(A) $\frac{\sqrt{3}W}{4}$ (B) $\frac{\sqrt{3}W}{2}$

(C) $\sqrt{3}W$ (D) W

Comprehension 2

A 20 kg floodlight in a park is supported at the end of a horizontal beam of negligible mass that is hinged to a pole, as shown in figure. A cable at an angle of $30°$ with the beam helps to support the light. Based on the above facts, answer the following questions.



4. The tension in the cable is

(A) 329 N (B) 239 N
(C) 932 N (D) 392 N

5. The horizontal force exerted on the beam by the pole is

(A) 339 N, left (B) 393 N, left
(C) 339 N, right (D) 393 N, right

6. The vertical force exerted on the beam by the pole is

(A) zero (B) 196 N, up
(C) 392 N, down (D) 196 N, down

Comprehension 3

A hemispherical bowl of radius $R = 0.1$ m is rotating about its own axis (which is vertical) with an angular velocity ω. A particle of mass 10^{-2} kg on the frictionless inner surface of the bowl is also rotating with the same ω. The particle is at a height h from the bottom of the bowl. Based on the above facts, answer the following questions.

7. The value of ω in terms of h and R is

(A) $\sqrt{\frac{g}{R}}$ (B) $\sqrt{\frac{g}{h}}$
(C) $\sqrt{\frac{g}{R+h}}$ (D) $\sqrt{\frac{g}{R-h}}$

8. The minimum value of ω needed, in order to have a non-zero value of h

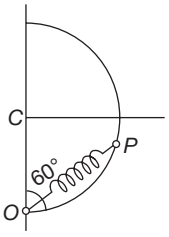
(A) 8.8 rads^{-1}
(B) 7.9 rads^{-1}
(C) 9.9 rads^{-1}
(D) None of the above seems to be correct

9. It is desired to measure g (acceleration due to gravity) using the set-up by measuring h accurately. Assuming that R and ω are known precisely and that the least count in the measurement of h is 10^{-4} m, the minimum possible error Δg in the measured value of g is

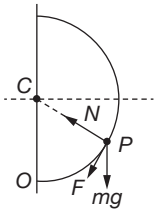
(A) 7.98×10^{-3} ms^{-2} (B) 9.78×10^{-3} ms^{-2}
(C) 8.79×10^{-3} ms^{-2} (D) 9.87×10^{-3} ms^{-2}

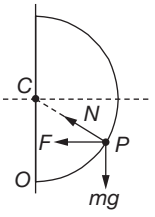
Comprehension 4

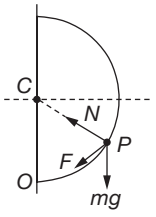
A smooth semicircular wire track of radius R is fixed in a vertical plane (shown in figure). One end of a massless spring of natural length $\frac{3R}{4}$ is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle $60°$ with the vertical. The spring constant is $K = \frac{mg}{R}$. Consider the instant when the ring is making an angle $60°$ with the vertical. The spring is released. Based on the above facts, answer the following questions.

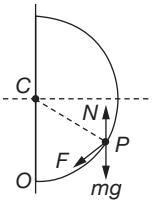


10. The free body diagram of the ring for the situation talked is

(A)


(B)


(C)


(D)


11. The tangential acceleration of the ring

(A) $\frac{\sqrt{3}}{4}g$ (B) $\frac{3\sqrt{3}}{8}g$

(C) $\frac{\sqrt{3}}{2}g$ (D) $\frac{5\sqrt{3}}{8}g$

12. The normal reaction of the ring

(A) $\frac{1}{8}$ mg (B) $\frac{1}{4}$ mg

(C) $\frac{3}{8}$ mg (D) $\frac{1}{2}$ mg

Comprehension 5

Two blocks of mass $M_1 = 10$ kg and $m_2 = 5$ kg connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rads^{-1} about a vertical axis passing through its centre O. The masses are placed along the diameter of table on either side of the centre O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table. Based on the above facts, answer the following questions.

13. The frictional force on m_1

(A) 18 N, outwards (B) 18 N, inwards

(C) 36 N, outwards (D) 36 N, inwards

14. The minimum angular speed of the turn table, so that the masses will slip from this position

(A) 12.67 rads^{-1} (B) 11.67 rads^{-1}

(C) 16.27 rads^{-1} (D) 17.67 rads^{-1}

15. How should the masses be placed from O with the spring remaining taut so that there is no frictional force acting on the mass m_1 ?

(A) m_2 at a distance of 0.1 m

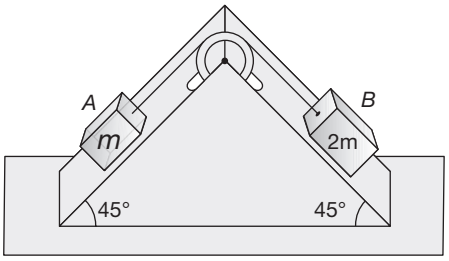
(B) m_1 at a distance of 0.2 m

(C) m_2 at a distance of 0.2 m

(D) m_1 at a distance of 0.3 m

Comprehension 6

Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless, inextensible string and a frictionless pulley as shown in figure. The wedge is inclined at $45°$ to the horizontal on both sides. The coefficient of friction between block A and the wedge is $\frac{2}{3}$ and that between block B and the wedge is $\frac{1}{3}$. If the system A and B is released from rest. Based on the above facts, answer the following questions.



16. The acceleration of A

(A) $\frac{g}{\sqrt{2}}$ (B) $\frac{g}{\sqrt{3}}$

(C) $\frac{g}{2}$ (D) zero

17. Tension in the string

(A) $\frac{\sqrt{2}}{3}$ mg (B) $\frac{2\sqrt{2}}{3}$ mg

(C) $\sqrt{2}$ mg (D) $\frac{4\sqrt{2}}{3}$ mg

18. The magnitude and direction of the force of friction acting on A

(A) $\frac{Mg}{\sqrt{2}}$, down the incline

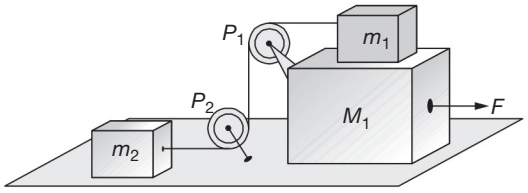
(B) $\frac{mg}{2\sqrt{2}}$, up the incline

(C) $\frac{mg}{3\sqrt{2}}$, down the incline

(D) $\frac{mg}{3\sqrt{2}}$, up the incline

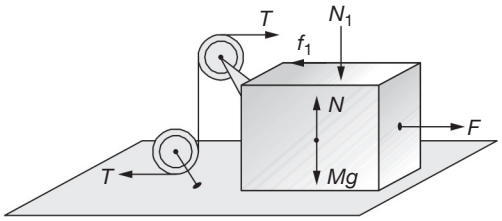
Comprehension 7

In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and strings are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to the mass M. Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. Take $g = 10$ ms^{-2}. Based on the above facts, answer the following questions.

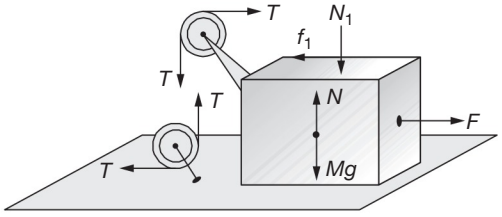


19. A free body diagram of mass M, clearly showing all the forces

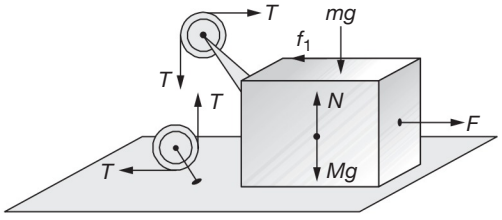
(A)



(B)



(C)



(D) None of these

20. For a particular force F it is found that $f_1 = 2f_2$.

(A) $f_1 = 15$ N, $f_2 = 30$ N

(B) $f_1 = 30$ N, $f_2 = 60$ N

(C) $f_1 = 30$ N, $f_2 = 60$ N

(D) $f_1 = 30$ N, $f_2 = 15$ N

21. The value of F such that condition $f_1 = 2f_2$ is fulfilled is

(A) 60 N (B) 30 N

(C) 45 N (D) 15 N

22. Tension in the string is

(A) 15 N (B) 16 N

(C) 17 N (D) 18 N

23. Accelerations of the masses

(A) $a_M = \frac{2}{5}\ \text{ms}^{-2},\ a_{m_1} = \frac{3}{5}\ \text{ms}^{-2},\ a_{m_2} = \frac{3}{5}\ \text{ms}^{-2}$

(B) $a_M = \frac{3}{5}\ \text{ms}^{-2},\ a_{m_1} = \frac{3}{5}\ \text{ms}^{-2},\ a_{m_2} = \frac{2}{5}\ \text{ms}^{-2}$

(C) $a_M = a_{m_1} = a_{m_2} = \frac{3}{5}\ \text{ms}^{-2}$

(D) $a_M = a_{m_1} = a_{m_2} = 0$

Comprehension 8

A particle moves in the xy plane under the action of a force $\vec{F}$ such that the value of its linear momentum $\vec{P}$ at any time t is $p_x = 2\cos t$ and $p_y = 2\sin t$. Based on the above facts, answer the following questions.

24. The magnitude of the force is

(A) 1 unit (B) 2 units

(C) 3 units (D) 4 units

25. What is the angle between $\vec{F}$ and $\vec{p}$?

(A) 0° (B) 30°

(C) 45° (D) 90°

26. The magnitude of the momentum is

(A) 1 unit (B) 2 units

(C) 3 units (D) 4 units

Comprehension 9

A block of mass m slides down an inclined plane of elevation angle α. The incline is fixed end to end in an elevator of base length l. Assume that initially the particle is at the top of the incline. Based on the above facts, answer the following questions.

27. When the elevator accelerates up with acceleration a_0, then the acceleration of the particle with respect to the incline is

(A) $g\sin\alpha$ (B) $(g + a_0)\sin\alpha$

(C) $a_0\sin\alpha$ (D) $g\sin\alpha + a_0\cos\alpha$

28. Consider that the elevator falls freely, in such a situation, the force exerted by the block on the elevator is

(A) $mg\sin\alpha$ (B) $mg\cos\alpha$

(C) zero (D) None of these

29. The time taken by the particle to reach the bottom of the incline, when the elevator accelerates up with acceleration a_0 is T. Then

(A) $T = \sqrt{\frac{2l}{g\sin\alpha}}$ (B) $T = \sqrt{\frac{4l}{(g + a_0)\sin(2\alpha)}}$

(C) $T = \sqrt{\frac{2l}{a_0\sin(2\alpha)}}$ (D) $T = \sqrt{\frac{l}{(g + a_0)\sin(2\alpha)}}$

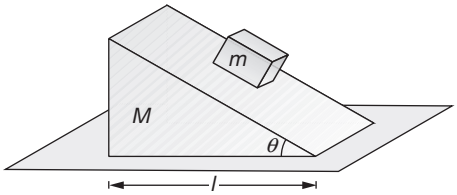
30. If the elevator moves with constant velocity, then the time taken by the block to reach the bottom of the incline is

(A) $\sqrt{\frac{2l}{g\sin(2\alpha)}}$ (B) $\sqrt{\frac{2l}{g\cos\alpha}}$

(C) $\sqrt{\frac{2l}{g}}$ (D) $\sqrt{\frac{4l}{g\sin(2\alpha)}}$

Comprehension 10

A block of mass m slides down a smooth incline of mass M and length l, solely as a result of the force of gravity. The incline is placed on a smooth horizontal table as shown in figure. Let us denote the coordinate system relative to the table as S and the coordinate system relative to the incline as S'.



31. The acceleration of m in the S' frame is

(A) $\frac{(M+m)g\sin\theta}{M+m\sin^2\theta}$ (B) $\frac{(M+m)g\sin\theta}{m-M\sin^2\theta}$

(C) $\frac{(M-m)g\sin\theta}{M+m\sin^2\theta}$ (D) $\frac{(M+m)g\sin\theta}{M+m\sin\theta}$

32. The acceleration of the incline in the S frame

(A) $\left(\frac{mg\sin\theta\cos\theta}{M-m\sin^2\theta}\right)$ (B) $\left(\frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$

(C) $\left(\frac{Mg\sin\theta\cos\theta}{M-m\sin^2\theta}\right)$ (D) $\left(\frac{Mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$

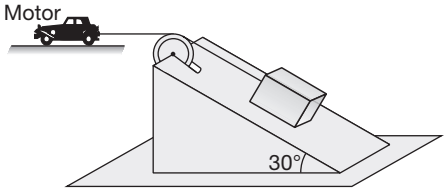
33. The force exerted by the small m on the wedge of mass M

(A) $mg\cos\theta$ (B) $\frac{Mmg\cos\theta}{M+m\sin^2\theta}$

(C) $\frac{mg}{\cos\theta}$ (D) None of these

Comprehension 11

A heavy block of mass 250 kg is being pulled up an inclined plane of angle $30°$ by means of a wire passing over a pulley of radius 5 cm and connected to a motor as shown in the figure. The power output of the motor is 5 kW and the block moves up with a constant velocity of $2\ \text{ms}^{-1}$. Based on the information provided, answer the following questions.



34. The coefficient of friction between the block and the inclined surface is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{5}}$

35. Contact force between the inclined plane and heavy block

(A) 1250 N (B) 25000 N

(C) 5000 N (D) 2500 N

36. Tension in the string

(A) 250 N (B) 1250 N

(C) 2500 N (D) 5000 N

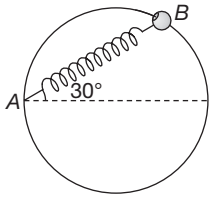
37. Frictional force acts on the block

(A) 12.5 N (B) 125 N

(C) 1250 N (D) 12500 N

Comprehension 12

A bead of mass m is attached to one end of a spring of natural length R and spring constant $k=\frac{(\sqrt{3}+1)mg}{R}$. The other end of the spring is fixed at point A on a smooth vertical ring of radius R as shown in figure. Based on the information provided, answer the following questions.



38. The normal reaction at B just after it is released to move is

(A) $\frac{mg}{2}$ (B) $\sqrt{3}\,mg$

(C) $3\sqrt{3}\,mg$ (D) $\frac{3\sqrt{3}\,mg}{2}$

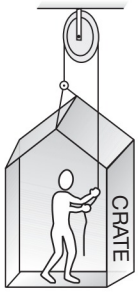
39. Tangential acceleration of the bead just after it is released is

(A) $\frac{g}{2}$ (B) $\frac{g}{4}$

(C) $\frac{2g}{3}$ (D) $\frac{3g}{4}$

Comprehension 13

A painter of mass 100 kg is raising himself and the crate (such an arrangement is called Bosun's chair) on which he stands as shown. When he pulls the rope the force exerted by him on the crate's floor is 450 N. If the crate weighs 25 kg then, based on the information provided, answer the following questions.



40. The acceleration of the system (painter + crate) is

(A) 22 ms^{-2}, up (B) 2 ms^{-2}, down

(C) 2 ms^{-2}, down (D) 22 ms^{-2}, down

41. The tension in the cable supporting the cabin is

(A) 500 N (B) 1000 N

(C) 1250 N (D) 750 N

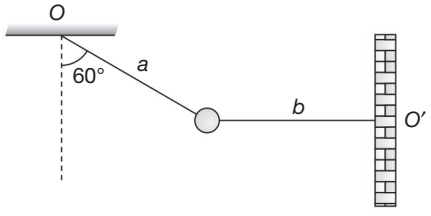
42. Assuming the painter to be of mass M, crate to be of mass m, then the system will be in equilibrium when the painter holds the rope with a force given by

(A) $(m+M)g$ (B) $(M-m)g$

(C) $\frac{1}{2}(M+m)g$ (D) $\frac{1}{2}(M-m)g$

Comprehension 14

A particle of mass m is suspended by the two strings a and b each of length l as shown. Based on this simple information, answer the following questions.



43. In equilibrium, tension in the string a is

(A) $2mg$ (B) $\sqrt{3}mg$

(C) mg (D) $3mg$

44. In equilibrium, tension in the string b is

(A) $2mg$ (B) $\sqrt{3}mg$

(C) mg (D) $3mg$

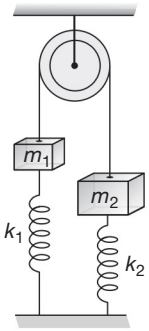
45. If the string a is now burned then the speed of the ball just before it collides with the wall is

(A) $\sqrt{\frac{gl}{2}}$ (B) $\sqrt{2gl}$

(C) $\sqrt{gl}$ (D) $\frac{\sqrt{gl}}{2}$

Comprehension 15

The system shown in the figure is in equilibrium. The masses m_1 and m_2 are 2 kg and 8 kg respectively, the springs have spring constants $k_1 = 50\ Nm^{-1}$ and $k_2 = 70\ Nm^{-1}$ and the compression in second spring is 0.5 m. Assuming both the springs to have the same natural length, answer the following questions.



46. The extension in first spring is

(A) 1.3 m (B) –0.5 m

(C) 0.5 m (D) 0.9 m

47. The tension in the string is

(A) 45 N (B) 40 N

(C) 50 N (D) 38 N

48. The acceleration of the mass m_1, when spring k_1 is cut, is

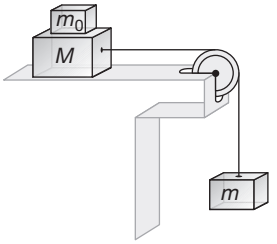
(A) zero (B) 2.5 ms^{-2}

(C) 3 ms^{-2} (D) None of these

Comprehension 16

A block of mass M is placed on a horizontal surface and it is tied through an inextensible string to a block of mass m,

as shown in figure. A block of mass m_0 is also placed on M. Based on the information provided, answer the following questions.



49. If there is no friction between any two surfaces in contact, then

(A) a downward acceleration of the block m is $\dfrac{mg}{m+m_0+M}$

(B) the acceleration of m_0 is zero

(C) the tension in the string is $Mg < T < mg$

(D) All the above

50. If a friction force exist between block M and the horizontal surface with the coefficient of friction μ, then the minimum value of μ (call it as $\mu_{\min}$) for which the block m remains stationary is

(A) $\dfrac{m}{M}$ (B) $\dfrac{m}{M+m_0}$

(C) $\dfrac{M+m_0}{M}$ (D) $\dfrac{M}{M+m_0}$

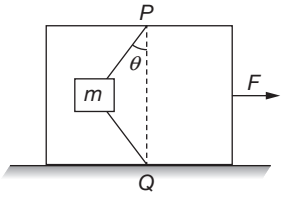
51. If $\mu < \mu_{\min}$ (the minimum friction required to keep the block m stationary), then the downward acceleration of m is

(A) $\left[\dfrac{m-\mu M}{m+M}\right]g$ (B) $\left[\dfrac{m-\mu(m_0+M)}{m+m_0+M}\right]g$

(C) $\left[\dfrac{m-\mu(m_0+M)}{m+M}\right]g$ (D) $\left[\dfrac{m-\mu M}{m+m_0+M}\right]g$

Comprehension 17

$ABCD$ is a vertical frame of mass M. Two strings between block of mass m and frame are ideal and lies in vertical plane. The frame $ABCD$ is placed on horizontal smooth surface and a horizontal force F is applied as shown. Points P and Q are in same vertical line and two strings are of equal length.



Based on the above facts, answer the following questions.

52. Minimum force F required to make both string tight is

(A) $Mg\tan\theta$ (B) $Mg\cot\theta$

(C) $(m+M)g\tan\theta$ (D) $(m+M)g\cot\theta$

53. If the lower string is just tight then tension in the upper string is

(A) $\dfrac{mg}{\sin\theta}$ (B) $\dfrac{mg}{\cos\theta}$

(C) $mg\sin\theta$ (D) $mg\cos\theta$

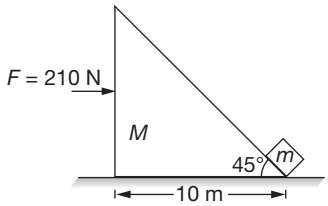
54. The value of force F for which the tension in upper string becomes twice than tension in lower string is

(A) $3(m+M)g\tan\theta$ (B) $2(m+M)g\tan\theta$

(C) $4(m+M)g\tan\theta$ (D) None of these

Comprehension 18

A block of mass $m = 2$ kg kept on a wedge of mass $M = 9$ kg and a horizontal force of 210 N is applied on the wedge as shown. All the surfaces are smooth and $g = 10\ \text{ms}^{-2}$. If initially, when the force is applied, the block m is at the bottom of wedge and if the base length of wedge is 10 m.



Based on the above facts, answer the following questions.

55. Acceleration of wedge when mass m is moving on it

(A) $\dfrac{210}{11}\ \text{ms}^{-2}$ (B) $\dfrac{10}{\sqrt{2}}\ \text{ms}^{-2}$

(C) $20\ \text{ms}^{-2}$ (D) $10\sqrt{2}\ \text{ms}^{-2}$

56. Speed of block m with respect to wedge when it reaches the top of wedge is

(A) $\dfrac{210}{11}\ \text{ms}^{-1}$ (B) $\dfrac{10}{\sqrt{2}}\ \text{ms}^{-1}$

(C) $10\sqrt{2}\ \text{ms}^{-1}$ (D) $20\ \text{ms}^{-1}$

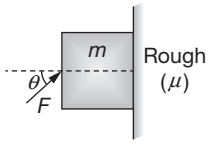
57. Distance moved by wedge when block m leaves the wedge from it top is

(A) $10\sqrt{2}$ m (B) $20\sqrt{2}$ m

(C) 20 m (D) 40 m

Comprehension 19

An external force F is applied at an angle θ with the horizontal as shown on the block of mass m. The coefficient of friction between block and wall is μ.



Based on the above facts, answer the following questions.

58. The minimum value of force f required to keep the block at rest is

(A) $\frac{mg}{\mu\cos\theta}$ (B) $\frac{mg}{\sin\theta+\mu\cos\theta}$

(C) $\frac{mg}{\sin\theta-\mu\cos\theta}$ (D) $\frac{mg}{\mu\tan\theta}$

59. The maximum value of force F up to which block remains at rest is

(A) $\frac{mg}{\mu\cos\theta}$ (B) $\frac{mg}{\sin\theta+\mu\cos\theta}$

(C) $\frac{mg}{\sin\theta-\mu\cos\theta}$ (D) $\frac{mg}{\mu\tan\theta}$

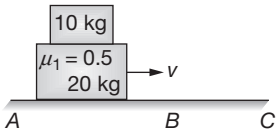
60. The value of force F for which friction force between block and wall is zero

(A) mg (B) $\frac{mg}{\sin\theta}$

(C) $\frac{mg}{\cos\theta}$ (D) $\frac{mg}{\tan\theta}$

Comprehension 20

Two blocks of masses 10 kg and 20 kg are moving together with a constant speed of 20 ms^{-1} and 10 kg block is not slipping on 20 kg when blocks are moving in region AB which is horizontal and smooth.



The coefficient of friction between the blocks is $\mu_1 = 0.5$. The horizontal surface BC is rough and coefficient of friction between 20 kg block and surface BC is μ_2. Taking $g = 10\ ms^{-2}$, answer the following questions.

61. When the blocks are moving on the smooth horizontal region AB of surface, then

(A) frictional force on 10 kg block is zero

(B) frictional force on 10 kg block is 50 N

(C) frictional force on 10 kg block is less than 50 N but non zero

(D) None of these

62. When this system of blocks enters the rough horizontal surface BC then still 10 kg block is not slipping on 20 kg block, then

(A) maximum value of μ_2 is 0.5

(B) maximum value of μ_2 is 0.75

(C) total kinetic friction is 100 N acting on 20 kg block for the above maximum value of μ_2

(D) total frictional force on 20 kg block is 150 N

63. If the value of $\mu_2 = 0.2$, then in 15 seconds after the blocks have entered the rough surface

(A) The distance travelled by them in rough surface is 75 m

(B) The distance travelled by them in rough surface is 100 m

(C) Total force applied by 10 kg on 20 kg block is 102 N after 15 seconds

(D) None of these

Comprehension 21

A block is rotating in a circular track and angular speed of the block is given $\omega = at^2$, where ω is in $rads^{-1}$ and t is in seconds, a is a constant and equal to 2 $rads^{-3}$. Mass of the block is 1 kg. Radius of circular track is 1 m. Based on the above facts, answer the following questions.

64. Net force acting on the block at $= 2$ s is

(A) 8 N (B) $8\sqrt{17}$ N

(C) 32 N (D) 4 N

65. Angle between velocity vector and acceleration vector is

(A) less than 90° (B) greater than 90°

(C) equal to 90° (D) data is insufficient

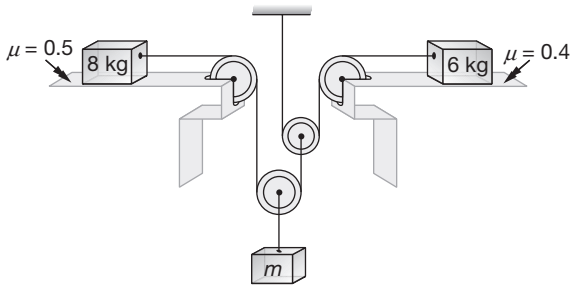
66. Time at which rate of change of speed is equal to $\frac{1}{\sqrt{2}}$ times the net acceleration is

(A) $\frac{1}{2}$ s (B) 1 s

(C) 2 s (D) 4 s

Comprehension 22

All pulleys and strings shown in figure are light and there is no friction in any string and pulley.



Based on the above facts, answer the following questions.

67. Maximum mass m to be hanged so that system remain in equilibrium

(A) 8 kg (B) 9.6 kg
(C) 6.4 kg (D) 10 kg

68. Mass m to be hanged so that none of the block remains in equilibrium must satisfy

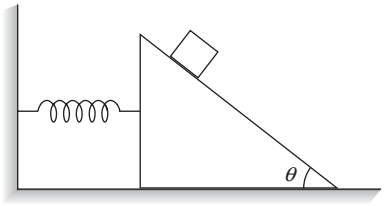
(A) $m > 8$ kg (B) $m > 9.6$ kg
(C) $m > 10$ kg (D) $m > 12$ kg

69. Acceleration of hanging block if mass of hanging block is 9 kg

(A) $\frac{9}{31}$ ms^{-2} (B) $\frac{10}{31}$ ms^{-2}
(C) $\frac{10}{41}$ ms^{-2} (D) $\frac{11}{32}$ ms^{-2}

Comprehension 23

In the situation shown below, a block of mass m is kept over wedge of mass M. Spring constant of spring shown in figure is k. All surfaces are frictionless.



Based on the above facts, answer the following questions.

70. Compression in spring is equal to

(A) $\frac{mg}{k}$ (B) $\frac{mg\cos\theta}{k}$
(C) $\frac{mg\sin 2\theta}{2k}$ (D) $\frac{mg\cos^2\theta}{2k}$

71. Force on ground due to wedge is

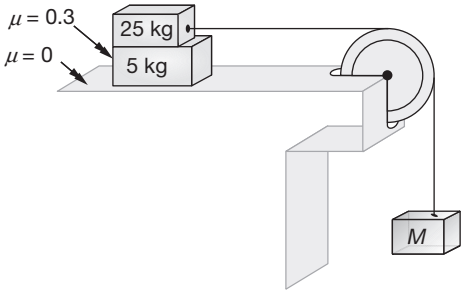
(A) Mg (B) $(M+m)g$
(C) $(m\cos\theta + M)g$ (D) $(m\cos^2\theta + M)g$

72. Net force on block is equal to

(A) $mg\sin\theta$ (B) $mg(\cos\theta + 1)$
(C) $mg\cos\theta$ (D) mg

Comprehension 24

Two blocks of masses 25 kg and 5 kg are placed on horizontal table as shown in the figure. A massless string passes over a frictionless pulley whose one end is connected to 25 kg block and other end is connected to block M. The coefficient of friction between two blocks is $\mu = 0.3$ and between 5 kg block and ground is zero. The system is released from rest.



Based on the above facts, answer the following questions. $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$

73. If $M = 45$ kg force of friction between the blocks will be

(A) 80 N (B) 60 N
(C) 30 N (D) 75 N

74. Value of M for which there is slipping between the blocks is

(A) > 90 kg (B) $M > 0$
(C) < 90 kg (D) None of these

75. If $M = 45$ kg, acceleration of 5 kg block will be

(A) 6 ms^{-2} (B) 5 ms^{-2}
(C) 8 ms^{-2} (D) 10 ms^{-2}

Comprehension 25

A block of mass m revolving in a circular path of radius r with a non-uniform speed of particle. Speed of particle

is given by $v = kt$, where k is constant. Time starts from $t = 0$ s. Based on the above facts, answer the following questions.

76. At $t = t_1$, magnitude of tangential acceleration is equal to magnitude of centripetal acceleration. Then t_1 is equal to

(A) $\frac{r}{k}$ (B) rk

(C) $\sqrt{\frac{r}{k}}$ (D) $\sqrt{\frac{k}{r}}$

77. Magnitude of net force on the particle at $t = t_1$ is

(A) $2mk$ (B) $\sqrt{2}mk$

(C) mk (D) $\frac{mk}{2}$

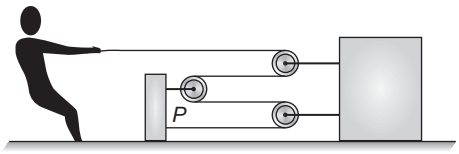
78. The angle between net acceleration vector and radius vector at $t = \frac{2}{\sqrt{3}}t_1$ is

(A) 53° (B) 37°

(C) 127° (D) 137°

Comprehension 26

A man of mass 50 kg is standing on a platform of mass 30 kg kept on a horizontal surface. Mass of block is 160 kg. Distance between block and pulley P is 6 m. There is no friction between block and platform. The man pulls the string with constant force 40 N.



Based on the above facts, answer the following questions.

79. Assuming no friction between platform and surface, time taken by block to reach pulley is

(A) $2\sqrt{3}$ s (B) 2 s

(C) $3\sqrt{2}$ s (D) 3 s

80. Length of string pulled by man in 1 s is

(A) 2 m (B) 6 m

(C) 8 m (D) 12 m

81. The minimum coefficient between platform and the surface so that it doesn't slip on the surface

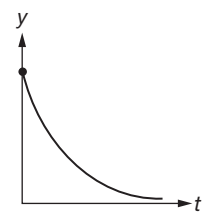
(A) 0.05 (B) 0.1

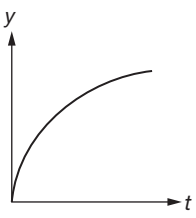
(C) 0.53 (D) None of these

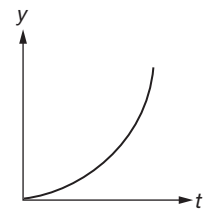
Comprehension 27

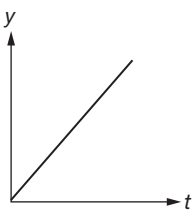
It is observed that sometimes the frictional force opposing the motion of a body is not constant but increases with speed of the body. One of the familiar example is the air resistance on a freely falling body. For small speeds this resisting force is approximately proportional to the speed. The equations of motion are not directly applied in such cases if variable force. Based on the above facts, answer the following questions.

82. The variation of the position of a body falling freely through air can be best represented graphically by (consider $y = 0$ at the highest point)

(A) 

(B) 

(C) 

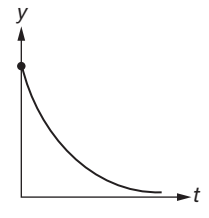
(D) 

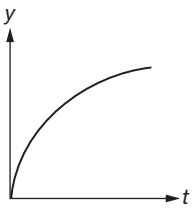
83. A 5 kg mass freely falling experiences a retarding force of $5v$ where v is the speed in ms^{-1} has

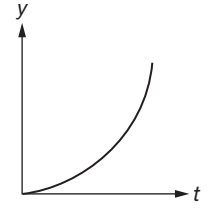
(A) 5 (B) 10

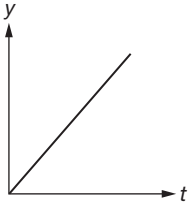
(C) 15 (D) 20

84. The variation of velocity with time is best represented by

(A) 

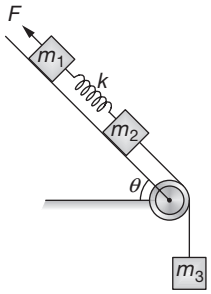
(B) 

(C) 

(D) 

Comprehension 28

As shown two block of mass $m_1 = 2$ kg and $m_2 = 4$ kg are connected by an ideal spring of force constant $k = 1000\ \text{Nm}^{-1}$ and the system is kept on a smooth inclined plane inclined at $\theta = 30°$ with horizontal. A third block of mass $m_3 = 1$ kg is attached to m_2 by an ideal string. A force $F = 15$ N is applied on m_1 and the system is released from rest. Assume that the spring was initially unstretched. $\left(g = 10\ \text{ms}^{-2}\right)$.



Based on the above facts, answer the following questions.

85. The common acceleration of blocks m_1, m_2 and m_3 is

(A) $\frac{25}{7}\ \text{ms}^{-2}$ (B) $\frac{35}{7}\ \text{ms}^{-2}$

(C) $\frac{15}{7}\ \text{ms}^{-2}$ (D) $\frac{7}{25}\ \text{ms}^{-2}$

86. Maximum elongation in spring is

(A) 1 cm (B) 2. 4 cm

(C) 3.6 cm (D) 4.8 cm

87. Acceleration of m_3 when spring has maximum elongation is (approx.)

(A) $1.2\ \text{ms}^{-2}$ (B) $2\ \text{ms}^{-2}$

(C) $1\ \text{ms}^{-2}$ (D) $1.5\ \text{ms}^{-2}$

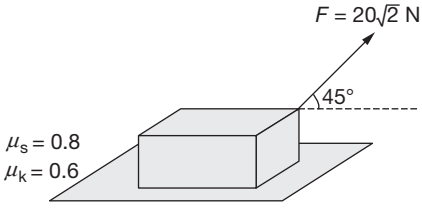
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

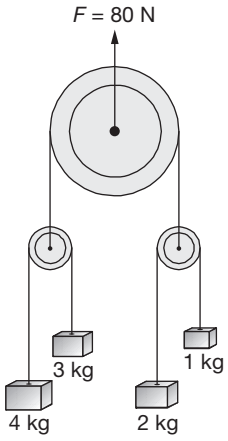
	p	q	r	s	t
A	●	●	○	●	●
B	○	○	○	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. In the diagram shown in figure a light string is attached to the mass and is pulled by a constant force $F = 20\sqrt{2}$ N which makes an angle of 45° with the horizontal. Taking $g = 10\ \text{ms}^{-2}$, match the quantities in COLUMN-I with their respective match(es) in COLUMN-II.



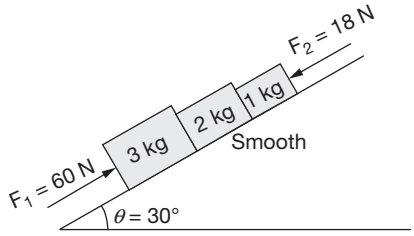
COLUMN-I	COLUMN-II
(A) Normal reaction, in newton.	(p) $20\sqrt{2}$
(B) Force of friction, in newton.	(q) 20
(C) Acceleration of block, in cms^{-2}.	(r) 12
(D) Tension in the string, in newton.	(s) 200

2. In the diagram shown in figure, all pulleys are smooth and massless and strings are light. Match the following. (Take $g = 10\ \text{ms}^{-2}$)



COLUMN-I	COLUMN-II
(A) Acceleration of 1 kg block	(p) up
(B) Acceleration of 2 kg block	(q) 5 ms^{-2}
(C) Acceleration of 3 kg block	(r) down
(D) Acceleration of 4 kg block	(s) $\frac{10}{3}$ ms^{-2}
	(t) Stationary
	(u) 10 ms^{-2}

3. In the diagram shown in figure, match the following $(g = 10\ ms^{-2})$.

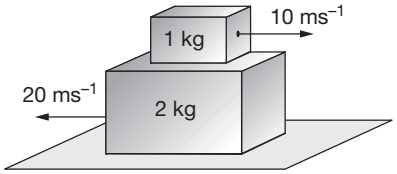


COLUMN-I	COLUMN-II
(A) Acceleration of 1 kg block, in ms^{-2}.	(p) 29
(B) Net force on 3 kg block, in N.	(q) 39
(C) Force exerted on 2 kg by 1 kg, in N.	(r) 2
(D) Force exerted on 3 kg by 2 kg, in N.	(s) 6
	(t) 25

4. Velocity of five particles A, B, C, D and E varies with time t as, $\vec{v}_A = \left(8t\hat{i} + 6\hat{j}\right)\ ms^{-1}$, $\vec{v}_B = \left(4\hat{i} + 3\hat{j}\right)\ ms^{-1}$, $\vec{v}_C = \left(6\hat{i} - 4t\hat{j}\right)$, $\vec{v}_D = \left(-3t\hat{i} + 5\hat{j}\right) ms^{-1}$ and $\vec{v}_E = \left(2\hat{i} + 5t\hat{j}\right) ms^{-1}$. Match the pseudo force as observed and specified in COLUMN-I to the respective directions given in COLUMN-II.

COLUMN-I	COLUMN-II
(A) On A as observed by B.	(p) Along positive x-direction
(B) On B as observed by C.	(q) Along negative x-direction
(C) On C as observed by D.	(r) Along positive y-direction
(D) On D as observed by E.	(s) Along negative y-direction
(E) On E as observed by A.	(t) Zero

5. In the arrangement shown in figure the coefficient of friction between 1 kg and 2 kg block is $\mu = 0.4$, and that between 2 kg block and group is 0.6. Match the following.

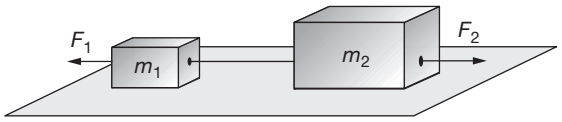


COLUMN-I	COLUMN-II
(A) 11	(p) absolute acceleration of 1 kg block, in ms^{-2}
(B) 22	(q) absolute acceleration of 2 kg block, in ms^{-2}
(C) 4	(r) relative acceleration between the two, in ms^{-2}
(D) 15	(s) net force on 2 kg block, in N
	(t) net force on 1 kg block, in N

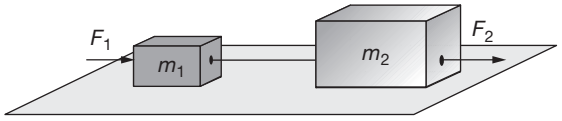
6. One or more of the entries of COLUMN-II may be the cause of linear motions described in COLUMN-I. Give the appropriate match(es) for the entries of COLUMN-I from COLUMN-II.

COLUMN-I	COLUMN-II
(A) Object moving along $+x$ axis with constant velocity	(p) Net force on the object must act along $+x$ axis
(B) Object moving along $+x$ axis under varying acceleration	(q) At least one force must act along $+x$ axis
(C) Object moving along $+x$ axis under constant acceleration	(r) No net force may act on the object
(D) Object moving along $+x$ axis	(s) No force may be acting along $+x$ axis

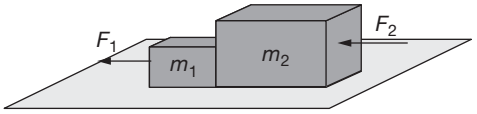
7. Two blocks having masses m_1 and m_2 are placed on smooth horizontal surfaces (as shown) for the different horizontal combination of the respective forces F_1 and F_2 applied on m_1 and m_2. Further it is given that $m_1F_2 > m_2F_1$, a_1 and a_2 are the accelerations of m_1 and m_2 if F_1 and F_2 would have acted on them separately.



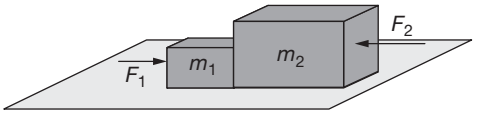
CASE-I



CASE-II



CASE-III

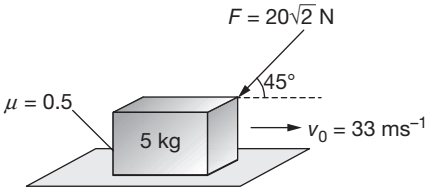


CASE-IV

If T_1, T_2, N_1 and N_2 be the respective tensions and the normal reactions in Case-I, II, III and IV, then match T_1, T_2, N_1 and N_2 from **COLUMN-I** to their values in **COLUMN-II**.

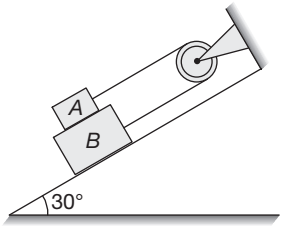
COLUMN-I	COLUMN-II
(A) T_1	(p) $\frac{m_1m_2}{m_1+m_2}\left(\frac{F_2}{m_2}-\frac{F_1}{m_1}\right)$
(B) N_1	(q) $\frac{m_1m_2}{m_1+m_2}(a_2+a_1)$
(C) T_2	(r) $\frac{m_1m_2}{m_1+m_2}\left(\frac{F_1}{m_1}+\frac{F_2}{m_2}\right)$
(D) N_2	(s) $\frac{m_1m_2}{m_1+m_2}(a_2-a_1)$

8. A block of mass $m = 5$ kg is moving right with velocity $v_0 = 33$ ms^{-1} on a rough surface. A force $F = 20\sqrt{2}$ is applied as shown $(g = 10$ ms$^{-2})$



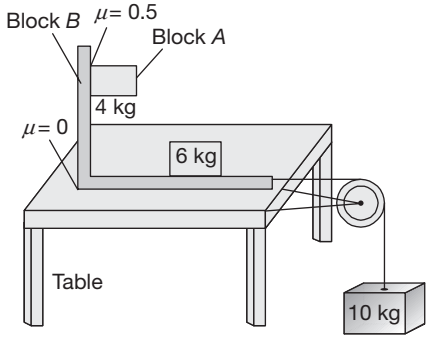
COLUMN-I	COLUMN-II
(A) at $t = 2.5$ s, friction force is	(p) zero
(B) at $t = 4.5$ s, friction force is	(q) 55 N
(C) at $t = 2.5$ s, net retardating force is	(r) 35 N
(D) at $t = 4.5$ s, net force on block is	(s) 15 N
	(t) 20 N

9. In the arrangement shown, a block A of mass $m_A = 1$ kg is sliding on a block B of mass $m_B = 4$ kg. Friction is absent between all surfaces in contact. Based on this information and $g = 10$ ms^{-2}, match the items of COLUMN-I with their respective values in COLUMN-II.



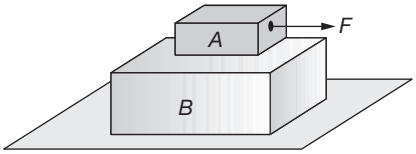
COLUMN-I	COLUMN-II
(A) Acceleration of A, in ms^{-2}	(p) $5\sqrt{3}$
(B) Thrust on the pulley, in N	(q) $25\sqrt{3}$
(C) Force exerted by A on B, in N	(r) 3
(D) Force exerted by B on incline, in N	(s) 8
	(t) 16

10. Configuration of L shaped block B of mass 6 kg and two other blocks are shown. System is set free to move. At any moment, match the following parameters. $\left(g = 10\ ms^{-2}\right)$.



COLUMN-I	COLUMN-II
(A) Friction force between block A and B	(p) 50 N
(B) Tension in the string	(q) 70 N
(C) Normal reaction on block A by block B	(r) 60 N
(D) Normal reaction on block B by table	(s) 10 N
	(t) 20 N

11. A block A of mass 10 kg lies on a block B of mass 25 kg resting on a smooth horizontal surface. The coefficient of static friction between A and B is $\mu = 0.5$. A force F is applied on the block A, as shown.

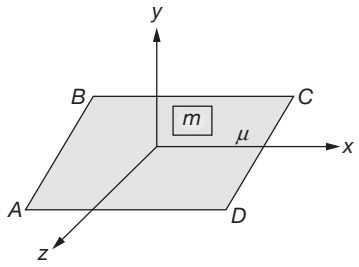


COLUMN-I	COLUMN-II
(A) Force of friction on B due to A, in N.	(p) 1
(B) Acceleration of $A\ \left(\text{in } ms^{-2}\right)$ for $F = 35$ N.	(q) 20
(C) Acceleration of $A\left(\text{in } ms^{-2}\right)$, for $F = 250$ N.	(r) towards left
(D) Acceleration of $B\left(\text{in } ms^{-2}\right)$, for $F = 250$ N.	(s) 2
	(t) 50
	(u) towards right

12. If the net force acting on a system is represented by $\vec{F}$ and its momentum is $\vec{p}$, then match the quantities in COLUMN-I with the quantities in COLUMN-II.

COLUMN-I	COLUMN-II
(A) If $\vec{F}$ is constant	(p) $\vec{p}$ may change its direction
(B) If $\vec{F}$ is changing in magnitude	(q) $\vec{p}$ must change its magnitude
(C) If $\vec{F}$ is changing in direction	(r) $\vec{p}$ may not change its direction
(D) If $\vec{F}$ is zero	(s) $\vec{p}$ must not change its direction

13. $ABCD$ is a horizontal surface on which a block of mass $m = 20$ kg is placed and coefficient of friction between this body and surface is $\mu = 0.5$ as shown. y-axis is vertical axis and x-z plane is horizontal. The surface starts moving with an acceleration which depends on time as $\vec{a} = \left(3t\hat{i} + 10\hat{j} + 4t\hat{k}\right)\ ms^{-2}$.



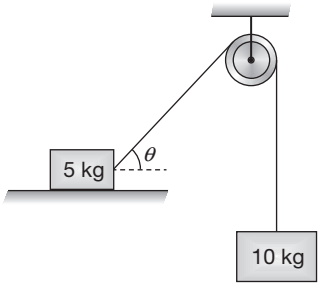
Assuming that at $t = 0$, the block is at rest with respect to the surface. If, t is time when block starts slipping, a is the acceleration of surface when block starts slipping. f_l is limiting friction when block starts slipping, F is total force applied by surface on block at $t = 1$ s, then match the quantities in **COLUMN-I** with their respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) t	(p) 200
(B) a	(q) $100\sqrt{17}$
(C) f_1	(r) $10\sqrt{2}$
(D) F	(s) 2

14. Match the force or set of forces in the left hand column to a characteristic of force or set of forces in the right-hand column.

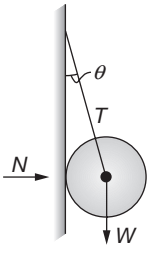
COLUMN-I (Force)	COLUMN-II (Characteristic)
(A) Electric (Electrostatic)	(p) Proportional to $\frac{1}{(\text{distance})^2}$
(B) Gravity	(q) Always attracts
(C) Both Electric and Gravity	(r) Always repels
(D) Neither electric nor gravity	(s) Attracts or repels
	(t) Central force

15. In the arrangement shown pulley is ideal and string is massless. 5 kg block is moving on the horizontal which is smooth. When θ is 37° acceleration of 5 kg and 10 kg blocks are a_1 and a_2. Tension in string connecting blocks is T and the total force applied by this string on pulley is F. Taking, $g = 10\ \text{ms}^{-2}$, match the quantities in COLUMN-I to their respective values (in SI units) in COLUMN-II.



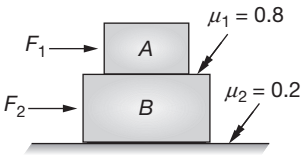
COLUMN-I	COLUMN-II
(A) a_1	(p) 5.6
(B) a_2	(q) 43.75
(C) T	(r) 7
(D) F	(s) 78.26

16. A metal sphere is hung by a string fixed to a wall. The forces acting on the sphere are shown. Match the following:



COLUMN-I	COLUMN-II
(A) $\vec{T} =$	(p) $(-\vec{W} - \vec{N})$
(B) $T^2 =$	(q) $W\tan\theta$
(C) $N =$	(r) $(N^2 + W^2)$
(D) $\vec{N} + \vec{T} + \vec{W} =$	(s) zero
	(t) None of these

17. A block A of mass 5 kg is placed on another block B of mass 10 kg placed on a horizontal rough surface. Horizontal forces F_1 and F_2 act on blocks A and B, respectively as shown in figure. Coefficient of friction between two blocks is 0.8 while between block B and horizontal surface is 0.2. Let f_1 denote friction force acting on block B due to A and f_2 denote friction force on horizontal surface. Then match the following.

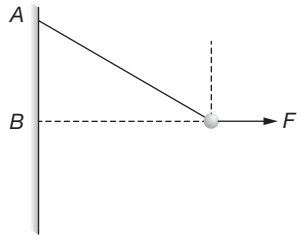


COLUMN-I	COLUMN-II
(A) If $F_1 = 35$ N, $F_2 = 0$ then f_1	(p) Kinetic
(B) If $F_1 = 50$ N, $F_2 = 0$ then f_2	(q) Static
(C) If $F_1 = 0$, $F_2 = 30$ then f_1	(r) zero
(D) If $F_1 = 10$ N, $F_2 = 50$ N then f_1	(s) Towards right

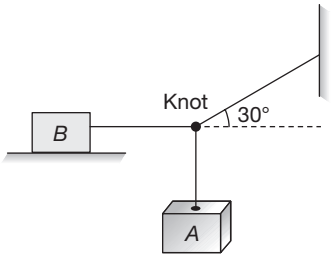
INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

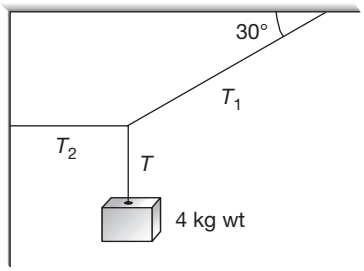
1. One end of a 0.5 m long string is fixed to a point A and the other end is fastened to a small object of weight 8 N. The object is pulled aside by a horizontal force F, until it is 0.3 m from the vertical through A. Find the magnitudes of the tension T in the string and the force F. Give your answer in newton.



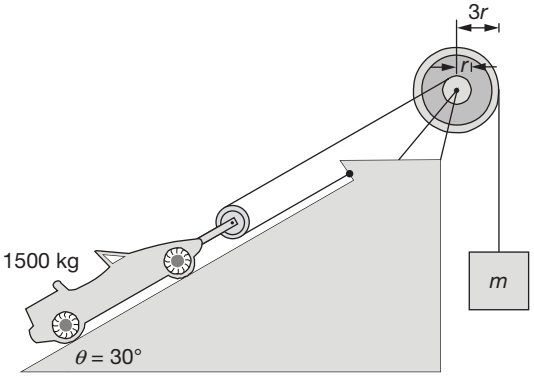
2. Block B in figure weighs 692 N. The coefficient of static friction between block and table is 0.25. Assuming that the cord between B and the knot is horizontal, find the maximum weight of block A, in kgwt for which the system will be stationary. Take $3 = (1.73)^2$ and $g = 10\ \text{ms}^{-2}$.



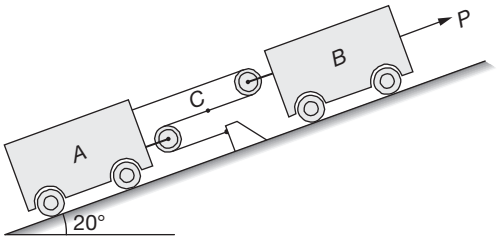
3. In the arrangement shown, determine the tensions, in newton, T_1, T_2 and T_3 in the strings. Take $g = 10\ \text{ms}^{-2}$ and $\sqrt{3} = 1.7$.



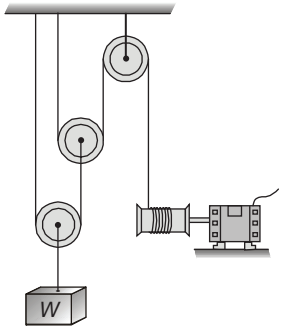
4. Find the mass m of the counterweight needed to balance the 1500 kg car on the incline shown in figure. Assume all pulleys are frictionless and massless.



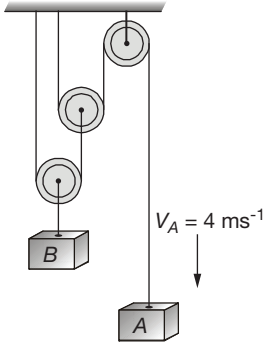
5. Under the action of force P, the constant acceleration of block B is $6\ \text{ftsec}^{-2}$ up the incline. For the instant when the velocity of B is $3\ \text{ftsec}^{-1}$ up the incline, determine the velocity of B relative to A, the acceleration of B relative to A in fts^{-2} and the absolute velocity of point C in fts^{-1} of the cable.



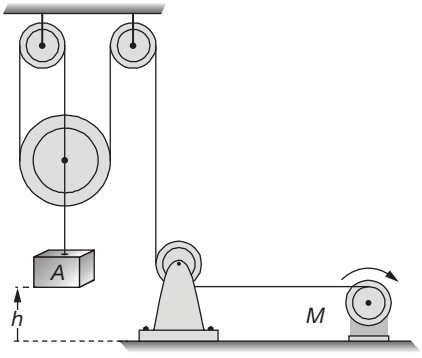
6. Determine the vertical rise h, in mm, of the load W during $5\ \text{s}$, if the hoisting drum wraps cable around it at the constant rate of $320\ \text{mms}^{-1}$.



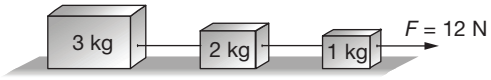
7. In the arrangement shown, find the speed, in cms^{-1}, of B, if A is moving downwards with a speed of $v_A = 4\ \text{ms}^{-1}$ at the instant shown.



8. In the arrangement shown, find the speed of cylinder A, if the rope is drawn towards the motor M at a constant rate of $9\ \text{ms}^{-1}$.



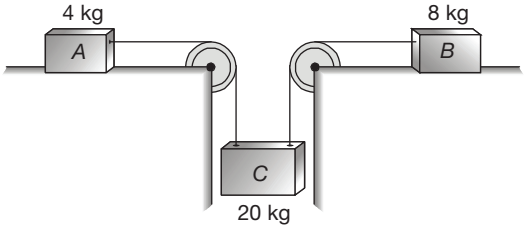
9. In the arrangement shown in figure. The strings are light and inextensible. The surface over which blocks are placed is smooth. Find:



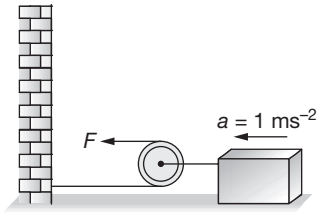
(a) the acceleration of each block, in ms^{-2}

(b) the tension in each string, in newton.

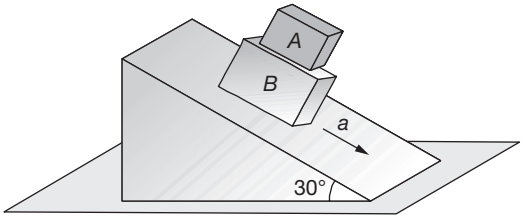
10. Consider the situation shown in figure the block B moves on a frictionless surface, while the coefficient of friction between A and the surface on which it moves is 0.2. Find the acceleration, in ms^{-2}, with which the masses move and also the tension, T_1 and T_2 in the left and the right string, in newton. Take $g = 10\ \text{ms}^{-2}$.



11. A block of mass 200 kg is set into motion on a frictionless horizontal surface with the help of frictionless pulley and a rope system as shown in figure. What horizontal force F, in newton, should be applied to produce in the block an acceleration of $1\ \text{ms}^{-2}$?

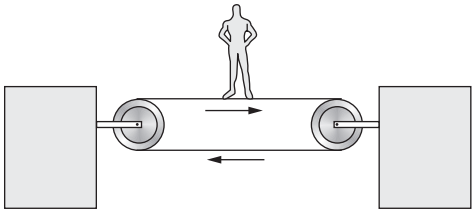


12. Determine the normal force, in newton, the 10 kg crate A exerts on the smooth block B if the block is given an acceleration of $a = 2\ \text{ms}^{-2}$ down the fixed incline. Also, find the acceleration of the crate, in ms^{-2}, for $\theta = 30°$. Take $g = 10\ \text{ms}^{-2}$.



13. The coefficient of friction between the road and the tyre of a vehicle is $\frac{4}{3}$. Find the maximum incline of the road (in degree to the nearest two digit integer) so that once hard brakes are applied and the wheel starts skidding, the vehicle going down at a speed of $10\ \text{ms}^{-1}$ is stopped within $5\ \text{m}\left(g = 10\ \text{ms}^{-2}\right)$.

14. Figure shows a man of mass $M = 65\ \text{kg}$ standing stationary with respect to a horizontal conveyor belt accelerating at $1\ \text{ms}^{-2}$. Calculate the net force, in newton, on the man. If the coefficient of static friction between the man's shoes and the belt is 0.2, upto what acceleration, in cms^{-2}, of the belt can the man continue to be stationary relative to the belt? Take $g = 10\ \text{ms}^{-2}$.



15. An initially stationary box full of sand is to be pulled across a floor by means of a cable in which the tension should not exceed 1200 N. The coefficient of friction between the box and the floor is $\frac{1}{2\sqrt{2}}$.

(a) What should be the angle, in degree, between the cable and the horizontal in order to pull the greatest possible amount of sand?

(b) What is the combined weight of the sand and box, in newton, in that situation?

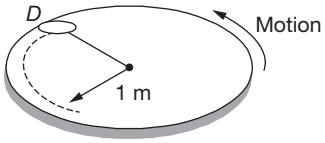
Take $\sin(10°)\cos(10°) = \frac{1}{6}$.

16. A 30 kg mass is initially at rest on the floor of a truck. The coefficient of static friction between the mass and the floor of truck is 0.3 and coefficient of kinetic friction is 0.2. Initially the truck is travelling due east at constant speed. Find the magnitude and direction of the friction force, in newton, acting on the mass if

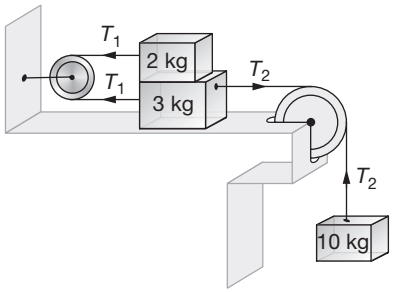
(a) the truck accelerates at $1.8\ \text{ms}^{-2}$ eastward.

(b) the truck accelerates at $3.8\ \text{ms}^{-2}$ westward.

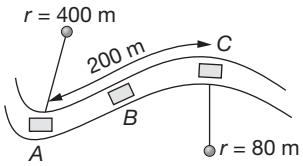
17. The 4 kg disk D is attached to the end of a cord as shown in figure. The other end of the cord is tied at the centre of a platform. If the platform is rotating rapidly and the disk is placed on it and released from rest as shown, determine the time, in seconds, it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is 100 N and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$. Take $g = 10\ \text{ms}^{-2}$.



18. Coefficient of kinetic friction between 3 kg and 2 kg block is 0.3. The horizontal table surface is smooth. Find the acceleration of block of mass 10 kg. (in ms^{-2})

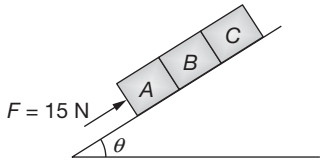


19. A 1500 kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 kmph at A to a speed of 50 kmph as it passes C. The radius of curvature of the road at A is 400 m and at C is 80 m. The total horizontal force exerted by the road on tyres at position C is _____ N.

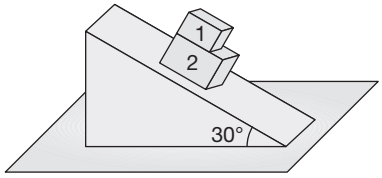


20. What is the minimum radius of a circle along which a cyclist can ride with a velocity $18\ \text{kmhr}^{-1}$ if the coefficient of friction between the tyres and the road is $\mu = 0.5$ $\left(\text{take } g = 10\ \text{ms}^{-2}\right)$

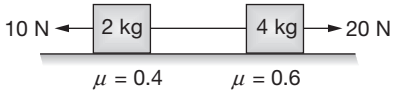
21. Three blocks of identical masses are placed at rest on a smooth inclined plane with force acting on block A parallel to the inclined plane. Find the contact force between block B and C.



22. Two blocks 1 and 2 of mass 2 kg and 4 kg are kept over a frictionless inclined surface (angle of inclination $= 30°$). Coefficient of friction between two blocks is $\mu = 0.2$. Calculate the frictional force on block 1 due to 2 (in newton).



23. Two blocks of mass $2\ \text{kg}$ and $4\ \text{kg}$ are connected through a massless inextensible string. Coefficient of friction between $2\ \text{kg}$ block and ground is 0.4 and between $4\ \text{kg}$ block and ground is 0.6. Two forces $F_1 = 10\ \text{N}$ and $F_2 = 20\ \text{N}$ are applied on the block as shown in figure. Calculate the friction force (in N) acting on $4\ \text{kg}$ block.

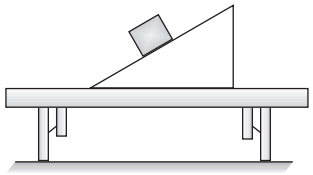


24. A light string fixed at one end to a clamp on ground passes over a fixed pulley and hangs at the other side. It makes an angle of $30°$ with the ground. A monkey of mass $5\ \text{kg}$ climbs up the rope. The clamp can tolerate a vertical force of $40\ \text{N}$ only. Find the maximum acceleration in upward direction with which the monkey can climb safely is (Neglect friction and take $g = 10\ \text{ms}^{-2}$)

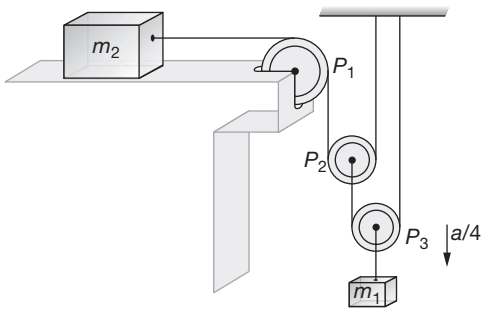
25. A car begins to move at time $t = 0$ and then accelerates along a straight track with a velocity given by $V(t) = 2t^2\ \text{ms}^{-1}$ for $0 \le t \le 2$, where t is time in second. After the end of acceleration, the car continues to move at a constant speed. A small block initially at rest on the floor of the car begins to slip at $t = 1\ \text{s}$ and stops slipping at $t = 3\ \text{s}$. The coefficient of static and kinetic friction between the block and the floor are μ_s and μ_k respectively. Find the value of $\dfrac{3\mu_s}{\mu_k}$.

26. A 40 kg wooden crate is being pushed across a wooden floor with a force of $160\ \text{N}$. If $\mu_k = 0.3$, find the acceleration of the crate. $\left(g = 10\ \text{ms}^{-2}\right)$.

27. A block of mass $0.2\ \text{kg}$ is kept on wedge of mass $0.6\ \text{kg}$ which is kept on rough table. Assuming wedge to be stationary, and the block to be slipping down with constant speed $2\ \text{ms}^{-1}$, calculate the normal force on the wedge due to table.

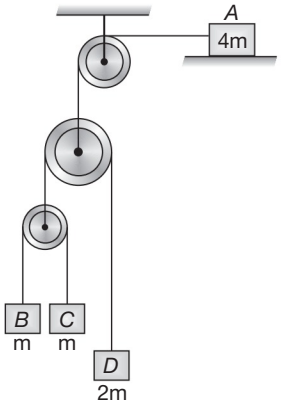


28. Pulleys are ideal and string are massless. The masses of blocks are $m_1 = 4\ \text{kg}$ and $m_2 = 1\ \text{kg}$ as shown. If all surfaces are smooth then the acceleration of m_2 in ms^{-2} is $\left(g = 10\ \text{ms}^{-2}\right)$

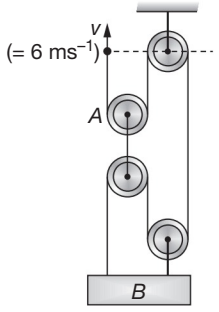


29. A block of mass 5 kg is placed on bus moving with acceleration $2\ \text{ms}^{-2}$. The pseudo force acting on block as seen by a man on ground is _____.

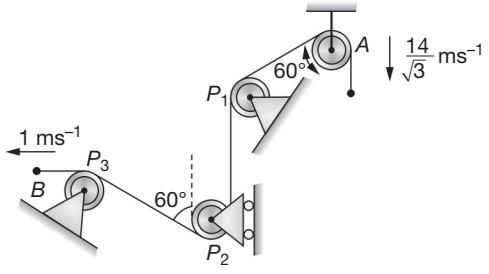
30. In the given arrangement, strings and pulleys are light and all surface are frictionless. Assuming at $t = 0$, system is released from rest, find the speed of block A in decameter per second at $t = 2$ sec.



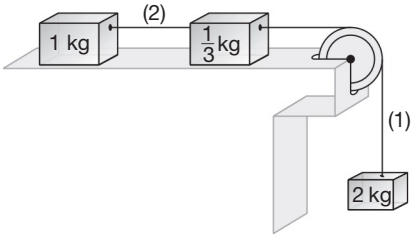
31. In the arrangement shown in figure, the end A of light inextensible string is pulled with constant velocity $v = 6\ \text{ms}^{-1}$. The velocity of block B is (in ms^{-1})



32. Figure shows a string passing through two fixed pulley P_1 and P_3 and a pulley P_2 free to move vertically. One end of string is attached with ring A. Calculate the velocity of pulley P_2 at the instant shown in ms^{-1}.

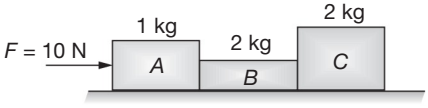


33. Find the tension in string 1. All the surfaces are frictionless. Strings are light and frictionless. $(\text{Take } g = 10\ \text{ms}^{-2})$.



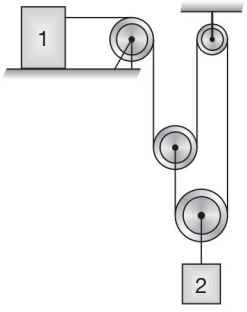
34. A block of mass 1 kg is just fit in a groove in a platform kept horizontally. The groove is along +ve x-axis. The platform is given an acceleration $\vec{a} = 2\hat{i} + 3\hat{j}\ \text{ms}^{-2}$. If block is not slipping on platform, then find the friction force acting on block (in newton).

35. Calculate the magnitude of the force exerted by A on B.

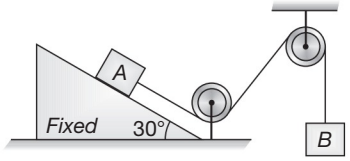


All surfaces are frictionless.

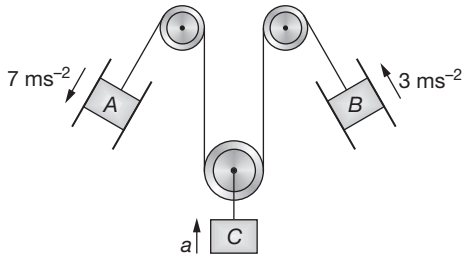
36. Two blocks 1 and 2 of mass 2 kg and 4 kg are kept connected as shown in figure. All pulley and string are massless and surfaces are frictionless. Calculate the acceleration of block 1 at the instant shown.



37. Two blocks A and B of equal mass m are connected through a massless string and arranged as shown in figure. Friction is absent everywhere. When the system is released from rest, then calculate the tension in string. Where m is 2 kg. $(g = 10\ \text{ms}^{-2})$

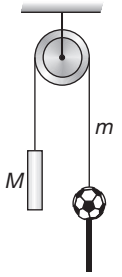


38. Assuming all the surfaces to be frictionless, find the acceleration of the block C in ms^{-2} shown in figure.

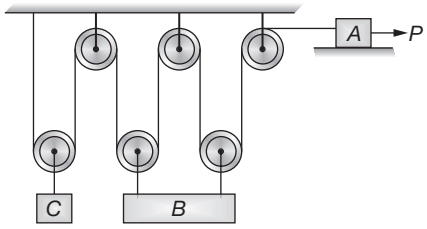


39. A block of mass 2 kg is placed on rough horizontal surface (coefficient of friction $= 0.2$) and is pulled by horizontal force $F = (2t)$ N where t is time in sec. Find the velocity of block at $t = 4$ seconds.

40. In the arrangement shown in figure, the mass of the rod $M = 2$ kg exceeds the mass $m = 0.5$ kg of the ball. The mass has an opening permitting it to slide along the thread with some friction. The mass of the pulley and the friction in its axle are negligible. At the initial moment ball was located opposite to the lower end of the rod. When set free, both bodies began moving with constant accelerations. Find the friction force between the ball and the thread if $t = 2$ second after the beginning of motion of the ball, the ball got opposite the upper end of the rod. The rod length equals $l = 0.3$ m. (in 10^{-1}).



41. Three blocks A, B and C of mass m each are arranged in pulley mass system as shown. Coefficient of friction between block A and horizontal surface is equal to 0.5 and a force P acts on A in the direction shown. Calculate $\frac{P}{mg}$ so that block 'C' doesn't move.



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1. **[Online April 2019]**

If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area $1\ m^2$ with a speed $10^4\ ms^{-1}$, the pressure exerted by the gas molecules will be of the order of

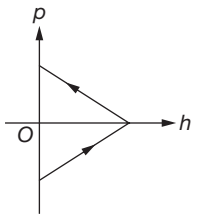
(A) $1\ Nm^{-2}$ (B) $2\ Nm^{-2}$

(C) $3\ Nm^{-2}$ (D) $4\ Nm^{-2}$

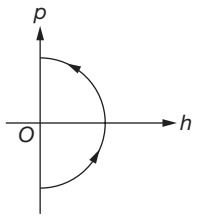
2. **[Online April 2019]**

A ball is thrown vertically up (taken as $+z$-axis) from the ground. The correct momentum height $(p\text{-}h)$ diagram is

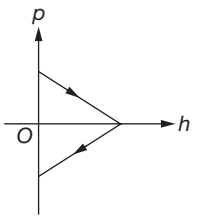
(A)



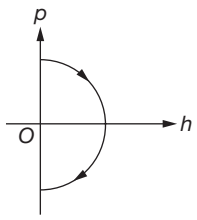
(B)



(C)



(D)



3. **[Online April 2019]**

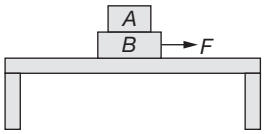
A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is

(A) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (B) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

(C) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (D) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

4. **[Online April 2019]**

Two blocks A and B of masses $m_A = 1\text{ kg}$ and $m_B = 3\text{ kg}$ are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is [Take $g = 10\text{ ms}^{-2}$]

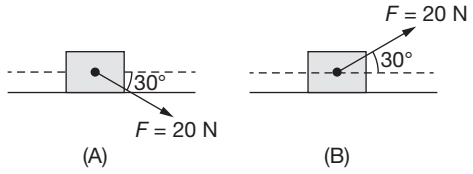


(A) 40 N (B) 12 N

(C) 16 N (D) 8 N

5. **[Online April 2019]**

A block of mass 5 is kg (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20\text{ N}$, making an angle of $30°$ with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be $\left(g = 10\text{ ms}^{-2}\right)$



(A) 0.4 ms^{-2} (B) 3.2 ms^{-2}

(C) 0 ms^{-2} (D) 0.8 ms^{-2}

6. **[Online April 2019]**

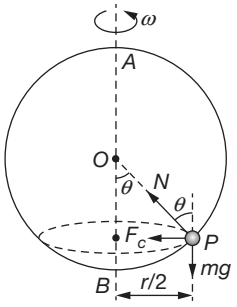
A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio $\frac{k_1}{k_2}$ of the corresponding force constants, k_1 and k_2 will be

(A) n^2 (B) $\frac{1}{n^2}$

(C) n (D) $\frac{1}{n}$

7. **[Online April 2019]**

A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to

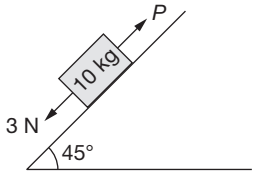


(A) $\frac{\left(g\sqrt{3}\right)}{r}$ (B) $\frac{2g}{r}$

(C) $\frac{2g}{\left(r\sqrt{3}\right)}$ (D) $\frac{\sqrt{3}g}{2r}$

8. **[Online January 2019]**

A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward? (Take $g = 10\text{ ms}^{-2}$)



(A) 25 N (B) 32 N

(C) 18 N (D) 23 N

9. **[Online January 2019]**

A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the roof at some point, the rope deviated at an angle of $45°$ at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is $\left(g = 10\text{ ms}^{-2}\right)$

(A) 100 N (B) 200 N

(C) 70 N (D) 140 N

10. **[Online January 2019]**

A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during

time interval T so that its momentum changes from p to $3p$. Here k is a constant. The value of T is

(A) $\sqrt{\frac{2k}{p}}$ (B) $2\sqrt{\frac{p}{k}}$

(C) $\sqrt{\frac{2p}{k}}$ (D) $2\sqrt{\frac{k}{p}}$

11. [Online January 2019]

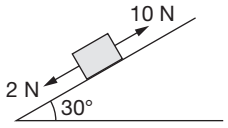
A particle moves from the point $\left(2.0\hat{i}+4.0\hat{j}\right)$ m, at $t=0$, with an initial velocity $\left(5.0\hat{i}+4.0\hat{j}\right)$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i}+4.0\hat{j}\right)$ ms^{-2}. What is the distance of the particle from the origin at time 2 s ?

(A) $20\sqrt{2}$ m (B) 15 m

(C) $10\sqrt{2}$ m (D) 5 m

12. [Online January 2019]

A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is [Take $g=10$ ms^{-2}]

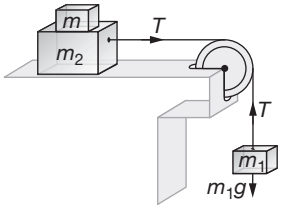


(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{\sqrt{3}}{4}$ (D) $\frac{2}{3}$

13. [2018]

Two masses $m_1=5$ kg and $m_2=10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure.



The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is

(A) 18.3 kg (B) 27.3 kg

(C) 43.3 kg (D) 10.3 kg

14. [Online 2018]

A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

(A) $\sqrt{1-\frac{1}{n^2}}$ (B) $1-\frac{1}{n^2}$

(C) $\frac{1}{2-n^2}$ (D) $\sqrt{\frac{1}{1-n^2}}$

15. [Online 2018]

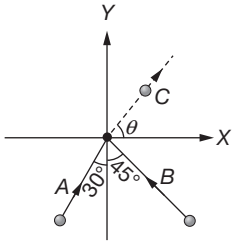
A body of mass 2 kg slides down with an acceleration of 3 ms^{-2} on a rough inclined plane having a slope of 30°. The external force required to take the same body up the plane with the same acceleration will be $\left(g=10\text{ ms}^{-2}\right)$

(A) 6 N (B) 14 N

(C) 4 N (D) 20 N

16. [Online 2017]

Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path, C makes with the X-axis is given by

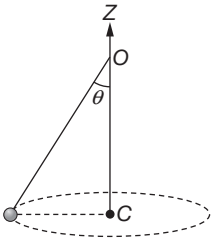


(A) $\tan\theta=\frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$ (B) $\tan\theta=\frac{1-\sqrt{3}}{1+\sqrt{2}}$

(C) $\tan\theta=\frac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$ (D) $\tan\theta=\frac{1-\sqrt{2}}{\sqrt{2}\left(1+\sqrt{3}\right)}$

17. [Online 2017]

A conical pendulum of length 1 m makes an angle $\theta=45°$ w.r.t. Z-axis and moves in a circle in the XY plane. The radius of the circle is 0.4 m and its center is vertically below O. The speed of the pendulum, in its circular path, will be $\left(\text{Take } g=10\text{ ms}^{-2}\right)$



(A) $0.4\ \text{ms}^{-1}$ (B) $2\ \text{ms}^{-1}$

(C) $0.2\ \text{ms}^{-1}$ (D) $4\ \text{ms}^{-1}$

18. [Online 2016]

A rocket is fired vertically from the earth with an acceleration of $2g$, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction $\mu_{\min}$ between the mass and the inclined surface such that the mass does not move is

(A) $\tan 2\theta$ (B) $\tan\theta$

(C) $3\tan\theta$ (D) $2\tan\theta$

19. [Online 2016]

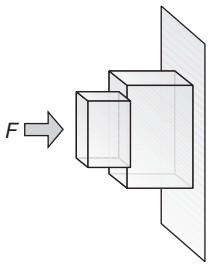
A particle of mass m is acted upon by a force F given by the empirical law $F=\frac{R}{t^2}v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot

(A) $\log v(t)$ against $\frac{1}{t}$ (B) $v(t)$ against t^2

(C) $\log v(t)$ against $\frac{1}{t^2}$ (D) $\log v(t)$ against t

20. [2015]

Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



(A) 120 N (B) 150 N

(C) 100 N (D) 80 N

21. [Online 2015]

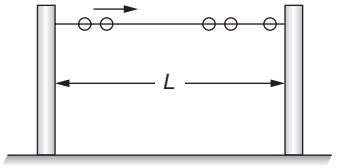
A block of mass $m=10$ kg rests on a horizontal table. The coefficient of friction between the block and the table is 0.05. When hit by a bullet of mass 50 g moving with speed v, that gets embedded in it, the block moves and comes to stop after moving a distance of 2 m on the table. If a freely falling object were to acquire speed $\frac{v}{10}$ after being dropped from height H, then neglecting energy losses and taking $g=10\ \text{ms}^{-2}$, the value of H is close to

(A) 0.2 km (B) 0.3 km

(C) 0.04 km (D) 0.5 km

22. [Online 2015]

A large number (n) of identical beads, each of mass m and radius r are strung on a thin smooth rigid horizontal rod of length $L(L\gg r)$ and are at rest at random positions. The rod is mounted between two rigid supports (shown in figure). If one of the beads is now given a speed v, the average force experienced by each support after a long time is (assume all collisions are elastic)



(A) $\frac{mv^2}{L-nr}$ (B) $\frac{mv^2}{L-2nr}$

(C) $\frac{mv^2}{2(L-nr)}$ (D) zero

23. [2014]

A block of mass m is placed on a surface with a vertical cross section given by $y=\frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(A) $\frac{1}{2}m$ (B) $\frac{1}{6}m$

(C) $\frac{2}{3}m$ (D) $\frac{1}{3}m$

24. [2012]

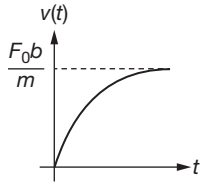
Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

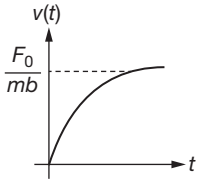
(A) $m_1 : m_2$ (B) $r_1 : r_2$

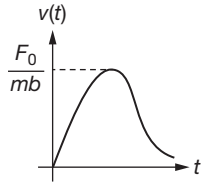
(C) $1:1$ (D) $m_1 r_1 : m_2 r_2$

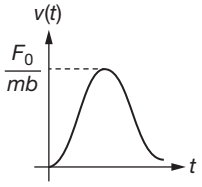
25. [2012]

A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves?

(A) 

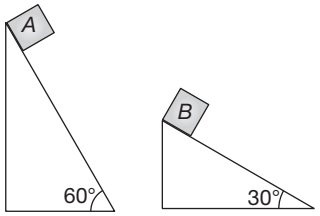
(B) 

(C) 

(D) 

26. [2010]

Two fixed frictionless inclined planes making an angle $30°$ and $60°$ with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(A) $4.9\ \text{ms}^{-2}$ in vertical direction

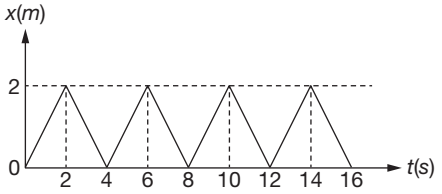
(B) $4.9\ \text{ms}^{-2}$ in horizontal direction

(C) $9.8\ \text{ms}^{-2}$ in vertical direction

(D) zero

27. [2010]

The figure shows the position-time $(x-t)$ graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(A) 0.2 Ns (B) 0.4 Ns

(C) 0.8 Ns (D) 1.6 Ns

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. [JEE (Advanced) 2016]

A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h(<l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of $30°$ with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio $\frac{h}{l}$ and the frictional force f at the bottom of the stick are $\left(g = 10\ \text{ms}^{-2}\right)$

(A) $\frac{h}{l} = \frac{\sqrt{3}}{16}$, $f = \frac{16\sqrt{3}}{3}$ N

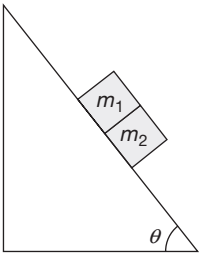
(B) $\frac{h}{l} = \frac{3}{16}$, $f = \frac{16\sqrt{3}}{3}$ N

(C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}$, $f = \frac{8\sqrt{3}}{3}$ N

(D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}$, $f = \frac{16\sqrt{3}}{3}$ N

2. [JEE (Advanced) 2014]

A block of mass $m_1 = 1$ kg another mass $m_2 = 2$ kg are placed together (shown in figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List-I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$.



In List-II expressions for the friction on the block m_2 are given. Match the correct expression of the friction in List-II with the angles given in List-I and choose the correct option. The acceleration due to gravity is denoted by g.

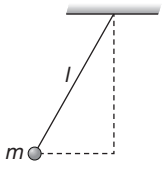
(Useful information $\tan(5.5°) \approx 0.1$; $\tan(11.5°) \approx 0.2$; $\tan(16.5°) \approx 0.3$)

List-I	List-II
P. $\theta = 5°$	1. $m_2 g \sin\theta$
Q. $\theta = 10°$	2. $(m_1 + m_2) g \sin\theta$
R. $\theta = 15°$	3. $\mu m_2 g \cos\theta$
S. $\theta = 20°$	4. $\mu(m_1 + m_2) g \cos\theta$

(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3
(C) P-2, Q-2, R-2, S-4 (D) P-2, Q-2, R-3, S-3

3. [IIT-JEE 2011]

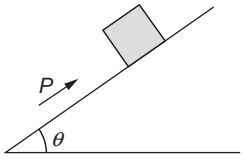
A ball of mass (m) 0.5 kg is attached to the end of a string having length (l) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rads^{-1}) is

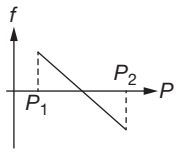


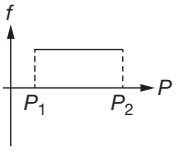
(A) 9 (B) 18
(C) 27 (D) 36

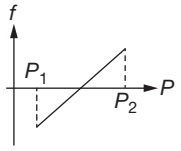
4. [IIT-JEE 2010]

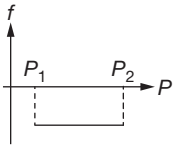
A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like



(A) 

(B) 

(C) 

(D) 

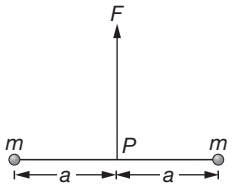
5. [IIT-JEE 2009]

A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$
(C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

6. [IIT-JEE 2007]

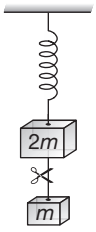
Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance a from the centre P (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is



(A) $\frac{F}{2m}\frac{a}{\sqrt{a^2 - x^2}}$ (B) $\frac{F}{2m}\frac{x}{\sqrt{a^2 - x^2}}$
(C) $\frac{F}{2m}\frac{x}{a}$ (D) $\frac{F}{2m}\frac{\sqrt{a^2 - x^2}}{x}$

7. [IIT-JEE 2006]

System shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of mass $2m$ and m just after the string is cut will be



(A) $\frac{g}{2}$ upwards, g downwards

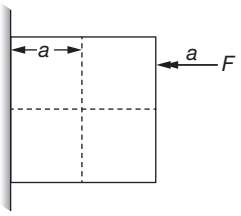
(B) g upwards, $\frac{g}{2}$ downwards

(C) g upwards, $2g$ downwards

(D) $2g$ upwards, g downwards

8. [IIT-JEE 2005]

A block of mass m is at rest under the action of force F against a wall as shown in figure. Which of the following statement is incorrect?



(A) $f = mg$ [where f is the frictional force]

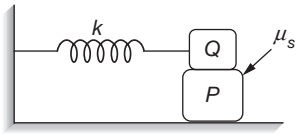
(B) $F = N$ [where N is the normal force]

(C) F will not produce torque

(D) N will not produce torque

9. [IIT-JEE 2004]

A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks?

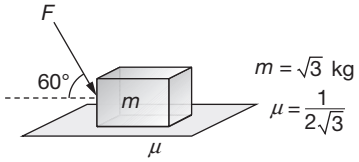


(A) $\frac{kA}{2}$ (B) kA

(C) $\mu_s mg$ (D) zero

10. [IIT-JEE 2003]

The maximum value of the force F such that the block shown in the arrangement, does not move is

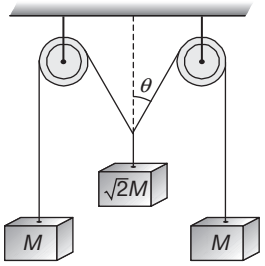


(A) 20 N (B) 10 N

(C) 12 N (D) 15 N

11. [IIT-JEE 2001]

The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be

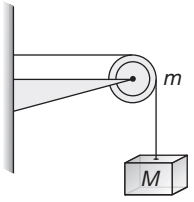


(A) 0° (B) 30°

(C) 45° (D) 60°

12. [IIT-JEE 2001]

A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by

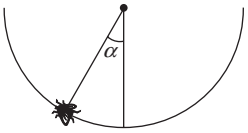


(A) $\sqrt{2}Mg$ (B) $\sqrt{2}mg$

(C) $\sqrt{(M+m)^2+m^2}\,g$ (D) $\sqrt{(M+m)^2+M^2}\,g$

13. [IIT-JEE 2001]

An insect crawls up a hemispherical surface very slowly as shown in figure. The coefficient of friction between the surface and the insect is $\frac{1}{3}$. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



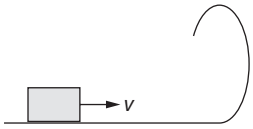
(A) $\cot\alpha = 3$ (B) $\tan\alpha = 3$

(C) $\sec\alpha = 3$ (D) $\operatorname{cosec}\alpha = 3$

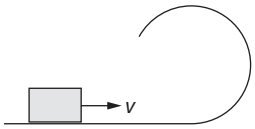
14. [IIT-JEE 2001]

A small block is shot into each of the four tracks as shown in the figure. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

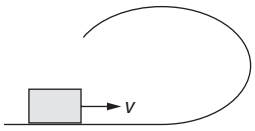
(A)



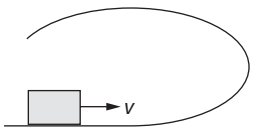
(B)



(C)



(D)



15. [2000, 2M]

A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$

(C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

16. [IIT-JEE 1994]

A block of mass 0.1 kg is held against a wall by applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, then the magnitude of frictional force acting on the block is

(A) 2.5 N (B) 0.98 N

(C) 4.9 N (D) 0.49 N

17. [IIT-JEE 1992]

A car is moving in a circular horizontal track of radius 10 m with a constant speed of $10\ \text{ms}^{-1}$. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1 m. The angle made by the rod with the track is

(A) 0° (B) 30°

(C) 45° (D) 60°

18. [IIT-JEE 1990]

When a bicycle is in motion and pedalled, the force of friction exerted by ground on the two wheels is such that it acts

(A) In the backward direction on the front wheel and in the forward direction on the rear wheel.

(B) In the forward direction on the front wheel and in the backward direction on the rear wheel.

(C) In the backward direction on both the front and the rear wheels.

(D) In the forward direction on both the front and the rear wheels.

19. [IIT-JEE 1980]

A block of mass 2 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is

(A) 9.8 N (B) 0.7×9.8 N

(C) $9.8\times\sqrt{3}$ N (D) $0.7\times9.8\times\sqrt{3}$ N

20. [IIT-JEE 1980]

A ship of mass 3×10^7 kg, initially at rest, is pulled by a force of 5×10^4 N through a distance of 3 m. Assuming that the resistance due to water is negligible, the speed of the ship is

(A) $1.5\ \text{ms}^{-1}$ (B) $60\ \text{ms}^{-1}$

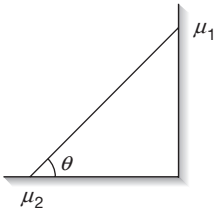
(C) $0.1\ \text{ms}^{-1}$ (D) $5\ \text{ms}^{-1}$

Multiple Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2014]

In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_2 \tan\theta = \dfrac{mg}{2}$

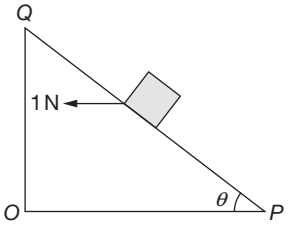
(B) $\mu_1 \neq 0$, $\mu_2 = 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

(C) $\mu_1 \neq 0$, $\mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

2. [IIT-JEE 2012]

A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10\text{ ms}^{-2}$)



(A) $\theta = 45°$.

(B) $\theta > 45°$ and a frictional force acts on the block towards P.

(C) $\theta > 45°$ and a frictional force acts on the block towards Q.

(D) $\theta < 45°$ and a frictional force acts on the block towards Q.

3. [IIT-JEE 1986]

A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\theta(|\theta| < \phi)$, the tension in the string and the velocity of the bob are T and V respectively. The following relations hold good under the above conditions

(A) $T\cos\theta = Mg$

(B) $T - Mg\cos\theta = \dfrac{MV^2}{L}$

(C) The magnitude of the tangential acceleration of the bob $|a_T| = g\sin\theta$.

(D) $T = Mg\cos\theta$

4. [IIT-JEE 1986]

A reference frame attached to the earth

(A) is an inertial frame by definition.

(B) cannot be an inertial frame because of revolution of earth round the sun.

(C) is an inertial frame because Newton's laws are applicable in this frame.

(D) cannot be an inertial frame because of rotation of earth about its own axis.

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. [JEE (Advanced) 2018]

A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4\text{ kg}$ is at rest on this surface. An impulse of 1 Ns is applied to the block at time $t = 0$, so that it starts moving along the X-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4\text{ s}$. The displacement of the block, in metres, at $t = \tau$ is ..... $\left(\text{Take, } e^{-1} = 0.37\right)$ [Round off your answer to nearest integer].

2. [IIT-JEE 2011]
A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\,\mu$, then N is

Assertion and Reasoning Type Problems

For the following Assertion-Reason type questions given below, choose the correct option:

(A) Both statement I & II are correct and statement II is correct explanation of statement I.
(B) Both statement I & II are correct and statement II is not correct explanation of statement I.
(C) Statement I is true and statement II is false.
(D) Statement II is correct and statement I is false.

1. [JEE (Advanced) 2008]
Statement-I: It is easier to pull a heavy object than to push it on a level ground.
Statement-II: The magnitude of frictional force depends on the nature of the two surfaces in contact.

2. [JEE (Advanced) 2007]
Statement-I: A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.
Statement-II: For every action there is an equal and opposite reaction.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Impulse Momentum)

1. 150 N
2. 4.5 kgms^{-1}
3. (a) 20 Ns
   (b) 8 kN
   (c) 16 kN
4. 15 N
5. 600 N

Test Your Concepts-II (Based on Constraints)

1. 3 ms^{-1}
2. 1.5 ms^{-1}
3. 0
4. $-\dfrac{L^2v_A^2}{\left(L^2-y^2\right)^{3/2}}$
5. 4 ms^{-2}
6. $\dfrac{v}{h}\sin^2\theta$
7. 2
8. 2 ms^{-2}
9. 0.4 ms^{-1} (down)
10. $\dfrac{s+\sqrt{2}x}{x+\sqrt{2}s}v_A$
11. $v_A\sqrt{\dfrac{2x^2+h^2}{x^2+h^2}}$
12. $-\dfrac{3y}{2\sqrt{y^2+b^2}}v_A$
13. 1.8 ms^{-1}, down
14. 45 cms^{-1}, upwards
15. 1 ms^{-1}, upwards ; 0.5 ms^{-2}, downwards
16. 0.25 ms^{-1}
17. (a) 0
    (b) 5 ms^{-2}
18. 6 m
19. 0.5 ms^{-1}
20. $2v_0$, downwards
21. 4.36 ms^{-2}
22. $\sqrt{31}a_0$
23. $u\sec\theta$
24. $\dfrac{2v\sin^2\left(\dfrac{\alpha}{2}\right)}{\cos\alpha}$
25. 0.5 ms^{-1}, 0.75 ms^{-2}, 1 ms^{-1}

Test Your Concepts-IV (Based on Newton's Laws of Motion: Accelerated Systems)

1. $4\sqrt{5}$ ms^{-1}
2. $\dfrac{mg}{2}\sin(2\theta)$
3. $\left(\dfrac{m_1\sin\alpha-2m_2}{m_1+4m_2}\right)g$, $2\left(\dfrac{m_1\sin\alpha-2m_2}{m_1+4m_2}\right)g$
4. 4.21 N, 5.79 ms^{-2}, downwards; 6.84 ms^{-2}, upwards
5. $\dfrac{2g(2\eta-\sin\theta)}{4\eta+1}$
6. $\sqrt{\dfrac{2l(\eta+4)}{3g(2-\eta)}}$
7. (a) 4.1 ms^{-2}
   (b) Tension in the string between A and B = 4.56 N
   Tension in the string between B and C = 18.2 N
8. (a) 4 ms^{-2}
   (b) 4 N
9. 4 N
10. 5.5 ms^{-2}
12. $\dfrac{mg\cos\alpha\sin\alpha}{\left(m\sin\alpha+\dfrac{M}{\sin\alpha}\right)}$, $\dfrac{mg\cos\alpha}{m\sin\alpha+\dfrac{M}{\sin\alpha}}$
13. 2.5 ms^{-2}
14. 2.34 ms^{-2}, 1.558 ms^{-2} and 81.8 N

15. $\dfrac{4F-(M+m)g}{M+m}$

17. 20 cm

18. 10 ms^{-1}

19. $\left(\dfrac{5m-M}{25m-M}\right)g$, $5\left(\dfrac{5m-M}{25m-M}\right)g$

20. $\dfrac{2m_1m_3g}{(m_2+m_3)(m_1+m_2)+m_2m_3}$

21. 735 N, 245 N, mode (b)

Test Your Concepts-V (Based on Equilibrium)

1. $\dfrac{2Mg}{\sqrt{3}}$, $\dfrac{Mg}{\sqrt{3}}$

2. $2\left[(\sqrt{3}+1)\hat{i}+(4+\sqrt{3})\hat{j}\right]$ N

3. mg

4. $\dfrac{4}{\sqrt{3}}$ N , $\dfrac{20}{\sqrt{3}}$ N

5. $\dfrac{mg}{A}$

6. $5\sqrt{3}$ N

7. $10\sqrt{2}$ N , $5\sqrt{2}$ N

Test Your Concepts-VI (Based on Non-inertial Frames: Pseudo Force)

1. $(g+a)\sin\theta$, down the plane

2. $(M_1+M_2+M_3)\dfrac{M_3}{M_2}g$

3. $a_0\sqrt{\dfrac{m}{k}}$

4. Acceleration of A is $\dfrac{g}{8}$ in horizontal direction and $\dfrac{5g}{8}$ in vertical direction.
Acceleration of B is $\dfrac{g}{2}$ in horizontal direction (leftwards) and acceleration of C is $\dfrac{g}{8}$ in horizontal direction (rightwards).

5. 5.66 ms^{-2}

6. $\dfrac{3g}{4}$

7. 30° , 20 N

8. $\tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$, $10\sqrt{7}$ N

9. $mg\cos^2\theta$

11. $\dfrac{m_0+m}{\cot\theta-1}$

Test Your Concepts-VII (Based on Friction)

1. $0.134r$

2. $\dfrac{1}{\sqrt{3}}$

3. 2.7 ms^{-2}

4. 3.16 s

5. $\dfrac{mg(\sin\theta+\mu\cos\theta)}{\sqrt{\mu^2+1}}$

6. 8.5 m

7. $a_A = 1.489$ ms^{-2}
$a_B = 0.744$ ms^{-2}
$T = 107.74$ N

8. $g(\sin\theta-\sqrt{2}\mu_K\cos\theta)$

9. $\sqrt{\dfrac{2F}{\rho}-\mu_K gL}$

10. (a) 6.63 ms^{-2}
(b) 0.33 m

11. (a) $\sqrt{\dfrac{22(\mu_2-\mu_1)gl}{10}}$
(b) $\sqrt{\dfrac{20l}{11(\mu_2-\mu_1)g}}$

13. 2.77 m

14. $a_A = 21$ ms^{-2}, $a_B = 1.6$ ms^{-2}

15. 1.33 ms^{-2}

16. (a) $\dfrac{F}{2\sqrt{3}}$
(b) 15.27 N

17. $\dfrac{(M+m)}{\mu-1}$, $\mu>1$

18. $\dfrac{(\mu+1)g}{\dfrac{2}{M_3}+\dfrac{1}{2M_1}+\dfrac{1}{2M_2}}$

19. 0.06 ms^{-2}

21. $a_A = a_C = \dfrac{g\left(m_3-\mu\left(2m_2+m_1\right)\right)}{m_1+m_3}$

22. 0.288

Test Your Concepts-VIII (Based on Circular Motion)

1. $v=\omega\sqrt{L^2-a^2}$

2. (a) $\dfrac{mv^2}{R}$

(b) $\dfrac{\mu mv^2}{R}$

(c) $\dfrac{\mu v^2}{R}$

(d) $v_0e^{-2\pi\mu}$

3. $\sqrt{\dfrac{2m_1\mu g}{(m_1-m_2)R}}$, 30 N

4. (a) $\sqrt{\dfrac{\mu g}{L}}$

(b) $\left(\left(\dfrac{\mu g}{L}\right)^2-\alpha^2\right)^{\frac{1}{4}}$

5. (a) 275.4 N

(b) 877.3 N

6. (a) 74°

(b) No

7. $\dfrac{3\pi a}{u}$

9. $4.08\ ms^{-1} \le v \le 15\ ms^{-1}$

10. $\omega_{min}=\sqrt{\dfrac{g(1-\mu\tan\theta)}{b\tan\theta(\mu+\tan\theta)}}$,

$\omega_{max}=\sqrt{\dfrac{g(1+\mu\tan\theta)}{b\tan\theta(\tan\theta-\mu)}}$

12. $\dfrac{v_0}{\left(1+\dfrac{\mu v_0 t}{R}\right)}$

13. $n_{max}=\dfrac{1}{2\pi}\sqrt{\dfrac{g(\sin\theta+\mu\cos\theta)}{r(\cos\theta-\mu\sin\theta)}}$,

$n_{min}=\dfrac{1}{2\pi}\sqrt{\dfrac{g(\sin\theta-\mu\cos\theta)}{r(\cos\theta+\mu\sin\theta)}}$

Single Correct Choice Type Questions

1. B	**2.** B	**3.** C	**4.** D	**5.** A	**6.** C	**7.** C	**8.** C	**9.** D	**10.** B
11. D	**12.** C	**13.** D	**14.** C	**15.** C	**16.** A	**17.** C	**18.** C	**19.** B	**20.** D
21. C	**22.** B	**23.** A	**24.** D	**25.** D	**26.** C	**27.** C	**28.** B	**29.** D	**30.** B
31. B	**32.** D	**33.** C	**34.** B	**35.** B	**36.** A	**37.** A	**38.** D	**39.** C	**40.** A
41. C	**42.** C	**43.** A	**44.** C	**45.** D	**46.** D	**47.** D	**48.** D	**49.** A	**50.** B
51. C	**52.** C	**53.** A	**54.** C	**55.** A	**56.** D	**57.** A	**58.** B	**59.** D	**60.** C
61. B	**62.** D	**63.** D	**64.** D	**65.** A	**66.** A	**67.** D	**68.** A	**69.** C	**70.** A
71. A	**72.** B	**73.** C	**74.** C	**75.** C	**76.** D	**77.** D	**78.** D	**79.** C	**80.** D
81. C	**82.** D	**83.** C	**84.** C	**85.** C	**86.** B	**87.** C	**88.** D	**89.** D	**90.** D
91. B	**92.** D	**93.** D	**94.** B	**95.** A	**96.** B	**97.** B	**98.** B	**99.** C	**100.** B
101. B	**102.** C	**103.** B	**104.** C	**105.** C	**106.** C	**107.** A	**108.** C	**109.** D	**110.** A
111. C	**112.** B	**113.** D	**114.** D	**115.** C	**116.** A	**117.** B	**118.** D	**119.** D	**120.** D
121. B	**122.** C	**123.** A	**124.** C	**125.** B	**126.** A	**127.** A	**128.** A	**129.** A	**130.** A
131. C	**132.** C	**133.** B	**134.** C	**135.** C	**136.** A	**137.** C	**138.** C	**139.** D	**140.** A

141. C	**142.** C	**143.** C	**144.** C	**145.** A	**146.** C	**147.** B	**148.** B	**149.** D	**150.** D
151. A	**152.** A	**153.** C	**154.** C	**155.** B	**156.** C	**157.** D	**158.** B	**159.** B	**160.** B
161. B	**162.** D	**163.** D	**164.** D	**165.** C	**166.** A	**167.** C	**168.** C	**169.** A	**170.** C
171. B	**172.** B	**173.** B	**174.** B	**175.** D	**176.** C	**177.** C	**178.** C	**179.** D	**180.** D
181. A	**182.** A	**183.** C	**184.** B	**185.** A	**186.** B	**187.** A	**188.** C	**189.** D	**190.** C
191. B	**192.** D	**193.** A	**194.** A	**195.** C	**196.** A	**197.** B	**198.** A	**199.** B	**200.** D
201. A	**202.** A	**203.** B	**204.** D	**205.** D	**206.** D	**207.** B	**208.** B	**209.** C	**210.** B
211. C	**212.** D	**213.** B	**214.** D	**215.** D	**216.** B	**217.** B	**218.** B	**219.** D	**220.** C
221. C	**222.** C								

Multiple Correct Choice Type Questions

1. A, D	**2.** B, D	**3.** A, C	**4.** A, B, C	**5.** A
6. A, C, D	**7.** B, D	**8.** A, B, C	**9.** B, C, D	**10.** B, D
11. A, B, D	**12.** A, C	**13.** A, D	**14.** B, D	**15.** A, D
16. A	**17.** D	**18.** A, C, D	**19.** B, C	**20.** D
21. A, D	**22.** B, C	**23.** A, C	**24.** A, B, C, D	**25.** A, B
26. C	**27.** B, D	**28.** A, B, D	**29.** B, D	**30.** A, B, C, D
31. B, D	**32.** A, B	**33.** A, B	**34.** B, C, D	**35.** C, D
36. A, C, D	**37.** A, B	**38.** A, C	**39.** A, D	**40.** A, D
41. A, C	**42.** A, B, C, D	**43.** A, C	**44.** A, D	**45.** A, B
46. A, B, C	**47.** B, C, D	**48.** B, C	**49.** A, C	**50.** B, D
51. A, C	**52.** A, B	**53.** A, B, C	**54.** B, C, D	**55.** A, B
56. A, B, C	**57.** A, C	**58.** A, D	**59.** A, C, D	**60.** A, B, D
61. B, C	**62.** B, D	**63.** A, D	**64.** B, D	**65.** A, B, C
66. A, B, C				

Reasoning Based Questions

1. B	**2.** A	**3.** D	**4.** A	**5.** B	**6.** B	**7.** A	**8.** D	**9.** B	**10.** B
11. D	**12.** D	**13.** A	**14.** B	**15.** C	**16.** A	**17.** A	**18.** D	**19.** D	**20.** A
21. D	**22.** D	**23.** A	**24.** B	**25.** D					

Linked Comprehension Type Questions

1. B	**2.** A	**3.** B	**4.** D	**5.** C	**6.** A	**7.** D	**8.** C	**9.** B	**10.** C
11. D	**12.** C	**13.** D	**14.** B	**15.** C	**16.** D	**17.** B	**18.** C	**19.** B	**20.** D
21. A	**22.** D	**23.** C	**24.** B	**25.** D	**26.** B	**27.** B	**28.** C	**29.** B	**30.** D
31. A	**32.** B	**33.** B	**34.** B	**35.** D	**36.** C	**37.** C	**38.** D	**39.** A	**40.** C
41. D	**42.** D	**43.** A	**44.** B	**45.** B	**46.** C	**47.** A	**48.** A	**49.** B	**50.** B
51. C	**52.** C	**53.** B	**54.** A	**55.** C	**56.** C	**57.** D	**58.** B	**59.** C	**60.** B
61. A	**62.** A	**63.** B	**64.** B	**65.** A	**66.** B	**67.** A	**68.** B	**69.** C	**70.** C
71. D	**72.** A	**73.** C	**74.** D	**75.** A	**76.** C	**77.** B	**78.** B	**79.** B	**80.** B
81. D	**82.** C	**83.** B	**84.** B	**85.** A	**86.** B	**87.** A			

Matrix Match/Column Match Type Questions

1. A → (q)	B → (r)	C → (s)	D → (p)	
2. A → (p, u)	B → (t)	C → (r, s)	D → (q, r)	
3. A → (r)	B → (s)	C → (t)	D → (q)	
4. A → (t)	B → (r)	C → (p)	D → (s)	E → (q)
5. A → (q)	B → (s)	C → (p, t)	D → (r)	
6. A → (r, s)	B → (p, s)	C → (p, s)	D → (p, r, s)	
7. A → (q, r)	B → (p, s)	C → (p, s)	D → (q, r)	
8. A → (r)	B → (t)	C → (q)	D → (p)	
9. A → (r)	B → (t)	C → (p)	D → (q)	
10. A → (s)	B → (p)	C → (t)	D → (q)	
11. A → (t, u)	B → (p, u)	C → (q, u)	D → (s, u)	
12. A → (p, q, r)	B → (p, q, r)	C → (p, q)	D → (s)	
13. A → (s)	B → (r)	C → (p)	D → (q)	
14. A → (p, s, t)	B → (p, q, t)	C → (p, t)	D → (r)	
15. A → (r)	B → (p)	C → (q)	D → (s)	
16. A → (p)	B → (r)	C → (q)	D → (s)	
17. A → (q, s)	B → (p, s)	C → (r)	D → (r)	

Integer/Numerical Answer Type Questions

1. 6	**2.** 10	**3.** 40	**4.** 125	**5.** 4
6. 400	**7.** 100	**8.** 3	**9.** (a) 2, (b) 6	**10.** 32
11. 100	**12.** 90	**13.** 16	**14.** 200	**15.** (a) 20, (b) 3600
16. (a) 54, (b) 59	**17.** 5	**18.** 6	**19.** 4220	**20.** 5
21. 5	**22.** 0	**23.** 18	**24.** 6	**25.** 4
26. 1	**27.** 8	**28.** 8	**29.** 0	**30.** 1
31. 2	**32.** 4	**33.** 8	**34.** 2	**35.** 8
36. 0	**37.** 5	**38.** 2	**39.** 4	**40.** 1
41. 5				

ARCHIVE: JEE MAIN

1. B	**2.** D	**3.** C	**4.** C	**5.** D	**6.** D	**7.** C	**8.** B	**9.** A	**10.** B
11. A	**12.** B	**13.** B	**14.** B	**15.** D	**16.** A	**17.** B	**18.** B	**19.** A	**20.** A
21. C	**22.** B	**23.** B	**24.** B	**25.** B	**26.** A	**27.** C			

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Single Correct Choice Type Problems

1. D	**2.** D	**3.** D	**4.** A	**5.** B	**6.** B	**7.** A	**8.** D	**9.** A	**10.** A
11. C	**12.** D	**13.** A	**14.** A	**15.** A	**16.** B	**17.** C	**18.** A	**19.** A	**20.** C

Multiple Correct Choice Type Problems

1. C, D **2.** A, C **3.** B, C **4.** B, D

Matrix Match/Column Match Type Questions

1. 6 **2.** 5

Assertion and Reasoning Type Problems

1. B **2.** B

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HINTS AND EXPLANATIONS

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Single Correct Choice Type Questions

15. Ratio $= \dfrac{\pi r^2}{4\pi r^2}$

Hence, the correct answer is (A).

16. $y = \dfrac{b}{a}x + c$

Compare with $y = mx + c$, we get, $m = \dfrac{b}{a}$.

Hence, the correct answer is (B).

28. $h = -\dfrac{b}{2a} = \dfrac{4}{2\times 3} = \dfrac{2}{3}$

$k = -\left(\dfrac{b^2 - 4ac}{4a}\right) = -\left(\dfrac{16 - 4\times 1\times 3}{4\times 3}\right) = -\dfrac{1}{3}$

Hence, the correct answer is (C).

31. $180° = \pi$ radian

Hence, the correct answer is (B).

49. $\sin(n\pi) = 0$, $n = 0, 1, 2, \ldots\ldots$

Hence, the correct answer is (C).

50. $\cos(180 - \theta) = -\cos\theta$

Hence, the correct answer is (A).

89. $\dfrac{dy}{dx} = \dfrac{1}{2}(2x^2+1)^{\frac{1}{2}-1} \times \dfrac{d}{dx}(2x^2+1)$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{4x}{2\sqrt{2x^2+1}} = \dfrac{2x}{\sqrt{2x^2+1}}$

Hence, the correct answer is (B).

90. $\dfrac{dy}{dx} = e^{\sqrt{2x}} \times \dfrac{d}{dx}\left(\sqrt{2x}\right) = \dfrac{2e^{\sqrt{2x}}}{2\sqrt{2x}} = \dfrac{e^{\sqrt{2x}}}{\sqrt{2x}}$

Hence, the correct answer is (A).

96. $y = 3\sin^2 x\cos x - 6\sec x\sec x\tan x$

$\Rightarrow \quad y = 3\left(\dfrac{\sqrt{3}}{2}\right)^2 \times \dfrac{1}{2} - 6\times 2\times 2\times\sqrt{3}$

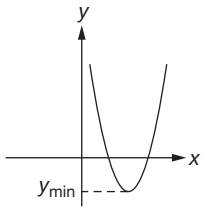
$\Rightarrow \quad y = \dfrac{9}{4} - 24\sqrt{3} = \dfrac{9 - 96\sqrt{3}}{4}$

Hence, the correct answer is (A).

97. $\dfrac{dy}{dx} = \dfrac{d\sin(2x^2)}{d(2x^2)} \times \dfrac{d(2x^2)}{dx} = \cos(2x^2)\times 4x$

Hence, the correct answer is (A).

98. $y = 2x^2 - x + 1$



$\Rightarrow \quad h = -\left(-\dfrac{1}{4}\right) = \dfrac{1}{4}$

$\Rightarrow \quad k = -\left(\dfrac{1 - 4\times 2}{4\times 8}\right) = -\dfrac{7}{8}$

Hence, the correct answer is (C).

99. $\dfrac{dy}{dx} = 2\sin x\cos x - 4\tan x\sec^2 x$

$\Rightarrow \quad \dfrac{dy}{dx} = 2\times\dfrac{1}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}} - 4\times T\times 2\times 2$

$\Rightarrow \quad \dfrac{dy}{dx} = 1 - 16 = -15$

Hence, the correct answer is (D).

100. $\dfrac{dy}{dx} = 3x^2 + 2$

at $x = 1$

$\Rightarrow \quad \left(\dfrac{dy}{dx}\right)\Big|_{x=1} = 3$

Hence, the correct answer is (D).

101. $y = \dfrac{1+x}{e^x}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{1+x}{e^x}\right)$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{e^x\left[\dfrac{d}{dx}(1+x)\right] - (1+x)\dfrac{d}{dx}(e^x)}{e^{2x}}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{e^x - (1+x)e^x}{e^{2x}} = -\dfrac{x}{e^x}$

Hence, the correct answer is (B).

103. $\dfrac{dy}{dx} = 3x^2 + 2$

Hence, the correct answer is (D).

104. $\frac{dy}{dx}=\frac{e^x-(1+x)e^x}{e^xe^x}=\frac{-x}{e^x}$

Hence, the correct answer is (B).

114. $\int_0^1(3x^2-4x+1)dx=\left(3\frac{x^3}{3}-\frac{4x^2}{2}+x\right)\Big|_0^1=1-\frac{4}{2}+1=0$

Hence, the correct answer is (A).

115. Let, $I=\int_0^{\pi/2}\sin(2\theta)\,d\theta=\left(\frac{-\cos(2\theta)}{2}\right)\Big|_0^{\pi/2}$

$\Rightarrow \quad I=-\left(\frac{\cos\pi-\cos 0}{2}\right)=-\left[\frac{-1-1}{2}\right]=1$

Hence, the correct answer is (B).

116. $\int_3^5\frac{1}{2x+3}dx=\left(\frac{\ln(2x+3)}{2}\right)\Big|_3^5=\frac{\ln(13)-\ln 9}{2}=\frac{1}{2}\ln\left(\frac{13}{9}\right)$

Hence, the correct answer is (B).

117. $\int_0^1(3-2x)^{-2}\,dx=\left(\frac{(3-2x)^{-2+1}}{(-2+1)\times 3}\right)\Big|_0^1=-\frac{1}{3}\left(1-\frac{1}{3}\right)=-\frac{2}{9}$

Hence, the correct answer is (B).

118. Let, $I=\int\sqrt{1+\cos x}\,dx=\int\left(\sqrt{1+2\cos\frac{x}{2}-1}\right)dx$

$\Rightarrow \quad I=\int\sqrt{2}\cos\frac{x}{2}dx$

$\Rightarrow \quad I=\sqrt{2}\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)}+C$

$\Rightarrow \quad I=2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$

Hence, the correct answer is (C).

119. $\int_0^2 2t\,dt=(t^2)\Big|_0^2=4$

Hence, the correct answer is (B).

120. Let $I=\int_{\pi/6}^{\pi/2}\sin x\,dx=\left(-\cos x\right)\Big|_{\pi/6}^{\pi/2}$

$\Rightarrow \quad I=\left[-\cos\left(\frac{\pi}{2}\right)\right]-\left[-\cos\left(\frac{\pi}{6}\right)\right]$

$\Rightarrow \quad I=0+\frac{\sqrt{3}}{2}$

Hence, the correct answer is (C).

121. $\int\frac{dt}{(6t-1)}=\frac{1}{6}\log(6t-1)+C$

Hence, the correct answer is (A).

122. $\int(4\cos t+t^2)dt=\int 4\cos t\,dt+\int t^2\,dt=4\sin t+\frac{t^3}{3}+C$

Hence, the correct answer is (C).

CHAPTER 2: MEASUREMENTS AND GENERAL PHYSICS

Test Your Concepts-I (Based on Principle of Homogeneity: Verification)

1. $[Q]=\dfrac{(ML^2)(MLT^{-2})(L^2T^{-2})}{(ML^2T^{-2})(L^3)}$

$\Rightarrow \quad [Q]=MT^{-2}$

So, dimensions of Q are 1 in mass, zero in length and -2 in time.

2. Yes, like work and torque, pressure and stress, surface tension and spring constant, angular momentum and Planck's constant etc.

3. $ML^2T^{-2}K^{-1}$, $M^{-1}L^3T^{-2}$

4. $[V]=L^3T^{-1}$

$$\left[\frac{\pi}{8}\frac{r^4}{\eta}\frac{P}{\ell}\right]=\left(\frac{L^4}{\cancel{M}\,\cancel{L^{-1}}\,T^{-1}}\right)\left(\frac{\cancel{M}\,\cancel{L^{-1}}\,T^{-2}}{L}\right)=L^3T^{-1}$$

6. $$\left[\frac{EJ^2}{M^5G^2}\right]=\frac{(ML^2T^{-2})(ML^2T^{-1})^2}{(M^5)(M^{-1}L^3T^{-2})^2}=M^0L^0T^0$$

7. $[x]=[\text{Capacitance}]=\left[\dfrac{Q}{V}\right]=\left[\dfrac{Q}{\frac{W}{Q}}\right]$

$\Rightarrow \quad [x]=M^{-1}L^{-2}T^2Q^2$

$[z]=[B]=\left[\dfrac{F}{qv}\right]=MT^{-1}Q^{-1}$

Since $x=3yz^2$

$\Rightarrow \quad [x]=[y][z^2]$

$\Rightarrow \quad M^{-1}L^{-2}T^2Q^2=[y](M^2T^{-2}Q^{-2})$

$\Rightarrow \quad [y]=M^{-3}L^{-2}T^4Q^4$

8. Yes, $\dfrac{e^2}{2h\varepsilon_0 c}$ is actually the Fine Structure Constant, which happens to be a pure number having a value $\dfrac{1}{137}$.

9. Here $[\mu]=\dfrac{\text{Velocity of light in air}}{\text{Velocity of light in glass}}$

Since $[\mu]=$ a dimensionless number $=M^0L^0T^0$

$\Rightarrow \quad [A]=[\mu]=$ a dimensionless number $=M^0L^0T^0$

Also $\left[\dfrac{B}{\lambda^2}\right]=[\mu]$

$\Rightarrow \quad [B]=[\mu][\lambda^2]=(M^0L^0T^0)L^2=L^2$

Hence A, being dimensionless, has no units and SI unit of B is m^2.

10. Since trigonometric functions are dimensionless, so

$[\tan\theta]=M^0L^0T^0$

But $[v]=LT^{-1}$

$\therefore$ Dimensions of LHS $\neq$ Dimensions of RHS

Hence the given relation is dimensionally wrong.

This relation can be corrected by dividing RHS by the speed u of the rainfall. So the corrected relation is $\tan\theta=\dfrac{v}{u}$.

Test Your Concepts-II (Based on Principle of Homogeneity: Conversion)

1. $$n_2=n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2}$$

$$\Rightarrow \quad n_2=4.18\left(\frac{1\text{ kg}}{\alpha\text{ kg}}\right)\left(\frac{1\text{ m}}{\beta\text{ m}}\right)^2\left(\frac{1\text{ s}}{\gamma\text{ s}}\right)^{-2}$$

$$\Rightarrow \quad n_2=4.18\alpha^{-1}\beta^{-1}\gamma^2$$

So, $4.18\text{ J}=4.18\ \alpha^{-1}\beta^{-2}\gamma^2$ new units

2. $100\text{ W}=100\text{ Js}=10^9\text{ erg}$ $\qquad \{\because 1\text{ J}=10^7\text{ erg}\}$

3. 10 poise.

4. $F=20\text{ N}$, $E=200\text{ J}$, $v=5\text{ ms}^{-1}$

Since $E=FL$

$$\Rightarrow \quad L=\frac{E}{F}=\frac{200}{20}=10\text{ m}$$

Since $v=LT^{-1}$

$\Rightarrow \quad 5=10\text{ T}^{-1}$

$\Rightarrow \quad T=2\text{ s}$

Further $F=MLT^{-2}$

$\Rightarrow \quad 20=M(10)(2)^{-2}$

$$\Rightarrow \quad M=\frac{80}{10}=8\text{ kg}$$

5. $n_2 = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^{-3}$

$\Rightarrow \quad n_2 = 8\left(\frac{1\text{ g}}{20\text{ g}}\right)\left(\frac{1\text{ cm}}{5\text{ cm}}\right)^{-3}$

$\Rightarrow \quad n_2 = 8\times\frac{1}{20}\times 5\times 5\times 5$

$\Rightarrow \quad n_2 = 50$

So, $8\text{ gcc}^{-1} = 50$ new units

6. $1\text{ hp} = 746\text{ W}$

7. $[p] = ML^{-1}T^{-2}$ and $[\sigma] = MT^{-2}$

Since $[v] = LT^{-1}$, $[\rho] = ML^{-3}$, $[v] = T^{-1}$

So, let

$$\rho v^4 = ML^{-3}L^4T^{-4}$$

$\Rightarrow \quad \rho v^4 = MLT^{-4}$

$\Rightarrow \quad \frac{\rho v^4}{v^3} = \frac{MLT^{-4}}{T^{-3}} = MLT^{-1}$

$\Rightarrow \quad p = \rho v^4 v^3$

Similarly,

$$\rho v^3 = MT^{-3}$$

$\Rightarrow \quad \frac{\rho v^3}{v} = \frac{MT^{-3}}{T^{-1}} = MT^{-2}$

So, $p \propto \rho v^4 v^3$ and $\sigma \propto \frac{\rho v^3}{v}$

8. Speed of light = 1 new unit of length/s

Time $= 8\text{ min }20\text{ s} = 8\times 60 + 20 = 500\text{ s}$

Distance between the earth and the sun is

$$d = \text{Speed of light} \times \text{time} = 1\times 500$$

$\Rightarrow \quad d = 500$ New units of length.

9. $F = MLT^{-2}$

$\Rightarrow \quad M = FT^2L^{-1} = \frac{FT^2}{L}$

$\Rightarrow \quad M = \frac{(1000\text{ N})(100\text{ s})^2}{(1000\text{ m})}$

$\Rightarrow \quad M = 10^4\text{ kg}$

10. Given that,

$S = 2\text{ cal cm}^{-2}\text{ min}$

But as $1\text{ cal} = 4.18\text{ J}$, $1\text{ cm} = 10^{-2}\text{m}$ and $1\text{ min} = 60\text{ s}$

$$S = \frac{2\times 4.18\text{ J}}{\left(10^{-2}\text{m}\right)^2(60\text{ s})} \approx 1.4\times 10^3\,\frac{\text{J}}{\text{m}^2\text{s}}$$

$\Rightarrow \quad S = 1.4\text{ kWm}^{-2}$

Test Your Concepts-III (Based on Principle of Homogeneity: Dependence)

1. $T \propto d^a r^b S^c$

$\Rightarrow \quad T = kd^a r^b S^C$ …(1)

where k is a dimensionless constant.

Taking dimensions on both sides of (1) and applying the Principle of Homogeneity we get

$$T = (ML^{-3})^a (L)^b (MT^{-2})^c$$

$\Rightarrow \quad T = M^{a+c}L^{-3a+b}T^{-2c}$

So, we get $-2c = 1$

$\Rightarrow \quad c = -\frac{1}{2}$

$$a + c = 0$$

$\Rightarrow \quad a = \frac{1}{2}$

$$-3a + b = 0$$

$\Rightarrow \quad b = \frac{3}{2}$

$\Rightarrow \quad T = kd^{\frac{1}{2}}r^{\frac{3}{2}}S^{-\frac{1}{2}}$

$\Rightarrow \quad T = k\sqrt{\frac{r^3 d}{S}}$

2. $t \propto p^a d^b E^c$

$\Rightarrow \quad t = kp^a d^b E^c$

where k is a dimensionless constant

$\Rightarrow \quad T = (ML^{-1}T^{-2})^a (ML^{-3})^b (ML^2T^{-2})^c$

$\Rightarrow \quad T = M^{a+b+c}L^{-a-3b+2c}T^{-2a-2c}$

Using Principle of Homogeneity we get

$$a + b + c = 0 \quad \ldots(1)$$

$$-a - 3b + 2c = 0 \quad \ldots(2)$$

$$-2a - 2c = 1 \quad \ldots(3)$$

From (3), we get

$$a + c = -\frac{1}{2}$$

Substituting in (1)

$$-\frac{1}{2} + b = 0$$

$\Rightarrow \quad b = \frac{1}{2}$

Substituting in (2), we get

$$-a - \frac{3}{2} + 2c = 0$$

$\Rightarrow \quad -a + 2c = \frac{3}{2}$ …(4)

Adding (3) and (4), we get

$$-3a = \frac{5}{2}$$

$$\Rightarrow \quad a = -\frac{5}{6}$$

Substituting in (3), to get

$$-2\left(-\frac{5}{6}\right) - 2c = 1$$

$$\Rightarrow \quad \frac{5}{3} - 2c = 1$$

$$\Rightarrow \quad 2c = \frac{5}{3} - 1$$

$$\Rightarrow \quad 2c = \frac{2}{3}$$

$$\Rightarrow \quad c = \frac{1}{3}$$

Hence we get

$$a = -\frac{5}{6},\ b = \frac{1}{2},\ c = \frac{1}{3}.$$

$$\Rightarrow \quad T = kp^{-\frac{5}{6}} d^{\frac{1}{2}} E^{\frac{1}{3}}$$

3. $T \propto m^a f^b \ell^c$

$T = km^a f^b \ell^c$

$T = M^a (MLT^{-2})^b L^c$

$\Rightarrow \quad T = M^{a+b} L^{b+c} T^{-2b}$

Using Principle of Homogeneity, we get

$a + b = 0$ …(1)

$b + c = 0$ …(2)

$-2b = 1$ …(3)

$$\Rightarrow \quad b = -\frac{1}{2},\ c = \frac{1}{2},\ a = \frac{1}{2}$$

Hence $T = km^{\frac{1}{2}} f^{-\frac{1}{2}} \ell^{\frac{1}{2}}$

$$\Rightarrow \quad T = k\sqrt{\frac{ml}{f}}$$

4. $v_T \propto W^a \eta^b r^c$

$\Rightarrow \quad v_T = kW^a \eta^b r^c$

$\Rightarrow \quad LT^{-1} = (MLT^{-2})^a (ML^{-1}T^{-1})^b L^c$

$\Rightarrow \quad LT^{-1} = M^{a+b} L^{a-b+c} T^{-2a-b}$

Using Principle of Homogeneity, we get

$a + b = 0$ …(1)

$a - b + c = 1$ …(2)

$-2a - b = -1$ …(3)

Adding (1) and (3), we get

$a = 1$

$\Rightarrow \quad b = -1$

$\Rightarrow \quad 1 - (-1) + c = 1$ $\{\because \text{ of } (2)\}$

$\Rightarrow \quad c = -1$

So, $a = 1,\ b = -1,\ c = -1$

$$\Rightarrow \quad v_T = k\left(\frac{W}{\eta r}\right)$$

In Fluid Mechanics, this law is also called the Stokes' Law with the value of $k = 6\pi$.

5. Let the rate of flow of volume of the liquid be denoted by V.

Then $[V] = \left[\frac{\text{Volume}}{\text{Time}}\right] = L^3T^{-1}$

Now, $V \propto \eta^a r^b p^c$

$\Rightarrow \quad V = k\eta^a r^b p^c$

where k is a dimensionless constant and p is the pressure gradient. So

$$[p] = \left[\frac{\text{Pressure}}{\text{Length}}\right] = ML^{-2}T^{-2}$$

$\Rightarrow \quad L^3T^{-1} = (ML^{-1}T^{-1})^a (L)^b (ML^{-2}T^{-2})^c$

$\Rightarrow \quad L^3T^{-1} = M^{a+c} L^{-a+b-2c} T^{-a-2c}$

Using Principle of Homogeneity, we get

$a + c = 0$ …(1)

$-a + b - 2c = 3$ …(2)

$-a - 2c = -1$ …(3)

From (1) and (3), we get

$c = 1$

$\Rightarrow \quad a = -1$ $\{\because \text{ of } (1)\}$

$\Rightarrow \quad -(-1) + b - 2(1) = 3$ $\{\because \text{ of } (2)\}$

$\Rightarrow \quad b = 4$

So, we get

$a = -1,\ b = 4,\ c = 1$

$$\Rightarrow \quad V = k\left(\frac{r^4}{\eta}\right)p$$

This expression is also called the Poiseulle's Equation, which governs the flow of a liquid through a horizontal tube across the ends of which a pressure gradient (p) is maintained. Also we shall see in hydromechanics that for this case $k = \frac{\pi}{8}$. So

$$V = \frac{\pi}{8}\left(\frac{r^4}{\eta}\right)p \qquad \{\text{Poiseulle's Equation}\}$$

CHAPTER 2

6. $\omega_c \propto \eta^a \rho^b d^c$

$\omega_c = k\eta^a \rho^b d^c$

where k is a dimensionless constant

$\Rightarrow \quad T^{-1} = [ML^{-1}T^{-1}]^a (ML^{-3})^b (L)^c$

$\Rightarrow \quad T^{-1} = M^{a+b} L^{-a-3b+c} T^{-a}$

Using the Principle of Homogeneity, we get

$a + b = 0$ …(1)

$-a - 3b + c = 0$ …(2)

$-a = -1$ …(3)

$\Rightarrow \quad a = 1,\ b = -1,\ c = -2$

So, we get

$$\omega_c = k\left(\frac{\eta}{\rho d^2}\right)$$

7. No it would not be possible to obtain dimensionally a relation for h without the additional experimental information that $h \propto \frac{1}{r}$. This is because without the information $h \propto \frac{1}{r}$, we will get four variables to be calculated from three equations which is just impossible. So, we shall be writing

$h \propto r^{-1} S^a d^b g^c$

$\Rightarrow \quad h = kr^{-1} S^a d^b g^c$

where k is a dimensionless constant

$\Rightarrow \quad L = L^{-1} (MT^{-2})^a (ML^{-3})^b (LT^{-2})^c$

$\Rightarrow \quad L = M^{a+b} L^{-1-3b+c} T^{-2a-2c}$

Using Principle of Homogeneity, we get

$a + b = 0$ …(1)

$-1 - 3b + c = 1$ …(2)

$-2a - 2c = 0$ …(3)

So, $(2① + ③)$ gives

$2b - 2c = 0$ …(4)

Again $2② + ④$ gives

$b = -1$

$\Rightarrow \quad a = 1$ $\{\because$ of (1)$\}$

$\Rightarrow \quad c = -1$

So, $a = 1,\ b = -1$ and $c = -1$

$\Rightarrow \quad h = kr^{-1} S d^{-1} g^{-1}$

$$\Rightarrow \quad h = k\left(\frac{S}{rgd}\right)$$

8. To show that the planet obeys Kepler's Third Law of planetary motion, we have to prove that

$T^2 \propto r^3$

Now, according to the problem, we have

$T \propto r^a M_S^b G^c$

$\Rightarrow \quad T = kr^a M_S^b G^c$

where k is a dimensionless constant.

$\Rightarrow \quad T = L^a M^b (M^{-1} L^3 T^{-2})^c$

$\Rightarrow \quad T = M^{b-c} L^{a+3c} T^{-2c}$

Using the Principle of Homogeneity, we get

$b - c = 0$ …(1)

$a + 3c = 0$ …(2)

$-2c = 1$ …(3)

$$\Rightarrow \quad c = -\frac{1}{2},\ b = -\frac{1}{2},\ a = \frac{3}{2}$$

So, we get

$$T = kr^{\frac{3}{2}} M_S^{-\frac{1}{2}} G^{-\frac{1}{2}}$$

$$\Rightarrow \quad T^2 = \left(\frac{k^2}{M_S G}\right) r^3$$

Since the product $M_S G$ is constant, so

$T^2 \propto r^3$

9. We have $\rho = ML^{-3},\ g = LT^{-2},\ \nu = T^{-1}$

Solving for M, L and T in terms of ρ, g and ν, we get

$M = \rho g^3 \nu^6,\ L = g\nu^{-2},\ T = \nu^{-1}$

$\Rightarrow \quad [\text{Force}] = MLT^{-2} = (\rho g^3 \nu^6)(g\nu^{-2})(\nu^2)$

$= \rho g^4 \nu^6$

10. Let the quantity be Q then,

$Q = f(v,\ F,\ T)$

Assuming that the function is the product of power functions of v, F and T,

$Q = Kv^x F^y T^z$ …(1)

where K is a dimensionless constant of proportionality. The above equation dimensionally becomes

$[Q] = (LT^{-1})^x (MLT^{-2})^y (T)^z$

i.e., $[Q] = [M^y][L^{x+y} T^{-x-2y+z}]$ …(2)

(a) Q = mass

i.e., $[Q] = [M]$

So Equation (2) becomes

$M = M^y L^{x+y} T^{-x-2y+z}$

its dimensional correctness requires

$y = 1,\ x + y = 0$ and $-x - 2y + z = 0$

which on solving yields

$x = -1;\ \ y = 1$ and $z = 1$

Substituting it in equation (1), we get

$Q = Kv^{-1}FT$

(b) $Q =$ energy i.e., $[Q] = ML^2T^{-2}$

So equation (2) becomes

$ML^2T^{-2} = M^yL^{x+y}T^{-x-2y+z}$

which in the light of principle of homogeneity yields

$y = 1,\ x + y = 2$ and $-x - 2y + z = -2$

which on solving yields

$x = y = z = 1$

So equation (1) becomes

$Q = KvFT$

Test Your Concepts-IV (Based on Errors, Significant Figures, Vernier Calliper and Screw Gauge)

1. $R = \dfrac{V}{\mathrm{I}} = \dfrac{8}{2} = 4\ \Omega$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta \mathrm{I}}{\mathrm{I}}$$

$$\Rightarrow \quad \%\frac{\Delta R}{R} = \left(\frac{0.5}{8} + \frac{0.2}{2}\right)100$$

$$\Rightarrow \quad \%\frac{\Delta R}{R} = \left(\frac{50}{8} + \frac{20}{2}\right) = 16.25\%$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{16.25}{100}$$

$$\Rightarrow \quad \Delta R = \frac{16.25}{100} \times 4$$

$$\Rightarrow \quad \Delta R = 0.65\ \Omega$$

So, $R = 4\ \Omega \pm 16.25\%$ or $R = (4 \pm 0.65)\Omega$

2. $d = \dfrac{m}{lbh}$

Actual value of density is given by

$$d = \frac{39.3}{(5.12)(2.56)(0.37)}\ \mathrm{gcc}^{-1}$$

$$\Rightarrow \quad d = 8.1\ \mathrm{gcc}^{-1}$$

Now $\dfrac{\Delta d}{d} = \dfrac{\Delta m}{m} + \dfrac{\Delta l}{l} + \dfrac{\Delta b}{b} + \dfrac{\Delta h}{h}$

$$\Rightarrow \quad \frac{\Delta d}{d} = \frac{0.1}{39.3} + \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37}$$

$$\Rightarrow \quad \frac{\Delta d}{d} \times 100\% = \frac{10}{39.3} + \frac{1}{5.12} + \frac{1}{2.56} + \frac{1}{0.37}$$

$$\Rightarrow \quad \frac{\Delta d}{d} \times 100\% = 0.25 + 0.19 + 0.39 + 2.7$$

$$\Rightarrow \quad \frac{\Delta d}{d} \times 100\% = 3.6\%$$

So, $d = 8.1\ \mathrm{gcc}^{-1} \pm 3.6\%$

3. Least Count $= 1$ MSD $- 1$ VSD

Since N VSD $= (N - 1)$MSD

$$\Rightarrow \quad 1\ \mathrm{VSD} = \left(\frac{N-1}{N}\right)\mathrm{MSD}$$

$$\Rightarrow \quad \text{Least Count} = 1\ \mathrm{MSD} - \left(\frac{N-1}{N}\right)\mathrm{MSD}$$

$$\Rightarrow \quad \text{Least Count} = \left(\frac{N - N + 1}{N}\right)\mathrm{MSD}$$

$$\Rightarrow \quad \text{Least Count} = \frac{1}{N}\mathrm{MSD}$$

Since 1 MSD = 1 mm

So, least count $= \dfrac{1}{N}$ mm

$$\Rightarrow \quad \text{Least count} = \frac{1}{10\mathrm{N}}\ \mathrm{cm}$$

4. Length of cylinder,

$h = 4.54$ cm {3 significant figures}

Radius of cylinder,

$r = 1.75$ cm {3 significant figures}

Volume of cylinder V,

$$\Rightarrow \quad V = \pi r^2 h = 3.14 \times (1.75)^2 \times 4.54\ \mathrm{cm}^2$$

$$\Rightarrow \quad V = 43.657775\ \mathrm{cm}^3 = 43.7\ \mathrm{cm}^3$$

{Rounded off upto 3 significant figures}

5.

Column A	Column B
17236	5
510 m	3
270	2
4.20	3
7042.6	5
0.017	2
6.1×10^{14}	2

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6.

Measured Value	Rounded off value to Three Significant Figures
7.364	7.36
8.3251	8.33
9.445	9.44
15.75	15.8
7.367	7.37
9.4450	9.44
15.7500	15.8

7. (a) $11.439 \cong 11.4$, because the least number of significant figures after the decimal is 1.

(b) $2.43 \cong 2.4$, because the least number of significant figures after the decimal is 1.

(c) $44.064 \cong 44$, because the least number of significant figures is 2.

(d) $107.843 \cong 108$, because the least number of significant figures is 3 (not 2), because $1100\ \text{ms}^{-1}$ has four significant figures.

8. (a) Least count

$$\text{L.C.} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

$$\Rightarrow \quad \text{L.C.} = \frac{1}{10}\ \text{mm} = 0.1\ \text{mm}$$

$$\Rightarrow \quad \text{L.C.} = 0.01\ \text{cm}$$

(b) So, the length,

$$L = N + n(\text{L.C.}) = (10.2 + 3 \times 0.01)\ \text{cm}$$

$$\Rightarrow \quad L = 10.23\ \text{cm}$$

9. $\text{L.C.} = \frac{1}{100} = 0.01\ \text{mm}$

Linear scale reading $= 6$ (pitch) $= 6\ \text{mm}$

Circular scale reading $= n(\text{L.C.}) = 40 \times 0.01 = 0.4\ \text{mm}$

So, total reading $= (6 + 0.4) = 6.4\ \text{mm}$

10. Here zero of vernier scale lies to the right of zero of main scale, hence, it has positive zero error.

Further, $N = 0$, $x = 5$

So, least count or Vernier constant is 0.01 cm

Hence, Zero error $= N + x \times \text{V.C.}$

$\Rightarrow$ Zero error $= 0 + 5 \times 0.01$

$\Rightarrow$ Zero error $= 0.05\ \text{cm}$

So, zero correction $= -0.05\ \text{cm}$

Hence the actual length will be 0.05 cm less than the measured length.

Single Correct Choice Type Questions

1. Maximum possible error

$$\left|\frac{\Delta H}{H} \times 100\right|_{\max} = 2\frac{\Delta \text{I}}{\text{I}} \times 100 + \frac{\Delta R}{R} \times 100 + \frac{\Delta t}{t} \times 100$$

$\{\because\ H = \text{I}^2 Rt\}$

Hence, the correct answer is (D).

2. The MKS unit of $\eta = \frac{\text{kg}}{\text{ms}}$ is $kgm^{-1}s^{-1}$

The CGS of $\eta = \frac{\text{g}}{\text{cms}}$ is $gcm^{-1}s^{-1}$

$$\frac{\eta_{\text{MKS}}}{\eta_{\text{CGS}}} = \frac{\text{kg}}{\text{ms}} \times \frac{\text{cms}}{\text{g}}$$

$$\frac{\eta_{\text{MKS}}}{\eta_{\text{CGS}}} = \frac{10^3\ \text{g} \times \text{cm} \times \text{s}}{100\ \text{cm} \times \text{s} \times \text{g}} = 10$$

Hence, the correct answer is (A).

3. $i = K \tan\theta$

Since $\tan\theta$ is a dimensionless physical quantity and hence the unit of k is equivalent to that of current i.e. ampere

Hence, the correct answer is (B).

4. Impulse = change in momentum

So, the dimensions of impulse and momentum are the same.

Hence, the correct answer is (C).

5. Coefficient of viscosity

$$\eta = \frac{\text{tangential force}}{\text{contact area} \times \text{velocity gradient}}$$

$$\Rightarrow \quad \eta = \frac{\text{newton}}{\text{m}^2 \times \text{m}/\text{s}} = \frac{\text{newton s}}{\text{m}^2}$$

$$\eta = \frac{\text{kg ms}}{\text{s}^2\text{m}^2} = \frac{\text{kg}}{\text{ms}}$$

Hence, the correct answer is (B).

6. Pressure correction $P' = \frac{an^2}{V^2}$

$$\Rightarrow \quad a = \frac{P'V^2}{n^2} = \frac{(\text{Pressure correction})(\text{Volume})^2}{(\text{amount of gas})^2}$$

$$\Rightarrow \quad a = \frac{(\text{Nm}^{-2})(\text{m}^3)^2}{\text{mol}^2}$$

$$\Rightarrow \quad a = \frac{\text{Nm}^4}{\text{mol}^2} = \frac{\text{kgmm}^4}{\text{s}^2\text{mol}^2} \qquad \{\because\ 1\ \text{N} = 1\ \text{kgms}^{-2}\}$$

$\Rightarrow \quad a = \dfrac{\text{kgm}^5}{\text{s}^2\text{mol}^2} = \dfrac{\text{kgm}^5}{\text{s}^2}$ (as mol is a number)

Hence, the correct answer is (C).

7. $\dfrac{1}{\sqrt{\mu\varepsilon}}$ is the velocity of electromagnetic waves in a medium. Hence its dimensions are equivalent to those of velocity.

Hence, the correct answer is (A).

8. There are 6 significant figures in the number 108.023 as all the figures define a particular measurement. In the number 00.19 there are only two significant figures as the zeroes on the left of decimal point and the zeroes on the right of non-zero numbers after the decimal point are not significant.

Hence, the correct answer is (A).

9. $n = \dfrac{1}{\text{area} \times \text{time}} = \dfrac{1}{L^2T}$

$[n_1 - n_2] = \dfrac{1}{\text{Volume}} = \dfrac{1}{L^3}$

$[x_2 - x_1] = L$

$\Rightarrow \quad D = -\dfrac{n(x_2 - x_1)}{n_2 - n_1} = \dfrac{L^{-2}T^{-1}L}{L^{-3}} = L^2T^{-1}$

Hence, the correct answer is (A).

10. $\dfrac{L}{R}$ and RC are the time constants of LR and RC circuits respectively. Hence their dimensions are equal to those of time.

Hence, the correct answer is (A).

11. $V = d^a E^b$

Writing dimensions on two sides of the equation

$[LT^{-1}] = [ML^{-1}T^{-2}]^b [ML^{-3}]^a$

$\Rightarrow \quad LT^{-1} = M^{a+b}L^{-3a-b}T^{-2b}$

Comparing the dimensions on two sides, we get

$a + b = 0$

$-3a - b = 1$

$-2b = -1$

Solving for a and b, we get

$b = \dfrac{1}{2}$ and $a = -\dfrac{1}{2}$

Hence, the correct answer is (A).

12. All the three represent the electrostatic potential energy stored in a condenser

$U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2}qV$

The dimensions of energy and work are the same

So, $[U] = \left[\dfrac{1}{2}CV^2\right] = \left[\dfrac{1}{2}\dfrac{q^2}{C}\right] = \left[\dfrac{1}{2}qV\right] = ML^2T^{-2}$

Hence, the correct answer is (B).

13. $\text{Mass} = \dfrac{\text{Force}}{\text{Acceleration}} = \dfrac{F}{LT^{-2}} = FL^{-1}T^2$

Hence, the correct answer is (A).

14. $[C^2LR] = \left[C^2L^2\dfrac{R}{L}\right] = \left[(LC)^2\left(\dfrac{R}{L}\right)\right]$

and we know that frequency of LC circuits is given by $f = \dfrac{1}{2\pi}\dfrac{1}{\sqrt{LC}}$, i.e., the dimension of LC is equal to $[T^2]$ and $\left[\dfrac{L}{R}\right]$ gives the time constant of L-R circuit so the dimension of $\dfrac{L}{R}$ is equal to $[T]$. By substituting the above dimension in the give formula $\left[(LC)^2\left(\dfrac{R}{L}\right)\right] = [T^2]^2[T^{-1}] = [T^3]$.

Hence, the correct answer is (B).

15. From the principle of dimensional homogeneity $[\alpha t] =$ dimensionless

$\Rightarrow \quad [\alpha] = \left[\dfrac{1}{t}\right] = T^{-1}$

Similarly $[x] = \dfrac{[v_0]}{[\alpha]}$

$\Rightarrow \quad [v_0] = [x][\alpha] = LT^{-1}$

Hence, the correct answer is (A).

16. From the dimensional homogeneity $[x^2] = [B]$

$\Rightarrow \quad [B] = L^2$

Also $[U] = \dfrac{[A]\left[x^{\frac{1}{2}}\right]}{[x^2] + [B]}$

$\Rightarrow \quad ML^2T^{-2} = \dfrac{[A]L^{\frac{1}{2}}}{L^2}$

$\Rightarrow \quad [A] = ML^{\frac{7}{2}}T^{-2}$

Now $[AB] = \left(ML^{\frac{7}{2}}T^{-2}\right) \times L^2 = ML^{\frac{11}{2}}T^{-2}$

Hence, the correct answer is (B).

17. Let $x =$ length, then

$[X] = [L]$ and $[dx] = [L]$

By principle of dimensional homogeneity

$\left[\dfrac{x}{a}\right] =$ dimensionless

$\Rightarrow \quad [a] = [x] = [L]$

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By substituting dimensions of each quantity on both sides we get

$$\frac{L}{\sqrt{L^2}} = L^n$$

$\Rightarrow \quad n = 0$

Hence, the correct answer is (C).

18. Here, $\frac{2\pi ct}{\lambda}$ as well as $\frac{2\pi x}{\lambda}$ are dimensionless (angle)

i.e. $\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right] = M^0L^0T^0$

So

(i) unit of ct is same as that of λ

(ii) unit of x is same as that of λ

(iii) $\left[\frac{2\pi c}{\lambda}\right] = \left[\frac{2\pi x}{\lambda t}\right]$ and

(iv) $\frac{x}{\lambda}$ is unit less but this is not the case with $\frac{c}{\lambda}$

Hence, the correct answer is (D).

19. Since, measurement of a physical $P = nu$ = constant

$\Rightarrow \quad n_1u_1 = n_2u_2$ or $n \propto \frac{1}{u}$

Hence, the correct answer is (D).

20. $n_1 = 100$, $M_1 = 1$ g, $L_1 = 1$ cm, $T_1 = 1$ sec, and $M_2 = 1$ kg, $L_2 = 1$ metre, $T_2 = 1$ minute

By substituting these values in the following conversion formula

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

where $a = 1,\ b = 1,\ c = -2$

$$n_2 = 100\left(\frac{1\text{ g}}{1\text{ kg}}\right)^1\left(\frac{1\text{ cm}}{1\text{ meter}}\right)^1\left(\frac{1\text{ s}}{1\text{ minute}}\right)^{-2}$$

$$n_2 = 100\left(\frac{1\text{ g}}{10^3\text{ g}}\right)^1\left(\frac{1\text{ cm}}{10^2\text{ cm}}\right)^1\left(\frac{1\text{ s}}{60\text{ s}}\right)^{-2} = 3.6$$

Hence, the correct answer is (C).

21. Relation between centigrade and Fahrenheit

$$\frac{K - 273}{5} = \frac{F - 32}{9}$$

According to problem $\frac{X - 273}{5} = \frac{X - 32}{9}$

$\Rightarrow \quad X = 574.25$

Hence, the correct answer is (B).

22. Because 1 newton = 10^5 dyne

Hence, the correct answer is (D).

23. We know that the dimensional formula of Young's modulus is $ML^{-1}T^{-2}$ and its C.G.S. unit is $\text{gcm}^{-1}\text{sec}^{-2}$ and M.K.S. unit is $\text{kgm}^{-1}\text{sec}^{-2}$.

Since, $n_2 = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^{-1}\left(\frac{T_1}{T_2}\right)^{-2}$

$\Rightarrow \quad n^2 = \left(\frac{1\text{ g}}{1\text{ kg}}\right)^1\left(\frac{1\text{ cm}}{1\text{ m}}\right)^{-1}\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2}$

So, conversion factor

$$\frac{n_2}{n_1} = \left[\frac{1\text{ g}}{10^{-3}\text{ g}}\right]^1\left[\frac{1\text{ cm}}{10^2\text{ cm}}\right]^{-1}\left[\frac{1\text{ s}}{1\text{ s}}\right]^{-2} = \frac{1}{10} = 0.1$$

Hence, the correct answer is (C).

24. $[P] = [ML^2T^{-3}]$

Using the relation

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

$$n_2 = (1\times10^6)\left(\frac{1\text{ kg}}{10\text{ kg}}\right)^1\left(\frac{1\text{ m}}{1\text{ dm}}\right)^2\left(\frac{1\text{ s}}{1\text{ min}}\right)^{-3}$$

{As 1 MW = 10^6 W }

$\Rightarrow \quad n_2 = 10^6\left(\frac{1\text{ kg}}{10\text{ kg}}\right)\left(\frac{10\text{ dm}}{1\text{ dm}}\right)^2\left(\frac{1\text{ s}}{60\text{ s}}\right)^{-3} = 2.16\times10^{12}$

Hence, the correct answer is (A).

25. $v_2 = v_1\frac{\alpha^2}{\beta}$

$\Rightarrow \quad [L_2T_2^{-1}] = [L_1T_1^{-1}]\frac{\alpha^2}{\beta}$ …(1)

$a_2 = a_1\alpha\beta$

$\Rightarrow \quad [L_2T_2^{-2}] = [L_1T_1^{-2}]\alpha\beta$ …(2)

and $F_2 = \frac{F_1}{\alpha\beta}$

$\Rightarrow \quad [M_2L_2T_2^{-2}] = [M_1L_1T_1^{-2}]\times\frac{1}{\alpha\beta}$ …(3)

Dividing equation (3) by equation (2) we get

$$M_2 = \frac{M_1}{(\alpha\beta)\alpha\beta} = \frac{M_1}{\alpha^2\beta^2}$$

Squaring equation (1) and dividing by equation (2) we get

$$L_2 = L_1\frac{\alpha^3}{\beta^3}$$

Dividing equation (1) by equation (2) we get

$$T_2 = T_1 \frac{\alpha}{\beta^2}$$

Hence, the correct answer is (B).

26. Let the new system be designated as 2 and the old system be designated as 1. Then

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^a \left(\frac{L_2}{L_1}\right)^b \left(\frac{T_2}{T_1}\right)^c$$

For velocity

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^0 \left(\frac{L_2}{L_1}\right)^1 \left(\frac{T_2}{T_1}\right)^{-1}$$

$$\frac{n_2}{n_1} = (100)(100)^{-1} = 1$$

So, no change for velocity and hence, OPTION (A) is incorrect

For Force

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^1 \left(\frac{L_2}{L_1}\right)^1 \left(\frac{T_2}{T_1}\right)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{10}\right)(100)(100)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{1}{1000}$$

So, OPTION (B) is correct

For Energy

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^1 \left(\frac{L_2}{L_1}\right)^2 \left(\frac{T_2}{T_1}\right)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{10}\right)(100)^2(100)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{1}{10}$$

So, OPTION (C) is incorrect

For Pressure

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^1 \left(\frac{L_2}{L_1}\right)^{-1} \left(\frac{T_2}{T_1}\right)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{10}\right)(100)^{-1}(100)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{1}{10^7}$$

So, OPTION (D) is also incorrect.

Hence, the correct answer is (B).

27. $[E] = [ML^2T^{-2}]$

$1 \text{ eluoj} = [100 \text{ kg}] \times [1 \text{ km}]^2 \times [100 \text{ sec}]^{-2}$

$\Rightarrow 1 \text{ eluoj} = 100 \text{ kg} \times 10^6 \text{ m}^2 \times 10^{-4} \text{ sec}^{-2}$

$\Rightarrow 1 \text{ eluoj} = 10^4 \text{ kg m}^2 \times \text{sec}^{-2} = 10^4 \text{ Joule}$

Hence, the correct answer is (A).

28. $1 \text{ gcms}^{-1} = 10^{-3} \text{ kg} \times 10^{-2} \text{ m} \times \text{s}^{-1}$

$\Rightarrow 1 \text{ gmcs}^{-1} = 10^{-5} \text{ kg} \times \text{m} \times \text{s}^{-1} = 10^{-5} \text{ Ns}$

Hence, the correct answer is (D).

29. Given $m = \text{mass} = [M]$, $\eta = \text{coefficient of rigidity} = [ML^{-1}T^{-2}]$, $L = \text{length} = [L]$.

By substituting the dimensions of these quantities we can check the accuracy of the given formulae

$$[T] = 2\pi\left(\frac{[M]}{[\eta][L]}\right)^{1/2} = \left[\frac{M}{ML^{-1}T^{-2}L}\right]^{1/2} = T$$

Hence, the correct answer is (D).

30. Let $m \propto c^x G^y h^z$

$\Rightarrow m = Kc^x G^y h^z$

By substituting the dimension of each quantity in both sides and using the Principle of Homogeneity

$$[M^1 L^0 T^0] = K[LT^{-1}]^x [M^{-1} L^3 T^{-2}]^y [ML^2 T^{-1}]^z$$

$$[M^1 L^0 T^0] = [M^{-y+z} L^{x+3y+2x} T^{-x-2y-z}]$$

By equating the power of M, L and T on both sides, we get

$$-y + z = 1, \; x + 3y + 2z = 0 \text{ and } -x - 2y - z = 0$$

Solving above three equations we get,

$$x = \frac{1}{2}, \; y = -\frac{1}{2} \text{ and } z = \frac{1}{2}$$

$\Rightarrow m \propto c^{1/2} G^{-1/2} h^{1/2}$

Hence, the correct answer is (B).

31. $[P^x Q^y C^z] = M^0 L^0 T^0$

By substituting the dimension of each quantity in the given expression

$$[ML^{-1}T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z = M^0 L^0 T^0$$

$$\Rightarrow [M^{x+y} L^{-x+z} T^{-2x-3y-z}] = M^0 L^0 T^0$$

Equating the power of M, L and T in both sides, we get

$$x + y = 0, \; -x + z = 0 \text{ and } -2x - 3y - z = 0$$

Solving the above equations, we get

$$x = 1, \; y = -1, \; z = 1$$

Hence, the correct answer is (B).

32. Writing dimensions of both sides

$$[L^3] = [L^2]^\alpha [LT^{-1}]^\beta [T]^\gamma$$

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$\Rightarrow \quad [L^3T^0] = [L^{2\alpha+\beta}T^{\gamma-\beta}]$

By comparing powers of both sides $2\alpha+\beta=3$ and $\gamma-\beta=0$

which gives $\beta=\gamma$ and $\alpha=\frac{1}{2}(3-\beta)$,

i.e. $\alpha \neq \beta = \gamma$

Hence, the correct answer is (B).

33. Let $A=\frac{KA_0V\lambda^x}{r}$

By substituting the dimension of each quantity in both sides

$\Rightarrow \quad [L]=\frac{[L]\cdot[L^3][L^x]}{[L]}$

$\Rightarrow \quad L^{-2}=L^x$

$\Rightarrow \quad x=-2$

$\Rightarrow \quad A\propto\lambda^{-2}$

$\Rightarrow \quad A\propto\frac{1}{\lambda^2}$

Hence, the correct answer is (B).

34. Volume $=a^3=(7.023)^3=373.715\ \text{m}^3$

In significant figures, the volume of cube will be $373.7\ \text{m}^3$ because the side of the cube has four significant figures.

Hence, the correct answer is (C).

35. Unit of luminous intensity is candela.

Hence, the correct answer is (C).

36. Assume the thickness of window pane to be 3 mm. If t is the time taken by light to penetrate it, then

$$t=\frac{\text{Thickness of Window Pane}}{\text{Speed of Light}}$$

$\Rightarrow \quad t=\frac{3\times10^{-3}}{3\times10^8}=10^{-11}\ \text{s}$

Hence, the correct answer is (C).

37. Latent Heat $=\frac{\text{Heat energy}}{\text{Mass}}$

So, unit of latent heat is Jkg^{-1}.

Hence, the correct answer is (C).

38. $E=\sigma T^4$

$\Rightarrow \quad [E]=[\sigma][T^4]$

$\Rightarrow \quad \frac{ML^2T^{-2}}{L^2T}=[\sigma]K^4$

$\Rightarrow \quad [\sigma]=MT^{-3}K^{-4}$

Hence, the correct answer is (B).

39. $F=(\text{mass})(\text{acceleration})$

$\Rightarrow \quad F=MLT^{-2}$

$\Rightarrow \quad T^2=\frac{ML}{F}$

$\Rightarrow \quad T=M^{\frac{1}{2}}L^{\frac{1}{2}}F^{-\frac{1}{2}}$

Hence, the correct answer is (A).

40. $70\ \text{dyne cm}^{-1}=70\frac{\text{dyne}}{\text{cm}}=\frac{70\times10^{-5}\ \text{N}}{10^{-2}\ \text{m}}$

$\Rightarrow \quad 70\ \text{dynecm}^{-1}=7\times10^{-2}\ \text{Nm}^{-1}$

Hence, the correct answer is (B).

41. Temperature is a fundamental quantity independent of all expressed.

Hence, the correct answer is (D).

42. $[\text{Density}]=ML^{-3}$

Since $n_2=n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$

where $a=1$, $b=-3$, $c=0$ and $n_1=8$, $n_2=?$

$\Rightarrow \quad n_2=8\left(\frac{1\ \text{g}}{20\ \text{g}}\right)\left(\frac{1\ \text{cm}}{5\ \text{cm}}\right)^{-3}$

$\Rightarrow \quad n_2=8\times\frac{1}{20}\times5\times5\times5$

$\Rightarrow \quad n_2=50$

So, New Density is 50 new units

Hence, the correct answer is (B).

43. $[\text{ergm}^{-1}]=[\text{force}]$

Hence, the correct answer is (A).

44. $[\text{Pressure}]=\left[\frac{\text{Force}}{\text{Area}}\right]=\text{ML}^{-1}T^{-2}$

Hence, the correct answer is (D).

45. $[F]=MLT^{-2}$

$\Rightarrow \quad [F]=(10\ \text{g})(10\ \text{cm})(0.1\ \text{s})^{-2}$

$\Rightarrow \quad [F]=10^4\ \text{gcms}^{-2}=0.1\ \text{N} \qquad \{\because 1\ \text{N}=10^5\ \text{dyne}\}$

Hence, the correct answer is (A).

46. $R=\frac{\rho\ell}{A}$

$\Rightarrow \quad \rho=\frac{RA}{l}=\frac{VA}{Il}=\frac{WA}{q(Il)}=\frac{WAt}{q^2l}$

$\Rightarrow \quad [\rho]=\frac{(ML^2T^{-2})(L^2)(T)}{(Q^2)(L)}=ML^3T^{-1}Q^{-2}$

Hence, the correct answer is (A).

47. SI unit of pressure is pascal.
Hence, the correct answer is (C).

48. $\left[\frac{\pi}{3}(a^2-b^2)h\right]=L^3=\text{Volume}$
Hence, the correct answer is (D).

49. $[\text{Angular momentum}]=[mvr]=ML^2T^{-1}$
$[\text{joulesecond}]=ML^2T^{-1}$
Hence, the correct answer is (D).

50. $x=at+bt^2+c$
$\Rightarrow \quad [x]=[at]=[bt^2]=[c]$
$\Rightarrow \quad [a]=mhr^{-1},\ [b]=mhr^{-2},\ [c]=m$
Hence, the correct answer is (B).

51. $\frac{1}{\sqrt{\mu_0\varepsilon_0}}=c=\text{Velocity of Light in vacuum.}$
Hence, the correct answer is (A).

52. 109.832×0.6107 must have 4 significant figures.
Hence, the correct answer is (C).

53. $[\tau]=[\text{Energy}]=ML^2T^{-2}$
Hence, the correct answer is (A).

54. $ML^2T^{-3}=\frac{ML^2T^{-2}}{T}=\frac{[\text{Work}]}{[\text{Time}]}=[\text{Power}]$
Hence, the correct answer is (D).

55. $\left[\frac{L}{p}\right]=\frac{ML^2T^{-1}}{MLT^{-1}}=M^0L^1T^0$
Hence, the correct answer is (A).

56. $e\propto\frac{1}{\sqrt{n}}$
$\Rightarrow \quad \frac{e_1}{e_2}=\sqrt{\frac{n_2}{n_1}}=\sqrt{4}=2$
$\Rightarrow \quad e_2=\frac{e_1}{2}=\frac{x}{2}$
Hence, the correct answer is (D).

57. $[v]=[at]=\left[\frac{b}{t+c}\right]$ and $[t]=[c]$
$\Rightarrow \quad [c]=T$
$\Rightarrow \quad [t+c]=T$
$\Rightarrow \quad \frac{[b]}{T}=LT^{-1}$
$\Rightarrow \quad [b]=L$ and $[a]=LT^{-2}$
So, $[a]=LT^{-2},\ [b]=L,\ [c]=T$
Hence, the correct answer is (C).

58. $[mc^2]=[\text{Energy}]=ML^2T^{-2}$
Hence, the correct answer is (C).

59. kgms^{-1} is the unit of momentum.
Hence, the correct answer is (B).

60. $1\text{ micron}=10^{-6}\text{ m}$
Hence, the correct answer is (C).

61. $\sigma=\frac{E}{T^4}$
where E is energy per second per unit area. So,
$\sigma=\text{watt}\ m^{-2}K^{-4}$
Hence, the correct answer is (C).

62. Ns is unit of impulse. Though $1\text{ Ns}=1\text{ kgms}^{-1}$ still it is not a unit of momentum.
Hence, the correct answer is (B).

64. $F=\frac{Gm_1m_2}{r^2}$
$\Rightarrow \quad [G]=\left[\frac{Fr^2}{m_1m_2}\right]=M^{-1}L^3T^{-2}$
Hence, the correct answer is (C).

65. $[KE]=ML^2T^{-2}$
$\Rightarrow \quad [\text{KE}]_{\text{new}}=(2M)(2L)^2T^{-2}$
$\Rightarrow \quad [KE]_{\text{new}}=8(ML^2T^{-2})=8(KE)_{\text{old}}$
Hence, the correct answer is (A).

66. See drawbacks of dimensional analysis.
Hence, the correct answer is (A).

67. $Y=\frac{\text{Stress}}{\text{Strain}}$
$\Rightarrow \quad [Y]=\frac{[\text{Stress}]}{[\text{Strain}]}=\frac{\left[\frac{\text{Force}}{\text{Area}}\right]}{M^0L^0T^0}$
$\Rightarrow \quad [Y]=FA^{-1}D^0$
Hence, the correct answer is (B).

68. 1 decapoise = 10 poise
$MKS=10(CGS)$
Hence, the correct answer is (B).

69. $[\text{Light year}]=L$
Hence, the correct answer is (D).

71. Torque and work have same dimensions.
Hence, the correct answer is (B).

72. $\left[\frac{t}{p}\right]=M^0L^0T^0$

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$\Rightarrow \quad [t]=[p]$

So, unit of t and p are the same.
Hence, the correct answer is (D).

73. $1\text{ eV}=1.6\times10^{-19}\text{ J}$
Hence, the correct answer is (C).

74. $F\propto D^aV^bA^c$

$MLT^{-2}=(ML^{-3})^a(LT^{-1})^b(L^2)^c$

$\Rightarrow \quad MLT^{-2}=M^aL^{-3a+b+2c}T^{-b}$

$\Rightarrow \quad a=1,\ -3a+b+2c=1,\ -b=-2$

$\Rightarrow \quad -3+2+2c=1$

$\Rightarrow \quad c=1$

So, $F=DV^2A^1$

$\Rightarrow \quad F=AV^2D$

Hence, the correct answer is (A).

75. $X=\varepsilon_0L\left(\dfrac{\Delta V}{\Delta t}\right)$

$X=\left(\dfrac{C}{L}\right)(L)\left(\dfrac{\Delta V}{\Delta t}\right) \qquad \left\{\because\ \varepsilon_0=\dfrac{\text{Capacitance}}{\text{Length}}\right\}$

$\Rightarrow \quad X=C\left(\dfrac{\Delta V}{\Delta t}\right)$

$\Rightarrow \quad X=\dfrac{C\Delta V}{\Delta t}=\dfrac{\Delta Q}{\Delta t}=\text{Current}$

So, X happens to be the current.
Hence, the correct answer is (D).

76. $\left[\dfrac{L}{R}\right]=[\text{Time}]$
Hence, the correct answer is (D).

77. $l\propto c^\alpha g^\beta p^\gamma$

$\Rightarrow \quad L=(LT^{-1})^\alpha(LT^{-2})^\beta(ML^{-1}T^{-2})^\gamma$

$\Rightarrow \quad L=M^\gamma L^{\alpha+\beta-\gamma}T^{-\alpha-2\beta-2\gamma}$

$\Rightarrow \quad \gamma=0,\ \alpha+\beta=1 \text{ and } -\alpha-2\beta=0$

$\Rightarrow \quad -\beta=1$

$\Rightarrow \quad \beta=-1 \text{ and } \alpha=2$

So, $l=c^2g^{-1}p^0$

Also, we could have done this by taking dimensional formulae for velocity and acceleration and eliminating t.
Hence, the correct answer is (A).

79. $v=G^aM^bR^c$

$\Rightarrow \quad LT^{-1}=(M^{-1}L^3T^{-2})^a(M)^b(L)^c$

$\Rightarrow \quad LT^{-1}=M^{-a+b}L^{3a+c}T^{-2a}$

$\Rightarrow \quad -a+b=0,\ 3a+c=1,\ -2a=-1$

$\Rightarrow \quad a=b,\ 3a+c=1,\ a=\dfrac{1}{2}$

$\Rightarrow \quad b=\dfrac{1}{2},\ c=-\dfrac{1}{2},\ a=\dfrac{1}{2}$

$\Rightarrow \quad v=\dfrac{G^{\frac{1}{2}}M^{\frac{1}{2}}}{R^{\frac{1}{2}}}=\sqrt{\dfrac{GM}{R}}$

Hence, the correct answer is (B).

80. $T=2\pi\sqrt{\dfrac{l}{g}}$

$\Rightarrow \quad T^2=4\pi^2\left(\dfrac{l}{g}\right)$

$\Rightarrow \quad g=4\pi^2\left(\dfrac{l}{T^2}\right)$

$\Rightarrow \quad \dfrac{\Delta g}{g}=\dfrac{\Delta l}{l}+2\left(\dfrac{\Delta T}{T}\right)$

$\Rightarrow \quad \dfrac{\Delta g}{g}\times100=1\%+2(3\%)=7\%$

Hence, the correct answer is (A).

83. $\nu=\dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

$\Rightarrow \quad T^{-1}=\dfrac{1}{L}\sqrt{\dfrac{MLT^{-2}}{[m]}}$

$\Rightarrow \quad T^{-2}=\dfrac{1}{L^2}\dfrac{MLT^{-2}}{[m]}$

$\Rightarrow \quad [m]=ML^{-1}T^0$

Hence, the correct answer is (A).

86. $g=\dfrac{GM}{r^2}$

$M=\dfrac{4}{3}\pi r^3\rho$

$\Rightarrow \quad g=\dfrac{G\left(\frac{4}{3}\pi r^3\rho\right)}{r^2}$

$\Rightarrow \quad \rho=\dfrac{3g}{4\pi Gr}$

Hence, the correct answer is (C).

87. $\left[\dfrac{1}{2}\varepsilon_0E^2\right]=\left[\dfrac{B^2}{2\mu_0}\right]=[\text{Pressure}]=ML^{-1}T^{-2}$

Hence, the correct answer is (B).

88. (A), (B) and (C) all have units same as energy per unit volume.
Hence, the correct answer is (D).

89. $[\alpha t^2] = M^0L^0T^0$

$\Rightarrow \quad [\alpha] = M^0L^0T^{-2}$

Hence, the correct answer is (B).

90. Magnetic Moment = (Current) (Area)

$\Rightarrow \quad [\text{Magnetic moment}] = AL^2$

Hence, the correct answer is (C).

91. Surface Tension $\propto E^aV^bT^c$

$\Rightarrow \quad MT^{-2} = (ML^2T^{-2})^a (LT^{-1})^b (T)^c$

$\Rightarrow \quad MT^{-2} = M^aL^{2a+b}T^{-2a-b+c}$

$\Rightarrow \quad a = 1,\ 2a + b = 0,\ -2a - b + c = -1$

$\Rightarrow \quad b = -2$ and $-2 + 2 + c = -2$

$\Rightarrow \quad a = 1,\ b = -2,\ c = -2$

So, the desired relation is $EV^{-2}T^{-2}$ OR

$$\text{Surface tension} = \frac{\text{Work Done}}{\text{Change in Surface Area}}$$

$$\Rightarrow \quad \sigma = \frac{E}{l^2} = \frac{E}{(VT)^2} = EV^{-2}T^{-2}$$

Hence, the correct answer is (A).

92. $$[Y] = \left[\frac{X}{Z^2}\right] = \left[\frac{\text{Capacitance}}{(\text{Magnetic Induction})^2}\right]$$

$$\Rightarrow \quad [Y] = \frac{M^{-1}L^{-2}Q^2T^2}{M^2Q^{-2}T^{-2}} = M^{-3}L^{-2}Q^4T^4$$

Hence, the correct answer is (B).

95. $[\lambda t] = M^0L^0T^0$

$\Rightarrow \quad [\lambda] = M^0L^0T^{-1}$

Hence, the correct answer is (B).

97. $$[\text{Mobility}] = \left[\frac{\text{Velocity}}{\text{Electric Field}}\right]$$

$$\Rightarrow \quad [\mu] = \frac{ms^{-1}}{NC^{-1}}$$

But $1\ \text{N} = 1\ \text{kgms}^{-2}$

$$\Rightarrow \quad [\mu] = \frac{\text{ms}^{-1}}{\text{kgms}^{-2}\text{C}^{-1}} = \text{Cskg}^{-1}$$

Hence, the correct answer is (A).

100. $\tan\theta = \dfrac{v^2}{rg}$ is both numerically and dimensionally correct.
Hence, the correct answer is (A).

102. Amplification factor is the ratio of identical physical quantities, so it has no units.
Hence, the correct answer is (D).

103. $$\frac{\Delta Y}{Y} = a\frac{\Delta M}{M} + b\frac{\Delta L}{L} + c\frac{\Delta T}{T}$$

$$\Rightarrow \quad \frac{\Delta Y}{Y}(\%) = (\alpha a + \beta b + \gamma c)\%$$

Hence, the correct answer is (C).

104. Since $T = 2\pi\sqrt{\dfrac{l}{g}}$

$$\Rightarrow \quad T^2 = 4\pi^2\left(\frac{l}{g}\right)$$

So, the relation $T^2 = \dfrac{l}{g}$ is just dimensionally correct, but numerically incorrect.
Hence, the correct answer is (C).

106. $1\ \text{MSD} = \dfrac{1}{10}(1\ \text{cm}) = 1\ \text{mm}$

$10\ \text{VSD} = 8\ \text{MSD}$

$1\ \text{VSD} = \dfrac{8}{10}\ \text{MSD}$

Least count $= 1\ \text{MSD} - 1\ \text{VSD}$

$$\Rightarrow \quad \text{LC} = 1\ \text{mm} - \frac{8}{10}\ \text{mm}$$

$$\Rightarrow \quad \text{LC} = \frac{2}{10}\ \text{mm}$$

$$\Rightarrow \quad \text{LC} = \frac{2}{100}\ \text{cm} = 0.02\ \text{cm}$$

Hence, the correct answer is (B).

107. $50\ \text{VSD} = 49\ \text{MSD}$

$$1\ \text{VSD} = \frac{49}{50}\ \text{MSD}$$

$$\Rightarrow \quad \text{VC} = 1\ \text{MSD} - 1\ \text{VSD}$$

$$\Rightarrow \quad \text{VC} = 1\ \text{MSD} - \frac{49}{50}\text{MSD}$$

$$\Rightarrow \quad \text{VC} = \frac{1}{50}\ \text{MSD}$$

$\Rightarrow \quad 1\ \text{MSD} = 50\ (\text{VC}) = 50\ (0.001\ \text{cm})$

$\Rightarrow \quad 1\ \text{MSD} = 0.05\ \text{cm} = 0.5\ \text{mm}$

$\Rightarrow \quad 1\ \text{MSD} = 0.5\ \text{mm}$

Hence, the correct answer is (B).

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108.

Device	Least Count
1. Metre Scale	1 mm
2. Vernier Calliper	0.1 mm
3. Screw Gauge	0.01 mm
4. Spherometer	0.01 mm

Hence, the correct answer is (D).

109. $\text{LC} = \frac{1}{N}\ \text{MSD}$

$$0.005\ \text{cm} = \frac{1}{N}(0.1\ \text{cm})$$

$$\Rightarrow \quad N = \frac{0.1}{0.005} = \frac{1000}{50}$$

$$\Rightarrow \quad N = 20$$

Hence, the correct answer is (B).

110. $\text{LC} = 1\ \text{MSD} - 1\ \text{VSD}$

$$\frac{1}{10\,N} = \frac{1}{10} - 1\ \text{VSD} \qquad \left\{\because 1\ \text{MSD} = \frac{1}{10}\ \text{cm}\right\}$$

$$\Rightarrow \quad 1\ \text{VSD} = \frac{1}{10} - \frac{1}{10\,N}$$

$$\Rightarrow \quad 1\ \text{VSD} = \frac{1}{10}\left(1 - \frac{1}{N}\right)$$

$$\Rightarrow \quad 1\ \text{VSD} = 1\ \text{MSD}\left(1 - \frac{1}{N}\right)$$

$$\Rightarrow \quad N\ \text{VSD} = (N-1)\text{MSD}$$

Hence, the correct answer is (C).

111. $100\ \text{VSD} = 99\ \text{MSD}$

$$\Rightarrow \quad 1\ \text{VSD} = \frac{99}{100}\ \text{MSD}$$

$$\text{LC} = 1\ \text{MSD} - 1\ \text{VSD}$$

$$\Rightarrow \quad \text{LC} = 1\ \text{MSD} - \frac{99}{100}\ \text{MSD}$$

$$\Rightarrow \quad \text{LC} = 0.01\ \text{MSD}$$

$$\Rightarrow \quad \text{LC} = 0.01\ (1\ \text{mm}) = 0.01\ \text{mm}$$

Hence, the correct answer is (C).

112. Error can be, at the maximum equal to the least count of the device used.

Hence, the correct answer is (D).

113. $$\text{Error} \propto \frac{1}{\sqrt{\text{Number of readings}}}$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \sqrt{\frac{100}{400}} = \frac{1}{2}$$

Hence, the correct answer is (B).

114. Vernier constant is actually the least count of the device.

Hence, the correct answer is (D).

115. The fraction of a degree of an angle is measured with the help of angular vernier, which is provided in spherometers and sextants used to measure angular displacements.

Hence, the correct answer is (C).

131. As $x = at^2 + b$ and therefore $[a] = [LT^{-2}]$ and $[b] = [L]$.

$$\Rightarrow \quad [ab] = [L^2T^{-2}]$$

Hence, the correct answer is (A).

136. Let time period depends on mass (m), amplitude (A) and constant (k) as $T \propto m^\alpha a^\beta k^\gamma$.

$$M^0L^0[T] \propto M^\alpha L^\beta \left[\frac{ML^2T^{-2}}{L^3}\right]^\gamma$$

$$\Rightarrow \quad M^0L^0[T] \propto M^{\alpha+\gamma}L^{\beta-\gamma}+T^{-2\gamma}$$

Equating dimensions both sides

$$\alpha + \gamma = 0$$

$$\Rightarrow \quad \alpha = -\gamma$$

$$\Rightarrow \quad \alpha = \frac{1}{2}$$

$$\beta - \gamma = 0$$

$$\Rightarrow \quad \beta = \gamma$$

$$\Rightarrow \quad \beta = -\frac{1}{2}$$

$$\Rightarrow \quad -2\gamma = 1$$

$$\Rightarrow \quad \gamma = -\frac{1}{2}$$

$$\Rightarrow \quad t \propto m^{1/2}a^{-1/2}k^{1/2}$$

$$\Rightarrow \quad t \propto \sqrt{\frac{m}{ak}}$$

$$\Rightarrow \quad t \propto \frac{1}{\sqrt{a}}$$

Hence, the correct answer is (A).

137. Dimensionally

$$F = MLT^{-2}$$

In C.G.S. system

$$1\ \text{dyne} = 1\ \text{g}\ 1\ \text{cm}\ (1\ \text{s})^{-2}$$

In new system

$$1x = (10\text{ g})(10\text{ cm})(0.1\text{ s})^{-2}$$

$$\Rightarrow \quad \frac{1\text{ dyne}}{1x} = \frac{1\text{ g}}{10\text{ g}} \times \frac{1\text{ cm}}{10\text{ cm}} \left(\frac{10\text{ s}}{1\text{ s}}\right)^{-2}$$

$$\Rightarrow \quad 1\text{ dyne} = \frac{1}{10{,}000} \times 1x$$

$$\Rightarrow \quad 10^4\text{ dyne} = 1x$$

$$\Rightarrow \quad 10x = 10^5\text{ dyne} = 1\text{ N}$$

$$\Rightarrow \quad x = \frac{1}{10}\text{ N}$$

Hence, the correct answer is (A).

138. $[F] = \dfrac{[A-x]}{[B][t]}$

$$\Rightarrow \quad [B] = \frac{[L]}{[MLT^{-2}][T]} = [M^{-1}T^{1}]$$

Hence, the correct answer is (C).

139. Dimension of $[\eta] \equiv \left[\dfrac{Fr}{Av}\right] \equiv \dfrac{[MLT^{-2}][L]}{[L^2][LT^{-1}]}$

$$[\eta] \equiv [ML^{-1}T^{-1}]$$

Dimensions of $\mu = Fr^n$

$$[\mu] = [MLT^{-2}]L^n$$

$$\Rightarrow \quad [\mu] = ML^{n+1}T^{-2}$$

Let η depend on mass m mean speed v and constant μ as

$$\eta \propto m^a v^b \mu^c$$

$$\Rightarrow \quad [ML^{-1}T^{-1}] \propto M^a [LT^{-1}]^b [ML^{n+1}T^{-2}]^c$$

$$\Rightarrow \quad [ML^{-1}T^{-1}] \propto M^{a+c} L^{b+c(n+1)} T^{-b-2c}$$

Equating dimensions on both sides, we get

$$a + c = 1$$

$$\Rightarrow \quad c = 1 - a$$

$$\Rightarrow \quad b + c(n+1) = -1$$

$$\Rightarrow \quad b = -[1 + c(n+1)]$$

$$\Rightarrow \quad -(b + 2c) = -1$$

$$\Rightarrow \quad b = 1 - 2c$$

$$\Rightarrow \quad 1 - 2c = -[1 + c(n+1)]$$

$$\Rightarrow \quad 2c - 1 = 1 + c(n+1)$$

$$\Rightarrow \quad 2c - c(n+1) = 2$$

$$\Rightarrow \quad c[2 - n - 1] = 2$$

$$\Rightarrow \quad c[1 - n] = 2$$

$$\Rightarrow \quad c = -\frac{2}{n-1}$$

Since, $a = 1 - c$

$$\Rightarrow \quad a = 1 + \frac{2}{n-1} = \frac{n-1+2}{n-1}$$

$$\Rightarrow \quad a = \frac{n+1}{n-1}$$

and $b = 1 - 2c = 1 + \dfrac{4}{n-1} = \dfrac{n-1+4}{n-1}$

$$\Rightarrow \quad b = \frac{n+3}{n-1}$$

$$\Rightarrow \quad \eta \propto m^{\frac{n+1}{n-1}} v^{\frac{n+3}{n-1}} \mu^{-\frac{2}{n-1}}$$

Hence, the correct answer is (B).

140. If number of observations are large, then average deviation

$$\Rightarrow \quad \text{Possible error} \propto \frac{1}{\text{No. of observation}}$$

Hence, the correct answer is (A).

141. According to Faraday's law, electric potential is

$$V \equiv \frac{d\phi}{dt} \qquad \ldots(1)$$

and according to photo-electric effect, we have

$$eV = h\nu$$

$$\Rightarrow \quad V = \left(\frac{h}{e}\right)\nu \qquad \ldots(2)$$

From equations (1) and (2) we can see that magnetic flux (ϕ) and $\dfrac{h}{e}$ have the same dimensions.

Hence, the correct answer is (A).

142. $$\left[\frac{E^2}{\mu_0}\right] = \left[\frac{\varepsilon_0 E^2}{\varepsilon_0 \mu_0}\right] = \left[\frac{\text{energy/volume}}{(1/\text{speed of light})^2}\right]$$

$$\Rightarrow \quad \left[\frac{E^2}{\mu_0}\right] = \left[\frac{\text{energy (speed)}^2}{\text{volume}}\right]$$

$$\Rightarrow \quad \left[\frac{E^2}{\mu_0}\right] = \left[\frac{ML^2T^{-2}L^2T^{-2}}{L^3}\right] = [MLT^{-4}]$$

Hence, the correct answer is (B).

143. Since, according to Wien's Law,

$$\lambda_m T = b$$

$$\Rightarrow \quad b^4 = \lambda_m^4 T^4 \text{ and } \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \sigma T^4$$

$$\Rightarrow \quad \sigma = \frac{\text{Energy}}{(\text{Area} \times \text{Time})T^4}$$

$$\Rightarrow \quad \sigma b^4 = \left(\frac{\text{Energy}}{\text{Area} \times \text{Time}}\right)\lambda_m^4$$

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$$\Rightarrow \quad [\sigma b^4] = \frac{[ML^2T^{-2}]}{[L^2][T]}[L^4] = [ML^4T^{-3}]$$

Hence, the correct answer is (B).

150. Because constant involved (G) is not dimensionless.
Hence, the correct answer is (D).

151. $[\alpha x] = M^0L^0T^0$

$\Rightarrow \quad [\alpha] = M^0L^{-1}T^0$

$\Rightarrow \quad [Bx] = [T]$

$\Rightarrow \quad [B] = M^0L^{-1}T^1$

$\Rightarrow \quad [\beta t] = M^0L^0T^0$

$\Rightarrow \quad [\beta] = M^0L^0T^{-1}$

Hence, the correct answer is (D).

152. Since, we have $[P] = ML^2T^{-3}$

So, unit of $P = 20 \text{ kg} \times (10 \text{ m})^2 \times (5 \text{ sec})^{-3}$

$\Rightarrow \quad P = 16 \text{ W}$

Hence, the correct answer is (A).

153. $\left[\frac{ma}{K}\right] = [l] = b$

Hence, the correct answer is (C).

154. $\frac{\Delta Y}{Y} = \frac{2\Delta D}{D} + \frac{\Delta l}{l}$

$\Rightarrow \quad \frac{\Delta Y}{Y} = 2\left(\frac{0.01}{0.4}\right) + \left(\frac{0.05}{0.8}\right)$

$\Rightarrow \quad \frac{\Delta Y}{Y} = 2 \times 0.025 + 0.0625$

$\Rightarrow \quad \frac{\Delta Y}{Y} = 0.05 + 0.0625 = 0.1125$

$\Rightarrow \quad \Delta Y = 2 \times 10^{11} \times 0.1125 = 0.225 \times 10^{11}$

So, $(2 \pm 0.2) \times 10^{11} \text{ Nm}^{-2}$
Hence, the correct answer is (B).

155. $[-\alpha t^2 + \beta t + \gamma] = M^0L^0T^0$

$\Rightarrow \quad [\alpha] = T^{-2}, [\beta] = T^{-1}, [\gamma] = 1$

Hence, the correct answer is (A).

156. $T = (ML^{-1}T^{-2})^{\alpha}(ML^{-3})^{\beta}(MT^{-2})^{\gamma}$

$\Rightarrow \quad \alpha + \beta + \gamma = 0 \quad \ldots(1)$

$\Rightarrow \quad -\alpha - 3\beta = 0 \quad \ldots(2)$

$\Rightarrow \quad -2\alpha - 2\gamma = 1 \quad \ldots(3)$

Hence, the correct answer is (A).

157. $T^2 = 4\pi^2 \frac{l}{g}$

$\Rightarrow \quad g = \frac{4\pi^2 l}{T^2}$

$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\frac{\Delta T}{T} \times 100$

$\Rightarrow \quad \frac{\Delta g}{g} = y + 2x$

Hence, the correct answer is (C).

158. Since, $F = \frac{q^2}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad ke^2 = Fr^2$

Energy $E = \frac{hc}{\lambda}$

$\Rightarrow \quad hc = E\lambda$

So, $\frac{e^2}{4\pi\varepsilon_0 hc} = \frac{Fr^2}{hc} = \frac{Fr^2}{E\lambda} = [M^0L^0T^0]$

Hence, the correct answer is (A).

159. $\frac{GIM^2}{E^2} = \frac{[M^{-1}L^3T^{-2}][M^1L^1T^{-1}][M^2]}{[M^1L^2T^{-2}]^2} = [T]$

Hence, the correct answer is (A).

160. $\frac{\Delta f_{net}}{f_{net}^2} = \frac{\Delta f_1}{f_1^2} + \frac{\Delta f_2}{f_2^2}$

Hence, the correct answer is (C).

161. $\frac{d^2y}{dx^2} = \frac{d^2l}{dt^2} = \text{Acceleration}$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. For light travelling in a medium, we have

$$\mu = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}$$

where μ is the refractive index of the medium. So, we have that the given relation in question is dimensionally correct but numerically incorrect.
Hence, (A) and (C) are correct.

2. $[y] = [At^2] = [Bt^3]$

$\Rightarrow \quad [A] = LT^{-2}$ and $[B] = LT^{-3}$

Now, $[A^3] = L^3T^{-6}$ and $[B^2] = L^2T^{-6}$

$\Rightarrow \quad \frac{[A^3]}{[B^2]} = L$

Hence, (A), (B) and (C) are correct.

3. All are the drawbacks of dimensional analysis.
Hence, (A), (B), (C) and (D) are correct.

4. Refractive index, poisson's ratio and specific gravity are dimensionless.
Hence, (A), (B) and (D) are correct.

5. $\left[\frac{2\pi ct}{\lambda}\right] = M^0L^0T^0$ …(1)

$\Rightarrow \quad [ct] = [\lambda]$ {(A) is correct}

$\left[\frac{2\pi x}{\lambda}\right] = M^0L^0T^0$ …(2)

$\Rightarrow \quad [x] = [\lambda]$ {(B) is correct}

From (1) and (2)

$$\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right]$$

$$\Rightarrow \quad \left[\frac{2\pi c}{\lambda}\right] = \left[\frac{2\pi x}{\lambda t}\right]$$

So, (C) is also correct and the above result also shows that (D) is incorrect.
Hence, the correct answer is (C).

6. Dyne cm^{-2} is the unit of pressure, stress and Young's Modulus.
Hence, (A), (B) and (C) are correct.

8. [Renold's number] = [Coefficient of friction] = $M^0L^0T^0$

[Curie] = [Frequency] = $M^0L^0T^{-1}$

[Latent heat] = [Gravitational potential] = $M^0L^2T^{-2}$

Hence, (A), (B) and (C) are correct.

9. $L = G^x c^y h^z$

$\Rightarrow \quad L = (M^{-1}L^3T^{-2})^x (LT^{-1})^y (ML^2T^{-1})^z$

$\Rightarrow \quad L = M^{-x+z}L^{3x+y+2z}T^{-2x-y-z}$

Using the Principle of Homogeneity, we get

$-x + z = 0$ …(1)

$3x + y + 2z = 1$ …(2)

$-2x - y - z = 0$ …(3)

Solving equations (1), (2) and (3), we get

$$x = \frac{1}{2},\ y = -\frac{3}{2},\ z = \frac{1}{2}$$

Hence, (B) and (C) are correct.

10. Please remember that dimensional correctness does not guarantee numerical correctness of a physical relation.
Hence, (A), (B) and (C) are correct.

11. Electrical unit of energy is kWhr . So (A) is correct
Also

$$(\text{volt})^2(\text{sec})(\text{ohm})^{-1} \equiv \left(\frac{V^2}{R}\right)t$$

where R is the resistance, t is the time and V is the electric potential difference. Now since we know that $W = \left(\frac{V^2}{R}\right)t$. So, we get (B) as the another correct option.
We have (pascal) $(\text{foot})^2$ as the unit of force, so (C) is incorrect
Finally, we have

$(\text{weber})(\text{ampere}) \equiv (\phi_B)(I) = LI^2 \qquad \{\because \phi_B = LI\}$

Now since $\frac{1}{2}LI^2$ = Energy associated with the magnetic field of a solenoid, so (D) is also correct.
Hence, (A), (B) and (D) are correct.

12. $x = \left[\frac{E}{B}\right] = [\text{velocity}] = \left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right] = \left[\frac{\ell}{CR}\right]$

So, all x, y and z have dimensional formula same as that of velocity.
Hence, (A), (B) and (C) are correct.

13. [Light year] = [Wavelength] = [Radius of gyration] = L
Hence, (B) and (C) are correct.

14. The units parsec, light year and micron represent length.
Hence, (A), (B) and (C) are correct.

15. [Impulse] = [Linear momentum] = MLT^{-1}
[Planck's constant] = [Angular momentum] = ML^2T^{-1}
[Young's modulus] = [Pressure] = $ML^{-1}T^{-2}$
Hence, (A), (B) and (D) are correct.

16. Energy per unit volume represents pressure or stress or modulus of elasticity.
Hence, (B), (C) and (D) are correct.

17. $\left[\frac{B^2}{2\mu_0}\right] = \begin{bmatrix}\text{Magnetic}\\ \text{Energy}\\ \text{Density}\end{bmatrix} = [\text{Pressure}] = ML^{-1}T^{-2}$

Hence, (A) and (B) are correct.

25. Since $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$\Rightarrow \quad f^{-1} = v^{-1} + u^{-1}$

$\Rightarrow \quad \Delta(f^{-1}) = \Delta(u^{-1}) + \Delta(v^{-1})$

$\Rightarrow \quad (-1)f^{-2}\Delta f = (-1)u^{-2}\Delta u + (-1)v^{-2}\Delta v$

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$$\Rightarrow \quad \frac{\Delta f}{f^2}=\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2} \qquad \ldots(1)$$

So, (A) is correct.

From (1), $\frac{\Delta f}{f}=f\left(\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right)$

$$\Rightarrow \quad \frac{\Delta f}{f}=\left(\frac{uv}{u+v}\right)\left(\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right)$$

So, (C) is also correct

Hence, (A) and (C) are correct.

30. $F'=\left(\frac{1}{\alpha\beta}\right)F$

$$\Rightarrow \quad m'a'=\left(\frac{1}{\alpha\beta}\right)ma$$

$$\Rightarrow \quad m'=\left(\frac{1}{\alpha\beta}\right)\left(\frac{a}{a'}\right)m$$

But $\frac{a}{a'}=\frac{1}{\alpha\beta}$

$$\Rightarrow \quad m'=\left(\frac{1}{\alpha^2\beta^2}\right)m$$

So, (B) is correct

Further

$$a'=(\alpha\beta)a$$

$$\Rightarrow \quad \frac{v'}{t'}=(\alpha\beta)\left(\frac{v}{t}\right)$$

$$\Rightarrow \quad t'=\left(\frac{1}{\alpha\beta}\right)\left(\frac{v'}{v}\right)t$$

$$\Rightarrow \quad t'=\left(\frac{1}{\alpha\beta}\right)\left(\frac{\alpha^2}{\beta}\right)t$$

$$\Rightarrow \quad t'=\left(\frac{\alpha}{\beta^2}\right)t$$

So, (C) is correct

Further

$$v'=\left(\frac{\alpha^2}{\beta}\right)v$$

$$\Rightarrow \quad \frac{l'}{t'}=\left(\frac{\alpha^2}{\beta}\right)\left(\frac{l}{t}\right)$$

$$\Rightarrow \quad l=\left(\frac{\alpha^2}{\beta}\right)\left(\frac{t'}{t}\right)l$$

$$\Rightarrow \quad l'=\left(\frac{\alpha^2}{\beta}\right)\left(\frac{\alpha}{\beta^2}\right)l$$

$$\Rightarrow \quad l'=\left(\frac{\alpha^3}{\beta^3}\right)l$$

So, (A) is also correct

Finally, we have

$$p'=m'v'$$

$$\Rightarrow \quad p'=\left(\frac{1}{\alpha^2\beta^2}\right)m\left(\frac{\alpha^2}{\beta}\right)v$$

$$\Rightarrow \quad p'=\left(\frac{1}{\beta^3}\right)mv=\left(\frac{1}{\beta^3}\right)p$$

So, (D) is also correct.

Hence, (A), (B), (C) and (D) are correct.

Reasoning Based Questions

1. Light year is distance travelled by light in vacuum in 1 year.

The wavelength is the distance between two consecutive crests or troughs of a wave.

The dimensions of both light year and wavelength is $\left[M^0LT^0\right]$ so, both represent distances.

Hence, the correct answer is (A).

2. $[c]=LT^{-1}=3\times10^8\,\text{ms}^{-1}$

and $[g]=LT^{-2}=10\,\text{ms}^{-2}$

$$\Rightarrow \quad \frac{c}{g}=\frac{LT^{-1}}{LT^{-2}}=T=\frac{3\times10^8}{10}=3\times10^7\,\text{s}$$

$$\Rightarrow \quad T=3\times10^7\,\text{s}$$

Hence, the correct answer is (B).

3. Surface energy is defined as the work done per unit surface area.

So, dimensional formula of surface energy is

$$\frac{ML^2T^{-2}}{L^2} \text{ i.e., } MT^{-2}$$

Hence, the correct answer is (D).

4. $g=\frac{GM}{R_e^2}=\frac{G}{R_e^2}\times\frac{4}{3}\pi R_e^3\rho \qquad \left\{\because M=\frac{4}{3}\pi R_e^3\rho\right\}$

$$\Rightarrow \quad g=\frac{4G\pi R_e\rho}{3}$$

$$\Rightarrow \quad \rho=\frac{3g}{4G\pi R_e}$$

Hence, the correct answer is (A).

5. 1 micron = 10^{-6} m

This measurement is used to measure the microscopic distance (length of biological cell) by the microscope.

Hence, the correct answer is (A).

6. Electrostatic potential is given by

$$V=\frac{\text{Work done}}{\text{Charge}} \qquad \ldots(1)$$

$\Rightarrow \quad V \propto$ work done

Hence, Statement 1 is true.

From Equation (1),

$$[V] = \frac{[\text{Work done}]}{[\text{Charge}]}$$

$$\Rightarrow \quad [V] = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$$

Hence, statement 2 is true. But statement 2 is not the correct explanation to statement 1.

Hence, the correct answer is (B).

7. The distance travelled in a particular second has dimensions same as that of velocity and not as distance. Statement-2 is an obvious correct statement.

Hence, the correct answer is (D).

8. Since $E = \frac{1}{2}mv^2$

$$\Rightarrow \quad \frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v} = 1\% + 2(2\%) = 5\%$$

Hence, the correct answer is (A).

9. Dimensions of $[\varepsilon_o] = M^{-1}L^{-3}T^4A^2$

Dimensions of $[\mu_o] = MLT^{-2}A^{-2}$

$$\Rightarrow \quad \left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right] = \frac{1}{\sqrt{(M^{-1}L^{-3}T^4A^2)(MLT^{-2}A^{-2})}}$$

$$\Rightarrow \quad \left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right] = \frac{1}{\sqrt{L^{-2}T^2}} = \frac{L}{T} = \text{Velocity}$$

Also, $\frac{1}{\sqrt{\mu_o\varepsilon_o}} = \frac{1}{\sqrt{\frac{\mu_o}{4\pi} \times 4\pi\varepsilon_o}} = \sqrt{\frac{9\times10^9}{10^{-7}}}$

$$\Rightarrow \quad \frac{1}{\sqrt{\mu_o\varepsilon_o}} = \sqrt{9\times10^{16}} = 3\times10^8 \text{ ms}^{-1} = c$$

Hence, the correct answer is (A).

10. The percentage of moisture in air is measured by hygrometer and the degree of hotness of a body is measured by the thermometer. Hence, statement 1 is false and statement 2 is true.

Hence, the correct answer is (D).

11. $A = 4\pi r^2$

Fractional error $\frac{\Delta A}{A} = 2\frac{\Delta r}{r}$

(Error will not be involved in constant 4π)

$$\Rightarrow \quad \frac{\Delta A}{A} \times 100 = 2 \times 0.3\% = 0.6\%$$

So, $\frac{\Delta A}{A} = 4\frac{\Delta r}{r}$ is false

Hence, the correct answer is (C).

12. Pressure gradient $= \frac{dP}{dx} = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$

Which is different with the dimension of pressure. Since, like quantities can be added or subtracted, so, we cannot subtract the pressure from pressure gradient.

Hence, the correct answer is (D).

13. Trigonometrical ratio have no dimensions. So, method of finding dimensions cannot be utilised for deriving formulae which are dependent on trigonometrical ratios.

Hence, the correct answer is (A).

14. The physical quantities which do not depend on the other quantities are called fundamental quantities. Also, length, mass and time are not derived quantities but are fundamental quantities.

Hence, the correct answer is (C).

15. According, the Principle of Homogeneity of dimensions, an equation representing the physical quantity is true, only when the dimensions on LHS and RHS must be the same. The dimensions of a physical quantity are the powers to which the fundamental units are raised to get the required physical quantity.

Hence, the correct answer is (B).

16. Nuclear cross section is measured in unit called barn. In SI system the value of 1 barn $= 10^{-28}\text{ m}^2$. Therefore, Statement-1 is true and Statement-2 is false.

Hence, the correct answer is (C).

17. The given relation is

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\Rightarrow \quad f^2 = \frac{T}{4l^2m}, \; T \text{ being tension.}$$

$$\Rightarrow \quad [m] = \left[\frac{T}{4\ell^2f^2}\right] = \frac{[MLT^{-2}]}{\left[L^2\frac{1}{T^2}\right]} = \frac{[MLT^{-2}]}{[L^2T^{-2}]}$$

$$= \frac{[M]}{[L]} = \frac{\text{Mass}}{\text{Length}} = \text{ linear mass density.}$$

Hence, the correct answer is (C).

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18. Avogadro number is the number of atoms/molecules/ions/nuclei/particles present per mole of a sample.
So, it has unit mol^{-1} and hence is not dimensionless.
Hence, the correct answer is (D).

19. Statement 1 is true because it is an obvious statement known to us.
However Statement 2 is incorrect because dimensional correctness may or may not establish numerical correctness of a physical relation.
Hence, the correct answer is (C).

20. AU is an astronomical unit. It is the mean distance between earth and sun.

$$1\ \text{AU} = 1.496 \times 10^{11}\ \text{m} = 1.5 \times 10^{11}\ \text{m}$$

Å is angstrom unit $1\ \text{Å} = 10^{-10}\ \text{m}$
Hence, the correct answer is (A).

21. Suppose $m = kv^a \rho^b g^c$

$$\Rightarrow \quad ML^0T^0 = (LT^{-1})^a (ML^{-3})^b (LT^{-2})^c$$

$$\Rightarrow \quad ML^0T^0 = M^b L^{a-3b+c} T^{-a-2c}$$

According to Principle of Homogeneity, we get

$$b = 1 \qquad \ldots(1)$$

$$a - 3b + c = 0 \qquad \ldots(2)$$

$$-a - 2c = 0 \qquad \ldots(3)$$

$$\Rightarrow \quad c = -3 \text{ and } a = 6$$

So, $a = 6$, $b = 1$, $c = -3$

$$\Rightarrow \quad m = k\frac{v^6 \rho}{g^3}$$

Hence, the correct answer is (B).

22. Pressure is given by

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Distance}}$$

$$\Rightarrow \quad \text{Pressure} = \frac{\text{Energy}}{\text{Volume}} = \text{Energy density}$$

Thus, pressure has the dimensions of energy density.
Hence, the correct answer is (A).

23. Since, the graph is a straight line therefore $P \propto Q$.

$$\Rightarrow \quad P = \text{constant} \times Q$$

$$\Rightarrow \quad \frac{P}{Q} = \text{constant} = \text{slope of the straight line.}$$

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Please keep in mind that acceleration due to gravity just represents the gravitational **field** or gravitational **intensity**.
Hence, (A) and (C) are correct.

2. Torque and moment of **force** have same dimensional formula.
Hence, the correct answer is (B).

3. Remember that actually, force is negative energy **gradient**.
Hence, the correct answer is (C).

4. Surface tension, surface **energy** and spring constant all have same dimensional formula.
Hence, the correct answer is (B).

5. Pressure, stress, **energy** density, modulus of elasticity all have same dimensional formula.
Hence, the correct answer is (A).

6. $[V] = [b]$
Hence, the correct answer is (B).

7. $[p] = \left[\frac{a}{V^2}\right]$

$$\Rightarrow \quad [a] = [pV^2]$$

Hence, the correct answer is (C).

8. All pV, pb and $\frac{ab}{V^2}$ has dimensional formula same as that of RT.
Hence, the correct answer is (C).

9. As discussed already, $\left[\frac{ab}{V^2}\right] = [RT]$

$$\Rightarrow \quad \left[\frac{ab}{RT}\right] = [V^2] = L^6$$

Hence, the correct answer is (D).

10. $[RT] = [pV] = [\text{Energy}]$
Hence, the correct answer is (A).

11. $[c] = c = LT^{-1} \qquad \ldots(1)$

$$[G] = G = M^{-1}L^3T^{-2} \qquad \ldots(2)$$

$$[h] = h = ML^2T^{-1} \qquad \ldots(3)$$

$$\Rightarrow \quad \frac{hc}{G} = M^2$$

$$\Rightarrow \quad M = c^{\frac{1}{2}} G^{-\frac{1}{2}} h^{\frac{1}{2}}$$

Hence, the correct answer is (A).

12. $\frac{Gh}{c^3} = \frac{L^5T^{-3}}{L^3T^{-3}}$

$$\Rightarrow \quad L^2 = Ghc^{-3}$$

$$\Rightarrow \quad L = c^{-\frac{3}{2}} G^{\frac{1}{2}} h^{\frac{1}{2}}$$

Hence, the correct answer is (B).

13. Put value of L (in terms of c, G and h) in (1), we get

$$T = c^{-\frac{5}{2}} G^{\frac{1}{2}} h^{\frac{1}{2}}$$

Hence, the correct answer is (C).

14. Least count of metre stick $= 1 \text{ mm} = 0.1 \text{ cm}$

$$\%\text{age error} = \frac{\Delta L}{L} \times 100\% = \frac{0.1}{50} \times 100\% = 0.2\%$$

Hence, the correct answer is (B).

15. $$\frac{\Delta m}{m} \times 100\% = \frac{0.1 \times 10^{-3}}{1} \times 100\% = 0.01\%$$

Hence, the correct answer is (D).

16. $$\frac{\Delta T}{T} \times 100\% = \frac{0.2}{4 \times 60} \times 100\% = 0.083\%$$

Hence, the correct answer is (C).

17. **Angular Momentum**, Planck's constant and energy per unit **frequency** have same dimensional formula.
Hence, the correct answer is (C).

18. Frequency, angular frequency, angular **speed** and **speed** gradient all have same dimensional formula.
Hence, the correct answer is (A).

19. **Electric** field strength and **electric** potential gradient have same dimensional formula.
Gravitational field strength and **gravitational** potential gradient have same dimensional formula.
Hence, (A) and (D) are correct.

20. Latent heat, gravitational potential, kinetic energy per unit mass, gravitational potential energy per unit mass all have same dimensional formula.
Hence, (A), (B) and (C) are correct.

21. Angle, Strain and Fine structure constant all are dimensionless.
Hence, (A) and (C) are correct.

22. $[D] = [T^2]$, $\frac{[C][T]}{[T^2]} = [F]$

$$\Rightarrow \quad C = [MLT^{-1}]$$

Hence, the correct answer is (B).

23. $\frac{[B]}{[T]} = [F]$

$$\Rightarrow \quad [B] = [MLT^{-1}]$$

Hence, the correct answer is (C).

24. $$\frac{1}{\sqrt{D}} = \frac{1}{[T^2]^{1/2}} = \frac{1}{[T]}$$

Hence, the correct answer is (A).

25. **The correct answer is (A).**

26. **The correct answer is (B).**

27. **The correct answer is (D).**

Combined solution to 25, 26, 27

Let $\omega \propto V^a$

$$\Rightarrow \quad \omega \propto R^b$$
$$\Rightarrow \quad \omega \propto M^c$$
$$\Rightarrow \quad \omega \propto l^d$$
$$\Rightarrow \quad \omega = kV^a R^b m^c l^d$$

Equating power of same base in dimensional formula of L.H.S. and R.H.S and solving.

$$a = \frac{2}{3},\ b = -\frac{1}{3},\ c = -\frac{1}{3},\ d = -\frac{1}{3}$$

So, $\omega \propto V^{2/3}$

$$\Rightarrow \quad \omega \propto R^{-1/3}$$

31. **The correct answer is (B).**

32. **The correct answer is (D).**

33. **The correct answer is (A).**

Combined solution to 31, 32, 33

E	V	F
ML^2T^{-2}	LT^{-1}	MLT^{-2}

$$\Rightarrow \quad [S] = MT^{-2} = \frac{(MLT^{-2})^2}{(ML^2T^{-2})} = \frac{F^2}{E}$$

$$\Rightarrow \quad [L] = ML^2T^{-1} = \frac{(ML^2T^{-2})^2}{(LT^{-1})(MLT^{-2})} = \frac{E}{VF}$$

$$\Rightarrow \quad [t] = T = \frac{ML^2T^{-2}}{(LT^{-1})(MLT^{-2})} = \frac{E}{VF}$$

34. **The correct answer is (A).**

35. **The correct answer is (B).**

36. **The correct answer is (C).**

Combined solution to 34, 35, 36

$$T = kp^a d^b E^c$$

$$\Rightarrow \quad T = 1(ML^{-1}T^{-2})^a (ML^{-3})^b (ML^2T^{-2})^c$$
$$\Rightarrow \quad a + b + c = 0 \qquad \ldots(1)$$
$$\Rightarrow \quad -a - 3b + 2c = 0 \qquad \ldots(2)$$
$$\Rightarrow \quad -2a - 2c = 1 \qquad \ldots(3)$$

40. Quantities having same dimensions can only be added and in $\ln(x)$ and $\sin(x)$, x is always dimensionless.
Hence, the correct answer is (B).

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43. Quantities having same dimensions can only be added & in $\ln(x)$ and $\sin(x)$, x is always dimensionless. **Hence, the correct answer is (C).**

44. **The correct answer is (A).**

45. **The correct answer is (B).**

46. **The correct answer is (B).**

Combined solution to 44, 45, 46

$$[a] = [P][V]^2 = ML^4T^{-2}$$

$$[b] = [V] = L^3$$

$$\Rightarrow \quad [R] = \frac{[P][V]}{[T]} = ML^2T^{-2}K^{-1}$$

Matrix Match/Column Match Type Questions

1. A → (q)
B → (p)
C → (r)
D → (t)

$$\left[\frac{L}{R}\right] = [\text{Inductive Time Constant}] = T$$

$$[CR] = [\text{Capacitive Time Constant}] = T$$

$$\left[\frac{E}{B}\right] = [\text{Velocity}] = LT^{-1}$$

$$\left[\frac{1}{\mu_0\varepsilon_0}\right] = [(\text{Velocity})^2] = L^2T^{-2}$$

2. A → (p, q)
B → (r, t)
C → (p, q)
D → (s)
Stress, pressure and energy density have same dimensional formula $ML^{-1}T^{-2}$. So,
(A) → (p, q)
Strain, Angle, Fine structure constant are dimensionless physical quantities. So,
(B) → (r, t)
Modulus of elasticity has the dimensional formula same as that of stress, pressure and energy density. So,
(C) → (p, q)
Torque has dimensional formula, same as that of energy i.e., ML^2T^{-2}.

3. A → (s)
B → (p)
C → (r)
D → (q)

$$[\text{Specific Heat}] = L^2T^{-2}K^{-1}$$

$$[\text{Boltzmann's Constant}] = ML^2T^{-2}K^{-1}$$

$$[\text{Wein's Constant}] = LK$$

$$[\text{Stefan's Constant}] = MT^{-3}K^{-4}$$

4. A → (q, r)
B → (q)
C → (p, s, t)
D → (q)

$$[\text{Impulse}] = MLT^{-1} = [\text{Linear Momentum}]$$

$$[\text{Planck's Constant}] = ML^2T^{-1}$$

$$[\text{Angular Momentum}] = ML^2T^{-1}$$

$$[\text{Energy per unit frequency}] = \left[\frac{E}{\nu}\right] = [h]$$

5. A → (r, s)
B → (p, q)
C → (t)
D → (p, q)
Stress, bulk modulus have unit Nm^{-2}.
Force constant, surface tension and surface energy have unit Nm^{-1} and $1\ \text{Nm}^{-1} = 1\ \text{kgs}^{-2}$.
Torque has a unit Nm

6. A → (p, t)
B → (r)
C → (q)
D → (r)
Angle and fine structure constant is dimensionless

7. A → (r)
B → (s)
C → (p, q, r)
D → (t)

$$[\text{Latent Heat}] = \left[\frac{\text{Energy}}{\text{Mass}}\right] = L^2T^{-2}$$

$$[\text{Mean Square Velocity}] = \left[\frac{k_BT}{m}\right] = L^2T^{-2}$$

However unit of latent heat is Jkg^{-1}.

$$[\text{Specific Heat}] = \left[\frac{1}{m}\left(\frac{\Delta Q}{\Delta T}\right)\right] = L^2T^{-2}K^{-1}$$

9. A → (p, q)
B → (r, s)
C → (r, s)
D → (r, s)

$$\left[\frac{GM_eM_s}{r}\right] = \text{joule}$$

$$\Rightarrow \quad [GM_eM_s] = (\text{joule})(\text{metre})$$

Since joule $=$ (coulomb)(volt)

$\Rightarrow \quad [GM_eM_s] =$ (volt)(coulomb)(metre)

Also, $[GM_eM_s] = ML^3T^{-2}$

$\Rightarrow \quad [GM_eM_s] =$ (kilogram)(metre)^3(second)$^{-2}$

Now, $\left[\frac{3RT}{M}\right] = \left[\frac{F^2}{q^2B^2}\right] = \left[\frac{GM_e}{R_e}\right] = v^2$

Also $v^2 = \left[\frac{\text{Energy}}{\text{Mass}}\right] = \left[\frac{CV^2}{\text{Mass}}\right] =$ (farad)(volt)^2(kg)$^{-1}$

10. A → (r)
B → (p)
C → (p)
D → (q)

(A) $y = \frac{F}{A}\frac{\ell}{\Delta\ell}$

(B) $V = -\frac{Gm}{r}$

(C) $L = \frac{Q}{m}$

(D) $G = \frac{Fr^2}{Gm_1m_2}$

17. A → (r)
B → (p, q)
C → (p)
D → (s)

$\eta = \frac{F}{6\pi rv}$

$\Rightarrow \quad [\eta] = ML^{-1}T^{-1}$

[Angle] = [Strain] = 1

$[\text{Stress}] = \frac{[F]}{[A]} = ML^{-1}T^{-2}$

Integer/Numerical Answer Type Questions

1. $\left[\frac{EJ^2}{M^5G^2}\right] = \frac{(ML^2T^{-2})(ML^2T^{-1})^2}{(M^5)(M^{-1}L^3T^{-2})^2} = M^0L^0T^0$

2. $\left[\frac{e^2}{2h\varepsilon_0 c}\right] = M^0L^0T^0$

$\Rightarrow \quad x = 2$

3. Since we know that $V = \frac{\pi}{8}\frac{r^4}{\eta}p$, so we get $x = 4$, $y = 1$ and $z = 1$.

4. $F = 20$ N, $E = 200$ J, $v = 5$ ms^{-1}

Since $E = FL$

$\Rightarrow \quad L = \frac{E}{F} = \frac{200}{20} = 10$ m

Since $v = LT^{-1}$

$\Rightarrow \quad 5 = 10\ T^{-1}$

$\Rightarrow \quad T = 2$ s

Further $F = MLT^{-2}$

$\Rightarrow \quad 20 = M(10)(2)^{-2}$

$\Rightarrow \quad M = \frac{80}{10} = 8$ kg

5. $n_2 = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2}$

$\Rightarrow \quad n_2 = 4.18\left(\frac{1\text{ kg}}{\alpha\text{ kg}}\right)\left(\frac{1\text{ m}}{\beta\text{ m}}\right)^2\left(\frac{1\text{ s}}{\gamma\text{ s}}\right)^{-2}$

$\Rightarrow \quad n_2 = 4.18\alpha^{-1}\beta^{-1}\gamma^2$

So, 4.18 J $= 4.18\ \alpha^{-1}\beta^{-2}\gamma^2$ new units

6. To show that the planet obeys Kepler's Third Law of planetary motion, we have to prove that

$T^2 \propto r^3$

Now, according to the problem, we have

$T \propto r^aM_S^bG^c$

$\Rightarrow \quad T = kr^aM_S^bG^c$

where k is a dimensionless constant.

$\Rightarrow \quad T = L^aM^b(M^{-1}L^3T^{-2})^c$

$\Rightarrow \quad T = M^{b-c}L^{a+3c}T^{-2c}$

Using the Principle of Homogeneity, we get

$b - c = 0$ …(1)
$a + 3c = 0$ …(2)
$-2c = 1$ …(3)

$\Rightarrow \quad c = -\frac{1}{2}, \ b = -\frac{1}{2}, \ a = \frac{3}{2}$

So, we get

$T = kr^{\frac{3}{2}}M_S^{-\frac{1}{2}}G^{-\frac{1}{2}}$

By experiments, we know that

$k = 2\pi$

$\Rightarrow \quad T^2 = 4\pi^2\left(\frac{r^3}{GM}\right)$

So, $a = 2$, $b = 3$, $c = 1$, $d = 1$

CHAPTER 2

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1. $\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right]=\left[\frac{\varepsilon_0}{\sqrt{\mu_0\varepsilon_0}}\right]=[LT^{-1}]\times[\varepsilon_0]$

Since, $F=\frac{q^2}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad [\varepsilon_0]=\frac{[AT]^2}{[MLT^{-2}]\times[L^2]}$

$\Rightarrow \quad \left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right]=[LT^{-1}]\times[A^2M^{-1}L^{-3}T^4]=[M^{-1}L^{-2}T^3A^2]$

Hence, the correct answer is (D).

2. $[p]=MLT^{-1}=[I^x h^y S^z]$

$\Rightarrow \quad MLT^{-1}=M^xL^{2x}(ML^2T^{-1})^y(MT^{-2})^z$

$\Rightarrow \quad MLT^{-1}=M^{x+y+z}L^{2x+2y}T^{-y-2z}$

Applying Principle of Homogeneity, we get

$x+y+z=1$

$2(x+y)=1$

$\Rightarrow \quad x+y=\frac{1}{2}$

$\Rightarrow \quad z=\frac{1}{2}$

$y+2z=1$

$\Rightarrow \quad y=0$

$\Rightarrow \quad x=\frac{1}{2}$

$\Rightarrow \quad [p]=I^{\frac{1}{2}}h^0S^{\frac{1}{2}}$

Hence, the correct answer is (D).

3. $T=2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow \quad g=4\pi^2\frac{l}{T^2}$

$\Rightarrow \quad \frac{\Delta g}{g}=\frac{\Delta l}{l}+\frac{2\Delta T}{T}=\left(\frac{0.1}{55}+\frac{2\times1}{30}\right)\times100$

$\Rightarrow \quad \frac{\Delta g}{g}=6.8\%$

Hence, the correct answer is (A).

4. Since $\rho=\frac{m}{V}$

So, maximum percentage error in ρ will be given by

$$\frac{\Delta\rho}{\rho}\times100\%=\left(\frac{\Delta m}{m}\right)\times100\%+3\left(\frac{\Delta L}{L}\right)\times100\% \quad \ldots(1)$$

This is only possible when error is small, which is not the case in this question.

However, if we apply equation (1), then we get

$\Delta\rho=3100 \text{ kgm}^{-3}$

Let us now calculate the error, without using the approximation, by calculating the minimum and maximum values of the density. Now,

$\rho_{\min}=\frac{m_{\min}}{V_{\max}}=\frac{9.9}{(0.11)^3}=7438 \text{ kgm}^{-3}$ and

$\rho_{\max}=\frac{m_{\max}}{V_{\min}}=\frac{10.1}{(0.09)^3}=13854.6 \text{ kgm}^{-3}$

$\Rightarrow \quad \Delta\rho=(\rho_{\max}-\rho_{\min})=6416.6 \text{ kgm}^{-3}$

So, correct answer should be (C) but not (B).

Hence, the correct answer is (C).

5. $5.29\times7=37.0 \text{ cm}^2$

Answer should be in 3 significant digits.

Hence, the correct answer is (B).

6. $X=5YZ^2$

$\Rightarrow \quad Y\propto\frac{X}{Z^2}$ …(1)

Since $X=C=\frac{Q^2}{E}=\frac{[A^2T^2]}{[ML^2T^{-2}]}$

$\Rightarrow \quad X=[M^{-1}L^{-2}T^4A^2]$

Also, $Z=B=\frac{F}{IL}$

$\Rightarrow \quad Z=[MT^{-2}A^{-1}]$

So, from equation (1), we get

$$Y=\frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}$$

$\Rightarrow \quad Y=[M^{-3}L^{-2}T^8A^4]$

Hence, the correct answer is (D).

7. $[R] = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$

Hence, the correct answer is (C).

8. $\text{LC} = \dfrac{0.5}{100} = 0.005 \text{ mm}$

Zero error, $e = -3 \times 0.005 = -0.015$ mm

Thickness $= (5.5 + 48 \times 0.005 + 0.015)$ mm

$\Rightarrow$ Thickness $= 5.755$ mm

Hence, the correct answer is (C).

9. $[T] = [G]^a [h]^b [c]^c$

$\Rightarrow \quad T' = [M^{-1}L^3T^{-2}]^a [ML^2T^{-1}]^b [LT^{-1}]^c$

Applying Principle of Homogeneity, we get

$-a + b = 0$

$\Rightarrow \quad a = b \qquad \ldots(1)$

$3a + 2b + c = 0$

$\Rightarrow \quad 5a + c = 0 \qquad \ldots(2)$

$-2a - b - c = 1$

$\Rightarrow \quad 3a + c = -1 \qquad \ldots(3)$

From (2) and (3), we get

$a = \dfrac{1}{2}, \; b = \dfrac{1}{2}, \; c = -\dfrac{5}{2}$

$\Rightarrow \quad [T] = \sqrt{\dfrac{Gh}{c^5}}$

Hence, the correct answer is (A).

10. Since $\rho = 128 \text{ kgm}^{-3}$

$\Rightarrow \quad \rho = \dfrac{128 \text{ kg}}{1m^3}$

$\Rightarrow \quad \rho = \dfrac{128}{\left(\dfrac{100}{25}\right)^3} \dfrac{1000}{50} = \dfrac{128}{4^3} \times 20 = 40$

Hence, the correct answer is (C).

11. Volume of cylinder is given by

$V = \dfrac{\pi D^2 h}{4}$

$\Rightarrow \quad V = \dfrac{\pi}{4} \times 34.2 \times (12.6)^2 = 4264.39 \text{ cm}^3$

Since, $\dfrac{\Delta V}{V} = \dfrac{2\Delta D}{D} + \dfrac{\Delta h}{h}$

$\Rightarrow \quad \dfrac{\Delta V}{V} = \dfrac{2 \times 0.1}{12.6} + \dfrac{0.1}{34.2}$

$\Rightarrow \quad \Delta V = 80.157$

Reducing the answers to appropriate significant figures, then

$V = 4260 \pm 80 \text{ cm}^3$

Hence, the correct answer is (C).

12. Since $\left[\dfrac{x^2}{\alpha kT}\right] = M^0L^0T^0$

$\Rightarrow \quad [x^2] = [\alpha kT]$

$\Rightarrow \quad [\alpha ML^2T^{-2}] = L^2$

$\Rightarrow \quad [\alpha] = M^{-1}T^2$

Also $[\alpha\beta] = MLT^{-2}$

$\Rightarrow \quad M^{-1}T^{+2}[\beta] = MLT^{-2}$

$\Rightarrow \quad [\beta] = M^2LT^{-4}$

Hence, the correct answer is (B).

13. $[V] = L^1T^{-1}$

$[A] = L^1T^{-2}$

$[F] = M^1L^1T^{-2}$

Since $Y = \dfrac{\text{Force}}{\text{Area}}$

$\Rightarrow \quad [Y] = M^1L^{-1}T^{-2}$

$\Rightarrow \quad M^1L^{-1}T^{-2} = [F]^\alpha [A]^\beta [V]^\gamma$

$\Rightarrow \quad M^1L^{-1}T^{-2} = M^\alpha L^{\alpha+\beta+\gamma} T^{-2\alpha-2\beta-\gamma}$

By Principle of Homogeneity, we get

$\alpha = 1, \; \beta = 2, \; \gamma = -4$

Hence, the correct answer is (C).

14. Since the least count is given by

$$\text{L.C.} = \frac{\text{Pitch}}{\text{Number of division on circular scale}}$$

$\Rightarrow \quad 5 \times 10^{-6} = \dfrac{10^{-3}}{N}$

$\Rightarrow \quad N = 200$

Hence, the correct answer is (A).

CHAPTER 2

15. $\left[\frac{l}{rcv}\right]=\left[\frac{l}{TV}\right]=?$

Since $W=\frac{1}{2}lI^2$

$\Rightarrow \quad [ML^2T^{-2}]=[lA^2]$

$\Rightarrow \quad [l]=[ML^2T^{-2}A^{-2}]$

Since $V=\frac{W}{q}$

$\Rightarrow \quad [v]=\frac{ML^2T^{-2}}{AT}=ML^2T^{-3}A^{-1}$

$\Rightarrow \quad \left[\frac{l}{rcv}\right]=\frac{ML^2T^{-2}A^{-2}}{TML^2T^{-3}A^{-1}}=[A^{-1}]$

Hence, the correct answer is (A).

16. Density of a material is given by

$$\rho=\frac{m}{V}=\frac{m}{l^3}$$

For maximum error in ρ

$$\frac{d\rho}{\rho}=\frac{dm}{m}+3\frac{dl}{l}$$

$\Rightarrow \quad \frac{d\rho}{\rho}\times 100=\frac{dm}{m}\times 100+3\frac{dl}{l}\times 100=1.5+(3\times 1)=4.5\%$

Hence, the correct answer is (C).

17. Least count $=\frac{0.25}{5\times 100}$ cm $=5\times 10^{-4}$ cm

Thickness of wire is

$$D=4\times\frac{0.25}{5}\text{ cm}+30\times\text{L.C.}$$

$\Rightarrow \quad D=4\times 0.05\text{ cm}+30\times 5\times 10^{-4}\text{ cm}$

$\Rightarrow \quad D=0.20\text{ cm}+0.0150\text{ cm}=0.2150\text{ cm}$

Hence, the correct answer is (B).

18. As we know $\frac{\Delta S}{S}=2\times\frac{\Delta r}{r}$ and $\frac{\Delta V}{V}=3\times\frac{\Delta r}{r}=\frac{3}{2}\times\frac{\Delta S}{S}$

$\Rightarrow \quad \frac{\Delta V}{V}=\frac{3}{2}\alpha$

Hence, the correct answer is (A).

19. For planck length, $l=kG^p\hbar^q c^r$

$$[M^0LT^0]=[M^{-1}L^3T^{-2}]^p[ML^2T^{-1}]^q[LT^{-1}]^r$$

$$[M^0LT^0]=\left[M^{-p+q}L^{(3p+2q+r)}T^{-(2p+q+r)}\right]$$

On comparing powers of M, L and T from both sides, we get

$$-p+q=0,\ 3p+2q+r=1,\ -(2p+q+r)=0$$

On solving these equations, $p=q=\frac{1}{2}$, $r=\frac{-3}{2}$

$\Rightarrow \quad l=\left(\frac{G\hbar}{c^3}\right)^{1/2}$ {Take, $k=1$}

Hence, the correct answer is (C).

20. Relative error in A is given by

$$\frac{\Delta A}{A}=\frac{3\Delta P}{P}+\frac{2\Delta Q}{Q}+\frac{1}{2}\frac{\Delta R}{R}+\frac{\Delta S}{S}$$

The maximum percentage error in the value of A will be

$$\frac{\Delta A}{A}\times 100=3\times 0.5+2\times 1+\frac{1}{2}\times 3+1.5=6.5\%$$

Hence, the correct answer is (A).

21. From Kepler's law

$$T^2=\frac{4\pi^2}{GM}r^3$$

$\Rightarrow \quad M=\left(\frac{4\pi^2}{G}\right)\frac{r^3}{T^2}$

$\Rightarrow \quad \frac{\Delta M}{M}=2\frac{\Delta T}{T}+3\frac{\Delta r}{r}$

Since, $\frac{\Delta r}{r}\approx 0$

$\Rightarrow \quad \left|\frac{\Delta M}{M}\right|=2\frac{\Delta T}{T}=2\times 10^{-2}$

Hence, the correct answer is (C).

22. Let $m=kT^xC^yh^z$ where k is a dimensionless constant.

$\Rightarrow \quad [ML^0T^0]=[T]^x[LT^{-1}]^y[ML^2T^{-1}]^z$

$\Rightarrow \quad [ML^0T^0]=[M^zL^{y+2z}T^{x-y-z}]$

$\Rightarrow \quad z=1,\ y+2z=0$ and $x-y-z=0$

Solving, we get, $x=-1$, $y=-2$, $z=1$; on putting values we get

$$[M]=[T^{-1}C^{-2}h]$$

Hence, the correct answer is (C).

23. Here, $P = a^{1/2}b^2c^3d^{-4}$

$$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$$

$$\Rightarrow \left(\frac{\Delta P}{P}\times 100\right)\% = \left(\frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}\right)\times 100\%$$

So, percentage relative error in the measurement of P is

$$\frac{\Delta P}{P}\times 100\% = \left(\frac{1}{2}\times 2 + 2\times 1 + 3\times 3 + 4\times 5\right)\% = 32\%$$

Hence, the correct answer is (D).

24. Here, $t_1 = 90\text{ s}$, $t_2 = 91\text{ s}$, $T_3 = 95\text{ s}$, $T_4 = 92\text{ s}$

L.C. $= 1\text{ s}$

Mean of the measurements, $\langle t\rangle = \dfrac{\Sigma t_i}{N}$

$$\langle t\rangle = \frac{90+91+95+92}{4} = 92\text{ s}$$

Mean deviation $= \dfrac{\Sigma|\bar{t} - t_i|}{N} = \dfrac{2+1+3+0}{4} = 1.5\text{ s}$

Since the least count of the instrument is 1 s, so reported mean time $= (92 \pm 2)\text{ s}$.

Hence, the correct answer is (A).

25. Electrical conductivity $= [M^{-1}L^{-3}T^3\text{I}^2]$

Hence, the correct answer is (B).

26. Given, A, B, C and D have different dimensions.

Also, $AD = C\ln(BD)$

log is the dimensionless, so $[B] = \dfrac{1}{[D]}$

Also, $[AD] = [C]$

(A) $\left[\dfrac{C}{BD}\right] = \dfrac{[C]}{1} = [C]$

$$\left[\frac{AD^2}{C}\right] = \frac{[AD][D]}{[C]} = [D]$$

So, $\dfrac{C}{BD} - \dfrac{AD^2}{C} = C - D$ which is not meaningful.

(B) $[B^2C^2] = [B^2][A^2D^2] = A^2[BD]^2 = [A^2]$

$\Rightarrow$ $(A^2 - B^2C^2)$ is meaningful.

(C) $\left[\dfrac{A}{B}\right] = [AD] = [C]$

$\Rightarrow$ $\left(\dfrac{A}{B} - C\right)$ is meaningful.

(D) $\left(\dfrac{A-C}{D}\right)$ is not meaningful as A and C both have different dimensions.

Hence, (A) and (D) are correct.

27. Here, capacitance $C = ke^x a_0^y h^z c^a$

$$[C] = [M^{-1}L^{-2}A^2T^4]$$

$$[e] = [AT],\ [a_0] = [L]$$

$$[c] = [L^1T^{-1}],\ [h] = [M^1L^2T^{-1}]$$

$$\Rightarrow [M^{-1}L^{-2}A^2T^4] = [AT]^x[L]^y[M^1L^2T^{-1}]^z[L^1T^{-1}]^a$$

Comparing both sides, we get

$$x = 2,\ z = -1,\ y + 2z + a = -2,\ x - z - a = 4$$

On solving these equations, we get

$$x = 2,\ y = 1,\ z = -1,\ a = -1$$

Also, $[C] = u$ so $u = \dfrac{e^2a_0}{hc}$

Hence, the correct answer is (C).

28. $[e] = [\text{IT}]$, $[m] = [M]$, $[c] = [LT^{-1}]$

$[h] = [ML^2T^{-1}]$, $[\mu_0] = [ML\text{I}^{-2}T^{-2}]$

If $\mu_0 = ke^am^bc^ch^d$, then

$$[ML\text{I}^{-2}T^{-2}] = [\text{IT}]^a[M]^b[LT^{-1}]^c[ML^2T^{-1}]^d$$

By equating powers, we get $a = -2$, $b + d = 1$

$$c + 2d = 1,\ a - c - d = -2$$

Solving these equations, we get

$$a = -2;\ b = 0;\ c = -1;\ d = 1$$

$$\Rightarrow [\mu_0] = \left[\frac{h}{ce^2}\right]$$

Hence, the correct answer is (C).

29. Let $\left(\dfrac{\dot{Q}}{A}\right)$ is derived quantity which is derived from three fundamental quantities η, $\left(\dfrac{S\Delta\theta}{h}\right)$ and $\left(\dfrac{1}{\rho g}\right)$

By using Principle of Homogeneity of dimensions, we get

$$\left[\frac{\dot{Q}}{A}\right] = [\eta]^x\left[\frac{S\Delta\theta}{h}\right]^y\left[\frac{1}{\rho g}\right]^z$$

$$\Rightarrow \left[\frac{\dot{Q}}{A}\right] = [M^1T^{-3}];\ [\eta] = [M^1L^{-1}T^{-1}]$$

$$\Rightarrow \left[\frac{S\Delta\theta}{h}\right] = [L^1T^{-2}];\ \left[\frac{1}{\rho g}\right] = [M^{-1}L^2T^2]$$

CHAPTER 2

$\Rightarrow \quad [M^{-1}L^0T^{-3}] = [M^1L^{-1}T^{-1}]^x [M^0L^1T^{-2}]^y [M^{-1}L^2T^2]^z$

Comparing both sides, we get

$x + 0 - z = 1,\ -x + y + 2z = 0$ and $-x - 2y + 2z = -3$

Solving these equations, we get

$x = 1,\ y = 1,\ z = 0$

$\Rightarrow \quad \frac{\dot{Q}}{A} = \eta \frac{S\Delta\theta}{h}$

Hence, the correct answer is (A).

30. Measured value of the length of rod $= 3.50$ cm

So, least count of the measuring instrument must be 0.01 cm

$\Rightarrow$ L.C. = 0.1 mm

For, vernier scale, 10 MSD = 1 cm = 10 mm

$\Rightarrow$ 1 MSD = 1 mm

Also, 9 MSD = 10 VSD

L.C. = 1 MSD – 1 VSD = (I – 0.9) mm = 0.1 mm

Hence, the correct answer is (C).

31. According to Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\Rightarrow \quad \varepsilon_0 = \frac{1}{4\pi}\frac{q_1q_2}{Fr^2}$$

$$\Rightarrow \quad [\varepsilon_0] = \frac{[AT][AT]}{[MLT^{-2}][L]^2} = [M^{-1}L^{-3}T^4A^2]$$

Hence, the correct answer is (C).

32. $R = \frac{V}{I}$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta l}{I}$$

The percentage error in R is

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta l}{I} \times 100 = 3\% + 3\% = 6\%$$

Hence, the correct answer is (D).

33. (i) All the non-zero digits are significant

(ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

(iii) If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.

(iv) The power of 10 is irrelevant to the determination of significant figures.

According to the above rules

23.023 has 5 significant figures

0.0003 has 1 significant figures

2.1×10^{-3} has 2 significant figures

Hence, the correct answer is (B).

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. $m = \left(\frac{4\pi R^3}{3}\right)\rho$

Taking log both sides, we get

$$\ln(m) = \ln\left(\frac{4\pi}{3}\right) + \ln(\rho) + 3\ln(R)$$

Differentiating with respect to time, we get

$$0 = 0 + \frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt}$$

$$\Rightarrow \quad \left(\frac{dR}{dt}\right) = -R \times \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$$

$$\Rightarrow \quad \frac{dR}{dt} = v$$

$$\Rightarrow \quad v = -R \propto \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$$

$$\Rightarrow \quad v \propto R$$

Hence, the correct answer is (A).

2. Distance covered by the stone

$$L = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times t_1^2$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{L}{5}}$$

Time taken by sound to reach the top is

$$t_2 = \frac{L}{v} = \frac{L}{300}$$

So total time taken is

$$T = \sqrt{\frac{L}{5}} + \frac{L}{300}$$

Time interval between dropping a stone and receiving the sound of impact is

$$dT = \frac{1}{\sqrt{5}}\frac{1}{2}L^{-1/2}dL + \left(\frac{1}{300}dL\right)$$

$$\Rightarrow \quad dT = \frac{1}{2\sqrt{5}}\frac{1}{\sqrt{20}}dL + \frac{dL}{300} = 0.01$$

$$\Rightarrow \quad dL\left(\frac{1}{20} + \frac{1}{300}\right) = 0.01$$

$$\Rightarrow \quad dL\left(\frac{15}{300}\right) = 0.01$$

$$\Rightarrow \quad dL = \frac{3}{16}$$

$$\Rightarrow \quad \frac{dL}{L} \times 100 = \frac{3}{16} \times \frac{1}{20} \times 100 = \frac{15}{16} \simeq 1\%$$

Hence, the correct answer is (A).

3. **For C_1:** 10 VSD = 9 MSD = 9 mm

$$\Rightarrow \quad 1 \text{ VSD} = 0.9 \text{ mm}$$

Since least count for vernier calliper 1 is

$$(\text{LC})_1 = 1 \text{ MSD} - 1 \text{ VSD}$$

$$\Rightarrow \quad (\text{LC})_1 = 0.1 \text{ mm} = 0.01 \text{ cm}$$

$$\Rightarrow \quad \begin{pmatrix}\text{Reading}\\ \text{of } C_1\end{pmatrix} = \begin{pmatrix}\text{Main}\\ \text{Scale}\\ \text{Reading}\end{pmatrix} + \begin{pmatrix}\text{Coinciding}\\ \text{Divisions}\end{pmatrix}\begin{pmatrix}\text{Least}\\ \text{Count}\end{pmatrix}$$

$$\Rightarrow \quad (\text{Reading of } C_1) = 2.8 \text{ cm} + (7)(0.01) \text{ cm} = 2.87 \text{ cm}$$

For C_2: 10 VSD = 11 MSD = 11 mm

$$\Rightarrow \quad 1 \text{ VSD} = 1.1 \text{ mm}$$

Least count for vernier calliper 2 is

$$(\text{LC})_2 = 1 \text{ MSD} - 1 \text{ VSD} = -0.1 \text{ mm} = -0.01 \text{ cm}$$

$$\Rightarrow \quad (\text{Reading of } C_2) = (2.8 + 0.1 - 0.07) \text{ cm} = 2.83 \text{ cm}$$

Hence, the correct answer is (C).

4. 1 MSD = 5.15 cm – 5.10 cm = 0.05 cm

$$1 \text{ VSD} = \frac{2.45 \text{ cm}}{50} = 0.049 \text{ cm}$$

$$\Rightarrow \quad \text{LC} = 1 \text{ MSD} - 1 \text{ VSD} = 0.01 \text{ cm}$$

Since

$$\begin{pmatrix}\text{Diameter}\\ \text{of}\\ \text{Cylinder}\end{pmatrix} = \begin{pmatrix}\text{Main}\\ \text{Scale}\\ \text{Reading}\end{pmatrix} + \begin{pmatrix}\text{Vernier}\\ \text{Scale}\\ \text{Reading}\end{pmatrix}\begin{pmatrix}\text{Least}\\ \text{Count}\end{pmatrix}$$

$$\Rightarrow \quad D = 5.10 + (24)(0.001) = 5.124 \text{ cm}$$

Hence, the correct answer is (B).

5. $\Delta d = \Delta l = \dfrac{0.5}{100}$ mm = 0.005 mm

Since, $Y = \dfrac{4MLg}{\pi l d^2}$

$$\Rightarrow \quad \left(\frac{\Delta Y}{Y}\right)_{\max} = \left(\frac{\Delta l}{l}\right) + 2\left(\frac{\Delta d}{d}\right)$$

$$\Rightarrow \quad \left(\frac{\Delta l}{l}\right) = \frac{0.5/100}{0.25} = 0.02 \text{ and } \frac{2\Delta d}{d} = \frac{(2)(0.5/100)}{0.5} = 0.02$$

$$\Rightarrow \quad \frac{\Delta l}{l} = 2\frac{\Delta d}{d}$$

Hence, the correct answer is (C).

6. Least count of screw gauge $= \dfrac{0.5}{50} = 0.01 \text{ mm} = \Delta r$

Diameter, $r = 2.5 \text{ mm} + 20 \times \dfrac{0.5}{50} = 2.70 \text{ mm}$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{0.01}{2.70}$$

$$\Rightarrow \quad \frac{\Delta r}{r} \times 100 = \frac{1}{2.7}$$

Now, density, $d = \dfrac{m}{V} = \dfrac{m}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

where r is the diameter of sphere

$$\Rightarrow \quad \frac{\Delta d}{d} \times 100 = \left\{\frac{\Delta m}{m} + 3\left(\frac{\Delta r}{r}\right)\right\} \times 100$$

$$\Rightarrow \quad \frac{\Delta d}{d} = \frac{\Delta m}{m} \times 100 + 3 \times \left(\frac{\Delta r}{r}\right) \times 100$$

$$\Rightarrow \quad \frac{\Delta d}{d} = 2\% + 3 \times \frac{1}{2.7} = 3.11\%$$

Hence, the correct answer is (C).

7. Least count of vernier callipers

$$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$\Rightarrow \quad \text{LC} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

CHAPTER 2

20 divisions of vernier scale $= 16$ divisions of main scale

$\Rightarrow \quad 1\ VSD = \frac{16}{20}\ \text{mm} = 0.8\ \text{mm}$

$\Rightarrow \quad LC = 1\ MSD - 1\ VSD = 1\ \text{mm} - 0.8\ \text{mm} = 0.2\ \text{mm}$

Hence, the correct answer is (D).

8. $T = 2\pi\sqrt{\frac{l}{g}}$, where $T = \frac{t}{n}$

$\Rightarrow \quad \frac{t}{n} = 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow \quad g = \frac{(4\pi^2)(n^2)l}{t^2}$

% error in g is

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} + \frac{2\Delta t}{t}\right) \times 100\%$$

$\Rightarrow \quad E_{\text{I}} = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{128}\right) \times 100 = 0.3125\%$

$\Rightarrow \quad E_{\text{II}} = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{64}\right) \times 100 = 0.46875\%$

$\Rightarrow \quad E_{\text{III}} = \left(\frac{0.1}{20} + \frac{2 \times 0.1}{36}\right) \times 100 = 1.055\%$

Hence E_1 is minimum

Hence, the correct answer is (B).

9. $Y = \frac{FL}{Al} = \frac{4FL}{\pi d^2 l} = \frac{(4)(1 \times 9.8)(2)}{\pi(0.4 \times 10^{-3})^2(0.8 \times 10^{-3})}$

$\Rightarrow \quad Y = 2 \times 10^{11}\ \text{Nm}^{-2}$

Further $\frac{\Delta Y}{Y} = 2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)$

$\Rightarrow \quad \Delta Y = \left\{2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)\right\} Y$

$\Rightarrow \quad \Delta Y = \left\{2 \times \frac{0.01}{0.4} + \frac{0.05}{0.8}\right\} \times 2 \times 10^{11}$

$\Rightarrow \quad \Delta Y = 0.225 \times 10^{11}\ \text{Nm}^{-2}$

$\Rightarrow \quad \Delta Y = 0.2 \times 10^{11}\ \text{Nm}^{-2}$ {by rounding off}

$\Rightarrow \quad (Y + \Delta Y) = (2 + 0.2) \times 10^{11}\ \text{Nm}^{-2}$

Hence, the correct answer is (B).

10. Length of air column in resonance is odd integer multiple of $\frac{\lambda}{4}$.

Hence, the correct answer is (A).

11. Least count $LC = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

$\Rightarrow \quad LC = \frac{0.5}{50} = 0.01\ \text{mm}$

Now, diameter of ball is

$$D = (2 \times 0.5\ \text{mm}) + (25 - 5)(0.01) = 1.2\ \text{mm}$$

Hence, the correct answer is (A).

12. $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{n}$

In answer (D) error in Δg is minimum and number of observations made are maximum. Hence, in this case error in g will be minimum.

Hence, the correct answer is (D).

13. Dipole moment = (charge) × (distance)

Electric flux = (electric field) × (area)

Hence, the correct answer is (D).

14. Density, $\rho = \frac{M}{V} = \frac{M}{\pi r^2 L}$

$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$

$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{0.003}{0.3} + 2 \times \frac{0.005}{0.5} + \frac{0.06}{6}$

$\Rightarrow \quad \frac{\Delta\rho}{\rho} = 0.01 + 0.02 + 0.01 = 0.04$

$\Rightarrow \quad \%RE = \frac{\Delta\rho}{\rho} \times 100 = 0.04 \times 100 = 4\%$

Hence, the correct answer is (D).

15. $\left[\frac{\alpha Z}{k\theta}\right] = \left[M^0 L^0 T^0\right]$

$[\alpha] = \left[\frac{k\theta}{Z}\right]$

Further $[P] = \left[\frac{\alpha}{\beta}\right]$

$\Rightarrow \quad [\beta] = \left[\frac{\alpha}{P}\right] = \left[\frac{k\theta}{ZP}\right]$

Dimensions of $k\theta$ are that to energy. Hence,

$$[\beta]=\left[\frac{ML^2T^{-2}}{LML^{-1}T^{-2}}\right]=\left[M^0L^2T^0\right]$$

Hence, the correct answer is (A).

16. $V=l^3=\left(1.2\times10^{-2}\text{ m}\right)^3=1.728\times10^{-6}\text{ m}^3$

Since the length (l) has two significant figures, the volume (V) will also have two significant figures.

Hence, the correct answer is (A).

17. $X=\varepsilon_0 L\left(\frac{\Delta V}{\Delta t}\right)$

$$X=\left(\frac{C}{L}\right)(L)\left(\frac{\Delta V}{\Delta t}\right) \qquad \left\{\because \varepsilon_0=\frac{\text{Capacitance}}{\text{Length}}\right\}$$

$$\Rightarrow \quad X=C\left(\frac{\Delta V}{\Delta t}\right)$$

$$\Rightarrow \quad X=\frac{C\Delta V}{\Delta t}=\frac{\Delta Q}{\Delta t}=\text{Current}$$

So, X happens to be the current.

Hence, the correct answer is (D).

18. $\left[\frac{1}{2}\varepsilon_0 E^2\right]=\left[\frac{B^2}{2\mu_0}\right]=[\text{Pressure}]=ML^{-1}T^{-2}$

Hence, the correct answer is (B).

19. $[Y]=\left[\frac{X}{Z^2}\right]=\left[\frac{\text{Capacitance}}{(\text{Magnetic Induction})^2}\right]$

$$\Rightarrow \quad [Y]=\frac{M^{-1}L^{-2}Q^2T^2}{M^2Q^{-2}T^{-2}}=M^{-3}L^{-2}Q^4T^4$$

Hence, the correct answer is (B).

20. Substituting the dimensions of mass $[M]$, length $[L]$ and coefficient of rigidity $[ML^{-1}T^{-2}]$ we get $T=2\pi\sqrt{\frac{M}{\eta L}}$ is the right formula for time period of oscillations.

Hence, the correct answer is (D).

21. RC has dimensions of time and V has the dimensions of $L\frac{dI}{dt}$

Hence, $\left[\frac{L}{RCV}\right]=\left[\frac{1}{T}\times\frac{T}{A}\right]=\frac{1}{\text{current}}$

Hence, the correct answer is (B).

22. $\left[\frac{EJ^2}{M^5G^2}\right]=\frac{[ML^2T^{-2}]\times[ML^2T^{-1}]^2}{[M^5]\times[M^{-1}L^3T^{-2}]^2}=[M^0L^0T^0]$

Hence, the correct answer is (B).

Multiple Correct Choice Type Problems

1. Angular momentum $J=mvr$

$$\Rightarrow \quad [J]=\frac{ML^2}{T}$$

Since m and J are dimensionless, so

$$T=L^2$$

Since, $p=mv=\frac{ML}{T}$

Since M is dimensionless, so

$$[p]=\left[\frac{ML}{T}\right]=\frac{L}{L^2}=L^{-1}$$

Similarly, $(\text{Energy})=\frac{ML^2}{T^2}=\frac{L^2}{L^2L^2}=L^{-2}$

$$(\text{Power})=\frac{ML^2}{T^2T}=\frac{L^2}{L^2L^2L^2}=L^{-4}$$

$$(\text{Force})=\frac{ML}{T^2}=\frac{L}{L^2L^2}=L^{-3}$$

Hence, (A), (C) and (D) are correct.

2. $[k_BT]=[\text{Energy}]=[FL]$

$[\varepsilon]=\left[\frac{Q^2}{FL^2}\right]$ and $[n]=L^{-3}$

where Q is charge, F is force, L is length

Hence, (B) and (D) are correct.

3. Error in T

$$T_{\text{mean}}=\frac{0.52+0.56+0.57+0.54+0.59}{5}=0.556\approx0.56\text{ s}$$

CHAPTER 2

and $\Delta T_{mean} = \dfrac{0.04+0.00+0.01+0.02+0.03}{5} = 0.02$ s

Percentage error in T is given by

$$\frac{\Delta T}{T}\times 100\% = \frac{0.02}{0.56}\times 100 = 3.57\%$$

Percentage Error in r is given by

$$\frac{\Delta r}{r}\times 100\% = \frac{1}{10}\times 100 = 10\%$$

Since, $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$

$$\Rightarrow \quad T^2 = 4\pi^2\frac{7}{5}\left(\frac{R-r}{g}\right)$$

$$\Rightarrow \quad g = \frac{28\pi^2}{5}\left(\frac{R-r}{T^2}\right)$$

So, error in measurement of g is

$$\frac{\Delta g}{g} = \left(\frac{\Delta R+\Delta r}{R-r}\right)+2\frac{\Delta T}{T} = \frac{2}{50}+2\times 0.0357$$

$$\Rightarrow \quad \frac{\Delta g}{g}\times 100 \approx 11\%$$

Hence, (A), (B) and (D) are correct.

4. $M \propto h^a c^b G^c$

$$M^1 \propto \left(ML^2T^{-1}\right)^a\left(LT^{-1}\right)^b\left(M^{-1}L^3T^{-2}\right)^c$$

$$\propto M^{a-c}L^{2a+b+3c}T^{-a-b-2c}$$

$a-c=1$ …(1)

$2a+b+3c=0$ …(2)

$a+b+2c=0$ …(3)

On solving (1), (2), (3)

$$a=\frac{1}{2},\ b=+\frac{1}{2},\ c=-\frac{1}{2}$$

$$\Rightarrow \quad M \propto \sqrt{c}$$

In the same way we can find that,

$$L \propto h^{1/2}c^{-3/2}G^{1/2}$$

$$L \propto \sqrt{h},\ L \propto \sqrt{G}$$

Hence, (A), (C) and (D) are correct.

5. (A) Energy of inductor

$$\frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}I^2$$

Energy of capacitor is

$$\frac{1}{2}CV^2 = \frac{1}{2}\varepsilon_0\left(\frac{A}{d}\right)V^2$$

So, $\mu_0\dfrac{A}{l}I^2$ and $\varepsilon_0\dfrac{A}{d}V^2$ have same dimensions

$$\Rightarrow \quad \mu_0 I^2 \text{ and } \varepsilon_0 V^2 \text{ have same dimension}$$

(C) $I = [\varepsilon_0 CV] = \left[\left(\dfrac{C}{l}\right)\left(\dfrac{l}{t}\right)V\right] = \left[\dfrac{CV}{t}\right] = \left[\dfrac{q}{t}\right] = I$

$$\left\{\because\ C = 4\pi\varepsilon_0 r,\ CV = q,\ c = \frac{l}{t}\right\}$$

Hence, (A) and (C) are correct.

6. For Vernier Callipers

1 MSD $= \dfrac{1}{8}$ cm

5 VSD $= 4$ MSD

$$\Rightarrow \quad 1 \text{ VSD} = \frac{4}{5}\text{ MSD} = \frac{4}{5}\times\frac{1}{8} = \frac{1}{10}\text{ cm}$$

Least count of vernier calliper $= 1$ MSD $- 1$ VSD

$$LC = \frac{1}{8}\text{ cm} - \frac{1}{10}\text{ cm} = 0.025\text{ cm}$$

(a) and (b)

Pitch of screw gauge $= 2\times 0.025 = 0.05$ cm

Least count of screw gauge $= \dfrac{0.05}{100}$ cm $= 0.005$ mm

(c) and (d)

Least count of linear scale of screw gauge $= 0.05$

Pitch $= 0.05\times 2 = 0.1$ cm

Least count of screw gauge $= \dfrac{0.1}{100}$ cm $= 0.01$ mm

Hence, (B) and (C) are correct.

7. $T = \dfrac{40\text{ s}}{20} = 2$ s

Further, $t = nT = 20T$

$$\Rightarrow \quad \Delta t = 20\Delta T$$

$$\Rightarrow \quad \frac{\Delta t}{t} = \frac{\Delta T}{T}$$

$$\Rightarrow \quad \Delta T = \frac{T}{t}\Delta t = \left(\frac{2}{40}\right)(1) = 0.05\text{ s}$$

Further, $T = 2\pi\sqrt{\dfrac{l}{g}}$

$$\Rightarrow \quad T \propto g^{-1/2}$$

$$\Rightarrow \quad \frac{\Delta T}{T}\times 100 = -\frac{1}{2}\times\frac{\Delta g}{g}\times 100$$

% error in determination of g is

$$\frac{\Delta g}{g}\times 100 = -200\times\frac{\Delta T}{T}$$

$$\frac{\Delta g}{g} = -\frac{200\times 0.05}{2} = -5\%$$

Hence, (A) and (C) are correct.

9. (A) $L = \frac{\phi}{i}$ or henry $= \frac{\text{weber}}{\text{ampere}}$

(B) $e = -L\left(\frac{di}{dt}\right)$

$$\Rightarrow \quad L = -\frac{e}{(di/dt)} \text{ or henry} = \frac{\text{volt-second}}{\text{ampere}}$$

(C) $U = \frac{1}{2}Li^2$

$$\Rightarrow \quad L = \frac{2U}{i^2} \text{ or henry} = \frac{\text{joule}}{(\text{ampere})^2}$$

(D) $U = \frac{1}{2}Li^2 = i^2Rt$

$$\Rightarrow \quad L = Rt \text{ or henry} = \text{ohm-second}$$

Hence, (A), (B), (C) and (D) are correct.

10. [Renold's Number] = [Coefficient of friction] = $M^0L^0T^0$

[Curie] = [Frequency] = $M^0L^0T^{-1}$

[Latent Heat] = [Gravitational Potential] = $M^0L^2T^{-2}$

Hence, (A), (B) and (C) are correct.

Linked Comprehension Type Problems

1. In terms of dimension, $F_e = F_m$

$$\Rightarrow \quad qE = qvB$$

$$\Rightarrow \quad E = vB$$

$$[E] = [B][LT^{-1}]$$

Hence, the correct answer is (C).

2. $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\Rightarrow \quad c^2 = \frac{1}{\mu_0\varepsilon_0}$$

$$\Rightarrow \quad \mu_0 = \varepsilon_0^{-1}c^{-2}$$

$$\Rightarrow \quad [\mu_0] = [\varepsilon_0]^{-1}[L^{-2}T^2]$$

Hence, the correct answer is (D).

3. $r = \frac{1-a}{1+a}$

$$\ln r = \ln(1-a) - \ln(1+a)$$

Differentiating, we get

$$\frac{dr}{r} = -\frac{da}{1-a} - \frac{da}{1+a}$$

or, we can write

$$\frac{\Delta r}{r} = -\left[\frac{\Delta a}{1-a} + \frac{\Delta a}{1+a}\right]$$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{-2\Delta a}{1-a^2}$$

$$\Rightarrow \quad \Delta r = -\left(\frac{2\Delta a}{1-a^2}\right)(r) = \frac{-2\Delta a}{(1+a)^2}$$

Hence, the correct answer is (B).

4. $N = N_0e^{-\lambda t}$

$$\ln N = \ln N_0 - \lambda t$$

Differentiating w.r.t. λ, we get

$$\frac{1}{N}\frac{dN}{d\lambda} = 0 - t$$

$$\Rightarrow \quad |d\lambda| = \frac{dN}{Nt} = \frac{40}{2000\times 1} = 0.02$$

Hence, the correct answer is (C).

5. N = Number of electrons per unit volume

$$\Rightarrow \quad [N] = [L^{-3}],\ [e] = [q] = [It] = [AT]$$

$$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$$

Substituting the dimensions, we can see that

$$\left[\sqrt{\frac{Ne^2}{m\varepsilon_0}}\right] = [T^{-1}]$$

Angular frequency has also the dimension $[T^{-1}]$

Hence, the correct answer is (C).

6. $\omega = 2\pi f = \frac{2\pi c}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{2\pi c}{\omega} = \frac{2\pi c}{\sqrt{\frac{Ne^2}{m\varepsilon_0}}}$$

Substituting the values, we get $\lambda \simeq 600$ nm

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

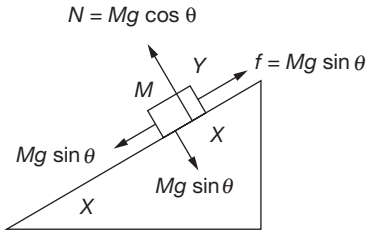
1. A → (p, q, s, t)
B → (q)
C → (s)
D → (s)

For (t)

A time varying magnetic field will produce a non-conservative electric field. Due to this electric field, a current starts flowing in the resistive loop and heat is produced.

2. A → (p, t)
B → (q, s, t)
C → (p, r, t)
D → (q)

For (p)



Net force on Y due to

$$X = \sqrt{(Mg\cos\theta)^2 + (Mg\sin\theta)^2} = Mg$$

(B) As the inclined is fixed. So, gravitational P.E. of X is constant.

(C) As K.E. is constant and P.E. of Y is decreasing. So mechanical energy of $(X+Y)$ is decreasing.

For (q)

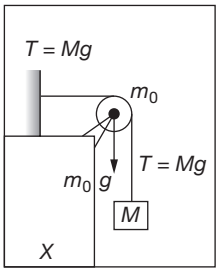
(A) Force on Y due to X will be greater than Mg which is equal to $(Mg + \text{repulsion force})$

(B) As the system is moving up, P.E. of X is increasing

(C) Mechanical energy of $(X+Y)$ is increasing

(D) Torque of the weight of Y about point $P = 0$

For (r)



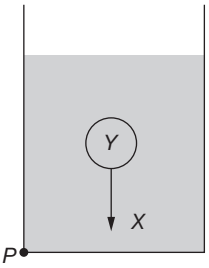
(A) Force on Y due to $X = \sqrt{\left[(M+m_0)g\right]^2 + (Mg)^2}$

(B) As the system moves down, gravitational P.E. of X decreases

(C) As the system moves down, total mechanical energy of $(X+Y)$ also decreases

(D) $\tau_P \neq 0$

For (s)



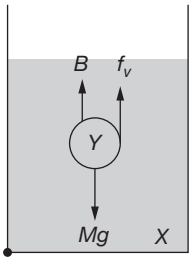
(A) Force on Y due to X = Buoyancy force which is less than mg

(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X increases.

(C) As there is no non-conservative force, so total mechanical energy of $X+Y$ remains conserved.

(D) $\tau_P \neq 0$

For (t)



(A) As the sphere is moving with constant velocity

$$B + f_v = Mg$$

so force on Y due to X is $B + f_v = Mg$

(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X will increase

(C) Increase in mechanical energy is the work done by friction force which is negative.

$$\Rightarrow \quad \Delta U = W_{fr} = -ve$$

(D) $\tau_P = 0$

3. A → (p, q)
B → (r, s)
C → (r, s)
D → (r, s)

$$\left[\frac{GM_eM_s}{r}\right] = \text{joule}$$

$$\Rightarrow \quad [GM_eM_s] = (\text{joule})(\text{metre})$$

Since joule $= (\text{coulomb})(\text{volt})$

$$\Rightarrow \quad [GM_eM_s] = (\text{volt})(\text{coulomb})(\text{metre})$$

Also, $[GM_eM_s] = ML^3T^{-2}$

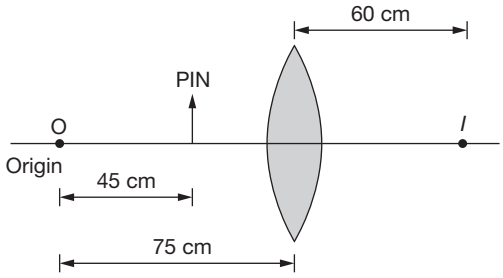
$$\Rightarrow \quad [GM_eM_s] = (\text{kilogram})(\text{metre})^3(\text{second})^{-2}$$

Now, $\left[\frac{3RT}{M}\right] = \left[\frac{F^2}{q^2B^2}\right] = \left[\frac{GM_e}{R_e}\right] = v^2$

Also $v^2 = \left[\frac{\text{Energy}}{\text{Mass}}\right] = \left[\frac{CV^2}{\text{Mass}}\right] = (\text{farad})(\text{volt})^2(\text{kg})^{-1}$

Integer/Numerical Answer Type Questions

1. Since there are four equal divisions in each cm, so least count is $\frac{1}{4}$ cm



Object distance i.e., distance of pin from lens is

$$u = -(75-45) = -30 \text{ cm}$$

$$\Rightarrow \quad \Delta u = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ cm}$$

Similarly, image distance is

$$v = +(135-75) = +60 \text{ cm}$$

$$\Delta v = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ cm}$$

Using Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f}$$

$$\Rightarrow \quad f = 20 \text{ cm}$$

Applying Error Analysis on the expression

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$$

$$\Rightarrow \quad \frac{\Delta f}{f} \times 100 = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) f \times 100\%$$

$$\Rightarrow \quad \%\text{age Error in } f \text{ is}$$

$$\frac{\Delta f}{f} \times 100 = \left[\frac{0.5}{(60)^2} + \frac{0.5}{(30)^2}\right] \times 20 \times 100$$

$$\Rightarrow \quad \frac{\Delta f}{f} \times 100 = 1.39\%$$

2. Given, $d = 0.5$ mm

$$Y = 2 \times 10^{11} \text{ Nm}^{-2}$$

$$\Rightarrow \quad l = 1 \text{ m}$$

$$\Rightarrow \quad \Delta l = \frac{Fl}{AY} = \frac{mgl}{\frac{\pi d^2}{4} Y}$$

$$\Rightarrow \quad \Delta l = \frac{1.2 \times 10 \times 1}{\frac{\pi}{4} \times (5 \times 10^{-4})^2 \times 2 \times 10^{11}} = 0.3 \text{ mm}$$

LC of vernier $= \left(1 - \frac{9}{10}\right)$ mm $= 0.1$ mm

So, 3rd division of vernier scale will coincide with main scale.

3. $E(t) = A^2 e^{-\alpha t}$ …(1)

$\alpha = 0.2 \text{ s}^{-1}$

$$\left(\frac{dA}{A}\right) \times 100 = 1.25\%$$

$$\left(\frac{dt}{t}\right) \times 100 = 1.50\%$$

$$\Rightarrow \quad (dt \times 100) = 1.5t$$

$$\Rightarrow \quad (dt \times 100) = 1.5 \times 5 = 7.5$$

$$\Rightarrow \quad \left(\frac{dE}{E}\right) \times 100 = \pm 2\left(\frac{dA}{A}\right) \times 100 \pm \alpha (dt \times 100)$$

Taking log on both sides of equation (1), we get

$$\log E = 2 \log A - \alpha t$$

$$\Rightarrow \quad \frac{dE}{E} = \pm 2 \frac{dA}{A} \pm \alpha dt$$

CHAPTER 2

$$\Rightarrow \quad \left(\frac{dE}{E}\right)\times 100 = \pm 2\left(\frac{dA}{A}\right)\times 100 \pm \alpha(dt\times 100)$$

$$\Rightarrow \quad \left(\frac{dE}{E}\right)\times 100 = \pm 2(1.25) \pm 0.2(7.5)$$

Since, during propagation, errors are always added up, so

$$\Rightarrow \quad \left(\frac{dE}{E}\right)\times 100 = \pm 2.5 \pm 1.5 = \pm 4\%$$

4. Let $d = k(\rho)^a (S)^b (f)^c$

where, k is a dimensionless constant. Then,

$$[L] = \left[\frac{M}{L^3}\right]^a \left[\frac{ML^2T^{-2}}{L^2T}\right]^b \left[\frac{1}{T}\right]^c$$

Equating the powers of M and L, we get

$$0 = a + b \qquad \ldots(1)$$

$$1 = -3a \qquad \ldots(2)$$

Solving these two equations, we get

$$b = \frac{1}{3}$$

$$\Rightarrow \quad n = 3$$

5. $Y = \dfrac{F/A}{\Delta l/l}$, where $\Delta l = 25\times 10^{-50}$ m

$$\Rightarrow \quad Y = \frac{F}{A}\left(\frac{l}{\Delta l}\right)$$

$$\Rightarrow \quad \frac{\Delta Y}{Y}\times 100 = \frac{10^{-5}}{25\times 10^{-5}}\times 100 = 4\%$$

CHAPTER 3: VECTORS

Test Your Concepts-I (Based on Addition, Subtraction and Resolution)

1. No, Yes

2. (a) $A_x = 4$ and $A_y = 6$ …(1)

$$\vec{A} + \vec{B} = \left(A_x\hat{i} + A_y\hat{j}\right) + \left(B_x\hat{i} + B_y\hat{j}\right)$$

$$\Rightarrow \quad \vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

So, $A_x + B_x = 10$ and $A_y + B_y = 9$ …(2)

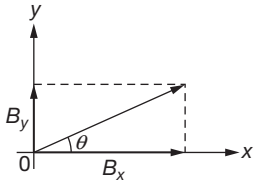
From equation sets (1) and (2), we get

$B_x = 6$ and $B_y = 3$

So, $\vec{B} = 6\hat{i} + 3\hat{j}$

(b) $|\vec{B}| = \sqrt{6^2 + 3^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$ m

(c) Let $\vec{B}$ make an angle θ with x-axis, then from the diagram



$$\tan\theta = \frac{B_y}{B_x} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right), \text{ with } x\text{-axis}$$

3. $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 6\hat{i} + 8\hat{j}$

$|\vec{A}| = \sqrt{4^2 + (-3)^2} = 5$

$|\vec{B}| = \sqrt{6^2 + 8^2} = 10$

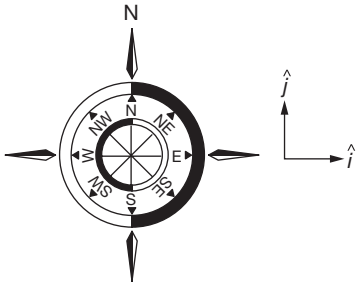
$\vec{A} + \vec{B} = 10\hat{i} + 5\hat{j}$

$\vec{A} - \vec{B} = -2\hat{i} - 11\hat{j}$

$\vec{B} - \vec{A} = 2\hat{i} + 11\hat{j}$

	$\vec{A}$	$\vec{B}$	$\vec{A}+\vec{B}$	$\vec{A}-\vec{B}$	$\vec{B}-\vec{A}$
Magnitude	5	10	$5\sqrt{5}$	$5\sqrt{5}$	$5\sqrt{5}$
Direction (θ with x-axis)	$\tan^{-1}\left(\frac{3}{4}\right)$ Below x-axis	$\tan^{-1}\left(\frac{4}{3}\right)$ Above x-axis	$\tan^{-1}\left(\frac{1}{2}\right)$ Above x-axis	$\tan^{-1}\left(\frac{11}{2}\right)$ Below negative x-axis	$\tan^{-1}\left(\frac{11}{2}\right)$ Above x-axis

4. $\Delta\vec{r}_1 = 12\hat{i}$, $\Delta\vec{r}_2 = 5\hat{j}$ and $\Delta\vec{r}_3 = 6\hat{k}$



Please note that vertically upwards does not mean north, it actually means "vertically upwards in space towards you", which happens to be the +z direction.

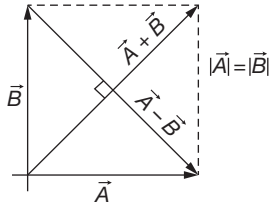
If $\Delta\vec{r}$ is the net displacement, then

$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$$

$$\Rightarrow \quad \Delta\vec{r} = 12\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\Rightarrow \quad \Delta\vec{r} = \sqrt{144 + 25 + 36} = \sqrt{205} \text{ m}$$

5. $|\vec{A}| = |\vec{B}|$



Since $|\vec{A} + \vec{B}|$ and $|\vec{A} - \vec{B}|$ are the two diagonals of a parallelogram whose adjacent sides are equal in magnitude. So, this parallelogram can either be a square or a rhombus and in both the cases, the diagonals are perpendicular to each other.

6. Already discussed in theory.

7. $\vec{F}_1 = 3\sqrt{2}\ NE = 3\sqrt{2}\left[\cos(45°)\hat{i} + \sin(45°)\hat{j}\right]$

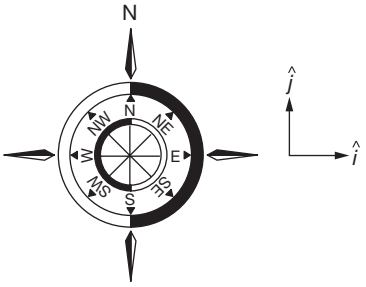
$$\Rightarrow \quad \vec{F}_1 = 3\hat{i} + 3\hat{j} \qquad \ldots(1)$$

$$\vec{F}_2 = 6\sqrt{2}\,SE = 6\sqrt{2}\left[\cos(45°)\hat{i} - \sin(45°)\hat{j}\right]$$

$$\Rightarrow \quad \vec{F}_2 = 6\hat{i} - 6\hat{j} \qquad \ldots(2)$$

$$\vec{F}_3 = \sqrt{2}\ NW = \sqrt{2}\left[-\cos(45°)\hat{i} + \sin(45°)\hat{j}\right]$$

$$\Rightarrow \quad \vec{F}_3 = -\hat{i} + \hat{j}$$



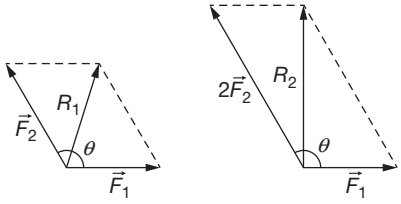
So, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 8\hat{i} - 2\hat{j}$

$\Rightarrow \quad |\vec{F}| = \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}$ kgf

8. $|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$ just implies that $\vec{a}$ and $\vec{b}$ are parallel.

9. Let θ be the angle between $\vec{F}_1$ and $\vec{F}_2$, then

$$R_1^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad \ldots(1)$$



When $\vec{F}_2$ is doubled i.e., made $2\vec{F}_2$, then too the angle between $\vec{F}_1$ and $2\vec{F}_2$ is still θ. But now the new resultant makes an angle of $90°$ with $\vec{F}_1$. So, using $\tan\beta = \dfrac{B\sin\theta}{A + B\cos\theta}$, we get

$$\tan\left(\frac{\pi}{2}\right) = \frac{2F_2\sin\theta}{F_1 + 2F_2\cos\theta}$$

$$\Rightarrow \quad \frac{2F_2\sin\theta}{F_1 + 2F_2\cos\theta} \to \infty \qquad \left\{\because \tan\left(\frac{\pi}{2}\right) \to \infty\right\}$$

$$\Rightarrow \quad F_1 + 2F_2\cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = -\frac{F_1}{2F_2} \qquad \ldots(2)$$

Negative sign gives an indication that θ is obtuse.
Substituting (2) in(1), we get

$$R_1^2 = F_1^2 + F_2^2 + 2F_1F_2\left(-\frac{F_1}{2F_2}\right)$$

$$\Rightarrow \quad R_1^2 = \cancel{F_1^2} + F_2^2 - \cancel{F_1^2}$$

$$\Rightarrow \quad R_1 = F_2$$

10. Change in velocity $= \Delta\vec{v} = \vec{v}_B - \vec{v}_A$

Since $|\vec{v}_B - \vec{v}_A| = \sqrt{v_B^2 + v_A^2 - 2v_Av_B\cos\theta}$

$$\Rightarrow \quad |\vec{v}_B - \vec{v}_A| = \sqrt{v^2 + v^2 - 2v^2\cos(40°)}$$

$$\Rightarrow \quad |\vec{v}_B - \vec{v}_A| = \sqrt{2}v\sqrt{1-\cos(40°)}$$

Since $1 - \cos(40°) = 2\sin^2(20°)$

$$\Rightarrow \quad |\vec{v}_B - \vec{v}_A| = \sqrt{2}v\sqrt{2\sin^2(20°)}$$

$$\Rightarrow \quad |\vec{v}_B - \vec{v}_A| = 2v\sin(20°)$$

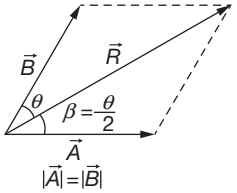
11. Given $|\vec{A}| = |\vec{B}|$

Let the resultant make an angle β with $\vec{A}$ and let the angle between $\vec{A}$ and $\vec{B}$ be θ. Then

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \quad \tan\beta = \frac{A\sin\theta}{A + A\cos\theta} \qquad \{\because A = B\}$$

$$\Rightarrow \quad \tan\beta = \frac{\sin\theta}{1+\cos\theta}$$



Since $\sin\theta = 2\sin\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{\theta}{2}\right)$

and $2\cos^2\left(\dfrac{\theta}{2}\right) = 1 + \cos\theta$

$$\Rightarrow \quad \tan\beta = \frac{\cancel{2}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\cancel{2}\cos^2\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow \quad \tan\beta = \tan\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \beta = \frac{\theta}{2}$$

Hence the resultant is equally inclined to $\vec{A}$ as well as $\vec{B}$.

12. (a) Since $|\vec{A}| = |\vec{B}| = 1$

$$\Rightarrow \quad |\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow \quad |\vec{A}+\vec{B}| = \sqrt{1+1+2\cos\theta} = \sqrt{2}\sqrt{1+\cos\theta}$$

$$\Rightarrow \quad |\vec{A}+\vec{B}| = 2\cos\left(\frac{\theta}{2}\right) \left\{\because 1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)\right\}$$

(b) Similarly,

$$|\vec{A}-\vec{B}| = \sqrt{1+1-2\cos\theta} = \sqrt{2}\sqrt{1-\cos\theta}$$

$$\Rightarrow \quad |\vec{A}-\vec{B}| = 2\sin\left(\frac{\theta}{2}\right) \quad \left\{\because\ 1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)\right\}$$

13. Let the two vectors be $\vec{A}$ and $\vec{B}$ inclined at an angle θ, then

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

According to the question, we have been given

$$R = A = B$$

$$\Rightarrow \quad A^2 = A^2 + A^2 + 2A^2\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

So, this is possible, when two vectors of equal magnitude are inclined to each other at an angle of $120°$.

14. Let the vector make an angle α with x-axis, β with y-axis and γ with z-axis. Then

$$\cos\alpha = \frac{x}{r} = \frac{1}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}$$

$$\cos\beta = \frac{y}{r} = \frac{1}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}$$

and $\cos\gamma = \frac{z}{r} = \frac{1}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}$

So, we simply conclude that

$$\alpha = \beta = \gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Hence the vector $\vec{r} = \hat{i}+\hat{j}+\hat{k}$ is equally inclined to all the three axis.

Test Your Concepts-II (Based on Dot Product)

1. Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, then

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

2. $\vec{v} = v_x\hat{i} + v_y\hat{j} = \hat{i} + \sqrt{3}\hat{j}$, $|\vec{v}| = 2$

$\vec{v}' = v_x'\hat{i} + v_y'\hat{j} = 2\hat{i} + 2\hat{j}$, $|\vec{v}| = 2\sqrt{2}$

Let θ be the angle between $\vec{v}$ and $\vec{v}'$, then

$$\cos\theta = \frac{\vec{v}\cdot\vec{v}'}{|\vec{v}||\vec{v}'|}$$

$$\Rightarrow \quad \cos\theta = \frac{2+2\sqrt{3}}{2(2\sqrt{2})}$$

$$\Rightarrow \quad \cos\theta = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} \qquad \ldots(1)$$

Since $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\Rightarrow \quad \cos(15°) = \cos(45°-30°) = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} \qquad \ldots(2)$$

So, from (1) and (2), we get

$\theta = 15°$

3. (a) Since $\vec{A} = 4\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{B} = -2\hat{i} - 5\hat{j} + 7\hat{k}$, so

$$|\vec{A}| = \sqrt{16+36+9} = \sqrt{61} \text{ and}$$

$$|\vec{B}| = \sqrt{4+25+49} = \sqrt{78}.$$

So, direction cosines for $\vec{A}$ are

$$(l_1, m_1, n_1) \equiv \left(\frac{4}{\sqrt{61}}, \frac{6}{\sqrt{61}}, -\frac{3}{\sqrt{61}}\right)$$

and that for $\vec{B}$ are

$$(l_2, m_2, n_2) \equiv \left(-\frac{2}{\sqrt{78}}, -\frac{5}{\sqrt{78}}, \frac{7}{\sqrt{78}}\right).$$

(b) Let θ, be the angle between $\vec{A}$ and $\vec{B}$, then

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

$$\Rightarrow \quad \cos\theta = \frac{-8-30-21}{\sqrt{61}\sqrt{78}}$$

$$\Rightarrow \quad \cos\theta = \frac{59}{\sqrt{61}\sqrt{78}} = \frac{59}{69}$$

$$\Rightarrow \quad \theta \cong 31.3°$$

4. Firstly we observe that $\vec{A} = \vec{B} + \vec{C}$, which means $\vec{A}$, $\vec{B}$ and $\vec{C}$ are representing the three sides of a triangle. Secondly, we observe that

$$\vec{A}\cdot\vec{C} = (3)(2) + (-2)(1) + (1)(-4) = 0$$

$$\Rightarrow \quad \vec{A} \perp \vec{C}$$

Hence the three vectors form a right angled triangle.

5. Projection of $\vec{A}$ along $\vec{r}$ is $A\cos\theta$. Since $\vec{A}\cdot\vec{r} = Ar\cos\theta$

$$\Rightarrow \quad A\cos\theta = \frac{\vec{A}\cdot\vec{r}}{|\vec{r}|} = \frac{50+48-54}{\sqrt{25+36+81}} = \frac{44}{\sqrt{142}}$$

6. $\vec{A} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} - 9\hat{j} + 6\hat{k}$

$$|\vec{A}| = \sqrt{36+4+9} = 7$$

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and $\vec{B} = \sqrt{4+81+36} = 11$

Now $\vec{A}\cdot\vec{B} = 12 - 18 + 18 = 12$

If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{12}{(7)(11)} = \frac{12}{77}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{12}{77}\right)$$

7. $\vec{u}$ = constant

$$\Rightarrow \quad u^2 = \text{constant}$$

$$\Rightarrow \quad \vec{u}\cdot\vec{u} = \text{constant} \qquad \ldots(1)$$

Taking derivative of both sides of (1), we get

$$\frac{d}{dt}(\vec{u}\cdot\vec{u}) = \vec{u}\cdot\frac{d\vec{u}}{dt} + \vec{u}\cdot\frac{d\vec{u}}{dt} = \frac{d}{dt}(\text{Constant})$$

$$\Rightarrow \quad 2\vec{u}\cdot\frac{d\vec{u}}{dt} = 0$$

$$\Rightarrow \quad \vec{u}\cdot\frac{d\vec{u}}{dt} = 0$$

Also, we can say that for a vector of constant magnitude $\vec{u} \perp \frac{d\vec{u}}{dt}$.

8. Let us reconstruct the question, for our convenience. According to the statement,

A force $F_1 = 5$ N acts along $\vec{A} = 6\hat{i} + 2\hat{j} + 3\hat{k}$

So,

$$\vec{F}_1 = 5(\hat{A}) = 5\left(\frac{6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{6^2+2^2+3^2}}\right) = \frac{5}{7}(6\hat{i}+2\hat{j}+3\hat{k})$$

A force $F_2 = 3$ N acts along $\vec{B} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

So, $\vec{F}_2 = 3(\hat{B}) = 3\left(\frac{3\hat{i}-2\hat{j}+6\hat{k}}{\sqrt{9+4+36}}\right) = \frac{3}{7}(3\hat{i}-2\hat{j}+6\hat{k})$

A force $F_3 = 1$ N acts along $\vec{C} = 2\hat{i} - 3\hat{j} - 6\hat{k}$

So, $\vec{F}_3 = 1(\hat{C}) = 1\left(\frac{2\hat{i}-3\hat{j}-6\hat{k}}{\sqrt{4+9+36}}\right) = \frac{1}{7}(2\hat{i}-3\hat{j}-6\hat{k})$

Now, the resultant of these three forces is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\Rightarrow \quad \vec{F} = \frac{1}{7}(30\hat{i}+10\hat{j}+15\hat{k}+9\hat{i}-6\hat{j}+18\hat{k}+2\hat{i}-3\hat{j}-6\hat{k})$$

$$\Rightarrow \quad \vec{F} = \frac{1}{7}(41\hat{i}+\hat{j}+27\hat{k}) \qquad \ldots(1)$$

Also $\Delta\vec{r} = \vec{r}_{\text{final}} - \vec{r}_{\text{initial}} = 3\hat{i} + 0\hat{j} + 2\hat{k} = 3\hat{i} + 2\hat{k}$

So,

$$W = \vec{F}\cdot\Delta\vec{r} = \frac{1}{7}[(41)(3)+0+(27)(2)] = \frac{177}{7}\text{ J}.$$

Test Your Concepts-III (Based on Cross Product, Scalar and Vector Triple Product)

1. (a) $\vec{\tau} = \vec{r}\times\vec{F}$

where $\vec{r} = (7\hat{i}-2\hat{j}+5\hat{k}) - (0\hat{i}+0\hat{j}+0\hat{k})$

$$\Rightarrow \quad \vec{r} = 7\hat{i} - 2\hat{j} + 5\hat{k}$$

So, $\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -2 & 5 \\ 3 & 5 & -2 \end{vmatrix}$

$$\Rightarrow \quad \vec{\tau} = -21\hat{i} + 29\hat{j} + 41\hat{k}$$

(b) In this case, we have

$$\vec{r} = (7\hat{i}-2\hat{j}+5\hat{k}) - (0\hat{i}-\hat{j}+0\hat{k})$$

$$\Rightarrow \quad \vec{r} = 7\hat{i} - \hat{j} + 5\hat{k}$$

$$\Rightarrow \quad \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -1 & 5 \\ 3 & 5 & -2 \end{vmatrix}$$

$$\Rightarrow \quad \vec{\tau} = -23\hat{i} + 29j + 38\hat{k}$$

2. $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$\Rightarrow \quad \vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

3. $|\vec{a}\times\vec{b}| = ab\sin\theta$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|^2 = a^2b^2\sin^2\theta$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|^2 = a^2b^2(1-\cos^2\theta)$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}| = \sqrt{a^2b^2 - a^2b^2\cos\theta}$$

Since $ab\cos\theta = \vec{a}\cdot\vec{b}$

$$\Rightarrow \quad |\vec{a}\times\vec{b}| = \sqrt{a^2b^2 - (\vec{a}\cdot\vec{b})^2}$$

4. $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \vec{a}\times\vec{a} - \vec{a}\times\vec{b} + \vec{b}\times\vec{a} - \vec{b}\times\vec{b}$

Since $\vec{a}\times\vec{a} = \vec{b}\times\vec{b} = \vec{0}$ and $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$, so

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = -\vec{a}\times\vec{b} - \vec{a}\times\vec{b}$$

$$\Rightarrow \quad (\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = -2(\vec{a}\times\vec{b}) = 2(\vec{b}\times\vec{a})$$

Since area of a parallelogram with diagonals

$\vec{d}_1 = \hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{d}_2 = 2\hat{i} - 3\hat{j} + 2\hat{k}$ is

$$\text{Area} = \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$$

$$\text{Now } \vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 2 & -3 & 2 \end{vmatrix}$$

$$\Rightarrow \quad \vec{d}_1 \times \vec{d}_2 = -13\hat{i} - 8\hat{j} + \hat{k}$$

$$\Rightarrow \quad |\vec{d}_1 \times \vec{d}_2| = \sqrt{(-13)^2 + (-8)^2 + (1)^2} = \sqrt{234}$$

So, Area $= \frac{1}{2}\sqrt{234}$ sq units

5. Area, with direction as the outward normal.

6. $\vec{A}\times\vec{B} = \vec{0}$ and $\vec{A}\cdot\vec{B} = 0$

$$\Rightarrow \quad AB\sin\theta = 0 \text{ and } AB\cos\theta = 0$$

Since $\sin\theta$ and $\cos\theta$ can never be zero simultaneously, so we have either $A = 0$ or $B = 0$. Hence for both the conditions to be met simultaneously either of the two vectors has to be a null vector.

7. Since $\vec{L} = \vec{r}\times\vec{p} = m(\vec{r}\times\vec{v})$ $\{\because \vec{p} = m\vec{v}\}$

$$\Rightarrow \quad \vec{L} = m\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{0.7v_0^2}{g} & \frac{0.2v_0^2}{g} & 0 \\ 0.7v_0 & -0.3v_0 & 0 \end{vmatrix}$$

$$\Rightarrow \quad \vec{L} = m\left[\hat{i}(0-0) - \hat{j}(0-0) + \hat{k}((0.7)(-0.3) - (0.7)(0.2))\right]$$

$$\Rightarrow \quad \vec{L} = -\frac{mv_0^3}{g}(0.35)\hat{k}$$

$$\Rightarrow \quad \vec{L} = -\left(\frac{7mv_0^3}{20g}\right)\hat{k}$$

$$\Rightarrow \quad |\vec{L}| = \frac{7\,mv_0^2}{20\,g}$$

8. Since, $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

For the particle to pass undeviated, $\vec{F} = \vec{0}$

$$\Rightarrow \quad \vec{E} = -\vec{v}\times\vec{B}$$

$$\Rightarrow \quad \vec{E} = \vec{B}\times\vec{v}$$

Let θ be the angle between $\vec{B}$ and $\vec{v}$, then

$$E = Bv\sin\theta \qquad \{\text{in magnitude}\}$$

$$\Rightarrow \quad v = \frac{E}{B\sin\theta}$$

For velocity to be minimum, we must have

$$\sin\theta = \text{MAXIMUM} = 1$$

$$\Rightarrow \quad \theta = 90°$$

Hence angle between B and v must be 90°, i.e., the $\vec{v}$ must be directed either along $+x$ axis or along $-x$ axis. However, for $\vec{E} = \vec{B}\times\vec{v}$ to be valid and $\vec{E}$ to be along $+y$ axis, we must have $\vec{v}$ along $+x$ axis, because $\hat{k}\times\hat{i} = \hat{j}$. So, $v_{\min} = \frac{E}{B}$.

9. Moment of force is actually the torque, so

$$\vec{\tau} = \vec{r}\times\vec{F}$$

where $\vec{r} = \left(-2\hat{i} + 3\hat{j} + 4\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$\Rightarrow \quad \vec{r} = -3\hat{i} + \hat{j} + \hat{k}$$

$$\text{So, } \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \quad \vec{\tau} = 4\hat{j} - 4\hat{k}$$

$$\Rightarrow \quad \vec{\tau} = 4\left(\hat{j} - \hat{k}\right)$$

10. $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$ …(1)

$\vec{b}\times(\vec{c}\times\vec{a}) = (\vec{b}\cdot\vec{a})\vec{c} - (\vec{b}\cdot\vec{c})\vec{a}$ …(2)

$\vec{c}\times(\vec{a}\times\vec{b}) = (\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}$ …(3)

On adding (1), (2) and (3), we get

$$\vec{a}\times(\vec{b}\times\vec{c}) + \vec{b}\times(\vec{c}\times\vec{a}) + \vec{c}\times(\vec{a}\times\vec{b}) = \vec{0}$$

Single Correct Choice Type Questions

1. $\vec{r} = \frac{1}{2}\vec{a}t^2$

$$\Rightarrow \quad \vec{r} = \frac{1}{2}(2\hat{i} + 8\hat{j} - 6\hat{k})(25)$$

$$\Rightarrow \quad \vec{r} = (\hat{i} + 4\hat{j} - 3\hat{k})25$$

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$\Rightarrow \quad \vec{r} = 25\hat{i} + 100\hat{j} - 75\hat{k}$

Hence, the correct answer is (A).

2. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

$\Rightarrow \quad 3\hat{i} + 2\hat{j} - \hat{k} + 3\hat{i} + 4\hat{j} - 5\hat{k} + A\hat{i} + A\hat{j} - A\hat{k} = \vec{0}$

$\Rightarrow \quad \hat{i}(6 + A) + \hat{j}(6 + A) + \hat{k}(-6 - A) = \vec{0}$

$\Rightarrow \quad (\hat{i} + \hat{j} - \hat{k})(6 + A) = \vec{0}$

$\Rightarrow \quad 6 + A = 0$

$\Rightarrow \quad A = -6$

Hence, the correct answer is (A).

3. $|\vec{A} \times \vec{B}| = 0$

$\Rightarrow \quad AB\sin\theta = 0$

$\Rightarrow \quad \theta = 0°$

Further

$\vec{A} \cdot (\vec{A} + \vec{B}) = \vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B}$

$\Rightarrow \quad \vec{A} \cdot (\vec{A} + \vec{B}) = A^2 + AB\cos 0$

$\Rightarrow \quad \vec{A} \cdot (\vec{A} + \vec{B}) = A^2 + AB$

Hence, the correct answer is (D).

4. $R = A = B$

Since

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad A^2 = A^2 + A^2 + 2A^2\cos\theta$

$\Rightarrow \quad \cos\theta = -\frac{1}{2}$

$\Rightarrow \quad \theta = 120°$

$\Rightarrow \quad \theta = \frac{2\pi}{3}$

Hence, the correct answer is (B).

5. For $\vec{a} + \vec{b}$ parallel to $\vec{a} - \vec{b}$

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \vec{0}$

$\Rightarrow \quad \vec{a} \times \vec{a} - \vec{b} \times \vec{b} + 2(\vec{b} \times \vec{a}) = \vec{0}$

$\Rightarrow \quad \vec{b} \times \vec{a} = \vec{0}$

$\Rightarrow \quad \vec{a} \parallel \vec{b}$

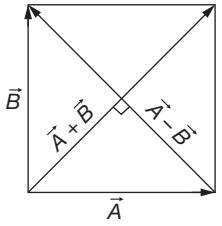
Hence, the correct answer is (D).

6. For $\vec{A} \parallel \vec{B}$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = +k = 3\,(k > 0)$$

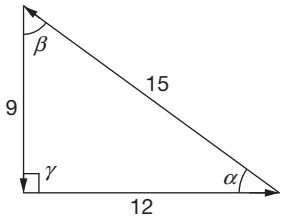
Hence, the correct answer is (A).

7. $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ always lie in the plane of $\vec{A}$ and $\vec{B}$. Further $\vec{A} \times \vec{B}$ is a vector normal to plane containing $\vec{A}$ and $\vec{B}$ i.e. $\vec{A} \times \vec{B}$ is perpendicular to $\vec{A}$ as well as $\vec{B}$. Hence $\vec{A} \times \vec{B}$ is perpendicular to $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$. For $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ to be mutually perpendicular. They must be the diagonals of a rhombus with sides $\vec{A}$ and $\vec{B}$ (diagonals of a rhombus are perpendicular) and then the sides must have equal magnitude. So, $(\vec{A} + \vec{B}) \perp (\vec{A} - \vec{B})$ when $|\vec{A}| = |\vec{B}|$.



Hence, the correct answer is (C).

8. From Lami's Theorem



$$\frac{\sin\alpha}{9} = \frac{\sin\beta}{12} = \frac{\sin\gamma}{15}$$

$\Rightarrow \quad \frac{\sin\alpha}{9} = \frac{\sin\beta}{12} = \frac{1}{15}$

$\Rightarrow \quad \sin\beta = \frac{4}{5}$

$\Rightarrow \quad \cos\beta = \frac{3}{5}$

$\Rightarrow \quad \theta = \pi - \cos^{-1}\left(\frac{3}{5}\right)$

Hence, the correct answer is (C).

9. $(\vec{a} \cdot \vec{b})^2 = a^2b^2$

$\Rightarrow \quad a^2b^2\cos^2\theta = a^2b^2$

$\Rightarrow \quad \cos\theta = 1$

$\Rightarrow \quad \theta = 0°$

$\Rightarrow \quad \vec{a} \parallel \vec{b}$

Hence, the correct answer is (A).

10. $a:b:c::2000:1732:1000$

$\Rightarrow \quad a:b:c::1:\dfrac{1.732}{2}:\dfrac{1}{2}$

$\Rightarrow \quad a:b:c::1:\dfrac{\sqrt{3}}{2}:\dfrac{1}{2}$

so according to law of sines the angles of the triangle in degrees are $90°$, $60°$ and $30°$.

Hence, the correct answer is (C).

11. $\vec{F}=\left(-3\hat{i}+\hat{j}+5\hat{k}\right)\text{N}$

$\Delta\vec{r}=7\hat{i}+3\hat{j}+\hat{k}-0\hat{i}-0\hat{j}-0\hat{k}$

$\Rightarrow \quad \Delta\vec{r}=\left(7\hat{i}+3\hat{j}+\hat{k}\right)$

$\vec{\tau}=\vec{r}\times\vec{F}$

$\Rightarrow \quad \vec{\tau}=\left(7\hat{i}+3\hat{j}+\hat{k}\right)\times\left(-3\hat{i}+\hat{j}+5\hat{k}\right)$

$$\Rightarrow \quad \vec{\tau}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 3 & 1\\ -3 & 1 & 5\end{vmatrix}$$

$\Rightarrow \quad \vec{\tau}=\hat{i}(15-1)-\hat{j}(35+3)+\hat{k}(7+9)$

$\Rightarrow \quad \vec{\tau}=14\hat{i}-38\hat{j}+16\hat{k}$

Hence, the correct answer is (A).

12. $\vec{C}=\vec{A}-\vec{B}$

$|\vec{C}|=A+B$

$\Rightarrow \quad (A+B)^2=A^2+B^2-2AB\cos\theta$

$\Rightarrow \quad A^2+B^2+2AB=A^2+B^2-2AB\cos\theta$

$\Rightarrow \quad \cos\theta=-1$

$\Rightarrow \quad \theta=180°$

$\Rightarrow \quad \theta=\pi$

Hence, the correct answer is (C).

13. A vector perpendicular to two vectors $\vec{A}$ and $\vec{B}$ is $\vec{A}\times\vec{B}$. If $\hat{n}$ is the unit vector. Then $\hat{n}=\pm\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

So, a vector perpendicular to $\vec{A}$ and $\vec{B}$ of magnitude R is $R\hat{n}$.

$$\Rightarrow \quad \vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 2 & -2 & 3\end{vmatrix}$$

$\Rightarrow \quad \vec{A}\times\vec{B}=8\hat{i}-4\hat{j}-8\hat{k}$

$\Rightarrow \quad \hat{n}=\pm\dfrac{1}{12}\left(8\hat{i}-4\hat{j}-8\hat{k}\right)$

$\Rightarrow \quad \vec{R}=R\hat{n}=\pm\dfrac{3}{12}\left(8\hat{i}-4\hat{j}-8\hat{k}\right)$

$\Rightarrow \quad \vec{R}=\pm\left(2\hat{i}-\hat{j}-2\hat{k}\right)$

Hence, the correct answer is (A).

14. $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \quad \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow \quad \vec{c}$ is in the plane of $\hat{i}$ and $\hat{j}$.

Hence, the correct answer is (A).

15. The forces must form a regular polygon of 12 sides. So angle between any two adjacent forces is $\dfrac{360}{12}=30°$

Hence, the correct answer is (B).

16. $|\vec{F}_1\times\vec{F}_2|=\vec{F}_1\cdot\vec{F}_2$

$\Rightarrow \quad F_1F\sin\theta=F_1F_2\cos\theta$

$\Rightarrow \quad \tan\theta=1$

$\Rightarrow \quad \theta=45°$

$|\vec{F}_1+\vec{F}_2|=\sqrt{F_1^2+F_2^2+2F_1F\cos(45°)}$

$\Rightarrow \quad |\vec{F}_1+\vec{F}_2|=\sqrt{F_1^2+F_2^2+\sqrt{2}F_1F_2}$

Hence, the correct answer is (D).

17. For coplanarity $\text{STP}=0$

$$\begin{vmatrix}2 & -1 & 1\\ 1 & 2 & -3\\ 3 & p & 5\end{vmatrix}=0$$

$2(10+3p)+1(5+9)+1(p-6)=0$

$20+6p+5+9+p-6=0$

$7p=28$

$\Rightarrow \quad p=\dfrac{28}{7}=4$

Hence, the correct answer is (C).

18. $\vec{R}=\overrightarrow{BA}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA}$

$\Rightarrow \quad \vec{R}=\overrightarrow{BA}+\overrightarrow{BA}$ $\quad\left\{\because \overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA}=\overrightarrow{BA}\right\}$

$\Rightarrow \quad \vec{R}=2\overrightarrow{BA}$

Hence, the correct answer is (D).

CHAPTER 3

19. No acceleration implies no net force on the body and hence resultant must be zero vector.

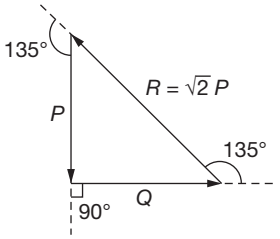
Hence, the correct answer is (A).

20. $A - B \le |\vec{A} + \vec{B}| \le A + B$

Hence, the correct answer is (C).

21. Since $\vec{P} + \vec{Q} + \vec{R} = \vec{0}$

$\Rightarrow$ $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are in same order forming a closed polygon/triangle (see figure)



We are not asked about the angles of triangle, but we are to find the angle between vectors. While finding angle between two vectors we must make sure that either their heads are coinciding or their tails are coinciding.

Hence, the correct answer is (A).

22. We know $\hat{A} \cdot \hat{A} = 1$, where $\hat{A}$ is any unit vector.

$\Rightarrow$ $\frac{d}{dt}(\hat{A} \cdot \hat{A}) = 0$

$\Rightarrow$ $\hat{A} \cdot \frac{d\hat{A}}{dt} + \hat{A} \cdot \frac{d\hat{A}}{dt} = 0$

$\Rightarrow$ $2\left(\hat{A} \cdot \frac{d\hat{A}}{dt}\right) = 0$

$\Rightarrow$ $\hat{A} \cdot \frac{d\hat{A}}{dt} = 0$

Hence, the correct answer is (A).

23. $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ …(1)

$\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ …(2)

Subtracting (2) from (1), we get

$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$

$\Rightarrow$ $\vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d}$

$\Rightarrow$ $\vec{a} \times (\vec{b} - \vec{c}) = -(\vec{b} - \vec{c}) \times \vec{d}$

$\Rightarrow$ $\vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = \vec{0}$

$\Rightarrow$ $\vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = \vec{0}$

$\Rightarrow$ $(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0}$

So, $(\vec{a} - \vec{d})$ and $(\vec{b} - \vec{c})$ are parallel

Hence, the correct answer is (B).

24. $(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = \vec{A} \cdot \vec{A} - \vec{B} \cdot \vec{B} = A^2 - B^2$

$\Rightarrow$ $(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$

$\Rightarrow$ $(\vec{A} + \vec{B}) \perp (\vec{A} - \vec{B})$

Hence, the correct answer is (C).

25. $W = \vec{F} \cdot \Delta\vec{r}$

$\Rightarrow$ $W = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1)$

$\Rightarrow$ $W = (-5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 0\hat{k})$

$\Rightarrow$ $W = -15 - 15$

$\Rightarrow$ $W = -30$ unit

Hence, the correct answer is (C).

26. For perpendicular vectors $\vec{a}$ and $\vec{b}$

$\vec{a} \cdot \vec{b} = 0$

$\Rightarrow$ $(2\hat{i} + 4\hat{j} - \hat{k}) \cdot (3\hat{i} - 2\hat{j} + x\hat{k}) = 0$

$\Rightarrow$ $6 - 8 - x = 0$

$\Rightarrow$ $-x = 2$

$\Rightarrow$ $x = -2$

Hence, the correct answer is (B).

27. $\vec{r}_f = 4\hat{i} + 4\hat{j} + 12\hat{k}$

$\vec{r}_i = \hat{i}$

$\Rightarrow$ $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$\Rightarrow$ $\Delta\vec{r} = 3\hat{i} + 4\hat{j} + 12\hat{k}$

$\Rightarrow$ $|\Delta\vec{r}| = \sqrt{3^2 + 4^2 + 12^2}$

$\Rightarrow$ $|\Delta\vec{r}| = 13$

Since $|\vec{v}| = \left|\frac{\Delta\vec{r}}{\Delta t}\right|$

$\Rightarrow$ $65 = \frac{|\Delta\vec{r}|}{\Delta t}$

$\Rightarrow$ $\Delta t = \frac{13}{65} = \frac{1}{5}$ s

Velocity vector is $\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$

$$\Rightarrow \quad \vec{v} = \frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{\left(\frac{1}{5}\right)}$$

$$\Rightarrow \quad \vec{v} = 15\hat{i} + 20\hat{j} + 60\hat{k}$$

Since particle has uniform velocity, so

$$\vec{r} = \vec{r}_0 + \vec{v}t$$

$$\Rightarrow \quad \vec{r} = \vec{i} + \left(15\hat{i} + 20\hat{j} + 60\hat{k}\right)2$$

$$\Rightarrow \quad \vec{r} = 31\hat{i} + 40\hat{j} + 120\hat{k}$$

Hence, the correct answer is (D).

28. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$

$$\Rightarrow \quad a^2 - b^2 = 0$$

$$\Rightarrow \quad |\vec{a}| = |\vec{b}|$$

Hence, the correct answer is (C).

29. Let $AC = x$ and the length of string between B and m_2 be y. Since A and B are fixed, so AO and BO are constants.

Let $AO = BO = l$

Since length of string is a constant

$$\Rightarrow \quad AC + CB + y = \text{constant}$$

$$\Rightarrow \quad 2\sqrt{l^2 + x^2} + y = \text{constant}$$

$$\Rightarrow \quad 2\frac{1}{2}(l^2 + x^2)^{\frac{1}{2}-1}\frac{d}{dt}(l^2 + x^2) + \frac{dy}{dt} = 0$$

$$\Rightarrow \quad \frac{2x}{\sqrt{l^2 + x^2}}\left(\frac{dx}{dt}\right) + \frac{dy}{dt} = 0$$

$$\Rightarrow \quad \frac{2x}{\sqrt{l^2 + x^2}}v_1 + v_2 = 0$$

Since $\cos\theta = \dfrac{x}{\sqrt{l^2 + x^2}}$

$$\Rightarrow \quad 2\cos\theta v_1 + v_2 = 0$$

$$\Rightarrow \quad \cos\theta = -\frac{v_2}{2v_1}$$

Negative sign indicates that v_1 and v_2 are directed opposite to each other.

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{v_2}{2v_1}\right)$$

Hence, the correct answer is (B).

30. Let $\vec{P} = \hat{i} - \hat{j} - 2\hat{k}$

$$\overrightarrow{PS} = 0\hat{i} + \hat{j} + 3\hat{k} - \left(\hat{i} - \hat{j} - 2\hat{k}\right)$$

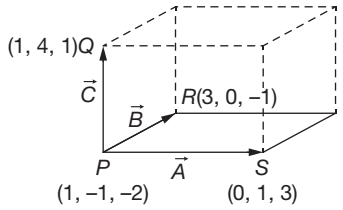
$$\Rightarrow \quad \overrightarrow{PS} = \vec{A} = -\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\overrightarrow{PR} = \left(3\hat{i} + 0\hat{j} - \hat{k}\right) - \left(\hat{i} - \hat{j} - 2\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{PR} = \vec{B} = 2\hat{i} + \hat{j} + \hat{k}$$

$$\overrightarrow{PQ} = \left(\hat{i} + 4\hat{j} + \hat{k}\right) - \left(\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{PQ} = \vec{C} = 5\hat{j} + 3\hat{k}$$



Volume of parallelepiped $= (\vec{A} \times \vec{B}) \cdot \vec{C}$

$$\Rightarrow \quad \text{Volume} = \begin{vmatrix} -1 & 2 & 5 \\ 2 & 1 & 1 \\ 0 & 5 & 3 \end{vmatrix}$$

$$\Rightarrow \quad \text{Volume} = (-1)(3-5) - 2(6-0) + 5(10-0)$$

$$\Rightarrow \quad \text{Volume} = 2 - 12 + 50$$

$$\Rightarrow \quad \text{Volume} = 40 \text{ unit}$$

Hence, the correct answer is (C).

31. $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2$

$$= A^2B^2\sin^2\theta + A^2B^2\cos^2\theta$$

$$= A^2B^2\left(\sin^2\theta + \cos^2\theta\right)$$

$$= A^2B^2$$

Hence, the correct answer is (D).

33. Displacement $(\vec{s}) = \left(-2\hat{i} + 3\hat{j} + 4\hat{k}\right) - \left(\hat{i} + 2.5\hat{k}\right)$

$$\Rightarrow \quad \vec{s} = \left(-3\hat{i} + 3\hat{j} + 1.5\hat{k}\right)\text{m}$$

$$\vec{F} = \left(\hat{i} + 4\hat{k}\right)N$$

Work done $= \vec{F} \cdot \vec{s}$

$$\Rightarrow \quad W = \left(-3\hat{i} + 3\hat{j} + 1.5\hat{k}\right) \cdot \left(\hat{i} + 4\hat{k}\right)$$

$$\Rightarrow \quad W = -3 + 0 + 6 = 3 \text{ J}$$

Hence, the correct answer is (C).

34. Let the required vector be $\vec{r}$.

$\Rightarrow \quad \vec{r} = \hat{i} - \hat{i} + 5\hat{j} - 2\hat{k} - 3\hat{i} - 6\hat{j} + 7\hat{k}$

$\Rightarrow \quad \vec{r} = -\hat{j} - 3\hat{i} + 5\hat{k}$

$\Rightarrow \quad \vec{r} = -3\hat{i} - \hat{j} + 5\hat{k}$

Hence, the correct answer is (C).

35. Direction of resultant will be given by $\tan\theta$, where θ is the angle which resultant makes with x-axis.

$\Rightarrow \quad \tan\theta = \frac{y}{x} = \frac{1}{2}$

$\Rightarrow \quad 2y - x = 0$

Hence, the correct answer is (B).

36. Moment of Force $= \vec{\tau} = \vec{r} \times \vec{F}$

$$\Rightarrow \quad \vec{\tau} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) \times \left(4\hat{i} + 5\hat{j}\right) = \begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ 1 & -2 & 3 \\ 4 & 5 & 0 \end{vmatrix}$$

$\Rightarrow \quad \vec{\tau} = \hat{i}(0 - 15) - \hat{j}(0 - 12) + \hat{k}(5 + 8)$

$\Rightarrow \quad \vec{\tau} = -15\hat{i} + 12\hat{j} + 13\hat{k}$

Hence, the correct answer is (A).

37. Area of $\Delta = \frac{1}{2}\left|(\vec{b} \times \vec{c})\right| = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|\vec{c} \times \vec{a}|$

$\Rightarrow \quad 3\Delta = \frac{1}{2}|\vec{b} \times \vec{c}| + \frac{1}{2}|\vec{a} \times \vec{b}| + \frac{1}{2}|\vec{c} \times \vec{a}|$

$\Rightarrow \quad 3\Delta = \frac{1}{2}|\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}|$

$\Rightarrow \quad \Delta = \frac{1}{6}\left|(\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}) + (\vec{c} \times \vec{a})\right|$

Hence, the correct answer is (C).

38. $\vec{a} \cdot \vec{b} = ab\cos\theta$

$\Rightarrow \quad 6 - 3 - 2 = \sqrt{(4 + 9 + 1)(9 + 1 + 4)}\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1}{14}$

Hence, the correct answer is (D).

39. $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$|\Delta\vec{v}| = \sqrt{v^2 + v^2 - 2v^2\cos(40°)}$

$\Rightarrow \quad |\Delta\vec{v}| = v\sqrt{2 - 2\cos(40°)}$

$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{2}v\sqrt{1 - \cos(40°)}$

$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{2}v\sqrt{2\sin^2(20°)} \quad \left\{\because 1 - \cos(2\theta) = 2\sin^2\theta\right\}$

$\Rightarrow \quad |\Delta\vec{v}| = 2v\sin(20°)$

Hence, the correct answer is (D).

40. The zero resultant can only be produced by three vectors when they form a closed triangle whose sides are in the same order. For a triangle to be formed the sum of any two sides must be greater than the third side.

Hence, the correct answer is (D).

41. $\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$

Hence, the correct answer is (B).

44. $\vec{A} \times \vec{B} = \vec{0}$

$\Rightarrow \quad \vec{A} \parallel \vec{B}$

$\vec{B} \times \vec{C} = \vec{0}$

$\Rightarrow \quad \vec{B} \parallel \vec{C}$

$\Rightarrow \quad \vec{A} \times \vec{C} = \vec{0}$

Hence, the correct answer is (C).

46. Scalar Triple product gives the volume of parallelopiped.

Hence, the correct answer is (B).

48. Impulse, $\vec{I} = \vec{F}t$

Hence, the correct answer is (B).

49. Electrostatic potential (i.e., the work done per unit charge) is a scalar quantity.

Hence, the correct answer is (B).

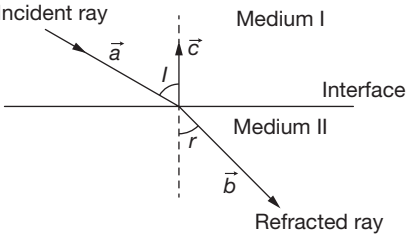
50. Kinetic Energy Gained = Work Done $= \vec{F} \cdot \Delta\vec{r}$

$\Rightarrow \quad K.E. = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) \cdot \left(10\hat{j}\right) = 50 \text{ J}$

Hence, the correct answer is (A).

51. According to the Law of Refraction i.e., Snell's Law, we have

$1\sin i = x\sin r$



where i and r are the angle of incidence and the angle of refraction respectively.

$$\sin i = \frac{\vec{a}\times\vec{c}}{|\vec{a}||\vec{c}|} = \vec{a}\times\vec{c} \qquad \{\because |\vec{a}| = |\vec{c}| = 1\}$$

$$\text{Also, } \sin r = \frac{\vec{b}\times\vec{c}}{|\vec{b}||\vec{c}|} = \vec{b}\times\vec{c} \qquad \{\because |\vec{b}| = |\vec{c}| = 1\}$$

Since both $\vec{a}\times\vec{c}$ and $\vec{b}\times\vec{c}$ have the same direction, so we have $\vec{a}\times\vec{c} = x(\vec{b}\times\vec{c})$

So, (C) is correct and not (B).

Hence, the correct answer is (C).

52. $A + B = 17$

$A - B = 7$

So, $A = 12$ and $B = 5$

Since $\vec{A} \perp \vec{B}$, so we have

$$R = \sqrt{A^2 + B^2} = 13$$

Hence, the correct answer is (D).

53. $|\vec{A}\times\vec{B}| = AB\sin\theta = (5\sqrt{3})(10)\sin(30°)$

$\Rightarrow \quad |\vec{A}\times\vec{B}| = 25\sqrt{3}$

Hence, the correct answer is (C).

54. Since, $\vec{v} = \vec{\omega}\times\vec{r}$

$$\Rightarrow \quad \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$$\Rightarrow \quad \vec{v} = \hat{i}(-24+6) - \hat{j}(18-5) + \hat{k}(-18+20)$$

$$\Rightarrow \quad \vec{v} = -18\hat{i} - 13\hat{j} + 2\hat{k}$$

Hence, the correct answer is (D).

55. All others are scalars.

Hence, the correct answer is (D).

56. $\vec{v} = \frac{d\vec{r}}{dt} = a\omega\left(-\sin(\omega t)\hat{i} + \cos(\omega t)\hat{j}\right)$

$$\vec{v}\cdot\vec{r} = a^2\omega(-\sin(\omega t)\cos(\omega t)) + \cos(\omega t)\sin(\omega t)$$

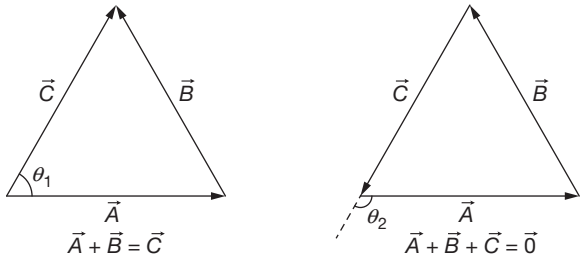
$\Rightarrow \quad \vec{v}\cdot\vec{r} = 0$

$\Rightarrow \quad \vec{v} \perp \vec{r}$

$\{\because$ dot product of $\perp$ vectors is always zero$\}$

Hence, the correct answer is (B).

57. Since $|\vec{A}| = |\vec{B}| = |\vec{C}|$, so $\theta_1 = \frac{\theta_2}{2}$



Hence, the correct answer is (B).

58. $P_x = 7$ and $P_y = 6$

$$P_x + Q_x = 11 \text{ and } P_y + Q_y = 9$$

$\Rightarrow \quad Q_x = 4$ and $Q_y = 3$

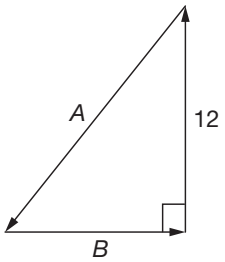
So, $|\vec{Q}| = \sqrt{Q_x^2 + Q_y^2} = 5$

Hence, the correct answer is (A).

60. Let the two vectors be $\vec{A}$ and $\vec{B}$, then

$$A + B = 18 \qquad \ldots(1)$$

Also, given that $R = 12$, and is perpendicular to one of these vectors. So, we can draw the situation as, where $\vec{B} \perp \vec{R}$.



From Pythagora's theorem, we get

$$A^2 - B^2 = 144$$

$\Rightarrow \quad (A+B)(A-B) = 144$

$\Rightarrow \quad A - B = 8 \qquad \ldots(2)$

So, $A = 13$, $B = 5$ OR $A = 5$, $B = 13$

Hence, the correct answer is (A).

61. Angular Momentum is always directed along the axis of rotation.

Hence, the correct answer is (C).

CHAPTER 3

62. $\vec{p} = 2\left[(\cos t)\hat{i} + (\sin t)\hat{j}\right]$

Since $\vec{F} = \dfrac{d\vec{p}}{dt}$

$\Rightarrow \quad \vec{F} = 2\left[(-\sin t)\hat{i} + (\cos t)\hat{j}\right]$

Since, $\vec{F} \cdot \vec{p} = 0$

So, $\vec{F} \perp \vec{p}$

Hence angle between $\vec{F}$ and $\vec{p}$ is 90°.

Hence, the correct answer is (B).

63. $P^2 = (30)^2 + (60)^2 + 2(30)(60)\cos(60°)$

$\Rightarrow \quad P = \sqrt{900 + 3600 + 1800}$

$\Rightarrow \quad P = \sqrt{6300}$

$\Rightarrow \quad P = 30\sqrt{7}$ N

Hence, the correct answer is (D).

64. $\vec{a} = \left[2\hat{i} + (6t)\hat{j} + \dfrac{2\pi^2}{9}\cos\left(\dfrac{\pi t}{3}\right)\hat{k}\right]$ ms^{-2}

$\Rightarrow \quad \dfrac{d\vec{v}}{dt} = 2\hat{i} + (6t)\hat{j} + \dfrac{2\pi^2}{9}\cos\left(\dfrac{\pi t}{3}\right)\hat{k}$

$\Rightarrow \quad \int_{\vec{v}(0)}^{\vec{v}} d\vec{v} = 2\hat{i}\int_0^t dt + 6\hat{j}\int_0^t t\,dt + \dfrac{2\pi^2}{9}\hat{k}\int_0^t \cos\left(\dfrac{\pi t}{3}\right)dt$

$\Rightarrow \quad \vec{v} - \vec{v}(0) = (2t)\hat{i} + (3t^2)\hat{j} + \dfrac{2\pi}{3}\sin\left(\dfrac{\pi t}{3}\right)\hat{k}$

$\Rightarrow \quad \vec{v} = (2t)\hat{i} + (3t^2)\hat{j} + \dfrac{2\pi}{3}\sin\left(\dfrac{\pi t}{3}\right)\hat{k} + \left(2\hat{i} + \hat{j}\right)$

$\Rightarrow \quad \vec{v} = 2(t+1)\hat{i} + (3t^2+1)\hat{j} + \dfrac{2\pi}{3}\sin\left(\dfrac{\pi t}{3}\right)\hat{k}$

$\Rightarrow \quad \dfrac{d\vec{r}}{dt} = (2t+2)\hat{i} + (3t^2+1)\hat{j} + \dfrac{2\pi}{3}\sin\left(\dfrac{\pi t}{3}\right)\hat{k}$

$\Rightarrow \quad \int_{\vec{r}(0)}^{\vec{r}} d\vec{r} = \hat{i}\int_0^2 (2t+2)dt + \hat{j}\int_0^2 (3t^2+1)dt +$

$$\frac{2\pi}{3}\hat{k}\int_0^2 \sin\left(\frac{\pi t}{3}\right)dt$$

$\Rightarrow \quad \vec{r} - \vec{r}(0) = \hat{i}\left(t^2 + 2t\right)\Big|_0^2 + \hat{j}\left(t^3 + t\right)\Big|_0^2 -$

$$2\hat{k}\left[\cos\left(\frac{\pi t}{3}\right)\Big|_0^2\right]$$

Since $\vec{r}(0) = \vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$, so

$\Rightarrow \quad \vec{r} = 8\hat{i} + 10\hat{j} - 2\hat{k}\left(\cos\left(\dfrac{2\pi}{3}\right) - \cos 0\right)$

$\Rightarrow \quad \vec{r} = 8\hat{i} + 10\hat{j} - 2\hat{k}\left(-\dfrac{1}{2} - 1\right)$

$\Rightarrow \quad \vec{r} = 8\hat{i} + 10\hat{j} + 3\hat{k}$

Hence, the correct answer is (B).

65. Electric field, magnetic moment and average velocity, all are vectors.

Hence, the correct answer is (A).

67. $\vec{P} \perp \vec{Q}$

$\Rightarrow \quad \vec{P} \cdot \vec{Q} = 0$

$\Rightarrow \quad a^2 - 2a - 3 = 0$

$\Rightarrow \quad a^2 - 3a + a - 3 = 0$

$\Rightarrow \quad a(a-3) + 1(a-3) = 0$

$\Rightarrow \quad (a+1)(a-3) = 0$

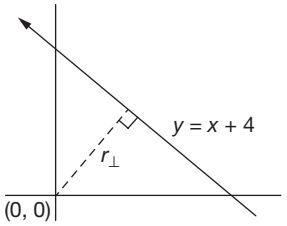
$\Rightarrow \quad a = -1$ OR $a = 3$

So, the positive value of a is 3

Hence, the correct answer is (A).

68. $L = mvr_\perp$

$v = 3\sqrt{2}$ unit along $y = x + 4$



Using the concept of Coordinate Geometry, we get

$$r_\perp = \frac{|0 - 0 + 4|}{\sqrt{1+1}}$$

$\Rightarrow \quad r_\perp = \dfrac{4}{\sqrt{2}}$ units

So, $|\vec{L}| = (5)(3\sqrt{2})\left(\dfrac{4}{\sqrt{2}}\right)$ unit

$\Rightarrow \quad |\vec{L}| = 60$ unit

Hence, the correct answer is (A).

69. Since (B) and (C) will purely depend upon the choice of origin and the axis selected, so we have (A) as the correct option.

Hence, the correct answer is (A).

70. Force is parallel to a line $y = \frac{3}{2}x + c$

The equation of given line can be written as

$$y = -\frac{k}{3}x + \frac{5}{3}$$

Work done will be zero, when force is perpendicular to the displacement i.e., the above two lines are perpendicular or

$$m_1 m_2 = -1$$

$$\Rightarrow \quad \left(\frac{3}{2}\right)\left(-\frac{k}{3}\right) = -1$$

$$\Rightarrow \quad k = 2$$

Hence, the correct answer is (A).

74. Given that $\left(\vec{F}_1 + \vec{F}_2\right) \cdot \vec{F}_1 = 0$

where $F_1 < F_2$

$$\Rightarrow \quad F_1^2 + F_1 F_2 \cos\theta = 0$$

Given that, $F_2 = 2F_1$

$$\Rightarrow \quad F_1^2 + 2F_1^2 \cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{2\pi}{3} \text{ radian} = 120^\circ$$

Hence, the correct answer is (A).

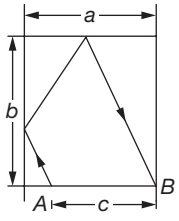
75. If $|\vec{R}| = |\vec{a} + \vec{b}|$ then, $(a - b) \le R \le (a + b)$

Hence, the correct answer is (C).

79. Component of $\vec{A}$ along $\vec{B} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} \cdot \vec{B}$

Hence, the correct answer is (A).

80. Let us resolve the velocity v imparted to the ball in components parallel with the sides of the table and consider the path of a ball as shown.



As seen from the diagram, we obtain the following two equations i.e.,

$$\frac{2a - c}{t} = v\cos\alpha \text{ and } \frac{2b}{t} = v\sin\alpha$$

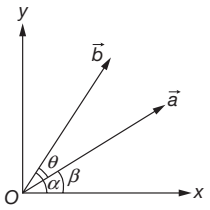
From these equations, we get

$$\cot\alpha = \frac{2a - c}{2b}$$

So, we get the angle α, at which the ball must be struck. It is important to note that the value for the velocity v which is imparted to the ball plays no part at all.

Hence, the correct answer is (C).

81.



$$\tan\beta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \beta = 30^\circ$$

$$\Rightarrow \quad \alpha = \theta + \beta = 53^\circ$$

$$\Rightarrow \quad \vec{b} = 10\cos 53\hat{i} + 10\sin 53\hat{j} = 6\hat{i} + 8\hat{j}$$

$$\Rightarrow \quad \vec{a} + \vec{b} = \left(6 + \sqrt{3}\right)\hat{i} + 9\hat{j}$$

Hence, the correct answer is (A).

82. Let $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

$$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

Given that $\vec{A} + \vec{B}$ is perpendicular to $\vec{A} - \vec{B}$, so

$$(\vec{A} + \vec{B})(\vec{A} - \vec{B}) = 0$$

$$\Rightarrow \quad (a_1 + b_1)(a_1 - b_1) + (a_2 + b_2)(a_2 - b_2) + (a_3 + b_3)(a_3 - b_3) = 0$$

$$\Rightarrow \quad a_1^2 + a_2^2 + a_3^2 = b_1^2 + b_2^2 + b_3^2$$

$$\Rightarrow \quad |\vec{A}| = |\vec{B}|$$

Cross product of $\vec{A}$ and $\vec{B}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ or $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

Hence, the correct answer is (D).

83. $x = 2t$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = 2$$

$$y = 2t^2$$

$$\Rightarrow \quad v_y = \frac{dy}{dt} = 4t$$

CHAPTER 3

$$\Rightarrow \quad \tan\theta = \frac{v_y}{v_x} = \frac{41}{2} = 2t$$

Differentiating with respect to time we get,

$$(\sec^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+\tan^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+4t^2)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{2}{1+4t^2}$$

So, $\frac{d\theta}{dt}$ at $t = 2$ s is given by

$$\left.\frac{d\theta}{dt}\right|_{t=2\text{ s}} = \frac{2}{1+4(2)^2} = \frac{2}{17} \text{ rads}^{-1}$$

Hence, the correct answer is (A).

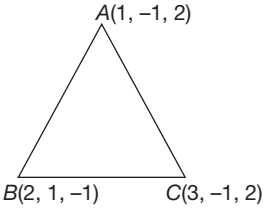
90. Vector in the direction of $\vec{A}$ and having same magnitude as B is $\vec{C} = B\hat{A}$

$$\vec{C} = B\left(\frac{\vec{A}}{A}\right)$$

$$\Rightarrow \quad \vec{C} = \frac{7}{3}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

Hence, the correct answer is (A).

91.



$$\overrightarrow{AB} = \hat{i}+2\hat{j}-3\hat{k}$$

$$\overrightarrow{BC} = \hat{i}-2\hat{j}+3\hat{k}$$

$$\overrightarrow{AB}\times\overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & -2 & 3 \end{vmatrix} = -6\hat{j}-4\hat{k}$$

$$\Rightarrow \quad |\overrightarrow{AB}\times\overrightarrow{BC}| = \sqrt{(-6)^2+(-4)^2} = \sqrt{52} \text{ unit}$$

$$\Rightarrow \quad \text{Area of } \Delta = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}| = \sqrt{13} \text{ unit}$$

Hence, the correct answer is (C).

92. Since, we know that

$$P = \vec{F}\cdot\vec{v} = \left(20\hat{i}-3\hat{j}+5\hat{k}\right)\cdot\left(6\hat{i}+20\hat{j}-3\hat{k}\right)$$

$$\Rightarrow \quad P = 120-60-15 = 45 \text{ W}$$

Hence, the correct answer is (A).

93. $A+B = 17$

and $A-B = 7$

$$\Rightarrow \quad A = 12$$

$$\Rightarrow \quad B = 5$$

Since $\theta = 90°$

$$\Rightarrow \quad R = \sqrt{A^2+B^2} = \sqrt{12^2+5^2} = 13 \text{ unit}$$

Hence, the correct answer is (D).

94. Since $\vec{R}$ is perpendicular to $\vec{a}$, therefore

$$\cos\theta = \frac{-a}{b}$$

$$\Rightarrow \quad R = \sqrt{b^2-a^2} \text{ and } S = \sqrt{3a^2+b^2}$$

Since, $2|\vec{R}| = |\vec{S}|$

$$\Rightarrow \quad 4b^2-4a^2 = 3a^2+b^2$$

$$\Rightarrow \quad 3b^2 = 7a^2$$

Hence, the correct answer is (A).

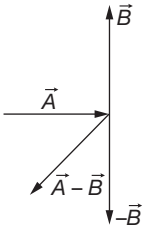
96. $\vec{v} = \frac{d\vec{r}}{dt} = 2bt\hat{i}+3ct^2\hat{j}$

$$\Rightarrow \quad \tan(60°) = \frac{v_x}{v_y} = \frac{2bt}{3ct^2}$$

$$\Rightarrow \quad t = \frac{2b}{3\sqrt{3}c}$$

Hence, the correct answer is (B).

97.



Hence, the correct answer is (D).

98. $2|\vec{A}| = |\vec{B}|$

$$\Rightarrow \quad |\vec{A}| = \frac{|\vec{B}|}{2}$$

Hence, the correct answer is (C).

99. $\frac{|\vec{R}|_{min}}{|R|_{max}} = \frac{1}{4} = \frac{|A-B|}{|A+B|}$

$\Rightarrow \quad A+B = 4|A-B|$

If $A > B$, then we have

$A+B = 4(A-B)$

$\Rightarrow \quad 3A = 5B$

$\Rightarrow \quad \frac{A}{B} = \frac{5}{3}$

If $B > A$, then we have

$A+B = 4(B-A)$

$\Rightarrow \quad \frac{A}{B} = \frac{3}{5}$

Hence, the correct answer is (C).

100. $\vec{a}\cdot\vec{b} = 0$ when $\vec{a} \perp \vec{b}$

$\Rightarrow \quad \left(2\hat{i}+3\hat{j}+8\hat{k}\right)\cdot\left(-4\hat{i}+4\hat{j}+\alpha\hat{k}\right) = 0$

$\Rightarrow \quad -8+12+8\alpha = 0$

$\Rightarrow \quad \alpha = \frac{-1}{2}$

Hence, the correct answer is (C).

101. $\vec{C} = \vec{A}+\vec{B} = \left(3\hat{i}+6\hat{j}-2\hat{k}\right)$

$\Rightarrow \quad \vec{C} = \frac{\vec{C}}{|C|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{3^2+6^2+(-2)^2}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$

Hence, the correct answer is (A).

102. Resultant can be zero if

$|F_1 - F_2| \le F_3 \le F_1 + F_2$

Hence, the correct answer is (D).

103. $\vec{F}_1 = F_1\hat{i}$ and $F_1\hat{i}\times(-4\hat{i}) = 0$

Hence, the correct answer is (D).

104. Component of $\vec{A}$ along $\vec{B}$ is

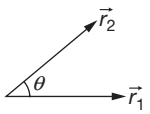
$\frac{\vec{A}\cdot\vec{B}}{B} = \frac{3+4}{\sqrt{2}} = \frac{7}{\sqrt{2}}$

In vector form, component of $\vec{A}$ along $\vec{B}$ is

$\frac{7}{\sqrt{2}}\hat{B} = \frac{7}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{7}{2}\left(\hat{i}+\hat{j}\right)$

Hence, the correct answer is (D).

105. Given that $|\vec{r}_1| = |\vec{r}_2| = l$

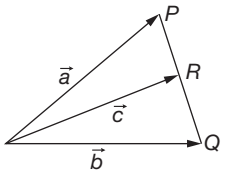


$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \quad |\Delta\vec{r}| = \sqrt{r_2^2 + r_1^2 - 2r_1r_2\cos\theta} = 2l\sin\left(\frac{\theta}{2}\right)$

Hence, the correct answer is (B).

106.



$\vec{a} + \overrightarrow{PR} = \vec{c}$ …(1)

and $\vec{b} + \overrightarrow{QR} = \vec{c}$ …(2)

Adding (1) and (2), we get

$\vec{a} + \vec{b} + \overrightarrow{QR} + \overrightarrow{PR} = 2\vec{c}$

Since, we have $\overrightarrow{PR} = -\overrightarrow{QR}$

$\Rightarrow \quad \vec{a} + \vec{b} = 2\vec{c}$

Hence, the correct answer is (A).

107. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -2\hat{i}+2\hat{k}$

Here, $\vec{a}\times\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so unit vector $\hat{n}$ along $\vec{a}\times\vec{b}$ is

$\hat{n} = \frac{-2\hat{i}+2\hat{k}}{\sqrt{(-2)^2+2^2}} = \frac{-\hat{i}+\hat{k}}{\sqrt{2}}$

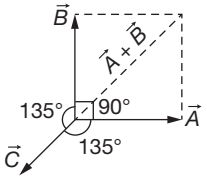
Hence, the correct answer is (B).

108. $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & 4 & 0 \end{vmatrix} = \hat{k}(8-3) = 5\hat{k}$

Hence, the correct answer is (A).

109. Let $|\vec{A}| = |\vec{B}| = a$ and $|\vec{C}| = \sqrt{2}a$ then,

$\vec{A}+\vec{B}+\vec{C} = 0$



$\Rightarrow \quad \vec{C} = -(\vec{A}+\vec{B})$

$\Rightarrow \quad C^2 = A^2 + B^2 + 2AB\cos\theta$

CHAPTER 3

$\Rightarrow \quad 2a^2 = a^2 + a^2 + 2a^2\cos\theta$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

Hence, the correct answer is (D).

110. $\vec{r} = b\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j}$

$\Rightarrow \quad \dfrac{d\vec{r}}{dt} = -b\omega\sin(\omega t)\hat{i} + a\omega\cos(\omega t)\hat{j}$

$\Rightarrow \quad \dfrac{d^2\vec{r}}{dt^2} = -b\omega^2\cos(\omega t)\hat{i} - a\omega^2\sin(\omega t)\hat{j}$

$\Rightarrow \quad a = -\omega^2\left(b\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j}\right)$

$\Rightarrow \quad -\omega^2\vec{r}$

Hence, the correct answer is (C).

111. $P = \sqrt{(30)^2 + (60)^2 + 2(30)(60)\cos(60°)}$

$\Rightarrow \quad P = \sqrt{900 + 3600 + 1800}$

$\Rightarrow \quad P = \sqrt{6300} = 30\sqrt{7}$

Hence, the correct answer is (D).

112. $\cos\gamma = \dfrac{5}{\sqrt{4^2 + 3^2 + 5^2}} = \dfrac{5}{\sqrt{50}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \gamma = \dfrac{\pi}{4}$

Hence, the correct answer is (B).

113. $\left(\vec{A}_1 + 2\vec{A}_2\right)\cdot\left(3\vec{A}_1 - 4\vec{A}_2\right)$

$= 3\left|\vec{A}_1\right|^2 + 2\left|\vec{A}_1\right|\left|\vec{A}_2\right|\cos\theta - 8\left|\vec{A}_2\right|^2$

$= \left(\left|\vec{A}_1\right|^2 + 2\left|\vec{A}_1\right|\left|\vec{A}_2\right|\cos\theta + \left|\vec{A}_2\right|^2\right) + 2\left|\vec{A}_1\right|^2 - 9\left|\vec{A}_2\right|^2$

$= 3^2 + 2\times 2^2 - 9\times 3^2 = 9 + 8 - 81 = -64$

Hence, the correct answer is (A).

114. Let the components of $\vec{A}$ make angles α, β and γ with x, y and z-axis respectively, then $\alpha = \beta = \gamma$.

Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad 3\cos^2\alpha = 1$

$\Rightarrow \quad \cos\alpha = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad A_x = A_y = A_z = A\cos\alpha = \dfrac{A}{\sqrt{3}}$

Hence, the correct answer is (A).

115. $\vec{A} = \hat{i} + \hat{j}$

$\cos\alpha = \dfrac{A_x}{|A|} = \dfrac{1}{\sqrt{2}} = \cos 45°$

$\Rightarrow \quad \alpha = 45°$

Hence, the correct answer is (B).

116. Magnitude of unit vector $= 1$

$\Rightarrow \quad \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

$\Rightarrow \quad c = \sqrt{0.11}$

Hence, the correct answer is (B).

117. $\dfrac{\vec{A}\cdot\vec{B}}{\left|\vec{i} + \vec{j}\right|} = \dfrac{\left(2\hat{i} + 3\hat{j}\right)\cdot\left(\hat{i} + \hat{j}\right)}{\sqrt{2}} = \dfrac{2+3}{\sqrt{2}} = \dfrac{5}{\sqrt{2}}$

Hence, the correct answer is (A).

118. For 17 N both the vector should be parallel i.e. angle between them should be zero.

For 7 N both the vectors should be antiparallel i.e. angle between them should be 180°.

For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90°.

Hence, the correct answer is (A).

119. $\vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$

$\Rightarrow \quad |\vec{A} + \vec{B}| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$

$\Rightarrow \quad \tan\theta = \dfrac{5}{10} = \dfrac{1}{2}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{1}{2}\right)$

Hence, the correct answer is (B).

120. $A = 3\ N$, $B = 2\ N$ then

$\Rightarrow \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow \quad R = \sqrt{9 + 4 + 12\cos\theta}$ …(1)

Now $A = 6\ N$, $B = 2\ N$ then

$2R = \sqrt{36 + 4 + 24\cos\theta}$ …(2)

From (1) and (2), we get

$\cos\theta = -\dfrac{1}{2}$

$\Rightarrow \quad \theta = 120°$

Hence, the correct answer is (D).

121. From the property of vector product, $\vec{C}$ is perpendicular to $\vec{A}$ as well as $\vec{B}$. Also $(\vec{A} + \vec{B})$ vector must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$

must be perpendicular to $(\vec{A}+\vec{B})$ also but the cross product $(\vec{A}\times\vec{B})$ gives a vector $\vec{C}$ which cannot be perpendicular to itself. So, the last statement is wrong.

Hence, the correct answer is (D).

122. Component of $\vec{v}$ along $\vec{a}$ is

$$\frac{\vec{v}\cdot\vec{a}}{|\vec{a}|}=\left(6\hat{i}+2\hat{j}-2\hat{k}\right)\cdot\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}=\frac{6+2-2}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{\vec{v}\cdot\vec{a}}{|\vec{a}|}=\frac{6}{\sqrt{3}}=2\sqrt{3}$$

In vector form, the component is $(2\sqrt{3})\hat{a}$.

$$\Rightarrow \quad (2\sqrt{3})\hat{a}=2\sqrt{3}\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}=2\left(\hat{i}+\hat{j}+\hat{k}\right)$$

Hence, the correct answer is (B).

123. $|7-5|\le F\le 7+5$

$\Rightarrow \quad 2\le F\le 12$

$\Rightarrow \quad F_{\min}=2$ newton

Hence, the correct answer is (D).

Multiple Correct Choice Type Question

1. A scalar, a vector and its magnitude do not depend on the orientation of the frame of reference.

Hence, (A), (B) and (C) are correct.

2. Think of $\hat{i}$, $\hat{j}$ and $\hat{k}$ to give an appropriate answer.

Hence, (C) and (D) are correct.

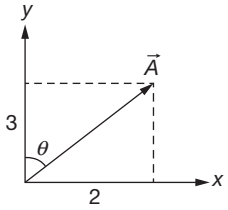
9. $\left|3\hat{i}+4\hat{j}\right|=\sqrt{3^2+4^2}=5$

Area of parallelogram $=|\vec{a}\times\vec{b}|$

Hence, (A) and (C) are correct.

10. $\tan\theta=\frac{2}{3}$

$$\Rightarrow \quad \cos\theta=\frac{3}{\sqrt{13}}$$



Hence, (B) and (C) are correct.

11. $\vec{p}=2\left[(\sin t)\hat{i}-(\cos t)\hat{j}\right]$

$$\Rightarrow \quad |\vec{p}|=2\sqrt{\sin^2 t+\cos^2 t}=2=\text{Constant}$$

Also, $\vec{F}=\frac{d\vec{p}}{dt}=2\left(\cos t\hat{i}+\sin t\hat{j}\right)$

$$\Rightarrow \quad \vec{F}\cdot\vec{p}=4(\sin t\cos t-\cos t\sin t)$$

$$\Rightarrow \quad \vec{F}\cdot\vec{p}=0$$

$$\Rightarrow \quad \vec{F}\perp\vec{p}$$

Hence, (A) and (C) are correct.

Reasoning Based Questions

1. Cross product of two vectors is perpendicular to the plane containing both the vectors.

Hence, the correct answer is (A).

2. $\cos\theta=\frac{\left(\hat{i}+\hat{j}\right)\cdot(\hat{i})}{\left|\hat{i}+\hat{j}\right||\hat{i}|}=\frac{1}{\sqrt{2}}$. Hence $\theta=45°$

Hence, the correct answer is (A).

3. $\frac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}}=\frac{AB\sin\theta}{AB\cos\theta}=\tan\theta$

Also, $\vec{A}\times\vec{B}$ is normal to the plane containing $\vec{A}$ as well as $\vec{B}$. So, $\vec{A}\times\vec{B}\perp(\vec{A}+\vec{B})$.

Hence, the correct answer is (B).

4. $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

$$\Rightarrow \quad A^2+B^2+2AB\cos\theta=A^2+B^2-2AB\cos\theta$$

$$\Rightarrow \quad 4AB\cos\theta=0$$

$$\Rightarrow \quad \cos\theta=0$$

$$\Rightarrow \quad \theta=90°$$

Also vector addition is commutative

Hence $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.

Hence, the correct answer is (B).

5. $\vec{v}=\vec{\omega}\times\vec{r}$ is the correct expression.

The expression $\vec{\omega}=\vec{v}\times\vec{r}$ is wrong.

Hence, the correct answer is (C).

6. For giving a zero resultant, it should be possible to represent the given vectors along the sides of a closed polygon and minimum number of sides of a polygon is three. If $\vec{A}+\vec{B}+\vec{C}=\vec{0}$, then this represents the sides of a triangle taken in same order, hence they must lie in one plane.

Hence, the correct answer is (B).

CHAPTER 3

7. Vector addition of two vectors is commutative i.e., $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.
Hence, the correct answer is (B).

9. Cross product of two vectors is anticommutative i.e., $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$.
Hence, the correct answer is (C).

11. If a vector quantity has zero magnitude then it is called a null vector. That quantity may have some direction even if its magnitude is zero.
Hence, the correct answer is (D).

12. Let $\vec{P}$ and $\vec{Q}$ are two vectors in opposite direction, then their sum $\vec{P}+(-\vec{Q})=\vec{P}-\vec{Q}$.
If $\vec{P}=\vec{Q}$ then sum equal to zero.
Hence, the correct answer is (A).

13. If two vectors are in opposite direction, then they cannot be like vectors.
Hence, the correct answer is (C).

14. If θ be the angle between two vectors $\vec{A}$ and $\vec{B}$, then their scalar product, $\vec{A}\cdot\vec{B}=AB\cos\theta$.
If $\theta=90°$, then $\vec{A}\cdot\vec{B}=0$,
i.e., if $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product will be zero.
Hence, the correct answer is (A).

15. We can multiply any vector by any scalar and the result is a vector, however dot product of a scalar with a vector is meaningless.
Hence, the correct answer is (B).

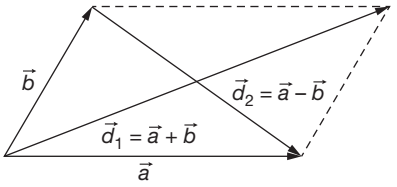
17. $\vec{A}\cdot\vec{B}=|\vec{A}||\vec{B}|\cos\theta=0$
$\vec{A}\times\vec{B}=|\vec{A}||\vec{B}|\sin\theta=0$
If $\vec{A}$ and $\vec{B}$ are not null vectors then it follows that $\sin\theta$ and $\cos\theta$ both should be zero simultaneously. But it cannot be possible so it is essential that one of the vector must be null vector.
Hence, the correct answer is (B).

19. The resultant of two vectors of unequal magnitude given by $R=\sqrt{A^2+B^2+2AB\cos\theta}$ cannot be zero for any value of θ.
Hence, the correct answer is (C).

20. $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{C}$
$\Rightarrow\quad AB\cos\theta_1=BC\cos\theta_2$
So, $A=C$, only when $\theta_1=\theta_2$
Hence, when angle between $\vec{A}$ and $\vec{B}$ is equal to angle between $\vec{B}$ and $\vec{C}$ only then $\vec{A}$ equal to $\vec{C}$.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Let $\vec{d}_1$ and $\vec{d}_2$ be the two diagonals of the parallelogram with sides $\vec{a}$ and $\vec{b}$, then



So, $\vec{d}_1=\vec{a}+\vec{b}=(2\vec{m}+\vec{n})+(\vec{m}-2\vec{n})$
$\Rightarrow\quad \vec{d}_1=3\vec{m}-\vec{n}$
Hence, the correct answer is (C).

2. Now $\vec{d}_2=\vec{a}-\vec{b}=(2\vec{m}+\vec{n})-(\vec{m}-2\vec{n})$
$\Rightarrow\quad \vec{d}_2=\vec{m}+3\vec{n}$
Hence, the correct answer is (B).

3. Since the dot product of a vector with itself is equal to the square of its magnitude, so we have
$d_1^2=\vec{d}_1\cdot\vec{d}_1$
$\Rightarrow\quad d_1^2=(3\vec{m}-\vec{n})\cdot(3\vec{m}-\vec{n})$
$\Rightarrow\quad d_1^2=9(\vec{m}\cdot\vec{m})+(\vec{n}\cdot\vec{n})-6(\vec{m}\cdot\vec{n})$
$\Rightarrow\quad d_1^2=9|\vec{m}|^2+|\vec{n}|^2-6|\vec{m}||\vec{n}|\cos(60°)$
Since $|\vec{m}|=|\vec{n}|=1$
$\Rightarrow\quad d_1^2=9+1-6(1)(1)\left(\frac{1}{2}\right)=7$
$\Rightarrow\quad d_1=\sqrt{7}$ units
Hence, the correct answer is (A).

4. Similarly, $d_2^2=\vec{d}_2\cdot\vec{d}_2$
$\Rightarrow\quad d_2^2=(\vec{m}+3\vec{n})\cdot(\vec{m}+3\vec{n})$
$\Rightarrow\quad d_2^2=|\vec{m}|^2+9|\vec{n}|^2+6(\vec{m}\cdot\vec{n})$
$\Rightarrow\quad d_2^2=1+9+6(1)(1)\left(\frac{1}{2}\right)=13$
$\Rightarrow\quad d_2=\sqrt{13}$ units
Hence, the correct answer is (D).

5. Finally, we know that area of the parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$ is

Area $= |\vec{a} \times \vec{b}|$

Now $\vec{a} \times \vec{b} = (2\vec{m} + \vec{n}) \times (\vec{m} - 2\vec{n})$

$\Rightarrow \quad \vec{a} \times \vec{b} = 2(\vec{m} \times \vec{m}) - 2(\vec{n} \times \vec{n}) - 4(\vec{m} \times \vec{n}) + (\vec{n} \times \vec{m})$

$\Rightarrow \quad \vec{a} \times \vec{b} = 5(\vec{n} \times \vec{m}) \qquad \{\because \ \vec{m} \times \vec{n} = -\vec{n} \times \vec{m}\}$

$\Rightarrow \quad |\vec{a} \times \vec{b}| = 5|\vec{n} \times \vec{m}|$

$\Rightarrow \quad |\vec{a} \times \vec{b}| = 5|\vec{n}||\vec{m}|\sin(60°)$

$\Rightarrow \quad |\vec{a} \times \vec{b}| = \dfrac{5\sqrt{3}}{2}$

$\Rightarrow$ Area $= |\vec{a} \times \vec{b}| = \dfrac{5\sqrt{3}}{2}$ sq. units

Hence, the correct answer is (C).

6. If θ be the angle between $\vec{d}_1$ and $\vec{d}_2$, then

$$\cos\theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1||\vec{d}_2|}$$

Now $\vec{d}_1 \cdot \vec{d}_2 = (3\vec{m} - \vec{n}) \cdot (\vec{m} + 3\vec{n})$

$\Rightarrow \quad \vec{d}_1 \cdot \vec{d}_2 = 3(\vec{m} \cdot \vec{m}) - 3(\vec{n} \cdot \vec{n}) - (\vec{n} \cdot \vec{m}) + 9(\vec{m} \cdot \vec{n})$

$\Rightarrow \quad \vec{d}_1 \cdot \vec{d}_2 = 3|\vec{m}|^2 - 3|\vec{n}|^2 + 8|\vec{m}||\vec{n}|\cos(60°)$

$\Rightarrow \quad \vec{d}_1 \cdot \vec{d}_2 = 3 - 3 + 8(1)(1)\left(\dfrac{1}{2}\right) = 4$

$\Rightarrow \quad \vec{d}_1 \cdot \vec{d}_2 = 4$

$\Rightarrow \quad \cos\theta = \dfrac{4}{\sqrt{7}\sqrt{13}}$

$\Rightarrow \quad \cos\theta = \dfrac{4}{\sqrt{91}}$

Hence, the correct answer is (D).

9. The diagonal are perpendicular either for a square or a rhombus and both have sides of equal magnitude.

Hence, the correct answer is (A).

10. $\vec{a} + \vec{b}$ lies in the plane containing $\vec{a}$ and $\vec{b}$, and $\vec{a} \times \vec{b}$ is a new vector perpendicular to $\vec{a}$ and $\vec{b}$. Hence angle between $\vec{a} + \vec{b}$ and $\vec{a} \times \vec{b}$ is $90°$.

Hence, the correct answer is (D).

11. x-component of $\vec{F}_2$ is $(F_2)_x = -10\cos(60°)$

$\Rightarrow \quad (F_2)_x = -5$ N

Hence, the correct answer is (B).

12. y-component of $\vec{F}_3$ is $(F_3)_y = -20\sin(60°)$

$\Rightarrow \quad (F_3)_y = -10\sqrt{3}$ N

Hence, the correct answer is (B).

13. $\vec{F}_1 = 5\sqrt{2}\left(\cos 45\hat{i} + \sin 45\hat{j}\right)$

$\Rightarrow \quad \vec{F}_1 = \left(5\hat{i} + 5\hat{j}\right)$ N

Hence, the correct answer is (C).

14. Since $B_x = -10\cos(30°) = -5\sqrt{3}$ m

and $D_x = 10\cos(30°) = 5\sqrt{3}$ m

$\Rightarrow \quad B_x + D_x = 0$

Hence, the correct answer is (C).

15. Since $A_x = 5\sqrt{2}\cos(45°) = 5$ m

and $C_x = -10\cos(80°) = -5$ m

$\Rightarrow \quad A_x + C_x = 0$

Hence, the correct answer is (A).

16. Since $A_y = 5\sqrt{2}\sin(45°) = 5$ m

$B_y = 10\sin(30°) = 5$ m

$C_y = -10\sin(60°) = -5\sqrt{3}$ m

$D_y = -10\sin(30°) = -5$ m

$\Rightarrow \quad A_y + B_y + C_y + D_y = 5\left(1 - \sqrt{3}\right)$ m

Hence, the correct answer is (A).

17. $\Sigma F_x = F_1 - F_3\cos 60°$

$\Rightarrow \quad \Sigma F_x = 10 - 20\left(\dfrac{1}{2}\right) = 0$

$\Sigma F_y = F_2 + F_3\sin(60°)$

$\Rightarrow \quad \Sigma F_y = 10\sqrt{3} + 20\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \Sigma F_y = 20\sqrt{3}$ N (along $+y$)

So, for ΣF_y to be zero, a force of $20\sqrt{3}$ must act along $-y$ direction.

Hence, the correct answer is (B).

18. Since $\vec{F}_1 = 10\hat{i}$, $\vec{F}_2 = 10\sqrt{3}\hat{j}$

and $\vec{F}_3 = 20\left(-\cos(60°)\hat{i} + \sin(60°)\hat{j}\right)$

CHAPTER 3

$\Rightarrow \quad \vec{F}_3 = -10\hat{i} + 10\sqrt{3}\hat{j}$

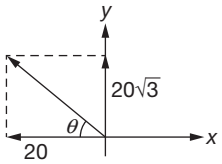
$\Rightarrow \quad \vec{F}_1 + \vec{F}_2 - \vec{F}_3 = 10\hat{i} + 10\sqrt{3}\hat{j} + 10\hat{i} - 10\sqrt{3}\hat{j}$

$\Rightarrow \quad \vec{F}_1 + \vec{F}_2 - \vec{F}_3 = 20\hat{i}$

Hence, the correct answer is (A).

19. $\vec{F}_2 + \vec{F}_3 - \vec{F}_1 = -20\hat{i} + 20\sqrt{3}\hat{j}$

$\Rightarrow \quad \left|\vec{F}_2 + \vec{F}_3 - \vec{F}_1\right| = \sqrt{20^2 + \left(20\sqrt{3}\right)^2} = 40 \text{ N}$



From diagram, $\tan\theta = \dfrac{20\sqrt{3}}{20}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60°$

So, $\left(\vec{F}_2 + \vec{F}_3 - \vec{F}_1\right)$ makes an angle of $120°$ with x-axis.

Hence, the correct answer is (B).

20. Since $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \quad \vec{a} + \vec{b} - \vec{c} = 0 = -2\vec{c}$

Hence, the correct answer is (C).

21. Since $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$ as well as $\vec{b}$, so $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$

Hence, the correct answer is (D).

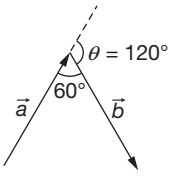
22. Since $\vec{b} = -(\vec{a} + \vec{c})$

$\Rightarrow \quad \vec{b} \cdot (\vec{a} + \vec{c}) = -\vec{b} \cdot \vec{b}$

$\Rightarrow \quad \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} = -b^2$

Hence, the correct answer is (A).

23.



So, angle between $\vec{a}$ and $\vec{b}$ is $120°$ (and not $60°$)

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p, r)
B → (p, s)
C → (q)
D → (p, s)

$\vec{A} \cdot \vec{B}$ is defined and is a scalar

$(\vec{A} \cdot \vec{B})\vec{C}$ is defined and is a vector

$(\vec{A} \cdot \vec{B}) \cdot \vec{C}$ is not defined, because dot product is always defined/taken between two vectors but not between a scalar and a vector

$(\vec{A} \times \vec{B}) \times \vec{C}$ is defined and is a vector.

2. A → (q, s)
B → (p)
C → (r)
D → (r)

$\vec{A} \times \vec{B}$ is a new vector perpendicular to $\vec{A}$ as well as $\vec{B}$.

So, $(\vec{A} \times \vec{B}) \cdot \vec{A} = 0$

Also, $\vec{A} \times \vec{A} = \vec{0}$ and $\vec{0} \times \vec{A} = \vec{0}$

3. A → (r)
B → (t)
C → (s)
D → (p)
E → (q)

For parallel vectors, $\vec{A} \times \vec{B} = \vec{0}$ and for perpendicular vectors $\vec{A} \cdot \vec{B} = 0$. So (A) → (r), (B) → (t)

$\vec{A} \times \vec{B} + \vec{C} \times \vec{A} = \vec{0}$

$\Rightarrow \quad \vec{A} \times \vec{B} - \vec{A} \times \vec{C} = \vec{0}$

$\Rightarrow \quad \vec{A} \times (\vec{B} - \vec{C}) = \vec{0}$

$\Rightarrow \quad \vec{B} - \vec{C} = k\vec{A}$ {$\because$ Vectors are parallel}

$\Rightarrow \quad \vec{B} = \vec{C} + k\vec{A}$

So, (C) → (s)

Now area of parallelogram is $|\vec{A} \times \vec{B}|$,

so (D) → (p)

Further component vector of $\vec{A}$ along $\vec{B}$ is $(A\cos\theta)\hat{B}$.

$\Rightarrow \quad (A\cos\theta)\hat{B} = \left(\dfrac{\vec{A} \cdot \vec{B}}{B}\right)\hat{B} = \left(\dfrac{\vec{A} \cdot \vec{B}}{B^2}\right)\vec{B}$

So, (E) → (q)

4. A → (r, s)
B → (s)
C → (t)
D → (p)
E → (q)
Resultant of two ordered vectors is given by triangle law of vector addition and is commutative in nature.
Dot product of two vectors is a scalar and is also commutative in nature
So, we have (A) → (r, s) and (B) → (s)

Further $\vec{A}\cdot\vec{B}=0$ and $|\vec{A}\times\vec{B}|=0$ is satisfied only when either of the two vectors is a null vector. So, (C) → (t).
As done already,

$$\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{A}\times\vec{C})+\vec{C}\times(\vec{A}\times\vec{B})=\vec{0}$$

So, (D) → (p)
Now for three vectors to form a closed polygon, we must have

$$\vec{A}+\vec{B}+\vec{C}=\vec{0}$$

$$\Rightarrow\quad \vec{A}+\vec{B}=-\vec{C}$$

$$\Rightarrow\quad \vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{C}=-\vec{C}\cdot\vec{C}$$

$$\Rightarrow\quad \vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{C}=-C^2 \qquad \ldots(1)$$

Similarly $\vec{A}+\vec{C}=-\vec{B}$

$$\Rightarrow\quad \vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}=-B^2 \qquad \ldots(2)$$

Again $\vec{B}+\vec{C}=-\vec{A}$

$$\Rightarrow\quad \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}=-A^2 \qquad \ldots(3)$$

Adding (1), (2) and (3), we get

$$2(\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A})=-(A^2+B^2+C^2)$$

$$\Rightarrow\quad \vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}=-\frac{1}{2}(A^2+B^2+C^2)$$

Hence $\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}=$ Negative
So, (E) → (q)

5. A → (s, t)
B → (r)
C → (q)
D → (p)

$$|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$$

$$\Rightarrow\quad \vec{A}\perp\vec{B}\text{, so (A)}\rightarrow\text{(s, t)}$$

$$\vec{A}-\vec{B}=\vec{A}+\vec{B}$$

$$\Rightarrow\quad \vec{B}=\vec{0}\text{, so (B)}\rightarrow\text{(r)}$$

$$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})=2(\vec{B}\times\vec{A}),$$

so (C) → (q)

$$(\vec{A}+\vec{B})\perp(\vec{A}-\vec{B})$$

$$\Rightarrow\quad |\vec{A}|=|\vec{B}|\text{, so (D)}\rightarrow\text{(p)}$$

10. A → (r)
B → (s)
C → (p)
D → (q)

$A:\vec{a}+\vec{b}+\vec{c}=0$ (polygon law)

$B:\vec{a}+\vec{b}=\vec{c}$ (triangle law)

$C:\vec{c}+\vec{b}=\vec{a}$ (triangle law)

$D:\vec{c}+\vec{a}=\vec{b}$

11. A → (r)
B → (q)
C → (s)
D → (p)

(A) $\vec{A}+\vec{B}=\vec{A}-\vec{B}$

$$\Rightarrow\quad B=0$$

(B) $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

$$\Rightarrow\quad \theta=\frac{\pi}{2}$$

(C) $|\vec{A}\times\vec{B}|=|\vec{A}\cdot\vec{C}|$

$$\Rightarrow\quad \theta=\frac{\pi}{4}$$

(D) $\vec{A}\times\vec{B}=\vec{A}\cdot\vec{C}$ (Not possible) because

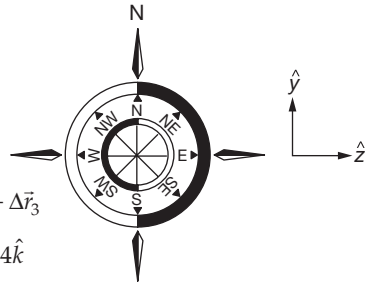
$\vec{A}\times\vec{B}$ is a vector and $\vec{A}\cdot\vec{C}$ is a scalar quantity.

Integer/Numerical Answer Type Questions

1. Given $\Delta\vec{r}_1=12\hat{i}$

$\Delta\vec{r}_2=3\hat{j}$ and

$\Delta\vec{r}_3=4\hat{k}$



$$\Rightarrow\quad \Delta\vec{r}=\Delta\vec{r}_1+\Delta\vec{r}_2+\Delta\vec{r}_3$$

$$\Rightarrow\quad \Delta\vec{r}=12\hat{i}+3\hat{j}+4\hat{k}$$

$$\Rightarrow\quad |\Delta\vec{r}|=\sqrt{144+9+16}=\sqrt{169}=13\text{ m}$$

$$\Rightarrow\quad |\Delta\vec{r}|=1300\text{ cm}$$

2. Since $R^2=F_1^2+F_2^2+2F_1F\cos\theta$

$$\Rightarrow\quad 3A^2+B^2=(A+B)^2+(A-B)^2+2(A^2-B^2)\cos\theta$$

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$\Rightarrow \quad 3A^2 + B^2 = 2A^2 + 2B^2 + 2(A^2 - B^2)\cos\theta$

$\Rightarrow \quad A^2 - B^2 = 2(A^2 - B^2)\cos\theta$

Since $A > B$, so we can cancel the common factor $A^2 - B^2$ from both the sides

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

3. Let the vectors be $\vec{A}$ and $\vec{B}$ inclined at an angle θ. Since $|\vec{A}| = |\vec{B}|$ and $R^2 = 3A^2$, so we get

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad 3A^2 = A^2 + A^2 + 2A^2\cos\theta$

$\Rightarrow \quad A^2 = 2A^2\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

4. $\vec{A} + \vec{B} = \vec{C}$

If θ is angle between $\vec{A}$ and $\vec{B}$, then

$\Rightarrow \quad C^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad A^2 + B^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

5. If θ be the angle between $\vec{v}_1$ and $\vec{v}_2$, then

$v_1^2 + v_2^2 + 2v_1v_2\cos\theta = v_1^2 + v_2^2 - 2v_1v_2\cos\theta$

$\Rightarrow \quad 4v_1v_2\cos\theta = 0$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

6. Let θ be angle between $\vec{F}_1$ and $\vec{F}_2$, then

$P^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad \ldots(1)$

Now, when $\vec{F}_2$ is reversed, angle between $\vec{F}_1$ and $-\vec{F}_2$ becomes $(180 - \theta)$. So,

$Q^2 = F_1^2 + F_2^2 + 2F_1F_2\cos(180 - \theta)$

$\Rightarrow \quad Q^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\theta \qquad \ldots(2)$

Adding (1) and (2), we get

$P^2 + Q^2 = 2(F_1^2 + F_2^2)$

$\Rightarrow \quad \frac{P^2 + Q^2}{F_1^2 + F_2^2} = 2$

7. Let the new vector be $\vec{r} = x\hat{i} + y\hat{j}$. Magnitude of $\vec{A}$ is given by

$|\vec{A}| = \sqrt{49 + 576} = \sqrt{625} = 25$

Now a vector having a magnitude of 25 units and parallel to $\vec{B} = 3\hat{i} + 4\hat{j}$ is

$\vec{r} = \pm 25\hat{B} = \pm 25\left(\frac{\vec{B}}{|\vec{B}|}\right)$

$\Rightarrow \quad \vec{r} = \pm 25\left(\frac{3\hat{i} + 4\hat{j}}{5}\right)$

$\Rightarrow \quad \vec{r} = \pm(15\hat{i} + 20\hat{j})$

So, for the vector to be in the first quadrant, both x and y must be positive. Hence

$x = 15$ and $y = 20$.

8. For vectors to be collinear, we must have

$\frac{x}{2} = \frac{4}{y} = -\frac{8}{4}$

$\Rightarrow \quad \frac{x}{2} = \frac{4}{y} = -2$

$\Rightarrow \quad x = -4$ and $y = -2$

So, $y - x = -2 - (-4) = 2$

9. Since we know that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

10. Let, $\vec{P} = \hat{i} - \hat{j} - 2\hat{k}$

$\Rightarrow \quad \overrightarrow{PS} = (0\hat{i} + \hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} - 2\hat{k})$

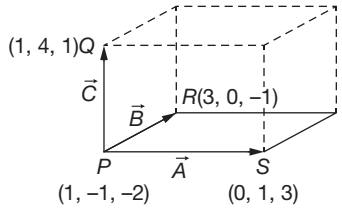
$\Rightarrow \quad \overrightarrow{PS} = \vec{A} = -\hat{i} + 2\hat{j} + 5\hat{k}$

$\overrightarrow{PR} = (3\hat{i} + 0\hat{j} - \hat{k}) - (\hat{i} - \hat{j} - 2\hat{k})$

$\Rightarrow \quad \overrightarrow{PR} = \vec{B} = 2\hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{PQ} = \left(\hat{i} + 4\hat{j} + \hat{k}\right) - \left(\hat{i} + \hat{j} + 2\hat{k}\right)$

$\Rightarrow \quad \overrightarrow{PQ} = \vec{C} = 5\hat{j} + 3\hat{k}$



Volume of parallelopiped $= (\vec{A} \times \vec{B}) \cdot \vec{C}$

$\Rightarrow \quad \text{Volume} = \begin{vmatrix} -1 & 2 & 5 \\ 2 & 1 & 1 \\ 0 & 5 & 3 \end{vmatrix}$

$\Rightarrow \quad \text{Volume} = (-1)(3-5) - 2(6-0) + 5(10-0)$

$\Rightarrow \quad \text{Volume} = 2 - 12 + 50$

$\Rightarrow \quad \text{Volume} = 40 \text{ unit}$

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1. Let $P = \left(2\vec{A}_1 + 3\vec{A}_2\right) \cdot \left(3\vec{A}_1 - 2\vec{A}_2\right)$

$\Rightarrow \quad P = 6\left|\vec{A}_1\right|^2 + 5\vec{A}_1 \cdot \vec{A}_2 - 6\left|\vec{A}_2\right|^2$

$\Rightarrow \quad P = (6 \times 9) + 5\vec{A}_1 \cdot \vec{A}_2 - (6 \times 25)$ …(1)

Since $25 = 9 + 25 + 2\vec{A}_1 \cdot \vec{A}_2$

$\Rightarrow \quad 2\vec{A}_1 \cdot \vec{A}_2 = -9$ …(2)

From (1) and (2), we get

$P = 54 - 22.5 - 150$

$\Rightarrow \quad P = -118.5$

Hence, the correct answer is (B).

2. Position vector of point G is

$\overrightarrow{OG} = \frac{a}{2}\left(\hat{i} + \hat{k}\right)$

Position vector of point H is

$\overrightarrow{OH} = \frac{a}{2}\left(\hat{j} + \hat{k}\right)$

$\Rightarrow \quad \overrightarrow{GH} = \overrightarrow{OH} - \overrightarrow{OG} = \frac{a}{2}\left(\hat{j} - \hat{i}\right)$

Hence, the correct answer is (A).

3. Clearly, $\left|\vec{A} + \vec{B}\right|^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad \left|\vec{A} + \vec{B}\right|^2 = 2A^2(1 + \cos\theta)$ …(1)

and $\left|\vec{A} - \vec{B}\right|^2 = A^2 + B^2 - 2AB\cos\theta$

$\Rightarrow \quad \left|\vec{A} - \vec{B}\right|^2 = 2A^2(1 - \cos\theta)$ …(2)

Since, $\left|\vec{A} + \vec{B}\right|^2 = n^2\left|\vec{A} - \vec{B}\right|^2$

$\Rightarrow \quad \cancel{2A^2}(1 + \cos\theta) = n^2\cancel{2A^2}(1 - \cos\theta)$

$\Rightarrow \quad 1 + \cos\theta = n^2 - n^2\cos\theta$

$\Rightarrow \quad (1 + n^2)\cos\theta = n^2 - 1$

$\Rightarrow \quad \cos\theta = \frac{n^2 - 1}{n^2 + 1}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$

Hence, the correct answer is (B).

4. Given that $\left|\vec{P}\right| = 2F$, $\left|\vec{Q}\right| = 3F$

Let θ be the angle between $\vec{P}$ and $\vec{Q}$, then

$R^2 = \left|\vec{P} + \vec{Q}\right|^2 = P^2 + Q^2 + 2PQ\cos\theta$

where $\vec{R}$ is the resultant of $\vec{P}$ and $\vec{Q}$

$\Rightarrow \quad \left|\vec{P} + \vec{Q}\right|^2 = 4F^2 + 9F^2 + 12F^2\cos\theta = R^2$ …(1)

Now, when $\vec{Q}$ is doubled, then angle between $\vec{P}$ and $2\vec{Q}$ is still θ, but new resultant of $\vec{P}$ and $2\vec{Q}$ (say $\vec{R}'$) has a value twice the previous resultant. So,

$\left|\vec{R}'\right| = 2R$

$\Rightarrow \quad R'^2 = 4R^2 = P^2 + (2Q)^2 + 2P(2Q)\cos\theta$

$\Rightarrow \quad 4R^2 = (2F)^2 + (6F)^2 + 24F^2\cos\theta$ …(2)

From (1) and (2), we get

$F^2 + 9F^2 + 6F^2\cos\theta = 4F^2 + 9F^2 + 12F^2\cos\theta$

$\Rightarrow \quad -3F^2 = 6F^2\cos\theta$

$\Rightarrow \quad \cos\theta = \left(-\frac{1}{2}\right)$

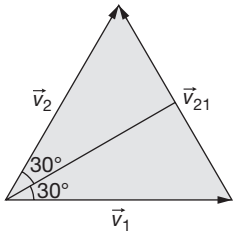
$\Rightarrow \quad \theta = 120°$

Hence, the correct answer is (D).

CHAPTER 3

5. From figure, $\vec{v}_1 + \vec{v}_{21} = \vec{v}_2$

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1$$



$$|\Delta\vec{v}| = |\vec{v}_{21}| = 2v\sin(30°)$$

$$\Rightarrow \quad |\Delta\vec{v}| = 2v \times \frac{1}{2} = v$$

Hence, the correct answer is (A).

6. If $\vec{C} = a\hat{i} + b\hat{j}$ then

$$\vec{A}\cdot\vec{C} = \vec{A}\cdot\vec{B}$$

$$\Rightarrow \quad a + b = 1 \qquad \ldots(1)$$

$$\vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{B}$$

$$\Rightarrow \quad 2a - b = 1 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$a = \frac{1}{3}, \; b = \frac{2}{3}$$

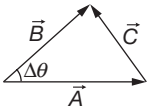
$$\Rightarrow \quad |\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$$

Hence, the correct answer is (B).

7. By triangle rule

$$\vec{A} + \vec{C} = \vec{B}$$

$$\Rightarrow \quad \vec{B} - \vec{A} = \vec{C}$$



$$\Rightarrow \quad |\vec{B} - \vec{A}| = |\vec{C}| = |\vec{B}|\sin\Delta\theta \qquad \{\because \Delta\theta \ll 1\}$$

$$\Rightarrow \quad |\vec{B} - \vec{A}| = |\vec{B}|\Delta\theta \qquad \{\because \text{for small } \Delta\theta, \sin\Delta\theta \simeq \Delta\theta\}$$

Again $|\vec{B}|\cos\Delta\theta = |\vec{A}|$

$$\Rightarrow \quad |\vec{B}| = |\vec{A}| \qquad \{\because \text{for small } \Delta\theta, \cos\Delta\theta \simeq 1\}$$

$$\Rightarrow \quad |\vec{B} - \vec{A}| = |\vec{B}|\Delta\theta = |\vec{A}|\Delta\theta$$

Hence, the correct answer is (C).

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Integer/Numerical Answer Type Questions

1. $\vec{A} = a\hat{i}$ and $\vec{B} = a\cos\omega t\hat{i} + a\sin\omega t\hat{j}$

$$\Rightarrow \quad \vec{A} + \vec{B} = (a + a\cos\omega t)\hat{i} + a\sin\omega t\hat{j} \text{ and}$$

$$\vec{A} - \vec{B} = (a - a\cos\omega t)\hat{i} + a\sin\omega t\hat{j}$$

Since, $|\vec{A} + \vec{B}| = \sqrt{3}|\vec{A} - \vec{B}|$

$$\Rightarrow \quad \sqrt{(a + a\cos\omega t)^2 + (a\sin\omega t)^2} = \sqrt{3}\sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$$

$$\Rightarrow \quad 2\cos\left(\frac{\omega t}{2}\right) = \pm\sqrt{3} \times 2\sin\left(\frac{\omega t}{2}\right)$$

$$\Rightarrow \quad \tan\left(\frac{\omega t}{2}\right) = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{\omega t}{2} = n\pi \pm \frac{\pi}{6}$$

$$\Rightarrow \quad \frac{\pi}{12}t = n\pi \pm \frac{\pi}{6}$$

$$\Rightarrow \quad t = (12n \pm 2) \text{ s}$$

$$\Rightarrow \quad t = 2 \text{ s}, 10 \text{ s}, 14 \text{ s and so on.}$$

CHAPTER 4: KINEMATICS I

Test Your Concepts-I (Based on Displacement, Velocity, Acceleration, Average Speed and Velocity)

1. $a = \dfrac{dv}{dt} = 2t - 1$

$\Rightarrow \quad \displaystyle\int_2^v dv = \int_0^t (2t-1)\,dt$

$\Rightarrow \quad v = t^2 - t + 2$

Since $v = \dfrac{dx}{dt}$

$\Rightarrow \quad \displaystyle\int_1^x dx = \int_0^t (t^2 - t + 2)\,dt$

$\Rightarrow \quad x = \dfrac{1}{3}t^3 - \dfrac{1}{2}t^2 + 2t + 1$

When $t = 6$ s,

$v = 32 \text{ ms}^{-1}$

$s = 67$ m

Since $v \neq 0$ at all the times, so

$d = 67 - 1 = 66$ m

2. Since $s = \dfrac{1}{2}at^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2s}{a}}$

So, average velocity $= \dfrac{s}{t} = \dfrac{s}{\sqrt{\frac{2s}{a}}} = \sqrt{\dfrac{as}{2}}$

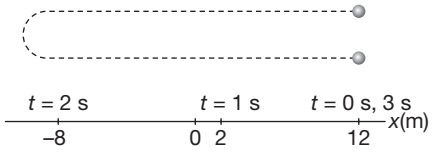
3. $|\vec{v}_{av}| = \dfrac{AB}{\Delta t} = \dfrac{2}{1} \text{ ms}^{-1} = 2 \text{ ms}^{-1}$

4. **Velocity:** $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(12 - 15t^2 + 5t^3)$

$\Rightarrow \quad v = -30t + 15t^2 \text{ ms}^{-1}$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. So,

$v = -30t + 15t^2 = 0$



$\Rightarrow \quad t(-30 + 15t) = 0$

$\Rightarrow \quad t = 0$ and 2 s

Position: The positions of the particle at $t = 0$ s, 1 s, 2 s and 3 s are

$$x\Big|_{t=0\text{ s}} = 12 - 15(0^2) + 5(0^3) = 12 \text{ m}$$

$$x\Big|_{t=1\text{ s}} = 12 - 15(1^2) + 5(1^3) = 2 \text{ m}$$

$$x\Big|_{t=2\text{ s}} = 12 - 15(2^2) + 5(2^3) = -8 \text{ m}$$

$$x\Big|_{t=3\text{ s}} = 12 - 15(3^2) + 5(3^3) = 12 \text{ m}$$

Using the above results, the path of the particle is shown. From this figure, the distance travelled by the particle during the time interval $t = 1$ s to $t = 3$ s is

$x_{\text{Tot}} = (2 + 8) + (8 + 12) = 30$ m

The average speed of the particle during the same time interval is

$v_{\text{avg}} = \dfrac{x_{\text{Tot}}}{\Delta t} = \dfrac{30}{3 - 1} = 15 \text{ ms}^{-1}$

5. (a) $c \equiv \dfrac{x}{t^2}$

Unit of c is ms^{-2}

Since, $b \equiv \dfrac{x}{t^3}$

Unit of b is ms^{-3}

(b) $x = 3t^2 - 2t^3$ …(1)

For x to be maximum, $\dfrac{dx}{dt} = 0$

$\Rightarrow \quad 6t - 6t^2 = 0$

$\Rightarrow \quad t = 0, 1$ s

Further $\dfrac{d^2x}{dt^2} = 6 - 12t$

At $t = 1\text{ s}$, $\dfrac{d^2x}{dt^2} = -6$

So, at $t = 1\text{ s}$, x is maximum.

(c) $x|_{t=0} = 0$

$x|_{t=1\text{ s}} = 1\text{ m}$

$x|_{t=4\text{ s}} = -80\text{ m}$

$\Rightarrow \quad x_{\text{Total}} = 1 + 1 + 80 = 82\text{ m}$

At $t = 4\text{ s}$, displacement $x = -80\text{ m}$

(d) From equation (1), we get

$v = 6t - 6t^2$ and $a = 6 - 12t$. So,

at $t = 0$, $v = 0$ and $a = 6\text{ ms}^{-2}$

at $t = 1\text{ s}$, $v = 0$ and $a = -6\text{ ms}^{-2}$

at $t = 2\text{ s}$, $v = -12\text{ ms}^{-1}$ and $a = -18\text{ ms}^{-2}$

at $t = 3\text{ s}$, $v = -36\text{ ms}^{-1}$ and $a = -30\text{ ms}^{-2}$

at $t = 4\text{ s}$, $v = -72\text{ ms}^{-1}$ and $a = -42\text{ ms}^{-2}$

6. $v = \dfrac{dx}{dt} - 3t^2 - 18t + 15$

$a = \dfrac{dv}{dt} = 6t - 18$

Acceleration at $t = 0$ is -18 ms^{-2} at $t = 3\text{ s}$ it is zero. Then it continuously goes on increasing. Hence acceleration and velocity both are maximum at $t = 10\text{ s}$. So,

$v_{\max} = 3 \times (10)^2 - 18(10) + 15 = 135\text{ ms}^{-1}$ and

$a_{\max} = 6 \times 10 - 18 = 42\text{ ms}^{-2}$

7. $\vec{v}_{av} = \dfrac{\vec{r}_f - \vec{r}_i}{\Delta t} = \dfrac{\left(5.1\hat{i} + 0.4\hat{j}\right) - \left(5\hat{i}\right)}{0.02} = 5\hat{i} + 20\hat{j}$

$|\vec{v}_{av}| = \sqrt{(20)^2 + (5)^2} = 20.6\text{ ms}^{-1}$

$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{20}{5}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{20}{5}\right) = 76°$

8. The displacement from A to C is

$\Delta x = x_C - x_A = -6 - (-8) = 2\text{ m}$

So, average velocity is

$v_{\text{avg}} = \dfrac{\Delta_x}{\Delta t} = \dfrac{2}{4+5} = 0.222\text{ ms}^{-1}$

The distances travelled from A to B and B to C are $x_{A\to B} = 8 + 3 = 11\text{ m}$ and $x_{B\to C} = 3 + 6 = 9\text{ m}$, respectively. Then, the total distance travelled is

$x_{\text{Total}} = x_{A\to B} + x_{B\to C} = 11 + 9 = 20\text{ m}$

$\left(v_{sp}\right)_{avg} = \dfrac{x_{\text{Total}}}{\Delta t} = \dfrac{20}{4+5} = 2.22\text{ ms}^{-1}$

9. (a) $a = \dfrac{dv}{dt} = 6t - 6$

At $t = 2\text{ s}$, $v = 0$

From $t = 0$ to $t = 2\text{ s}$, particle travels along negative x-direction. Then it moves towards positive x-axis. Since

$v = \dfrac{dx}{dt} = 3t^2 - 6t$

$\Rightarrow \quad \displaystyle\int_0^x dx = \int_0^t (3t^2 - 6t)\,dt$

$\Rightarrow \quad x = t^3 - 3t^2$

At $t = 2\text{ s}$, $x = -4\text{ m}$

At $t = 3.5\text{ s}$, $x = 6.125\text{ m}$

Distance travelled, $x_{\text{Total}} = 4 + 4 + 6.125 = 14.125\text{ m}$

(b) Average velocity $= \dfrac{6.125}{3.5} = 1.75\text{ ms}^{-1}$ and

Average speed $= \dfrac{14.125}{3.5} = 4.03\text{ ms}^{-1}$

10. **Position:** The position of the particle when $t = 6\text{ s}$ is

$x\Big|_{t=6\text{ s}} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27\text{ m}$

Total Distance Travelled: The velocity of the particle can be determined by applying.

$v = \dfrac{dx}{dt} = 4.5t^2 - 27t + 22.5$

The times when the particle stops are obtained by substituting $v = 0$. So,

$4.5t^2 - 27t + 22.5 = 0$

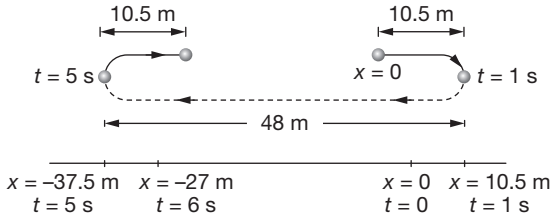
$\Rightarrow \quad t = 1\text{ s}$ and $t = 5\text{ s}$

The position of the particle at $t = 0\text{ s}$, 1 s and 5 s are

$x\Big|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$

$$x\Big|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ m}$$

$$x\Big|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ m}$$



From the particle's path, the total distance travelled is

$$x_{\text{total}} = (10.5) + [10.5 - (-37.5)] + (10.5)$$

$$x_{\text{total}} = 10.5 + 48 + 10.5 = 69 \text{ m}$$

You can also try obtaining the same result by using

$$x = \frac{\left|\int_0^1 v\,dt\right| + \left|\int_1^5 v\,dt\right|}{6} = 69 \text{ m}$$

11. According to the problem, we are given that

$$\frac{dv}{dx} = -3 \text{ ms}^{-1} \text{ per metre}$$

Since $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad a = \frac{dv}{dt} = v\frac{dx}{dt} = (10)(-3) = -30 \text{ ms}^{-2}$$

Test Your Concepts-II (Based on Constant Acceleration)

1. Since, $s_{n^{\text{th}}} = u + \frac{a}{2}(2n-1)$

$$\Rightarrow \quad x = u + \frac{a}{2}(2p-1) \qquad \ldots(1)$$

$$\Rightarrow \quad y = u + \frac{a}{2}(2q-1) \qquad \ldots(2)$$

$$\Rightarrow \quad z = u + \frac{a}{2}(2r-1) \qquad \ldots(3)$$

Subtracting equation (3) from equation (2), we get

$$y - z = \frac{a}{2}(2q - 2r)$$

$$\Rightarrow \quad q - r = \frac{y-z}{a}$$

$$\Rightarrow \quad (q-r)x = \frac{1}{a}(yx - zx) \qquad \ldots(4)$$

Similarly, we can show that

$$(r-p)y = \frac{1}{a}(zy - xy) \text{ and} \qquad \ldots(5)$$

$$(p-q)z = \frac{1}{a}(xz - yz) \qquad \ldots(6)$$

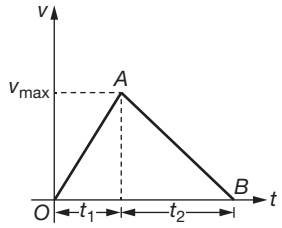
Adding equation (4), (5) and (6), we get

$$(q-r)x + (r-p)y + (p-q)z = 0$$

2. (a) Let the car accelerates for time t_1 and decelerates for time t_2. Then

$$t = t_1 + t_2 \qquad \ldots(1)$$

and corresponding velocity-time graph will be as shown in figure.



Since

$$v_{\max} = \alpha t_1$$

$$\Rightarrow \quad t_1 = \frac{v_{\max}}{\alpha} \qquad \ldots(2)$$

and $0 = v_{\max} + (-\beta)t_2$

$$\Rightarrow \quad t_2 = \frac{v_{\max}}{\beta} \qquad \ldots(3)$$

From equations (1), (2) and (3), we get

$$\frac{v_{\max}}{\alpha} + \frac{v_{\max}}{\beta} = t$$

$$\Rightarrow \quad v_{\max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$$

$$\Rightarrow \quad v_{\max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$$

(b) Total distance or displacement is the area under v-t graph

$$\Rightarrow \quad s = \frac{1}{2}v_{\max}t$$

$$\Rightarrow \quad s = \frac{1}{2}\left(\frac{\alpha\beta t}{\alpha+\beta}\right)t$$

$$\Rightarrow \quad \text{Distance} = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$$

CHAPTER 4

3. As acceleration is constant, from $s = ut + \frac{1}{2}at^2$, we have

$$x = \frac{1}{2}at^2 \qquad (\text{as } u = 0) \qquad \ldots(1)$$

Now, if it travels a distance y in next t s then the total distance travelled in $(2t)$ sec will be $x + y$, so

$$x + y = \frac{1}{2}a(2t)^2 \qquad \ldots(2)$$

Dividing equation (2) by (1),

$$\frac{x+y}{x} = 4$$

$$\Rightarrow \quad y = 3x$$

4. Let s be the distance covered by each car. Let the times taken by the two cars to complete the journey be t_A and t_B and their velocities at the finishing point be v_A and v_B respectively. According to the problem,

$$v_A - v_B = v \text{ and } t_B - t_A = t$$

Now, $\frac{v}{t} = \frac{v_A - v_B}{t_B - t_A}$

Where $v_A = \sqrt{2a_1 s}$ and $v_B = \sqrt{2a_2 s}$

$$t_A = \sqrt{\frac{2s}{a_1}} \text{ and } t_B = \sqrt{\frac{2s}{a_2}}$$

$$\Rightarrow \quad \frac{v}{t} = \frac{\sqrt{2a_1 s} - \sqrt{2a_2 s}}{\sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}}} = \frac{\sqrt{a_1} - \sqrt{a_2}}{\sqrt{\frac{1}{a_2}} - \sqrt{\frac{1}{a_1}}} = \sqrt{a_1 a_2}$$

$$\Rightarrow \quad v = t\sqrt{a_1 a_2}$$

5. Since the race ends in a dead heat i.e., the two cars reach their destination in the same time say t. If the length of the course be s. Then

$$s = v_1 t + \frac{1}{2}a_1 t^2 = v_2 t + \frac{1}{2}a_2 t^2$$

$$\Rightarrow \quad t\left[\frac{1}{2}(a_1 - a_2)t + (v_1 - v_2)\right] = 0$$

$$\Rightarrow \quad t = 0 \text{ or } t = -\frac{2(v_1 - v_2)}{(a_1 - a_2)}$$

But $t \neq 0$, as it corresponds to initial position. So,

$$t = -\frac{2(v_1 - v_2)}{(a_1 - a_2)}$$

Substituting t in the equation, $s = v_1 t + \frac{1}{2}a_1 t^2$, we get

$$s = -\frac{2v_1(v_1 - v_2)}{(a_1 - a_2)} + \frac{1}{2}a_1 \frac{4(v_1 - v_2)^2}{(a_1 - a_2)^2}$$

$$\Rightarrow \quad s = \frac{2(v_1 - v_2)}{(a_1 - a_2)^2}\left[-v_1(a_1 - a_2) + a_1(v_1 - v_2)\right]$$

$$\Rightarrow \quad s = \frac{2(v_1 - v_2)(v_1 a_2 - v_2 a_1)}{(a_1 - a_2)^2}$$

6. (a) $(15)^2 - v_1^2 = 2\left(\frac{5}{3}\right)(60) \qquad \{\because v^2 - u^2 = 2as\}$

$$\Rightarrow \quad v_1 = 5 \text{ ms}^{-1}$$

(b) Since $v = u + at$

$$\Rightarrow \quad 15 = v_1 + 6a \qquad \ldots(1)$$

Since $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad 60 = 6v_1 + \frac{1}{2} \times a \times (6)^2$$

$$\Rightarrow \quad 10 = v_1 + 3a \qquad \ldots(2)$$

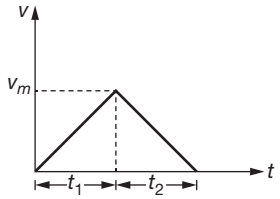
Solving equation (1) and (2), we get

$$a = \frac{5}{3} = 1.67 \text{ ms}^{-2}$$

(c) $v_1^2 = 2as$

$$\Rightarrow \quad s = \frac{v_1^2}{2a} = \frac{25}{\frac{10}{3}} = 7.5 \text{ m}$$

7. The corresponding v-t graph is shown in figure



Given $t_1 + t_2 = 4$ min $\qquad \ldots(1)$

Since, $\frac{v_m}{t_1} = x$ and $\frac{v_m}{t_2} = y$

$$\Rightarrow \quad t_1 = \frac{v_m}{x} \text{ and } t_2 = \frac{v_m}{y}$$

Since $s_1 = \left(\frac{0 + v_m}{2}\right)t_1$

and $s_2 = \left(\frac{v_m + 0}{2}\right)t_2$

$$\Rightarrow \quad \frac{1}{2}(t_1+t_2)(v_m)=4 \text{ km}$$

$$\Rightarrow \quad \frac{1}{2}\times 4\times v_m=4$$

$$\Rightarrow \quad v_m=2 \text{ kmmin}^{-1}$$

Substituting these values in equation (1), we get

$$\frac{2}{x}+\frac{2}{y}=4$$

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{y}=2$$

Please see that this problem can also be done by using the concept of graphs.

8. Let t_0 be the reaction time and a the magnitude of deceleration. Then in the first case,

$$0=\left(80.5\times\frac{5}{18}\right)^2-2a\left(56.7-80.5\times\frac{5}{18}\times t_0\right) \quad \ldots(1)$$

In the second case, we have

$$0=\left(48.3\times\frac{5}{18}\right)^2-2a\left(24.4-48.3\times\frac{5}{18}\times t_0\right) \quad \ldots(2)$$

Solving these two equations, we get

$$t_0=0.74 \text{ s} \text{ and } a=6.2 \text{ ms}^{-2}$$

9. $s=\frac{1}{2}at_0^2$ and $v=at_0$

Since t is measured from the beginning of motion, so we have for the particle to return to the initial position,

$$-s=v(t-t_0)-\frac{1}{2}a(t-t_0)^2$$

$$\Rightarrow \quad -\frac{1}{2}at_0^2=at_0(t-t_0)-\frac{1}{2}a(t-t_0)^2$$

$$\left\{\because\ s=\frac{1}{2}at_0^2 \text{ and } v=at_0\right\}$$

$$\Rightarrow \quad -\cancel{\frac{1}{2}at_0^2}=at_0t-at_0^2-\frac{1}{2}at^2-\cancel{\frac{1}{2}at_0^2}+at_0t$$

$$\Rightarrow \quad -\frac{1}{2}at^2+2at_0t-at_0^2=0$$

$$\Rightarrow \quad t^2-4t_0t-2t_0^2=0$$

$$\Rightarrow \quad t=\frac{4t_0\pm\sqrt{16t_0^2+8t_0^2}}{2}$$

$$\Rightarrow \quad t=\frac{4t_0+\sqrt{24}t_0}{2}$$

$$\Rightarrow \quad t=\frac{4t_0+2\sqrt{6}t_0}{2}$$

$$\Rightarrow \quad t=t_0+\sqrt{6}t_0$$

$$\Rightarrow \quad t=3.45t_0$$

10. Squaring the given equation, we get

$$v^2=4+4x$$

Now, comparing it with

$$v^2=u^2+2as$$

we get, $u=2 \text{ ms}^{-1}$ and $a=2 \text{ ms}^{-2}$

So, displacement at $t=2\ s$ is given by

$$s=ut+\frac{1}{2}at^2$$

$$\Rightarrow \quad s=(2)(2)+\frac{1}{2}(2)(2)^2$$

$$\Rightarrow \quad s=8 \text{ m}$$

11. Average Speed $=v_{av}=\dfrac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$$\Rightarrow \quad v_{av}=\frac{l+2l+3l}{t_1+t_2+t_3} \quad \ldots(1)$$

For *OA*

$l=\text{Area }\Delta(OAM)$

$$\Rightarrow \quad l=\frac{1}{2}v_{\max}t_1$$

$$\Rightarrow \quad t_1=\frac{2l}{v_{\max}} \quad \ldots(2)$$

For *AB*

$2l=\text{Area}(\text{ABNM})$

$$\Rightarrow \quad 2l=v_{\max}t_2$$

$$\Rightarrow \quad t_2=\frac{2l}{v_{\max}} \quad \ldots(3)$$

For *BC*

$3l=\text{Area }\Delta(BCN)$

$$\Rightarrow \quad 3l=\frac{1}{2}v_{\max}t_3$$

$$\Rightarrow \quad t_3=\frac{6l}{v_{\max}} \quad \ldots(4)$$

CHAPTER 4

Substituting (2), (3) and (4) in (1), we get

$$\frac{v_{av}}{v_{\max}} = \frac{3}{5}$$

12. Let a be the uniform acceleration of α-particle. According to the problem $s = 2$ m, $v = 9000$ ms^{-1} and $u = 1000$ ms^{-1}

Since $v^2 - u^2 = 2as$, so we get

$$a = \frac{v^2 - u^2}{2s} = \frac{(9000)^2 - (1000)^2}{2 \times (2)}$$

$$\Rightarrow \quad a = \frac{8 \times 10^7}{4} = 2 \times 10^7 \text{ ms}^{-2}$$

Let the particle remains in the tube for time t. Then

$$v = u + at$$

$$\Rightarrow \quad t = \frac{v - u}{a} = \frac{9000 - 1000}{2 \times 10^7} = 4 \times 10^{-4} \text{ s}$$

13. (a) Suppose the time of retardation of sports car be t hours. Thus the time during which the sports car moves with constant speed before being overtaken is also t. Since

$$s_{\text{total}} = 1 \text{ km}$$

$$\Rightarrow \quad \left(\frac{60 + 40}{2}\right)t + 40t = 1 \text{ km}$$

$$\Rightarrow \quad t = \frac{1}{90} \text{ h}$$

Time taken by police car to reach sports car is

$$2t = \frac{1}{45} \text{ h} = 80 \text{ s}$$

(b) Let the speed of the police car be v, when it overtakes the sports car. Then

$$\frac{v}{2} \times 2t = 1 \text{ km}$$

$$\Rightarrow \quad vt = 1 \text{ km}$$

$$\Rightarrow \quad \frac{v}{90} = 1 \text{ km}$$

$$\Rightarrow \quad v = 90 \text{ kmh}^{-1}$$

(c) Let a be the acceleration of police car, then

$$90 = \frac{a}{45}$$

$$\Rightarrow \quad a = 90 \times 45 \text{ kmh}^{-2}$$

If the retardation of the sports car be a', then

$$40 = 60 - \frac{a'}{90}$$

$$\Rightarrow \quad a' = 1800 \text{ kmh}^{-2}$$

Let t_1 be the required time, then

$$90 \times 45 \times t_1 = 60 - 1800t_1$$

$$\Rightarrow \quad t_1 = \frac{60}{5850} \text{ h} = 37 \text{ s}$$

14. (a) Time of acceleration, say t_1 is

$$t_1 = \frac{v_{\max}}{a_1} = \frac{90 \text{ kmh}^{-1}}{3 \text{ kmh}^{-1} \text{ per second}} = 30 \text{ s} = 0.5 \text{ min}$$

Time for deceleration, say t_2 is

$$t_2 = \frac{v_{\max}}{a_2} = \frac{90 \text{ kmh}^{-1}}{3 \text{ kmh}^{-1} \text{ per second}}$$

$$t_2 = 18 \text{ s} = 0.3 \text{ min}$$

Now we shall calculate the distance covered during acceleration and deceleration

Distance covered during acceleration is

$$s_1 = \left(\frac{0 + v_{\max}}{2}\right)t_1$$

$$\Rightarrow \quad s_1 = \frac{1}{2}\left(90 \times \frac{5}{18}\right) = 375 \text{ m}$$

Distance covered during deceleration is

$$s_3 = \left(\frac{v_{\max} + 0}{2}\right)t_2$$

$$\Rightarrow \quad s_3 = \frac{1}{2}\left(90 \times \frac{5}{18}\right)(18) = 225 \text{ m}$$

Distance covered with maximum speed is

$$s_2 = 50000 - 375 - 225 = 49400 \text{ m}$$

$$\Rightarrow \quad s_2 = 49.4 \text{ km}$$

So, total time for a non-stop train is

$$T = 0.5 \text{ min} + \frac{49.4 \times 60}{90} \text{ min} + 0.3 \text{ min}$$

$$\Rightarrow \quad T = 33.7 \text{ min}$$

(b) 12 intermediate stations mean that the train has to accelerate and decelerate at 13 stations (including the start). So, combined time of deceleration and acceleration is $t_1 + t_3 = 0.8 \times 13 \text{ min} = 10.4 \text{ min}$. Distance covered in acceleration and deceleration.

$$s = (375 + 225) \times 13 = 7800 \text{ m} = 7.8 \text{ km}$$

Time for uniform speed is $t_2 = (50 - 7.8)\left(\frac{60}{90}\right)$

$$\Rightarrow \quad t_2 = 28.13 \text{ min}$$

Time in motion $T = 10.4 + 28.13 = 38.53$ min

Halting time at 12 stations is $t_{halt} = 12 \times \frac{1}{2} = 6$ min

Toal time $T = t_1 + t_2 + t_3 + t_{halt}$

$\Rightarrow \quad T = (38.53 + 6)$ min $= 44.53$ min

15. Let x be the distance travelled by the truck when truck and car are side by side. The distance travelled by the car will be $(x + 150)$ as the car is 150 metre behind the truck.

Let the car overtake the truck at time t, then

For Truck

$$x = \frac{1}{2} \times (1.5)t^2 \qquad \ldots(1)$$

For Car

$$(x + 150) = \frac{1}{2} \times (2)t^2 \qquad \ldots(2)$$

From equations (1) and (2), we have

$$\frac{x + 150}{x} = \frac{2}{1.5}$$

$\Rightarrow \quad x = 450$ metre (truck)

and $x + 150 = 600$ metre (car)

Substituting the value of x in equation (1), we get

$$450 = \frac{1}{2}(1.5)t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{450 \times 2}{1.5}}$$

$$\Rightarrow \quad t = \sqrt{600} = 24.5 \text{ s}$$

16. Let x_1 and x_2 be the distances travelled by the car before they stop under deceleration.

Since $v^2 - u^2 = 2as$, so we get

$$(0)^2 - (10)^2 = 2(-2)x_1$$

and $(0)^2 - (12)^2 = 2(-2)x_2$

Solving we get $x_1 = 25$ m and $x_2 = 36$ m

Total distance covered by the two cars

$$x_1 + x_2 = 25 + 36 = 61 \text{ metre}$$

Distance between the two cars when they stop

$$x = 150 - 61 = 89 \text{ m}$$

17. Let car A take t_1 hour for first half and t_2 hour for second half.

Since first half distance = second half distance

$\Rightarrow \quad 30t_1 = 60t_2$

$\Rightarrow \quad t_1 = 2t_2$

further $t_1 + t_2 = 2$

$\Rightarrow \quad t_1 = \frac{4}{3}$ h and $t_2 = \frac{2}{3}$ h

Total distance $x = 30\left(\frac{4}{3}\right) + 60\left(\frac{2}{3}\right) = 80$ km

Let a be the acceleration of car B. Since $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad 80 = 0 \times 2 + \frac{1}{2} \times a \times (2)^2$$

$$\Rightarrow \quad a = 40 \text{ kmh}^{-2}$$

To have the **same velocity**, we have

In interval 1, $at_1' = v_1$

$$\Rightarrow \quad 40t_1' = 30$$

$$\Rightarrow \quad t_1' = \frac{3}{4} \text{ h} \qquad \left(t_1 < \frac{4}{3}\right)$$

and in interval 2, $at_2' = v_2$

$$40t_2' = 60$$

$$\Rightarrow \quad t_2' = \frac{3}{2} \text{ h} \qquad \left(\frac{4}{3} < t_2 < 2\right)$$

To overtake, the distance travelled by both the cars in the first half is the same

$$\Rightarrow \quad \frac{1}{2}(40)t^2 = 30t$$

$$t = \frac{3}{2}h\left(> \frac{4}{3}h\right)$$

So no over taking in first half.

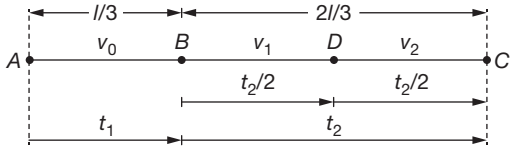
For second half, again we have $s_A = s_B$

$$\Rightarrow \quad 30\left(\frac{4}{3}\right) + 60\left(t - \frac{4}{3}\right) = \frac{1}{2}(40)t^2$$

$\Rightarrow \quad t = 1h\left(< \frac{4}{3}h\right)$ not possible and $t = 2$, i.e., at the end of the journey

Hence there is no overtaking during the entire journey.

18. The situation is shown in figure



Since $v_{av} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$$\Rightarrow \quad v_{av} = \frac{\frac{l}{3} + \frac{2l}{3}}{t_1 + t_2} \qquad \ldots(1)$$

CHAPTER 4

For *AB*

$$\frac{l}{3} = v_0 t_1$$

$$\Rightarrow \quad t_1 = \frac{l}{3v_0} \qquad \ldots(2)$$

For *BC*

$$\frac{2l}{3} = v_1\left(\frac{t_2}{2}\right) + v_2\left(\frac{t_2}{2}\right)$$

$$\Rightarrow \quad t_2 = \frac{4l}{3(v_1 + v_2)} \qquad \ldots(3)$$

Substituting (2) and (3) in (1), we get

$$v_{av} = \frac{3v_0(v_1 + v_2)}{4v_0 + v_1 + v_2}$$

19. Let the man take t s to get the door. The distance moved by the man in t s should therefore be 6 m more than the distance moved by the bus in the same time.

Distance moved by the man in t s is $x_{man} = 4t$

Distance moved by the bus in t s is

$$x_{bus} = \frac{1}{2} \times 1.2 \times t^2 = 0.6t^2$$

Since $x_{bus} = x_{man}$

$$\Rightarrow \quad 4t = 6 + 0.6t^2$$

Solving, we get

$$t = 4.387 \text{ s}$$

$$\Rightarrow \quad 2.27 \text{ s}$$

If the man is 10 m behind the door, then $4t = 10 + 0.6t^2$. This equation has imaginary roots.

20. For car A, $u_1 = 8 \text{ ms}^{-1}$, $a_1 = 1 \text{ ms}^{-2}$

For car B, $u_2 = 5 \text{ ms}^{-1}$, $a_2 = 1.1 \text{ ms}^{-2}$

Let x be the length of the track, then

$$x = 8t + \left(\frac{1}{2}\right)(1)t^2 \text{ and } x = 5t + \frac{1}{2}(1.1)t^2$$

$$\Rightarrow \quad t = 60 \text{ s}$$

Now, $x = 8 \times 60 + \left(\frac{1}{2}\right)(1)(60)^2$

$$\Rightarrow \quad x = 2280 \text{ m}$$

Further, before 10 s, i.e., in $60 - 10 = 50$ s

Car A has travelled a distance $s_A = 1650$ m and car B has travelled a distance $s_B = 1625$ m

21. $25 = 0 + (1.5)t_1$

$$\Rightarrow \quad t_1 = \frac{50}{3} \text{ s}$$

$$\Rightarrow \quad s_1 = 0 + \frac{1}{2}(1.5)\left(\frac{2500}{9}\right) = \frac{1875}{9} \text{ m}$$

$$\Rightarrow \quad s_1 = \frac{1875}{9} \text{ m}$$

Distance covered in the next 60 seconds is

$$s_2 = (25)(60) = 1500 \text{ m}$$

$$\text{Average Speed} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{1500 + \frac{1875}{9}}{\frac{50}{3} + 60}$$

$$\Rightarrow \quad v_{av} = \frac{1708.33}{76.67}$$

$$\Rightarrow \quad v_{av} = 22.3 \text{ ms}^{-1}$$

22. The corresponding v-t graph is shown in figure.

For accelerated interval,

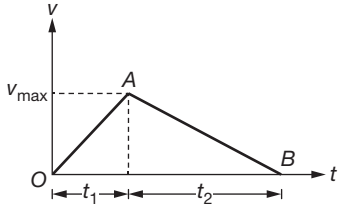
$$v_{max} = 0 + (0.2)t_1$$

$$\Rightarrow \quad t_1 = 5 \text{ v}_{max} \qquad \ldots(1)$$

For decelerated interval,

$$0 = v_{max} + (-0.1)t_2$$

$$\Rightarrow \quad t_2 = 10 \text{ v}_{max} \qquad \ldots(2)$$



Further,

$$s_1 + s_2 = 14$$

$$\Rightarrow \quad \left(\frac{0 + v_{max}}{2}\right)t_1 + \left(\frac{v_{max} + 0}{2}\right)t_2 = 14$$

$$\Rightarrow \quad \frac{1}{2}(t_1 + t_2)v_{max} = 14$$

$$\Rightarrow \quad \frac{1}{2}(15 \text{ v}_{max}^2) = 14$$

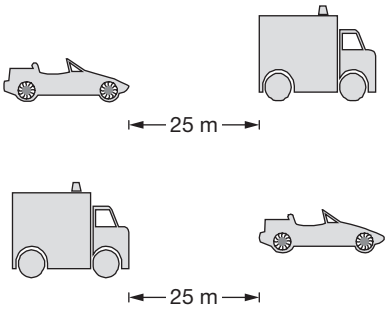
$$\Rightarrow \quad v_{max}^2 = \frac{28}{15}$$

$\Rightarrow \quad v_{max} = 1.37 \text{ ms}^{-1}$

This v_{max} happens to be less than the maximum attainable speed of 2.5 ms^{-1}. So,

$$t_{MIN} = t_1 + t_2 = 15 \; v_{max} = 20.6 \text{ ms}^{-1}$$

23.



(a) $s_{Car} = s_{Truck} + 25 + 25 + \begin{pmatrix}\text{length}\\ \text{of car}\end{pmatrix} + \begin{pmatrix}\text{length}\\ \text{of truck}\end{pmatrix}$

$\Rightarrow \quad 20t + \frac{1}{2}(0.6)t^2 = (20t) + 50 + 5 + 20$

$\Rightarrow \quad t = 15.8 \text{ s}$

(b) $s_{Car} = (20)(15.8) + \frac{1}{2}(0.6)(15.8)^2 = 391 \text{ m}$

(c) $v_{Car} = 20 + (0.6)(15.8) = 29.5 \text{ ms}^{-1}$

24. Velocity of train after 30 second,

$$v_T = at = 30 \times 3 \times 10^{-2} = 0.9 \text{ ms}^{-1}$$

Let v_C be the velocity of the car in opposite direction. Then, in the next 60 s,

$$348 = \left|\begin{matrix}\text{displacement}\\ \text{of train}\end{matrix}\right| + \left|\begin{matrix}\text{Displacement of the car}\\ \text{in opposite direction}\end{matrix}\right|$$

$\Rightarrow \quad 348 = (0.9)(60) + \frac{1}{2}(3 \times 10^{-2})(3600) + 60v_c$

$\Rightarrow \quad v_C = 4 \text{ ms}^{-1}$

Thus velocity of the car should be 4 ms^{-1}, opposite to the direction of motion of the train.

25. Let the initial velocity of the particle (i.e., at $t = 0$) be u and acceleration be a. Since at $t = 0$, displacement measured from a convenient fixed position is 2 m and at $t = 10$ s it is zero. So displacement from $t = 0$ to $t = 10$ s is -2 m. Since

$$s = ut + \frac{1}{2}at^2$$

$\Rightarrow \quad -2 = 10u + \frac{1}{2}a(10)^2$

$\Rightarrow \quad 5u + 25a = -1 \qquad \ldots(1)$

Further the particle reverses its direction of motion at $t = 6$ s, so

$$0 = u + 6a$$

$$5u + 30a = 0 \qquad \ldots(2)$$

Subtracting (1) from (2), we get

$$5a = 1$$

$\Rightarrow \quad a = \frac{1}{5}\text{ms}^{-2} = 0.2\text{ms}^{-2}$ and $u = -6a = -1.2 \text{ ms}^{-1}$

So, if v be the velocity at $t = 10$ s, then

$$v = u + at$$

$\Rightarrow \quad v = -1.2 + (0.2)(10) = 0.8 \text{ ms}^{-1}$

Test Your Concepts-III (Based on Variable Acceleration)

1. Since $a = \frac{dv}{dt}$

$\Rightarrow \quad 30 - \frac{v}{5} = \frac{dv}{dt}$

$\Rightarrow \quad \frac{dv}{\left(30 - \frac{v}{5}\right)} = dt$

$\Rightarrow \quad \int_0^{30} \frac{dv}{30 - \frac{v}{5}} = \int_0^t dt$

$\Rightarrow \quad -5\log_e\left(30 - \frac{v}{5}\right)\Big|_0^{30} = t\Big|_0^t$

$\Rightarrow \quad -5\log_e\left(\frac{24}{30}\right) = t$

$\Rightarrow \quad t = 5\log_e\left(\frac{5}{4}\right) \text{ s}$

2. $a = 2s$

$\Rightarrow \quad v\frac{dv}{ds} = 2s$

$\Rightarrow \quad vdv = 2s\,ds$

$\Rightarrow \quad \int_0^v vdv = \int_0^s 2s\,ds$

$\Rightarrow \quad \left(\frac{v^2}{2}\right)\Big|_0^v = \left(s^2\right)\Big|_0^s$

$\Rightarrow \quad \frac{v^2}{2} = s^2$

CHAPTER 4

$\Rightarrow \quad v = \pm\sqrt{2}s$

This is the desired velocity-displacement equation.

3. Since $v = v_0 - kx$

$$\Rightarrow \quad \frac{dx}{dt} = v_0 - kx$$

$$\Rightarrow \quad \int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{k}\log_e(v_0 - kx)\Big|_0^x = t\Big|_0^t$$

$$\Rightarrow \quad -\frac{1}{k}\log_e\left(\frac{v_0 - kx}{v_0}\right) = t$$

$$\Rightarrow \quad \frac{v_0 - kx}{v_0} = e^{-kt}$$

$$\Rightarrow \quad x = \frac{v_0}{k}(1 - e^{-kt}) \qquad \ldots(1)$$

Since $v = \frac{dx}{dt} = \frac{d}{dt}\left[\frac{v_0}{k}(1 - e^{-kt})\right]$

$$\Rightarrow \quad v = v_0 e^{-kt}$$

Also, $a = \frac{dv}{dt} = \frac{d}{dt}(v_0 e^{-kt})$

$$\Rightarrow \quad a = -kv_0 e^{-kt}$$

4. Given $a = -4s$

$$\Rightarrow \quad v\frac{dv}{ds} = -4s$$

$$\Rightarrow \quad \int_{v_0}^{0} v\,dv = -4\int_0^s s\,ds$$

$$\Rightarrow \quad -\frac{v_0^2}{2} = -4\frac{s^2}{2}$$

$$\Rightarrow \quad s = 2v_0$$

Students remain often confused that the equation $s = 2v_0$ is not dimensionally correct. Please do not miss to write dimensions of proportionality constant.

5. Since $a = \frac{dv}{dt}$

$$\Rightarrow \quad v\,dv = a\,dx \qquad \left\{\because\ a = v\frac{dv}{dx}\right\}$$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^x (8 - 2x)\,dx$$

$$\Rightarrow \quad \frac{v^2}{2}\Big|_0^v = (8x - x^2)\Big|_0^x$$

$$\Rightarrow \quad v = \sqrt{16x - 2x^2}\ \text{ms}^{-1} \qquad \ldots(1)$$

At $x = 2$ m,

$$v\Big|_{x=2\text{ m}} = \sqrt{16(2) - 2(2^2)} = \pm 2\sqrt{6}\ \text{ms}^{-1}$$

When the velocity is maximum then

$$\frac{dv}{dx} = 0$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{16 - 4x}{2\sqrt{16x - 2x^2}} = 0$$

$$\Rightarrow \quad 16 - 4x = 0$$

$$\Rightarrow \quad x = 4\ \text{m}$$

Substituting $x = 4$ m in (1), we get

$$|v_{MAX}| = \sqrt{16(4) - 2(16)} = \sqrt{64 - 32}$$

$$\Rightarrow \quad |v_{MAX}| = \sqrt{32} = \pm 4\sqrt{2}\ \text{m}$$

6. $a = \frac{dv}{dt}$

$$\Rightarrow \quad \frac{k}{v} = \frac{dv}{dt}$$

$$\Rightarrow \quad v\,dv = k\,dt$$

$$\Rightarrow \quad \int_{v_0}^{v} v\,dv = k\int_0^t dt$$

$$\Rightarrow \quad \frac{v^2}{2}\Big|_{v_0}^{v} = kt\Big|_0^t$$

$$\Rightarrow \quad v^2 - v_0^2 = 2kt$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + 2kt}$$

7. Initial Acceleration $= \frac{dv}{dt}\Big|_{t=0} = A - B(0) = A$

$$a = 0$$

$$\Rightarrow \quad A - Bv = 0$$

$$\Rightarrow \quad v = \frac{A}{B}$$

Since $\frac{dv}{dt} = A - Bv$

$$\Rightarrow \quad \int_0^v \frac{dv}{A - Bv} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{B}\log_e(A-Bv)\Big|_0^v = t$$

$$\Rightarrow \quad \log_e\left(\frac{A-Bv}{A}\right) = -Bt$$

$$\Rightarrow \quad A - Bv = Ae^{-Bt}$$

$$\Rightarrow \quad v = \frac{A}{B}(1-e^{-Bt})$$

Check-Point

At $t = 0$, $v = 0$

At $t \to \infty$, $v \to \frac{A}{B}$

8. $v = -4x^2$

$$\Rightarrow \quad \frac{dx}{dt} = -4x^2$$

$$\Rightarrow \quad \int_2^x x^{-2}dx = \int_0^t -4dt$$

$$\Rightarrow \quad -x^{-1}\Big|_2^x = -4t\Big|_0^t$$

$$\Rightarrow \quad t = \frac{1}{4}(x^{-1} - 0.5)$$

$$\Rightarrow \quad x = \frac{2}{8t+1}$$

$$v = \frac{dx}{dt} = \frac{(8t+1)(0)-(2)\frac{d}{dt}(8t+1)}{(8t+1)^2} = -\frac{16}{(8t+1)^2}$$

$$a = \frac{dv}{dt} = -\left(\frac{(8t+1)^2(0)-16\frac{d}{dt}(8t+1)^2}{(8t+1)^4}\right)$$

$$\Rightarrow \quad a = \frac{16(2)(8)(8t+1)}{(8t+1)^4}$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{256}{(8t+1)^3}\ \text{ms}^{-2}$$

9. (a) $\frac{dv}{dt} = 3 - 2t$

$$\Rightarrow \quad \int_{v_0}^v dv = \int_0^t (3-2t)dt$$

$$\Rightarrow \quad v = v_0 + 3t - t^2 \qquad \ldots(1)$$

Since $v = \frac{dx}{dt}$

$$\Rightarrow \quad \int_{x_0}^x dx = \int_0^t (v_0 + 3t - t^2)dt$$

$$\Rightarrow \quad x = x_0 + v_0 t + \frac{3}{2}t^2 - \frac{t^3}{3} \qquad \ldots(2)$$

Since $x\Big|_{t=0} = x\Big|_{t=5}$

$$\Rightarrow \quad x_0 = x_0 + 5v_0 + \frac{75}{2} - \frac{125}{3}$$

$$\Rightarrow \quad 5v_0 = \frac{125}{3} - \frac{75}{2}$$

$$\Rightarrow \quad v_0 = \frac{25}{3} - \frac{15}{2}$$

$$\Rightarrow \quad v_0 = \frac{50-45}{6} = \frac{5}{6}\ \text{ms}^{-1}$$

(b) From (1), we get velocity at $t = 5$ s

$$v = \frac{5}{6} + (3)(5) - (5)^2$$

$$\Rightarrow \quad v = \frac{5}{6} + 15 - 25$$

$$\Rightarrow \quad v = \frac{5+90-150}{6}$$

$$\Rightarrow \quad v = -\frac{55}{6}\ \text{ms}^{-1}$$

10. Since $dv = adt$ $\left\{\because a = \frac{dv}{dt}\right\}$

$$\Rightarrow \quad \int_0^v dv = 5\int_0^t e^t dt$$

$$\Rightarrow \quad v = 5(e^t - 1)\ \text{ms}^{-1}$$

Further $v = \frac{dx}{dt}$

$$\Rightarrow \quad dx = vdt$$

$$\Rightarrow \quad \int_0^x dx = 5\int_0^t (e^t - 1)dt$$

$$\Rightarrow \quad x = 5(e^t - t - 1)\ \text{m}$$

11. $v = 200x$

$$\Rightarrow \quad \frac{dv}{dx} = 200\frac{d}{dx}(x)$$

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$$\Rightarrow \quad \frac{dv}{dx} = 200$$

Since $a = \frac{dv}{dt} = v\left(\frac{dv}{dx}\right)$

$$\Rightarrow \quad a = (200x)(200)$$

$$\Rightarrow \quad a = 4 \times 10^4 x \text{ mms}^{-2}$$

When $x = 2000$ mm, then

$$a = \left(4 \times 10^4\right)(2000) \text{ mms}^{-2}$$

$$\Rightarrow \quad a = 80 \text{ kms}^{-2}$$

Further $v = 200x$

$$\Rightarrow \quad \frac{dx}{dt} = 200x$$

$$\Rightarrow \quad \int_{500}^{x} \frac{dx}{x} = 200\int_{0}^{t} t$$

$$\Rightarrow \quad \log_e\left(\frac{x}{500}\right) = 200t$$

$$\Rightarrow \quad t = \frac{1}{200}\log_e\left(\frac{x}{500}\right)$$

At $x = 2000$ mm,

$$t = \frac{1}{200}\log_e(4)$$

$$\Rightarrow \quad t = 6.93 \times 10^{-3} \text{ s}$$

$$\Rightarrow \quad t = 6.93 \text{ ms}$$

12. Please note that here acceleration is a function of time, i.e., acceleration is not constant. So, we cannot apply

$$\vec{v} = \vec{u} + \vec{a}t$$

Since $\vec{a} = \frac{d\vec{v}}{dt}$

$$\Rightarrow \quad d\vec{v} = \vec{a}dt$$

$$\Rightarrow \quad \int_{0}^{\vec{v}} d\vec{v} = \int_{0}^{2} \vec{a}dt$$

$$\Rightarrow \quad \vec{v} = \int_{0}^{2}\left(2t\hat{i} + 3t^2\hat{j}\right)dt = \left(t^2\hat{i} + t^3\hat{j}\right)\Big|_0^2$$

$$\Rightarrow \quad \vec{v} = \left(4\hat{i} + 8\hat{j}\right) \text{ ms}^{-1}$$

Hence, velocity of particle at time $t = 2$ s is $\left(4\hat{i} + 8\hat{j}\right)$ ms^{-1}

13. Since $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad -\left(g + kv^2\right) = v\frac{dv}{dx}$$

$$\Rightarrow \quad -\int_{v_0}^{v} \frac{v\,dv}{g + kv^2} = \int_{0}^{x} dx \qquad \ldots(1)$$

Let us calculate $\int \frac{v\,dv}{g + kv^2} = I$ (say)

Substitute $g + kv^2 = z$

$$\Rightarrow \quad 2kv\,dv = dz$$

$$\Rightarrow \quad v\,dv = \frac{dz}{2k}$$

So, $I = \frac{1}{2k}\int \frac{dz}{z}$

$$\Rightarrow \quad I = \frac{1}{2k}\log_e(z)$$

Substituting value of z, we get

$$I = \frac{1}{2k}\log_e\left(g + kv^2\right)$$

So, from (1), we get

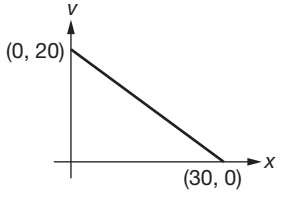
$$-\frac{1}{2k}\log_e\left(g + kv^2\right)\Big|_{v_0}^{v} = x\Big|_0^x$$

$$\Rightarrow \quad x = -\frac{1}{2k}\log_e\left(\frac{g + kv^2}{g + kv_0^2}\right) = \frac{1}{2k}\log_e\left(\frac{g + kv_0^2}{g + kv^2}\right)$$

Now, the particle achieves its maximum height, which is also the point of reversal of motion when $v = 0$. Thus,

$$h_{\max} = \frac{1}{2k}\log_e\left(\frac{g + kv_0^2}{g}\right)$$

14. The equation or the relation that predicts the motion specified in the problem is



$$v - 20 = \left(\frac{20 - 0}{0 - 30}\right)(x - 0)$$

$\Rightarrow \quad v = 20 - \frac{2}{3}x$

$\frac{dv}{dt} = -\frac{2}{4}\frac{dx}{dt} = -\frac{2}{3}v \qquad \ldots(1)$

At $x = 15$ m, $v = 20 - \frac{2}{3} \times 15 = 10$ ms^{-1}

So, acceleration $\frac{dv}{dt} = -\frac{2}{3} \times 10 = -\frac{20}{3}$ ms^{-2}

From equation (1), we get

$$\int_{20}^{V} \frac{dv}{v} = -\frac{2}{3}\int_{0}^{t} dt$$

$\Rightarrow \quad v = 20e^{-2/3t}$

$\Rightarrow \quad dx = 20e^{-2/3t}dt \qquad \left\{\because v = \frac{dx}{dt}\right\}$

$\Rightarrow \quad \int_{0}^{x} dx = 20\int_{0}^{t} e^{-2/3t}dt$

$\Rightarrow \quad x = 30\left(1 - e^{-2/3t}\right)$ m

Note that $x \to 30$ m when $t \to \infty$.

15. $dv = a\,dt \qquad \left\{\because a = \frac{dv}{dt}\right\}$

$\Rightarrow \quad \int_{0}^{v} dv = \int_{0}^{t} (12t - 3t^{1/2})dt$

$\Rightarrow \quad v\Big|_0^v = \left(6t^2 - 2t^{3/2}\right)\Big|_0^t$

$\Rightarrow \quad v = \left(6t^2 - 2t^{3/2}\right)$ ms^{-1}

Using this result and the initial condition $x_0 = 15\ m$ at $t = 0$ s,

$dx = v\,dt \qquad \left\{\because v = \frac{dx}{dt}\right\}$

$\Rightarrow \quad \int_{x_0}^{x} dx = \int_{0}^{t} (6t^2 - 2t^{3/2})dt$

$\Rightarrow \quad x\Big|_{15\text{ m}}^{x} = \left(2t^3 - \frac{4}{5}t^{5/2}\right)\Big|_0^t$

$\{\because$ at $t = 0,\ x_0 = 15$ m$\}$

$\Rightarrow \quad x = \left(2t^3 - \frac{4}{5}t^{5/2} + 15\right)$ m

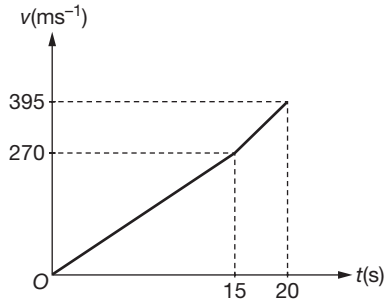
Test Your Concepts-IV (Based on Graph)

1. At $t = 15$ s, $v = 0 + (18)(15) = 270$ ms^{-1}

At $t = 20$ s, $v = 270 + (25)(20 - 15) = 395$ ms^{-1}

For stage 1, $v = 18t \quad 0 \le t \le 15$ s

For stage 2, $v = 270 + 25(t - 15) \quad 15 < t \le 20$ s



From v-t graph, we have

at $t = 15$ s, $s = \frac{1}{2}(15)(270) = 2025$ m

at $t = 20$ s,

$s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15)$

$\Rightarrow \quad s = 3687.5$ m

For stage 1, i.e.,

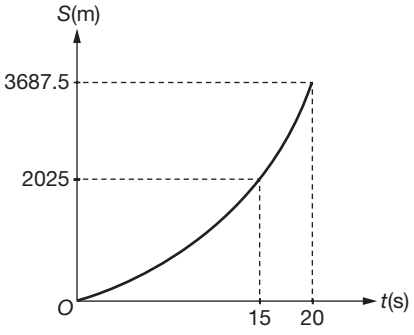
$0 \le t \le 15$ s, we have

$a_1 = 18$ and $v = 18t$

$\Rightarrow \quad s = s_0 + v_0 t + \frac{1}{2}a_1 t^2 = 0 + 0 + 9t^2 = 9t^2$

So, at $t = 15$ s, we have

$s = 9(15)^2 = 2025$ m



For stage 2, i.e.,

$15 \le t \le 20$ s, we have

$a_2 = 25$ and $v = 270 + 25(t - 15)$

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$$\Rightarrow \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2 =$$

$$\Rightarrow \quad s = 2025 + 270(t-15) + \frac{1}{2}(25)(t-15)^2$$

So, at $t = 20$ s, we have

$$v = 395 \text{ ms}^{-1} \text{ and } s = 3687.5 \text{ m}$$

2. For $0 \le x < 3$ m,

$$a = \frac{dv}{dt} = v\frac{dv}{dx} = (2x+4)(2) = (4x+8) \text{ ms}^{-2}$$

At $x = 0$ m and 3 m,

$$a\Big|_{x=0 \text{ m}} = 4(0) + 8 = 8 \text{ ms}^{-2}$$

$$a\Big|_{x=3 \text{ m}} = 4(3) + 8 = 20 \text{ ms}^{-2}$$

For $3m < x \le 6$ m,

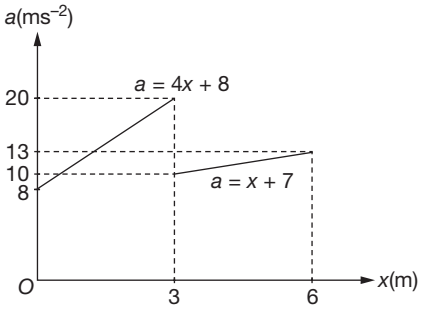
$$a = \frac{dv}{dt} = v\frac{dv}{dx} = (x+7)(1) = (x+7) \text{ ms}^{-2}$$

At $x = 3$ m and 6 m,

$$a\Big|_{x=3 \text{ m}} = 3 + 7 = 10 \text{ ms}^{-2}$$

$$a\Big|_{x=6 \text{ m}} = 6 + 7 = 13 \text{ ms}^{-2}$$

So, we observe that at $x = 3$ m, the acceleration suddenly switches from 20 ms^{-2} to 10 ms^{-2}. The a-x graph is shown as per the data calculated.



3. For $0 \le x < 225$ m,

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \quad a = \left(5\sqrt{x}\right)\left(\frac{5}{2\sqrt{x}}\right) = 12.5 \text{ ms}^{-2}$$

For $225 \text{ m} < x \le 525$ m,

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

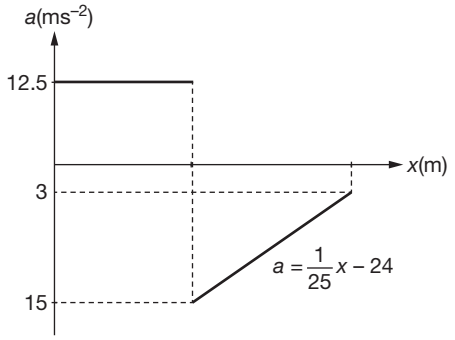
$$\Rightarrow \quad a = (-0.2x + 120)(-0.2) = (0.04x - 24) \text{ ms}^{-2}$$

At $x = 225$ m and 525 m

$$a\Big|_{x=225 \text{ m}} = 0.04(225) - 24 = -15 \text{ ms}^{-2}$$

$$a\Big|_{x=525 \text{ m}} = 0.04(525) - 24 = -3 \text{ ms}^{-2}$$

The a-x graph is shown in figure.



4. For $0 \le t < 6$, $dv = a\,dt$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{1}{6} t^2 dt$$

$$\Rightarrow \quad v = \frac{1}{18} t^3 \qquad \ldots(1)$$

Since $dx = v\,dt$ $\qquad \left\{\because v = \frac{dx}{dt}\right\}$

$$\Rightarrow \quad \int_0^x dx = \int_0^t \frac{1}{18} t^3 dt$$

$$\Rightarrow \quad x = \frac{1}{72} t^4$$

So, when $t = 6$ s, $v = 12 \text{ ms}^{-1}$, $x = 18$ m

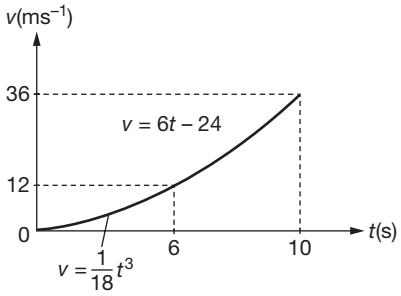
For $6 < t \le 10$, $dv = a\,dt$

$$\Rightarrow \quad \int_{12}^v dv = \int_6^t 6\,dt$$

$$\Rightarrow \quad v = 6t - 24$$

Since $dx = v\,dt$

$$\Rightarrow \quad \int_{18}^x dx = \int_6^t (6t - 24)\,dt$$



$x = 3t^2 - 24t + 54$

When $t = 10$ s, $v = 36$ ms^{-1} and $x = 114$ m

5. For $0 \le x < 200$ m, the initial condition is $v = 0$ at $x = 0$

$vdv = adx$

Since $a = v\dfrac{dv}{dx}$

$$\Rightarrow \int_0^v v\,dv = \int_0^x (0.1x + 5)\,dx$$

$$\frac{v^2}{2}\bigg|_0^v = (0.05x^2 + 5x)\bigg|_0^x$$

$v = \left(\sqrt{0.1x^2 + 10x}\right)$ ms^{-1}

At $x = 200$ m,

$$v\big|_{x=200\text{ m}} = \sqrt{0.1(200^2) + 10(200)}$$
$$= 77.46 \text{ ms}^{-1} = 77.5 \text{ ms}^{-1}$$

For $200 \text{ m} < x \le x_0$, the initial condition is

$v = 77.46$ ms^{-1} at $x = 200$ m

Again, since $vdv = adx$ $\quad \left\{\because a = \dfrac{vdv}{dx}\right\}$

$$\Rightarrow \int_{77.46\text{ ms}^{-1}}^{v} v\,dv = \int_{200\text{ m}}^{x} -15\,dx$$

$$\Rightarrow \frac{v^2}{2}\bigg|_{77.46\text{ ms}^{-1}}^{v} = -15x\bigg|_{200\text{ m}}^{x}$$

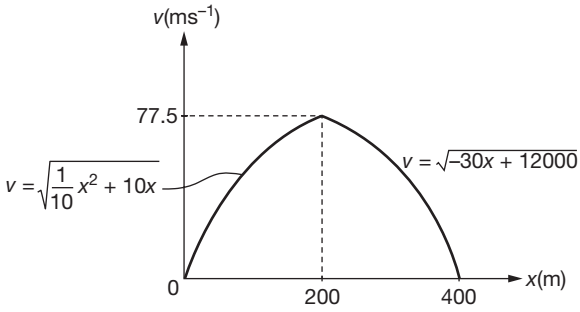
$\Rightarrow v = \sqrt{-30x + 12000}$ ms^{-1}

When $v = 0$ i.e., at $x = x_0$, we have

$0 = \sqrt{-30x_0 + 12000}$

$\Rightarrow x_0 = 400$ m

The v-x graph is shown in figure.



6. $x = t^3 - 3t^2 + 2t$

$$\Rightarrow v = \frac{dx}{dt} = 3t^2 - 6t + 2$$

$$\Rightarrow a = \frac{dv}{dt} = 6t - 6$$

To draw the x-t graph, let us first calculate the points of reversal of motion i.e., the points or the times when $v = 0$. So, $v = 0$ gives

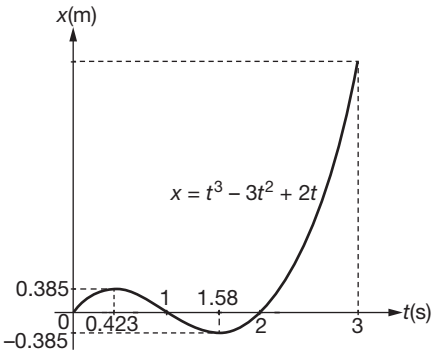
$3t^2 - 6t + 2 = 0$

$$\Rightarrow t = \left(\frac{6 - \sqrt{12}}{6}\right) \text{s and } t = \left(\frac{6 + \sqrt{12}}{6}\right) \text{s}$$

$\Rightarrow t = 1.577$ s and $t = 0.423$ s

$$x\big|_{t=1.577} = -0.386 \text{ m}$$

$$x\big|_{t=0.4226} = 0.385 \text{ m}$$



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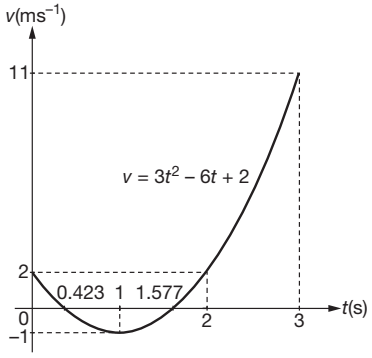
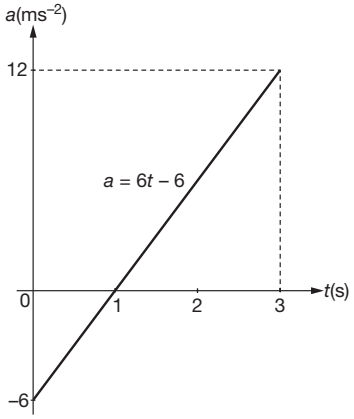




7. For $0 \le x < 625$ m , we observe

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \quad a = \left(0.6x^{3/4}\right)\left[\frac{3}{4}(0.6)x^{-1/4}\right] = \left(0.27x^{1/2}\right) \text{ ms}^{-2}$$

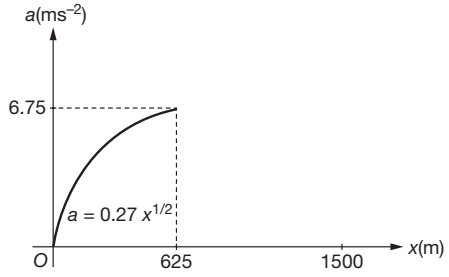
At $x = 625$ m ,

$$a\Big|_{x=625 \text{ m}} = 0.27\left(625^{1/2}\right) = 6.75 \text{ ms}^{-2}$$

For $625 \text{ m} < x < 1500$ m

$$a = \frac{dv}{dt} = v\frac{dv}{dx} = 75(0) = 0 \qquad \left\{\because \frac{dv}{dx} = 0\right\}$$

The a-x graph is shown in figure



8. From the graph, $a = 22.5 - \frac{22.5}{150}x$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^{60}\left(22.5 - \frac{22.5}{150}x\right)dx$$

$$\Rightarrow \quad \frac{v^2}{2} = (22.5)(60) - \left(\frac{22.5}{150}\right)\frac{(60)^2}{2}$$

$$\Rightarrow \quad v = 46.47 \text{ ms}^{-1}$$

9. **At** $x = 50$ m

Since $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad a = (20)\left(\frac{40}{100}\right) = 8 \text{ ms}^{-2}$$

At $x = 150$ m

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \quad a = (45)\left(\frac{10}{100}\right) = 4.5 \text{ ms}^{-2}$$

10. Since, $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad v\,dv = a\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \text{area under } a\text{-}x \text{ graph} \qquad \{\text{since } v_{\text{initial}} = 0\}$$

$$\Rightarrow \quad v = \sqrt{2(\text{area of } a\text{-}x \text{ graph})}$$

At $x = 40$ m , we have

$$v = \sqrt{2 \times 40 \times 2} = 12.7 \text{ ms}^{-1}$$

At $x = 90$ m , we have

$$v = \sqrt{2 \times (100 + 40 \times 4)} = 22.8 \text{ ms}^{-1}$$

At $x = 200$ m , we have

$$v = \sqrt{2 \times (100 + 400 + 150)} = 36.1 \text{ ms}^{-1}$$

11. (a) From the graph $a = 2t - 2$

$$\Rightarrow \quad \int_0^V dv = \int_0^t (2t - 2)\,dt$$

$$\Rightarrow \quad v = t^2 - 2t$$

(b) $$\int_0^x dx = \int_2^4 (t^2 - 2t)\,dt \qquad \left\{\because v = \frac{dx}{dt}\right\}$$

$$\Rightarrow \quad x = \left(\frac{t^3}{3} - t^2\right)\Big|_2^4 = \left(\frac{64}{3} - 16 - \frac{8}{3} + 4\right) \text{ m}$$

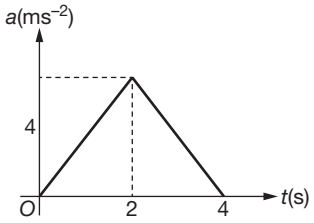
$$\Rightarrow \quad x = 6.67 \text{ m}$$

12. $dv = adt$

$$\Rightarrow \quad \int_{v_i}^{v_f} dv = \int_0^t a\,dt$$

$$\Rightarrow \quad \Delta v = \int_0^t adt$$

$\Rightarrow$ Change in velocity = Area under a-t graph



$$\Rightarrow \quad v_f - v_i = \frac{1}{2}(4)(4) = 8 \text{ ms}^{-1}$$

$$\Rightarrow \quad v_f = v_i + 8 = (2+8) \text{ ms}^{-1}$$

$$\Rightarrow \quad v_f = 10 \text{ ms}^{-1}$$

13. Displacement = Area under velocity-time graph

Hence, $x_{OA} = \frac{1}{2} \times 2 \times 10 = 10$ m

$$x_{AB} = 2 \times 10 = 20 \text{ m}$$

$$\Rightarrow \quad x_{OAB} = 10 + 20 = 30 \text{ m}$$

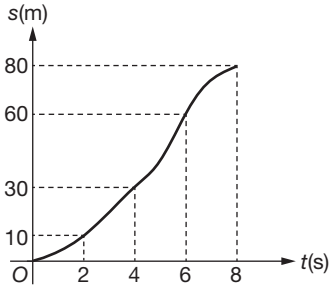
$$x_{BC} = \frac{1}{2} \times 2(10+20) = 30 \text{ m}$$

$$\Rightarrow \quad x_{OABC} = 30 + 30 = 60 \text{ m}$$

and $x_{CD} = \frac{1}{2} \times 2 \times 20 = 20$ m

$$\Rightarrow \quad x_{OABCD} = 60 + 20 = 80 \text{ m}$$

Between 0 to 2 s and 4 s to 6 s motion is accelerated, hence displacement-time graph is a parabola. Between 2 s to 4 s motion is uniform, so displacement-time graph will be a straight line. Between 6 s to 8 s motion is decelerated hence displacement-time graph is again a parabola but inverted in shape. At the end of 8 s velocity is zero, therefore, slope of displacement-time graph should be zero. The corresponding graph is shown in figure.



Test Your Concepts-V (Based on Motion Under Gravity)

1. (a) A particle thrown upwards has its velocity in opposite direction to its acceleration (g, downwards).

(b) When the particle is released from rest from a certain height, its velocity is zero, while acceleration is g downwards. Similarly, at the extreme position of a pendulum velocity is zero, while acceleration is not zero.

(c) In uniform circular motion velocity is perpendicular to its radial or centripetal acceleration.

2. Since $a = \frac{dv}{dt} = v\frac{dv}{dy}$

$$\Rightarrow \quad v\frac{dv}{dy} = -\frac{g_0 R^2}{(R+y)^2}$$

$$\Rightarrow \quad \int_u^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R+y)^2}$$

$$\Rightarrow \quad u^2 = 2g_0 R^2 \left(\frac{1}{R+y}\right)\Big|_\infty^0 = 2g_0 R$$

$$\Rightarrow \quad u = \sqrt{2g_0 R}$$

This velocity is also called as the escape velocity, v_e

Substituting the values, we get $v_e = 11.2 \text{ kms}^{-1}$

3. Let the particles meet t_0 second after the projection of the first particle. Further suppose that they meet at a height h from the ground

For first particle,

$$h = ut_0 - \frac{1}{2}gt_0^2 \qquad \ldots(1)$$

where u ms^{-1} is the velocity of the particle

For second particle,

$$h = u(t_0 - t) - \frac{1}{2}g(t_0 - t)^2 \qquad \ldots(2)$$

CHAPTER 4

From equations (1) and (2), we get

$$ut_0 - \frac{1}{2}gt_0^2 = u(t_0 - t) - \frac{1}{2}g(t_0 - t)^2$$

$$\Rightarrow \quad ut_0 - \frac{1}{2}gt_0^2 = ut_0 - ut - \frac{1}{2}gt_0^2 + gt_0t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad ut - gt_0t + \frac{1}{2}gt^2 = 0$$

$$\Rightarrow \quad u - gt_0 + \frac{1}{2}gt = 0$$

$$\Rightarrow \quad u - gt_0 + \frac{1}{2}gt = 0$$

$$\Rightarrow \quad t_0 = \left(\frac{t}{2} + \frac{u}{g}\right) \text{ second}$$

Velocity of first particle is v_0. So

$$\Rightarrow \quad v_1 = u - gt_0 = u - g\left(\frac{t}{2} + \frac{u}{g}\right)$$

$$\Rightarrow \quad v_1 = -\frac{1}{2}gt \text{ (downwards)}$$

Velocity of second particle is v_2. So

$$v_2 = u - g(t_0 - t) = u - g\left(\frac{t}{2} + \frac{u}{g} - t\right)$$

$$\Rightarrow \quad v_2 = -\frac{1}{2}gt \text{ (upwards)}$$

4. Let h be the maximum height. Then applying

$$v^2 - u^2 = 2as$$

Taking upward direction as positive, we get

$$v = 0,\ u = 100 \text{ ms}^{-1},\ a = -g = -10 \text{ ms}^{-2}$$

$$\Rightarrow \quad 0^2 - (100)^2 = 2(-10)h$$

$$\Rightarrow \quad h = 500 \text{ m}$$

Hence, velocity at height $h = 250$ m will be

$$v^2 - (100)^2 = 2(-10)(250)$$

$$\Rightarrow \quad v^2 = 10000 - 5000 = 5000$$

$$\Rightarrow \quad v = 10\sqrt{50} \text{ ms}^{-1}$$

5. Let u be the velocity of projection of the particle. Then

$$h = uT - \frac{1}{2}gT^2$$

$$\Rightarrow \quad uT = h + \frac{1}{2}gT^2$$

$$\Rightarrow \quad u = \frac{h}{T} + \frac{1}{2}gT = \frac{2h + gT^2}{2T}$$

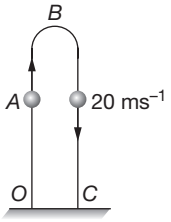
If H be the greatest height reached, then

$$0 = u^2 - 2gH$$

$$\Rightarrow \quad 0 = \left(\frac{2h + gT^2}{2T}\right)^2 - 2gH$$

$$\Rightarrow \quad H = \frac{\left(2h + gT^2\right)^2}{8gT^2}$$

6. In the graphs, $v_A = at_{O\to A} = (4)(5) = 20 \text{ ms}^{-1}$

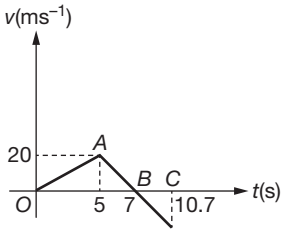


$$v_B = 0 = v_A - gt_{A\to B}$$

$$\Rightarrow \quad t_{A\to B} = \frac{v_A}{g} = \frac{20}{10} = 2 \text{ s}$$

$$\Rightarrow \quad t_{O\to A\to B} = (5 + 2)\text{s} = 7 \text{ s}$$

Now, $s_{O\to A\to B}$ = area under v-t graph between 0 to 7 s



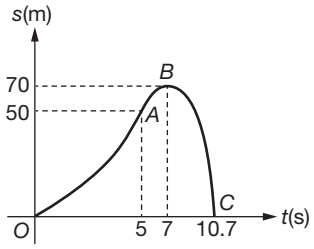
$$\Rightarrow \quad s_{O\to A\to B} = \frac{1}{2}(7)(20) = 70 \text{ m}$$

Further $|s_{OAB}| = |s_{BC}| = \frac{1}{2}g\left(t_{B\to C}\right)^2$

$$\Rightarrow \quad 70 = \frac{1}{2}(10)\left(t_{B\to C}\right)^2$$

$$\Rightarrow \quad t_{B\to C} = \sqrt{14} = 3.7 \text{ s}$$

$$\Rightarrow \quad t_{O\to A\to B\to C} = 7 + 3.7 = 10.7 \text{ s}$$



Also $s_{O\to A}$ = area under v-t graph between OA

$$s_{O\to A} = \frac{1}{2}(5)(20) = 50 \text{ m}$$

7. The second stone will catch up the first stone when the distance covered by it in $(t-n)$ second will be equal to the distance covered by the first stone in t second.
Distance covered by first stone in t second

$$s_1 = \frac{1}{2}gt^2 \qquad \ldots(1)$$

Distance covered by second stone in $(t-n)$ second

$$s_2 = u(t-n) + \frac{1}{2}g(t-n)^2 \qquad \ldots(2)$$

From equations (1) and (2), we have

$$\frac{1}{2}gt^2 = u(t-n) + \frac{1}{2}g(t-n)^2$$

$$\Rightarrow \quad \frac{1}{2}g\left[t^2 - (t-n)^2\right] = u(t-n)$$

$$\Rightarrow \quad \frac{1}{2}g\left[2tn - n^2\right] = u(t-n)$$

$$\Rightarrow \quad gtn - \frac{1}{2}gn^2 = ut - un$$

$$\Rightarrow \quad t(gn-u) = n\left(\frac{1}{2}gn - u\right)$$

$$\Rightarrow \quad t = \frac{n\left(\frac{1}{2}gn - u\right)}{(gn-u)}$$

The distance covered by the first stone in this time

$$s_1 = \frac{1}{2}g \times \left[\frac{n\left(\frac{1}{2}gn-u\right)}{(gn-u)}\right]^2 = \frac{gn^2}{8}\left(\frac{gn-2u}{gn-u}\right)^2$$

Thus the second stone will overtake the first stone at a distance

$$\frac{gn^2}{8}\left(\frac{gn-2u}{gn-u}\right)^2, \text{ below the top of the cliff.}$$

8. Here the speed of the body is attenuated due to air drag. The speed becomes zero under the effect of gravity. Due to air drag, let effective value of g be g'. Then

$$0 = u_0^2 - 2g'h$$

$$\Rightarrow \quad u_0^2 = 2g'h \qquad \ldots(1)$$

Here $g' = g + \left(\frac{cu_0^2}{m}\right)$ $\qquad \ldots(2)$

because cu_0^2 is the force with which air drags and produces a retardation of $\left(\frac{cu_0^2}{m}\right)$

From equation (1)

$$h = \frac{u_0^2}{2g'} = \frac{u_0^2}{2\left[g + \left(\frac{cu_0^2}{m}\right)\right]}$$

$$\Rightarrow \quad h = \frac{u_0^2}{2g\left(1 + \frac{cu_0^2}{mg}\right)} \qquad \ldots(3)$$

As the body comes down, it acquires the velocity u'

Now $u'^2 = 2gh$

$$\Rightarrow \quad h = \frac{u'^2}{2g} \qquad \ldots(4)$$

From equation (3) and (4), we get

$$\frac{u_0^2}{2g\left(1 + \frac{cu_0^2}{mg}\right)} = \frac{u'^2}{2g}$$

$$\Rightarrow \quad u'^2 = \frac{u_0^2}{\left(1 + \frac{cu_0^2}{mg}\right)}$$

$$\Rightarrow \quad u' = \frac{u_0}{\sqrt{1 + \frac{cu_0^2}{mg}}}$$

9. $a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

Take downward direction as positive, we get

$$v_1 = \sqrt{2gh_1} \text{ and } v_2 = -\sqrt{2gh_2}$$

$$\Rightarrow \quad a = -\frac{\sqrt{2gh_2} - \sqrt{2gh_1}}{\Delta t}$$

$$\Rightarrow \quad a = -\left(\frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\Delta t}\right)$$

Negative sign with a indicates that it is in the upward direction. So,

$$|a| = \frac{\sqrt{2(9.8)(4)} + \sqrt{2(9.8)(2)}}{12 \times 10^{-3}}$$

$$\Rightarrow \quad |a| = \frac{8.85 + 6.26}{12 \times 10^{-3}} = 1.26 \times 10^3 \text{ ms}^{-2}$$

10. For the motion of the rocket for first minute

$u_1 = 0$, $a_1 = 30$ msec^{-2}

$\Rightarrow$ $t_1 = 60$ sec.

Let v_1 be the velocity of the rocket after one minute (60 sec.), then

$$v_1 = u_1 + a_1 t_1 = 0 + 30 \times 60 = 1800 \text{ msec}^{-1}$$

Height h_1 attained is

$$h_1 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} \times 30 \times (60)^2$$

$\Rightarrow$ $h_1 = 54000$ metre

When the fuel is used up,

$u_2 = 1800$ msec^{-1}, $v_2 = 0$ and $a_2 = -10$ msec^{-2}

$$\Rightarrow \quad h_2 = \frac{(1800)^2}{2+10} = 162000 \text{ metre}$$

(a) Maximum height reached $= h_1 + h_2$

$\Rightarrow$ $h_{\max} = 54000 + 162000$

$\Rightarrow$ $h_{\max} = 216{,}000$ metre

(b) Let t_2 be the time for the journey when fuel is used up, then

$$t_2 = \frac{v_2 - u_2}{a_2} = \frac{1800}{10} = 180 \text{ sec.}$$

Again let t_3 be the time for descent, then using the formula $h = \frac{1}{2} g t_3^2$, we have

$$216000 = \frac{1}{2} \times 10 \times t_3^2$$

$\Rightarrow$ $t_3^2 = 43200$

$\Rightarrow$ $t_3 = (43200)^{1/2} = 207.8$ sec.

Hence total time elapsed is

$$T = t_1 + t_2 + t_3 = 60 + 180 + 207.8$$

$\Rightarrow$ $T = 447.8$ sec.

11. Take upward direction as positive, then

$$v^2 - u^2 = 2as$$

$\Rightarrow$ $(5^2) - u^2 = 2(-10)(15)$

$\Rightarrow$ $u^2 = 325 \text{ m}^2\text{s}^{-2}$

$\Rightarrow$ $u = 5\sqrt{13} \text{ ms}^{-1}$

(a) $h = \frac{u^2}{2g} = \frac{325}{20} = 16.25$ m

(b) $t = \frac{u}{g} = \frac{5\sqrt{13}}{10} = 1.8$ s

12. Let h be the height from the ground at which both balls collide. Suppose the two balls collide after t second. Here the distance travelled by the first ball in t second must be the same as the distance travelled by the second ball in $(t-2)$ second, because second ball is thrown after 2 second of the first.

For the first,

$u = 39.2$ msec^{-1}, $g = 9.8$ msec^{-2} and $s = h$

Using, $h = ut + \frac{1}{2} g t^2$, we get

$$h = 39.2t - \frac{1}{2} \times 9.8 \times t^2 \qquad \ldots(1)$$

For the second ball, we have

$$h = 39.2(t-2) - \frac{1}{2} \times 9.8 (t-2)^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$39.2t - \frac{1}{2} \times 9.8 \times t^2 = 39.2(t-2) - \frac{1}{2} \times 9.8 \times (t-2)^2$$

$$\Rightarrow \quad 39.2t - 4.9t^2 = 39.2t - 78.4 - 4.9t^2 + 19.6t - 19.6$$

Solving, we get $t = 5.02$ second

Putting this value of t in equation (1), we have

$h = 73.302$ metre

13. $s_1(t) = s_2(t - t_0)$

Taking upward direction as positive, we get

$$v_0 t - \frac{1}{2} g t^2 = v_0 (t - t_0) - \frac{1}{2} g (t - t_0)^2$$

On evaluation, we get

$$t = \frac{v_0}{g} + \frac{t_0}{2}$$

14. $$\sqrt{\frac{2h}{g}} - \sqrt{\frac{2 \times 5}{g}} = \sqrt{\frac{2 \times (h-25)}{g}}$$

$\Rightarrow$ $h = 45$ m

15. (a) $h_{\max} = \frac{v_{\text{net}}^2}{2g} + 28 + 2 = \frac{(20+10)^2}{2(10)} + 30 = 75$ m

(b) Using $s = ut + \frac{1}{2} a t^2$ {relative to lift}

$$\Rightarrow \quad -2 = 20t - \frac{1}{2}(10) t^2$$

$$-4 = 40t - 10t^2$$

$$\Rightarrow \quad 10t^2 - 40t - 4 = 0$$

$$\Rightarrow \quad t = \frac{40 \pm \sqrt{1600 + 160}}{20}$$

$$\Rightarrow \quad t = \frac{40 + 41.95}{20}$$

$$\Rightarrow \quad t \cong 4.1 \text{ s}$$

16. Let h be the height of the tower and v be the velocity of the ball at the top of tower. Since the boy catches the ball 3 second after the ball first passes him, the displacement of the ball during the time is zero.

Using the formula, $h = ut + \frac{1}{2}gt^2$, we have

$$0 = u(3) - \frac{1}{2}(9.8)(3)^2$$

Solving, we get $v = 14.7 \text{ ms}^{-1}$

So when the boy catches the ball, its velocity will be 14.7 ms^{-1} in the downward direction.

Now, using the formula,

$v^2 = u^2 + 2gh$, we have

$$(14.7)^2 = (24.5)^2 - 2 \times 9.8 \times h$$

Solving, we get $h = 19.6$ metre

17. $s_1(t) - s_2(t-1) = 10$

$$\Rightarrow \quad \frac{1}{2} \times 10 \times t^2 - \frac{1}{2} \times 10(t-1)^2 = 10$$

$$\Rightarrow \quad t = 1.5 \text{ s}$$

Test Your Concepts-VI (Based on Planar Motion)

1. $u_x = 0$, $a_x = 4 \text{ ms}^{-2}$ and $a_y = 2 \text{ ms}^{-2}$

$$x = u_x t + \frac{1}{2}a_x t^2 = \frac{1}{2}a_x t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2 \times 32}{4}} = 4 \text{ s}$$

(a) $y = u_y t + \frac{1}{2}a_y t^2 = (8)(4) + \frac{1}{2}(2)(4)^2 = 48$ m

(b) $v_x = a_x t = (4)(4) = 16 \text{ ms}^{-1}$

$$v_y = u_y + a_y t = 8 + (2)(4) = 16 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} \approx 22 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{16^2 + 16^2} = 16\sqrt{2} \text{ ms}^{-1}$$

2. $v_x = \frac{dx}{dt} = 5t$

$$\Rightarrow \quad \int_0^x dx = \int_0^t 5t\,dt \qquad \{\because \text{ at } t = 0,\ x = 0 \text{ and } y = 0\}$$

$$\Rightarrow \quad x = \frac{5t^2}{2}$$

$$\Rightarrow \quad y = 0.5x^2 = \frac{25t^4}{8}$$

$$\Rightarrow \quad v_y = \frac{dy}{dt} = \frac{25}{2}t^3$$

At $t = 1$ s, $x = \frac{5}{2}$ m and $y = \frac{25}{8}$ m

So, distance from the origin is

$$s = \sqrt{x^2 + y^2} \cong 4 \text{ m}$$

Now, $a_x = \frac{dv_x}{dt} = 5 \text{ ms}^{-2}$ and $a_y = \frac{dv_y}{dt} = \frac{75}{2}t^2$

At $t = 1$ s we have, $a_y = \frac{75}{2} \text{ ms}^{-2}$

$$\Rightarrow \quad a = \sqrt{a_x^2 + a_y^2} = 37.8 \text{ ms}^{-2}$$

3. $\frac{dx}{dt} = v_x = k\omega\cos(\omega t)$ and

$\frac{dy}{dt} = v_y = k\omega\sin(\omega t)$

Now, speed $v = \sqrt{\left(v_x^2 + v_y^2\right)} = k\omega$

$$\Rightarrow \quad s = vt = k\omega t$$

4. (a) $x = kt$ …(1)

$y = kt(1 - \alpha t)$ …(2)

Equation of trajectory is given by

$$y = kt - k\alpha t^2 = x - k\alpha\frac{x^2}{k^2} \quad \left\{\because \text{ from (1), } t = \frac{x}{k}\right\}$$

$$\Rightarrow \quad y = x - \alpha\frac{x^2}{k}$$

This equation happens to be a parabola.

(b) $v_x = \frac{dx}{dt} = k$ and

$$v_y = \frac{dy}{dt} = k - 2k\alpha t = k(1 - 2\alpha t)$$

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = k\hat{i} + k(1 - 2\alpha t)\hat{j}$$

$$\Rightarrow \quad v = \sqrt{k^2 + k^2(1 - 2\alpha t)^2} = k\sqrt{1 + (1 - 2\alpha t)^2}$$

Acceleration $\vec{a}$ is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = -2\alpha k\hat{j} = \text{constant}$$

CHAPTER 4

5. Since $v^2 = v_x^2 + v_y^2$

$$\Rightarrow \quad \frac{d}{dt}(v^2) = \frac{d}{dt}(v_x^2) + \frac{d}{dt}(v_y^2)$$

$$\Rightarrow \quad 2v\frac{dv}{dt} = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{v_x a_x + v_y a_y}{v} \qquad \ldots(1)$$

Now, if $\vec{a} = a_x\hat{i} + a_y\hat{j}$ and $\vec{v} = v_x\hat{i} + v_y\hat{j}$, then

$$\vec{a}\cdot\vec{v} = a_x v_x + a_y v_y \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{dv}{dt} = \frac{\vec{a}\cdot\vec{v}}{v} = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$$

6. $(y-40)^2 = 160x$

Differentiating w.r.t. time, we get

$$2(y-40)\frac{dy}{dt} = 160\frac{dx}{dt}$$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = \frac{1}{80}(y-40)v_y \qquad \ldots(1)$$

Since $y = 80$ m

$$\Rightarrow \quad v_x = \frac{1}{80}(80-40)(180) = 90 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{(90)^2 + (180)^2} = 201.25 \text{ ms}^{-1}$$

Again differentiating (1) w.r.t. time, we get

$$a_x = \frac{1}{80}\left[(y-40)a_y + v_y^2\right]$$

At $a_y = 0$ and $v_y = 180 \text{ ms}^{-1}$, we get

$$a_x = \frac{1}{80}(180)^2 = 405 \text{ ms}^{-2}$$

7. Here, acceleration $\vec{a} = (4\hat{i} + 2\hat{j}) \text{ ms}^{-2}$ is constant. So,

$$\vec{v} = \vec{u} + \vec{a}t \text{ and } \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

Substituting the proper values, we get

$$\vec{v} = (2\hat{i} + 3\hat{j}) + (2)(4\hat{i} + 2\hat{j}) = (10\hat{i} + 7\hat{j}) \text{ ms}^{-1} \text{ and}$$

$$\vec{s} = (2)(2\hat{i} + 3\hat{j}) + \frac{1}{2}(2)^2(4\hat{i} + 2\hat{j}) = (12\hat{i} + 10\hat{j}) \text{ m}$$

Therefore, velocity and displacement of particle at $t = 2$ s are $(10\hat{i} + 7\hat{j}) \text{ ms}^{-1}$ and $(12\hat{i} + 10\hat{j})$ m respectively.

8. $u_x = 20 \text{ kmh}^{-1} = \frac{50}{9} \text{ ms}^{-1}$

$$u_y = 12 \text{ kmh}^{-1} = \frac{10}{3} \text{ ms}^{-1}$$

Using, $s_y = u_y t + \frac{1}{2}a_y t^2$

Taking downward direction as positive, we get

$$s_y = 50 \text{ m}, \; u_y = -\frac{10}{3} \text{ ms}^{-1}, \; a_y = 10 \text{ ms}^{-2}$$

$$\Rightarrow \quad 50 = -\frac{10}{3}t + \frac{1}{2}(10)t^2$$

$$\Rightarrow \quad 10 = -\frac{2}{3}t + t^2$$

$$\Rightarrow \quad 3t^2 - 2t - 30 = 0$$

$$\Rightarrow \quad t = \frac{2 + \sqrt{4 + 360}}{2(3)} = \frac{2 + 19}{6}$$

$$\Rightarrow \quad t = \frac{21}{6} = \frac{7}{2} = 3.5 \text{ s}$$

Speed with which the bag strikes the ground is v given by

$$v^2 = u^2 + 2gh \text{ where } u^2 = u_x^2 + u_y^2$$

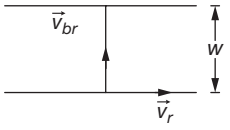
$$\Rightarrow \quad v = \sqrt{u_x^2 + u_y^2 + 2gh}$$

$$\Rightarrow \quad v = \sqrt{\frac{2500}{81} + \frac{100}{9} + (2)(10)(50)}$$

$$\Rightarrow \quad v \cong 32 \text{ ms}^{-1}$$

Test Your Concepts-VII (Based on Relative Velocity)

1. Done already.

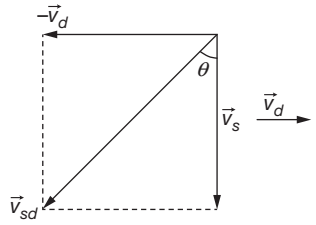


(a) $t = \frac{w}{v_{br}} = \frac{400}{10} = 40$ s

(b) $x = v_r t = 80$ m

2. v_s = Velocity of snow flakes $= 8 \text{ ms}^{-1}$,

v_d = Velocity of driver $= 50 \text{ kmh}^{-1} = 13.9 \text{ ms}^{-1}$



Since we know,

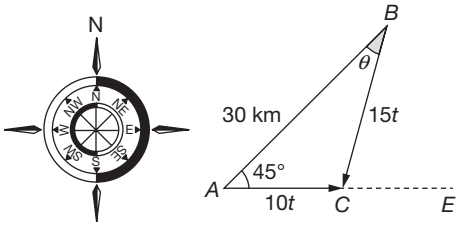
$\vec{v}_{sd} = \vec{v}_s - \vec{v}_d$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{13.9}{8}\right) = 60°$

3. Done already, see theory.

4. The situation is shown in figure

Let A and B be the positions of two ships respectively. The ship A at 9 am is 30 km south-west of B, i.e., $AB = 30$ km. AB makes an angle 45° with the east. A travels in the direction AC with velocity 10 kmhr^{-1}. Let B travels in the direction BC and interception takes place at point C after a time t. Then $AC = 10t$ and $BC = 15t$.



(a) Let BC makes an angle θ with BA

In triangle ABC, we have from Lami's Theorem

$\Rightarrow \quad \frac{10t}{\sin\theta} = \frac{15t}{\sin 45°}$

$\Rightarrow \quad \sin\theta = \frac{10t \sin 45°}{15t}$

$\Rightarrow \quad \sin\theta = \frac{10}{15} \times \frac{1}{\sqrt{2}} = 0.4714$

$\Rightarrow \quad \theta = 28°$

Now, $\angle BCA = 180° - (45° + 28°) = 107°$

(b) From triangle ABC, again using Lami's Theorem, we get

$\frac{15t}{\sin 45°} = \frac{30}{\sin 107°}$

$\Rightarrow \quad 15t = \frac{30 \sin 45°}{\sin 107°}$

$\Rightarrow \quad t = \frac{2 \sin 45°}{\sin 107°} = 2\left(\frac{0.7070}{0.9563}\right) = 1.48$ hour

5. Let t be the time after turning back of the motorboat when it again meets with the raft.

Throughout the journey raft moves with v_r (absolute velocity of river) while the boatman travels with $(v_{br} + v_r)$ for 1 hour (downstream) and with $(v_{br} - v_r)$ for t hours (upstream). Since,

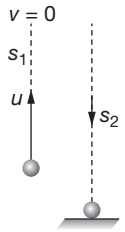
Displacement of raft = displacement of motorboat = 6 km

$\Rightarrow \quad v_r(1+t) = (v_{br} + v_r)(1) - (v_{br} - v_r)(t) = 6$ km

Solving, we get

$t = 1$ hr and $v_r = 3$ kmhr^{-1}

6. (a) If we consider elevator at rest, then relative acceleration of the bolt is $a_r = 9.8 + 1.2 = 11$ ms^{-1} (downwards).



After 2 second velocity of the elevator is $v = at = (1.2)(2) = 2.4$ ms^{-1}. Therefore initial velocity of the bolt is also 2.4 ms^{-1} and it gets accelerated with relative acceleration 11 ms^{-2}. With respect to elevator, initial velocity of bolt is zero and it has to travel 2.7 m with 11 ms^{-2}. Thus, time taken can be directly given as

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 2.7}{11}} = 0.7$ s

(b) Displacement of bolt relative to ground in $t = 0.7$ s is given by

$s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad s = (2.4)(0.7) + \frac{1}{2}(-9.8)(0.7)^2$

$\Rightarrow \quad s = -0.72$ m

Velocity of bolt will become zero after a time, say t_0. Then

$t_0 = \frac{u}{g} \qquad \{v = u - gt\}$

$\Rightarrow \quad t_0 = \frac{2.4}{9.8} = 0.245$ s

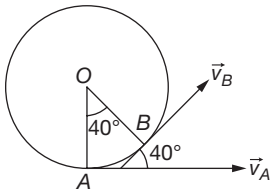
CHAPTER 4

Therefore, distance travelled by the bolt is $s_{\text{bolt}} = s_1 + s_2$

$$\Rightarrow \quad s_{\text{bolt}} = \frac{u^2}{2g} + \frac{1}{2}g(t - t_0)^2$$

$$\Rightarrow \quad s_{\text{bolt}} = \frac{(2.4)^2}{2(9.8)} + \frac{1}{2}(9.8)(0.7 - 0.245)^2 = 1.3 \text{ m}$$

7. Here, $|\vec{v}_A| = |\vec{v}_B| = v$



Change in velocity, $\Delta\vec{v} = \vec{v}_B - \vec{v}_A = \vec{v}_B + (-\vec{v}_A)$

Angle between $\vec{v}_A$ and $\vec{v}_B$ is $\theta = 40°$

Since

$$|\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{v_B^2 + v_A^2 + 2v_A v_B \cos(180 - \theta)}$$

$$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{v^2 + v^2 - 2v_2 \cos(40°)}$$

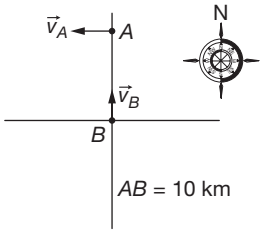
$$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{2}v\sqrt{1 - \cos(40°)}$$

$$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{2}v\sqrt{2}\sqrt{\sin^2\left(\frac{40°}{2}\right)}$$

$$\left\{\because\ 1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)\right\}$$

$$\Rightarrow \quad |\Delta\vec{v}| = 2v\sin(20°)$$

8. Ships A and B are moving with same speed 20 kmhr^{-1} in the directions shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find $\vec{v}_{BA}$

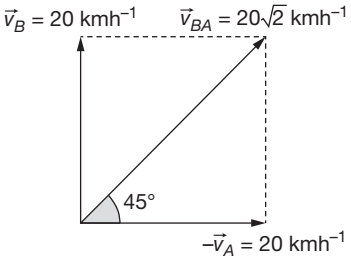


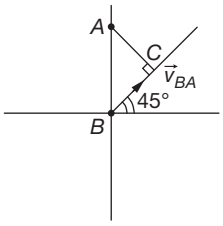
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Here, $|\vec{v}_{BA}| = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2} \text{ kmhr}^{-1}$

i.e., $\vec{v}_{BA}$ is $20\sqrt{2} \text{ kmhr}^{-1}$ at an angle of $45°$ from east towards north. Thus, the given problem can be simplified as:

A is at rest and B is moving with $\vec{v}_{BA}$ in the direction shown in figure.





Therefore, the minimum distance between the two is

$$s_{\min} = AC = AB\sin 45° = 10\left(\frac{1}{\sqrt{2}}\right) \text{ km} = 5\sqrt{2} \text{ km}$$

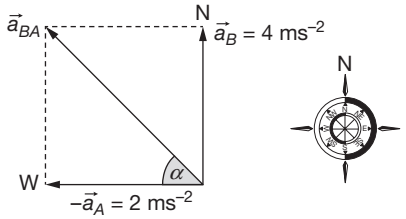
and the desired time is

$$t = \frac{BC}{|\vec{v}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} \qquad \{BC = AC = 5\sqrt{2} \text{ km}\}$$

$$\Rightarrow \quad t = \frac{1}{4} \text{ hr} = 15 \text{ minutes}$$

9. This is a case of two dimensional motion. Therefore, acceleration of car B with respect to car A is

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$



Here, $\vec{a}_B$ = acceleration of car $B = 4 \text{ ms}^{-2}$ (due north)

and $\vec{a}_A$ = acceleration of car $A = 2 \text{ ms}^{-2}$ (due east)

$$|\vec{a}_{BA}| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5} \text{ ms}^{-2}$$

and $\alpha = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$

Thus, $\vec{a}_{BA}$ is $2\sqrt{5} \text{ ms}^{-2}$ at an angle of $\alpha = \tan^{-1}(2)$ NW

i.e., at an angle $\alpha = \tan^{-1}(2)$ towards the north of west.

10. Time for which train A accelerates is t_1 (say). Then

$$80 = 0 + 6t_1$$

$$\Rightarrow \quad t_1 = \frac{40}{3} \text{ s}$$

Distance travelled is say x_1, given by

$$(80)^2 - 0^2 = 2(6)x_1$$

$$\Rightarrow \quad x_1 = \frac{6400}{12} = \frac{1600}{3} \text{ m}$$

In time t_1, train B must have moved forward by

$$x_2 = 60\left(\frac{40}{3}\right) = 800 \text{ m}$$

So, total remaining distance is

$$x = 6000 - (x_1 + x_2)$$

$$\Rightarrow \quad x = 6000 - \left(\frac{1600}{3} + 800\right)$$

$$\Rightarrow \quad x = \frac{14000}{3} \text{ m}$$

If the trains now meet in time t_2, then

$$t_2 = \frac{x}{v_{\text{rel}}} = \frac{\frac{14000}{3}}{140}$$

$$\Rightarrow \quad t_2 = \frac{100}{3} \text{ s}$$

$$\Rightarrow \quad t = t_1 + t_2 = \frac{140}{3} = 46.67 \text{ s}$$

So, the distance travelled by A, when both meet is

$$x_A = x_1 + v_A t_2 = \frac{1600}{3} + (80)\left(\frac{100}{3}\right)$$

$$\Rightarrow \quad x_A = \frac{9600}{3} = 3200 \text{ m}$$

11. Let a_r be the relative acceleration of lift upwards. Then

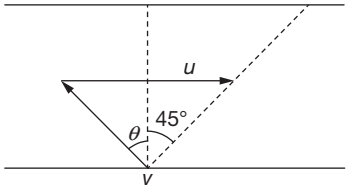
$$t = \frac{2u}{a_r} = \frac{2u}{a+g}$$

$$\Rightarrow \quad a = \frac{2u - gt}{t}$$

12. $v_x = u - v\sin\theta$ and $v_y = v\cos\theta$

Absolute velocity of boatman is along AB i.e., it makes an angle 45 degrees with x-axis. Hence

$$\tan 45° = \frac{v_y}{v_x} = 1$$



$$\Rightarrow \quad v_y = v_x$$

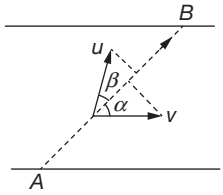
$$\Rightarrow \quad u - v\sin\theta = v\cos\theta$$

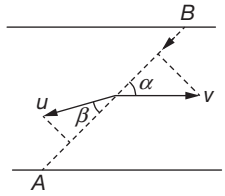
$$\Rightarrow \quad v = \frac{u}{\sin\theta + \cos\theta} = \frac{u}{\sqrt{2}\sin(\theta + 45°)}$$

The minimum value of v is when $\theta = 45°$. So

$$v_{\text{MIN}} = \frac{u}{\sqrt{2}}, \text{ for } \theta = 45°$$

13. In order that the moving launch is always on the straight line AB, the components of velocity of the current and of the launch in the direction perpendicular to AB should be equal, so,





$$u\sin\beta = v\sin\alpha \quad \ldots(1)$$

Also, $S = AB = (u\cos\beta + v\cos\alpha)t_1$ $\ldots(2)$

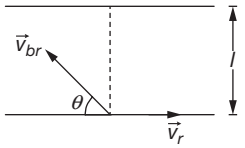
Further $BA = (u\cos\beta - v\cos\alpha)t_2$ $\ldots(3)$

$$t_1 + t_2 = t \quad \ldots(4)$$

Solving these equations, we get

$$u = 8 \text{ ms}^{-1} \text{ and } \beta = 12°$$

14. (a) $v_r > v_{br}$, the man cannot reach the point directly opposite to his starting point just by rowing. He will experience a non-zero drift and hence has to walk to reach the destination.



In the situation shown in figure, if t_1 is the time taken to cross the river, then

$$t_1 = \frac{l}{v_{br}\cos\theta} = \frac{120}{3\cos\theta} = \frac{40}{\cos\theta}$$

Drift, $x = (v_r - v_{br}\sin\theta)t = (4 - 3\sin\theta)\left(\frac{40}{\cos\theta}\right)$

If, v_w is the walking speed of the man $(= 1 \text{ ms}^{-1})$, then $t_2 = \frac{x}{v_w}$ is the time taken by him to walk and reach the point opposite to the point of start.

$$\Rightarrow \quad t_2 = \frac{40}{\cos\theta} = (4 - 3\sin\theta)$$

Total time $t = t_1 + t_2$

$$\Rightarrow \quad t = \frac{40}{\cos\theta}(5 - 3\sin\theta)$$

For t to be minimum, $\frac{dt}{d\theta} = 0$

$$\Rightarrow \quad -3\cos^2\theta + (5 - 3\sin\theta)\sin\theta = 0$$

$$\Rightarrow \quad 5\sin\theta = 3$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{3}{5}\right)$$

So, he should row at an angle $90° + \sin^{-1}\left(\frac{3}{5}\right)$ upstream.

(b) $t_{\min} = \left(\frac{40}{4/5}\right)\left(5 - 3\times\frac{3}{5}\right) = 50\left(\frac{16}{5}\right) = 160$ s

15. (a) Let the two meet at a distance s from ground. Then

$$s - 12 = 18t - \frac{1}{2}(9.8)t^2 \qquad \ldots(1)$$

and $s - 5 = 2t$ $\ldots(2)$

Solving these two equations, we get

$t = 3.65$ s and $s = 12.3$ m

(b) $v_b = 18 - (9.8)(3.65) = -17.8$ ms^{-1}

i.e., velocity of ball is 17.8 ms^{-1} (downwards) at the time of impact.

So, the relative velocity is 19.8 ms^{-1} (downwards)

16. $t = \frac{80}{1.6} = 50$ s

(a) $v_r = \frac{40}{50} = 0.8$ ms^{-1}

(b) $v = \sqrt{v_r^2 + v_{sr}^2} = \sqrt{(0.8)^2 + (1.6)^2} = 1.79$ ms^{-1}

(c) $\theta = \sin^{-1}\left(\frac{v_r}{v_{sr}}\right) = \sin^{-1}\left(\frac{0.8}{1.6}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

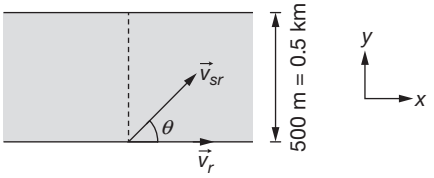
Hence the swimmer should head at $(90° + 30°) = 120°$ upstream.

17. Time taken by the pedestrian to cross the track, $t = \frac{d}{v}$

So, minimum distance required for cyclist is $s_{\min} = Vt = \frac{Vd}{v}$

18. (a) $(v_s)_y = (v_{sr})_y = 3\sin\theta$ kmh^{-1}

$$\Rightarrow \quad t = \frac{w}{(v_s)_y} = \frac{0.5}{3\sin\theta} \text{ hour} = \frac{10}{\sin\theta} \text{ minute}$$



(b) Time is shortest at $\sin\theta = 1$ or $\theta = 90°$

$t_{\min} = 10$ minute

(c) Since, $v_r > v_{sr}$, so he can not reach the other shore at the point directly opposite to his starting point. If he starts at angle θ as shown in figure he will reach a distance.

$$x = \left(\frac{w}{v_{sr}\sin\theta}\right)(v_r + v_{sr}\cos\theta)$$

For x to be minimum $\frac{v_r + v_{sr}\cos\theta}{\sin\theta}$ should be minimum

$$\Rightarrow \quad \frac{d}{d\theta}\left\{\frac{v_r + v_{sr}\cos\theta}{\sin\theta}\right\} = 0$$

$$\Rightarrow \quad -v_{sr}\sin^2\theta - (v_r + v_{sr}\cos\theta)(\cos\theta) = 0$$

$$\Rightarrow \quad v_r\cos\theta = -v_{sr}$$

$$\Rightarrow \quad \cos\theta = -\frac{v_{sr}}{v_r} = -\frac{3}{5}$$

$$\Rightarrow \quad \sin\theta = \frac{4}{5}$$

$$\Rightarrow \quad x_{\min} = \frac{\left(\frac{1}{2}\right)}{3\sin(127°)}(5 + 3\cos(127°))$$

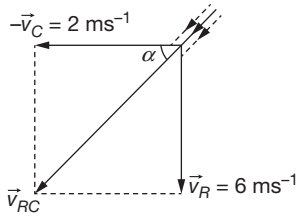
$$\Rightarrow \quad x_{\min} = \frac{\left(\frac{1}{2}\right)}{\left(3\times\frac{4}{5}\right)}\left(5 - \frac{9}{5}\right) = \frac{2}{3} \text{ km}$$

19. $\vec{v}_{RC}$ = velocity of rain w.r.t. cart $= \vec{v}_R - \vec{v}_C$

From figure, we observe that

$$\tan\alpha = \frac{6}{2}$$

$$\Rightarrow \quad \alpha = \tan^{-1}(3)$$



20. Time taken by the mirrors to collide is $T = \frac{d}{v+v} = \frac{d}{2v}$

Speed of the particle with respect to the approaching mirror is $(v+3v) = 4v$.

Time taken for the first trip is $t_1 = \frac{d}{4v}$

New separation between the mirrors just after first trip i.e., at $t_1 = \frac{d}{4v}$ is

$$x_1 = d - (v+v)t_1$$

$$\Rightarrow \quad x_1 = d - \frac{d}{2} = \frac{d}{2}$$

So, if t_2 is the time taken by the particle for the second trip, then

$$t_2 = \frac{\frac{d}{2}}{v+3v} = \frac{d}{8v}$$

New separation between the mirrors just after second trip i.e., at $t_2 = \frac{d}{8v}$ is

$$x_2 = \frac{d}{2} - (v+v)t_2$$

$$\Rightarrow \quad x_2 = \frac{d}{2} - (2v)\left(\frac{d}{8v}\right)$$

$$\Rightarrow \quad x_2 = \frac{d}{2} - \frac{d}{4} = \frac{d}{4}$$

So, if t_3 is the time taken by the particle for the third trip, then

$$t_3 = \frac{\frac{d}{4}}{v+3v} = \frac{d}{16v}$$

and so time taken for nth trip is $t_n = \frac{d}{2^{n+1}v}$

However, we observe that

$$t_1 + t_2 + t_3 + ... + t_n = T$$

$$\Rightarrow \quad \frac{d}{4v} + \frac{d}{8v} + \frac{d}{16v} + ... + \frac{d}{2^{n+1}v} = \frac{d}{2v}$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^n} = 1$$

$$\Rightarrow \quad \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} = 1 \qquad \left\{\because S_n = \frac{a(1-r^n)}{1-r}\right\}$$

$$\Rightarrow \quad 1-\left(\frac{1}{2}\right)^n = 1 \Rightarrow \left(\frac{1}{2}\right)^n = 0$$

$$\Rightarrow \quad n \to \infty$$

Total distance travelled by the particle is

$$s_{\text{total}} = vT = (3v)\left(\frac{d}{2v}\right) = \frac{3d}{2}$$

Single Correct Choice Type Questions

1.

MISCONCEPTION

Taking east direction as positive, we have

$u = +9$ ms^{-1} and $a = -3$ ms^{-2}

$$s_{5^{th}} = u + \frac{a}{2}(2(5)-1)$$

$$\Rightarrow \quad s_{5^{th}} = 9 - \frac{2}{2}(9) = 0$$

Actually this is not the correct answer, because we must observe that the particle reverses its direction of motion $(v = 0)$ at $t = 4.5$ s which lies in the specified interval. So, we will proceed other way round.

Since the total distance is

$$ds = v\,dt$$

$$\Rightarrow \quad s_{\text{total}} = \int v\,dt$$

$$\Rightarrow \quad s_{\text{total}} = \left|\int_4^{4.5} v\,dt\right| + \left|\int_{4.5}^{5} v\,dt\right|$$

Now $\int v\,dt = \int (u+at)\,dt = ut + \frac{1}{2}at^2$

CHAPTER 4

$$\Rightarrow \quad s_{\text{total}} = \left|9(4.5-4)+\frac{1}{2}(-2)\left|\left[\left(\frac{9}{2}\right)^2-(4)^2\right]\right.\right. + \left|9(5-4.5)+\frac{1}{2}(-2)\left[(5)^2-\left(\frac{9}{2}\right)^2\right]\right|$$

$$\Rightarrow \quad s_{\text{total}} = |4.5-4.25|+|4.5-4.25| = 0.5 \text{ m}$$

Hence, the correct answer is (B).

2. $a = \dfrac{dv}{dt} = 8-2t$

$$\Rightarrow \quad a = 8-2(5)$$

$$\Rightarrow \quad a = -2 \text{ ms}^{-2}$$

Hence, the correct answer is (D).

3. For constant speed, $a_T = 0$

$$\Rightarrow \quad a = a_C = a_N = \frac{V_T^2}{R}$$

$$\Rightarrow \quad 0.8g = \frac{v^2}{(200)}$$

$$\Rightarrow \quad v^2 = (0.8)(10)(200)$$

$$\Rightarrow \quad v = 40 \text{ ms}^{-1}$$

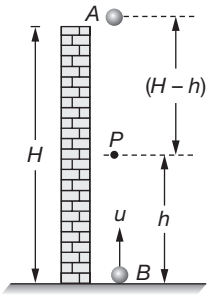
$$\Rightarrow \quad v = 40 \times \frac{18}{5} \text{ kmh}^{-1}$$

$$\Rightarrow \quad v = 144 \text{ kmh}^{-1}$$

Hence, the correct answer is (C).

4. Let the balls A and B collide at the point P a distance h from the ground. Let u be the velocity of launch of B and the launch velocity of A is zero. At the point P, final velocity of A is

$$v_A = \sqrt{2g(H-h)} \qquad \ldots(1)$$



and that of B is

$$v_B = \sqrt{u^2-2gh} \qquad \ldots(2)$$

Also, for the balls to collide at P, time taken by A to reach P equals time taken by B to reach P. So we have

$$(H-h) = \frac{1}{2}gt^2 \qquad \ldots(3)$$

$$h = ut - \frac{1}{2}gt^2 \qquad \ldots(4)$$

From (3), $t = \sqrt{\dfrac{2(H-h)}{g}}$

Substitute in (4), we get

$$h = u\sqrt{\frac{2(H-h)}{g}} - \frac{1}{2}g\frac{2(H-h)}{g}$$

$$\Rightarrow \quad h = u\sqrt{\frac{2(H-h)}{g}} - (H-h)$$

$$\Rightarrow \quad h = u\sqrt{\frac{2(H-h)}{g}} - H + h$$

$$\Rightarrow \quad u = H\sqrt{\frac{g}{2(H-h)}} \qquad \ldots(5)$$

Substitute (5) in (2), we get

$$v_B = \sqrt{\frac{gH^2}{2(H-h)} - 2gh} \qquad \ldots(6)$$

Since $v_A = 2v_B$

$$\Rightarrow \quad v_A^2 = 4v_B^2$$

$$\Rightarrow \quad 2g(H-h) = \frac{4H^2g}{2(H-h)} - 8gh$$

$$\Rightarrow \quad 2gH - 2gh = \frac{2H^2g}{H-h} - 8gh$$

$$\Rightarrow \quad H-h = \frac{H^2}{H-h} - 4h$$

$$\Rightarrow \quad \frac{H^2}{H-h} - H = 3h$$

$$\Rightarrow \quad \frac{H^2-H^2+Hh}{H-h} = 3h$$

$$\Rightarrow \quad \frac{H}{H-h} = 3$$

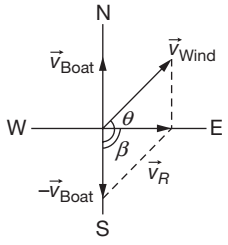
$$\Rightarrow \quad H = 3H-3h$$

$$\Rightarrow \quad 2H = 3h$$

$$\Rightarrow \quad h = \frac{2H}{3}$$

Hence, the correct answer is (D).

5. $\tan\beta = \dfrac{v_{wind}\sin\theta}{v_{boat} + v_{wind}\cos\theta}$



$\Rightarrow \quad \tan\beta = \dfrac{72\sin(135°)}{51 + 72\cos(135°)}$

$\Rightarrow \quad \tan\beta \to \infty$

$\Rightarrow \quad \beta \approx 90°$

i.e., direction of flag is towards east

Hence, the correct answer is (A).

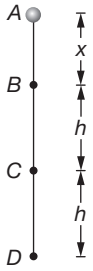
6. $L = \dfrac{1}{2}gt^2 - \dfrac{1}{2}g(t-T)^2$

Solve $t = \dfrac{T}{2} + \dfrac{L}{gT}$

Hence, the correct answer is (B).

7. Let the point B be situated at a distance x below A. If the particle reaches B in time t, then

$$x = \frac{1}{2}gt^2 \qquad \ldots(1)$$



For AC, $x + h = \dfrac{1}{2}g(t+2)^2$ …(2)

For AD, $x + 2h = \dfrac{1}{2}g(t+3)^2$ …(3)

Solving (1), (2) and (3), we get

$t = 0.5$ s

Hence, the correct answer is (B).

8. At the maximum height, $v = 0$

$\Rightarrow \quad 0^2 - u^2 = 2(-g)(h)$

$\Rightarrow \quad u = \sqrt{2gh}$

$\Rightarrow \quad u = \sqrt{2(10)(20)} = 20 \text{ ms}^{-1}$

If t be the time taken by the ball to reach the maximum height, then

$0 = u - gt$

$\Rightarrow \quad 0 = 20 - (10)t$

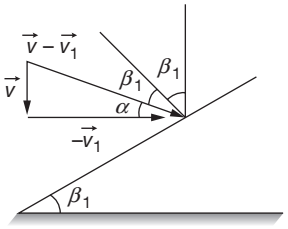
$\Rightarrow \quad t = 2$ s

So, time taken by the ball to return to juggler's hand is 4 s. To maintain a proper distance between them, the balls must be thrown up at an interval of

$$\Delta t = \frac{4}{4}s = 1 \text{ s}$$

Hence, the correct answer is (B).

10. Let hail fall along the vertical with a velocity $\vec{v}$. In the reference frame fixed to the car, the angle of incidence of hailstones on the windscreen is equal to the angle of reflection. The velocity of the hailstone before striking the windscreen is $\vec{v} - \vec{v}_1$ (by triangle law of vectors). Further the hailstones bounce vertically upwards (from the viewpoint of the driver) after reflection from wind screen of the car. If angle of incidence and hence the angle of reflection both equal β_1 then $\alpha + 2\beta_1 = 90°$.



Further $\tan\alpha = \dfrac{v}{v_1}$

$\Rightarrow \quad \tan(90 - 2\beta_1) = \dfrac{v}{v_1}$

$\Rightarrow \quad \dfrac{v}{v_1} = \cot(2\beta_1)$ {for 1st car}

Similarly for second car

$\Rightarrow \quad \dfrac{v}{v_2} = \cot(2\beta_2)$

$$\frac{v_1}{v_2} = \frac{\cot(2\beta_2)}{\cot(2\beta_1)} = \frac{\cot(30)}{\cot(60)} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3$$

Hence, the correct answer is (A).

11. Let v be the velocity of escalator and u be the velocity of the person. Then

$$t_1 = \frac{l}{u} \qquad \ldots(1)$$

Further $t_2 = \frac{l}{v}$ $\ldots(2)$

If t be the time taken by him to walk up the escalator, then

$$t = \frac{l}{u+v}$$

From (1) and (2)

$$u = \frac{l}{t_1} \text{ and } v = \frac{l}{t_2}$$

$$\Rightarrow \quad t = \frac{l}{u+v} = \frac{l}{\frac{l}{t_1}+\frac{l}{t_2}}$$

$$\Rightarrow \quad t = \frac{t_1 t_2}{t_1 + t_2}$$

Hence, the correct answer is (D).

12. Since both are falling under gravity, so relative acceleration of one stone w.r.t. other is zero. So,

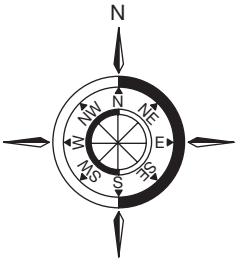
$$\Delta x = (u_1 + u_2)t = ut \qquad \{\because u_1 = 0\}$$

So, the graph is a straight line passing through the origin.

Hence, the correct answer is (C).

13. Velocity of wind is $10\ ms^{-1}$ from South to North i.e.

$$\vec{v}_{\text{wind}} = 10\ \hat{j}$$



But, to the cyclist it appears to blow from the East at $10\ \text{ms}^{-1}$. So, velocity of wind relative to the cyclist is $10\ \text{ms}^{-1}$ from East to West i.e.

$$\vec{v}_{\text{wind/cyclist}} = -10\ \hat{i}$$

Since $\vec{v}_{\text{wind/cyclist}} = \vec{v}_{\text{wind}} - \vec{v}_{\text{cyclist}}$

$$\Rightarrow \quad -10\ \hat{i} = 10\ \hat{j} - \vec{v}_{\text{cyclist}}$$

$$\Rightarrow \quad \vec{v}_{\text{cyclist}} = 10\ \hat{i} + 10\ \hat{j}$$

Hence, the correct answer is (B).

14. $$\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\sqrt{2a_1 s} - \sqrt{2a_2 s}}{\sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}}}$$

$$\Rightarrow \quad v = \left(\sqrt{a_1 a_2}\right)t$$

$$\Rightarrow \quad v = \sqrt{(2g)(8g)}t$$

$$\Rightarrow \quad v = 4gt$$

Hence, the correct answer is (A).

15. The graph shows the following v-s relation i.e.,

$$v = s$$

$$\Rightarrow \quad \frac{dv}{ds} = 1$$

Since $a = \frac{vdv}{ds}$

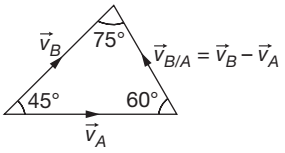
$$\Rightarrow \quad a = v(1)$$

$$\Rightarrow \quad a = v$$

So, a-v graph is again a straight line passing through the origin and inclined to the x-axis at an angle of $45°$.

Hence, the correct answer is (C).

16. According to Law of sines or Lami's Theorem



$$\Rightarrow \quad \frac{v_A}{\sin 75^o} = \frac{v_B}{\sin 60^o} = \frac{v_{B/A}}{\sin 45^o}$$

$$\Rightarrow \quad v_B = 717\ \text{km}\,h^{-1}$$

Hence, the correct answer is (C).

17. Since, $h = \frac{1}{2}gt^2$ $\ldots(1)$

and $\frac{9h}{16} = \frac{1}{2}g(t-1)^2$ $\ldots(2)$

$$\Rightarrow \quad \frac{3}{4} = \frac{t-1}{t}$$

$$\Rightarrow \quad 3t = 4t - 4$$

$$\Rightarrow \quad t = 4\ \text{s}$$

$$\Rightarrow \quad h = 80\ \text{m}$$

Hence, the correct answer is (C).

18. The graph shown is actually for

$$s = ut - \frac{1}{2}at^2$$

Lets see this, by making the LHS a perfect square.

$$\Rightarrow \quad s = -\frac{a}{2}\left(t^2 - \frac{2ut}{a}\right)$$

$$\Rightarrow \quad s = -\frac{a}{2}\left[t^2 - \left(\frac{2u}{a}\right)t + \frac{u^2}{a^2} - \frac{u^2}{a^2}\right]$$

$$\Rightarrow \quad s = -\frac{a}{2}\left[\left(t - \frac{u}{a}\right)^2 - \frac{u^2}{a^2}\right]$$

$$\Rightarrow \quad s - \frac{u^2}{2a} = -\frac{a}{2}\left(t - \frac{u}{a}\right)^2$$

$$\Rightarrow \quad S = -\frac{a}{2}T^2$$

where $S = \frac{s - u^2}{2a}$ and $T = t - \frac{u}{a}$ and the origin at $(T = 0,\ S = 0) \equiv \left(\frac{u}{a},\ \frac{u^2}{2a}\right)$

So, $\frac{u}{a} = 2.5$ and $\frac{u^2}{2a} = 100$

$$\Rightarrow \quad \frac{u^2}{2a} = \frac{u}{2}\left(\frac{u}{a}\right) = 100$$

$$\Rightarrow \quad \frac{u}{2}(2.5) = 100$$

$$\Rightarrow \quad u = \frac{200}{2.5} = 80 \text{ ms}^{-1}$$

and $a = \frac{80}{2.5} = 32 \text{ ms}^{-2}$

Hence, the correct answer is (C).

19. $s_1 = \frac{1}{2}at^2$

$$s_2 = (at)t - \frac{1}{2}a\frac{t^2}{4}$$

$$\Rightarrow \quad s_2 = at^2 - \frac{at^2}{8}$$

$$\Rightarrow \quad s_2 = \frac{7at^2}{8}$$

Hence, the correct answer is (A).

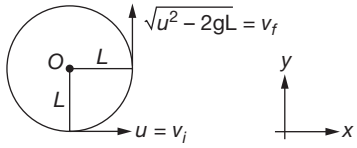
20. v_i be the initial velocity i.e. velocity at lowest point

$$\Rightarrow \quad \vec{v}_i = u\hat{i}$$

v_f be the final velocity i.e. velocity when the string becomes vertical. So, we have

$$v_f^2 - v_i^2 = 2(-g)L$$

$$\Rightarrow \quad v_f = \sqrt{u^2 - 2gL} \qquad \text{(in magnitude)}$$



$$\Rightarrow \quad \vec{v}_f = \left(\sqrt{u^2 - 2gL}\right)\hat{j}$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \left(\sqrt{u^2 - 2gL}\right)\hat{j} - u\hat{i}$$

$$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{u^2 - 2gL + u^2} = \sqrt{2\left(u^2 - gL\right)}$$

Hence, the correct answer is (D).

21. $v = \frac{dx}{dt} = 3at^2 + 2bt + c$

$$\Rightarrow \quad u = v\Big|_{t=0} = c \qquad \ldots(1)$$

$$\Rightarrow \quad a = \frac{dv}{dt} = 6at + 2b$$

$$\Rightarrow \quad a\Big|_{t=0} = 2b$$

$$\Rightarrow \quad \frac{a\Big|_{t=0}}{v\Big|_{t=0}} = \frac{2b}{c}$$

Hence, the correct answer is (B).

22. $a = \frac{P}{mv}$

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{P}{mv}$$

$$\Rightarrow \quad v^2 dv = \frac{P}{m}dx$$

$$\Rightarrow \quad \int_{v_1}^{v_2} v^2 dv = \frac{P}{m}\int_0^x dx \qquad \{\because P = \text{constant}\}$$

CHAPTER 4

$$\Rightarrow \quad \frac{v^3}{3}\Big|_{v_1}^{v_2} = \frac{P}{m}\left(x\Big|_0^x\right)$$

$$\Rightarrow \quad v_2^3 - v_1^3 = \frac{3Px}{m}$$

$$\Rightarrow \quad x = \frac{m}{3P}\left(v_2^3 - v_1^3\right)$$

Hence, the correct answer is (C).

23. $\frac{dv}{dt} = a - bv$

$$\Rightarrow \quad \int_0^v \frac{dv}{a-bv} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{b}\left[\log_e(a-bv) - \log_e(a)\right] = t$$

$$\Rightarrow \quad \frac{a-bv}{a} = e^{-bt}$$

$$\Rightarrow \quad v = \frac{a}{b}\left(1 - e^{-bt}\right)$$

At $t = 0$ i.e. initially, velocity must be zero (as stated in the Problem) and this condition is met only by Option (A).

Hence, the correct answer is (A).

24. Initial velocity of dropping = zero

Let v_1 be velocity at end of 10 s.

$$\Rightarrow \quad v_1 = gt$$

$$\Rightarrow \quad v_1 = 100 \text{ ms}^{-1}$$

Distance travelled during this time is

$$h_1 = \frac{v_1^2}{2g} = \frac{(100)^2}{2(10)}$$

$$\Rightarrow \quad h_1 = 500 \text{ m}$$

So, a remaining distance of $2495 - 500 = 1995\ m$ has to be travelled with a retardation of 2.5 ms^{-2}. Let the parachutist strike the ground with velocity v. Then

$$v^2 - v'^2 = 2a(h - h')$$

$$\Rightarrow \quad v^2 - (100)^2 = 2(-2.5)(1995)$$

$$\Rightarrow \quad v^2 = 10000 - 9975$$

$$\Rightarrow \quad v^2 = 25$$

$$\Rightarrow \quad v = 5 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

25. Since $s = \left(\frac{u+v}{2}\right)t$

$$4 = \left(\frac{1000 + 9000}{2}\right)t$$

$$\Rightarrow \quad t = \frac{8}{10000}$$

$$\Rightarrow \quad t = 8 \times 10^{-4} \text{ s}$$

Hence, the correct answer is (B).

26. $s_A = s_B$

$$8t + \frac{1}{2}(2)t^2 = 4t + \frac{1}{2}(4)t^2$$

$$\Rightarrow \quad 8 + t = 4 + 2t$$

$$\Rightarrow \quad t = 4 \text{ s}$$

Since $s_A = 8t + \frac{1}{2}(2)t^2$

$$\Rightarrow \quad s_A = 8(4) + (4)^2$$

$$\Rightarrow \quad s_A = 48 \text{ m}$$

Hence, the correct answer is (C).

27. $v_y = \frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt} = \frac{1}{2}v_x$

$$\Rightarrow \quad v_y = \frac{dy}{dt} = 2 - t \qquad \{\because v_x = 4 - 2t\}$$

$a_x = -2 \text{ ms}^{-2}$ and $a_y = -1 \text{ ms}^{-2}$

$u_x = v_x\big|_{t=0} = 4 \text{ ms}^{-1}$ and $u_y = v_y\big|_{t=0} = 2 \text{ ms}^{-1}$

Since we observe that a_x and a_y both are negative but u_x and u_y are positive, so motion is retarded initially and accelerated once the direction of motion is reversed.

Hence, the correct answer is (D).

28. Let the ball be at height h at time t and time $(t + \Delta t)$. Then

$$h = ut - \frac{1}{2}gt^2 \text{ and} \qquad \ldots(1)$$

$$h = u(t + \Delta t) - \frac{1}{2}g(t + \Delta t)^2 \qquad \ldots(2)$$

Equating (1) and (2), we get

$$t = \frac{2u - g\Delta t}{2g} \qquad \ldots(3)$$

Substituting (3) in (1), we get

$$h = \frac{4u^2 - g^2(\Delta t)^2}{8g}$$

$$\Rightarrow \quad u = \frac{1}{2}\sqrt{8gh + g^2(\Delta t)^2}$$

Hence, the correct answer is (C).

29. $v_x = \frac{dx}{dt} = 30\cos(6t)$ and

$$v_y = \frac{dy}{dt} = 30\sin(6t)$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = 30 \text{ ms}^{-1}$$

So, we observe that the speed is constant. Hence distance

$$x = vt$$

$$\Rightarrow \quad x = (30)(4) = 120 \text{ m}$$

Hence, the correct answer is (D).

30. Let the acceleration be A and initial velocity be U. Then

$$a = Up + \frac{1}{2}Ap^2 \qquad \ldots(1)$$

$$a + b = U(p+q) + \frac{1}{2}A(p+q)^2$$

$$\Rightarrow \quad b = Uq + \frac{1}{2}Aq^2 + Apq \qquad \ldots(2)$$

$q(1) - p(2)$ gives

$$qa - pb = \frac{1}{2}Ap^2q - \frac{1}{2}Aq^2p - Ap^2q$$

$$\Rightarrow \quad qa - pb = -\frac{1}{2}Aq^2p - \frac{1}{2}Ap^2q$$

$$\Rightarrow \quad pb - qa = \frac{1}{2}Apq(p+q)$$

$$\Rightarrow \quad A = \frac{2(pb-qa)}{pq(p+q)} = \frac{2(bp-aq)}{pq(p+q)}$$

Hence, the correct answer is (D).

31. $v_x = \frac{dx}{dt} = -a\omega\sin(\omega t)$

$$a_x = \frac{dv_x}{dt} = -a\omega^2\cos(\omega t)$$

Similarly

$$v_y = \frac{dy}{dt} = a\omega\cos(\omega t)$$

$$a_y = \frac{dv_y}{dt} = -a\omega^2\sin(\omega t)$$

So, acceleration is $\sqrt{a_x^2 + a_y^2}$

$$\Rightarrow \quad \text{Acceleration} = a\omega^2$$

Hence, the correct answer is (C).

32. **For** $0 \le t \le 2s$

$$a_1 = 2 \text{ ms}^{-2}$$

$$\Rightarrow \quad \frac{dv_1}{dt} = 2$$

$$\Rightarrow \quad \int_0^{v_1} dv_1 = 2\int_0^t dt$$

$$\Rightarrow \quad v_1 = 2t \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dx_1}{dt} = 2t$$

$$\Rightarrow \quad dx_1 = 2t\,dt$$

$$\Rightarrow \quad \int_0^{x_1} dx_1 = 2\int_0^2 t\,dt$$

$$\Rightarrow \quad x_1 = 2\frac{t^2}{2}\Big|_0^2$$

$$\Rightarrow \quad x_1 = 4 \text{ m}$$

For $2s \le t \le 4s$

$$a_2 = 1 \text{ ms}^{-2}$$

$$\Rightarrow \quad \frac{dv_2}{dt} = 1$$

$$\Rightarrow \quad \int_4^{v_2} dv^2 = \int_2^t dt \qquad \{\because \text{ of (1)}\}$$

$$\Rightarrow \quad v_2 - 4 = t - 2$$

$$\Rightarrow \quad v_2 = 2 + t \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{dx_2}{dt} = 2 + t$$

$$\Rightarrow \quad \int_4^{x_2} dx_2 = \int_2^4 (2+t)\,dt$$

$$\Rightarrow \quad x_2 - 4 = 2(2) + \frac{1}{2}(4^2 - 2^2)$$

$$\Rightarrow \quad x_2 - 4 = 4 + 6$$

CHAPTER 4

$\Rightarrow \quad x_2 = 14 \text{ m}$

Now $v\big|_{t=4} = 6 \text{ ms}^{-1}$

With this initial velocity and a retardation of 1.5 ms^{-2}, the particle will come to rest in a time t (say). Then

$0 = 6 - (1.5)t$

$\Rightarrow \quad t = 4 \text{ s}$

So, for a further 4 s, $\frac{dv_3}{dt} = -1.5$

$dv_3 = -1.5dt$

$\Rightarrow \quad \int_6^{v_3} dv_3 = -1.5\int_4^t dt$

$\Rightarrow \quad v_3 - 6 = -1.5(t-4)$

$\Rightarrow \quad v_3 - 6 = -1.5t + 6$

$\Rightarrow \quad v_3 = -1.5t + 12 \qquad \ldots(3)$

$\Rightarrow \quad \frac{dx_3}{dt} = -1.5t + 12$

$\Rightarrow \quad \int_{14}^{x} dx_3 = -1.5\int_4^8 tdt + 12\int_4^8 dt$

$\Rightarrow \quad x - 14 = -1.5\left(\frac{t^2}{2}\Big|_4^8\right) + 12\left(t\Big|_4^8\right)$

$\Rightarrow \quad x - 14 = -1.5(32-8) + 12(4)$

$\Rightarrow \quad x - 14 = 12$

$\Rightarrow \quad x = 26 \text{ m}$

Please be careful while taking the limits in each interval.

Hence, the correct answer is (B).

33. $80 \text{ kmh}^{-1}\ NE \equiv 80\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) = \vec{v}_A$

$60 \text{ kmh}^{-1}\ SE \equiv 60\left(\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right) = \vec{v}_B$

Since $\vec{v}_{A/B}$ = velocity of A relative to $B = \vec{v}_A - \vec{v}_B$

$\Rightarrow \quad \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = 10\sqrt{2}\hat{i} + 70\sqrt{2}\hat{j}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{7}\right).$

Hence, the correct answer is (D).

34. $s = \frac{t^2}{2} + \frac{t^3}{3}$

$v_T = \frac{ds}{dt} = t + t^2$

$\Rightarrow \quad a_T = 1 + 2t$

$\Rightarrow \quad a_T\big|_{t=2s} = 5 \text{ ms}^{-2}$

Now $a_C = \frac{v_T^2}{r} = \frac{\left(v_T\big|_{t=2}\right)^2}{r}$

$\Rightarrow \quad a_C = \frac{(6)^2}{3} = 12 \text{ ms}^{-2}$

Since $a^2 = a_C^2 + a_T^2$

$\Rightarrow \quad a^2 = 144 + 25$

$\Rightarrow \quad a = 13 \text{ ms}^{-2}$

Hence, the correct answer is (D).

35. $v = \sqrt{2gh}$

$\Rightarrow \quad v = \sqrt{2(10)5}$

$\Rightarrow \quad v = 10 \text{ ms}^{-1}$

$t = \sqrt{\frac{2h}{g}}$

$\Rightarrow \quad t = \sqrt{\frac{2(5)}{10}} = 1 \text{ s}$

So, total time to hit the bottom from the surface is $t' = 5 - 1 = 4$ s

$\Rightarrow \quad d = vt'$

$\Rightarrow \quad d = (10)(4)$

$\Rightarrow \quad d = 40 \text{ m}$

Hence, the correct answer is (D).

36. Whenever a particle separated from a body moving with some velocity and having some acceleration, then the particle separated always retains the velocity of the body but does not retains its acceleration. So, in this case we have the velocity of the balloon at $t = 8$ s is

$v = 0 + (1.25)(8) = 10 \text{ ms}^{-1}$ and

$s_{\text{balloon}} = \frac{1}{2}(1.25)(64) = 40 \text{ m}$

When released the stone has a velocity of 10 ms^{-1} up. So, for the stone, we have

$s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad -40 = (10)t + \frac{1}{2}(-10)t^2$

$\Rightarrow \quad -4 = t - \frac{t^2}{2}$

$\Rightarrow \quad -8 = 2t - t^2$

$\Rightarrow \quad t^2 - 2t - 8 = 0$

$\Rightarrow \quad t^2 - 4t + 2t - 8 = 0$

$\Rightarrow \quad t(t-4) + 2(t-4) = 0$

$\Rightarrow \quad t = -2 \text{ s} \text{ or } t = 4 \text{ s}$

Hence, the correct answer is (C).

37. $v = \sqrt{2gh}$ and $v' = \sqrt{2g(h+x)}$

$\Rightarrow \quad v' = \sqrt{2gh}\left(1 + \frac{x}{h}\right)^{\frac{1}{2}}$

Since $x \ll h$

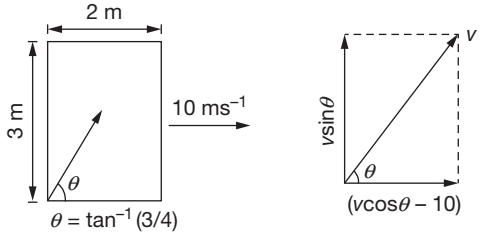
$\therefore \quad v' = v\left(1 + \frac{x}{2h}\right)$

$\therefore \quad$ Increase in velocity $= \frac{vx}{2h}$

Hence, the correct answer is (A).

38. $2 = (v\cos\theta - 10)t$ and

$3 = (v\sin\theta)t$



$\Rightarrow \quad t = \frac{1}{10}\left(\frac{3}{\tan\theta} - 2\right)$

$\Rightarrow \quad t = \frac{1}{10}\left(\frac{3}{\left(\frac{3}{4}\right)} - 2\right)$

$\Rightarrow \quad t = \frac{1}{10}(4-2)$

$\Rightarrow \quad t = 0.2 \text{ s}$

Hence, the correct answer is (A).

39. Since slope of v-t graph gives acceleration, so we observe that at all the instants acceleration of P is greater than Q.

Hence, the correct answer is (D).

40. Since, $t = \sqrt{\frac{2h}{g}}$ and $t' = \sqrt{\frac{2(h+x)}{g}}$

$\Rightarrow \quad t' = \sqrt{\frac{2h}{g}}\left(1 + \frac{x}{h}\right)^{\frac{1}{2}}$

$\Rightarrow \quad t' - t \approx \frac{xt}{2h}$

Hence, the correct answer is (C).

41. $a = -0.6 - 0.001\, v^2$

$v\frac{dv}{dx} = -0.6 - 0.001v^2$

$\Rightarrow \quad \int_{60}^{0} \frac{vdv}{0.6 + 0.001v^2} = -\int_{0}^{s} dx$

$\Rightarrow \quad \frac{1000}{2}\int_{60}^{0} \frac{2vdv}{v^2 + 600} = -s$

$\Rightarrow \quad s = -\frac{1000}{2}\log_e(v^2 + 600)\Big|_{60}^{0}$

$\Rightarrow \quad s = -500\log_e\left[\frac{600}{(60)^2 + 600}\right]$

$\Rightarrow \quad s = -500\log_e\left(\frac{600}{4200}\right)$

$\Rightarrow \quad s = 500\log_e(7)$

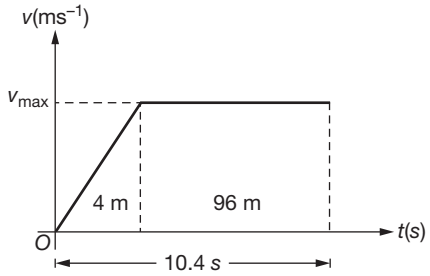
$\Rightarrow \quad s = 500(1.946)$

$\Rightarrow \quad s = 973 \text{ m}$

Hence, the correct answer is (C).

42. $s_1 = 4 = \frac{1}{2}(v_{max})t_1$

$s_2 = 96 = v_{max}t_2$



Since $t_1 + t_2 = 10.4$ s

$\Rightarrow \quad \frac{8}{v_{max}} + \frac{96}{v_{max}} = 10.4$

$$\Rightarrow \quad \frac{8+96}{v_{max}} = 10.4$$

$$\Rightarrow \quad v_{max} = \frac{104}{10.4} = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

43. $\vec{v} = \dot{\vec{r}} = \frac{d\vec{r}}{dt} = -a\omega\sin(\omega t)\hat{i} + a\omega\cos(\omega t)\hat{j} + b\hat{k}$

$$|\vec{v}| = \sqrt{a^2\omega^2 + b^2}$$

Distance moved by the particle in one full turn of helix is given by

$$s = |\vec{v}|T = \frac{2\pi}{\omega}\sqrt{a^2\omega^2 + b^2}$$

Hence, the correct answer is (B).

44. Area under a v-t graph gives the distance travelled. So

$$s = (20)(6) + \frac{1}{2}(20+45)3 + \frac{1}{2}(3)(45)$$

$$\Rightarrow \quad s = 120 + 165$$

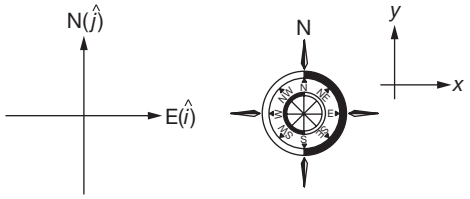
$$\Rightarrow \quad s = 285 \text{ m}$$

Hence, the correct answer is (C).

45. $\vec{v}_w = \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$

$$\vec{v}_m = (at)\hat{j}$$

$$\Rightarrow \quad \vec{v}_{wm} = \vec{v}_w - \vec{v}_m = \frac{v}{\sqrt{2}}\hat{i} + \left(\frac{v}{\sqrt{2}} - at\right)\hat{j}$$



It appears due east when, the y component vanishes. So

$$\frac{v}{\sqrt{2}} - at = 0$$

$$\Rightarrow \quad t = \frac{v}{\sqrt{2}a}$$

Hence, the correct answer is (C).

46. From graph

$$v_f^2 = (\text{final velocity})^2 = 900 \text{ km}^2\text{h}^{-2}$$

$$v_i^2 = (\text{initial velocity})^2 = 3600 \text{ km}^2\text{h}^{-2}$$

s = Distance covered = 0.6 km

Since, $v_f^2 - v_i^2 = 2as$ we get $a = -2250 \text{ kmh}^{-2}$

Hence, the correct answer is (C).

47. $a = \frac{v-u}{t} = \frac{10-25}{3} = -5 \text{ ms}^{-2}$ {Retardation}

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad s = 25(3) + \frac{1}{2}(-5)(9)$$

$$\Rightarrow \quad s = 75 - 22.5$$

$$\Rightarrow \quad s = 52.5 \text{ m}$$

Acceleration is constant, so (C) happens to be incorrect.

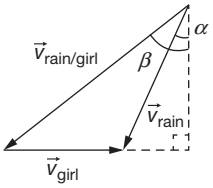
Hence, the correct answer is (A).

48. $v_{av} = \frac{l+l}{\frac{l}{2v_1} + \frac{l}{2v_2}}$

$$\Rightarrow \quad v_{av} = \frac{2v_1v_2}{v_1+v_2} = 48 \text{ kmh}^{-1}$$

Hence, the correct answer is (D).

49.



$$\tan\beta = \frac{v_{girl} + v_{rain}\sin\alpha}{v_{rain}\cos\alpha}$$

$$\Rightarrow \quad \tan\beta = \frac{8 + 10\sin\alpha}{10\cos\alpha}$$

Hence, the correct answer is (B).

50. The particle strikes the point B, when the velocity of the particle with respect to the platform is along AB i.e., the component of relative velocity along AD must be zero.

$$\Rightarrow \quad v - 2v\cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°$$

Hence, the correct answer is (C).

51. In this problem, we must check that the distance travelled by the elevator while accelerating plus the distance travelled by the elevator while decelerating must equal 100 m. If this is not satisfied then the remaining distance must have been travelled with a uniform speed equal to the maximum speed attained by the elevator at the end of accelerated motion.

Hence, the correct answer is (B).

52. Let v be the velocity of the river and u be the velocity of swimmer with respect to river. Then

$$t_1 = \frac{2d}{\sqrt{u^2 - v^2}} \qquad \ldots(1)$$

Further

$$t_2 = \frac{d}{u+v} + \frac{d}{u-v} = \frac{2ud}{u^2 - v^2} \qquad \ldots(2)$$

and $t_3 = \dfrac{2d}{u}$ $\qquad \ldots(3)$

From (1), (2) and (3), we get

$$t_1^2 = t_2 t_3$$

Hence, the correct answer is (D).

53. On passing through a plank of thickness x, the velocity of bullet becomes $\dfrac{19}{20}u$, where u is the initial velocity.

$$\Rightarrow \quad \left(\frac{19u}{20}\right)^2 - u^2 = 2ax \qquad \ldots(1)$$

Let the bullet further pass through n planks. Hence total number of planks $= (n+1)$

$$0^2 - u^2 = 2a(n+1)x \qquad \ldots(2)$$

Solving we get, $(n+1) \approx 11$

Hence, the correct answer is (B).

54. $v = \dfrac{6 \text{ km}}{2(1\ hr)} = 3 \text{ kmh}^{-1}$

Hence, the correct answer is (A).

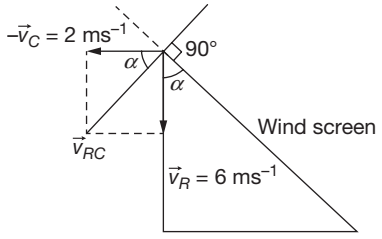
55. For rain drops to strike the wind screen normally, velocity of rain with respect to car $\vec{v}_{RC} = \vec{v}_R - \vec{v}_C$ should be perpendicular to the wind screen.

So, components of $\vec{v}_R$ and $-\vec{v}_C$ parallel to wind screen should cancel each other.

$\Rightarrow \quad 6\cos\alpha = 2\sin\alpha$

$\Rightarrow \quad \tan\alpha = 3$

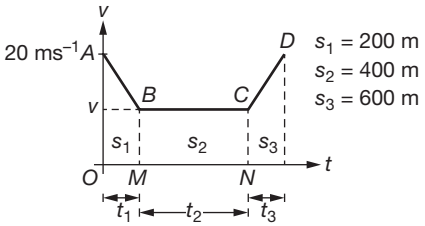
$\Rightarrow \quad \alpha = \tan^{-1}(3)$



Hence, the correct answer is (D).

56. **For *AB***

$$200 = \left(\frac{20+v}{2}\right)t_1 \qquad \ldots(1)$$



($\because$ 72 kmh^{-1} = 20 ms^{-1})

For *BC*

$$400 = vt_2 \qquad \ldots(2)$$

For *CD*

$$600 = \left(\frac{20+v}{2}\right)t_3 \qquad \ldots(3)$$

$\Rightarrow \quad t_1 = \dfrac{1}{3}t_3$

Since, $t_2 = t_1 + t_3 = 4t_1$

$\Rightarrow \quad 400 = 4vt_1$

$\Rightarrow \quad vt_1 = 100 \qquad \ldots(4)$

Divide (1) and (4), we have

$$2 = \frac{20+v}{2v}$$

$\Rightarrow \quad 4v = 20 + v$

$\Rightarrow \quad v = \dfrac{20}{3} \text{ ms}^{-1}$

$\Rightarrow \quad t_1 = 15\ s,\ t_2 = 60\ s,\ t_3 = 45\ s$

Hence, $t = t_1 + t_2 + t_3$

$\Rightarrow \quad t = 120 \text{ s} = 2 \text{ minute}$

Hence, the correct answer is (C).

57. From $O \rightarrow A$ we have acceleration varying linearly with time hence

$$\Rightarrow \quad a-0=\frac{5}{6}(t-0)$$

$$\Rightarrow \quad a=\frac{5}{6}t$$

$$\Rightarrow \quad \frac{dv}{dt}=\frac{5}{6}t$$

$$\Rightarrow \quad \int_0^{v_1} dv=\frac{5}{6}\int_0^{t} t\,dt$$

$$\Rightarrow \quad v_1=\frac{5}{12}t^2$$

Further $v_1=\frac{dx_1}{dt}=\frac{5}{12}t^2$

$$\Rightarrow \quad \int_0^{x_1} dx_1=\frac{5}{12}\int_0^{6} t^2dt \Rightarrow x_1=\frac{5}{36}t^3\Big|_0^6$$

$$\Rightarrow \quad x_1=\frac{5}{36}(6^3)=30 \text{ m}$$

For $A \rightarrow B$ the acceleration is constant and have a value of $5\ \text{ms}^{-2}$. Now velocity at point A is

$$v_1=\frac{5}{12}(36)=15\ \text{ms}^{-1}$$

$$\Rightarrow \quad x_2=v_1(6)+\frac{1}{2}(5)(6)^2$$

$$\Rightarrow \quad x_2=180 \text{ m}$$

So, total distance travelled is $x=30+180=210$ m

Hence, the correct answer is (B).

58. Let u be the initial velocity and a be the acceleration. Then

$$2=u(2)+\frac{1}{2}a(2)^2$$

$$\Rightarrow \quad u+a=1 \qquad \ldots(1)$$

$$4.2=u(2+4)+\frac{1}{2}a(2+4)^2$$

$$\Rightarrow \quad 4.2=6u+18a$$

$$\Rightarrow \quad 0.7=u+3a$$

$$\Rightarrow \quad u+3a=0.7 \qquad \ldots(2)$$

From (1) and (2), we get

$$2a=-0.3$$

$$a=-0.15\ \text{ms}^{-2}$$

and $u=1.15\ \text{ms}^{-1}$

$$v\Big|_{t=7s}=u+7a$$

$$\Rightarrow \quad v\Big|_{t=7s}=1.15-7(0.15) \Rightarrow v\Big|_{t=7s}=1.15-1.05$$

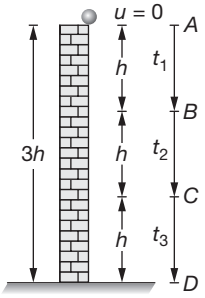
$$\Rightarrow \quad v\Big|_{t=7s}=0.1\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

59. For $A \rightarrow B$, $h=\frac{1}{2}gt_1^2$ …(1)

For $A \rightarrow C$, $2h=\frac{1}{2}g(t_1+t_2)^2$ …(2)

For $A \rightarrow D$, $3h=\frac{1}{2}g(t_1+t_2+t_3)^2$ …(3)



Please note that $u=0$ for all the intervals that start from A.

From (1), (2) and (3), we get

$$t_1:t_2:t_3=1:(\sqrt{2}-1):(\sqrt{3}-\sqrt{2})$$

Hence, the correct answer is (C).

60. Initially, when both are thrown up simultaneously, then acceleration of A w.r.t. B is zero (as both fall under the influence of gravity). So, relative position versus time plot is a straight line. Now the particle A reach the ground and stops, so its acceleration becomes zero and hence the relative acceleration of B w.r.t. A is g.

Therefore the relative position vs t graph is now parabolic. So, the variation plot is first linear and then parabolic.

Hence, the correct answer is (D).

61. Initially let the dog and cat be at A and B respectively with their velocities perpendicular to each other as shown

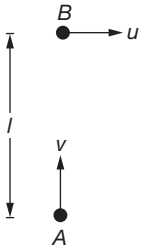


FIGURE 1: Positions of dog and cat at $t = 0$

The cat has a velocity constant in magnitude and direction but dog has a velocity continuously aimed at cat i.e. constant in magnitude but varying in direction.

Let at any instant t, v makes an angle θ with u. Then let us resolve u parallel to v and perpendicular to v as shown in Figure (2). Component parallel to v is $u\cos\theta$ and perpendicular to v is $u\sin\theta$

Relative velocity of dog towards cat is

$$(v - u\cos\theta) = -\frac{dx}{dt}$$

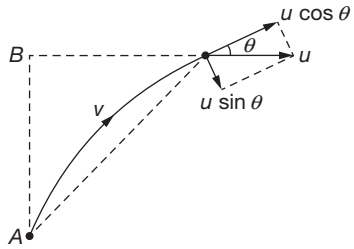


FIGURE 2: Positions of dog and cat at time t

The negative sign on RHS indicates that the distance between cat and dog decreases if the dog has to catch the cat.

$$\Rightarrow \quad -dx = (v - u\cos\theta)dt$$

$$\Rightarrow \quad -\int_{\ell}^{0} dx = \int_0^t v\,dt - \int_0^t (u\cos\theta)dt$$

$$\Rightarrow \quad l = \int_0^t v\,dt - \int_0^t (u\cos\theta)dt \qquad \ldots(1)$$

For dog to catch the cat we must have

$$\begin{pmatrix}\text{Horizontal}\\ \text{displacement}\\ \text{of CAT}\end{pmatrix} = \begin{pmatrix}\text{Horizontal}\\ \text{displacement}\\ \text{of DOG}\end{pmatrix}$$

i.e., $\quad ut = \int_0^t (v\cos\theta)dt \qquad \ldots(2)$

From (2) $\int_0^t (\cos\theta)dt = \frac{ut}{v}$

Substituting in (1), we get

$$l = vt - u\left(\frac{ut}{v}\right)$$

$$\Rightarrow \quad l = \frac{(v^2 - u^2)t}{v}$$

$$\Rightarrow \quad t = \frac{vl}{v^2 - u^2}$$

Also for dog to catch the cat $v > u$.

Hence, the correct answer is (A).

62. $v^2 \propto s$

$\Rightarrow$ acceleration is constant.

Hence, the correct answer is (A).

63. $AB = \left(100 \times \frac{5}{18}\right)t$ metre

$$CD = \left(60 \times \frac{5}{18}\right)t \text{ metre}$$

For BC, $v_C = v_B + at$

$$\Rightarrow \quad 60 \times \frac{5}{18} = 100 \times \frac{5}{18} + a(4)$$

$$\Rightarrow \quad 4a = -40 \times \frac{5}{18}$$

$$\Rightarrow \quad a = -\frac{50}{18} = -\frac{25}{9} \text{ ms}^{-2}$$

So the distance BC is

$$BC = 100 \times \frac{5}{18} \times 4 + \frac{1}{2}\left(-\frac{25}{9}\right)(4)^2$$

$$BC = \frac{1000}{9} - \frac{200}{9} = \frac{800}{9} \text{ m}$$

However $AB + BC + CD = 3000$ m

$$\Rightarrow \quad (160)\left(\frac{5}{18}\right)t + \frac{800}{9} = 3000$$

$$\Rightarrow \quad \frac{400}{9}t + \frac{800}{9} = 3000$$

$$\Rightarrow \quad 400t + 800 = 27000$$

$$\Rightarrow \quad 400t = 26200$$

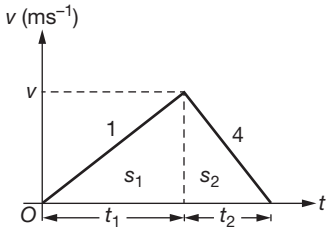
$$\Rightarrow \quad t = 65.5 \text{ s}$$

Hence, the correct answer is (C).

CHAPTER 4

64. If we take the maximum attainable velocity as 24 ms^{-1}, then to accelerate with 1 ms^{-2} (from rest), the time is

$t_1 = 24$ s $\quad \{\because\ 24 = 0 + (1)t_1\}$



During this time the distance covered is observed to be

$$s_1 = \frac{1}{2}a_1 t_1^2$$

$$\Rightarrow \quad s_1 = \frac{1}{2}(1)(24)^2 = 288 \text{ m} > 200 \text{ m}$$

which is not possible.

So, the car must attain a $v_{max} < 24$ ms^{-1}

Let that maximum velocity be v. Then

$t_1 + t_2 = t$ (say) and $s_1 + s_2 = 200$ m

$$\Rightarrow \quad \frac{v}{1} + \frac{v}{4} = t$$

$$\Rightarrow \quad v = \frac{4t}{5} \qquad \ldots(1)$$

Now $s_1 + s_2 = 200$

$$\Rightarrow \quad \frac{v^2}{2(1)} + \frac{v^2}{2(4)} = 200$$

$$\Rightarrow \quad \frac{16t^2}{50} + \frac{16t^2}{200} = 200$$

$$\Rightarrow \quad 16t^2\left(\frac{5}{200}\right) = 200$$

$$\Rightarrow \quad t^2 = 500$$

$$\Rightarrow \quad t \cong 22.4 \text{ s}$$

You could have directly applied the formula

$$s = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$$

where $s = 200$ m, $t = ?$

$\alpha = 1\ ms^{-2}$ and $\beta = 4\ ms^{-2}$

$\Rightarrow \quad t = 22.4$ s

Hence, the correct answer is (A).

65. Let the length of the escalator be ℓ. If u be the velocity of person, then

$$90 = \frac{l}{u} \qquad \ldots(1)$$

If v be the velocity of the escalator, then

$$60 = \frac{l}{v} \qquad \ldots(2)$$

If t be the time taken by him to walk up the moving escalator, then

$$t = \frac{l}{u+v} \qquad \ldots(3)$$

From (1), $u = \frac{l}{90}$

From (2), $v = \frac{l}{60}$

Substituting in (3), we get

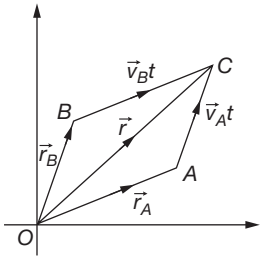
$$t = \frac{l}{\frac{l}{90} + \frac{l}{60}}$$

$$\Rightarrow \quad t = \frac{(60)(90)}{60+90}$$

$$\Rightarrow \quad t = 36 \text{ s}$$

Hence, the correct answer is (A).

66. Let the particles collide at time t.



For particles to collide at C

$$\vec{r}_B + \vec{v}_B t = \vec{r}_A + \vec{v}_A t$$

$$\Rightarrow \quad \vec{r}_B - \vec{r}_A = (\vec{v}_A - \vec{v}_B)t \qquad \ldots(1)$$

$$\Rightarrow \quad |\vec{r}_B - \vec{r}_A| = |\vec{v}_A - \vec{v}_B|t$$

$$\Rightarrow \quad t = \frac{|\vec{r}_B - \vec{r}_A|}{|\vec{v}_B - \vec{v}_A|} \qquad \ldots(2)$$

Substituting (2) in (1) and rearrange to get (D)

Hence, the correct answer is (D).

67. Since, $\frac{dv}{dt} = (g - cx^2)$

$$\Rightarrow \quad v\frac{dv}{dx} = g - cx^2$$

$$\Rightarrow \int_{v_0}^{0} v\,dv = \int_{0}^{x_m} \left(g - cx^2\right)dx$$

$$\Rightarrow -\frac{v_0^2}{2} = gx_m - \frac{cx_m^3}{3}$$

$$\Rightarrow \frac{cx_m^3}{3} = gx_m + \frac{v_0^2}{2}$$

$$\Rightarrow c = \frac{6gx_m + 3v_0^2}{2x_m^3}$$

Hence, the correct answer is (D).

68. $s_{n^{th}} = u + \frac{a}{2}(2n-1)$

$$\Rightarrow s_{n^{th}} = 0 + \frac{a}{2}(2n-1)$$

$$\Rightarrow s_{n^{th}} = \frac{a}{2}(2n-1)$$

$$\Rightarrow s_1 : s_2 : s_3 \equiv 1:3:5$$

Hence, the correct answer is (C).

69. $\vec{v} = \vec{u} + \vec{a}t$

$$\Rightarrow \vec{v} = \left(6\hat{i} + 8\hat{j}\right) + \left(0.8\hat{i} + 0.6\hat{j}\right)10$$

$$\Rightarrow \vec{v} = (6+8)\hat{i} + (8+6)\hat{j}$$

$$\Rightarrow \vec{v} = 14\hat{i} + 14\hat{j}$$

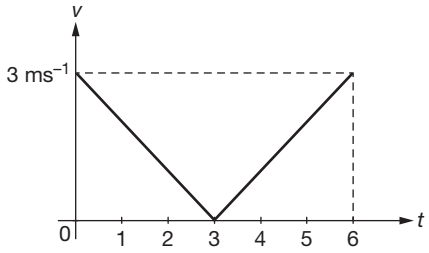
$$\Rightarrow |\vec{v}| = 14\sqrt{2}\ \text{ms}^{-2}$$

Hence, the correct answer is (C).

70. Distance moved by the car is the area under v-t graph. So,

$$s_{\text{total}} = \frac{1}{2}(3)(3) + \frac{1}{2}(3)(3)$$

$$\Rightarrow s_{\text{total}} = 9\ \text{m}$$



Hence, the correct answer is (C).

72. For OA, we have $a = \frac{5}{4}t$

$$\Rightarrow dv = \frac{5}{4}t\,dt$$

$$\Rightarrow v_A = \int dv = \frac{5}{4}\int_0^4 t\,dt = 10\ \text{ms}^{-1}$$

Otherwise, we can also observe that Δv = Area under a-t graph. So,

$$\Delta v = v = 10\ \text{ms}^{-1}$$

For AB, we have

$$v_B = v_A + at$$

$$\Rightarrow v_B = 10 + (5)(4) = 30\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

73. For constant acceleration, assuming $u = 0$, then

$v^2 \propto x$ or $s \propto t^2$

Hence, the correct answer is (D).

74. s = Area under the graph

$$\Rightarrow s = \frac{1}{2}(8)(2) + \frac{1}{2}(12)(3)$$

$$\Rightarrow s = 8 + 18$$

$$\Rightarrow s = 26\ \text{m}$$

Hence, the correct answer is (C).

75. Maximum acceleration is from C to D. So

$$a = \frac{60-20}{1-0.75}$$

$$\Rightarrow a = \frac{40}{0.25}$$

$$\Rightarrow a = 160\ \text{kmh}^{-2}$$

Hence, the correct answer is (B).

76. $\vec{v}_{\text{man}} = \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$

Let $\vec{v}_{\text{wind}} = a\hat{i} + b\hat{j}$

$$\Rightarrow \vec{v}_{\text{wind/man}} = \left(a - \frac{v}{\sqrt{2}}\right)\hat{i} + \left(b - \frac{v}{\sqrt{2}}\right)\hat{j}$$

$$\Rightarrow \tan\theta = \frac{b - \frac{v}{\sqrt{2}}}{a - \frac{v}{\sqrt{2}}} = \tan(270)$$

$$\Rightarrow a - \frac{v}{\sqrt{2}} = 0$$

CHAPTER 4

$$\Rightarrow \quad a = \frac{v}{\sqrt{2}}$$

$$\Rightarrow \quad \vec{v}_{\text{wind}} = \frac{v}{\sqrt{2}}\hat{i} + b\hat{j}$$

When the man doubles his speed

$$\vec{v}'_{\text{man}} = 2\left(\frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}\right) = \sqrt{2}\left(v\hat{i} + v\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}'_{\text{wind/man}} = \left(\frac{v}{\sqrt{2}} - \sqrt{2}v\right)\hat{i} + \left(b - \sqrt{2}v\right)\hat{j}$$

$$\Rightarrow \quad \tan\theta' = \frac{b - \sqrt{2}v}{\frac{v}{\sqrt{2}} - \sqrt{2}v} = \frac{2v - \sqrt{2}b}{v}$$

But $\theta' = 270 - \cot^{-1}(2)$

$$\Rightarrow \quad \tan\left[270 - \cot^{-1}(2)\right] = \frac{2v - \sqrt{2}b}{v}$$

$$\Rightarrow \quad \cot\left[\cot^{-1}(2)\right] = \frac{2v - \sqrt{2}b}{v}$$

$$\Rightarrow \quad 2v = 2v - \sqrt{2}b$$

$$\Rightarrow \quad b = 0$$

$$\therefore \quad \vec{v}_{\text{wind}} = \frac{v}{\sqrt{2}}\hat{i}$$

Hence, the correct answer is (B).

77. Distance between two cars leaving from station x is,

$$d = \left(\frac{1}{6}\right) \times 60 = 10 \text{ km}$$

Man meets the first car after time,

$$t_1 = \frac{60}{60 + 60} = \frac{1}{2} \text{ h}$$

He will meet the next car after time,

$$t_2 = \frac{10}{60 + 60} = \frac{1}{12} \text{ h}$$

In the remaining half an hour, number of cars he will meet again is, $n = \frac{1/2}{1/12} = 6$

So, total number of cars would be meet on route will be 7.

Hence, the correct answer is (B).

78. $$h = \frac{1}{2}gt^2 \qquad \ldots(1)$$

$$\frac{h}{2} = \frac{1}{2}g(t-1)^2 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$t = 2 \pm \sqrt{2} \text{ s}$$

But $2 - \sqrt{2} < 1$ s

$$\Rightarrow \quad t = 2 + \sqrt{2} \text{ s}$$

$$\Rightarrow \quad t = 3.41 \text{ s}$$

Hence, the correct answer is (B).

79. $$\frac{dv}{A - Bv} = dt$$

$$\Rightarrow \quad \int_0^v \frac{dv}{A - Bv} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{B}\log_e(A - Bv)\Big|_0^v = t$$

$$\Rightarrow \quad \log_e\left(\frac{A - Bv}{A}\right) = -Bt$$

$$\Rightarrow \quad \frac{A - Bv}{A} = e^{-Bt}$$

$$\Rightarrow \quad v = \frac{A}{B}\left(1 - e^{-Bt}\right)$$

Hence, the correct answer is (C).

80. Let the distance travelled by each car be s, then

$$v_1 - v_2 = v_0$$

$$\Rightarrow \quad \sqrt{2a_1 s} - \sqrt{2a_2 s} = v_0 \text{ and}$$

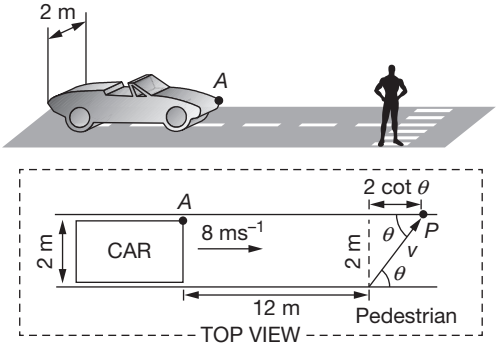
$$t_2 - t_1 = t_0$$

$$\Rightarrow \quad \sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}} = t_0$$

$$\Rightarrow \quad \frac{v_0}{t_0} = \frac{\sqrt{a_1} - \sqrt{a_2}}{\frac{1}{\sqrt{a_2}} - \frac{1}{\sqrt{a_1}}} = \sqrt{a_1 a_2}$$

Hence, the correct answer is (D).

81. For safely crossing the road the pedestrian must cross the road by the time the car travels a distance $(12 + 2\cot\theta)$ (see figure).



So $\frac{12+2\cot\theta}{8}=\frac{2}{v\sin\theta}$

$\Rightarrow \quad v=\frac{8}{6\sin\theta+\cos\theta}$

For v to be MINIMUM

$\Rightarrow \quad \frac{dv}{d\theta}=0$

$\Rightarrow \quad \tan\theta=6$ {Answer to PROBLEM 82}

$\Rightarrow \quad v=\frac{8}{\cos\theta(6\tan\theta+1)}=\frac{8}{\cos\theta(36+1)}$

$\Rightarrow \quad v=\frac{8}{37\left(\frac{1}{\sqrt{37}}\right)}$ $\{\because \tan\theta=6\}$

$\Rightarrow \quad v=\frac{8}{\sqrt{37}}\simeq\frac{8}{6}$

$\Rightarrow \quad v\simeq\frac{4}{3}\text{ ms}^{-1}$ {Answer to PROBLEM 81}

Hence, the correct answer is (A).

82. See PROBLEM 81.
Hence, the correct answer is (C).

83. For pedestrian to cross the road safely, the time taken by the pedestrian to reach P equals the time taken by the edge A of the car to reach P.
To reach P total distance $(12+2\cot\theta)$ is travelled by the car with a velocity of 8 ms^{-1}. Hence

$$t=\frac{12+2\cot\theta}{8}$$

$\Rightarrow \quad t=\frac{12+2\left(\frac{1}{6}\right)}{8}=\frac{\left(12+\frac{1}{3}\right)}{8}$

$\Rightarrow \quad t=\frac{37}{24}\text{ s}$

Hence, the correct answer is (B).

84. $a=\frac{v^2}{r}$

$\Rightarrow \quad \frac{a}{g}=\frac{v^2}{rg}$

$\Rightarrow \quad \frac{a}{g}=\frac{10^{14}}{(0.8)(10)}$

$\Rightarrow \quad \frac{a}{g}=1.25\times10^{13}$

Hence, the correct answer is (C).

85. Let the separation between the particles be 10 m at time t. Then $10=\frac{1}{2}gt^2-\frac{1}{2}g(t-1)^2$

$\Rightarrow \quad 2=t^2-(t-1)^2$

$\Rightarrow \quad 2=t^2-t^2-1+2t$

$\Rightarrow \quad 3=2t$

$\Rightarrow \quad t=1.5\text{ s}$

Hence, the correct answer is (B).

86. Taking downward direction as positive
CASE-1

$$h=ut_1+\frac{1}{2}gt_1^2 \qquad \ldots(1)$$

CASE-2

$$h=-ut_2+\frac{1}{2}gt_2^2 \qquad \ldots(2)$$

$(1)t_2+(2)t_1$ gives

$$h(t_1+t_2)=\frac{1}{2}gt_1^2t_2+\frac{1}{2}gt_2^2t_1$$

$\Rightarrow \quad h=\frac{1}{2}gt_1t_2 \qquad \ldots(3)$

If t is the time taken by it to fall freely to the ground from the same height, then

$$h=\frac{1}{2}gt^2 \qquad \ldots(4)$$

Equating (3) and (4), we get

$$t=\sqrt{t_1t_2}$$

Hence, the correct answer is (D).

87. $a=kv$

$v\frac{dv}{dx}=kv$ $\left\{\because a=\frac{dv}{dt}=v\frac{dv}{dx}\right\}$

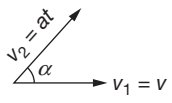
$\Rightarrow \quad \frac{dv}{dx}=k$

$\Rightarrow$ Slope of velocity-displacement graph is a constant
Hence, the correct answer is (C).

88. Velocity of 1st particle at time t is $v_1=v$
Velocity of 2nd particle at time t is $v_2=at$

Relative velocity $v_r=\sqrt{v_1^2+v_2^2-2v_1v_2\cos\alpha}$

$\Rightarrow \quad v_r^2=v^2+a^2t^2-2v(at)\cos\alpha$



CHAPTER 4

For v_r to be least or v_r^2 to be least we must have

$$\frac{d}{dt}\left(v_r^2\right)=0$$

$$\Rightarrow \quad 0+2a^2t-2av\cos\alpha=0$$

$$\Rightarrow \quad t=\frac{v\cos\alpha}{a}$$

Hence, the correct answer is (B).

89. $$v_r=\sqrt{v^2+a^2\left(\frac{v^2\cos^2\alpha}{a^2}\right)-2v\left(\frac{v\cos\alpha}{a}\right)a\cos\alpha}$$

$$\Rightarrow \quad v_r=\sqrt{v^2-v^2\cos^2\alpha}$$

$$\Rightarrow \quad v_r=v\sin\alpha$$

Hence, the correct answer is (A).

90. $v_A^2=2g(h+x)$ and

$$v_B^2=u^2+2gx$$

Since $v_A=v_B$

$$\Rightarrow \quad u^2+2gx=2gh+2gx$$

$$\Rightarrow \quad u^2=2gh$$

$$\Rightarrow \quad u=\sqrt{2gh}$$

Hence, the correct answer is (C).

91. $v_1+v_2=4$

$$v_1-v_2=\frac{4}{10}=0.4$$

$$\Rightarrow \quad 2v_1=4.4$$

$$\Rightarrow \quad v_1=2.2\ \text{ms}^{-1} \text{ and } v_2=1.8\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

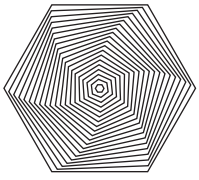
92. $$\frac{g}{2}(2n-1)=\frac{1}{2}g(3)^2$$

$$\Rightarrow \quad 2n=10$$

$$\Rightarrow \quad n=5\ \text{s}$$

Hence, the correct answer is (A).

93. $$t=\frac{a}{v-v\cos\left(\frac{2\pi}{n}\right)}=\frac{a}{2v\sin^2\left(\frac{\pi}{n}\right)}$$



where n is the number of sides in a symmetrical polygon. For this problem $n=6$

$$\Rightarrow \quad t=\frac{a}{2v\sin^2\left(\frac{\pi}{6}\right)}=\frac{2a}{v}$$

Hence, the correct answer is (A).

94. **METHOD I**

$$v=kx+v_0$$

Since $a=\frac{dv}{dt}$

$$\Rightarrow \quad a=k\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad a=kv=k^2x+kv_0$$

$$\Rightarrow \quad a=\alpha x+\text{constant}$$

METHOD II

Since $v=kx+v_0$

$$\Rightarrow \quad \frac{dv}{dx}=k$$

Now $a=\frac{dv}{dt}=v\left(\frac{dv}{dx}\right)$

$$\Rightarrow \quad a=(kx+v_0)k$$

$$\Rightarrow \quad a=k^2x+kv_0$$

$$\Rightarrow \quad a=\alpha x+\text{constant}$$

So, we observe that acceleration increases linearly with x.

Hence, the correct answer is (D).

95. Area under a-t graph equals the change in velocity (Δv)

$$\Rightarrow \quad \Delta v=\frac{1}{2}(3)(4)=6\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_f-v_i=6$$

$$\Rightarrow \quad v_f-2=6$$

$$\Rightarrow \quad v_f=8\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

96. From ground to maximum height, we have

$$h=\frac{u^2}{2g}=\frac{(20)^2}{2(9.8)}$$

$$\Rightarrow \quad h=\frac{400}{19.6}=20.4\ \text{m}$$

$$\Rightarrow \quad t_1=\frac{v}{g}=\frac{20.4}{9.8}=2.08\ \text{s}$$

Now the ball is caught 5 m above the point of launch in its downward motion. So, it is travelled 15 m down in time t_2 (say).

So, $15 = 0 + \frac{1}{2} g t_2^2$

$\Rightarrow \quad 15 = 4.9 t_2^2$

$\Rightarrow \quad t_2 = \sqrt{3.06}$

$\Rightarrow \quad t_2 = 1.75$ s

So, total time $T = t_1 + t_2$

$\Rightarrow \quad T = 2.08 + 1.75$

$\Rightarrow \quad T = 3.83$ s

Please observe that if the options given are not close to each other, then you can also take $g = 10$ ms^{-2} and the answer will be 3.73 s, which is very close to 3.8 s.
DO NOT APPLY THIS WHEN ANSWERS ARE VERY CLOSE IN VALUES.

Hence, the correct answer is (B).

97. At time t, the separation between the particles is

$$x = x_1 - x_2 = 2vt - \frac{1}{2} a t^2$$

For x to be MAXIMUM,

$$\frac{dx}{dt} = 0$$

$$2v - at = 0$$

$\Rightarrow \quad t = \frac{2v}{a}$

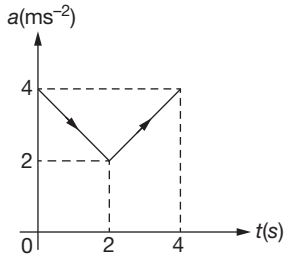
$$x_{MAX} = (2v)\left(\frac{2v}{a}\right) - \frac{1}{2} a \left(\frac{4v^2}{a^2}\right)$$

$\Rightarrow \quad x_{MAX} = \frac{4v^2}{a} - \frac{2v^2}{a} = \frac{2v^2}{a}$

Hence, the correct answer is (C).

98. Since, area under a-t graph is equal to the change in velocity, so

$$\Delta v = \frac{1}{2}(2+4)(2) + \frac{1}{2}(2+4)(2) = 12 \text{ ms}^{-1}$$



$\Rightarrow \quad \Delta v = v - u = 12$ ms^{-1}

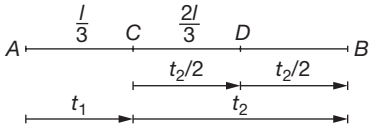
Since $u = 0$

$\Rightarrow \quad v = 12$ ms^{-1}

Hence, the correct answer is (B).

99. $v_{av} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$\Rightarrow \quad v_{av} = \frac{\frac{l}{3} + \frac{2l}{3}}{t_1 + t_2}$ …(1)



For AC, $\frac{l}{3} = 4t_1$

$\Rightarrow \quad t_1 = \frac{l}{12}$ …(2)

For CB, $\frac{2l}{3} = 2\left(\frac{t_2}{2}\right) + 6\left(\frac{t_2}{2}\right)$

$\Rightarrow \quad \frac{2l}{3} = t_2 + 3t_2$

$\Rightarrow \quad t_2 = \frac{l}{6}$ …(3)

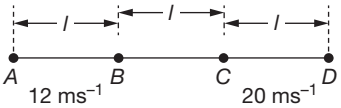
Substituting (2) and (3) in (1), we get

$$v_{av} = \frac{l}{\frac{l}{12} + \frac{2l}{12}} = \frac{l}{\left(\frac{3l}{12}\right)}$$

$\Rightarrow \quad v_{av} = 4$ ms^{-1}

Hence, the correct answer is (B).

100. Let velocities at A, B, C and D be v_A, v_B, v_C and v_D.



Since, $\ell = (v_{av})t$

For AB, $12 = \dfrac{v_A + v_B}{2} \Rightarrow v_A + v_B = 24$ …(1)

For CD, $20 = \dfrac{v_C + v_D}{2} \Rightarrow v_C + v_D = 40$ …(2)

For AB, $v_B^2 - v_A^2 = 2al$ …(3)

For CD, $v_D^2 - v_C^2 = 2al$ …(4)

$\Rightarrow \quad (v_B - v_A)24 = (v_D - v_C)40$

(Equating (3) & (4) and using (1) & (2))

$\Rightarrow \quad \dfrac{v_B - v_A}{v_D - v_C} = \dfrac{5}{3}$

Lets assume

$v_B - v_A = 5k$ …(5)

$v_D - v_C = 3k$ …(6)

Solving (1) and (5), we get

$v_B = \dfrac{24 + 5k}{2}$; $v_A = \dfrac{24 - 5k}{2}$ …(7)

Solving (2) and (6), we get

$v_D = \dfrac{40 + 3k}{2}$; $v_C = \dfrac{40 - 3k}{2}$ …(8)

Further for interval *BC*

$v_C^2 - v_B^2 = 2al$ …(9)

Equating (9) and (3) and using (7) and (8) and solving we get, a quadratic in k i.e.

$k^2 + 60k - 64 = 0$

We get $k = -30 + 2\sqrt{241}$

(Rejecting negative sign)

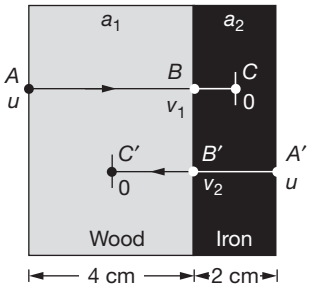
∴ Average velocity in interval BC is

$v_{av} = \dfrac{v_B + v_C}{2} = \dfrac{24 + 40 + 2k}{4}$

$\Rightarrow \quad v_{av} = \left(1 + \sqrt{241}\right)\text{ ms}^{-1}$

Hence, the correct answer is (C).

101. Let a_1 and a_2 be the retardations offered to the bullet by wood and iron respectively.



For A → B → C

$v_1^2 - u^2 = 2a_1(4)$, and

$0^2 - v_1^2 = 2a_2(1)$

Adding, we get

$-u^2 = 2(4a_1 + a_2)$ …(1)

For $A' \rightarrow B' \rightarrow C'$

$v_2^2 - u^2 = 2a_2(2)$, and

$0^2 - v_2^2 = 2a_1(2)$

Adding, we get

$-u^2 = 2(2a_1 + 2a_2)$ …(2)

Equating (1) and (2) and solving, we get

$4a_1 + a_2 = 2a_1 + 2a_2$

$\Rightarrow \quad a_2 = 2a_1$

Hence, the correct answer is (B).

102. $v_x = 30\text{ ms}^{-1}$

Since, $h = \dfrac{1}{2}gt^2$

$\Rightarrow \quad 80 = \dfrac{1}{2}(10)t^2$

$\Rightarrow \quad t = 4\text{ s}$

i.e., the ball hits the ground 4 s after the launch.

Now, since $v_y = gt = (10)(4) = 40\text{ ms}^{-1}$

$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2}$

$\Rightarrow \quad v = \sqrt{(30)^2 + (40)^2}$

$\Rightarrow \quad v = 50\text{ ms}^{-1}$

Hence, the correct answer is (B).

103. $v_1 = -\sqrt{2gd}$ and $v_2 = +\sqrt{2g\left(\dfrac{d}{2}\right)}$

Hence, the correct answer is (A).

104. Distance covered in the last one second i.e., distance covered in the nth second if the ball takes n seconds to hit the ground is

$x_1 = s_{n^{th}} = 0 + \dfrac{g}{2}(2n - 1)$ …(1)

Distance covered in the first three seconds is

$x_2 = \dfrac{1}{2}g(3)^2 = \dfrac{9g}{2}$ …(2)

Since $x_1 = x_2$

$\Rightarrow \quad \dfrac{g}{2}(2n - 1) = \dfrac{9g}{2}$

$\Rightarrow \quad 2n - 1 = 9$

$\Rightarrow \quad n = 5$ s

So, total time taken by the ball to hit the ground is 5 s . So,

$$h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad h = \frac{1}{2}(10)(5)^2 = 5(25) = 125 \text{ m}$$

$$\Rightarrow \quad h = 125 \text{ m}$$

Hence, the correct answer is (C).

105. Let $OP = k$

$OQ = kr$

$OR = kr^2$ and so on

then $v_P^2 = 2a(OP)$ ...(1)

$v_Q^2 = 2a(OQ)$ ...(2)

$v_R^2 = 2a(OR)$ ...(3)

and so on.

$$\Rightarrow \quad v_P^2 : v_Q^2 : v_R^2 : \ldots\ldots \equiv 1 : r : r^2 : \ldots\ldots$$

$$\Rightarrow \quad v_P : v_Q : v_R : \ldots\ldots \equiv 1 : r^{\frac{1}{2}} : r : \ldots\ldots$$

Hence, the correct answer is (B).

106. $0^2 - u^2 = 2(-g)H$

$$\Rightarrow \quad u = \sqrt{2gH} \qquad \ldots(1)$$

$$\Rightarrow \quad h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow \quad h = \left(\sqrt{2gH}\right)t - \frac{1}{2}gt^2$$

Quadratic in t solve t getting two values for time whose ratio is $\frac{t_1}{t_2} = \frac{1}{3}$ (given). Substituting values we get

$4h = 3H$

Hence, the correct answer is (A).

107. $v_b =$ velocity of boat $= v$ (say)

$v_r =$ velocity of river $= \eta v$

To start with, we must observe that velocity of river flow is η times greater than the velocity of boat. So, the boat has to drift and in this problem we are to minimise the drift. Let, the velocity of the boat make an angle θ with the river velocity. Then $v_x = v\cos\theta$ is the component of the velocity of boat along the river flow, whereas $v_y = v\sin\theta$ is the component of the velocity of boat perpendicular to river flow. Due to this the net velocity of boat downstream will become $v_r + v_x = \eta v + v\cos\theta = V$ (say)

If t is the time taken by the boat to cross the river of width L, then

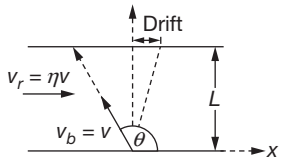
$$t = \frac{L}{v_y} = \frac{L}{v\sin\theta}$$

During this time the drift equals (say x)

$$x = Vt$$

$$x = (\eta v + v\cos\theta)\frac{L}{v\sin\theta}$$

$$x = (\eta\text{cosec}\theta + \cot\theta)L$$



To MINIMISE x, we must have

$$\frac{dx}{d\theta} = 0$$

$$\eta(-\text{cosec}\theta\cot\theta) - \text{cosec}^2\theta = 0$$

$$\Rightarrow \quad \eta\text{cosec}\theta\cot\theta = -\text{cosec}^2\theta$$

$$\Rightarrow \quad \eta\cot\theta = -\text{cosec}\theta$$

$$\Rightarrow \quad \eta\frac{\cos\theta}{\sin\theta} = -\frac{1}{\sin\theta}$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{\eta}$$

$$\Rightarrow \quad -\cos\theta = \frac{1}{\eta}$$

$$\Rightarrow \quad \sin\left(\theta - \frac{\pi}{2}\right) = \frac{1}{\eta}$$

$$\Rightarrow \quad \theta - \frac{\pi}{2} = \sin^{-1}\left(\frac{1}{\eta}\right)$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{\eta}\right)$$

Hence, the correct answer is (C).

108. Relative to P, the path of S should be along the line SP. So perpendicular to this path, relative velocity should be zero.

Hence, the correct answer is (C).

CHAPTER 4

Multiple Correct Choice Type Questions

1. $\frac{dv}{dt} = -\alpha v$ …(1)

$\Rightarrow \quad v\frac{dv}{dx} = -\alpha v$

$\Rightarrow \quad \frac{dv}{dx} = -\alpha$

$\Rightarrow \quad \int_{v_0}^{0} dv = -\alpha \int_{0}^{s} dx$

$\Rightarrow \quad 0 - v_0 = -\alpha s$

$\Rightarrow \quad s = \frac{v_0}{\alpha}$

Also from (1),

$\frac{dv}{v} = -\alpha dt$

$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_{0}^{t} dt$

$\Rightarrow \quad v = v_0 e^{-\alpha t}$ …(2)

$v = 0$, when $t \to \infty$

Also, $v = \frac{v_0}{2}$

$\Rightarrow \quad \frac{v_0}{2} = v_0 e^{-\alpha t}$

$\Rightarrow \quad e^{\alpha t} = 2$

$\Rightarrow \quad t = \frac{\log_e(2)}{\alpha} = \frac{7}{10\alpha} \qquad \{\because \log_e(2) = 0.7\}$

Hence, (A), (C) and (D) are correct.

2. $s = ut + \frac{1}{2}at^2$

Average velocity $= v_{av} = \frac{s}{t} = u + \frac{1}{2}at$

$\Rightarrow \quad v_{av} = \frac{2u + at}{2} = \frac{u + (u + at)}{2}$

$\Rightarrow \quad v_{av} = \frac{u + v}{2}$

Hence, (A) and (D) are correct.

3. Acceleration $= \frac{dv}{dt} = \dot{v} = 0 + k\dot{x} \qquad \left\{\because \dot{x} = \frac{dx}{dt} = v\right\}$

$\Rightarrow \quad \dot{v} = a = kv = k(v_0 + kx)$

Further,

$a = \frac{dv}{dt} = kv$

$\Rightarrow \quad \frac{dv}{dt} = kv$

$\Rightarrow \quad \frac{dv}{v} = kdt$

$\Rightarrow \quad \int_{v_0}^{v_1} \frac{dv}{v} = k\int_{0}^{t} dt$

$\Rightarrow \quad t = \frac{1}{k}\log_e\left(\frac{v_1}{v_0}\right)$

Since, $v = v_0 + kx$. Hence slope of velocity displacement curve is $\frac{dv}{dx} = k$

Hence, (A), (B) and (C) are correct.

4. The speed with which the lower end of the rod moves is

$v_x = \frac{dx}{dt}$

$\Rightarrow \quad v_x = \frac{dy}{dt}\left(\frac{dx}{dy}\right)$

Since, $x = \sqrt{l^2 - y^2}$

$\Rightarrow \quad \frac{dx}{dy} = \frac{-y}{\sqrt{l^2 - y^2}}$

$\Rightarrow \quad v_x = \frac{-y}{\sqrt{l^2 - y^2}}\frac{dy}{dt}$

$\Rightarrow \quad v_x = \frac{y}{\sqrt{l^2 - y^2}}\left(-\frac{dy}{dt}\right)$

$\Rightarrow \quad v_x = \frac{yv}{\sqrt{l^2 - y^2}}$

At $y = 0$, $v_x = 0$

Hence, (C) and (D) are correct.

5. Since $v^2 = v_x^2 + v_y^2$

$\Rightarrow \quad 2v\frac{dv}{dt} = 2v_x\left(\frac{2v_x}{dt}\right) + 2v_y\left(\frac{dv_y}{dt}\right)$

$\Rightarrow \quad \frac{dv}{dt} = \frac{v_x a_x + v_y a_y}{v} = \frac{\vec{a}\cdot\vec{v}}{v}$

This is also equal to the projection of $\vec{a}$ along $\vec{v}$.
Hence, (C) and (D) are correct.

6. $x_t = t^3 - 9t^2 + 6t$

$$v = \frac{dx_t}{dt} = (3t^2 - 18t + 6)\ \text{cms}^{-1}$$

For body to be at rest $v = 0$

$\Rightarrow \quad t = (3 \pm \sqrt{7})s$

$\Rightarrow \quad t_1 = (3 - \sqrt{7})\ s$ and $t_2 = (3 + \sqrt{7})\ s$

Displacement of particle in travelling from 1st zero of velocity to second zero is –74 cm and $v < 0$ for

$$(3 - \sqrt{7}) < t < (3 + \sqrt{7})$$

Hence, (A), (C) and (D) are correct.

7. The v-t graph for the two cases are shown here

For Case A, we have

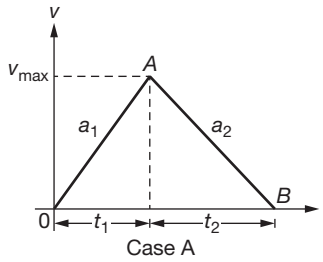
$$v_{\max} = a_1 t_1 = a_2 t_2 \qquad \ldots(1)$$

For Case B, we have

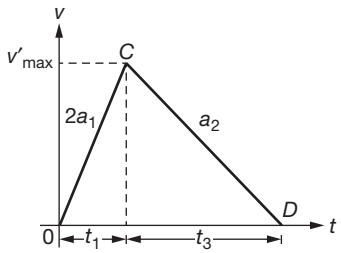
$$v'_{\max} = (2a_1)t_1 = a_2 t_3 \qquad \ldots(2)$$

From these equations, we get

$$v'_{\max} = 2v_{\max} \text{ and } t_3 = 2t_2$$



Case A



Case B

Also, $s_1 = (\text{Area under the curve } OAB)$

$$\Rightarrow \quad s_1 = \frac{1}{2} v_{\max} (t_1 + t_2)$$

and $s_2 = (\text{Area under the curve OCD})$

$$\Rightarrow \quad s_2 = \frac{1}{2} v'_{\max} (t_1 + t_3)$$

$$\Rightarrow \quad s_2 = \frac{1}{2}(2v_{\max})(t_1 + 2t_2)$$

So, the average velocity in Case A, is

$$v_1 = \frac{s_1}{t_1 + t_2} = \frac{v_{\max}}{2}$$

The average velocity in Case B, is

$$v_2 = \frac{s_2}{t_1 + t_3} = \frac{1}{2} v'_{\max} = v_{\max} \qquad \{\because\ v'_{\max} = 2v_{\max}\}$$

So, we observe that $v_2 = 2v_1$ and $2s_1 < s_2 < 4s_1$

Hence, (B) and (C) are correct.

8. $\Rightarrow \quad \frac{dv}{dt} = -\alpha v$

$$\Rightarrow \quad v\frac{dv}{dx} = -\alpha v$$

$$\Rightarrow \quad \int_{v_0}^{0} dv = -\alpha \int_{0}^{x_0} dx$$

$$\Rightarrow \quad -v_0 = -\alpha x_0$$

$$\Rightarrow \quad x_0 = \frac{v_0}{\alpha}$$

Further, $\frac{dv}{dt} = -\alpha v$

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_{0}^{t} dt$$

$$\Rightarrow \quad \log_e v\big|_{v_0}^{v} = -\alpha t\big|_0^t$$

$$\Rightarrow \quad v = v_0 e^{-\alpha t}$$

For $t \to \infty$, $v \to 0$

Hence particle continues to move for a long time span.

Hence, (A) and (B) are correct.

9. For AC, $v^2 - (7)^2 = 2al \qquad \ldots(1)$

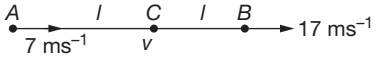
For CB, $(17)^2 - v^2 = 2al \qquad \ldots(2)$

From (1) and (2), we get

$$v^2 = \frac{(7)^2 + (17)^2}{2}$$

$$\Rightarrow \quad v^2 = 169$$

$$\Rightarrow \quad v = 13\ \text{ms}^{-1}$$



Let t_1 be the time taken to go from A to C and t_2 be the time taken to go from C to B. Then

$$l=\left(\frac{13+7}{2}\right)t_1 \qquad \text{(for } AC\text{)} \qquad \ldots(3)$$

and $$l=\left(\frac{13+17}{2}\right)t_2 \qquad \text{(for } CB\text{)} \qquad \ldots(4)$$

So, $10t_1=15t_2$

$$\Rightarrow \quad \frac{t_1}{t_2}=\frac{3}{2}$$

Also, we know that, for accelerated motion between two points at separation s, we have

$$s=\left(\frac{u+v}{2}\right)t=v_{av}t$$

So, from (1), we get

$(v_{av})_{AC}=10\ \text{ms}^{-1}$ and $(v_{av})_{CB}=15\ \text{ms}^{-1}$.

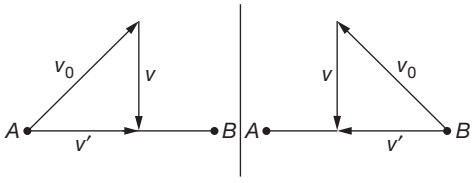
Hence, (A), (B) and (D) are correct.

10. When wind blows along the line AB,

$$t=t_{A\to B}+t_{B\to A}$$

$$\Rightarrow \quad t=\frac{l}{v+v_0}+\frac{l}{v_0-v}$$

$$\Rightarrow \quad t=\frac{2lv_0}{v_0^2-v^2}$$



If wind blows perpendicular to AB

$$t=t_{A\to B}+t_{B\to A}$$

In both the cases, we have

$$v'=\sqrt{v_0^2-v^2}$$

$$t_{A\to B}=\frac{l}{\sqrt{v_0^2-v^2}} \quad \text{and}$$

$$t_{B\to A}=\frac{l}{\sqrt{v_0^2-v^2}}$$

$$\Rightarrow \quad t=\frac{2l}{\sqrt{v_0^2-v^2}}$$

If the wind were not present then total time taken for the trip would have been $t=\dfrac{2l}{v_0}$

i.e. the total time for the trip increases because of the presence of wind.

Hence, (A), (B) and (D) are correct.

11. $a=4-2x$

$$\Rightarrow \quad v\frac{dv}{dx}=4-2x$$

$$\Rightarrow \quad vdv=(4-2x)dx$$

$$\Rightarrow \quad \int_0^v vdv=\int_0^x (4-2x)dx$$

$$\Rightarrow \quad \frac{v^2}{2}=4x-x^2$$

$$\Rightarrow \quad v^2=8x-2x^2 \qquad \ldots(1)$$

From (1), we observe that $v=0$, when $x=0$ and $x=4$ m.

Now, $a=0$

$$\Rightarrow \quad 4-2x=0$$

$$\Rightarrow \quad x=2\ \text{m}$$

So, this happens to be the mean position for the particle to oscillate.

Hence, (B) and (C) are correct.

12. Distance travelled by motor bike at $t=18\ s$

$$s_{\text{bike}}=s_1=\frac{1}{2}(18)(60)=540\ \text{m}$$

Distance travelled by car at $t=18\ s$

$$s_{\text{car}}=s_2=(18)(40)=720\ \text{m}$$

Therefore, separation between them at $t=18\ s$ is 180 m.

Let separation between them decreases to zero at time t beyond 18 s.

Hence, $s_{\text{bike}}=540+60t$ and $s_{\text{car}}=720+40t$

$$s_{\text{car}}-s_{\text{bike}}=0$$

$$\Rightarrow \quad 720+40t=540+60t$$

$$\Rightarrow \quad t=9\ s \ \text{beyond 18 s}$$

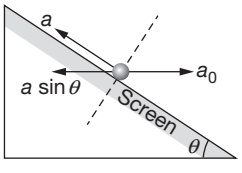
$$\Rightarrow \quad t=(18+9)\ s=27\ s \ \text{from start}$$

and distance travelled by both is $s_{\text{bike}}=s_{\text{car}}=1080$ m

Hence, (A) and (C) are correct.

13. $(a_S)_{HOR}=a_0-a\cos\theta$

$(a_S)_{VER}=a_0\sin\theta$



Hence, (B) and (C) are correct.

14. Since particle is decelerated, Hence

$$a = -\alpha\sqrt{v}$$

$$\Rightarrow \quad \frac{dv}{dt} = -\alpha\sqrt{v} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dv}{dx} \times \frac{dx}{dt} = -\alpha\sqrt{v}$$

$$\Rightarrow \quad \frac{v}{\sqrt{v}} dv = -\alpha dx$$

$$\Rightarrow \quad \sqrt{v}\, dv = -\alpha\, dx$$

Integrating both sides, we have

$$\int_{v_0}^{0} v^{1/2}\, dv = -\alpha \int_{0}^{x_0} dx$$

$$x_0 = \frac{2}{3}\frac{v_0^{3/2}}{\alpha}$$

Further, from (1), we have

$$\Rightarrow \quad v^{-1/2} dv = -\alpha dt$$

Integrating,

$$\int_{v_0}^{0} v^{-1/2} dv = -\alpha \int_{0}^{t_0} dt$$

$$\Rightarrow \quad t_0 = 2\frac{\sqrt{v_0}}{\alpha}$$

Hence, (A) and (D) are correct.

15. $v_{av} = \dfrac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$

$$\Rightarrow \quad 20 = \frac{s_1 + s_2 + s_3}{25}$$

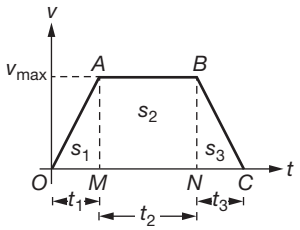
$$\Rightarrow \quad s_1 + s_2 + s_3 = 500 \qquad \ldots(1)$$

Since car accelerated from $0 \to v_{max}$ in time t_1 with acceleration of $5\ ms^{-2}$ and it decelerated from $v_{max} \to 0$ in time t_3 with a retardation of $5\ ms^{-2}$ hence $t_1 = t_3$ and $s_1 = s_3$. Hence,

$$t_2 = 25 - 2t_1 \qquad \ldots(2)$$

$$\Rightarrow \quad s_1 = s_3 = \frac{1}{2}(5)t_1^2 \text{ and } v_{max} = 5t_1$$

$$\Rightarrow \quad s_2 = v_{max} t_2 = 5t_1 t_2$$



Substituting values in (1) and using (2) we get, a quadratic in t_1 which on solving gives $t_1 = 5$ s.

Hence, $t_2 = 15$ s and $v_{max} = 25\ ms^{-1}$

Hence, (A), (B), (C) and (D) are correct.

17. $x_t = 8t^2 - 3t^3$

$$\Rightarrow \quad v_t = 16t - 9t^2$$

For particle to be at rest $v_t = 0$

$$\Rightarrow \quad 16t - 9t^2 = 0$$

$$\Rightarrow \quad t = 0 \text{ or } t = \frac{16}{9}\ s$$

Substituting $t = \dfrac{16}{9}\ s$ in $x_t = 8t^2 - 3t^3$, we get

$$x_t = 8.43 \text{ m}$$

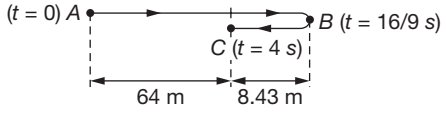
Now $v_{av} = \dfrac{x_{t=4} - x_{t=0}}{4 - 0}$

$$\Rightarrow \quad v_{av} = \frac{\left[8(4)^2 - 3(4)^3\right] - \left[8(0)^2 - 3(0)^3\right]}{4}$$

$$\Rightarrow \quad v_{av} = \frac{8(16) - 3(64)}{4} = -16\ ms^{-1}$$

Acceleration $= a = \dfrac{dv_t}{dt} = 16 - 18t$

$$\Rightarrow \quad a|_{t=4s} = 16 - 18(4) = -56\ ms^{-2}$$



$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Total Distance $= (64 + 8.43 + 8.43)\text{m} = 80.86$ m

$$\Rightarrow \quad \text{Average speed} = \frac{80.86}{4} = 20.21\ ms^{-1}$$

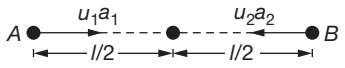
Hence, (A), (B), (C) and (D) are correct.

CHAPTER 4

18. In general, we have $|d\vec{r}| = dr$ and $v \neq \frac{dr}{dt}$. You can think about these cases by taking an example of circular motion.

Hence, (A), (B) and (D) are correct.

19.



$$\frac{l}{2} = u_1 t + \frac{1}{2}a_1 t^2 \qquad \ldots(1)$$

and $$-\frac{l}{2} = -u_2 t + \frac{1}{2}(-a_2)t^2$$

$$\Rightarrow \quad \frac{l}{2} = u_2 t + \frac{1}{2}a_2 t^2 \qquad \ldots(2)$$

Subtracting (1) & (2), we get

$$t = 2\left(\frac{u_2 - u_1}{a_1 - a_2}\right) \qquad \ldots(3)$$

Substituting (3) in (1) or (2) and rearranging, we get

$$l = \frac{4(u_2 - u_1)}{(a_1 - a_2)^2}(a_1 u_2 - a_2 u_1) \qquad \ldots(4)$$

Since the particles P & Q reach the other ends of A and B with equal velocities say v

For particle P

$$v^2 - u_1^2 = 2a_1 l \qquad \ldots(5)$$

For particle Q

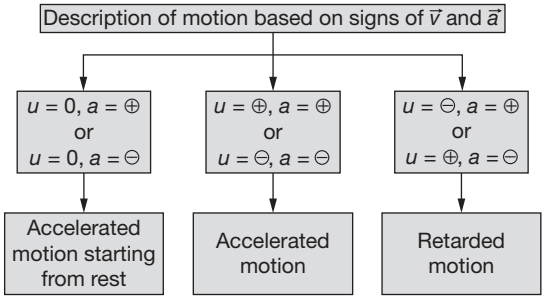
$$v^2 - u_2^2 = 2a_2 l \qquad \ldots(6)$$

Subtracting and then substituting value of ℓ and rearranging, we get

$$(u_2 + u_1)(a_1 - a_2) = 8(a_1 u_2 - a_2 u_1)$$

Hence, (A), (B) and (C) are correct.

20. My dear students, most of you forget to keep in mind the nature of motion, whether accelerated or decelerated, is just not predicted by checking the sign of $\vec{a}$. It also requires the nature of $\vec{v}$ to be defined or mentioned along with. So keep in mind the following table for describing an accelerated or a retarded motion.


Description of motion based on signs of $\vec{v}$ and $\vec{a}$
$u = 0, a = \oplus$ or $u = 0, a = \ominus$
$u = \oplus, a = \oplus$ or $u = \ominus, a = \ominus$
$u = \ominus, a = \oplus$ or $u = \oplus, a = \ominus$
Accelerated motion starting from rest
Accelerated motion
Retarded motion

Hence, (B), (C) and (D) are correct.

21. Since, $x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$

$$\Rightarrow \quad x = 2 + \frac{y}{2}$$

$$\Rightarrow \quad y = 2x - 4 \qquad \ldots(1)$$

This is the equation of a straight line

Now, $\frac{dx}{dt} = 2 + 8t$ and $\frac{dy}{dt} = 4 + 16t$

$$\Rightarrow \quad a_x = \frac{dv_x}{dt} = 8 \text{ and } a_y = \frac{dv_y}{dt} = 16$$

$$\Rightarrow \quad \vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\Rightarrow \quad \vec{a} = 8\hat{i} + 16\hat{j}$$

$$\Rightarrow \quad \vec{a} = \text{constant}$$

So, the motion is uniformly accelerated along a straight line.

Hence, (C) and (D) are correct.

22. The separation r, between the particle at time t is

$$r^2 = (4 - 2t)^2 + (2t)^2 \qquad \ldots(1)$$

This separation will first decrease and then increase.

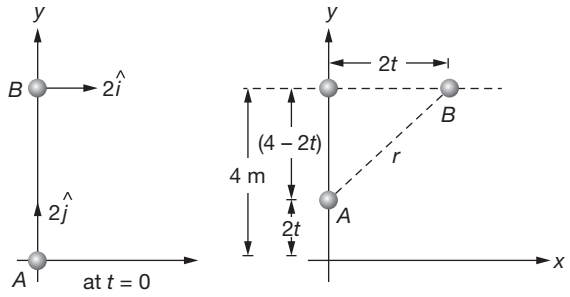
This r is minimum, when

$$\frac{d}{dt}(r^2) = 0$$

$$\Rightarrow \quad 2(4 - 2t)(-2) + 2(2t)(2) = 0$$

$$\Rightarrow \quad -8 + 4t + 4t = 0$$

$$\Rightarrow \quad t = 1 \text{ s}$$



At this time, we have

$$r_{\min}^2 = (4 - 2)^2 + (2)^2$$

$$\Rightarrow \quad r_{\min}^2 = 8$$

$$\Rightarrow \quad r_{\min} = 2\sqrt{2} \text{ m}$$

Hence, (B), (C) and (D) are correct.

23. This problem can be answered if you think about the situation when the particle reverses its direction of motion, because in that case $a \neq 0$ but $v = 0$, but this is not an always condition.

So, a can be non-zero when $v = 0$. Another answer is (D), because this situation is just like when the v is constant, then $a = 0$.

Hence, (B) and (D) are correct.

24. At time T, the particle reverses its direction of motion, because $v = 0$ and also $a =$ constant. So, (A) and (B) are correct. Also from the graph we observe that the initial and the final speeds of the particle have same value. So, (C) happens to be correct too. Finally, displacement of the particle is actually the area under the v-t graph which is zero. So, (D) is also correct. You can also think about this graph for a body through up with an initial velocity under the influence of gravity. So it will return to the point of start, hence giving zero displacement.

Hence, (A), (B), (C) and (D) are correct.

26. **METHOD I**

Since $v = \sqrt{x}$

$\Rightarrow \quad v^2 = x$

Comparing with

$v^2 - 0^2 = 2ax$

we observe that $u = 0$ and $2a = 1$

$\Rightarrow \quad u = 0$ and $a = \frac{1}{2}\ \text{ms}^{-2}$

Please keep this thing in mind that whenever you observe $v^2 \propto x$ OR $v \propto \sqrt{x}$ OR $x \propto t^2$ OR $t \propto \sqrt{x}$ OR $v \propto t$, then **acceleration is constant** or the motion is **uniformly accelerated**.

METHOD II

$v = \sqrt{x}$

$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{1}{2}x^{\frac{1}{2}-1}\right)\frac{dx}{dt}$

$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{1}{2\sqrt{x}}\right)v$

But $v = \sqrt{x}$

$\Rightarrow \quad \frac{dv}{dt} = \frac{1}{2}$

$\Rightarrow \quad a = \frac{1}{2}\ \text{ms}^{-2}$

Hence, (A), (B) and (C) are correct.

28. Distance covered is $x|_{t\to\infty} - x|_{t=0} = x_0$

However, since the time of motion is infinite, so average speed is zero, because

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$\Rightarrow \quad \text{Average Speed} = \frac{x_0}{\infty} = 0$

Hence, (B) and (C) are correct.

29. $\frac{dv}{dt} = -v^2 + 2v - 1$

Terminal velocity is a constant velocity and hence when terminal velocity is attained, then acceleration is zero.

$\Rightarrow \quad \frac{dv}{dt} = 0$

$\Rightarrow \quad -v^2 + 2v - 1 = 0$

$\Rightarrow \quad (v-1)^2 = 0$

$\Rightarrow \quad v = 1\ \text{ms}^{-1}$

Also, $\frac{dv}{dt} = -v^2 + 2v - 1 = -(v-1)^2$

$\Rightarrow \quad \int_0^v \frac{dv}{(v-1)^2} = -\int_0^t dt$

$\Rightarrow \quad -\left(\frac{1}{v-1}\right)\Big|_0^v = -t$

$\Rightarrow \quad \frac{1}{v-1} + 1 = t$

$\Rightarrow \quad \frac{1}{v-1} = t - 1$

$\Rightarrow \quad v = \frac{t}{t-1}$

When acceleration is one fourth of its initial value, i.e. $a = -\frac{1}{4}$, then we have

$-v^2 + 2v - 1 = -\frac{1}{4}$

$\Rightarrow \quad -4v^2 + 8v - 3 = 0$

$\Rightarrow \quad v = \frac{-(8) \pm \sqrt{64 - 4(-4)(-3)}}{2(-4)}$

$\Rightarrow \quad v = \frac{-8 \pm 4}{-8} = \frac{1}{2}\ \text{ms}^{-1},\ \frac{3}{2}\ \text{ms}^{-1}$

Hence, (A), (B) and (D) are correct.

30. In x-t graph at maxima and minima. $\frac{dx}{dt} = 0$. Hence particle comes to rest for 6 times and average velocity for total period is negative.

Hence, (A) and (D) are correct.

CHAPTER 4

31. Since, $v\frac{dv}{dx} = -\alpha v$

$$\Rightarrow \int_{v_0}^{0} dv = -\alpha \int_{0}^{x_0} dx$$

$$\Rightarrow x_0 = \frac{v_0}{\alpha}$$

So, option (A) is correct.

Also, $\frac{dv}{dt} = -\alpha v$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_{0}^{t} dt$$

$$\Rightarrow v = v_0 e^{-\alpha t}$$

$$\Rightarrow v = 0 \text{ for } t \to \infty$$

So, option (C) is also correct.

Hence, (A) and (C) are correct.

32. Equation of straight line is

$$v^2 = 15x + 25$$

Differentiating with respect to x, we get

$$2v\frac{dv}{dx} = 15$$

$$\Rightarrow v\frac{dv}{dx} = a = \frac{15}{2} \text{ ms}^{-2}$$

$$\Rightarrow a = 7.5 \text{ ms}^{-2}$$

Since, $v = u + at = 5 + (7.5)(1) = 12.5 \text{ ms}^{-1}$

$$\Rightarrow v = 12.5 \text{ ms}^{-1}$$

Hence, (B), (C) and (D) are correct.

35. During accelerated motion

$$v = \alpha t_1 \text{ and } v^2 = 2\alpha x$$

During decelerated motion

$$0 = v - \beta t_2$$

$$\Rightarrow \alpha t_1 = \beta t_2$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\beta}{\alpha}$$

Also, $0^2 = v^2 - 2\beta y$

$$\Rightarrow v^2 = 2\beta y$$

$$\Rightarrow 2\alpha x = 2\beta y$$

$$\Rightarrow \frac{x}{y} = \frac{\beta}{\alpha}$$

Hence, (A), (B) and (C) are correct.

36. Acceleration is the slope of v-t graph, which is negative between 5 s and 6 s.

Between $t = 0$ and $t = 4$ s the particle changes its direction of motion at 1 s and 3 s.

Hence, (C) and (D) are correct.

38. $v_{\max} = \left(\frac{a_1 a_2}{a_1 + a_2}\right) t = \left(\frac{2\times 4}{2+4}\right)(6) = 8 \text{ ms}^{-1}$

and $s = \frac{1}{2}\left(\frac{a_1 a_2}{a_1 + a_2}\right) t^2 = \frac{1}{2}\left(\frac{2\times 4}{2+4}\right)(6)^2 = 24 \text{ m}$

Hence, (A) and (C) are correct.

39. $v = 4t - t^2$

$$\Rightarrow \frac{ds}{dt} = 4t - t^2$$

$$\Rightarrow \int ds = \int_{0}^{5} (4t - t^2)\, dt$$

$$\Rightarrow s = 2\times 5^2 - \frac{5^3}{3}$$

$$\Rightarrow s = \frac{150 - 125}{3} = \frac{25}{3} \text{ m}$$

Average velocity $= \frac{25}{3\times 5} \text{ ms}^{-1} = \frac{5}{3} \text{ ms}^{-1}$

$$a = \frac{dv}{dt} = (4 - 2t) \text{ ms}^{-2}$$

$$\Rightarrow a = 4 \text{ ms}^{-2} \text{ at } t = 0$$

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. Positive slope indicates that acceleration is increasing uniformly with time and is not uniform.

Hence, the correct answer is (D).

2. Acceleration is the rate of change of velocity i.e., $a = \frac{dv}{dt}$.

Hence, the correct answer is (D).

3. A body has no relative motion with respect to itself. Hence if a frame of reference of the body is fixed, then the bodies will be always at relatively rest in this frame of reference. So, (A) is correct

Hence, the correct answer is (A).

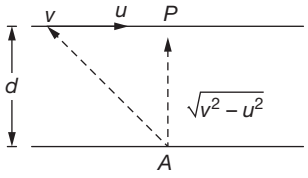
4. As $\vec{v} = \frac{d\vec{r}}{dt}$, hence statement-I is correct.

In most of the cases known instantaneous velocity and average velocity are not equal. They can be equal only when the particle moves with a constant uniform velocity.

Hence, the correct answer is (C).

5. To an observer on the balloon, the ground is a moving frame.
Hence, the correct answer is (D).

6. Time to cross the river $t = \dfrac{d}{\sqrt{v^2 - u^2}}$



Hence, the correct answer is (A).

8. Suppose motion of 1 is viewed from 2 then as 2 is stationary. If 1 is to meet it, then 1 must come straight towards 2 i.e., the relative velocity must be along the line joining 1 and 2.
Hence, the correct answer is (A).

9. $a_r = g - g = 0$

$\Rightarrow \quad \dfrac{dv_r}{dt} = 0$

$\Rightarrow \quad v_r = \text{constant (say } k_1)$

$\Rightarrow \quad \dfrac{dx_r}{dt} = k_1$

$\Rightarrow \quad x_r = k_1 t$

Hence, the correct answer is (A).

11. At the highest point of the graph velocity (slope of x-t curve) changes suddenly from a positive to a negative value. This is physically impossible.
Hence, the correct answer is (D).

12. Area under acceleration - time graph gives change in velocity and not average velocity.
Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. **The correct answer is (B).**

2. **The correct answer is (D).**

3. **The correct answer is (C).**

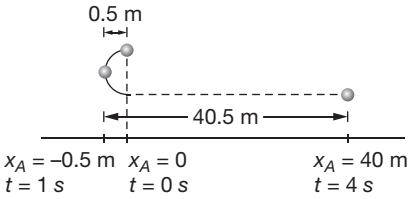
Combined solution to 1, 2, 3

For A

$dv_A = a_A dt$

$\Rightarrow \quad \displaystyle\int_0^{v_A} dv_A = \int_0^t (6t - 3)\,dt$

$\Rightarrow \quad v_A = 3t^2 - 3t$



For B

$dv_B = a_B dt$

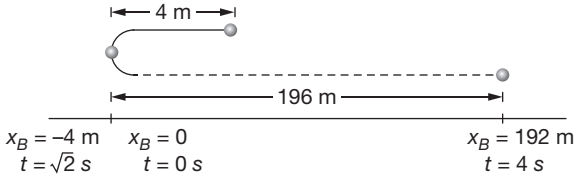
$\Rightarrow \quad \displaystyle\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)\,dt$

$\Rightarrow \quad v_B = 4t^3 - 8t$

Let us now calculate the times when A and B are at rest.
The particle A is at rest $(v_A = 0)$, when

$3t^2 - 3t = 0$

$\Rightarrow \quad t = 0 \text{ s and } t = 1 \text{ s}$



The particle B is at rest $(v_B = 0)$, when

$4t^3 - 8t = 0$

$\Rightarrow \quad t = 0 \text{ s and } t = \sqrt{2} \text{ s}$

The position of particles A and B can be determined using

$v = \dfrac{dx}{dt}$, so

$dx_A = v_A dt$

$\Rightarrow \quad \displaystyle\int_0^{x_A} dx_A = \int_0^t (3t^2 - 3t)\,dt$

$\Rightarrow \quad x_A = t^3 - \dfrac{3}{2}t^2$

Similarly

$dx_B = v_B dt$

$\Rightarrow \quad \displaystyle\int_0^{x_B} dx_B = \int_0^t (4t^3 - 8t)\,dt$

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$\Rightarrow \quad x_B = t^4 - 4t^2$

The positions of particle A at $t = 1$ s and 4 s are

$$x_A\big|_{t=1\text{ s}} = 1^3 - \frac{3}{2}(1^2) = -0.5 \text{ m}$$

$$x_A\big|_{t=4\text{ s}} = 4^3 - \frac{3}{2}(4^2) = 40 \text{ m}$$

Particle A has travelled a total distance given by

$$d_A = 2(0.5) + 40 = 41 \text{ m}$$

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$x_B\big|_{t=\sqrt{2}} = \left(\sqrt{2}\right)^4 - 4\left(\sqrt{2}\right)^2 = -4 \text{ m}$$

$$x_B\big|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ m}$$

Particle B has travelled a total distance given by

$$d_B = 2(4) + 192 = 200 \text{ m}$$

At $t = 4$ s the distance between A and B is

$$\Delta x_{AB} = 192 - 40 = 152 \text{ m}$$

4. Taking, $+x$ direction as positive, we get

$u = 40 \text{ ms}^{-1}$ and $a = -10 \text{ ms}^{-2}$

So, this a acts as retardation and the velocity of the body is first reduced to zero, say at time t_0.
Since

$$v = u + at$$

$\Rightarrow \quad 0 = 40 + (-10)t_0$

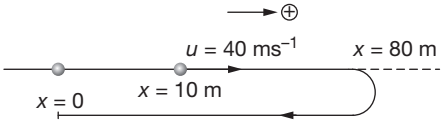
$\Rightarrow \quad t_0 = 4$ s

Hence, the correct answer is (B).

5. $0^2 - u^2 = 2ax_0$

$\Rightarrow \quad -(40)^2 = 2(-10)x_0$

$\Rightarrow \quad x_0 = \dfrac{1600}{20} = 80 \text{ m}$



So, the maximum x-coordinate of the particle, along the $+x$ direction is

$$x_{\text{MAX}} = x_{\text{Initial}} + x_0$$

$\Rightarrow \quad x_{\text{MAX}} = 10 + 80 = 90 \text{ m}$

Hence, the correct answer is (C).

6. Further, at the origin, $s = -10$ m. So, using

$v^2 - u^2 = 2as$, we get

$$v^2 - (40)^2 = 2(-10)(-10)$$

$\Rightarrow \quad v^2 = 1800$

$\Rightarrow \quad v = 30\sqrt{2} \text{ ms}^{-1}$, along negative x direction

Hence, the correct answer is (B).

7. Also, using $v = u + at$, we get

$$t = \frac{v - u}{a} = \frac{-30\sqrt{2} - 40}{-10}$$

$\Rightarrow \quad t = \left(4 + 3\sqrt{2}\right)$ s

Hence, the correct answer is (B).

8. $a = 2t - 4$

$$\Rightarrow \quad \frac{dv}{dt} = 2t - 4$$

$$\Rightarrow \quad \int_0^v dv = 2\int_0^t t\,dt - 4\int_0^t dt$$

$$\Rightarrow \quad v = \left(2\frac{t^2}{2} - 4t\right)\Big|_0^t$$

$\Rightarrow \quad v = t^2 - 4t$ …(1)

Now, $v = 0$

$\Rightarrow \quad t^2 - 4t = 0$

$\Rightarrow \quad t = 0$ (initially) OR $t = 4$ s

Hence, the correct answer is (D).

9. The velocity of the particle is maximum, when

$$\frac{dv}{dt} = 0$$

$\Rightarrow \quad a = 0$

$\Rightarrow \quad 2t_0 - 4 = 0$

$\Rightarrow \quad t_0 = 2$ s

and $v_{\text{max}} = v\big|_{t_0 = 2\text{ s}} = 2^2 - 4(2)$

$\Rightarrow \quad v_{\text{max}} = -4 \text{ ms}^{-1}$

Hence, the correct answer is (B).

10. Since we have calculated

$$v = t^2 - 4t$$

$\Rightarrow \quad v = t^2 - 4t + 4 + 4$

$\Rightarrow \quad v = (t - 2)^2 + 4$

$\Rightarrow \quad v-4=(t-2)^2$

This happens to be the equation of the parabola with origin at the point (2, 4)

Hence, the correct answer is (D).

11. Initially, at $t=0$, the body falls from rest, so $v=0$ at $t=0$ and hence $\left.\frac{dv}{dt}\right|_{t=0}=6\ \text{ms}^{-2}$

Hence, the correct answer is (A).

12. Acceleration is zero (when terminal velocity is attained), so

$0=6-3v_T$

$\Rightarrow \quad v_T=2\ \text{ms}^{-1}$

Hence, the correct answer is (C).

13. $\frac{dv}{dt}=6-3\ \text{v}$

$\Rightarrow \quad \int_0^v \frac{dv}{6-3v}=\int_0^t dt$

$\Rightarrow \quad -\frac{1}{3}\log_e(6-3v)\Big|_0^v=t$

$\Rightarrow \quad \log_e\left(\frac{6-3v}{6}\right)=-3t$

$\Rightarrow \quad v=2\left(1-e^{-3t}\right)$

Hence, the correct answer is (B).

14. $a=\frac{dv}{dt}=6e^{-3t}$

Hence, the correct answer is (C).

15. $v_x=a\omega\cos(\omega t)$

$v_y=a\omega\sin(\omega t)$

Since, $v^2=v_x^2+v_y^2$

$\Rightarrow \quad v=a\omega$

Hence, the correct answer is (A).

16. $x=a\sin(\omega t)=2a\sin\left(\frac{\omega t}{2}\right)\cos\left(\frac{\omega t}{2}\right)$

$y=2a\sin^2\left(\frac{\omega t}{2}\right)$

$\Rightarrow \quad \sin\left(\frac{\omega t}{2}\right)=\sqrt{\frac{y}{2a}}$

$\Rightarrow \quad \cos\left(\frac{\omega t}{2}\right)=\sqrt{1-\frac{y}{2a}}$

$\Rightarrow \quad x=2a\sqrt{\frac{y}{2a}}\sqrt{1-\frac{y}{2a}}$

$\Rightarrow \quad x=\sqrt{2ay}\sqrt{1-\frac{y}{2a}}$

Hence, the correct answer is (B).

17. $a_x=\frac{dv_x}{dt}=-a\omega^2\sin(\omega t)$

$a_y=\frac{dv_y}{dt}=a\omega^2\cos(\omega t)$

Since acceleration $=\sqrt{a_{x^2}+a_{y^2}}$

$\Rightarrow \quad$ Acceleration $=a\omega^2$

Hence, the correct answer is (B).

18. Since displacement is zero, so average velocity is also zero.

Hence, the correct answer is (A).

19. From A to B, let time taken be t_1, then

$t_1=\frac{d}{v}$

From B to A, let time taken be t_2, then

$t_2=\frac{d}{\left(v-\frac{v}{3}\right)}=\frac{3d}{2v}$

So, total time taken is

$T=t_1+t_2$

$\Rightarrow \quad T=\frac{d}{v}+\frac{3d}{2v}=\frac{5d}{2v}$

Hence, the correct answer is (D).

20. Average speed $(v_{av})=\frac{\text{Total Distance}}{\text{Total Time}}$

$\Rightarrow \quad v_{av}=\frac{2d}{T}$

$\Rightarrow \quad v_{av}=\frac{2d}{\left(\frac{5d}{2v}\right)}$

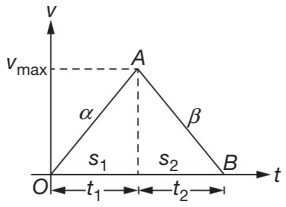
$\Rightarrow \quad v_{av}=\frac{4v}{5}$

Hence, the correct answer is (D).

21. According to the problem, we have

$s_1+s_2=\frac{1}{2}v_{\max}t$

CHAPTER 4



$\Rightarrow \quad 100 = \frac{1}{2} v_{max} (20)$

$\Rightarrow \quad v_{max} = 10 \text{ ms}^{-1}$

Hence, the correct answer is (B).

22. Now $v_{max} = \alpha t_1 = \beta t_2$

$\Rightarrow \quad 10 = 2t_1$

$\Rightarrow \quad t_1 = 5 \text{ ms}^{-1}$

Now $t_1 + t_2 = 20$ s

$\Rightarrow \quad t_2 = 15$ s

Hence, the correct answer is (C).

23. Also $v_{max} = \beta t_2$

$\Rightarrow \quad 10 = \beta(15)$

$\Rightarrow \quad \beta = \frac{2}{3} \text{ ms}^{-2}$

Hence, the correct answer is (B).

24. Average speed $= \frac{\text{Total Distance}}{\text{Total Time}}$

$\Rightarrow \quad v_{av} = \frac{100}{20} = 5 \text{ ms}^{-1}$

Hence, the correct answer is (A).

25. $s_1 = \frac{1}{2} v_{max} t_1$

$\Rightarrow \quad s_1 = \frac{1}{2}(10)(5) = 25$ m

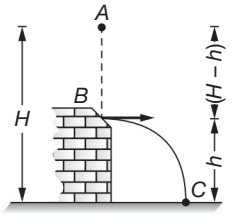
Hence, the correct answer is (A).

26. $s_2 = 100 - 25 = 75$ m

Hence, the correct answer is (B).

27. Let time taken by body to go from A to B be t_1 and that to go from B to C be t_2. Then

$$t_1 = \sqrt{\frac{2(H-h)}{g}} \text{ and } t_2 = \sqrt{\frac{2h}{g}}$$



So, total time taken is

$$t = t_1 + t_2 = \sqrt{\frac{2}{g}} \left[\sqrt{H-h} + \sqrt{h} \right]$$

For t to be maximum $\frac{dt}{dh} = 0$, we have

$$\frac{h}{H} = \frac{1}{2}$$

Hence, the correct answer is (C).

28. Substituting $h = \frac{H}{2}$ from PROBLEM 27, in expression for t we get value of t_{max}

Hence, the correct answer is (A).

29. **The correct answer is (C).**

30. **The correct answer is (A).**

Combined solution to 29, 30

Take downward direction as positive, we get

$$-h = -ut_1 + \frac{1}{2} g t_1^2 \qquad \ldots(1)$$

$$-h = -ut_2 + \frac{1}{2} g t_2^2 \qquad \ldots(2)$$

Subtracting (2) from (1), we get

$$0 = u(t_2 - t_1) + \frac{1}{2} g (t_1^2 - t_2^2)$$

$$\Rightarrow \quad u = \frac{1}{2} g (t_1 + t_2) \qquad \ldots(3)$$

{ANSWER TO PROBLEM 30}

Substituting value of u from (3) in (1), we get

$$h = \frac{g t_1 t_2}{2}$$

31. $v^2 - u^2 = 2(g)(-h)$

At $h = h_{max}$

$v = 0$

$$\Rightarrow \quad h_{max} = \frac{u^2}{2g}$$

$$\Rightarrow \quad h_{max} = \frac{1}{8} g (t_1 + t_2)^2$$

Hence, the correct answer is (C).

32. $v^2 - u^2 = 2g\left(-\frac{h}{2}\right)$

$\Rightarrow \quad v^2 = \frac{1}{4}g^2(t_1+t_2)^2 - gh$

But $h = \frac{1}{2}gt_1t_2$ {From PROBLEM 29}

$\Rightarrow \quad v^2 = \frac{1}{4}g^2(t_1+t_2)^2 - \frac{1}{2}g^2t_1t_2$

$\Rightarrow \quad v^2 = \frac{1}{4}g^2\left(t_1^2+t_2^2+2t_1t_2-2t_1t_2\right)$

$\Rightarrow \quad v = \frac{1}{2}g\sqrt{t_1^2+t_2^2}$

Hence, the correct answer is (C).

33. Let v be the velocity of the particle at height $\left(\frac{H}{2}\right)$

$\Rightarrow \quad v^2 = u^2 + 2(-g)\frac{H}{2} = u^2 - gH$

$\Rightarrow \quad v^2 = \frac{g^2}{4}(t_1+t_2)^2 - g\left\{\frac{g}{8}(t_1+t_2)\right\}$

$\Rightarrow \quad v^2 = \frac{g^2}{8}(t_1+t_2)^2$

$\Rightarrow \quad v = \frac{g}{2\sqrt{2}}(t_1+t_2)$

Hence, the correct answer is (C).

34. Slope of line $= -\frac{2}{3}$

Equation of line is $(v-20) = -\frac{2}{3}(s-0)$

$\Rightarrow \quad v = 20 - \frac{2}{3}s$ …(1)

Velocity at s = 15 m i.e.

$$v = \left.\frac{ds}{dt}\right|_{s=15\ m} = 20 - \frac{2}{3}(15) = 10\ \text{ms}^{-1}$$

Differentiate (1) with respect to time

acceleration $= \frac{dv}{dt} = -\frac{2}{3}\frac{ds}{dt}$

$\therefore \quad \left.\frac{dv}{dt}\right|_{s=15\ m} = -\frac{2}{3}\left.\frac{ds}{dt}\right|_{s=15\ m} = -\frac{20}{3}\ \text{ms}^{-2}$

Hence, the correct answer is (D).

35. From Solution to Problem 60, we have

$v = 20 - \frac{2}{3}s$ {See equation (1)}

$\Rightarrow \quad \frac{ds}{dt} = 20 - \frac{2}{3}s$

$\frac{ds}{20 - \frac{2}{3}s} = dt$

$\Rightarrow \quad \int_0^{30}\frac{ds}{\left(20-\frac{2}{3}s\right)} = \int_0^t dt$

$\Rightarrow \quad -\frac{1}{\left(\frac{2}{3}\right)}\log_e\left(20-\frac{2}{3}s\right)\Big|_0^{30} = t$

$\Rightarrow \quad -\frac{3}{2}\left[\log_e(0) - \log_e(20)\right] = t$

$\Rightarrow \quad t \to \infty$ $\{\because \log_e 0 \to \infty\}$

Hence, the correct answer is (D).

36. Since, we know that time interval is

$$t = \frac{\text{Relative Displacement}}{\text{Relative Velocity}} = \frac{\Delta l_r}{v_r}$$

Relative Displacement is the distance travelled by each bus in 5 minutes, so

$$\Delta l_r = \frac{5}{60}\times 60 = 5\ \text{km}$$

Also, relative velocity of the buses is

$$v_r = 60 + 60 = 120\ \text{kmhr}^{-1}$$

$\Rightarrow \quad t = \frac{5}{120}\ \text{hr} = \frac{5}{120}\times 60\ \text{min} = 2.5\ \text{min}$

Hence, the correct answer is (B).

37. Distance = speed × time $= 60\times\frac{2.5}{60} = 2.5$ km

Hence, the correct answer is (A).

38. Since each bus takes 30 min to reach another city, therefore total number of buses it meets during the journey is

$$N = \frac{30}{2.5} - 1 = 11$$

(Note that the 12th bus just starts the motion)
Hence, the correct answer is (D).

39. Average velocity is defined as

$$v_{av} = \frac{\text{Displacement}}{\text{Time Interval}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

CHAPTER 4

$$\Rightarrow \quad v_{av} = \frac{\left[12(8)-(8)^2\right]-0}{8-0} = 4 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

40. To calculate the average speed, we must first find the instant of reversal of motion, i.e. when $v = 0$. Since

$$v = \frac{dx}{dt} = 12 - 2t$$

$$\Rightarrow \quad v = 0$$

$$\Rightarrow \quad t = 6 \text{ s}$$

So distance travelled by the particle from $t = 0$ to $t = 8$ s is

$$l = |36 - 0| + |32 - 36| = 40 \text{ m}$$

So average speed is

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad \text{Average Speed} = \frac{40}{8} = 5 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

41. Average acceleration is the rate of change of velocity.
Hence, option (C) is correct.

42. $a = 2t - 4$

$$\Rightarrow \quad \frac{dv}{dt} = 2t - 4$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t (2t-4)\,dt$$

$$\Rightarrow \quad v = t^2 - 4t$$

The particle comes to rest when $v = 0$

$$\Rightarrow \quad t = 4 \text{ s}$$

Hence, the correct answer is (B).

43. The speed of the particle is maximum when

$$\frac{dv}{dt} = 2t - 4 = 0$$

$$\Rightarrow \quad t = 2 \text{ s}$$

Hence, the correct answer is (C).

44. Since, we have

$$\Rightarrow \quad \int_0^x dx = \int_0^4 (t^2 - 4t)\,dt$$

$$\Rightarrow \quad x = \frac{4^3}{3} - 2 \times 4^2 = \frac{64}{3} - 32 = -\frac{32}{3}$$

$$\text{Distance} = \frac{32}{3} \text{ m}$$

Hence, the correct answer is (D).

48. $x = 6t^2 - t^3$

$$\Rightarrow \quad \frac{dx}{dt} = v = 12t - 3t^2$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{d^2x}{dt^2} = a = 12 - 6t$$

$$\Rightarrow \quad \frac{dv}{dt} = 0 \text{ for v to be maximum}$$

$$\Rightarrow \quad 12 - 6t$$

$$\Rightarrow \quad t = 2 \text{ s}$$

Hence, the correct answer is (B).

49. The position of the particle at different times are given here.

Position (x)	At Time (t)
$x_3 = 25$ m	$t = 3$ s
$x_4 = 32$ m	$t = 4$ s
$x_5 = 27$ m	$t = 5$ s

So, the distance travelled by the particle in the said interval i.e. from $t = 3$ s to $t = 5$ s is

$$\ell = |32 - 25| + |27 - 32| = 12 \text{ m}$$

Hence, the correct answer is (C).

50. The position of the particle at different times are given here

Position (x)	At Time (t)
$x_0 = 0$ m	$t = 0$ s
$x_4 = 32$ m	$t = 4$ s
$x_6 = 0$ m	$t = 6$ s

So, the distance travelled by the particle in the said interval i.e. from $t = 0$ s to $t = 6$ s is

$$l = |32 - 0| + |0 - 32| = 64 \text{ m}$$

Since,

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\text{Av. Speed} = \frac{64}{6} = \frac{32}{3}\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

51. Since $a = -6t$

$$\Rightarrow \quad \frac{dv}{dt} = -6t$$

$$\Rightarrow \quad \int_{27}^{v} dv = -\int_{0}^{t} 6t\,dt$$

$$\Rightarrow \quad v - 27 = -3t^2$$

$$\Rightarrow \quad v = (27 - 3t^2)\ \text{ms}^{-1}$$

$$\Rightarrow \quad \frac{dx}{dt} = 27 - 3t^2$$

$$\Rightarrow \quad \int_{0}^{x} dx = \int_{0}^{t} (27 - 3t^2)\,dt$$

$$\Rightarrow \quad x = 27t - t^3$$

Now, at $x = 26$ m, let the time be $t = t_0$ (say). So, we have

$$26 = 27t_0 - t_0^3$$

$$\Rightarrow \quad t_0^3 - 27t_0 + 26 = 0$$

$$\Rightarrow \quad (t_0 - 1)(t_0^2 + t_0 - 26) = 0$$

$$\Rightarrow \quad t_0 = 1\ \text{s}$$

$$\Rightarrow \quad t_0 = \left(\frac{\sqrt{105} - 1}{2}\right)\ \text{s}$$

The earlier time gives the value of time when particle is at $x = 26$ m on forward journey and former time gives the value of time when particle is at $x = 26$ m on return journey.
However, in the second case, the particle would have covered more than 26 m.
So particle covers a distance of 26 m at $t = 1$ s
Hence, velocity of the particle at this time is

$$v = [27 - 3(1)^2]\ \text{ms}^{-1} = 24\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

52. $\frac{dv}{dt} = -6t$

For maximum velocity, we have $\frac{dv}{dt} = 0$

$$\Rightarrow \quad t = 0$$

Since, $\frac{d^2v}{dt^2} = -6 < 0$

So $t = 0$ corresponds to maxima and hence

$$v_{\max} = 27 - 3(0)^2 = 27\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

53. $\frac{dx}{dt} = 27 - 3t^2$

For maximum displacement, we have

$$\frac{dx}{dt} = 0$$

$$\Rightarrow \quad 27 - 3t^2 = 0$$

$$\Rightarrow \quad t = 3\ \text{s}$$

Since, $\frac{d^2x}{dt^2} = -6t$ and

$$\left.\frac{d^2x}{dt^2}\right|_{t=3} = -18 < 0$$

Hence $t = 3$ corresponds to maxima of position. So

$$x_{\max} = 27(3) - (3)^3 = 81 - 27 = 54\ \text{m}$$

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (s, u)
B → (s, t)
C → (s, u)
D → (s, t)
E → (q, r)
F → (s, u)
G → (p, s)
Between $t = 0$ and $t = 1$ s, motion is decelerated $(a \neq 0)$.
(A) → (s, u)
Between $t = 1$ s and $t = 2$ s, motion is accelerated $(a \neq 0)$.
So, (B) → (s, t)
Between $t = 2$ s and $t = 3$ s, motion is decelerated $(a \neq 0)$.
So, (C) → (s, u)
Between $t = 3$ s and $t = 4$ s, motion is accelerated $a \neq 0$.
So, (D) → (s, t)
Between $t = 4$ s and $t = 5$ s, motion is uniform i.e., $v \neq 0$ and $a = 0$.
So, (E) → (q, r)
Between $t = 5$ s and $t = 6$ s, motion is decelerated $(a \neq 0)$.
So, (F) → (s, u)
At $t = 1$ s and at $t = 3$ s the motion reverses its direction, so $v = 0$ and simultaneously $a \neq 0$.
So, (G) → (p, s)

CHAPTER 4

2. A → (q)
B → (r)
C → (s)
D → (p)

$$x = -5t^2 + 20t + 10$$

$$\Rightarrow \quad \frac{dx}{dt} = -10t + 20$$

$$\Rightarrow \quad v = 20 - 10t \qquad \ldots(1)$$

Now $v = 0$ at $t = 2$ s, so the distance travelled by the particle from $t = 0$ to $t = 3$ s is

$$x = \left|\int_0^2 v\,dt\right| + \left|\int_2^3 v\,dt\right|$$

Now $\int v\,dt = (20 - 10t)\,dt$

$$\Rightarrow \quad \int v\,dt = 20t - 5t^2$$

$$\Rightarrow \quad x = \left|(20t - 5t^2)\big|_0^2\right| + \left|(20t - 5t^2)\big|_2^3\right|$$

$$\Rightarrow \quad x = |40 - 20| + |80 - 80 - 40 + 20|$$

$$\Rightarrow \quad x = |20| + |-20|$$

$$\Rightarrow \quad x = 40 \text{ m}$$

So, average speed is

$$v_{av} = \frac{x}{t} = \frac{40}{4} = 10 \text{ ms}^{-1}$$

Displacement of the particle from $t = 0$ to $t = 4$ s is

$$\Delta x = \int_0^4 v\,dt = \int_0^4 (20 - 10t)\,dt$$

$$\Rightarrow \quad \Delta x = (20t - 5t^2)\big|_0^4$$

$$\Rightarrow \quad \Delta x = 80 - 80 = 0$$

So, average velocity is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{0}{4} = 0 \text{ ms}^{-1}$$

From (1), $a = \frac{dv}{dt} = -10 \text{ ms}^{-2}$

$$u = v|_{t=0} = 20 \text{ ms}^{-1}$$

Since, initially $u = \oplus$ and $a = \ominus$, so a happens to be retardation for the motion from $t = 0$ to $t = 2$ s (till its velocity becomes zero momentarily i.e., the particle reverses its direction of motion).

Velocity at $t = 4$ s is

$$v|_{t=4\text{ s}} = 20 - 10(4) = -20 \text{ ms}^{-1}$$

Hence, speed at $t = 4$ s is 20 ms^{-1}

3. A → (q)
B → (r)
C → (p, s)
D → (t)

$\frac{d\vec{r}}{dt}$ = Velocity

$\frac{d|\vec{v}|}{dt}$ = Tangential Acceleration

$\frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$ = Acceleration

$\left|\frac{d\vec{r}}{dt}\right|$ = Instantaneous speed

4. A → (r, t)
B → (p)
C → (q)
D → (s)

$$x = (t - 3)^2$$

$$\Rightarrow \quad x = t^2 - 6t + 9$$

$$\Rightarrow \quad v = \frac{dx}{dt} = 2t - 6$$

At the point of reversal, $v = 0$, so

$$2t - 6 = 0$$

$$\Rightarrow \quad t = 3 \text{ s}$$

t	0	1	2	3	4	5	6
x	9	4	1	0	1	4	9
v	–6	–4	–2	0	2	4	6
a	2	2	2	2	2	2	2
Nature of Motion	Decele-rating	Decele-rating	Decele-rating	Accele-rating	Accele-rating	Accele-rating	Accele-rating

Displacement of the particle from $t = 0$ to $t = 6$ s is zero.
Distance travelled by the particle from $t = 0$ to $t = 6$ is $9 + 9 = 18$ m

Average speed of the particle is

$$v_{av} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad v_{av} = \frac{18}{6} = 3 \text{ ms}^{-1}$$

Average Velocity of the particle is

$$|\vec{v}_{av}| = \frac{\text{Displacement}}{\text{Time}} = 0$$

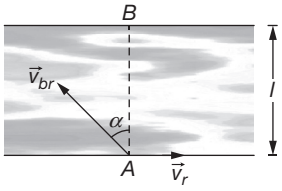
Acceleration of the particle over the entire duration of motion is 2 ms^{-2}.

5. A → (t)
B → (s)
C → (p)
D → (q)
E → (r)

Given, that $v_{br} = 4 \text{ kmhr}^{-1}$ and $v_r = 2 \text{ kmhr}^{-1}$

$$\Rightarrow \quad \alpha = \sin^{-1}\left(\frac{v_r}{v_{br}}\right) = \sin^{-1}\left(\frac{2}{4}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

Hence, to reach the point directly opposite to starting point he should head the boat at an angle of $30°$ with AB or $90° + 30° = 120°$ with the river flow.



Time taken by the boatman to cross the river for zero drift condition is

$$t_1 = \frac{l}{v_{br}\cos\alpha}$$

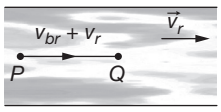
where $l = 4 \text{ km}$, $v_{br} = 4 \text{ kmh}^{-1}$ and $\alpha = 30°$

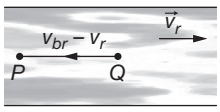
$$\Rightarrow \quad t_1 = \frac{4}{4\cos(30°)} = \frac{2}{\sqrt{3}} \text{ hr}$$

$$\Rightarrow \quad t_1 = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ minute}$$

For shortest time $\theta = 0°$ i.e., the man must row the boat perpendicular to the river flow. So $\beta = 90°$

and $t_{\min} = t_2 = \dfrac{l}{v_{br}\cos(0°)} = \dfrac{4}{4} = 1 \text{ hr} = 60 \text{ minute}$





Hence, he should head his boat perpendicular to the river current for crossing the river in shortest time and this shortest time t_2 of 60 minute.

$$t = t_{P\to Q} + t_{Q\to P}$$

$$\Rightarrow \quad t = \frac{PQ}{v_{br} + v_r} + \frac{QP}{v_{br} + v_r}$$

$$\Rightarrow \quad t = \frac{2}{4+2} + \frac{2}{4-2} = \frac{4}{3} \text{ hr}$$

$$\Rightarrow \quad t = 80 \text{ minute}$$

9. A → (r)
B → (p)
C → (r)
D → (s)

$$v_i = +10 \text{ ms}^{-1} \text{ and } v_f = 0$$

$$\Rightarrow \quad \Delta v = v_f - v_f = -10 \text{ ms}^{-1}$$

$$\Rightarrow \quad a_{av} = \frac{\Delta v}{\Delta t} = \frac{-10}{6} = \frac{-5}{3} \text{ ms}^{-2}$$

Total displacement = area under v-t graph (with sign) and acceleration = slope of v-t graph.

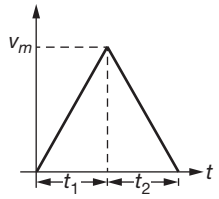
10. A → (r)
B → (p)
C → (s)
D → (q)

After 2 s velocity of balloon and hence the velocity of the particle will be $20 \text{ ms}^{-1} (= at)$ and its height from the ground will be $20 \text{ m} \left(= \frac{1}{2}at^2\right)$. Now g, will start acting on the particle.

11. A → (p)
B → (q)
C → (s)
D → (r)

$$\frac{v_m}{t_1} = \alpha \text{ and } \frac{v_m}{t_2} = \beta$$

Also $x + y = \dfrac{1}{2} \times v_m \times (t_1 + t_2)$



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$$\Rightarrow \quad (x+y)=\frac{1}{2}\times v_m\times\left(\frac{v_m}{\alpha}+\frac{v_m}{\beta}\right)$$

$$\Rightarrow \quad v_m=\sqrt{\frac{2(x+y)\alpha\beta}{(\alpha+\beta)}}$$

Now, $v_{av}=\dfrac{x+y}{t_1+t_2}=\dfrac{x+y}{\frac{v_m}{\alpha}+\frac{v_m}{\beta}}=\dfrac{\alpha\beta}{\alpha+\beta}\cdot\dfrac{x+y}{v_m}$

12. A → (q)
B → (p, q)
C → (p, r)
D → (r, s)

$$v=\frac{dx}{dt}$$

$$\Rightarrow \quad a=\frac{dv}{dt}$$

$$\Rightarrow \quad v=\int_0^t a\,dt$$

$$\Rightarrow \quad x=\int_0^t v\,dt$$

14. A → (r)
B → (q)
C → (s)
D → (p)

$A \rightarrow v=Kt$

$$\Rightarrow \quad a=\frac{dv}{dt}=K$$

$B \rightarrow v=Kt^2$

$$\Rightarrow \quad a=\frac{dv}{dt}=2Kt$$

$C \rightarrow K_1 t \rightarrow$ First part of graph

$$\Rightarrow \quad a=\frac{dv}{dt}=K_1$$

$\Rightarrow \quad v=K_2 \rightarrow$ Second part of graph

$$\Rightarrow \quad a=\frac{dv}{dt}=0$$

$D \rightarrow v=v_0e^{-Kt}$

$$\Rightarrow \quad a=\frac{dv}{dt}=(-v_0K)e^{-Kt}$$

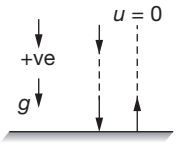
$$\Rightarrow \quad a=-a_0e^{-Kt}$$

$$\Rightarrow \quad a_0=v_0K$$

15. A → (r)
B → (s)
C → (p)
D → (q)
In motion M : slope of s-t graph is positive and increasing. Therefore, velocity of the particle is positive and increasing. Hence, it is A type motion. Similarly, N, P and Q can be observed from the slope.

20. A → (s)
B → (r)
C → (q)
D → (p)
Acceleration is constant and is equal to acceleration due to gravity, which is acting vertically downwards i.e. in the positive direction, so (p) represents the a-t graph for the motion.

Since $v=u+at$ and $s=\Delta y=ut+\frac{1}{2}gt^2$



Before collision, $v=gt \quad \{\because u=0\}$

So, before collision velocity is increasing linearly with time and is increasing. However, after collision velocity decreases with time linearly and is negative as in (r).
Displacement before collision is $y=\frac{1}{2}gt^2$ and positive, whereas after collision, the displacement is $y=-v_0t+\frac{1}{2}gt^2$ and is positive (a parabola opening upwards). Here is the tricky part, as we have taken the origin at the point from where the ball is dropped and in both the cases of upward and downward motion, the displacement is downwards and hence displacement is positive in both the cases.
The distance-time graph is positive and always increasing.

22. A → (p, q)
B → (p, q)
C → (r)
D → (p, q)
With constant positive acceleration, speed increases, when velocity is positive and speed decreases, when velocity is negative.
Similarly, with constant negative acceleration speed increases, when velocity is negative and speed decreases, when velocity is positive.

23. A → (q)
B → (r)
C → (s)
D → (p)
Solve using relative velocity diagram.

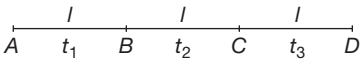
25. A → (q)
B → (r)
C → (r)
D → (p, r)
Particle will change the direction of motion
when $\frac{dx}{dt} = 0$

$\Rightarrow \quad -20 + 10t = 0$

$\Rightarrow \quad t = 2\text{ s}$

Integer/Numerical Answer Type Questions

1. Let velocities at A, B, C and D be v_A, v_B, v_C and v_D respectively.



For AB

$$l = \left(\frac{v_A + v_B}{2}\right)t_1 \qquad \ldots(1)$$

For BC

$$l = \left(\frac{v_B + v_C}{2}\right)t_2 \qquad \ldots(2)$$

For CD

$$l = \left(\frac{v_C + v_D}{2}\right)t_3 \qquad \ldots(3)$$

From (1), (2) and (3), we have

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{2l}\left(v_A + v_B - v_B - v_C + v_C + v_D\right)$$

$$\Rightarrow \quad \frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{2l}\left(v_A + v_D\right) \qquad \ldots(4)$$

For AD

$$3l = \left(\frac{v_A + v_D}{2}\right)\left(t_1 + t_2 + t_3\right)$$

$$\Rightarrow \quad \frac{1}{2l}\left(v_A + v_D\right) = \frac{3}{t_1 + t_2 + t_3} \qquad \ldots(5)$$

So, from (4) and (5), we get

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

$$\Rightarrow \quad k = 3$$

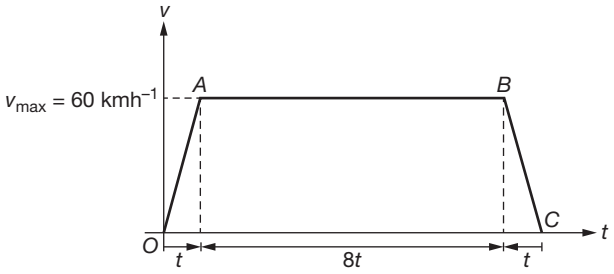
2. Since time to accelerate from zero to 60 kmh^{-1} is equal to the time taken to decelerate from 60 kmh^{-1} to zero. Hence, we can say both OA and BC are identical intervals. So, total distance travelled from O to A to B to C is

$$l = l_1 + l_2 + l_1 = \frac{1}{2}v_{\max}t + (v_{\max})8t + \frac{1}{2}v_{\max}t$$

Since,

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad v_{av} = \frac{l_1 + l_2 + l_1}{t + 8t + t}$$



$$\Rightarrow \quad v_{av} = \frac{\frac{1}{2}v_{\max}t + (v_{\max})8t + \frac{1}{2}v_{\max}t}{10t}$$

$$\Rightarrow \quad v_{av} = \frac{9}{10}v_{\max} = \frac{9}{10}(60\text{ kmh}^{-1}) = 54\text{ kmh}^{-1}$$

$$\Rightarrow \quad v_{av} = 54 \times \frac{5}{18}\text{ ms}^{-1} = 15\text{ ms}^{-1}$$

3. $u = 27\text{ ms}^{-1}$ at $t_0 = 0\text{ s}$. Since

$dv = adt$

$$\Rightarrow \quad \int_{27}^{v} dv = \int_{0}^{t} -6t\,dt$$

$$\Rightarrow \quad v = (27 - 3t^2)\text{ ms}^{-1} \qquad \ldots(1)$$

At $v = 0$, from equation …(1)

$$0 = 27 - 3t^2$$

$$\Rightarrow \quad t = 3\text{ s}$$

So, it will stop at $t = 3\text{ s}$

Since $dx = vdt$

$$\Rightarrow \quad \int_{0}^{x} dx = \int_{0}^{3} (27 - 3t^2)\,dt$$

$$\Rightarrow \quad x = (27t - t^3)\Big|_0^3 = 27(3) - (3)^3 = 54\text{ m}$$

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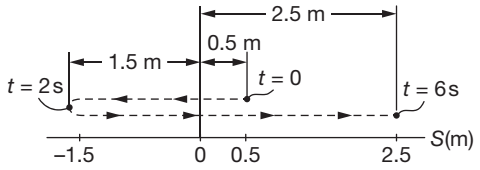
4. $x_{\text{total}} = (0.5 + 1.5 + 1.5 + 2.5) = 6$ m

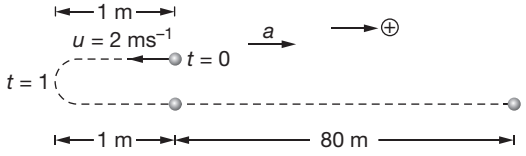


$t = (2 + 4) = 6$ s

$(v_{sp})_{avg} = \dfrac{x_{\text{total}}}{t} = \dfrac{6}{6} = 1$ ms^{-1}

5. (a) $80 = \left(\dfrac{u + 18}{2}\right)10$

$\Rightarrow \quad 16 = u + 18$



$\Rightarrow \quad u = -2$ ms^{-1}

(b) $18 = -2 + a(10)$

$\Rightarrow \quad a = 2$ ms^{-2}

(c) $v^2 - u^2 = 2as$

$\Rightarrow \quad 0^2 - (2)^2 = 2(-2)s$

$\Rightarrow \quad s = 1$ m

$0 = -2 + (2)t$

$\Rightarrow \quad t = 1$ s

(d) $s_{\text{total}} = 80 + |-1| + |1| = 82$ m

6. For normal driver, the car moves a distance of $x_0 = vt = 44(0.75) = 33$ m before he or she reacts and decelerates the car. The stopping distance can be obtained using

$v^2 - u^2 = 2a(x - x_0)$,

where $x = ?$, $x_0 = 33$ m, $a = -2$ ms^{-2}, $a = 44$ ms^{-1}

$\Rightarrow \quad 0^2 - 44^2 = 2(-2)(x - 33)$

$\Rightarrow \quad x = 517$ m

For a drunk driver, the car moves a distance of $x_0' = vt = 44(3) = 132$ m before he or she reacts and decelerates the car. The stopping distance can be obtained again by using

$v^2 - u^2 = 2a(x' - x_0')$

$\Rightarrow \quad 0^2 - 44^2 = 2(-2)(x' - 132)$

$x' = 616$ m

7. (a) Let the bus overtake the car after a distance x. When the two meet, the time taken (say t) is the same. So, for bus,

$x = 0 + \dfrac{1}{2}5t^2 \qquad \left\{\because s = ut + \dfrac{1}{2}at^2\right\}$

For car, $x = 50t$

$\Rightarrow \quad \dfrac{5}{2}t^2 = 50t$

$\Rightarrow \quad t = 20$ s

$\Rightarrow \quad x = (50)(20) = 1000$ m

(b) Let the bus be moving with a velocity v.

So, $v^2 - 0 = 2(5)(1000) \qquad \{\because v^2 - u^2 = 2as\}$

$\Rightarrow \quad v = 100$ ms^{-1}

8. Velocity of truck is $V = 72$ kmh^{-1} $= 20$ ms^{-1}

Time to overtake the truck, 0.6 km down the road is

$t = \dfrac{d}{V} = \dfrac{0.6 \times 10^3}{20} = 30$ s

If u be the initial velocity of the car, then

$600 = u(30) + \dfrac{1}{2}(1)(30)^2$

Solving it we have $u = 5$ ms^{-1}

Velocity of the car at the instant of overtaking is

$v = 5 + (1)(30) = 35$ ms$^{-1} \qquad \{\because v = u + at\}$

The relative velocity of car with respect to truck is

$v_r = 35 - 20 = 15$ ms^{-1}

The distance between them at $t = 50$ s, i.e., 20 s after overtaking is

$s = (15)(20) = 300$ m

9. Since $s = \left(\dfrac{u + v}{2}\right)t$, so for decelerated interval,

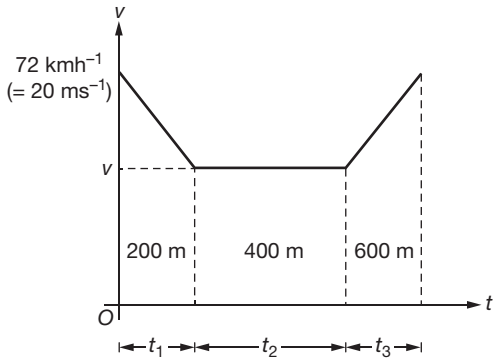
$200 = \left(\dfrac{20 + v}{2}\right)t_1 \qquad \ldots(1)$

For uniform interval,

$400 = vt_2 \qquad \ldots(2)$

and again for accelerated interval, we have

$600 = \left(\dfrac{20 + v}{2}\right)t_3 \qquad \ldots(3)$



Given $t_2 = t_1 + t_3$

$$\Rightarrow \quad \frac{400}{v} = \frac{400}{20+v} + \frac{1200}{20+v}$$

$$\Rightarrow \quad \frac{400}{v} = \frac{1600}{20+v}$$

$$\Rightarrow \quad 20 + v = 4v$$

$$\Rightarrow \quad v = \frac{20}{3} \text{ ms}^{-1}$$

$$\Rightarrow \quad t_1 = \frac{400}{20+\frac{20}{3}} = \frac{1200}{80} \text{ s} = 15 \text{ s}$$

$$\Rightarrow \quad t_2 = \frac{400}{v} = \frac{400}{\frac{20}{3}} = \frac{1200}{20} = 60 \text{ s}$$

$$\Rightarrow \quad t_3 = \frac{1200}{20+v} = \frac{1200}{20+\frac{20}{3}} = \frac{3600}{80} \text{ s} = 45 \text{ s}$$

$$\Rightarrow \quad t = (15 + 60 + 45) \text{ s} = 120 \text{ s}$$

$$\Rightarrow \quad t = 2 \text{ minute}$$

10. Let the truck overtake the car at time $t = 0$ and the car overtake the truck at time t.

For truck, $s = 10t$, and for car, $s = \frac{1}{2}2t^2$

$$10t = \frac{1}{2}(2)t^2$$

$$\Rightarrow \quad t = 10 \text{ s}$$

$$\Rightarrow \quad s = \frac{1}{2}(2)(10)^2 = 100 \text{ m}$$

and $v = (2)(10) = 20 \text{ ms}^{-1}$

11. Let u be the initial velocity of the bullet.

According to the problem $v = \frac{u}{2}$, when $s = 3$ cm

$$\Rightarrow \quad \left(\frac{u}{2}\right)^2 - u^2 = 2a(3)$$

$$\Rightarrow \quad a = \frac{u^2}{8} \qquad \ldots(1)$$

Let the bullet travels a distance x cm before coming to rest so,

$$0 - u^2 = 2ax$$

$$\Rightarrow \quad u^2 = 2\left(\frac{u^2}{8}\right)x$$

$$\Rightarrow \quad x = 4 \text{ cm}$$

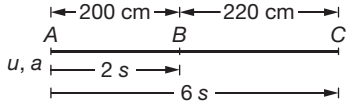
So the bullet will move $4 - 3 = 1$ cm further before coming to rest

12. Let the particle start with initial velocity u and a uniform acceleration a from point A.

For AB

$$200 = u(2) + \frac{1}{2}a(2)^2$$

$$\Rightarrow \quad u + a = 100 \qquad \ldots(1)$$



For AC

$$(200 + 220) = u(2+4) + \frac{1}{2}a(2+4)^2$$

$$\Rightarrow \quad 420 = 6u + 18a$$

$$\Rightarrow \quad u + 3a = 70 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$a = -15 \text{ cms}^{-2} \text{ and } u = 115 \text{ cms}^{-1}$$

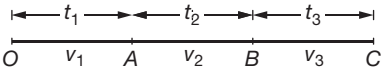
If v be the velocity at the end of seventh second from the start, then using $v = u + at$, we get

$$\Rightarrow \quad v = 115 + (-15)(7)$$

$$\Rightarrow \quad v = 10 \text{ cms}^{-1}$$

13. **For OB**

$$a = \frac{v_2 - v_1}{(t_2 + t_1) - 0} = \frac{v_2 - v_1}{t_2 + t_1} \qquad \ldots(1)$$



For AC

$$a = \frac{v_3 - v_2}{(t_3 + t_2 + t_1) - t_1} = \frac{v_3 - v_2}{t_3 + t_2} \qquad \ldots(2)$$

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From (1) and (2), equating the values of a, we get

$$\left(\frac{v_1 - v_2}{v_2 - v_3}\right)\left(\frac{t_3 + t_2}{t_2 + t_1}\right) = 1$$

14. For $0 \le x < 1000$ m, the initial condition is $v = 0$ at $x = 0$

Since $vdv = adx$

$$\Rightarrow \int_0^v v dv = \int_0^x 6 dx \qquad \{\because a = 6 \text{ ms}^{-2} \text{ from graph}\}$$

$$\Rightarrow \frac{v^2}{2} = 6x$$

$$\Rightarrow v = 2\sqrt{3x} \qquad \ldots(1)$$

When $x = 1000$ m, we get

$$v = 2\sqrt{3000} = 20\sqrt{30} \text{ ms}^{-1}$$

For $1000 \text{ m} < x \le x_0$, the initial condition is

$v = 20\sqrt{30}$ ms^{-1} at $x = 1000$ m

Since $vdv = adx$

$$\Rightarrow \int_{20\sqrt{30}}^v v dv = \int_{1000}^x -4 dx$$

$$\Rightarrow \frac{v^2}{2}\Big|_{20\sqrt{30}}^v = -4x\Big|_{1000}^x$$

$$\Rightarrow \frac{v^2}{2} - \frac{12000}{2} = -4(x - 1000)$$

$$\Rightarrow v^2 - 12000 = -8x + 8000$$

$$\Rightarrow v^2 = 20000 - 8x$$

$$\Rightarrow v = \left(\sqrt{20000 - 8x}\right) \text{ ms}^{-1}$$

When $v = 0$, at $x = x_0$, then

$$\Rightarrow 0 = \sqrt{20000 - 8x_0}$$

$$\Rightarrow x_0 = 2500 \text{ m}$$

15. The velocity v of the balloon when it has risen to a height H is given by

$$v^2 = 0 + 2\left(\frac{g}{8}\right)H$$

taking upward direction positive

$$\Rightarrow v = \sqrt{\frac{gH}{4}} = \frac{\sqrt{gH}}{2} \text{ msec}^{-1}$$

This will also be the velocity of the stone in upward direction when it is dropped.

Taking upward direction positive for the stone,

$$-H = \left(\frac{\sqrt{gH}}{2}\right)t - \frac{1}{2}gt^2 \qquad \{\because g = -g \text{ and } h = -H\}$$

$$\Rightarrow gt^2 - \sqrt{gH}\,t - 2H = 0$$

$$\Rightarrow t = \frac{\sqrt{gH} \pm \sqrt{gH + 8gH}}{2g}$$

$$\Rightarrow t = \frac{\sqrt{gH} \pm 3\sqrt{gH}}{2g}$$

$$\Rightarrow t = 2\sqrt{\frac{H}{g}} \qquad \{\text{taking positive value}\}$$

$$\Rightarrow x = 2$$

16. For the first kilometre of the journey,

$$v^2 - u^2 = 2ax$$

where $v = 10$ ms^{-1}, $u = 2$ ms^{-1}, $a = ?$, $s = 1000$ m

$$\Rightarrow 100 - 4 = 2a(1000)$$

$$\Rightarrow a = 0.048 \text{ ms}^{-2}$$

For the second kilometre, again applying

$$v^2 - u^2 = 2as$$

where $v = ?$, $u = 10$ ms^{-1}, $a = 0.048$ ms^{-2} and $s = 1000$ m

$$\Rightarrow v = 14 \text{ ms}^{-1}$$

For the whole journey, $v_0 = 2$ ms^{-1}, $v = 14$ ms^{-1} and 0.048 ms^{-2}. So,

$$v = k + at$$

$$\Rightarrow 14 = 2 + 0.048t$$

$$\Rightarrow t = 250 \text{ s}$$

17. $t = \frac{AC}{v_1} + \frac{CB}{v_2}$

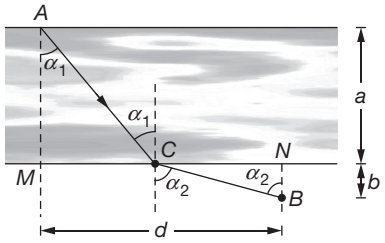
where $AC = a\sec\alpha_1$ and $CB = b\sec\alpha_2$

$$\Rightarrow t = \frac{a\sec\alpha_1}{v_1} + \frac{b\sec\alpha_2}{v_2}$$

For t_1 to be MINIMUM, we have

$$\frac{dt}{d\alpha_1} = 0$$

$$\Rightarrow \frac{a}{v_1}\sec\alpha_1\tan\alpha_1 + \frac{b}{v_2}\sec\alpha_2\tan\alpha_2\left(\frac{d\alpha_2}{d\alpha_1}\right) = 0 \qquad \ldots(1)$$



Please note that the question itself asks us to adjust the values of α_1 and α_2. So both α_1 and α_2 are variables.

Now, since $MC + CN = d$ = constant

$$\Rightarrow \quad a\tan\alpha_1 + b\tan\alpha_2 = \text{constant} \qquad \ldots(2)$$

Taking derivative of (2) w.r.t. α_1 on both sides, we get

$$a\sec^2\alpha_1 + b\sec^2\alpha\left(\frac{d\alpha_2}{d\alpha_1}\right) = 0$$

$$\Rightarrow \quad \frac{d\alpha_2}{d\alpha_1} = -\frac{a\sec^2\alpha_1}{b\sec^2\alpha_2} \qquad \ldots(3)$$

Substituting (3) in (1), we get

$$\frac{a}{v_1}\cancel{\sec\alpha_1}\tan\alpha_1 + \frac{b}{v_2}\cancel{\sec\alpha_2}\tan\alpha_2\left(-\frac{a}{b}\frac{\cancel{\sec^2}\,\alpha_1}{\cancel{\sec^2}\,\alpha_2}\right) = 0$$

$$\frac{\tan\alpha_1}{v_1} = \frac{\tan\alpha_2\sec\alpha_1}{v_2\sec\alpha_2}$$

$$\frac{1}{v_1} = \frac{\sin\alpha_1}{\cancel{\cos\alpha_1}} = \frac{1}{v_2}\left(\frac{\sin\alpha_2}{\cancel{\cos\alpha_2}}\right)\left(-\frac{\cancel{\cos\alpha_2}}{\cancel{\cos\alpha_1}}\right)$$

$$\Rightarrow \quad \frac{\sin\alpha_1}{\sin\alpha_2} = \frac{v_1}{v_2}$$

Since, $v_1 = 3\sqrt{3}$ ms^{-1}, $\alpha_1 = 30°$ and $\alpha_2 = 60°$

$$\Rightarrow \quad \frac{\sin(30°)}{\sin(60°)} = \frac{3\sqrt{3}}{v_2}$$

$$\Rightarrow \quad v_2 = 9 \text{ ms}^{-1}$$

18. v_m = absolute velocity of man,

v_{me} = velocity of man w.r.t. escalator,

v_e = velocity of escalator

If the length of the elevator by ℓ, then from the given condition,

$$v_m = \frac{l}{90} \text{ ms}^{-1}$$

$$v_e = \frac{l}{60} \text{ ms}^{-1}$$

Time taken by the person to walk up in the moving escalator is,

$$t = \frac{l}{v_e + v_m} = \frac{l}{\frac{l}{90} + \frac{l}{60}} = \frac{90\times 60}{90+60} = 36 \text{ s}$$

Here we observe that the time t does not depend on ℓ the length of escalator.

19. Initial relative velocity of approach of A and B is

$$u_r = (144 - 108) \text{ kmh}^{-1} = 36 \text{ kmh}^{-1} = 10 \text{ ms}^{-1}$$

The driver of train A applies brakes, so if a is the retardation, then retardation of A w.r.t. B is $a_r = -a$. With this retardation he just manages to avoid the collision, so

$$v_r^2 - u_r^2 = 2a_r l_r$$

$$\Rightarrow \quad O^2 - (10)^2 = 2(-a)(1000)$$

$$\Rightarrow \quad a = \frac{100}{2000} = \frac{1}{20} \text{ ms}^{-2} = 5 \text{ cms}^{-2}$$

If this retardation is produced for a duration on t (say), then

$$v_r = u_r + a_r t$$

$$\Rightarrow \quad 0 = (10) - \left(\frac{1}{20}\right)t$$

$$\Rightarrow \quad t = 200 \text{ s}$$

20. For convenience, let us take $\hat{i}$ along east, $\hat{j}$ along north and $\hat{k}$ vertically downwards.

Now given, $\vec{v}_b = 8\hat{i}$, $\vec{v}_{sb} = 12\hat{j} + 2\hat{k}$

So, absolute velocity of submarine is

$$\vec{v}_s = \vec{v}_{sb} + \vec{v}_b = 8\hat{i} + 12\hat{j} + 2\hat{k} \qquad \{\because \vec{v}_{sb} = \vec{v}_s - \vec{v}_b\}$$

Further $\vec{v}_{hs} = -4\hat{i} - 5\hat{k}$

$$\Rightarrow \quad \vec{v}_h = \vec{v}_{hs} + \vec{v}_s = 4\hat{i} + 12\hat{j} - 3\hat{k} \qquad \{\because \vec{v}_{hs} = \vec{v}_h - \vec{v}_s\}$$

$$\Rightarrow \quad |\vec{v}_h| = \sqrt{(4)^2 + (12)^2 + (3)^2} = 13 \text{ ms}^{-1}$$

Since, $\vec{v}_{hb} = \vec{v}_h - \vec{v}_b = -4\hat{i} + 12\hat{j} - 3\hat{k}$

$$\Rightarrow \quad |\vec{v}_{hb}| = \sqrt{(-4)^2 + (12)^2 + (-3)^2} = 13 \text{ ms}^{-1}$$

21. (a) Distance travelled by the particle between $t = 0$ to 10 s is given by

$$s = \text{Area of } \Delta OAB = \frac{1}{2}\times 10\times 12 = 60 \text{ m}$$

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$\Rightarrow \quad \text{Average speed} = \frac{60}{10} = 6 \text{ ms}^{-1}$$

CHAPTER 4

(b) Acceleration of the particle during journey OA is given by

$v = u + at$ where $v = 12\ \text{ms}^{-1}$, $u = 0$ and $t = 5\ \text{s}$

$\Rightarrow \quad a = +2.4\ \text{ms}^{-2}$

Similarly, acceleration of the particle during journey AB is given by

$v = u + at$ where $v = 0$, $u = 12\ \text{ms}^{-1}$ and $t = 5\ \text{s}$

$\Rightarrow \quad a = -2.4\ \text{ms}^{-2}$

Velocity of the particle after 2 s from the start is given by

$$v = u + at = 0 + 2.4 \times 2 = 4.8\ \text{ms}^{-1}$$

So, distance covered by the particle between $t = 2$ to 5 s (in 3 s) is given by

$$s_1 = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad s_1 = (4.8)(3) + \frac{1}{2}(2.4)(3)^2 = 25.2\ \text{m}$$

Distance covered by the particle from $t = 5$ s to 6 s (in 1 s) is given by

$$s_2 = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad s_2 = (12)(1) + \frac{1}{2}(-2.4)(1)^2 = 10.8\ \text{m}$$

Total distance travelled from $t = 2$ s to 6 s is

$$s = s_1 + s_2 = 25.2 + 10.8 = 36\ \text{m}$$

Average speed in the interval $t = 2$ s to 6 s

$$v_{av} = \frac{\text{total distance covered}}{\text{total time taken}} = \frac{36}{4} = 9\ \text{ms}^{-1}$$

22. $\dfrac{dx}{dt} = v_0 - \dfrac{v_0 t}{5}$

$$\Rightarrow \quad x = v_0 t - \frac{v_0 t^2}{10}$$

where x can be either +10 cm or –10 cm

For $x = +10$ cm, we have $10t - t^2 = 10$

$$\Rightarrow \quad t = \left(\frac{10 \pm \sqrt{60}}{2}\right)\ \text{s}$$

Hence we have

$$t_1 = \left(\frac{10 - \sqrt{60}}{2}\right)\ \text{s} \qquad \{\text{First Instant}\}$$

$$\Rightarrow \quad t_2 = \left(\frac{10 + \sqrt{60}}{2}\right)\ \text{s} \qquad \{\text{Second Instant}\}$$

For $x = -10$ cm, we have $10t - t^2 = -10$

$$\Rightarrow \quad t = \left(\frac{10 \pm \sqrt{140}}{2}\right)\ \text{s}$$

$$\Rightarrow \quad t_3 = \left(\frac{10 + \sqrt{140}}{2}\right)\ \text{s} \qquad \{\text{Third Instant}\}$$

However, we will not get fourth instant as time cannot be negative.

Time interval between the second and the third instant is $\Delta t = t_3 - t_2 \approx 2\ \text{s}$

23. Initial velocity is $v_i = 0$ and final velocity of the athlete is $v_f = 36 \times \dfrac{5}{18} = 10\ \text{ms}^{-1}$.

Since the particle is moving with uniform acceleration, so, we have

$$v_{av} = \langle v \rangle = \frac{v_i + v_f}{2} = 5\ \text{ms}^{-1}$$

25. $h = ut - \dfrac{1}{2}gt^2$

$$\Rightarrow \quad gt^2 - 2ut + 2h = 0$$

$$\Rightarrow \quad t_1 t_2 = \frac{2h}{g} \text{ and } t_1 + t_2 = \frac{2u}{g} = T$$

Since $(t_2 - t_1)^2 = (t_1 + t_2)^2 - 4t_1 t_2$

$$\Rightarrow \quad 16 = 64 - 4\left(\frac{2h}{g}\right)$$

$$\Rightarrow \quad h = 60\ \text{m}$$

26. $(\Delta S)_{\min} = \left(\sqrt{a^2 + \dfrac{a^2}{4}}\right) \times 2 = a\sqrt{5}$

$$\Rightarrow \quad t_{\min} = \frac{(\Delta s)_{\min}}{v} = \frac{a\sqrt{5}}{v}$$

$$\Rightarrow \quad n = 5$$

28. From graph, we have

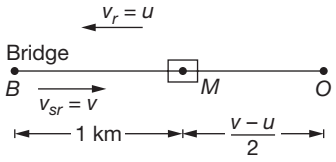
$$v = t + 2$$

$$\Rightarrow \quad x_2 - x_0 = \frac{1}{2} \times (2 + 4) \times 2 = 6\ \text{m}$$

$$\Rightarrow \quad x_2 = 9\ \text{m}$$

29. Let $v_r = u$ and $v_{sr} = v$, then

Time taken by swimmer to go from M to O and O to B is equal to the time taken by cork to go from M to B.



$$\frac{1}{2}+\frac{1+\frac{v-u}{2}}{v+u}=\frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{2}+\frac{2+v-u}{2(v+u)}=\frac{1}{u}$$

$$\Rightarrow \quad \frac{(v+u+2+v-u)}{2(v+u)}=\frac{1}{u}$$

$$\Rightarrow \quad (2v+2)u=2(v+u)$$

$$\Rightarrow \quad 2vu+2u=2v+2u$$

$$\Rightarrow \quad u=1 \text{ kmh}^{-1}$$

30. $s_{A/B}=v_{A/B}t+\frac{1}{2}a_{A/B}t^2$

31. Displacement in last two seconds is

$$s=\Delta x=\frac{1}{2}(1)(2)^2=2 \text{ m}$$

$$\Rightarrow \quad v_{av}=\frac{\Delta x}{\Delta t}=1 \text{ ms}^{-1}$$

32. $0=16^2-2as \qquad \{\because v^2=u^2+2as\}$

$$\Rightarrow \quad a=\frac{16\times 16}{2\times 0.4}=320 \text{ ms}^{-2}$$

$$\Rightarrow \quad t=\frac{v}{a}=\frac{16}{320}=5\times 10^{-2} \text{ s}$$

$$\Rightarrow \quad x=5$$

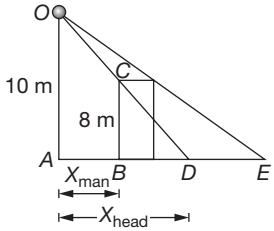
33. $a=\frac{v_f-v_i}{\Delta t}=\frac{35+25}{0.01}=6000 \text{ ms}^{-2}$

$$\Rightarrow \quad a=6 \text{ kms}^{-2}$$

34. $[v_{av}]_x=\frac{\int_{x_1}^{x_2} v\,dx}{x_2-x_1}=\frac{\int_0^x \sqrt{u^2+2ax}\,dx}{x}=5.6 \text{ ms}^{-1}$

35. $x_{man}=vt$

$$\Rightarrow \quad \frac{OA}{AD}=\frac{BC}{BD}$$



$$\Rightarrow \quad \frac{10 \text{ m}}{x_{man}}=\frac{2 \text{ m}}{x_{head}-x_{man}}$$

$$\Rightarrow \quad x_{head}=\frac{5}{4}x_{man}=\frac{5}{4}vt$$

Velocity of shadow of head $=10 \text{ ms}^{-1}$

36. Relative displacement = relative velocity × time

$$\Rightarrow \quad 8\times 3=(10-2)t$$

$$\Rightarrow \quad t=3 \text{ s}$$

37. $v=\int_0^4 a\,dt=\int_0^2 a\,dt+\int_2^4 a\,dt$

$$\Rightarrow \quad v=\int_0^2 (2-t)\,dt+\int_2^4 (t-2)\,dt$$

$$\Rightarrow \quad v=4 \text{ ms}^{-1}$$

38. $v=u+at$

$$\Rightarrow \quad v=6-\frac{1}{2}\times 10=1 \text{ ms}^{-1}$$

39. $a=\frac{30-20}{1.5}=\frac{36.7-30}{\Delta t}$

$$\Rightarrow \quad \Delta t=\frac{6.7\times 1.5}{10}\approx 1 \text{ s}$$

40. Area under curve is

$$A=\frac{1}{2}\times 10\times 30=150 \qquad \ldots(1)$$

Also, area under a-x curve is equal to

$$A=\frac{v^2-u^2}{2} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{1}{2}(v^2-u^2)=150$$

$$\Rightarrow \quad v^2=u^2+300$$

$$\Rightarrow \quad v^2=(10)^2+300$$

$$\Rightarrow \quad v=\sqrt{400}=20 \text{ ms}^{-1}$$

CHAPTER 4

41. Speed of bike is

$$v_b = 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}$$

Speed of jeep is

$$v_j = 90 \times \frac{5}{18} = 25 \text{ ms}^{-1}$$

Relative velocity of jeep w. r. t. bike is

$$v_{jb} = 25 - 20 = 5 \text{ ms}^{-1}$$

Distance covered by bike in 10 s is

$$s_b = 20 \times 10 = 200 \text{ m}$$

Time taken by Jeep to cover 200 m with velocity 5 ms^{-1} is

$$t = \frac{200}{5} = 40 \text{ s}$$

Therefore distance covered by police jeep in 40 s is

$$s_j = 40 \times 25 = 1000 \text{ m} = 1 \text{ km}$$

42. Time when velocity of ball is zero

$$0 = 9.8 \times gt$$

$$\Rightarrow \quad t = \frac{9.8}{9.8} = 1 \text{ s}$$

So, total time taken by ball to come back is 2 s.
Distance travelled by trolley in 2 s is

$$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1 \times 2^2 = 2 \text{ m}$$

So, ball will fall 2 m behind the boy.

43. $t = \dfrac{2\ell}{v-u} = \dfrac{2 \times 3600}{2}$

$$\Rightarrow \quad t = 3600 \text{ s}$$

$$\Rightarrow \quad t = 1 \text{ h}$$

44. At B, velocity is 2 ms^{-1}

Slope of line AB is $v = -2 \text{ ms}^{-1}$

So, speed of particle at $t = 12.5$ s is 2 ms^{-1}

45. $v_{avg} = \dfrac{2v_0(v_1 + v_2)}{2v_0 + v_1 + v_2}$

$$\Rightarrow \quad v_{avg} = \frac{2 \times 3(4.5 + 7.5)}{6 + 4.5 + 7.5} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_{avg} = \frac{6 \times 12}{18} \text{ ms}^{-1} = 4 \text{ ms}^{-1}$$

46. $vdv = ads$

$$\Rightarrow \quad \frac{v^2}{2} = \text{Area of a-s graph}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{1}{2}$$

$$\Rightarrow \quad v = 1 \text{ ms}^{-1}$$

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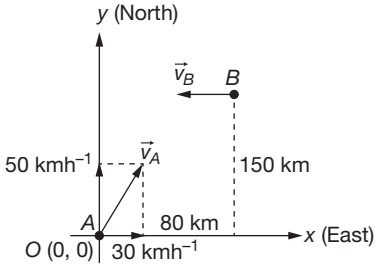
1. The arrangement of the ships is shown in the Figure (not drawn to scale). Given that



$$\vec{v}_A = 30\hat{i} + 50\hat{j} \text{ kmhr}^{-1}, \vec{r}_A = 0\hat{i} + 0\hat{j}$$

$$\vec{v}_B = (-10\hat{i}) \text{kmhr}^{-1}, \vec{r}_B = (80\hat{i} + 150\hat{j}) \text{ km}$$

$$\Rightarrow \quad \vec{r}_{BA} = (80\hat{i} + 150\hat{j}) \text{ km}$$

$$\Rightarrow \quad \vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i} = -40\hat{i} - 50\hat{j}$$

The time after which distance between the ships is minimum is given by

$$t = \frac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{|\vec{v}_{BA}|^2}$$

$$\Rightarrow \quad t = \frac{(80\hat{i} + 150\hat{j}) \cdot (-40\hat{i} - 50\hat{j})}{|-40\hat{i} - 50\hat{j}|^2}$$

$$\Rightarrow \quad t = \left(\frac{3200 + 7500}{4100}\right) \text{hr}$$

$$\Rightarrow \quad t = \frac{10700}{4100} \text{hr} = 2.6 \text{ hr}$$

Hence, the correct answer is (D).

2. Since $a = \text{constant}$, $u = 0$

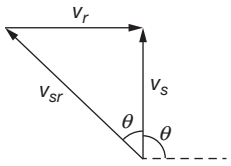
$$\Rightarrow \quad v = at$$

Since particle starts from the origin

$$\Rightarrow \quad x = \frac{1}{2}at^2$$

So, (I), (II) and (IV) are correct graphs.
Hence, the correct answer is (C).

3. Drawing the relative velocity diagram, we get



$$\sin\theta = \frac{v_r}{v_{sr}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30^\circ$$

$$\Rightarrow \quad \phi = 90 + \theta = 120^\circ$$

Hence, the correct answer is (D).

4. $\vec{r} = 15t^2\hat{i} + 4\hat{j} - 20t^2\hat{j}$

$$\Rightarrow \quad \frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$$

$$\Rightarrow \quad \frac{d^2\vec{r}}{dt^2} = 30\hat{i} - 40\hat{j}$$

$$\Rightarrow \quad \left|\frac{d^2\vec{r}}{dt^2}\right| = 50 \text{ ms}^{-2}$$

Hence, the correct answer is (A).

5. $x = at + bt^2 - ct^3$

Velocity $\dot{x} = \frac{dx}{dt}$ is given by

$$\dot{x} = a + 2bt - 3ct^2$$

Acceleration $\ddot{x} = \frac{d^2x}{dt^2}$ is given by

$$\ddot{x} = 2b - 6ct$$

For $\ddot{x} = 0$, $t = +\frac{b}{3c}$

$$\Rightarrow \quad v = \dot{x} = a + 2b\left(\frac{+b}{3c}\right) - 3c\left(\frac{b^2}{3c \times 3c}\right)$$

$$\Rightarrow \quad v = -\frac{b^2}{3c} + \frac{2b^2}{3c} + a = a + \frac{b^2}{3c}$$

Hence, the correct answer is (B).

6. $v^2 = u^2 - 2as$

$$\Rightarrow \quad v^2 = (1)^2 - (2)\left(\frac{2.5 \times 10^{-2}}{20 \times 10^{-3}}\right)\left(\frac{20}{100}\right)$$

$$\Rightarrow \quad v^2 = 1 - \frac{1}{2}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{2}} \text{ ms}^{-1} = 0.7 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

7. $v = \frac{dx}{dt} = b\sqrt{x}$ …(1)

$$\Rightarrow \quad \int_0^x \frac{dx}{\sqrt{x}} = \int_0^\tau b\,dt$$

$$\Rightarrow \quad 2\sqrt{x} = b\tau \qquad \ldots(2)$$

$$\Rightarrow \quad v = b\left(\frac{b\tau}{2}\right) = \frac{b^2\tau}{2}$$

Hence, the correct answer is (C).

8. Since $\frac{dx}{dt} = ky$ and $\frac{dy}{dt} = kx$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow \quad y\,dy = x\,dx$$

$$\Rightarrow \quad \int y\,dy = \int x\,dx$$

$$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + \text{constant}$$

$$\Rightarrow \quad y^2 - x^2 = \text{constant}$$

$$\Rightarrow \quad y^2 = x^2 + \text{constant}$$

Hence, the correct answer is (C).

9. $t_A = t_B - t$

$$\Rightarrow \quad v_A = a_1(t_B - t) = a_2 t_B + v \qquad \ldots(1)$$

$$\Rightarrow \quad S = \frac{1}{2}a_1(t_B - t)^2 = \frac{1}{2}a_2 t_B^2$$

$$\Rightarrow \quad t_B\left(1 - \sqrt{\frac{a_2}{a_1}}\right) = t \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v = t\sqrt{a_1 a_2}$$

Hence, the correct answer is (D).

10. $v_x = \frac{dx}{dt} = -a\omega\sin(\omega t)$

$$v_y = \frac{dy}{dt} = a\omega\cos(\omega t)$$

$$v_z = \frac{dz}{dt} = a\omega$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{2}a\omega$$

Hence, the correct answer is (B).

CHAPTER 4

11. Since displacement is the area under v-t graph. So

$$\text{Area} = \frac{1}{2}\times 2\times 2 + 2\times 2 + 3\times 1$$

$$\Rightarrow \quad \text{Displacement} = 9 \text{ m}$$

Hence, the correct answer is (A).

12. When moving in same direction

$$t_1 = \frac{\ell_1 + \ell_2}{v_1 - v_2}$$

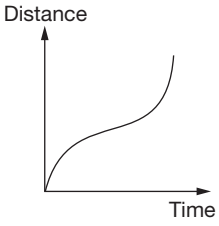
When moving in opposite direction

$$t_2 = \frac{\ell_1 + \ell_2}{v_1 + v_2}$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{v_1 + v_2}{v_1 - v_2} = \frac{80+30}{80-30} = \frac{11}{5}$$

Hence, the correct answer is (B).

13. In OPTIONS (A), (C) and (D), the given graphs represent uniformly decelerated motion of a particle in a straight line with positive initial velocity. Distance-time graph of such a motion is shown here.



Hence, the correct answer is (B).

14. Distance travelled by car in 15 s is

$$s_1 = \frac{1}{2}\times AC \times OC = \frac{1}{2}(45)(15) = \frac{675}{2} \text{ m}$$

Distance travelled by scooter in 15 s is

$$s_2 = v \times t = 30\times 15 = 450 \text{ m}$$

Required difference in distance is $(s_2 - s_1) = \Delta s$

$$\Rightarrow \quad \Delta s = 450 - \frac{675}{2} = \frac{225}{2} = 112.5 \text{ cm}$$

Let the car catch scooter in time t, then

$$\frac{675}{2} + 45(t-15) = 30t$$

$$\Rightarrow \quad 337.5 + 45t - 675 = 30t$$

$$\Rightarrow \quad 15t = 337.5$$

$$\Rightarrow \quad t = 22.5 \text{ s}$$

Hence, the correct answer is (D).

15. Using, $v^2 = u^2 - 2as$, we get

$$0 = u^2 - 2as$$

$$\Rightarrow \quad s = \frac{u^2}{2a}$$

$$\Rightarrow \quad \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$$

$$\Rightarrow \quad s_2 = \left(\frac{u_2}{u_1}\right)^2 s_1$$

$$\Rightarrow \quad s_2 = (2)^2(40) = 160 \text{ m}$$

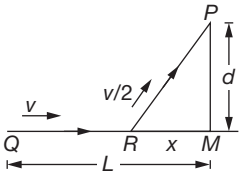
Hence, the correct answer is (C).

16. Time taken by car to reach at location P from location Q is

$$t = \frac{QR}{v} + \frac{RP}{(v/2)}$$

$$\Rightarrow \quad t = \frac{(L-x)}{v} + \frac{2\sqrt{d^2+x^2}}{v}$$

For t to be minimum, we have



$$\frac{dt}{dx} = 0$$

$$\Rightarrow \quad \frac{dt}{dx} = \frac{1}{v}(0-1) + 2\times\left(\frac{1}{2}\right)\frac{1}{v}\times\frac{2x}{\sqrt{d^2+x^2}}$$

$$\Rightarrow \quad \frac{dt}{dx} = \frac{-1}{v} + \frac{2x}{v\sqrt{d^2+x^2}}$$

Since, for minimum value of t

$$\frac{dt}{dx} = 0$$

$$\Rightarrow \quad -\frac{1}{v} + \frac{2x}{v\sqrt{d^2+x^2}} = 0$$

$$\Rightarrow \quad 1 = \frac{2x}{\sqrt{d^2+x^2}}$$

$$\Rightarrow \quad 4x^2 = d^2 + x^2$$

$$\Rightarrow \quad 3x^2 = d^2$$

$$\Rightarrow \quad x = \frac{d}{\sqrt{3}}$$

Hence, the correct answer is (B).

17. Velocity of the body going upwards is given by $v = v_0 - gt$, where v_0 is the initial velocity.

Hence, the graph between velocity and time should be a straight line with negative slope (g) and intercept v_0.

Also, during the whole motion, acceleration of the body is constant i.e., slope should be constant.

Hence, the correct answer is (C).

18. Here, acceleration is given by, $a = -c$

$$\Rightarrow \quad \frac{dv}{dt} = -c$$

$$\Rightarrow \quad \frac{dx}{dt} \cdot \frac{dv}{dx} = -c$$

$$\Rightarrow \quad v\frac{dv}{dx} = -c$$

$$\Rightarrow \quad vdv = -cdx$$

$$\Rightarrow \quad \frac{v^2}{2} = -cx + k$$

$$\Rightarrow \quad x = -\frac{v^2}{2c} + \frac{k}{c}$$

Hence, the correct answer is (A).

20. Acceleration of car, $a_C = 4\ \text{ms}^{-2}$

Acceleration of bus, $a_B = 2\ \text{ms}^{-2}$

Initial separation between the bus and car is

$$s_{CB} = 200\ \text{m}$$

Acceleration of car with respect to bus is

$$a_{CB} = a_C - a_B = 2\ \text{ms}^{-2}$$

Initial velocity of car w.r.t. bus is

$$u_{CB} = 0\,,\ t = ?$$

Since, $s_{CB} = u_{CB}t + \frac{1}{2}a_{CB}t^2$

$$\Rightarrow \quad 200 = (0)t + \frac{1}{2}(2)t^2$$

$$\Rightarrow \quad t^2 = 200$$

$$\Rightarrow \quad t = 10\sqrt{2}\ \text{s}$$

Hence, the correct answer is (C).

21. Using $h = ut + \frac{1}{2}at^2$

For stone 1, we have

$$y_1 = 10t - \frac{1}{2}gt^2$$

and for stone 2, we have

$$y_2 = 40t - \frac{1}{2}gt^2$$

Relative position of the second stone with respect to the first,

$$\Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 - 10t + \frac{1}{2}gt^2$$

$$\Rightarrow \quad \Delta y = 30t$$

After 8 seconds, stone 1 reaches ground, i.e., $y_1 = -240$ m

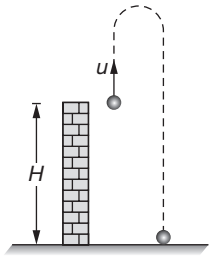
$$\Rightarrow \quad \Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 + 240$$

Therefore, it will be a parabolic curve till other stone reaches ground.

Hence, the correct answer is (A).

22. Time taken by the particle to reach the top most point is,

$$t = \frac{u}{g} \qquad \ldots(1)$$



Time taken by the particle to reach the ground is

$$t' = nt$$

Since, $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad -H = u(nt) - \frac{1}{2}g(nt)^2$$

Using (1), we get

$$-H = u \times n\left(\frac{u}{g}\right) - \frac{1}{2}gn^2\left(\frac{u}{g}\right)^2$$

$$\Rightarrow \quad -2gH = 2nu^2 - n^2u^2$$

$$\Rightarrow \quad 2gH = nu^2(n-2)$$

Hence, the correct answer is (D).

23. $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\Rightarrow \quad \frac{1}{\sqrt{v}}dv = -2.5dt$$

CHAPTER 4

On integrating, within limits i.e., $v_1 = 6.25\ \text{ms}^{-1}$ to $v_2 = 0$, we get

$$\int_{6.25}^{0} v^{-1/2}dv = -2.5\int_{0}^{t} dt$$

$$\Rightarrow \quad 2\sqrt{v}\Big|_{6.25}^{0} = -(2.5)t$$

$$\Rightarrow \quad t = \frac{-2\sqrt{6.25}}{-2.5} = 2\ \text{s}$$

Hence, the correct answer is (B).

24. Here, $\vec{v} = K\left(y\hat{i} + x\hat{j}\right)$

$$\Rightarrow \quad \vec{v} = Ky\hat{i} + Kx\hat{j} \qquad \ldots(1)$$

Since, $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$ …(2)

Equating equations (1) and (2), we get

$$\frac{dx}{dt} = Ky \text{ and } \frac{dy}{dt} = Kx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{Kx}{Ky} = \frac{x}{y} \qquad \ldots(3)$$

Integrating both sides of the above equation, we get

$$\int ydy = \int xdx$$

$$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + \text{constant}$$

$$\Rightarrow \quad y^2 = x^2 + \text{constant}$$

Hence, the correct answer is (A).

25. $\vec{v} = \vec{u} + \vec{a}t$

$$\Rightarrow \quad \vec{v} = \left(3\hat{i} + 4\hat{j}\right) + \left(0.4\hat{i} + 0.3\hat{j}\right) \times 10$$

$$\Rightarrow \quad \vec{v} = (3+4)\hat{i} + (4+3)\hat{j}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2}\ \text{units}$$

Hence, the correct answer is (B).

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Single Correct Choice Type Problems

1. Given line have positive intercept but negative slope. So its equation can be written as

$$v = -mx + v_0 \qquad \ldots(1)$$

where $m = \tan\theta = \frac{v_0}{x_0}$

By differentiating with respect to time we get

$$\frac{dv}{dt} = -m\frac{dx}{dt} = -mv$$

Now substituting the value of v from equation (1) we get

$$\frac{dv}{dt} = -m(-mx + v_0) = m^2x - mv_0$$

$$\Rightarrow \quad a = m^2x - mv_0$$

i.e. the graph between a and x should have positive slope but negative intercept on a-axis. So graph (A) is correct.

Hence, the correct answer is (A).

2. Distance travelled in t^{th} second is,

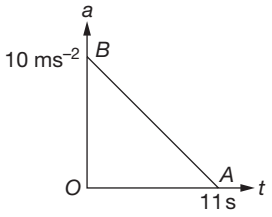
$$s_t = u + \frac{a}{2}(2t-1)$$

Since, $u = 0$, so

$$\Rightarrow \quad \frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$$

Hence, the correct answer is (C).

3. The area under acceleration time graph gives change in velocity. As acceleration is zero at the end of 11 s



$$\Rightarrow \quad v_{\max} = \text{Area of } \Delta OAB$$

$$\Rightarrow \quad v_{\max} = \frac{1}{2} \times 11 \times 10 = 55\ ms^{-1}$$

Hence, the correct answer is (B).

4. $v_1 = -\sqrt{2gd}$ and $v_2 = +\sqrt{2g\left(\frac{d}{2}\right)}$

Hence, the correct answer is (A).

5. $\left|\text{Average Velocity}\right| = \left|\dfrac{\text{Displacement}}{\text{time}}\right| = \dfrac{\text{AB}}{\text{time}} = \dfrac{2}{1} = 2 \text{ ms}^{-1}$

Hence, the correct answer is (B).

6. **For Q**

Since acceleration due to gravity has no component along AB. So motion of the particle along AB is non-accelerated motion with uniform velocity v. If t_Q is the time taken by particle Q to go from A to B, then

$$t_Q = \frac{AB}{v} \qquad \ldots(1)$$

For P

Since motion of particle P from A to C is accelerated and that from C to B is retarded. So from A to C the horizontal component of velocity gradually increases to attain a maximum value at the lowest point which further decreases gradually to attain the same value v at point B.

That is the motion from A to B to C experiences a horizontal velocity which has a value more than v. So $t_Q > t_P$

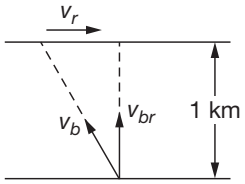
Hence, the correct answer is (A).

7. v_b = velocity of boat in still water $= 5 \text{ kmh}^{-1}$

v_{br} = velocity of boat w.r.t. river

v_r = velocity of river

$$v_{br} = \frac{1 \text{ km}}{\left(\frac{15}{60}\right)h} = 4 \text{ kmh}^{-1}$$



By Pythagoras Theorem

$$v_r^2 + v_{br}^2 = v_b^2$$

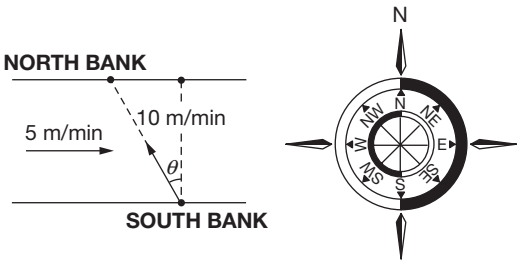
$$\Rightarrow \quad v_r^2 = v_b^2 - v_{br}^2$$

$$\Rightarrow \quad v_r^2 = 5^2 - 4^2$$

$$\Rightarrow \quad v_r = 3 \text{ kmh}^{-1}$$

Hence, the correct answer is (B).

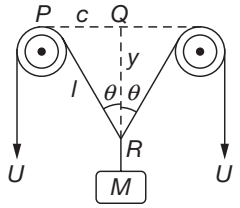
8.



To swim across the river to a point directly opposite in the shortest time, the man should swim at an angle $\theta = \sin^{-1}\left(\dfrac{5}{10}\right) = 30°$ towards the West of North.

Hence, the correct answer is (C).

9. In the right angle ΔPQR, $l^2 = c^2 + y^2$



c = constant, l and y are variable

Differentiating this equation with respect to time, we get

$$2l\frac{dl}{dt} = 0 + 2y\frac{dy}{dt}$$

or $$\left(-\frac{dy}{dt}\right) = \frac{l}{y}\left(-\frac{dl}{dt}\right)$$

Here, $-\dfrac{dy}{dt} = v_M$

$$\Rightarrow \quad \frac{l}{y} = \frac{1}{\cos\theta}$$

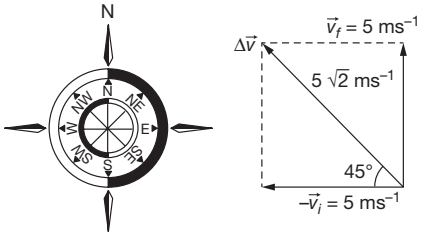
and $$\frac{-dl}{dt} = U$$

Hence, $v_M = \dfrac{U}{\cos\theta}$

Hence, the correct answer is (C).

10. $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\Delta\vec{v} = 5\sqrt{2} \text{ ms}^{-1}$ in North-West direction.



$$\vec{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} \text{ ms}^{-2} \text{ (in North-West direction)}$$

Hence, the correct answer is (C).

CHAPTER 4

Multiple Correct Choice Type Problems

1. $x = a\cos(pt)$

$y = b\sin(pt)$

$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow$ Trajectory is an Ellipse

$\Rightarrow \quad \ddot{x} = -ap^2\cos(\omega t)$

$\Rightarrow \quad \ddot{x} = -p^2 x$

Similarly $\ddot{y} = -p^2 y$

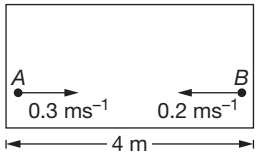
So, acceleration is directed towards the focus.

Hence, (A), (B) and (C) are correct.

2. The body starts from rest at $x = 0$ and then again comes to rest at $x = 1$. It means initially acceleration is positive and then negative.
So, we can conclude that α cannot remains positive for all t in the interval $0 \le t \le 1$ i.e., α must change sign during the motion.
Hence, (A) and (D) are correct.

Integer/Numerical Answer Type Questions

1. Let us consider motion of two balls with respect to rocket.



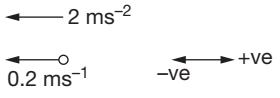
Motion of ball *A* relative to rocket

Maximum distance of ball A from left wall is

$$S_A = \frac{u^2}{2a} = \frac{0.3 \times 0.3}{2 \times 2} = \frac{0.09}{4} \approx 0.02 \text{ m} \quad \{\because 0 = u^2 - 2aS\}$$

So, collision of two balls will take place very near to left wall.

Motion of ball *B* relative to rocket



$$S = ut + \frac{1}{2}at^2$$

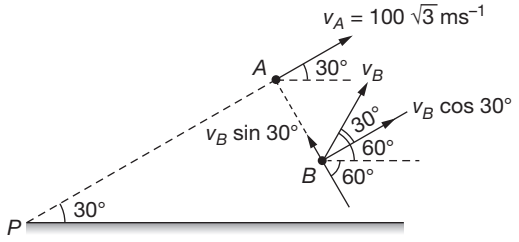
$$\Rightarrow \quad -4 = -0.2t - \left(\frac{1}{2}\right)2t^2$$

Solving this equation, we get

$t = 1.9$ s

Nearest Integer $= 2$ s

2. Relative velocity of B with respect to A is perpendicular to line PA. Therefore, along the line PA, velocity components of A and B should be same.



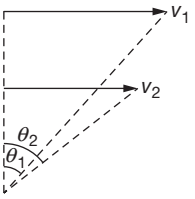
$$\Rightarrow \quad v_A = 100\sqrt{3} = v_B \cos 30°$$

$$\Rightarrow \quad v_B = 200 \text{ ms}^{-1}$$

$$\Rightarrow \quad t_0 = \frac{500}{v_B \sin(30)} = \frac{500}{200 \times \frac{1}{2}} = 5 \text{ s}$$

Assertion and Reasoning Type Problems

1.



Statement-II, is the mathematical definition of relative velocity. However, it does not explain Statement-I correctly. The correct explanation of Statement-I is due to visual perception of motion. The object appears to be moving faster, when its angular velocity is greater w.r.t. observer.

Hence, the correct answer is (B).

CHAPTER 5: KINEMATICS II

Test Your Concepts-I (Based on Curvilinear Motion)

1. $\vec{v} = v_x\hat{i} + v_y\hat{j}$ and $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{v}\cdot\vec{a} = v_xa_x + v_ya_y$

Further $v = \sqrt{v_x^2 + v_y^2}$

$\Rightarrow \quad v^2 = v_x^2 + v_y^2$

$\Rightarrow \quad 2v\dfrac{dv}{dt} = 2v_x\dfrac{dv_x}{dt} + 2v_y\dfrac{dv_y}{dt}$

$\Rightarrow \quad \dfrac{dv}{dt} = \dfrac{v_xa_x + v_ya_x}{v}$ …(1)

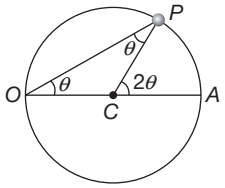
$\Rightarrow \quad \dfrac{dv}{dt} = \dfrac{\vec{a}\cdot\vec{v}}{v} = \dfrac{v_xa_x + v_ya_y}{v} = a_T$

Because from our knowledge of vectors we know that component of $\vec{A}$ along $\vec{B}$ is $\dfrac{\vec{A}\cdot\vec{B}}{B}$. So, $\dfrac{\vec{a}\cdot\vec{v}}{v}$ is the component of $\vec{a}$ along $\vec{v}$ and this happens to be the tangential acceleration, a_T.

2. Let, O be a point on a circle and P be the position of the particle at any time t, such that

$\angle POA = \theta$

Then $\angle PCA = 2\theta$



Here, C is the centre of the circle

Angular velocity of P about O is

$\omega_0 = \dfrac{d\theta}{dt}$

Angular velocity of P about C is

$\omega_c = \dfrac{d}{dt}(2\theta) = 2\dfrac{d\theta}{dt}$

$\Rightarrow \quad \omega_c = 2\omega_0$

3. The angular speed is

$\omega = \dfrac{v}{r} = \dfrac{2}{0.25} = 8 \text{ rads}^{-1}$

4. The tangential acceleration of the particle is

$a_T = \dfrac{dv}{dt} = \dfrac{4-2}{4} = 0.5 \text{ ms}^{-2}$

The angular acceleration is

$\alpha = \dfrac{a_T}{r} = \dfrac{0.5}{0.25} = 2 \text{ rads}^{-2}$

5. $a = \sqrt{a_T^2 + a_N^2} = \sqrt{\left(\dfrac{dv}{dt}\right)^2 + \left(\dfrac{v^2}{R}\right)^2}$

$\Rightarrow \quad a = \sqrt{(8)^2 + \left(\dfrac{256}{50}\right)^2} = 9.5 \text{ ms}^{-2}$

6. Since $y = u_yt + \dfrac{1}{2}a_yt^2$, where $u_y = 0$ and $a_y = g$, $y = h$.

So $h = \dfrac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(20)}{10}} = 2 \text{ s}$

Further since $x = u_xt + \dfrac{1}{2}a_xt^2$, where $u_x = v$ (say), $a_x = 0$, $x = 10$ m, so

$10 = vt = v(2)$

$\Rightarrow \quad v = 5 \text{ ms}^{-1}$

Since $a = \dfrac{v^2}{R}$

$\Rightarrow \quad a = \dfrac{(5)^2}{0.5} = \dfrac{25}{0.5} = 50 \text{ ms}^{-2}$

$\Rightarrow \quad a = 50 \text{ ms}^{-2}$

7. $1.5t + 0.7t = 2\pi R = 14\pi$

$\Rightarrow \quad t = \dfrac{14\pi}{2\cdot 2} = \dfrac{14\left(\dfrac{22}{7}\right)}{2\cdot 2} = 20 \text{ s}$

$a = \dfrac{v_B^2}{R} = \dfrac{(1\cdot 5)^2}{7} = \dfrac{2\cdot 25}{7} = 0\cdot 32 \text{ ms}^{-2}$

8. When $t = 3$ s, the motorcycle travels at a speed of

$v = \dfrac{1}{5}(3^2) = 1.8 \text{ ms}^{-1}$

The tangential acceleration is

$a_T = \dot{v} = \dfrac{2t}{5} = (0.4t) \text{ ms}^{-2}$. When $t = 3$ s,

$a_T = 0.4(3) = 1.2 \text{ ms}^{-2}$

Since, $a_N = \frac{v^2}{r} = \frac{(1.8)^2}{50} = 0.0648 \text{ ms}^{-2}$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(1.2)^2 + (0.0648)^2} \cong 1.2 \text{ ms}^{-2}$$

9. Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_T = 0$. Thus,

$$a = a_N = \frac{v^2}{r}$$

$$\Rightarrow \quad 7.5 = \frac{v^2}{200}$$

$$\Rightarrow \quad v = 38.7 \text{ ms}^{-1}$$

10. Since the automobile is travelling at a constant speed,

$$a_T = 0$$

Thus, $a_N = a = 5 \text{ ms}^{-2}$

Since $a_N = \frac{v^2}{r}$, so we get

$$v = \sqrt{a_N r} = \sqrt{800(5)} \cong 63 \text{ ms}^{-1}$$

11. $a_T = \left(\frac{2000 \text{ km}}{h^2}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 = 0.1543 \text{ ms}^{-2}$

$$v = \left(\frac{60 \text{ km}}{h}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 16.67 \text{ ms}^{-1}$$

$$a_N = \frac{v^2}{r} = \frac{16.67^2}{600} = 0.463 \text{ ms}^{-2}$$

$$\Rightarrow \quad a = \sqrt{a_T^2 + a_N^2} = \sqrt{(0.1543)^2 + (0.463)^2} = 0.488 \text{ ms}^{-2}$$

12. Since the car is travelling with a constant speed, its tangential component of acceleration is zero, i.e., $a_T = 0$. Thus,

$$a = a_N = \frac{v^2}{r}$$

$$\Rightarrow \quad 3 = \frac{25^2}{r}$$

$$\Rightarrow \quad r = 208 \text{ m}$$

13. Here, the car's tangential component of acceleration of $a_T = -3 \text{ ms}^{-2}$. Thus,

$$a = \sqrt{a_T^2 + a_N^2}$$

$$\Rightarrow \quad 5 = \sqrt{(-3)^2 + a_N^2}$$

$$\Rightarrow \quad a_N = 4 \text{ ms}^{-2}$$

Since $a_N = \frac{v^2}{r}$

$$4 = \frac{20^2}{r}$$

$$\Rightarrow \quad r = 100 \text{ m}$$

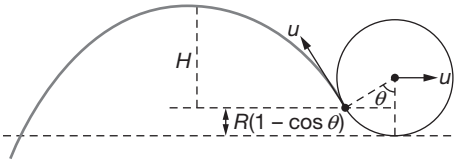
14. Velocity of the particle at an angle θ as it leaves the rim is

$$\vec{v} = (u - u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$$

$$\Rightarrow \quad \vec{v} = u(1 - \cos\theta)\hat{i} + (u\sin\theta)\hat{j}$$

Maximum height from ground attained by a particle

$$h = R(1 - \cos\theta) + H = R(1 - \cos\theta) + \frac{u^2 \sin^2\theta}{2g} \quad \ldots(1)$$



For h to be maximum, we have

$$\frac{dh}{d\theta} = 0$$

$$\Rightarrow \quad \sin\theta\left(\frac{u^2}{g}\cos\theta + R\right) = 0$$

$$\Rightarrow \quad \text{Either } \sin\theta = 0 \text{ or } \cos\theta = -\frac{Rg}{u^2} = -\frac{3}{5}$$

$$\Rightarrow \quad \theta = 0° \text{ or } \theta = \cos^{-1}\left(-\frac{3}{5}\right)$$

Also we observe that $\left[\frac{d^2h}{d\theta^2}\right] < 0$; for $\theta = \cos^{-1}\left(-\frac{3}{5}\right)$ and $\frac{d^2h}{d\theta^2} > 0$; for $\theta = 0°$

So h is maximum, when

$$\cos\theta = -\frac{3}{5}$$

$$\Rightarrow \quad h_{\max} = \frac{3}{2}\left(1 + \frac{3}{5}\right) + \frac{25}{20}\frac{16}{25} = 3.2 \text{ m}$$

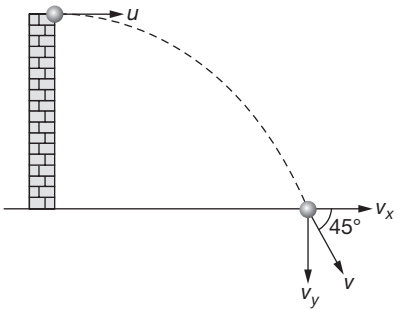
Test Your Concepts-II (Based on Horizontal Projectile)

1. $u_y = 0$ and $a_y = g = 9.8 \text{ ms}^{-2}$

Since $y = u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad y = (0)(3) + \frac{1}{2}(9.8)(3)^2$$

$$\Rightarrow \quad y = 44.1 \text{ m}$$



Further, $v_y = u_y + a_y t = 0 + (9.8)(3)$

$\Rightarrow \quad v_y = 29.4 \text{ ms}^{-1}$

As the resultant velocity v makes an angle of $45°$ with the horizontal so

$$\tan 45° = \frac{v_y}{v_x}$$

$\Rightarrow \quad 1 = \dfrac{29.4}{v_x}$

$\Rightarrow \quad v_x = 29.4 \text{ ms}^{-1}$

Therefore, the speed with which the body was projected (horizontally) is 29.4 ms^{-1}.

2. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times(36-16.4)}{9.8}} = 2 \text{ s}$

Since $x = ut$

$\Rightarrow \quad u = \dfrac{x}{t} = \dfrac{40}{2} = 20 \text{ ms}^{-1}$

3. $u = \sqrt{2gh}$

$\Rightarrow \quad u = \sqrt{(2)(10)(5)} = 10 \text{ ms}^{-1}$

Let the water strike the ground in time t. Then

$$x = ut = 10t \qquad \ldots(1)$$

Further since $y = u_y t + \dfrac{1}{2} a_y t^2$

$\Rightarrow \quad 2.5 = \dfrac{1}{2}(10)t^2 \qquad \{\because u_y = 0\}$

$\Rightarrow \quad t = 0.7 \text{ s}$

$\Rightarrow \quad x = (10)(0.7) \text{ m}$

$\Rightarrow \quad x = 7 \text{ m}$

4. Let the ball clear the net at time t_1. Then

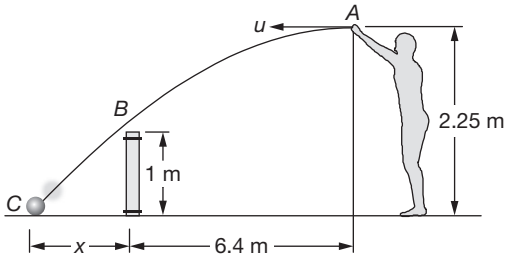
$$\Delta y = 2.25 - 1 = 1.25 \text{ m}$$

Since $\Delta y = u_y t + \dfrac{1}{2} a_y t^2$

$\Rightarrow \quad 1.25 = 0 + \dfrac{1}{2}(10)t_1^2$

$\Rightarrow \quad t_1^2 = 0.25$

$\Rightarrow \quad t_1 = 0.5 \text{ s}$



Let the ball clear the net and then strike the ground in total time t_2 (say). Then

$$2.25 = 0 + \frac{1}{2}(10)t_2^2$$

$\Rightarrow \quad t_2 = 0.67 \text{ s}$

Further the net is at a horizontal distance of

$$7 = u(0.5)$$

$\Rightarrow \quad u = 14 \text{ ms}^{-1}$

Now for the ball to fall after the net, let the horizontal distance be x. Then total horizontal distance covered by the ball from the beginning is $(7 + x)$, in a time $t_2 = 0.67$ s. So,

$$(7 + x) = (14)(0.67)$$

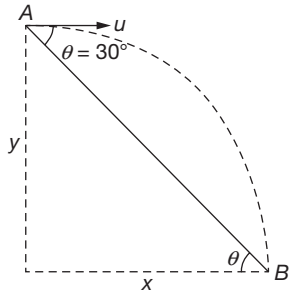
$\Rightarrow \quad 7 + x = 9.4$

$\Rightarrow \quad x = 2.4 \text{ m}$

5. $y = \dfrac{1}{2} g t^2$,

$$x = ut$$

$$\tan\theta = \frac{y}{x} = \frac{gt}{2u}$$



CHAPTER 5

$$\Rightarrow \quad u = \frac{gt}{2\tan\theta} = \frac{(10)(4)}{(2)\left(\frac{1}{\sqrt{3}}\right)} = 20\sqrt{3} \text{ ms}^{-1}$$

$$\Rightarrow \quad u \cong 34 \text{ ms}^{-1}$$

6. Let the ball clear the point C at time t_1. Then

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad (1+3.9) = 0 + \frac{1}{2}(9.8)t_1^2 \qquad \{\because u_y = 0\}$$

$$\Rightarrow \quad 4.9 = \frac{1}{2}(9.8)t_1^2$$

$$\Rightarrow \quad t_1 = 1 \text{ s}$$

Since $BC = u_x t$

$$\Rightarrow \quad 6 = u(1)$$

$$\Rightarrow \quad u = 6 \text{ ms}^{-1}$$

Let the ball hit the ground at D in time t.
Then

$$6 + x = 6t \qquad \ldots(1)$$

Further $(1+3.9+14.7) = \frac{1}{2}(9.8)t^2$

$$\Rightarrow \quad t = 2 \text{ s}$$

$$\Rightarrow \quad 6 + x = 12$$

$$\Rightarrow \quad x = 6 \text{ m}$$

7. $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 0.8}{10}} = 0.4$ s

$$u_{\text{MIN}} = \frac{(120-30+10)\times 10^{-3}}{0.4} = 0.25 \text{ ms}^{-1}$$

and $u_{\text{MAX}} = \frac{(120+30-10)\times 10^{-3}}{0.4} = 0.35 \text{ ms}^{-1}$

$$\Rightarrow \quad u_{MIN} = 25 \text{ cms}^{-1} \text{ and } u_{MAX} = 35 \text{ cms}^{-1}$$

Test Your Concepts-III (Based on Oblique Projectile)

1. Since, $R = H$

$$\Rightarrow \quad \frac{u^2\sin(2\theta)}{g} = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = 4$$

$$\Rightarrow \quad \tan\theta = 4$$

$$\Rightarrow \quad \theta = \tan^{-1}(4)$$

$$\Rightarrow \quad \theta = 76°$$

2. Maximum horizontal range is obtained when $\theta = 45°$, so

$$R_{\max} = \frac{u^2}{g}$$

Maximum height attained i.e., at $\theta = 45°$ is

$$H_{\max} = \frac{u^2\sin^2(45°)}{2g} = \frac{u^2}{4g} = \frac{R_{\max}}{4}$$

$$\Rightarrow \quad R_{\max} = 4H_{\max}$$

3. There are two launch angles θ and $90° - \theta$ for which the horizontal range R is same. So,

$$H_1 = \frac{u^2\sin^2\theta}{2g} \text{ and}$$

$$H_2 = \frac{u^2\sin^2(90°-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$\Rightarrow \quad H_1 + H_2 = \frac{u^2}{2g}(\sin^2\alpha + \cos^2\alpha) = \frac{u^2}{2g}$$

Clearly the sum of the maximum heights for these two launch angles is independent of them

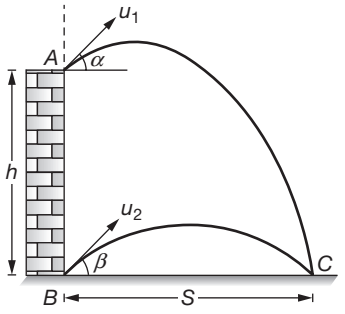
4. Done in theory.

5. Let, height of tower be h.

$$-h = (u_1\sin\alpha)t - \frac{1}{2}gt^2 \text{ and}$$

$$0 = (u_2\sin\beta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad (u_1\sin\alpha)t + h = (u_2\sin\beta)t$$



Also, $S = (u_1\cos\alpha)t = (u_2\cos\beta)t$

$$\Rightarrow \quad t = \frac{S}{u_1\cos\alpha} = \frac{S}{u_2\cos\beta}$$

$$\Rightarrow \quad u_1 \sin\alpha\left(\frac{S}{u_1\cos\alpha}\right)+h=\left(\frac{u_2\sin\beta}{u_2\cos\beta}\right)S$$

$$\Rightarrow \quad h+S\tan\alpha=S\tan\beta$$

$$\Rightarrow \quad h=S(\tan\beta-\tan\alpha)$$

6. (a) Since $v_y^2-u_y^2=2a_y y$

$$\Rightarrow \quad u_y^2=v_y^2-2a_y y$$

where

$$v_y=6.1 \text{ ms}^{-1}$$

$$a_y=-9.8 \text{ ms}^{-1}$$

$$y=9.1 \text{ m}$$

$$\Rightarrow \quad u_y=14.7 \text{ ms}^{-1}$$

Since, at maximum height $v_y=0$

$$\Rightarrow \quad 0^2-u_y^2=2(-g)H$$

$$\Rightarrow \quad H=\frac{u_y^2}{2g}=\frac{(14.7)^2}{2(9.8)}\cong 11 \text{ m}$$

(b) Range $R=\dfrac{2u_x u_y}{g}=\dfrac{2\times 7.6\times 14.7}{9.8}\approx 23$ m

(c) At the instant when the ball hits the ground, we have

$$\text{Speed}=\sqrt{u_y^2+u_x^2}=\sqrt{(14.7)^2+(7.6)^2}\approx 17 \text{ ms}^{-1}$$

$$\theta=\tan^{-1}\left(\frac{u_y}{u_x}\right) \qquad \{\text{with horizontal}\}$$

$$\Rightarrow \quad \theta=\tan^{-1}\left(\frac{14.7}{7.6}\right)\approx 63°$$

7. $\text{Range}=\dfrac{u^2\sin 2\theta}{g}=\dfrac{(20)^2\sin 90°}{9.8}=40.82$ m

Since $T=\dfrac{2u\sin\theta}{g}=\dfrac{(2)(20)\sin(45°)}{9.8}=2.9$ s

So, speed of player is

$$v=\frac{\Delta x}{T}=\frac{50-40.82}{2.9}=3.16 \text{ ms}^{-1}$$

{towards the coach}

8. $t=\dfrac{2v_r}{g}=\dfrac{2\times 20}{9.8}=4.1$ s

If s be the distance travelled by the car in this time then

$$s=ut$$

$$\Rightarrow \quad s=\left(30\times\frac{5}{18}\right)(4.1)=34 \text{ m}$$

If R be the range of the projectile, then

$$R=\frac{2}{g}(u_x)(u_y)$$

$$\Rightarrow \quad R=\frac{2}{g}(u)(v_r)=\frac{2}{9.8}\left(30\times\frac{5}{18}\right)(20)=34 \text{ m}$$

Now since we observe both R and s to be the same i.e., 34 m , so the projectile will land on the vehicle at the tube location.

9. $u_x=30\cos 30°=15\sqrt{3} \text{ ms}^{-1}$

$$u_y=30\sin 30°=15 \text{ ms}^{-1}$$

(a) Since $y=u_y t+\dfrac{1}{2}a_y t^2$

$$\Rightarrow \quad -50=15t-\frac{1}{2}(10)t^2$$

$$\Rightarrow \quad t=5 \text{ s}$$

(b) $x=u_x t=75\sqrt{3} \text{ m}\cong 130$ m

(c) Since $v^2=u^2+2gh$

$$\Rightarrow \quad v^2=(30)^2+2\times 10\times 50$$

$$\Rightarrow \quad v=43.6 \text{ ms}^{-1}$$

10. **METHOD I**

Let the balls collide at time t (say), then

For ball A, $x_A=(v_0\cos\theta)t$ …(1)

For ball B, $x_B=(kv_0\cos\phi)t$ …(2)

Further at the point of collision, we have

$$x_A=x_B$$

$$\Rightarrow \quad \cos\theta=k\cos\phi$$

$$\Rightarrow \quad k=\frac{\cos\theta}{\cos\phi}$$

METHOD II

The balls will collide if component of relative velocity in horizontal direction is zero, or

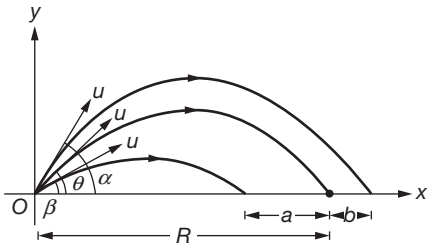
$$kv_0\cos\phi=v_0\cos\theta$$

$$\Rightarrow \quad k=\frac{\cos\theta}{\cos\phi}$$

11. Let θ be the proper angle of projection. Then the actual range is

$$R=\frac{u^2\sin(2\theta)}{g}$$

CHAPTER 5



For the first projectile,

$$\frac{u^2\sin(2\theta)}{g} - a = \frac{u^2\sin(2\alpha)}{g} \qquad \ldots(1)$$

For the second projectile

$$\frac{u^2\sin(2\theta)}{g} + b = \frac{u^2\sin(2\beta)}{g} \qquad \ldots(2)$$

Using $(1)b + (2)a$, we get

$$(a+b)\left(\frac{u^2\sin(2\theta)}{g}\right) = \frac{bu^2\sin(2\alpha)}{g} + \frac{au^2\sin(2\beta)}{g}$$

$$\Rightarrow \quad \sin(2\theta) = \frac{b\sin(2\alpha) + a\sin(2\beta)}{a+b}$$

$$\Rightarrow \quad 2\theta = \sin^{-1}\left(\frac{b\sin(2\alpha) + a\sin(2\beta)}{a+b}\right)$$

$$\Rightarrow \quad \theta = \frac{1}{2}\sin^{-1}\left(\frac{b\sin(2\alpha) + a\sin(2\beta)}{a+b}\right)$$

12. Let the velocities of the two particles be parallel at time t. If the angles made by the respective velocities be θ and ϕ respectively, then for them being parallel,

$$\theta = \phi$$

$$\Rightarrow \quad \tan\theta = \tan\phi$$

$$\Rightarrow \quad \frac{u\sin\alpha - gt}{u\cos\alpha} = \frac{v\sin\beta - gt}{v\cos\beta}$$

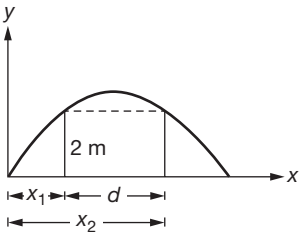
$$\Rightarrow \quad uv\sin\alpha\cos\beta - gt(v\cos\beta) = uv\cos\alpha\sin\beta - gt(u\cos\alpha)$$

$$\Rightarrow \quad uv\sin(\alpha-\beta) = gt(v\cos\beta - u\cos\alpha)$$

$$\Rightarrow \quad t = \frac{uv\sin(\alpha-\beta)}{(v\cos\beta - u\cos\alpha)}$$

13. Since $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

Now $y = 2$ m, $x = ?$, $u = 10$ ms^{-1}, $\theta = 45°$



$$\Rightarrow \quad 2 = x - \frac{10\times x^2}{(2)(100)\left(\frac{1}{2}\right)}$$

$$\Rightarrow \quad 2 = x - \frac{x^2}{10}$$

$$\Rightarrow \quad x^2 - 10x + 20 = 0$$

$$\Rightarrow \quad x = \frac{10+\sqrt{100-80}}{2} \text{ OR } x = \frac{10-\sqrt{100-80}}{2}$$

$$\Rightarrow \quad d = \frac{10+\sqrt{20}}{2} - \frac{10-\sqrt{20}}{2} = \sqrt{20} = 4.5 \text{ m}$$

So, $x_1 = \dfrac{10-\sqrt{20}}{2} = 2.75$ m

14. $t_A = \dfrac{2u_{Ay}}{g}$, $t_B = \dfrac{2u_{By}}{g}$

$$\Rightarrow \quad t_A^2 - t_B^2 = \frac{4}{g^2}\left(u_{Ay}^2 - u_{By}^2\right) = \frac{4}{g^2}(2gh)$$

$$\Rightarrow \quad g = \frac{8h}{t_A^2 - t_B^2}$$

15. Range $= 800$ mm $= 0.8$ m

$$\Rightarrow \quad \frac{u^2\sin(2\theta)}{g} = 0.8 \text{ m} \qquad \ldots(1)$$

Maximum height $H = 500$ mm $= 0.5$ m

$$\Rightarrow \quad \frac{u^2\sin^2\theta}{2g} = 0.5 \text{ m} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\theta = 68.2° \text{ and } u = 3.374$$

$$\Rightarrow \quad v = u\cos\theta = 1.253 \text{ ms}^{-1}$$

16. Let us take the x-y coordinate system so that its origin coincides with point A.

***x*-Motion:** Here, $x = 20 - 0 = 20$ m

Since $x = u_x t + \dfrac{1}{2}a_x t^2$

$$\Rightarrow \quad 20 = (v_A\cos 30°)t \qquad \ldots(1)$$

***y*-Motion:** Here, $y = 10 - 1.8 = 8.2$ m

$$(v_A)_y = v_A \sin 30° \text{ and } a_y = -g = -9.8 \text{ ms}^{-2}$$

Since

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 8.2 = (v_A \sin 30°)t + \frac{1}{2}(-9.8)(t)^2$$

Thus, $8.2 = \left(\dfrac{20 \sin 30°}{\cos 30°}\right) - 4.9t^2 \qquad \left\{\because t = \dfrac{20}{V_A \cos(30°)}\right\}$

$$\Rightarrow \quad t = 0.83 \text{ s}$$

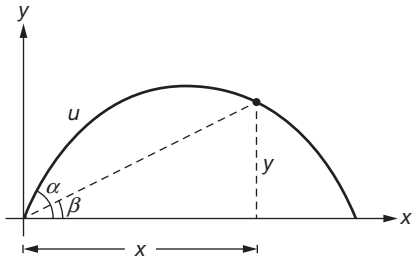
So that

$$v_A = \frac{20}{\cos 30°(0.83)} \cong 28 \text{ ms}^{-1}$$

17. $\tan\beta = \dfrac{y}{x} = \dfrac{(u\sin\alpha)t - \frac{1}{2}gt^2}{(u\cos\alpha)t}$

$$\Rightarrow \quad u(\sin\alpha - \cos\alpha\tan\beta) = \frac{gt}{2}$$

$$\Rightarrow \quad u = \frac{gt}{2(\sin\alpha - \cos\alpha\tan\beta)} = \frac{gt\cos\beta}{2\sin(\alpha - \beta)}$$



18. Since $\tan\beta = \dfrac{u\sin\alpha - gt}{u\cos\alpha}$

$$\Rightarrow \quad \frac{\sin\beta}{\cos\beta} = \frac{u\sin\alpha - gt}{u\cos\alpha}$$

$$\Rightarrow \quad u\sin\beta\cos\alpha = u\sin\alpha\cos\beta - gt\cos\beta$$

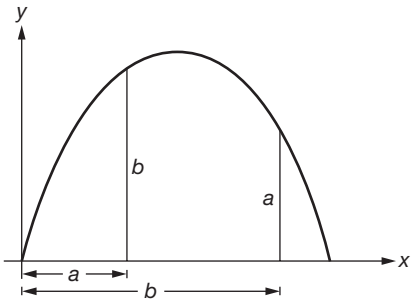
$$\Rightarrow \quad u(\sin\alpha\cos\beta - \sin\beta\cos\alpha) = gt\cos\beta$$

$$\Rightarrow \quad u\sin(\alpha - \beta) = gt\cos\beta$$

19. Since $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

For the particle clearing the wall of height b at distance a, we have $x = a$, $y = b$. So

$$b = a\tan\theta\left(1 - \frac{a}{R}\right) \qquad \ldots(1)$$



For the particle clearing the wall of height a at a distance b, we have $x = b$, $y = a$. So

$$a = b\tan\theta\left(1 - \frac{b}{R}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{b}{a} = \frac{a}{b}\left(\frac{R-a}{R-b}\right) \qquad \{\text{Divide (1) by (2)}\}$$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{R-a}{R-b}$$

$$\Rightarrow \quad Rb^2 - b^3 = Ra^2 - a^3$$

$$\Rightarrow \quad R(b^2 - a^2) = b^3 - a^3$$

$$\Rightarrow \quad R = \frac{b^3 - a^3}{b^2 - a^2} = \frac{(a^2 + b^2 + ab)(b-a)}{(a+b)(b-a)}$$

$$\Rightarrow \quad R = \frac{a^2 + b^2 + ab}{a+b} \qquad \ldots(3)$$

Substituting (3) in (1) or (2), we get

$$b = a\tan\theta\left(1 - \frac{a(a+b)}{a^2 + ab + b^2}\right)$$

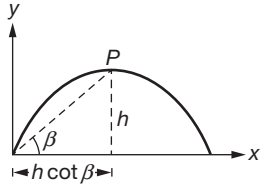
$$\Rightarrow \quad b = a\tan\theta\left(\frac{b^2}{a^2 + ab + b^2}\right)$$

$$\Rightarrow \quad \tan\theta = \frac{a^2 + ab + b^2}{ab}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{a^2 + ab + b^2}{ab}\right)$$

20. Coordinates of the point P are $(h\cot\beta, h)$.

Substituting in $y = x\tan\theta - \dfrac{gx^2}{2u^2}(1 + \tan^2\theta)$



$$\Rightarrow \quad h=(h\cot\beta)\tan\theta-\frac{gh^2\cot^2\beta}{2u^2}(1+\tan^2\theta)$$

This is quadratic in $\tan\theta$. For θ to be real discriminant of this equation ≥ 0, which gives

$$u\geq\sqrt{gh(1+\operatorname{cosec}\beta)}$$

$$\Rightarrow \quad u_{MIN}=\sqrt{gh(1+\operatorname{cosec}\beta)}$$

21. Let u be the velocity of projection in both the cases. In the first case, we have

$$T_1=\frac{2u\sin\alpha}{g} \text{ and} \qquad \ldots(1)$$

$$R_1=(u\cos\alpha+v)T_1$$

$$\Rightarrow \quad R_1=\frac{2u^2\sin\alpha\cos\alpha}{g}+\frac{2uv\sin\alpha}{g} \qquad \ldots(2)$$

In the second case, we have

$$T_2=\frac{2u\sin\alpha}{g}=T_1 \qquad \ldots(3)$$

$$R_2=(u\cos\alpha-v)T_2$$

$$\Rightarrow \quad R_2=\frac{2u^2\sin\alpha\cos\alpha}{g}-\frac{2uv\sin\alpha}{g} \qquad \ldots(4)$$

From (2) and (4), we get

$$R_1-R_2=\frac{4uv\sin\alpha}{g} \qquad \ldots(5)$$

and $\dfrac{R_1}{R_2}=\dfrac{u\cos\alpha+v}{u\cos\alpha-v}$

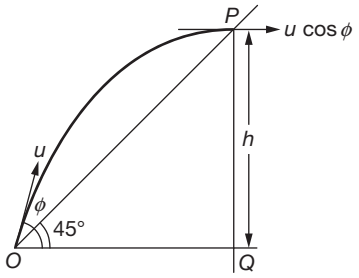
$$\Rightarrow \quad u=\frac{v(R_1+R_2)}{\cos\alpha(R_1-R_2)} \qquad \ldots(6)$$

From equations (5) and (6), we have

$$\tan\alpha=\frac{g(R_1-R_2)^2}{4v^2(R_1+R_2)}$$

Test Your Concepts-IV (Based on Projectile on an Inclined Plane)

1. (a) Let the particle be projected from O with velocity u and strike the plane at a point P horizontally. Then



$$PQ=OQ$$

$$\Rightarrow \quad \text{Maximum height}=\frac{\text{Horizontal range}}{2}$$

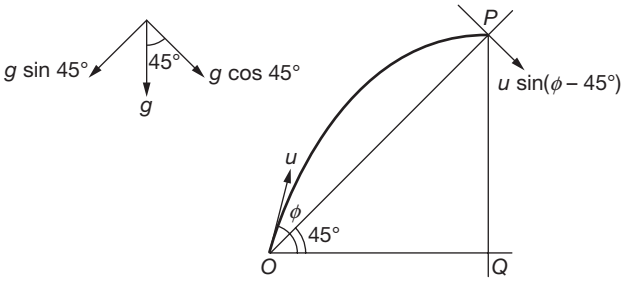
$$\Rightarrow \quad \frac{u^2\sin^2\phi}{2g}=\frac{u^2\sin 2\phi}{2g}=\frac{u^2\sin\phi\cos\phi}{g}$$

$$\Rightarrow \quad \tan\phi=2$$

(b) At time $t=T=\dfrac{2u\sin(\phi-45^\circ)}{g\cos 45^\circ}$

component of velocity along the plane is zero.

$$\Rightarrow \quad 0=u\cos(\phi-45^\circ)-(g\sin 45^\circ)t$$



$$\Rightarrow \quad u\cos(\phi-45^\circ)=(g\sin 45^\circ)\left(\frac{2u\sin(\phi-45^\circ)}{g\cos 45^\circ}\right)$$

$$\Rightarrow \quad 2\tan(\phi-45^\circ)=\cot 45^\circ=1$$

$$\Rightarrow \quad 2\left(\frac{\tan\phi-\tan 45^\circ}{1+\tan\phi\tan 45^\circ}\right)=1$$

$$\Rightarrow \quad \tan\phi=3$$

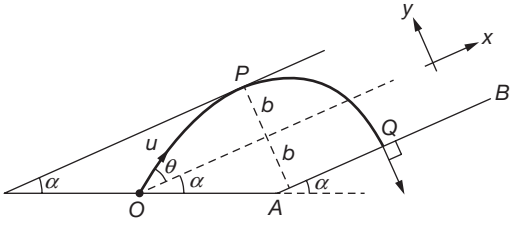
2. Consider the motion of the particle from O to P. The velocity v_y at P is zero.

$$v_y^2=u_y^2+2a_yy$$

$$\Rightarrow \quad 0=(u\sin\theta)^2-2(g\cos\alpha)b$$

$$\Rightarrow \quad b=\frac{u^2\sin^2\theta}{2g\cos\alpha} \qquad \ldots(1)$$

Now, consider the motion of the particle from O to Q. The particle strikes the point Q at 90° to AB, i.e., its velocity along x-direction is zero



Using $v_x = u_x + a_x t$, we get

$$0 = u\cos\theta - (g\sin\alpha)t$$

$$\Rightarrow \quad t = \frac{u\cos\theta}{g\sin\alpha} \qquad \ldots(2)$$

For motion along y-direction, we have

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad -b = u\sin\theta\left(\frac{u\cos\theta}{g\sin\alpha}\right) + \frac{1}{2}(-g\cos\alpha)\left(\frac{u\cos\theta}{g\sin\alpha}\right)^2 \qquad \ldots(3)$$

From equations (1) and (3), we get

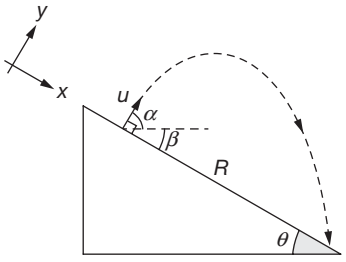
$$-\frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \frac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$$

$$\Rightarrow \quad -\frac{\sin^2\theta}{2\cos\alpha} = \frac{\sin\theta\cos\theta}{\sin\alpha} - \frac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$$

Solving, we get

$$\tan\theta = (\sqrt{2}-1)\cot\alpha$$

3. From the theory for motion of projectile down an inclined plane, we observe that here $\alpha = 90 - \theta$ and $\beta = \theta$.

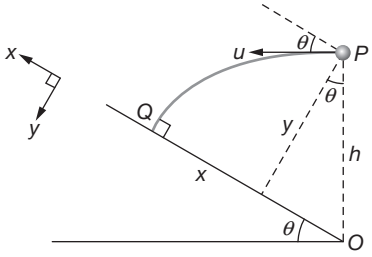


So, we get from, $R = \dfrac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$

$$R = \frac{2u^2\sin(90)\cos(90-\theta)}{g\cos^2\theta}$$

$$\Rightarrow \quad R = \frac{2u^2\tan\theta\sec\theta}{g}$$

4. $u_x = u\cos\theta$, $u_y = u\sin\theta$, $a_x = -g\sin\theta$, $a_y = g\cos\theta$



At Q, we have $v_x = 0$

$$\Rightarrow \quad u_x + a_x t = 0$$

$$\Rightarrow \quad t = \frac{u\cos\theta}{g\sin\theta} \qquad \ldots(1)$$

$$y = h\cos\theta$$

$$\Rightarrow \quad u_y t + \frac{1}{2}a_y t^2 = h\cos\theta$$

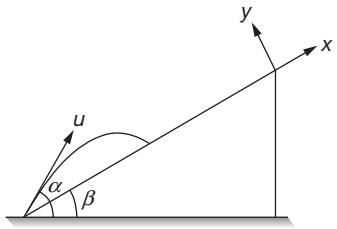
$$\Rightarrow \quad (u\sin\theta)\left(\frac{u\cos\theta}{g\sin\theta}\right) + \frac{1}{2}(g\cos\theta)\left(\frac{u\cos\theta}{g\sin\theta}\right)^2 = h\cos\theta$$

Solving this equation we get,

$$u = \sqrt{\frac{2gh}{2+\cot^2\theta}}$$

5. Particle will retrace its path if it strikes at right angles to the plane i.e.,

$$v_x = 0 \text{ at } T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$



Further, $R = \dfrac{u^2}{g\cos^2\beta}\left[\sin(2\alpha-\beta) - \sin\beta\right]$ $\qquad \ldots(1)$

Substituting in $v_x = u_x + a_x t$, we get

$$0 = u\cos(\alpha-\beta) - g\sin\beta\left\{\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\right\} \qquad \ldots(2)$$

Eliminating α from equations (1) and (2) we get,

$$u = \sqrt{\frac{gR(1+3\sin^2\beta)}{2\sin\beta}}$$

Alternatively, same kind of problem has already been done in theory portion.

6. Consider motion of B along the plane
initial velocity along the incline is $u\cos(\alpha+\beta)$
acceleration is $g\sin\alpha$

$$\Rightarrow \quad OP = u\cos(\alpha+\beta)t + \frac{1}{2}g\sin(\alpha)t^2 \qquad \ldots(1)$$

For motion of particle A along the plane, initial velocity is zero and acceleration is $g\sin\alpha$

$$\Rightarrow \quad OP = \frac{1}{2}(g\sin\alpha)t^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$u\cos(\alpha+\beta)t = 0$$

So, either $t = 0$ or $\alpha+\beta = \frac{\pi}{2}$

Thus, the condition for the particles to collide again is

$$\alpha+\beta = \frac{\pi}{2}$$

Single Correct Choice Type Questions

1. $\omega = \frac{(v_\perp)_{rel}}{r_\perp} = \left[\frac{\sqrt{3}\sin(60°)-8\sin(30°)}{2.5}\right]$

$$\Rightarrow \quad \omega = 1 \text{ rads}^{-1}$$

Hence, the correct answer is (D).

2. Since $0 = (v\sin\theta)t + \frac{1}{2}(-a)t^2$

$$\Rightarrow \quad t = \frac{2v\sin\theta}{a}$$

Also, $h = (v\cos\theta)t + \frac{1}{2}gt^2$

$$\Rightarrow \quad h = \frac{2v^2}{a}\sin\theta\left(\cos\theta + \frac{g}{a}\sin\theta\right)$$

Hence, the correct answer is (D).

3. $v_H = \sqrt{\frac{2}{5}}\left(v_{\frac{H}{2}}\right) \qquad \ldots(1)$

Since speed at a point at height h is

$$v = \sqrt{u^2-2gh}$$

Since $H = \frac{u^2\sin^2\theta}{2g}$

$$\Rightarrow \quad v_H = \sqrt{u^2-2gh} = u\cos\theta \text{ and}$$

$$v_{\frac{H}{2}} = \sqrt{u^2-2g\left(\frac{H}{2}\right)} = \sqrt{u^2-gH}$$

$$\Rightarrow \quad v_{\frac{H}{2}} = \sqrt{u^2-\frac{u^2\sin^2\theta}{2}}$$

From (1)

$$v_H^2 = \frac{2}{5}(v)_{\frac{H}{2}}^2$$

$$\Rightarrow \quad u^2\cos^2\theta = \frac{2}{5}\left(u^2 - \frac{u^2\sin^2\theta}{2}\right)$$

$$\Rightarrow \quad 5\cos^2\theta = 2-\sin^2\theta$$

$$\Rightarrow \quad 5-5\sin^2\theta = 2-\sin^2\theta$$

$$\Rightarrow \quad 4\sin^2\theta = 3$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = 60°$$

Hence, the correct answer is (D).

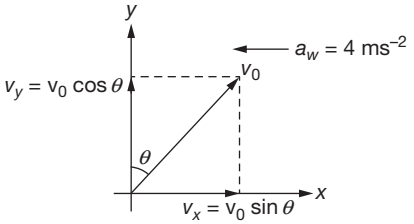
4. If T be the time of flight of the ball, then

$$T = \frac{2v_y}{g} = \frac{2(v_0\cos\theta)}{g}$$

$$\Rightarrow \quad T = \frac{2(20\cos\theta)}{10} = 4\cos\theta \qquad \ldots(1)$$

For the ball to return to the boy's hand, we have $x = 0$.

$$\Rightarrow \quad 0 = v_xT + \frac{1}{2}a_xT^2$$



$$\Rightarrow \quad 0 = (v_0\sin\theta)T + \frac{1}{2}(-4)T^2$$

$$\Rightarrow \quad 0 = 20\sin\theta - 2T$$

$$\Rightarrow \quad T = 10\sin\theta \qquad \ldots(2)$$

From (1) and (2), we get

$$4\cos\theta = 10\sin\theta$$

$$\Rightarrow \quad \cot\theta = 2.5$$

$$\Rightarrow \quad \theta = \cot^{-1}(2.5)$$

Hence, the correct answer is (D).

5. Given $a_N = a_T$

$$\Rightarrow \quad \frac{v^2}{R} = \frac{dv}{dt}$$

$$\Rightarrow \quad v^{-2}dv = \frac{dt}{R}$$

$$\Rightarrow \quad \int_{v_0}^{v} v^{-2}dv = \frac{1}{R}\int_0^t dt$$

$$\Rightarrow \quad \frac{v^{-2+1}}{-2+1}\Big|_{v_0}^{v} = \frac{1}{R}t\Big|_0^t$$

$$\Rightarrow \quad \frac{1}{v_0} - \frac{1}{v} = \frac{t}{R}$$

$$\Rightarrow \quad v = \frac{v_0 R}{R - v_0 t}$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{v_0 R}{R - v_0 t}$$

$$\Rightarrow \quad \int_0^{2\pi R} dr = v_0 R \int_0^t \frac{dt}{R - v_0 t}$$

$$\Rightarrow \quad 2\pi R = v_0 R\left[-\frac{1}{v_0}\log_e\left(R - v_0 t\right)\Big|_0^t\right]$$

$$\Rightarrow \quad 2\pi R = -R\log_e\left(\frac{R - v_0 t}{R}\right)$$

$$\Rightarrow \quad -2\pi = \log_e\left(\frac{R - v_0 t}{R}\right)$$

$$\Rightarrow \quad \frac{R - v_0 t}{R} = e^{-2\pi}$$

$$\Rightarrow \quad t = \frac{R}{v_0}\left(1 - e^{-2\pi}\right)$$

Hence, the correct answer is (D).

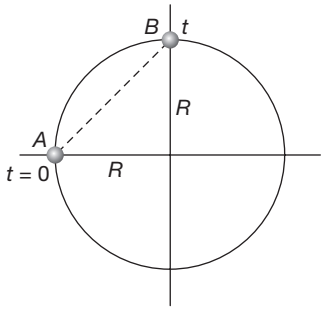
6. $R_{\max} = \dfrac{v^2}{g}$ at $\theta = 45°$

$$\text{Maximum Area} = \pi\left(R_{\max}\right)^2 = \frac{\pi v^4}{g^2}$$

Hence, the correct answer is (B).

7. $\vec{v}_{av} = \dfrac{\Delta\vec{x}}{\Delta t} = \dfrac{\text{Displacement}}{\text{Time lapsed}}$

$$\Rightarrow \quad |\vec{v}_{av}| = \frac{|\text{Displacement}|}{\text{Time lapsed}} = \frac{\sqrt{2}R}{t}$$



where $t = \sqrt{\dfrac{2\theta}{\alpha}}$ $\qquad \left\{\because\ \theta = \dfrac{1}{2}\alpha t^2\right\}$

Since $\theta = \dfrac{\pi}{2}$ radian and $\alpha = \dfrac{\pi}{4}$ rads^{-2}

$$\Rightarrow \quad t = \sqrt{\frac{2\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{4}\right)}} = 2 \text{ s}$$

$$\Rightarrow \quad |\vec{v}_{av}| = \frac{\sqrt{2}R}{2} = \frac{\sqrt{2}\sqrt{2}}{2} = 1 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

8. $R_\phi = R_{90-\phi} = 4\sqrt{h_\phi h_{90-\phi}}$

Hence, the correct answer is (A).

9. Considering the vertical motion, if t be the time taken by the particle to go from A to B, then

$$h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

During this time the particle must make an integral number of rotations (say n).
Now, considering the horizontal motion, then

$$ut = (2\pi r)n$$

$$\Rightarrow \quad n = \frac{u}{2\pi r}\sqrt{\frac{2h}{g}}$$

Hence, the correct answer is (A).

10. $R = (a+b) = \dfrac{u^2\sin(2\alpha)}{g} = \dfrac{u^2}{g}$ $\qquad \left\{\because \alpha = 45^o\right\}$

$$\Rightarrow \quad u^2 = g(a+b) \qquad \ldots(1)$$

Since, $y = a\tan\alpha - \dfrac{ga^2}{2u^2\cos^2\alpha}$

$$\Rightarrow \quad h = a\tan\alpha - \frac{ga^2}{2g(a+b)\cos^2\alpha}$$

$$\Rightarrow \quad h = a\tan 45 - \frac{ga^2}{2g(a+b)\cos^2 45}$$

$$\Rightarrow \quad h = \frac{ab}{a+b}$$

Hence, the correct answer is (D).

CHAPTER 5

11. $y = \dfrac{gx^2}{2u^2}$

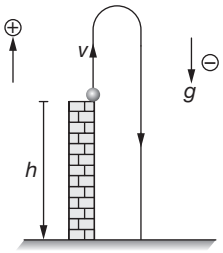
$\Rightarrow \quad 20 - 12 = \dfrac{(9.8)(30)^2}{2u^2}$

$\Rightarrow \quad u^2 = \dfrac{(9.8)900}{8}$

$\Rightarrow \quad u = 23.5 \text{ ms}^{-1}$

Hence, the correct answer is (A).

12. Taking upward direction as $\oplus$, we get



$$-h = vt + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \quad t^2 - \frac{2v}{g}t - \frac{2h}{g} = 0$$

$$\Rightarrow \quad t = \frac{1}{2}\left(\frac{2v}{g} + \sqrt{\frac{4v^2}{g^2} + 4\left(\frac{2h}{g}\right)}\right)$$

$$\Rightarrow \quad t = \frac{1}{2}\left(\frac{2v}{g} + \frac{2v}{g}\sqrt{1 + \frac{2hg}{v^2}}\right)$$

$$\Rightarrow \quad t = \frac{v}{g}\left(1 + \sqrt{1 + \frac{2hg}{v^2}}\right)$$

Hence, the correct answer is (A).

13. At maximum height, radius of curvature is

$$R = \frac{v_T^2}{a_C} = \frac{(u\cos\theta)^2}{g} = \frac{u^2\cos^2\theta}{g} = \frac{u_x^2}{g}$$

$$\Rightarrow \quad R = \frac{\left(72 \times \dfrac{5}{18}\right)^2}{10}$$

$$\Rightarrow \quad R = \frac{400}{10} = 40 \text{ m}$$

Hence, the correct answer is (C).

14. Given $a_y = \dfrac{d^2y}{dt^2} = \alpha$ and $\dfrac{d^2x}{dt^2} = 0$

Since $y = \beta x^2$

$$\Rightarrow \quad \frac{dy}{dt} = 2\beta x\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = 2\beta\left[x\left(\frac{d^2x}{dt^2}\right) + \left(\frac{dx}{dt}\right)^2\right]$$

$$\Rightarrow \quad \alpha = 2\beta v_x^2$$

$$\Rightarrow \quad v_x = \sqrt{\frac{\alpha}{2\beta}}$$

Hence, the correct answer is (B).

15. At maximum height, we have

$$u\cos\theta = \frac{u}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°$$

So, $R = \dfrac{\sqrt{3}u^2}{2g}$

Hence, the correct answer is (C).

16. Since the projectile is at same height at times t_1 and t_2. As studied, the sum of these times equals the time of flight of the projectile. So,

$$T = t_1 + t_2 = 3 + 5 = 8 \text{ s}$$

Hence, the correct answer is (D).

17. $h^2 = r^2 - x^2$

$$\tan\theta = \frac{h}{x}$$

$$h = r\sin\theta$$

$$\Rightarrow \quad x = h\cot\theta$$

$$\Rightarrow \quad \frac{dx}{dt} = -h\,\text{cosec}^2\theta\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \quad \frac{dx}{dt} = -(8000)\,\text{cosec}^2(60)(-0.0.25)$$

$$\Rightarrow \quad \frac{dx}{dt} = (8000)\left(\frac{4}{3}\right)(0.025)$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{800}{3} \text{ ms}^{-1}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{800}{3} \times \frac{18}{5}$$

$$\Rightarrow \quad v = 960 \text{ kmh}^{-1}$$

Hence, the correct answer is (B).

18. Taking upward direction as positive, we get

$$-80 = 12t + \frac{1}{2}(-10)t^2$$

$$\Rightarrow \quad -80 = 12t - 5t^2$$

$$\Rightarrow \quad 5t^2 - 12t - 80 = 0$$

$$\Rightarrow \quad t = \frac{12 \pm \sqrt{144 + 1600}}{2(5)}$$

$$\Rightarrow \quad t \cong 5.5 \text{ s}$$

Hence, the correct answer is (B).

19. $4 = (u\cos\alpha)t$ …(1)

$$2 = \frac{u^2 \sin^2 \alpha}{2g} \qquad \ldots(2)$$

$$-3 = (u\sin\alpha)t - \frac{1}{2}gt^2 \qquad \ldots(3)$$

From (2)

$$u\sin\alpha = 2\sqrt{g} \qquad \ldots(4)$$

Substituting in (3), we get

$$-3 = 6.3t - 4.9t^2$$

$$\Rightarrow \quad 4.9t^2 - 6.3t - 3 = 0$$

$$\Rightarrow \quad t = \frac{6.3 \pm \sqrt{39.7 - 4(4.9)(-3)}}{2(4.9)}$$

$$\Rightarrow \quad t = \frac{6.3 \pm \sqrt{98.5}}{9.8}$$

$$\Rightarrow \quad t = \frac{6.3 + 9.9}{9.8} = 8.1 \text{ s}$$

So, from (1), $4 = (u\cos\alpha)(8.1)$

$$\Rightarrow \quad u\cos\alpha = \frac{4}{8} \cong \frac{1}{2} \qquad \ldots(5)$$

From (4) and (5), we get

$$u^2 \sin^2 \alpha + u^2 \cos^2 \alpha = 40 + \frac{1}{4}$$

$$\Rightarrow \quad u = \sqrt{40.25} = 6.3 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

20. $\tan\theta = \tan\alpha + \tan\beta$

$$\Rightarrow \quad \tan\alpha + \tan\beta = \tan 60$$

$$\Rightarrow \quad \tan\alpha + \tan\beta = \sqrt{3}$$

Hence, the correct answer is (D).

21. The radius of curvature at the instant the projectile has a velocity v making an angle β with the horizontal is

$$R = \frac{v^2}{g\cos\beta}$$

where $\beta = 30°$ and v has to be found. Since horizontal motion is non-accelerated motion, so

$$v_x = u_x$$

$$\Rightarrow \quad v\cos\beta = u\cos\theta$$

$$\Rightarrow \quad v\cos(30°) = 20\cos(60°)$$

$$\Rightarrow \quad v\left(\frac{\sqrt{3}}{2}\right) = 20\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad v = \frac{20}{\sqrt{3}} \text{ ms}^{-1}$$

$$\Rightarrow \quad R = \frac{\left(\frac{20}{\sqrt{3}}\right)^2}{g\cos(30°)} = \frac{\frac{400}{3}}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{80}{3\sqrt{3}} = 15.4 \text{ m}$$

Hence, the correct answer is (C).

22. If T be the time of flight, then

$$T = \frac{2u\sin(\alpha - 30°)}{g\cos(30°)}$$

For the projectile to hit the plane normally, $v_x = 0$

$$\Rightarrow \quad u_x + a_x T = 0$$

$$\Rightarrow \quad u\cos(\alpha - 30°) - g\sin(30°)T = 0$$

$$\Rightarrow \quad u\cos(\alpha - 30°) = g\sin(30°)\left(\frac{2u\sin(\alpha - 30°)}{g\cos(30°)}\right)$$

$$\Rightarrow \quad \tan(\alpha - 30°) = \frac{1}{2}\cot(30°)$$

$$\Rightarrow \quad \tan(\alpha - 30°) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad (\alpha - 30°) = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad \alpha = 30° + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

In general, for the particle to hit the incline normally, we have

$$2\tan(\alpha - \beta) = \cot\beta$$

OR

$$\cot(\alpha - \beta) = 2\tan\beta$$

Hence, the correct answer is (D).

23. $y = 9x^2$

$$\Rightarrow \quad \frac{dy}{dt} = v_y = 18x\frac{dx}{dt} = 18x\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad \frac{dy}{dt} = v_y = 6x$$

CHAPTER 5

$\Rightarrow \quad a_y = \dfrac{d^2y}{dt^2} = \dfrac{dv_y}{dt} = 6\left(\dfrac{dx}{dt}\right) = 6\left(\dfrac{1}{3}\right)$

$\Rightarrow \quad a_y = 2\ ms^{-2}$ along y-axis

Hence $\vec{a}_y = \left(2\ ms^{-2}\right)\hat{j}$

Hence, the correct answer is (D).

24. $\omega_{\text{centre}} = \dfrac{d\theta}{dt}$

$\omega_{\text{circumference}} = \dfrac{d}{dt}\left(\dfrac{\theta}{2}\right) = \dfrac{1}{2}\omega_{\text{centre}}$

$\Rightarrow \quad \omega_{\text{centre}} = 2\omega_{\text{circumference}}$

$\Rightarrow \quad \omega_{\text{centre}} = 4\ \text{rads}^{-1}$

Now $v = R\omega_{\text{centre}} = (2)(4) = 8\ \text{ms}^{-1}$

Hence, the correct answer is (D).

25. Since we know that ranges are equal for complimentary angles i.e., ϕ and $90-\phi$. Also, we know that

$R_\phi = R_{90-\phi} = 4\sqrt{H_\phi H_{90-\phi}}$

$\Rightarrow \quad R_\phi = R_{90-\phi} = 4\sqrt{(4)(16)}$

$\Rightarrow \quad R_\phi = R_{90-\phi} = 32\ \text{m}$

Hence, the correct answer is (D).

26. When the particle is moving in a circular path with uniform speed, then the tangential acceleration is zero. However, radial acceleration is non-zero.
Hence, the correct answer is (C).

27. $y = \alpha x - \beta x^2$

Comparing with standard equation of projectile

$y = x\tan\theta - \dfrac{ax^2}{2u^2\cos^2\theta}$ we get

$\alpha = \tan\theta$ and $\beta = \dfrac{a}{2u^2}\sec^2\theta$

Since $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \quad \beta = \dfrac{a}{2u^2}(1+\alpha^2)$

$\Rightarrow \quad u = \sqrt{\dfrac{a}{2\beta}(1+\alpha^2)}$

Hence, the correct answer is (A).

28. For a horizontal projectile, we have

$y = \dfrac{gx^2}{2u^2}$. So,

$h = \dfrac{g(250)^2}{2v^2}$ …(1)

Similarly,

$4h = \dfrac{gx^2}{2(v/2)^2}$ …(2)

Dividing (1) & (2), we get

$x = 250\ m$

Hence, the correct answer is (A).

29. $x^2 + y^2 = l^2$

$\Rightarrow \quad 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

$\Rightarrow \quad v_b = \dfrac{x}{y}\dfrac{dx}{dt} = 10\ \text{ms}^{-1}$ $\quad (x = y \text{ at } 45^\circ)$

Hence, the correct answer is (D).

30. $b = a\tan\alpha - \dfrac{ga^2}{2u^2\cos^2\alpha}$

and $a = b\tan\alpha - \dfrac{gb^2}{2u^2\cos^2\alpha}$

$\Rightarrow \quad \dfrac{a\tan\alpha - b}{b\tan\alpha - a} = \dfrac{a^2}{b^2}$

$\Rightarrow \quad ab^2\tan\alpha - b^3 = ba^2\tan\alpha - a^3$

$\Rightarrow \quad ab^2\tan\alpha - ba^2\tan\alpha = b^3 - a^3$

$\Rightarrow \quad ab(b-a)\tan\alpha = (b-a)(b^2 + a^2 + ab)$

$\Rightarrow \quad \tan\alpha = \dfrac{b^2 + a^2 + ab}{ab}$

$\Rightarrow \quad \tan\alpha = \dfrac{b^2 + a^2 + 2ab + ab - 2ab}{ab}$

$\Rightarrow \quad \tan\alpha = \dfrac{(b-a)^2 + 3ab}{ab}$

$\Rightarrow \quad \tan\alpha = 3 + \dfrac{(b-a)^2}{ab}$

Since, $(b-a)^2 > 0$

$\Rightarrow \quad \tan\alpha > 3$

$\Rightarrow \quad (\tan\alpha)_{\min} = 3$

Hence, the correct answer is (B).

31. $H_1 = \dfrac{u^2\sin^2\alpha}{2g}$

$H_2 = \dfrac{u^2\sin^2(90-\alpha)}{2g} = \dfrac{u^2\cos^2\alpha}{2g}$

{Because range is same for complimentary angles}

Further $H_1 - H_2 = \dfrac{H_1 + H_2}{2}$

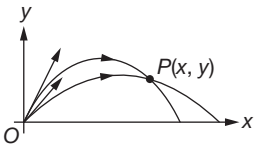
$\Rightarrow \quad H_1 = 3H_2$

$\Rightarrow \quad \tan\alpha = \sqrt{3}$

$\Rightarrow \quad \alpha = 60^o$

Hence, the correct answer is (C).

32.



Let the particles pass through the common point $P(x,y)$ at times t_1 and t_2.

$$u_1 t_1 = u_2 t_2 \qquad \ldots(1)$$

$$v_1 t_1 - \frac{1}{2} g t_1^2 = v_2 t_2 - \frac{1}{2} g t_2^2 \qquad \ldots(2)$$

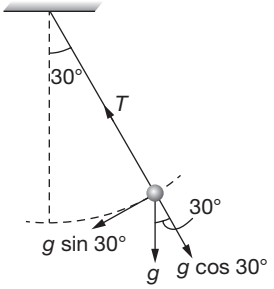
Rearranging and solving equations (1) and (2) to get value of $(t_1 - t_2)$.

$\Rightarrow \quad (t_1 - t_2) = \dfrac{2}{g}\left(\dfrac{v_1 u_2 - v_2 u_1}{u_1 + u_2}\right)$

Hence, the correct answer is (D).

33. Rate of change of speed is

$\dfrac{dv}{dt}$ = tangential acceleration



$\Rightarrow \quad \dfrac{dv}{dt} = g\sin(30°)$

$\Rightarrow \quad \dfrac{dv}{dt} = 10\left(\dfrac{1}{2}\right)\ \text{ms}^{-2} = 5\ \text{ms}^{-2}$

Hence, the correct answer is (B).

34. $T_0 = \dfrac{2u\sin\theta}{g}$ (is time of flight when no drag exists)

$$T = \frac{u\sin\theta}{g + \frac{g}{10}} + \frac{u\sin\theta}{g - \frac{g}{10}}$$

$$T = \frac{u\sin\theta}{g}\left(\frac{10}{11} + \frac{10}{9}\right)$$

$$T = \frac{2u\sin\theta}{g}\left[5\left(\frac{20}{99}\right)\right]$$

$$T = T_0 \frac{100}{99}$$

$\Rightarrow \quad$ Percentage increase $= \left(\dfrac{100}{99} - 1\right) \times 100 = \dfrac{100}{99}\%$

$\Rightarrow \quad$ Percentage increase $\simeq 1\%$

Hence, the correct answer is (A).

35. Coordinates of the point P are $(R, -h)$. Since the point P lies on the trajectory, so $(R, -h)$ must satisfy

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

$\Rightarrow \quad -h = R\tan\theta - \dfrac{gR^2}{2(2ag)}(1 + \tan^2\theta)$

$\Rightarrow \quad R^2\tan^2\theta - 4aR\tan\theta + (R^2 - 4ah) = 0$

For θ to be real, we must have

DISCRIMINANT ≥ 0

$(4aR)^2 - 4R^2(R^2 - 4ah) \geq 0$

$\Rightarrow \quad 4a^2 \geq R^2 - 4ah$

$\Rightarrow \quad R^2 \leq 4a(a + h)$

$\Rightarrow \quad R \leq 2\sqrt{a(a+h)}$

$\Rightarrow \quad R_{MAX} = 2\sqrt{a(a+h)}$

Hence, the correct answer is (D).

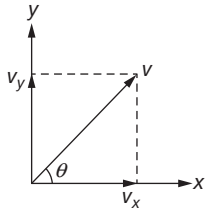
36. $\vec{r} = (2t)\hat{i} + (2t^2)\hat{j}$

$\Rightarrow \quad x = 2t$ and $y = 2t^2$

$\Rightarrow \quad \dfrac{dx}{dt} = v_x = 2$ and $\dfrac{dy}{dt} = v_y = 4t$

Since θ is the angle which $\vec{v} = v_x\hat{i} + v_y\hat{j}$ makes with the positive x-axis, so

$$\tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t \qquad \ldots(1)$$



Differentiating both sides w.r.t. t we get

$$\sec^2\theta\left(\frac{d\theta}{dt}\right)=2$$

$$\Rightarrow \quad \frac{d\theta}{dt}=\frac{2}{\sec^2\theta}=\frac{2}{1+\tan^2\theta}$$

From (1),

$$\tan\theta=2t$$

$$\Rightarrow \quad \frac{d\theta}{dt}=\frac{2}{1+4t^2}$$

So, $\left.\frac{d\theta}{dt}\right|_{t=0.5\text{ s}}=\frac{2}{1+4(0.25)}=1\text{ rads}^{-1}$

Hence, the correct answer is (D).

37. Done in theory.
Hence, the correct answer is (B).

38. **For Q**
Since acceleration due to gravity has no component along AB. So motion of the particle along AB is non-accelerated motion with uniform velocity v. If t_Q is the time taken by particle Q to go from A to B, then

$$t_Q=\frac{AB}{v} \qquad \ldots(1)$$

For P
Since motion of particle P from A to C is accelerated and that from C to B is retarded. So from A to C the horizontal component of velocity gradually increases to attain a maximum value at the lowest point which further decreases gradually to attain the same value v at point B.
That is the motion from A to B to C experiences a horizontal velocity which has a value more than v. So $t_Q > t_P$.

Hence, the correct answer is (A).

39. Tangential acceleration, for a simple pendulum is

$$a_T=g\sin\theta$$

So, at $\theta=30°$, we have

$$a_T=g\sin(30°)=\frac{g}{2}=5\text{ ms}^{-2}$$

Further we know that $a_C=\ell\omega^2$

$$\Rightarrow \quad a_C=\left(\frac{200}{1000}\right)(9)^2$$

$$\Rightarrow \quad a_C=\left(\frac{2}{10}\right)81=16.2\text{ ms}^{-2}$$

Since the total acceleration is

$$a=\sqrt{a_C^2+a_T^2}$$

$$\Rightarrow \quad a=\sqrt{(16.2)^2+(5)^2}$$

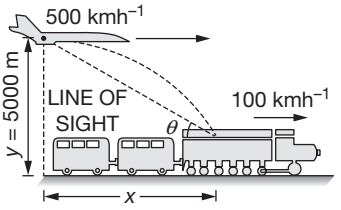
$$\Rightarrow \quad a=17\text{ ms}^{-2}$$

Hence, the correct answer is (D).

40. Velocity of train relative to bomber is u_r, then

$$u_r=500-100=400\text{ kmh}^{-1}$$

$$\Rightarrow \quad u_r=400\times\frac{5}{18}\text{ ms}^{-1}=\frac{1000}{9}\text{ ms}^{-1}$$



Since, $y=\frac{gx^2}{2u_r^2}$

$$\Rightarrow \quad 5000=\frac{gx^2}{2\left(\frac{1000}{9}\right)^2}$$

$$\Rightarrow \quad x=3513.65\text{ m}$$

Further $\tan\theta=\frac{y}{x}$

$$\Rightarrow \quad \tan\theta=\frac{5000}{3513.65}$$

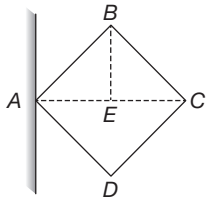
$$\Rightarrow \quad \tan\theta=1.423$$

$$\Rightarrow \quad \theta\simeq 55^o$$

Hence, the correct answer is (A).

41. Let $AC=x$ and $BE=y$. Then

$$BE^2+AE^2=\ell^2$$



$$\Rightarrow \quad y^2+\left(\frac{x}{2}\right)^2=\ell^2$$

$$\Rightarrow \quad 2y\left(\frac{dy}{dt}\right)+\frac{x}{2}\left(\frac{dx}{dt}\right)=0$$

$$\Rightarrow \quad -\frac{dy}{dt}=\frac{1}{2}\left(\frac{x}{2y}\right)\left(\frac{dx}{dt}\right)$$

When the rhombus is a square, then $x = 2y$

$$\Rightarrow \quad v_B=\frac{1}{2}v_c=\frac{v}{2}$$

Hence, the correct answer is (B).

42. $v=\sqrt{v_r^2+v_\theta^2}$

Here $v_r=\frac{dr}{dt}=2\ \text{ms}^{-1}$

Since $v_\theta=r\cdot\frac{d\theta}{dt}=(2\times1)(4)=8\ \text{ms}^{-1}$

$$\Rightarrow \quad v=\sqrt{64+4}=\sqrt{68}=8.25\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

43. $a_T=\frac{v^2-u^2}{2\ell}=\frac{(15)^2-(5)^2}{2(200)}=0.5\ \text{ms}^{-2}$

$$a_C=\frac{v_{\text{final}}}{r}=\frac{(15)^2}{\left(\frac{400}{\pi}\right)} \qquad \left\{\because\ l=\frac{2\pi r}{4}=200\right\}$$

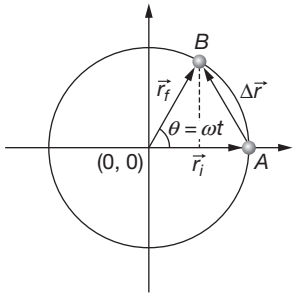
$$\Rightarrow \quad a_C=1.8\ \text{ms}^{-2}$$

$$\Rightarrow \quad a=\sqrt{(1.8)^2+(0.5)^2}$$

$$\Rightarrow \quad a=1.9\ \text{ms}^{-2}$$

Hence, the correct answer is (C).

44. For a particle moving in a circle with constant angular velocity ω, the figure shows the discussion at time t.



Initial position vector of the particle is

$$\vec{r}_i=a\hat{i}$$

Final position vector of the particle at time t is

$$\vec{r}_f=(a\cos\theta)\hat{i}+(a\sin\theta)\hat{j}$$

So, displacement of the particle is

$$\overrightarrow{AB}=\Delta\vec{r}=\vec{r}_f-\vec{r}_i$$

$$\Rightarrow \quad \Delta\vec{r}=(a\cos\theta-a)\hat{i}+(a\sin\theta)\hat{j}$$

$$\Rightarrow \quad |\Delta\vec{r}|=a\sqrt{(\cos\theta-1)^2+\sin^2\theta}$$

$$\Rightarrow \quad |\Delta\vec{r}|=a\sqrt{\cos^2\theta+1-2\cos\theta+\sin^2\theta}$$

Since $\cos^2\theta+\sin^2\theta=1$

$$\Rightarrow \quad |\Delta\vec{r}|=a\sqrt{1+1-2\cos\theta}=\sqrt{2}a\sqrt{1-\cos\theta}$$

Further $1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)$

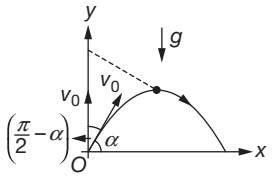
$$\Rightarrow \quad |\Delta\vec{r}|=\sqrt{2}a\sqrt{2\sin^2\left(\frac{\theta}{2}\right)}=2\sin\left(\frac{\theta}{2}\right)$$

Since $\theta=\omega t$

$$\Rightarrow \quad |\Delta\vec{r}|=2a\sin\left(\frac{\omega t}{2}\right)$$

Hence, the correct answer is (D).

45. Since both bodies are moving under influence of gravity hence relative acceleration is zero i.e. both move with constant velocity relative to each other



$\vec{v}_{12}$ = Relative velocity of 1 with respect to 2

$$\vec{v}_{12}=\vec{v}_1-\vec{v}_2$$

$$|\vec{v}_{12}|=\sqrt{v_1^2+v_2^2-2v_1v_2\cos\left(\frac{\pi}{2}-\alpha\right)}$$

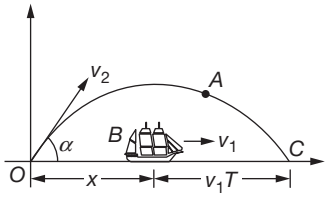
$$|\vec{v}_{12}|=v_0\sqrt{2(1-\sin\alpha)}=2v_0\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$

$$|\vec{r}_{12}|=|\vec{v}_{12}|t$$

$$|\vec{r}_{12}|=2v_0t\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$

Hence, the correct answer is (C).

46.



For shell to hit the boat, time taken by the boat to go from B to C with uniform velocity v_1 equals the time taken by shell to go from O to A to C i.e. $\frac{2v_2\sin\alpha}{g}=T$. Further, we observe

$$x+v_1T=R=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha)$$

$$\Rightarrow\quad x=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha-v_1)$$

Hence, the correct answer is (C).

47. When range is maximum, the $h=\frac{R}{4}$. So $h=100\ m$. Velocity of the projectile is minimum at the maximum height. So this point has coordinates (200, 100)
Hence, the correct answer is (B).

48. $y=x\tan\theta-\frac{gx^2}{2u^2\cos^2\theta}$

$$y=mx-\frac{gx^2}{2u^2}(1+m^2),\ \text{where}\ m=\tan\theta$$

$$\Rightarrow\quad\left(\frac{gx^2}{2u^2}\right)m^2-xm+\left(\frac{gx^2}{2u^2}+y\right)=0\qquad\ldots(1)$$

Equation (1) is a quadratic in m with two roots m_1 and m_2.

These two roots m_1 and m_2 of equation (1) give us the two firing angles for the two trajectories shown (in the statement) such that the projectiles pass through a common point A. This point A will approach the envelope E as the two roots approach equality i.e. the DISCRIMINANT for quadratic in m must equal zero.

$$x^2-4\left(\frac{gx^2}{2u^2}\right)\left(\frac{gx^2}{2u^2}+y\right)=0$$

$$\Rightarrow\quad 1-\left(\frac{2g}{u^2}\right)\left(\frac{gx^2}{2u^2}+y\right)=0$$

$$\Rightarrow\quad \frac{u^2}{2g}-\frac{gx^2}{2u^2}-y=0$$

$$\Rightarrow\quad y=\frac{u^2}{2g}-\frac{gx^2}{2u^2}$$

is the equation of the envelope E.
Hence, the correct answer is (A).

49. For both the particles launched simultaneously to hit the point C, we have

$$t_A=t_B$$

$$\frac{2u\sin\theta}{g}=\sqrt{\frac{2h}{g}}$$

$$\Rightarrow\quad \frac{2(10)(\sin(60°))}{10}=\sqrt{\frac{2h}{10}}$$

$$\Rightarrow\quad 2\frac{\sqrt{3}}{2}=\sqrt{\frac{2h}{10}}$$

$$\Rightarrow\quad \sqrt{3}=\sqrt{\frac{2h}{10}}$$

$$\Rightarrow\quad h=15\ \text{m}$$

Hence, the correct answer is (C).

50. $y=x\tan\alpha-\frac{gx^2}{2u^2\cos^2\alpha}$

For y to be maximum

$$\Rightarrow\quad \frac{dy}{d\alpha}=0$$

$$\Rightarrow\quad \tan\alpha=\frac{u^2}{gx}$$

Hence, the correct answer is (C).

51. $y=x\tan\alpha-\frac{gx^2}{2u^2\cos^2\alpha}$

$$\Rightarrow\quad y=x\tan\alpha-\frac{gx^2}{2u^2}(1+\tan^2\alpha)$$

$$\Rightarrow\quad y_{\max}=\frac{u^2}{2g}-\frac{gx^2}{2u^2}=h\qquad\left\{\because\tan\alpha=\frac{u^2}{gx}\right\}$$

Hence, the correct answer is (D).

52. Motion of one projectile as seen from other is a straight line inclined at acute angle with the horizontal.
Hence, the correct answer is (D).

53. $(u_y)_1=10\sin(30°)=5\ \text{ms}^{-1}$

$$(u_y)_2=\frac{10}{\sqrt{3}}\sin(60°)=5\ \text{ms}^{-1}$$

Since $(u_y)_1=(u_y)_2$ and time of flight is $T=\frac{2u_y}{g}$

$$\Rightarrow\quad T_1=T_2$$

i.e., both the particles strike the ground at the same instant.
So, the maximum separation between them is the difference of their horizontal ranges.

$$\Rightarrow\quad \Delta x=R_1-R_2$$

$$\Rightarrow \quad \Delta x = \frac{(20)^2 \sin(60°)}{g} - \frac{\left(\frac{20}{\sqrt{3}}\right)^2 \sin(120°)}{g}$$

$$\Rightarrow \quad \Delta x = 20\sqrt{3} - \frac{20\sqrt{3}}{3}$$

$$\Rightarrow \quad \Delta x = 20\sqrt{3}\left(1 - \frac{1}{3}\right)$$

$$\Rightarrow \quad \Delta x = \frac{40}{\sqrt{3}} = 23 \text{ m}$$

Hence, the correct answer is (C).

54. Since horizontal components of both are the same, so relative horizontal velocity $(v_{12})_x$ is zero and hence $\vec{v}_{12}$ (velocity of 1 w.r.t. 2) has only a vertical component please note that the angles θ_1 and θ_2 are with the vertical and hence the horizontal component of velocities are $u_1 \sin\theta_1$ and $u_2 \sin\theta_2$ which are actually equal.

Hence, the correct answer is (D).

55. Since $y = \frac{gx^2}{2u^2}$ …(1)

When $x = nb$ and $y = nh$, then

$$n = \frac{2hu^2}{gb^2} \qquad \{\text{Put values in (1)}\}$$

Hence, the correct answer is (B).

56. **METHOD I**

$$y = ax - bx^2$$

Compare with

$$y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

we get

$$\tan\theta = a \text{ and } \frac{g}{2u^2\cos^2\theta} = b$$

$$H_{MAX} = \frac{u^2\sin^2\theta}{2g} = \left(\frac{g}{2b\cos^2\theta}\right)\frac{\sin^2\theta}{2g}$$

$$\Rightarrow \quad H_{MAX} = \frac{\tan^2\theta}{4b} = \frac{a^2}{4b}$$

and $\theta = \tan^{-1}(a)$

METHOD II

At maximum height

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad a - 2bx = 0$$

$$\Rightarrow \quad x = \frac{a}{2b}$$

$$\Rightarrow \quad y_{MAX} = y\Big|_{x=\frac{a}{2b}} = a\left(\frac{a}{2b}\right) - b\left(\frac{a^2}{4b^2}\right)$$

$$\Rightarrow \quad y_{MAX} = \frac{a^2}{4b}$$

Now $\frac{dy}{dx} = a - 2bx$

So, the slope at the point of launch $(x = 0)$ is $\tan\theta$. Hence

$$\frac{dy}{dx}\Big|_{x=0} = a = \tan\theta$$

$$\Rightarrow \quad \theta = \tan^{-1}(a)$$

Hence, the correct answer is (C).

57. $\frac{dx}{dt} = 8\pi\sin(2\pi t)$

$$\Rightarrow \quad dx = 8\pi\sin(2\pi t)\,dt$$

$$\Rightarrow \quad \int_8^x dx = 8\pi\int_0^t \sin(2\pi t)\,dt$$

$$\Rightarrow \quad x - 8 = -8\pi\frac{\cos(2\pi t)}{2\pi}\Big|_0^t$$

$$\Rightarrow \quad x - 8 = 4\cos(2\pi t - 1)$$

$$\Rightarrow \quad x - 8 = 4\cos(2\pi t) - 4$$

$$\Rightarrow \quad x - 4 = 4\cos(2\pi t) \qquad \ldots(1)$$

Further since, $\frac{dy}{dt} = 8\pi\cos(2\pi t)$

$$\Rightarrow \quad dy = 8\pi\cos(2\pi t)\,dt$$

$$\Rightarrow \quad \int_0^y dy = 8\pi\int_0^t \cos(2\pi t)\,dt$$

$$\Rightarrow \quad y = \frac{8\pi\sin(2\pi t)}{2\pi}\Big|_0^t$$

$$\Rightarrow \quad y = 4\sin(2\pi t) \qquad \ldots(2)$$

From (1) and (2), we have

$$(x-4)^2 + y^2 = 4$$

This is the equation of a circle with centre at $(4, 0)$ and radius 2.

Hence, the correct answer is (B).

CHAPTER 5

58. Done Earlier.
Hence, the correct answer is (D).

59. $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{u_y^2}{2g}$

Since u_y and a_y remain the same as previous, so H remains same.

Also, $T = \dfrac{2u\sin\theta}{g} = \dfrac{2u_y}{g}$

So, T also remains the same

Now, $R_{new} = u_x T + \dfrac{1}{2}a_x T^2$

$\Rightarrow \quad R_{new} = R + \dfrac{1}{2}\left(\dfrac{g}{2}\right)\left(\dfrac{4u_y^2}{g^2}\right) = R + \dfrac{H}{2}$

Hence, the correct answer is (B).

60. Since $\dfrac{dx}{dt} = \dfrac{dy}{dt} = c$

$\Rightarrow \quad \dfrac{d^2x}{dt^2} = \dfrac{d^2y}{dt^2} = 0$

Further $z = ax^3 + by^2$

$\Rightarrow \quad \dfrac{dz}{dt} = 3ax^2\dfrac{dx}{dt} + 2by\dfrac{dy}{dt}$

$\Rightarrow \quad \dfrac{dz}{dt} = 3acx^2 + 2bcy \qquad \left\{\because \dfrac{dx}{dt} = c = \dfrac{dy}{dt}\right\}$

$\Rightarrow \quad \dfrac{d^2z}{dt^2} = 6acx\left(\dfrac{dx}{dt}\right) + 2bc\left(\dfrac{dy}{dt}\right)$

$\Rightarrow \quad \dfrac{d^2z}{dt^2} = 6ac^2x + 2bc^2$

Now acceleration of particle is

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$\Rightarrow \quad \vec{a} = \left(6ac^2x + 2bc^2\right)\hat{k}$

Hence, the correct answer is (B).

61. Since, $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow \quad b = a\tan\alpha - \dfrac{ga^2}{2u^2\cos^2\alpha}$

$\Rightarrow \quad a\tan\alpha - b = \dfrac{ga^2}{2u^2\cos^2\alpha}$

$\Rightarrow \quad \dfrac{u^2}{g} = \dfrac{a^2\sec^2\alpha}{2(a\tan\alpha - b)} \qquad \ldots(1)$

Further, maximum height h is

$$h = \frac{u^2\sin^2\alpha}{2g}$$

$\Rightarrow \quad h = \dfrac{a^2\sec^2\alpha\sin^2\alpha}{4(a\tan\alpha - b)}$

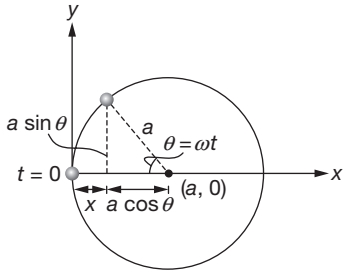
$\Rightarrow \quad h = \dfrac{a^2\tan^2\alpha}{4(a\tan\alpha - b)}$

Hence, the correct answer is (A).

62. In all the cases, we observe that the horizontal component of the velocity of the food packet is same as the horizontal component of the velocity of the aeroplane and due to this, at all the instants, both have the same horizontal displacements.
Hence, the correct answer is (D).

63. $x + a\cos\theta = a$

$\Rightarrow \quad x = a(1 - \cos\theta) = a[1 - \cos(\omega t)]$



and $y = a\sin\theta = a\sin(\omega t)$

Also we observe that at $t = 0$, $|\vec{r}| = a$

Hence, the correct answer is (C).

64. Kinetic energy is minimum at the maximum height. So,

$$K_{MIN} = \frac{1}{2}mu_x^2$$

$$\frac{(K_{MIN})_1}{(K_{MIN})_2} = \frac{(u_x)_1^2}{(u_x)_2^2} = \frac{4}{1}$$

$\Rightarrow \quad \dfrac{(u_x)_1}{(u_x)_2} = 2 \qquad \ldots(1)$

Further, $H = \dfrac{u_y^2}{2g}$

Since, $\dfrac{H_1}{H_2} = \dfrac{9}{1}$

$$\Rightarrow \quad \frac{H_1}{H_2}=\frac{(u_y)_1^2}{(u_y)_2^2}=\frac{9}{1}$$

$$\Rightarrow \quad \frac{(u_y)_1}{(u_y)_2}=\frac{3}{1}$$

Now $R=\frac{2}{g}(u_x)(u_y)$

$$\Rightarrow \quad \frac{R_1}{R_2}=\frac{(u_x)_1}{(u_x)_2}\frac{(u_y)_1}{(u_y)_2}=(2)(3)$$

$$\Rightarrow \quad \frac{R_1}{R_2}=\frac{6}{1}$$

Hence, the correct answer is (C).

65. $\delta=\frac{1}{2}gt^2=\frac{gx^2}{2u^2\cos^2\alpha}$ $\left\{\because\ t=\frac{x}{u\cos\alpha}\right\}$

Hence, the correct answer is (D).

66. $y=\frac{x^2}{2}=\frac{\left(\frac{t^2}{2}\right)^2}{2}=\frac{t^4}{8}$ $\left\{\because\ x=\frac{t^2}{2}\right\}$

$$\Rightarrow \quad \frac{dy}{dt}=\frac{t^3}{2}$$

$$\Rightarrow \quad \left.\frac{dy}{dt}\right|_{t=2s}=4\ \text{ms}^{-1}$$

Similarly, for $x=\frac{t^2}{2}$

$$\frac{dx}{dt}=t$$

$$\Rightarrow \quad \left.\frac{dx}{dt}\right|_{t=2s}=2\ \text{ms}^{-1}$$

$$\Rightarrow \quad \left.\vec{v}\right|_{t=2s}=\left(4\hat{i}+2\hat{j}\right)\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

67. $v_x=\frac{dx}{dt}=8t+4$

$v_y=\frac{dy}{dt}=-3t^2+12$

$a_x=\frac{dv_x}{dt}=8$

$a_y=\frac{dv_y}{dt}=-6t$

$$\Rightarrow \quad \vec{a}=8\hat{i}-(6t)\hat{j}$$

$$\Rightarrow \quad \left.\vec{a}\right|_{t=1\text{ s}}=8\hat{i}-6\hat{j}$$

$$\Rightarrow \quad |\vec{a}|=10\ \text{ms}^{-2}$$

Hence, the correct answer is (B).

68. Since $v^2=v_x^2+v_y^2$

$$\Rightarrow \quad 2v\frac{dv}{dt}=2v_x\frac{dv_x}{dt}+2v_y\frac{dv_y}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt}=\frac{v_xa_x+v_ya_y}{v}=\frac{v_xa_x+v_ya_y}{\sqrt{v_x^2+v_y^2}}$$

$$\Rightarrow \quad \frac{dv}{dt}=\frac{(3)(2)+(4)(1)}{5}=2\ \text{ms}^{-2}$$

Hence, the correct answer is (C).

69. Since horizontal motion is non accelerated, so $u_x=v_x$

$$\Rightarrow \quad u\cos 60=v\cos 30$$

$$\Rightarrow \quad 10\left(\frac{1}{2}\right)=v\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad v=\frac{10}{\sqrt{3}}\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

70. $u_x=v_x$

$$u\sin\theta=v\sin\phi \qquad \{\theta \text{ and } \phi \text{ are with the vertical}\}$$

$$\Rightarrow \quad v=u\sin\theta\,\text{cosec}\,\phi=u\left(\frac{\text{cosec}\,\phi}{\text{cosec}\,\theta}\right)$$

Hence, the correct answer is (D).

71. At $h=0.4$ m, $\vec{v}=6\hat{i}+2\hat{j}$

$$\Rightarrow \quad v_x=6\ \text{ms}^{-1} \text{ and } v_y=2\ \text{ms}^{-1}$$

Since $v_x=u_x$

$$\Rightarrow \quad u_x=6\ \text{ms}^{-1}$$

Also, $v_y^2-u_y^2=2a_yy$

$$\Rightarrow \quad 2^2-u_y^2=2(-g)(0.4)$$

$$\Rightarrow \quad u_y^2=4+8$$

$$\Rightarrow \quad u_y=2\sqrt{3}\ \text{ms}^{-1}$$

If θ is the angle made by the initial velocity with the vertical, then

$$u_x=u\sin\theta=6 \text{ and } u_y=u\cos\theta=2\sqrt{3}$$

CHAPTER 5

$$\Rightarrow \quad \frac{u\sin\theta}{u\cos\theta} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60°$$

Hence, the correct answer is (B).

72. Since we know that $\ell = r\theta$

$$\Rightarrow \quad 100 = r\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad r = \frac{400}{\pi} \text{ m}$$

Since $a = \dfrac{v^2}{r}$

$$\Rightarrow \quad a = \frac{(20)^2}{\left(\frac{400}{\pi}\right)} = \pi \text{ ms}^{-2} \quad \left\{\because v = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}\right\}$$

$$\Rightarrow \quad a = 3.14 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

73. $\tan\beta = \dfrac{u\sin\theta - gt}{u\cos\theta}$

$$\Rightarrow \quad \tan(45°) = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\Rightarrow \quad u\sin\theta - 10 = u\cos\theta \qquad \ldots(1)$$

After three more second it is travelling horizontally i.e., at $t = 4$ s $\beta = 0$

$$\Rightarrow \quad u\sin\theta - g(4) = 0$$

$$\Rightarrow \quad u\sin\theta = 40 \qquad \ldots(2)$$

Substituting in (1), we get

$$u\cos\theta = 30 \qquad \ldots(3)$$

Squaring and adding, we get

$$u = 50 \text{ ms}^{-1}$$

$$\tan\theta = \frac{4}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

Hence, the correct answer is (C).

74. Since we know that

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\Rightarrow \quad y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

For the particle to pass through the point (30 m, 40 m), we have

$$40 = 30\tan\theta - \frac{g(30)^2}{2u^2}(1 + \tan^2\theta)$$

$$\Rightarrow \quad 900\tan^2\theta - 6u^2\tan\theta + (900 + 8u^2) = 0$$

For real values of θ, we must have

$$(6u^2)^2 - 4(900)(900 + 8u^2) \geq 0$$

$$36u^4 - 3600(900 + 8u^2) \geq 0$$

$$\Rightarrow \quad u^4 - 100(900 + 8u^2) \geq 0$$

$$\Rightarrow \quad u^4 - 800u^2 - 90000 \geq 0$$

$$\Rightarrow \quad u^4 - 800u^2 + (400)^2 - (400)^2 - 90000 \geq 0$$

$$\Rightarrow \quad (u^2 - 400)^2 - 250000 \geq 0$$

$$\Rightarrow \quad (u^2 - 400)^2 \geq 250000$$

$$\Rightarrow \quad (u^2 - 400) \geq 500$$

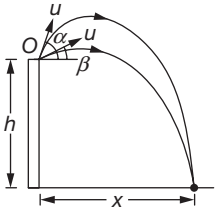
$$\Rightarrow \quad u^2 \geq 900$$

$$\Rightarrow \quad u \geq 30 \text{ ms}^{-1}$$

$$\Rightarrow \quad u_{MIN} = 30 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

75.



$$-h = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \qquad \ldots(1)$$

and $-h = x\tan\beta - \dfrac{gx^2}{2u^2\cos^2\beta}$ $\qquad \ldots(2)$

Equating (1) and (2), we get

$$x = \frac{2u^2}{g}\left(\frac{1}{\tan\alpha + \tan\beta}\right) \qquad \ldots(3)$$

Substituting (3) in (1), we get

$$h = \frac{2u^2}{g}\left(\frac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta}\right)$$

$$\Rightarrow \quad h = \frac{2u^2}{g}\cot(\alpha + \beta)$$

Hence, the correct answer is (D).

76. $y = \frac{1}{2}gt^2$ and $x = v_0 t$

$\Rightarrow \quad y = \frac{1}{2}(10)(3)^2$

$\Rightarrow \quad y = 45 \text{ m}$

Since $\tan(30°) = \frac{y}{x}$

$\Rightarrow \quad x = \frac{y}{\tan(30°)} = 45\sqrt{3} \text{ m}$

Since $x = v_0 t$

$\Rightarrow \quad 45\sqrt{3} = v_0(3)$

$\Rightarrow \quad v_0 = \frac{45\sqrt{3}}{3}$

$\Rightarrow \quad v_0 = 15\sqrt{3} \text{ ms}^{-1}$

$\Rightarrow \quad v_0 = 25.98 \text{ ms}^{-1}$

$\Rightarrow \quad v_0 \cong 26 \text{ ms}^{-1}$

Hence, the correct answer is (C).

77. Given $v_T = 10 \text{ ms}^{-1}$ and $a = 100 \text{ ms}^{-2}$

Also, $a_T = 60 \text{ ms}^{-2}$

Since $a^2 = a_C^2 + a_T^2$

$\Rightarrow \quad (100)^2 = a_C^2 + (60)^2$

$\Rightarrow \quad a_C = 80 \text{ ms}^{-2}$

$\Rightarrow \quad \frac{v_T^2}{r} = 80$

$\Rightarrow \quad r = \frac{(10)^2}{80} = \frac{100}{80}$

$\Rightarrow \quad r = 1.25 \text{ m}$

Hence, the correct answer is (D).

78. $y = 2 \text{ m}$, $x = 3 \text{ m}$

Since $y = \frac{gx^2}{2u^2}$

$\Rightarrow \quad 2 = \frac{(10)9}{2u^2}$

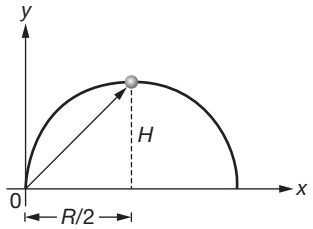
$\Rightarrow \quad u^2 = \frac{90}{4}$

$\Rightarrow \quad u \cong 4.7 \text{ ms}^{-1}$

Hence, the correct answer is (A).

79. Average velocity $= \frac{\text{Displacement}}{\text{Time}}$

$\Rightarrow \quad v_{av} = \frac{\sqrt{H^2 + \frac{R^2}{4}}}{\left(\frac{T}{2}\right)}$



where $H = \frac{u^2 \sin^2\theta}{2g}$

$T = \frac{2u\sin\theta}{g}$

$R = \frac{2u^2 \sin\theta\cos\theta}{g}$

$\Rightarrow \quad v_{av} = \frac{\sqrt{\frac{u^4\sin^4\theta}{4g^2} + \frac{u^4\sin^2\theta\cos^2\theta}{g^2}}}{\frac{u\sin\theta}{g}}$

$\Rightarrow \quad v_{av} = \frac{u}{2}\sqrt{\frac{\sin^4\theta}{\sin^2\theta} + \frac{4\sin^2\theta\cos^2\theta}{\sin^2\theta}}$

$\Rightarrow \quad v_{av} = \frac{u}{2}\sqrt{\sin^2\theta + 4\cos^2\theta}$

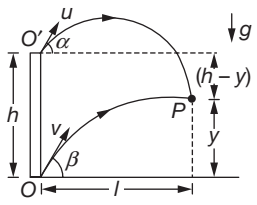
$\Rightarrow \quad v_{av} = \frac{u}{2}\sqrt{\sin^2\theta + \cos^2\theta + 3\cos^2\theta}$

$\Rightarrow \quad v_{av} = \frac{u}{2}\sqrt{1 + 3\cos^2\theta}$

Hence, the correct answer is (C).

80. Let the origin be located at the point O and O'.
For the particles to strike the object placed at P simultaneously

$u\cos\alpha = v\cos\beta$ ...(1)



Also,

$$y = l\tan\beta - \frac{gl^2}{2v^2\cos^2\beta} \qquad \ldots(2)$$

and $-(h-y) = l\tan\alpha - \frac{gl^2}{2u^2\cos^2\alpha}$ $\ldots(3)$

Subtract (3) from (2) and use (1) we get

$$h = l(\tan\beta - \tan\alpha)$$

Hence, the correct answer is (D).

81. $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Here $y = 5$ m, $x = 20$ m, $u = ?$, $\theta = 30°$

$$\Rightarrow \quad 5 = 20\tan(30°) - \frac{(10)(20)^2}{2u^2\left(\frac{3}{4}\right)}$$

$$\Rightarrow \quad 5 = \frac{20}{\sqrt{3}} - \frac{5\times400\times4}{3u^2}$$

$$\Rightarrow \quad \frac{5\times400\times4}{3u^2} = \frac{20}{\sqrt{3}} - 5$$

$$\Rightarrow \quad u^2 = \frac{5\times1600}{3\left(\frac{20}{\sqrt{3}} - 5\right)}$$

$$\Rightarrow \quad u^2 = \frac{5\times1600}{20\sqrt{3} - 15}$$

$$\Rightarrow \quad u^2 = \frac{8000}{19.64}$$

$$\Rightarrow \quad u^2 \cong 400$$

$$\Rightarrow \quad u = 20 \text{ ms}^{-1}$$

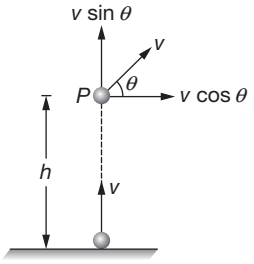
Hence, the correct answer is (A).

82. Relative acceleration between the particles is zero, because both are under influence of gravity.
If s be the relative separation between them at time t, then

$$s^2 = [h - (v - v\sin\theta)t]^2 + [(v\cos\theta)t]^2$$

For s to be MINIMUM or s^2 to be MINIMUM, we have

$$\frac{d}{dt}(s^2) = 0$$



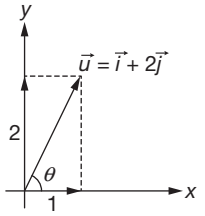
$$\Rightarrow \quad -2[h - (v - v\sin\theta)t][v - v\sin\theta] + 2(v^2\cos^2\theta)t = 0$$

$$\Rightarrow \quad t = \frac{h}{2v}$$

Hence, the correct answer is (D).

83. $\vec{u} = \hat{i} + 2\hat{j}$

$|\vec{u}| = \sqrt{5}$ at an angle $\theta = \tan^{-1}(2)$ with the horizontal



Since $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

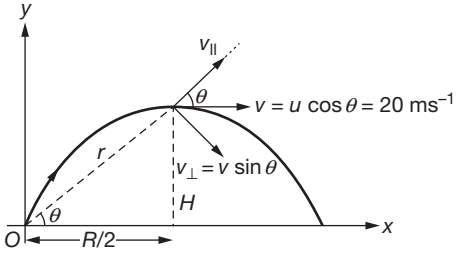
$$\Rightarrow \quad y = x\tan\theta - \frac{gx}{2u^2}(1 + \tan^2\theta)$$

$$\Rightarrow \quad y = 2x - \frac{(10)x^2}{2(5)}(1 + 4)$$

$$\Rightarrow \quad y = 2x - 5x^2$$

Hence, the correct answer is (D).

84. $H = \frac{u^2\sin^2\theta}{2g} = \frac{(20\sqrt{2})^2\sin^2(45°)}{2(10)} = 20$ m



Since at maximum height $x = \frac{R}{2}$, so we get

$$r^2 = \frac{R^2}{4} + H^2$$

where $R = \frac{u^2 \sin(2\theta)}{g} = 80$ m

$\Rightarrow \quad r = 20\sqrt{5}$ m

Now $\omega = \frac{v_\perp}{r} = \frac{v \sin\theta}{r}$

where $\sin\theta = \frac{H}{r}$

$\Rightarrow \quad \omega = \frac{vH}{r^2} = \frac{(20)(20)}{400(5)} = \frac{1}{5} = 0.2$ rads^{-1}

Hence, the correct answer is (B).

85. $x = \frac{2u^2 \sin(\alpha - \beta)\cos\alpha}{g\cos^2\beta}$ where $\alpha = 60°$ and $\beta = 30°$

$\Rightarrow \quad x = \frac{2(30)^2 \sin(30°)\cos(60°)}{g\cos^2(30°)}$

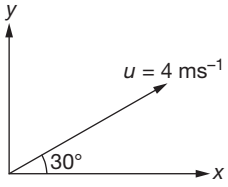
$\Rightarrow \quad x = \frac{(2)(900)(0.5)(0.5)}{(10)\left(\frac{3}{4}\right)} = 60$ m

Hence, the correct answer is (C).

86. Let us first calculate the components of the velocity of ball relative to the lift.

$u_x = 4\cos(30°) = 2\sqrt{3}$ ms^{-1} and

$u_y = 4\sin(30°) = 2$ ms^{-1}



The acceleration of the ball with respect to the lift is $(10 + 2) = 12$ ms^{-2} in the negative y-direction (i.e., vertically downwards)

So, the time of flight T is

$$T = \frac{2u_y}{a_y} = \frac{2(2)}{12} = \frac{1}{3}\text{ s}$$

Hence, the correct answer is (C).

87. Since $R = u_x T$

$\Rightarrow \quad R = (2\sqrt{3})\left(\frac{1}{3}\right) = \frac{2}{\sqrt{3}}$ m

Hence, the correct answer is (B).

88. $x_{\text{AIRCRAFT}} = x_{\text{BULLET}}$

$\Rightarrow \quad (277)t = (540\cos\theta)t$

$\{\because 1000 \text{ kmh}^{-1} = 277 \text{ ms}^{-1}\}$

$\Rightarrow \quad \cos\theta = \frac{277}{540}$

$\Rightarrow \quad \theta \cong 59°$

Just think that $\frac{277}{540} \cong \frac{270}{540} = \frac{1}{2}$, so θ must be an angle close to $60°$.

Hence, the correct answer is (C).

89. $v_x = 8t - 2$

$\Rightarrow \quad \frac{dx}{dt} = 8t - 2$

$\Rightarrow \quad \int_{14}^{x} dx = 8\int_{2}^{t} t\,dt - 2\int_{2}^{t} dt$

$\Rightarrow \quad x - 14 = 4(t^2 - 4) - 2(t - 2)$

$\Rightarrow \quad x - 14 = 4t^2 - 16 - 2t + 4$

$\Rightarrow \quad x = 4t^2 - 2t + 2$ …(1)

Since $v_y = 2$

$\Rightarrow \quad \frac{dy}{dt} = 2$

$\Rightarrow \quad \int_{4}^{y} dy = 2\int_{2}^{t} dt$

$\Rightarrow \quad y - 4 = 2(t - 2)$

$\Rightarrow \quad y = 2t$ …(2)

$\Rightarrow \quad t = \frac{y}{2}$

Substituting in (1), we get

$$x = 4\left(\frac{y}{2}\right)^2 - 2\left(\frac{y}{2}\right) + 2$$

$\Rightarrow \quad x = y^2 - y + 2$

Hence, the correct answer is (A).

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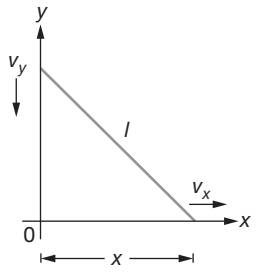
90. The situation discussed is shown at time t. Then we observe that at any instant

$$\ell^2 = x^2 + y^2 \qquad \ldots(1)$$



$$\Rightarrow \quad 0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\Rightarrow \quad \frac{dx}{dt} = -\frac{y}{x}\left(\frac{dy}{dt}\right)$$

From (1), $x = \sqrt{\ell^2 - y^2}$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{yv_0}{\sqrt{\ell^2 - y^2}} \qquad \left\{\because \frac{dy}{dt} = -v_0\right\}$$

So, the speed of the lower end decreases and vanishes when $y \to 0$.

Hence, the correct answer is (B).

91. Initial velocity, $\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

Final velocity, $\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

So, $\Delta\vec{v} = \vec{v} - \vec{u} = -(gt)\hat{j}$

$$\Rightarrow \quad \Delta\vec{v} = -(20)\hat{j}$$

$$\Rightarrow \quad |\Delta\vec{v}| = 20 \text{ ms}^{-1} \text{ along } -y \text{ direction}$$

Hence, the correct answer is (C).

92. To catch the ball, the horizontal relative velocity of the boy relative to the ball must be zero.

$$\Rightarrow \quad v - u\cos(90 - \alpha) = 0$$

$$\Rightarrow \quad v = u\sin\alpha$$

Hence, the correct answer is (B).

93. $\omega = 2000 \text{ rpm} = 2000 \times \frac{2\pi}{60} \text{ rads}^{-1}$

$$r = \frac{5}{2} \text{ m} = 2.5 \text{ m}$$

$$a = r\omega^2 = (2.5)\left(2000 \times \frac{2\pi}{60}\right)^2$$

$$\Rightarrow \quad a = (2.5)\frac{16\pi^2}{36}(10000)$$

$$\Rightarrow \quad a = 1.1 \times 10^5 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

94. $\vec{u} = a\hat{i} + b\hat{j}$ and

$$R = 2H \qquad \ldots(1)$$

Since $\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

$$\Rightarrow \quad u\cos\theta = u_x = a \text{ and}$$

$$u\sin\theta = u_y = b$$

Also from (1), we have

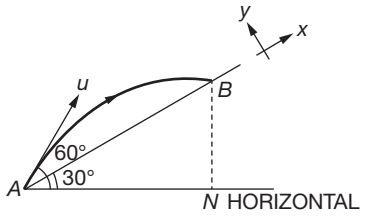
$$\frac{2}{g}(u_x)(u_y) = 2\left(\frac{u_y^2}{2g}\right)$$

$$\Rightarrow \quad 2ab = b^2$$

$$\Rightarrow \quad b = 2a$$

Hence, the correct answer is (C).

95. Time taken by the particle to go from A to N (along horizontal) is also t.



$$AN = (u\cos(60°))t = ut\cos(60°)$$

In ΔABN

$$\cos 30 = \frac{AN}{AB}$$

$$\Rightarrow \quad AB = \frac{AN}{\cos 30} = \frac{ut\cos(60°)}{\cos(30°)}$$

$$\Rightarrow \quad AB = \frac{ut}{\sqrt{3}}$$

Hence, the correct answer is (A).

96. $H = \frac{1}{2}g(2t)^2 = 2gt^2 \qquad \ldots(1)$

Let stone hit the wall at height h, then

$$h = H - \frac{1}{2}gt^2 \qquad \ldots(2)$$

From (1) and (2), we get

$$h = H - \frac{H}{4} = \frac{3H}{4}$$

Hence, the correct answer is (C).

97. $R = u\sqrt{\frac{2h}{g}} = 12\sqrt{\frac{10}{10}} = 12 \text{ m}$

$\Rightarrow \quad S = \sqrt{R^2 + r^2} = 13 \text{ m}$

Hence, the correct answer is (B).

98. For two angles of projection α and β ranges are same, so the angles must be complimentary i.e. $\alpha + \beta = \frac{\pi}{2}$.
Hence, the correct answer is (D).

99. As $\theta < 45°$, body never moves perpendicular to its initial direction of motion. Since $T = \frac{2u\sin\theta}{g}$

$\Rightarrow \quad T\left(=\frac{u}{g}\right) < t\left(=\frac{2u}{g}\right)$, which is impossible.

Hence, the correct answer is (D).

100. Since $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow \quad y = 80 \times \frac{3}{4} - \frac{10 \times 80 \times 80 \times 25}{2 \times 50 \times 50 \times 16} = 40 \text{ m}$

So, distance from point of projection is

$$r = \sqrt{(80)^2 + (40)^2} \text{ m} = 40\sqrt{5} \text{ m}$$

Hence, the correct answer is (C).

103. $\tan(45°) = \frac{20\sin(60°) - 10t}{20\cos(60°)}$

$\Rightarrow \quad 10t = 20\sin(60°) - 20\cos(60°)$

$\Rightarrow \quad t = (\sqrt{3} - 1) \text{ s}$

Hence, the correct answer is (D).

104. Since $y = (u\sin\theta)t - \frac{1}{2}gt^2$

$\Rightarrow \quad 15 = 52\left(\frac{5}{13}\right)t - 5t^2$

$\Rightarrow \quad 15 = 20t - 5t^2$

$\Rightarrow \quad t^2 - 4t + 3 = 0$

$\Rightarrow \quad (t-1)(t-3) = 0$

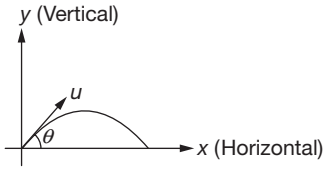
$\Rightarrow \quad t = 1 \text{ s and } t = 3 \text{ s}$

So, time for which the ball is at least 15 m above the ground is

$$\Delta t = 3 - 1 = 2 \text{ s}$$

Hence, the correct answer is (B).

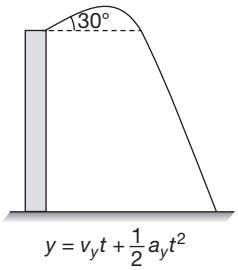
105. According to law of independence of directions motion of a body along three mutually perpendicular directions is independent of each other.



Hence, the correct answer is (D).

106. Since, $y = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow \quad 70 = -(50\sin 30°)t + \frac{1}{2} \times 10 \times t^2$



$\Rightarrow \quad 70 = -25t + 5t^2$

$\Rightarrow \quad t^2 - 5t - 14 = 0$

$\Rightarrow \quad t^2 - 7t + 2t - 14 = 0$

$\Rightarrow \quad (t-7)(t+2) = 0$

$\Rightarrow \quad t = 7 \text{ s}$ (as negative time is not possible)

Hence, the correct answer is (C).

107. Standard equation of projectile is

$$y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta}x^2 \qquad \ldots(1)$$

and the given equation is

$$y = \sqrt{3}x - \frac{g}{2}x^2 \qquad \ldots(2)$$

Comparing (1) and (2), we get

$$\tan\theta = \sqrt{3}$$

$\Rightarrow \quad \theta = 60°$

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and $2u^2\cos^2\theta = 2$

$\Rightarrow \quad u = \dfrac{1}{\cos\theta} = \dfrac{1}{\cos 60^\circ} = 2 \text{ ms}^{-1}$

Hence, the correct answer is (B).

108. $\dfrac{u_x}{u_y} = \cot(30^\circ) = \sqrt{3}$

$\Rightarrow \quad u_x = 80\sqrt{3} \text{ ms}^{-1}$

Since, $T = \dfrac{2u_y}{g}$

$\Rightarrow \quad T = 16 \text{ s}$

At $t = \dfrac{T}{4} = 4 \text{ s}$, $v_x = 80\sqrt{3} \text{ ms}^{-1}$

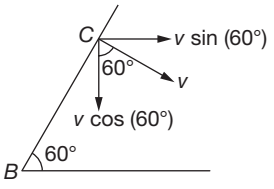
Since, $v_y = 80 - 10 \times 4 = 40 \text{ ms}^{-1}$

$\Rightarrow \quad v = \sqrt{(80\sqrt{3})^2 + (40)^2} = 140 \text{ ms}^{-1}$

Hence, the correct answer is (C).

109. Let v be the velocity at the time of collision.

Then, $u\sqrt{2}\cos(45^\circ) = v\sin(60^\circ)$



$\Rightarrow \quad (u\sqrt{2})\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\sqrt{3}v}{2}$

$\Rightarrow \quad v = \dfrac{2}{\sqrt{3}}u$

Hence, the correct answer is (C).

110. $T = \dfrac{2 \times 20\sin(37^\circ)}{10} = \dfrac{12}{5} \text{ s}$

Range is given by $R = (20\cos 37^\circ + 10)T$

$\Rightarrow \quad R = \dfrac{12}{5} \times \left(20 \times \dfrac{4}{5} + 10\right) = 26 \times \dfrac{12}{5} = 62.4 \text{ m}$

Hence, the correct answer is (D).

111. $H = \dfrac{u^2\cos^2\beta}{2g}$

$\Rightarrow \quad u\cos\beta = \sqrt{2gH}$

So, $t = \dfrac{T}{2} = \dfrac{u\cos\beta}{g} = \sqrt{\dfrac{2H}{g}}$

Hence, the correct answer is (B).

112. Since, time of flight does not depend on horizontal acceleration, so

$$T = \frac{2u\sin\theta}{g} = \frac{2(20)\left(\frac{1}{\sqrt{2}}\right)}{10} = 2\sqrt{2} \text{ s}$$

Since $x = u_x t + \dfrac{1}{2}a_x t^2$

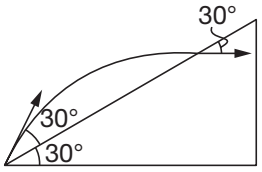
$\Rightarrow \quad R = u_x T + \dfrac{1}{2}a_x T^2$

$\Rightarrow \quad R = (20\cos 45^\circ)(2\sqrt{2}) + \dfrac{1}{2}(2)(2\sqrt{2})^2$

$\Rightarrow \quad R = 40 + 8 = 48 \text{ m}$

Hence, the correct answer is (B).

113.



Since, particle hits the inclined plane at an angle of 30°, which means that the particle hits the plane horizontally. So, this is the highest point of trajectory.

$\Rightarrow \quad t = \dfrac{T}{2} = \dfrac{u\sin\theta}{g} = \dfrac{20\sin(60^\circ)}{g}$

Hence, the correct answer is (A).

114. Since, $v_x = \dfrac{dx}{dt} = 2 \text{ ms}^{-1}$

$\Rightarrow \quad x = 2t \qquad \ldots(1)$

Also, $v_y = \dfrac{dy}{dt} = 5x$

$\Rightarrow \quad dy = x\,dt = 5(2t)\,dt \qquad \{\text{from (1)}\}$

$\Rightarrow \quad dy = 10t\,dt$

$\Rightarrow \quad y = \dfrac{10t^2}{2} = 5t^2 \qquad \ldots(2)$

From (1) and (2), we get

$$y = 5\left(\frac{x}{2}\right)^2$$

$\Rightarrow \quad y = \dfrac{5}{4}x^2$

Hence, the correct answer is (A).

115. Since, $\frac{u^2\sin^2\theta}{2g} = 5$ m

$\Rightarrow \quad \frac{(20)^2\sin^2\theta}{20} = 5$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 30°$

So, $R = \frac{u^2\sin(2\theta)}{g} = \frac{(20)^2\sin(60°)}{10} = 20\sqrt{3}$

Hence, the correct answer is (D).

117. Since, $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow \quad h = d\tan\theta - \frac{gd^2}{2u^2\cos^2\theta}$

$\Rightarrow \quad \frac{gd^2}{2u^2\cos^2\theta} = (d\tan\theta - h)$

$\Rightarrow \quad u^2 = \frac{gd^2}{2\cos^2\theta(d\tan\theta - h)}$

$\Rightarrow \quad u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

Hence, the correct answer is (B).

118. Apply formula of time of flight on inclined plane.

Hence, the correct answer is (B).

119. $T = \frac{2u\sin\theta}{g}$

$\Rightarrow \quad R = \frac{u^2\sin(2\theta)}{g} - \frac{2u^2\sin^2\theta}{g}$

R will be maximum when, $\frac{dR}{d\theta} = 0$

$\Rightarrow \quad 2\cos(2\theta) - 2\sin(2\theta) = 0$

$\Rightarrow \quad \tan(2\theta) = 1$

$\Rightarrow \quad 2\theta = \frac{\pi}{4}$

$\Rightarrow \quad \theta = \frac{\pi}{8}$ radian

Hence, the correct answer is (B).

120. $u = 20\ \text{ms}^{-1}$, $\theta = 45°$

Given $h = Ax - Bx^2$

Comparing with, $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

We get

$A = \tan\theta = \tan 45° = 1$

and $B = \frac{b}{2u^2\cos^2\theta} = \frac{10}{2(20)^2\left(\frac{1}{2}\right)} = \frac{1}{40}$

$\Rightarrow \quad \frac{A}{B} = \frac{40}{1}$

Hence, the correct answer is (D).

121. $v_x = \frac{dx}{dt} = 2 + 8t$

$\Rightarrow \quad a_x = \frac{dv_x}{dt} = 8$

$v_y = \frac{dy}{dt} = 5$

$\Rightarrow \quad a_y = \frac{dv_y}{dt} = 0$

Hence, the correct answer is (C).

122. $v_x = \frac{dx}{dt} = v_0$

$\Rightarrow \quad x = v_0 t$

Since, $v_y = \frac{dy}{dt} = a\omega\cos(\omega t)$

$\Rightarrow \quad y = \int_0^t a\omega\cos(\omega t)\,dt = \frac{a\omega\sin(\omega t)}{\omega}$

$\Rightarrow \quad y = a\sin(\omega t) = a\sin\left(\frac{\omega x}{v_0}\right)$

Hence, the correct answer is (C).

123. $\frac{\frac{1}{2}mu_1^2\cos^2\theta_1}{\frac{1}{2}mu_2^2\cos^2\theta_2} = \frac{4}{1}$

$\Rightarrow \quad \frac{u_1\cos\theta_1}{u_2\cos\theta_2} = 2$ …(1)

and $\frac{u_1^2\sin^2\theta_1}{u_2^2\sin^2\theta_2} = \frac{4}{1}$

$\Rightarrow \quad \frac{u_1\sin\theta_1}{u_2\sin\theta_2} = \frac{2}{1}$ …(2)

From equations (1) and (2), we get

$\frac{(u_1\sin\theta_1)(u_1\cos\theta_1)}{(u_2\sin\theta_2)(u_2\cos\theta_2)} = \frac{4}{1}$

CHAPTER 5

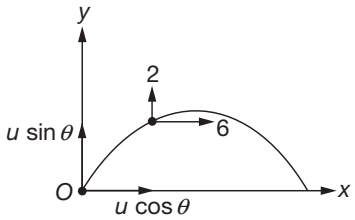
$$\Rightarrow \quad \frac{\frac{gR_1}{2}}{\frac{gR_2}{2}} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{4}{1}$$

Hence, the correct answer is (B).

124. $2 = \sqrt{u^2 \sin^2\theta - 2hg}$

$$\Rightarrow \quad 4 = u^2 \sin^2\theta - 2(0.4)(10)$$



$$\Rightarrow \quad u^2 \sin^2\theta = 12$$

$$\Rightarrow \quad u\sin\theta = 2\sqrt{3} \text{ and } u\cos\theta = 6$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30°$$

Hence, the correct answer is (C).

125. $u_x = 14.7 \text{ ms}^{-1}$,

$$u_y = 9.8 \times 2 = 19.6 \text{ ms}^{-1}$$

$$\Rightarrow \quad u = \sqrt{(14.7)^2 + (19.6)^2} = 24.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad \tan\theta = \frac{19.6}{14.7} = \frac{4}{3}$$

$$\Rightarrow \quad \theta = 53°$$

Hence, the correct answer is (D).

126. Since, both projectiles have equal height, so

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

So, time of flight of both will be same.

$$\Rightarrow \quad T_1 = T_2 = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$

Since Range is greater for second, i.e.

$$R_2 > R_1$$

Thus $(u_x)_2 > (u_x)_1$

$$\Rightarrow \quad u_2 > u_1$$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. Acceleration of the bead down the wire is $g\cos\theta$.

Hence $v^2 = 0^2 + 2(g\cos\theta)(2R\cos\theta)$

Where $P_1P_2 = 2R\cos\theta$

$$\Rightarrow \quad v = 2\left(\sqrt{gR}\right)\cos\theta$$

Since, $t = \frac{v}{a} = \frac{2\sqrt{gR}\cos\theta}{g\cos\theta} = 2\sqrt{\frac{R}{g}}$

Hence, (B), (C) and (D) are correct.

2. $$y = x\tan\theta\left(1 - \frac{x}{R}\right) \qquad \ldots(1)$$

where $x = (u\cos\theta)t$ and $R = (u\cos\theta)T$ …(2)

Also, we observe that

$$R = \frac{2u^2 \sin\theta\cos\theta}{g} \text{ and } H = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow \quad \frac{H}{R} = \frac{\sin^2\theta}{4\sin\theta\cos\theta} = \frac{1}{4}\tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{4H}{R} \qquad \ldots(3)$$

So, we get

$$y = \frac{4Hx}{R}\left(1 - \frac{x}{R}\right)$$

Substitute x and R from (2), we get

$$y = 4H\left(\frac{t}{T}\right)\left(1 - \frac{t}{T}\right)$$

Hence, (A) and (C) are correct.

3. $$\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j} \qquad \ldots(1)$$

Let the velocity at point Q, at time t be $\vec{v}$, then

$$\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

Since $\vec{v} \perp \vec{u}$, so

$$\vec{v} \cdot \vec{u} = 0$$

$$\Rightarrow \quad u^2 - ugt\sin\theta = 0$$

$$\Rightarrow \quad t = \frac{u}{g\sin\theta} = \left(\frac{u}{g}\right)\text{cosec}\,\theta$$

Speed of the particle at Q is $|\vec{v}|$, where

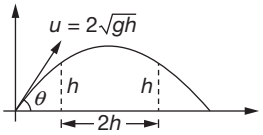
$$|\vec{v}| = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

Substituting the value of t, we get

$$|\vec{v}| = u\cot\theta$$

Hence, (C) and (D) are correct.

4.



$$h = 2\sqrt{hg}\,(\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad t^2 - 4\sqrt{\frac{h}{g}}(\sin\theta)t + \frac{2h}{g} = 0$$

$$\Rightarrow \quad t = \frac{4\sqrt{\frac{h}{g}} \pm \sqrt{16\left(\frac{h}{g}\right)\sin^2\theta - \frac{8h}{g}}}{2}$$

$$\Rightarrow \quad t = \frac{4\sqrt{\frac{h}{g}} \pm \sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}}{2}$$

$$\Rightarrow \quad t_1 = \frac{4\sqrt{\frac{h}{g}} - \sqrt{\frac{8h}{g}}\sqrt{2sin^2\theta - 1}}{2}$$

$$\Rightarrow \quad t_2 = \frac{4\sqrt{\frac{h}{g}} + \sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}}{2}$$

$$\Rightarrow \quad (t_2 - t_1) = \sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}$$

Further horizontal distance travelled during this time is $2h$

$$\Rightarrow \quad 2h = 2\sqrt{hg}\cos\theta(t_2 - t_1)$$

$$\Rightarrow \quad 2h = 2\sqrt{hg}\cos\theta\sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}$$

Solving we get $\cos\theta = \frac{1}{2}$

$\theta = 60°$ with horizontal or $30°$ with vertical

$$\Rightarrow \quad (t_2 - t_1) = 2\sqrt{\frac{h}{g}}$$

Hence, (A) and (D) are correct.

5. $\vec{u} = 3\hat{i}$ and $\vec{a} = -\hat{i} - 0.5\hat{j}$

$$\Rightarrow \quad u_x = 3\ \text{ms}^{-1},\ a_x = -1\ \text{ms}^{-2} \text{ and } a_y = -0.5\ \text{ms}^{-2}$$

Since u_x and a_x have opposite sign, so firstly the particle must reverse its direction of motion. Let this be done when the coordinate of the particle is x. Then

$$0^2 - u_x^2 = 2a_x x$$

$$\Rightarrow \quad -(3)^2 = 2(-1)x$$

$$\Rightarrow \quad x = 4.5\ \text{m}$$

Also, this will happen at say time t, then

$$v_x = u_x + a_x t \text{ gives, } 0 = 3 + (-1)t$$

$$\Rightarrow \quad t = 3\ \text{s}$$

At this time t, the y coordinate of the particle is given by

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad y = 0 + \frac{1}{2}(-0.5)(3)^2$$

$$\Rightarrow \quad y = -\frac{9}{4}m = -2.25\ \text{m}$$

Also, $v_y = 0 + a_y t = -(0.5)(3) = -1.5\ \text{ms}^{-1}$

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = 0\hat{i} + (-1.5)\hat{j} = -1.5\hat{j}\ \text{ms}^{-1}$$

and $\vec{r} = x\hat{i} + y\hat{j} = \left(4.5\hat{i} - 2.25\hat{j}\right)$ m

Hence, (A) and (D) are correct.

7. Since the particles have same ranges.

Hence $\alpha + \beta = \frac{\pi}{2}$

$$R = \frac{2u^2\sin\alpha\cos\alpha}{g} = \frac{2u^2\sin\beta\cos\beta}{g}$$

$$h_1 = \frac{u^2\sin^2\alpha}{2g}$$

$$h_2 = \frac{u^2\sin^2\beta}{2g} = \frac{u^2\cos^2\alpha}{2g}$$

$$\Rightarrow \quad R = 4\sqrt{h_1 h_2}$$

$$t_1 = \frac{2u\sin\alpha}{g}$$

$$t_2 = \frac{2u\sin\beta}{g} = \frac{2u\cos\alpha}{g}$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \tan\alpha \text{ and } \frac{h_1}{h_2} = \tan^2\alpha$$

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$\Rightarrow \quad \tan\alpha = \sqrt{\frac{h_1}{h_2}}$

Hence, (A), (B), (C) and (D) are correct.

8. $x = a\cos(pt)$

$y = b\sin(pt)$

$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow$ Trajectory is an Ellipse

$\Rightarrow \quad \ddot{x} = -ap^2\cos(\omega t)$

$\Rightarrow \quad \ddot{x} = -p^2 x$

Similarly $\ddot{y} = -p^2 y$

so acceleration is directed towards the focus.
Hence, (A), (B) and (C) are correct.

9. $0 = \sqrt{2gh}(\cos\alpha)t - \frac{1}{2}(g\cos\alpha)t^2$

$\Rightarrow \quad t = \sqrt{\frac{8h}{g}}$ after it strikes the incline at A

$s = \sqrt{2gh}(\sin\alpha)t + \frac{1}{2}(g\sin\alpha)t^2$

Substituting value of t, we get $s = 8h\sin\alpha$
Hence, (A) and (D) are correct.

10. $\vec{u} = 4\hat{i} + 3\hat{j} + 3\sqrt{3}\hat{k}$

$\Rightarrow \quad |\vec{u}| = 2\sqrt{13}$ ms^{-1}

Velocity at highest point is $\vec{v} = 4\hat{i} + 3\hat{j}$

{because at the highest point $v_Z = 0$}

$\Rightarrow \quad |\vec{v}| = 5$ ms^{-1}

Hence, (B) and (D) are correct.

11. Let shells collide in mid-air at point $P(x, y)$.

For the 1st shell

$x = (u\cos\alpha)T$ …(1)

$y = (u\sin\alpha)T - \frac{1}{2}gT^2$ …(2)

For the 2nd shell

$x = u\cos\beta(T - t)$ …(3)

$y = (u\sin\beta)(T - t) - \frac{1}{2}g(T - t)^2$ …(4)

Equating (1) and (3), we get

$T\cos\alpha = (T - t)\cos\beta$ {OPTION **(A)**}

$\Rightarrow \quad \frac{\cos\alpha}{\cos\beta} = 1 - \frac{t}{T}$

$\Rightarrow \quad \frac{\cos\alpha}{\cos\beta} < 1$

$\Rightarrow \quad \cos\alpha < \cos\beta$

$\Rightarrow \quad \alpha > \beta$ {OPTION **(B)**}

Equating (2) and (4), we get

$(u\sin\alpha)T - \frac{1}{2}gT^2 = (u\sin\beta)(T - t) - \frac{1}{2}g(T - t)^2$

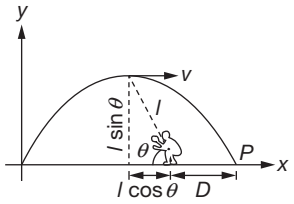
{OPTION **(D)**}

Hence, (A), (B) and (D) are correct.

13. Let the projectile reach the point of impact in time t. Then

$l\sin\theta = \frac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\frac{2\ell\sin\theta}{g}}$ …(1)



Further

$l\cos\theta + D = vt$

$\Rightarrow \quad D = vt - l\cos\theta$

$\Rightarrow \quad D = v\sqrt{\frac{2l\sin\theta}{g}} - l\cos\theta$ …(2)

For projectile to pass over the observer's head

$D > 0$

$\Rightarrow \quad v\sqrt{\frac{2l\sin\theta}{g}} > l\cos\theta$

$\Rightarrow \quad v^2\left(\frac{2l\sin\theta}{g}\right) > l^2\cos^2\theta$

$\Rightarrow \quad l < \frac{2v^2\tan\theta\sec\theta}{g}$

Hence, (B) and (C) are correct.

14. Since, we know that the range is same for complimentary angles i.e., $R_\theta = R_{90-\theta} = \frac{2u^2\sin\theta\cos\theta}{g}$

Then, $T_1 = \frac{2u\sin\theta}{g}$ and $T_2 = \frac{2u\cos\theta}{g}$

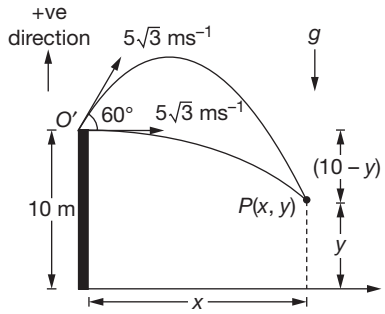
$\Rightarrow \quad T_1T_2 \propto R$ and

$$\frac{T_1}{T_2} = \tan\theta$$

Hence, (C) and (D) are correct.

15. Let the second shell reach the point $P(x, y)$ at time t_2 and the first shell be fired at a time

$$(t_2 - \Delta t) = t_1 .$$



For shell 1

$$x = (5\sqrt{3})t_1 \qquad \ldots(1)$$

$$-(10 - y) = -\frac{1}{2}gt_1^2 \qquad \ldots(2)$$

For shell 2

$$x = (5\sqrt{3}\cos 60)t_2 \qquad \ldots(3)$$

$$-(10 - y) = (5\sqrt{3}\sin 60)t_2 - \frac{1}{2}gt_2^2 \qquad \ldots(4)$$

Equating (1) and (3), we get

$$t_2 = 2t_1 \qquad \ldots(5)$$

Equating (2) and (4), we get

$$-\frac{1}{2}gt_1^2 = (5\sqrt{3}\sin 60)t_2 - \frac{1}{2}gt_2^2$$

$\Rightarrow \quad -\frac{1}{2}gt_1^2 = \frac{15}{2}(2t_1) - \frac{1}{2}g(2t_1)^2$

$\Rightarrow \quad -\frac{1}{2}gt_1^2 = 15t_1 - 2gt_1^2$

$\Rightarrow \quad -5t_1^2 = 15t_1 - 20t_1^2$

$\Rightarrow \quad 15t_1^2 = 15t_1$

$\Rightarrow \quad 15t_1^2 - 15t_1 = 0$

$\Rightarrow \quad 15t_1(t_1 - 1) = 0$

$\Rightarrow \quad t_1 = 1s$

$\Rightarrow \quad t_2 = 2s$ i.e. $\Delta t = 1s$

$\Rightarrow \quad x = 5\sqrt{3}\ m$ and $y = 5\ m$

Hence, (A), (B), (C) and (D) are correct.

16. Since $R = \frac{u^2\sin(2\theta)}{g}$

$\Rightarrow \quad R = kg^{-1}$ …(1)

$\Rightarrow \quad dR = -kg^{-2}dg$ …(2)

$\Rightarrow \quad \frac{dR}{R} = \frac{-kg^{-2}dg}{kg^{-1}}$

$\Rightarrow \quad \frac{dR}{R} = -\frac{dg}{g} = -\left(\frac{g_p - g_e}{g}\right)$

$\Rightarrow \quad$ Fractional Decrement $= \frac{dR}{R} = \frac{1}{291}$

Range at equator is $R_e = \frac{u^2\sin(2\theta)}{g_e}$

Range at pole is $R_p = \frac{u^2\sin(2\theta)}{g_p}$

$$R = \left(\begin{array}{c}\text{Original range with}\\ \text{no variation of } g\\ \text{taken into account}\end{array}\right) = \frac{u^2\sin(2\theta)}{g}$$

$$\frac{dR}{R} = \left(\begin{array}{c}\text{Fractional}\\ \text{Decrement}\end{array}\right) = \frac{R_e - R_p}{R}$$

$\Rightarrow \quad \frac{dR}{R} = \frac{u^2\sin(2\theta)\left(\frac{1}{g_e} - \frac{1}{g_p}\right)}{u^2\sin(2\theta)\frac{1}{g}}$

$\Rightarrow \quad \frac{dR}{R} = -g\left(\frac{1}{g_p} - \frac{1}{g_e}\right)$

Hence, (B) and (D) are correct.

17. For shell to hit the boat the time lapsed equals the time of flight of the shell $= \frac{2v_2\sin\alpha}{g}$

$\Rightarrow \quad$ Distance travelled by boat $= v_1\left(\frac{2v_2\sin\alpha}{g}\right)$

Distance of boat from the cannon at the instant the shell is fired is $x = R - \left(\frac{2v_1v_2\sin\alpha}{g}\right)$

CHAPTER 5

$$\Rightarrow \quad x=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha)-\frac{2v_1v_2\sin\alpha}{g}$$

$$\Rightarrow \quad x=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha-v_1)$$

and the distance of boat from the cannon when the shell hits the boat is equal to the range of the shell

$$R=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha)$$

Hence, (A), (B), (C) and (D) are correct.

18. $u_x=u\cos(45°)=40\ \text{ms}^{-1}$ and

$u_y=u\sin(45°)=40\ \text{ms}^{-1}$

At $t=2$ s,

$$v_x=u_x=40\ \text{ms}^{-1} \qquad \ldots(1)$$

$$v_y=u_y-gt=40-(10)(2)=20\ \text{ms}^{-1}$$

So, (B) is correct

Further $x=u_xt=80$ m

and $y=u_yt+\frac{1}{2}a_yt^2$

$$\Rightarrow \quad y=(40)(2)+\frac{1}{2}(-10)(4)$$

$$\Rightarrow \quad y=60\ \text{m}$$

Hence (D) is correct

So, displacement of the particle is $r=\sqrt{x^2+y^2}$

$$\Rightarrow \quad r=\sqrt{(80)^2+(60)^2}=100\ \text{m}$$

Hence (A) is also correct.

Also, we observe that if α is the angle made by v with the horizontal, then

$$\tan\alpha=\frac{v_y}{v_x}=\frac{20}{40}=\frac{1}{2}$$

Hence (C) is not correct.

Hence, (A), (B) and (D) are correct.

19. $x_0+ut=vt$

$$\Rightarrow \quad (6\hat{i}+8\hat{j})+(2t)\hat{i}=(a\hat{i}+b\hat{j})t$$

At $t=2$ s, we get

$$\Rightarrow \quad 10\hat{i}+8\hat{j}=2a\hat{i}+2b\hat{j}$$

$$\Rightarrow \quad 2a=10 \text{ and } 2b=8$$

$$\Rightarrow \quad a=5 \text{ and } b=4$$

Hence, (B) and (C) are correct.

20. $y=h-x\tan\alpha \qquad \ldots(1)$

Further $y=(v_0\sin\theta)t-\frac{1}{2}gt^2 \qquad \ldots(2)$

and $x=(v_0\cos\theta)t \qquad \ldots(3)$

Equating (1) and (2) and put value of x from (3) to get

$$\Rightarrow \quad t^2-\frac{2v_0}{g}(\cos\theta\tan\alpha+\sin\theta)t+\frac{2h}{g}=0$$

$$\Rightarrow \quad t^2-\left[\frac{2v_0\sin(\theta+\alpha)}{g\cos\alpha}\right]t+\frac{2h}{g}=0 \qquad \ldots(4)$$

For t to be MINIMUM

$$\frac{dt}{d\theta}=0$$

$$\Rightarrow \quad 2t\frac{dt}{d\theta}-\frac{2v_0}{g\cos\alpha}\frac{d}{d\theta}[t\sin(\alpha+\theta)]+0=0$$

$$\Rightarrow \quad 0-\frac{2v_0}{g\cos\alpha}\left[\sin(\alpha+\theta)\frac{dt}{d\theta}+t\cos(\alpha+\theta)\right]=0$$

$$\Rightarrow \quad 0-\frac{2v_0t\cos(\alpha+\theta)}{g\cos\alpha}=0 \qquad \left\{\because \frac{dt}{d\theta}=0\right\}$$

$$\Rightarrow \quad \cos(\theta+\alpha)=0 \qquad \{\because t\neq 0\}$$

$$\Rightarrow \quad \theta+\alpha=\frac{\pi}{2} \qquad \{\text{OPTION } \textbf{(A)}\}$$

Substituting $\theta+\alpha=\frac{\pi}{2}$ in (4), we get

$$t_{\min}^2-\frac{2v_0}{g\cos\alpha}t_{\min}+\frac{2h}{g}=0$$

$$\Rightarrow \quad t_{\min}=\frac{v_0-\sqrt{v_0^2-2gh\cos^2\alpha}}{g\cos\alpha} \qquad \{\text{OPTION } \textbf{(C)}\}$$

For $t_{\min}$ to be real $v_0^2>2gh\cos^2\alpha$

$$\Rightarrow \quad v_0>\sqrt{2gh}\cos\alpha$$

So if $v_0<\sqrt{2gh}\cos\alpha$, then projectile never reaches the roof. $\{\text{OPTION } \textbf{(D)}\}$

Hence, (A), (C) and (D) are correct.

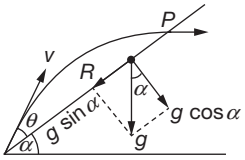
21. As studied and discussed in theory, we observe that all (A), (B), (C) and (D) are correct.

Hence, (A), (B), (C) and (D) are correct.

22. The vertical velocity at P is zero.

Hence $v\sin(\theta+\alpha)-gt=0$

$$\Rightarrow \quad t=\frac{v\sin(\theta+\alpha)}{g} \qquad \ldots(1)$$



Since displacement perpendicular to the plane is zero hence

$$0 = (v\sin\theta)t + \frac{1}{2}(-g\cos\alpha)t^2$$

$$\Rightarrow \quad t = \frac{2v\sin\theta}{g\cos\alpha} \qquad \ldots(2)$$

Equating (1) & (2) and rearranging, we get

$$\frac{v\sin(\theta+\alpha)}{g} = \frac{2v\sin\theta}{g\cos\alpha}$$

$$\Rightarrow \quad 2\sin\theta = \cos\alpha(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$

$$\Rightarrow \quad 2\sin\theta = \cos^2\alpha\sin\theta + \cos\theta\sin\alpha\cos\alpha$$

$$\Rightarrow \quad (2-\cos^2\alpha)\sin\theta = \cos\theta\sin\alpha\cos\alpha$$

$$\Rightarrow \quad \tan\theta = \frac{\sin\alpha\cos\alpha}{2-\cos^2\alpha}$$

$$\Rightarrow \quad \tan\theta = \frac{\sin\alpha\cos\alpha}{(1-\cos^2\alpha)+1} = \frac{\sin\alpha\cos\alpha}{1+\sin^2\alpha}$$

$$\Rightarrow \quad \tan\theta = \frac{\tan\alpha}{1+2\tan^2\alpha}$$

Hence, (A) and (D) are correct.

23. Time of ascent $= 2+1 = 3$ s

If α is the angle of projection then

$$\frac{u\sin\alpha}{g} = 3$$

$$\Rightarrow \quad u\sin\alpha = 30 \qquad \ldots(1)$$

If $\beta = 30°$ is the angle which projectile makes with horizontal at $t = 2$ s and v is the velocity at this instant of time then

$$v\cos\beta = u\cos\alpha \qquad \ldots(2)$$

and $v\sin\beta = u\sin\alpha - g(2)$

$$\Rightarrow \quad v\sin\beta = 30 - 20 = 10$$

$$\Rightarrow \quad v\sin 30 = 10$$

$$\Rightarrow \quad v = 20 \text{ ms}^{-1}$$

Substituting $\beta = 30°$ and $v = 20$ ms^{-1} in (2), we get

$$v\frac{\sqrt{3}}{2} = u\cos\alpha$$

$$u\cos\alpha = 10\sqrt{3} \qquad \ldots(3)$$

Squaring (1) & (3) and adding

$$u^2 = 1200$$

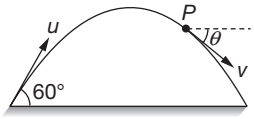
$$\Rightarrow \quad u = 20\sqrt{3} \text{ ms}^{-1}$$

and $\tan\alpha = \sqrt{3}$

$$\Rightarrow \quad \alpha = 60° \text{ with horizontal or } 30° \text{ with vertical}$$

Hence, (B) and (C) are correct.

24. At the instant shown, we have



$$v\cos\theta = u\cos(60°) = \frac{u}{2}$$

$$\Rightarrow \quad v = \frac{u}{2\cos\theta} \qquad \ldots(1)$$

The radius of curvature is given by

$$R = \frac{v^2}{g\cos\theta} = \frac{u^2}{4g\cos^3\theta}$$

$$\Rightarrow \quad R_{\min} = \frac{u^2}{4g}, \text{ when } \theta = 0°$$

Since, $R = \frac{8}{3\sqrt{3}}R_{\min}$

$$\Rightarrow \quad \frac{u^2}{4g\cos^3\theta} = \frac{8}{3\sqrt{3}}\left(\frac{u^2}{4g}\right)$$

$$\Rightarrow \quad \cos^3\theta = \frac{3\sqrt{3}}{8} = \left(\frac{\sqrt{3}}{2}\right)^3$$

$$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = 30°$$

So, from equation (1), we get

$$v = \frac{u}{\sqrt{3}}$$

Hence, (B) and (C) are correct.

25. $y = 3x + 4$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{3dx}{dt}$$

$$\Rightarrow \quad v_y = 3v_x = 3 \qquad \{\because v_2 = 1 \text{ ms}^{-1}\}$$

CHAPTER 5

$$\Rightarrow \quad \frac{dy_y}{at} = \frac{3dv_x}{dt}$$

$$\Rightarrow \quad a_y = 3a_x = 3 \qquad \{\because\ a_x = 1\ \text{ms}^{-1}\}$$

nothing can be said about velocity and acceleration as the given values are at particular instant.

Hence, (A) and (B) are correct.

26. $x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$

$$\Rightarrow \quad y = 2 + 2x$$

So, the particle moves along the straight line.

Since, $\frac{dx}{dt} = 2 + 8t$ and $\frac{dy}{dt} = 4 + 16t$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = 8 \text{ and } \frac{d^2y}{dt^2} = 16$$

$$\Rightarrow \quad \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = (2+8)\hat{i} + (4+16t)\hat{j}$$

$$\Rightarrow \quad \vec{a} = 8\,\hat{i} + 16\,\hat{j} = \text{(constant)}$$

Hence, (A) and (B) are correct.

27. $\vec{v} = 2t\,\hat{i} + t^2\,\hat{j}$

$$\Rightarrow \quad \frac{d\vec{v}}{dt} = 2\hat{i} + 2t\,\hat{j}$$

$$\Rightarrow \quad |\vec{a}_{net}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

Since, tangential acceleration is defined as the rate of change of speed, which is

$$|\vec{v}| = \sqrt{(2t)^2 + (t^2)^2} = \sqrt{4t^2 + t^4}$$

So, $a_T = \frac{d|\vec{v}|}{dt} = \frac{1}{2\sqrt{4t^2 + t^4}}(8t + 4t^3)$

$$\Rightarrow \quad a_T = \frac{1}{2 \times \sqrt{5}} \times 12 = \frac{6}{\sqrt{5}}\ \text{ms}^{-2}$$

Since, we know that, for a particle moving in a curvilinear path with variable speed, the particle possesses both tangential acceleration (a_T) and centripetal acceleration (a_C), so we have

$$a_{\text{net}}^2 = a_C^2 + a_T^2$$

$$\Rightarrow \quad a_C^2 = a_{\text{net}}^2 - a_T^2$$

$$\Rightarrow \quad a_C^2 = \left(2\sqrt{2}\right)^2 - \left(\frac{6}{\sqrt{5}}\right)^2 = 8 - \frac{36}{5} = \frac{4}{5}$$

$$\Rightarrow \quad a_C = \frac{2}{\sqrt{5}}\ \text{ms}^{-2}$$

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. $R = H$

$$\Rightarrow \quad \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$\Rightarrow \quad \tan\theta = 4$$

$$R_{\max}\big|_{\theta = 45^\circ} = \frac{u^2}{g}$$

Hence, the correct answer is (B).

2. In case of non-uniform circular motion, net acceleration will not be directed towards centre.

Hence, the correct answer is (D).

3. Direction of acceleration changes.

Hence, the correct answer is (D).

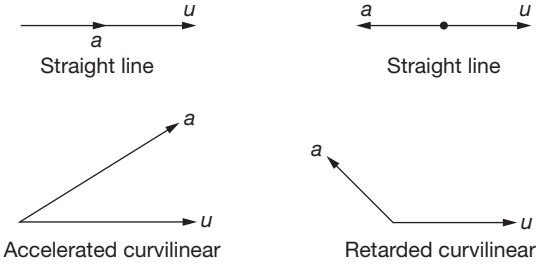
4. The particle will move in a parabolic path till acceleration (due to gravity) is constant. For this the particle should be near the surface of the Earth and air resistance should be negligible.

Hence, the correct answer is (D).

5. In uniform circular motion direction of acceleration is always along the radial direction. As particle is rotating so radial vector keeps on changing.

Hence, the correct answer is (D).

6.



Hence, the correct answer is (A).

7. In uniform circular motion, the magnitude of velocity and acceleration remains same, but due to change in direction of motion, the direction of velocity and acceleration changes. Furthermore, the centripetal acceleration is given by $a = \omega^2 r$.

Hence, the correct answer is (B).

8. For the observer sitting in train, the initial horizontal velocity of coin and train are same, thus observer will find its path as a straight line. Whereas the observer on the ground will see the coin to follow a parabolic path (as of a horizontal projectile).

Hence, the correct answer is (D).

9. $u_x = 0$, $a_x = 0$, $u_y = u\sin\theta$ and $a_y = -g$

$\Rightarrow \quad v_x = u_x + a_x T = u_x \qquad \left\{\text{where } T = \dfrac{2u\sin\theta}{g}\right\}$

Also $v_y = u\sin\theta - gt$

$\Rightarrow \quad v_y = u\sin\theta - g\left(\dfrac{2u\sin\theta}{g}\right)$

$\Rightarrow \quad v_y = -u\sin\theta$

Hence, the correct answer is (A).

10. $H = \dfrac{u^2\sin^2\theta}{2g}$ i.e., it is independent of mass of projectile.

Hence, the correct answer is (A).

11. As $T = \dfrac{2u\sin\theta}{g}$

$\Rightarrow \quad T' = \dfrac{2nu\sin\theta}{g}$

$\Rightarrow \quad T' = nT$

$R = u\cos\theta T$

$\Rightarrow \quad R' = (nu\cos\theta)(nT)$

$\Rightarrow \quad R' = n^2 R$

Hence, the correct answer is (C).

12. Since acceleration due to gravity acts vertically downwards, so it has no component along the horizontal and hence does not affect the horizontal velocity.
Hence, the correct answer is (A).

13. Since, $R_{\max} = \dfrac{u^2}{g} \qquad \{\text{For } \theta = 45°\}$

$\Rightarrow \quad H = \dfrac{u^2\sin^2(45°)}{2g} = \dfrac{u^2}{4g}$

So, H is 25% of range.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. $x = kt$, $y = kt(1-\alpha t)$

Equation of trajectory,

$$y = kt - k\alpha t^2 = k\left(\frac{x}{k}\right) - k\alpha\left(\frac{x^2}{k^2}\right)$$

$\Rightarrow \quad y = x - \alpha\dfrac{x^2}{k} \rightarrow$ parabola

Hence, the correct answer is (C).

2. $v_x = \dfrac{dx}{dt} = k$

$v_y = \dfrac{dy}{dt} = k - 2k\alpha t$

$v_y = k(1-2\alpha t)$

$\Rightarrow \quad \vec{v} = \hat{i}v_x + \hat{j}v_y = \hat{i}k + \hat{j}k(1-2\alpha t)$

$\Rightarrow \quad |\vec{v}| = v = \sqrt{k^2 + k^2(1-2\alpha t)^2}$

$\Rightarrow \quad v = k\sqrt{1+(1-2\alpha t)^2}$

$\Rightarrow \quad v^2 = k^2\left[1+(1-2\alpha t)^2\right]$

For v to be MINIMUM OR v^2 to be MINIMUM, we have

$$\frac{d}{dt}(v^2) = 0$$

$\Rightarrow \quad t = \dfrac{1}{2\alpha}$

Hence, the correct answer is (B).

3. Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \hat{j}(-2\alpha)k = -(2k\alpha)\hat{j} = \text{constant}$$

Hence, the correct answer is (D).

4. Let t_0 be the instant when $\vec{v}$ and $\vec{a}$ make an angle $\dfrac{\pi}{4}$ with each other; the condition is

$$\vec{v}\cdot\vec{a} = va\cos\left(\frac{\pi}{4}\right)$$

$\Rightarrow \quad 0 + k(1-2\alpha t)(-2\alpha k) = k\sqrt{1-(1-2\alpha t)^2}\,(-2\alpha k)\cdot\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \sqrt{2}(1-2\alpha t_0) = \sqrt{1+(1-2\alpha t_0)^2}$

$\Rightarrow \quad t_0 = \dfrac{1}{\alpha}$

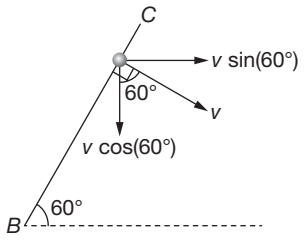
Hence, the correct answer is (A).

5. Let v be the velocity at the time of collision.

Then, $u\sqrt{2}\cos 45° = v\sin 60°$

$\Rightarrow \quad (u\sqrt{2})\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\sqrt{3}v}{2}$

CHAPTER 5



$\Rightarrow \quad v = \dfrac{2}{\sqrt{3}}u$

Hence, the correct answer is (D).

6. Substituting the proper values in $v_y = u_y + a_x t$, we get,

$$-v\cos 60° = \left(u\sqrt{2}\sin 45°\right) - gt$$

$\Rightarrow \quad t = \dfrac{u}{g}\left(\dfrac{\sqrt{3}+1}{\sqrt{3}}\right)$

Hence, the correct answer is (C).

7. Since $\vec{v} = 20\hat{i} + 10\hat{j}$

$\Rightarrow \quad v_x = u_x = 20 \text{ ms}^{-1}$

Since, $v_y = 10 \text{ ms}^{-1}$ and $v_y^2 - u_y^2 = 2a_y y$

$\Rightarrow \quad (10)^2 - u_y^2 = 2(-10)(15)$

$\Rightarrow \quad u_y^2 = 400$

$\Rightarrow \quad u_y = 20 \text{ ms}^{-1}$

So, $u = \sqrt{u_x^2 + u_y^2} = 20\sqrt{2} \text{ ms}^{-1}$

Hence, the correct answer is (B).

8. Since $\tan\theta = \dfrac{u_y}{u_x} = \dfrac{20}{20} = 1$

$\Rightarrow \quad \theta = 45° = \dfrac{\pi}{4}$ radian

Hence, the correct answer is (B).

9. $H = \dfrac{u_y^2}{2g} = \dfrac{400}{20} = 20 \text{ m}$

$$R = \frac{u^2\sin(2\theta)}{g} = \frac{\left(20\sqrt{2}\right)^2\sin(90)}{10}$$

$\Rightarrow \quad R = 80 \text{ m}$

Now, the coordinate where the maximum height is attained is $\left(\dfrac{R}{2}, H\right)$. So, we get $(40, 20)$ m as the answer.

Hence, the correct answer is (D).

10. For minimum horizontal velocity, i.e., for the particle just to land at A, we have

$x = 3a$ and $y = 4a$

$\Rightarrow \quad 4a = g\dfrac{(3a)^2}{2u_{MIN}^2}$ $\qquad \left\{\because\ y = \dfrac{gx^2}{2u^2}\right\}$

$\Rightarrow \quad u_{MIN}^2 = \dfrac{9}{8}ag$

$\Rightarrow \quad u_{MIN} = \sqrt{\dfrac{9}{8}ag}$

Hence, the correct answer is (A).

11. Since $y = \dfrac{1}{2}a_y t^2$

$\Rightarrow \quad 4a = \dfrac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\dfrac{8a}{g}}$

Hence, the correct answer is (D).

12. For maximum horizontal, velocity i.e., when the particle lands at B, we have

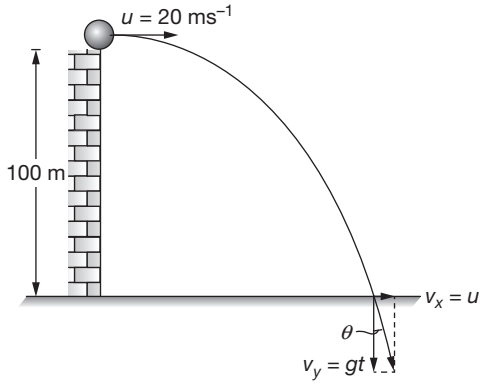
$x = 3a + 2a = 5a$ and $y = 4a$

So, $4a = \dfrac{g(5a)^2}{2u_{MAX}^2}$

$\Rightarrow \quad u_{MAX} = \sqrt{\dfrac{25}{8}ag}$

Hence, the correct answer is (B).

13. $100 = \dfrac{1}{2}gt^2$ $\qquad \left\{\because\ y = \dfrac{1}{2}gt^2\right\}$



$\Rightarrow \quad t = \sqrt{\frac{200}{10}} = \sqrt{20} = 2\sqrt{5}$ s

Hence, the correct answer is (C).

14. $x = ut = 20\left(2\sqrt{5}\right)$

$\Rightarrow \quad x = 40\sqrt{5}$ m

Hence, the correct answer is (B).

15. $\tan\theta = \frac{v_x}{v_y}$

Since $v_x = u$ and $v_y = gt$

$\Rightarrow \quad \tan\theta = \frac{u}{gt} = \frac{20}{(10)\left(2\sqrt{5}\right)}$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{5}}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Hence, the correct answer is (C).

16. $x = 6t \qquad y = 8t - 5t^2$

Compare with equations of motion of the projectile i.e.

$x = (u\cos\theta)t$ and $y = (u\sin\theta)t - \frac{1}{2}gt^2$, we get

$u_x = u\cos\theta = 6$ and $u_y = u\sin\theta = 8$

and $g = 10$ ms^{-2} (acting along $-y$ axis)

Hence, the correct answer is (C).

17. Since acceleration has no component along x-axis, so $u_x = \text{constant} = 6$ ms^{-1}.

Hence, the correct answer is (C).

18. $u_y = u\sin\theta = 8$ ms^{-1}

Hence, the correct answer is (C).

19. The velocity of projection is

$u = \sqrt{u_x^2 + u_y^2}$

$\Rightarrow \quad u = \sqrt{6^2 + 8^2}$

$\Rightarrow \quad u = 10$ ms^{-1}

Hence, the correct answer is (D).

20. $t_{\text{ascent}} = \frac{u_y}{g}$

$\Rightarrow \quad t_{\text{ascent}} = \frac{8}{10} = 0.8$ s

Hence, the correct answer is (D).

21. $H = \frac{u_y^2}{2g}$

$\Rightarrow \quad H = \frac{(8)^2}{2(10)}$

$\Rightarrow \quad H = 3.2$ m

Hence, the correct answer is (D).

22. $R = \frac{2}{g}\left(u_x\right)\left(u_y\right)$

$\Rightarrow \quad R = \frac{2}{10}(6)(8)$

$\Rightarrow \quad R = 9.6$ m

Hence, the correct answer is (D).

23. $H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$

$\Rightarrow \quad 8 = \frac{u_y^2}{2g}$

$\Rightarrow \quad u_y = 4\sqrt{g}$

Hence, the correct answer is (D).

24. Since, $R = \frac{2}{g}\left(u_x\right)\left(u_y\right)$

$\Rightarrow \quad 24 = \frac{2}{g}\left(u_x\right)\left(4\sqrt{g}\right)$

$\Rightarrow \quad u_x = 3\sqrt{g}$

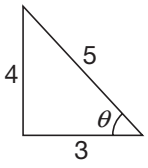
Hence, the correct answer is (B).

25. $u = \sqrt{u_x^2 + u_y^2}$

$\Rightarrow \quad u = 5\sqrt{g}$

Hence, the correct answer is (D).

26.



$\tan\theta = \frac{u_y}{u_x}$

$\Rightarrow \quad \tan\theta = \frac{4\sqrt{g}}{3\sqrt{g}}$

$\Rightarrow \quad \tan\theta = \frac{4}{3}$

$\Rightarrow \quad \sin\theta = \frac{4}{5} = 0.8$

Hence, the correct answer is (B).

27.

Along x	Along y
$a_x = 6t$	$a_y = 8t$
$\frac{dv_x}{dt} = 6t$	$\frac{dv_y}{dt} = 8t$
$\int_0^{v_x} dv_x = \int_0^3 6t\,dt$	$\int_0^{v_y} dv_y = \int_0^3 8t\,dt$
$v_x = \frac{6t^2}{2}\Big\|_0^3 = 27\text{m}$	$v_y = \frac{8t^2}{2}\Big\|_0^3 = 36\text{ m}$

$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{27^2 + 36^2}$

$\Rightarrow \quad v = \sqrt{729 + 1296} = 45\text{ ms}^{-1}$

Hence, the correct answer is (A).

28. $|\vec{a}| = 10\text{ ms}^{-2}$

Since, we know that,

$$s = \frac{1}{2}at^2$$

$\Rightarrow \quad s = \frac{1}{2} \times 10 \times 9$

$\Rightarrow \quad s = 45\text{ m}$

Hence, the correct answer is (D).

29. Path is a straight line.

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (r)
B → (t)
C → (q)
D → (p)
E → (s)

$$\vec{u} = u\left(\cos(45°)\hat{i} + \sin(45°)\hat{j}\right)$$

$\Rightarrow \quad \vec{u} = 20\hat{i} + 20\hat{j}\text{ ms}^{-1}$ and $\vec{a} = -10\hat{j}$

Since

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$\Rightarrow \quad \vec{r} = \left(20\hat{i} + 20\hat{j}\right)t + \frac{1}{2}\left(-10\hat{j}\right)t^2$

$\Rightarrow \quad \vec{r} = (20t)\hat{i} + (20 - 5t)\hat{j}$

So, at $t = 1\text{ s}$, we get

$$\vec{r} = 20\hat{i} + 15\hat{j}$$

$\Rightarrow$ Average Velocity at $t = 1\text{ s}$ is

$$\vec{v}_{av} = \frac{\vec{r}}{t} = 20\hat{i} + 15\hat{j}$$

$\Rightarrow \quad |\vec{v}_{av}| = \sqrt{400 + 225} = 25\text{ ms}^{-1}$

Further $\vec{v} = \vec{u} + \vec{a}t$

$\Rightarrow \quad \vec{v} = 20\hat{i} + 20\hat{j} - \left(10\hat{j}\right)t$

$\Rightarrow \quad \vec{v} = 20\hat{i} + (20 - 10t)\hat{j}$

$\Rightarrow \quad \Delta\vec{v} = \vec{v} - \vec{u} = -(10t)\hat{j}$

$\Rightarrow \quad \Delta\vec{v} = -20\hat{j}$ {at $t = 2\text{ s}$}

So, average acceleration at $t = 1\text{ s}$, 2 s, 3 s and 4 s

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = -10\hat{j}$$ {as expected}

Radius of curvature at $t = 0$ is

$$r = \frac{u^2}{(g\cos\theta)}$$

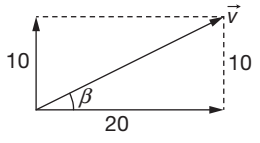
$\Rightarrow \quad r = \frac{\left(20\sqrt{2}\right)^2}{\left(\frac{10}{\sqrt{2}}\right)} = 80\sqrt{2}\text{ m}$

Radius of curvature at $t = 1\text{ s}$ is

$$r = \frac{v^2}{g\cos\beta}$$

where v is the velocity at $t = 1\text{ s}$ and β is the angle which $v(\text{at } t = 1\text{ s})$ makes with the x-axis

Since $v = 20\hat{i} + 10\hat{j}$, so $\tan\beta = \frac{10}{20}$



$\Rightarrow \quad \cos\beta = \frac{20}{\sqrt{20^2 + 10^2}} = \frac{20}{10\sqrt{5}} = \frac{2}{\sqrt{5}}$

So, $r = \dfrac{\left(\sqrt{20^2+10^2}\right)^2}{(10)\left(\dfrac{2}{\sqrt{5}}\right)}$

$\Rightarrow \quad r = \dfrac{500\sqrt{5}}{20}$

$\Rightarrow \quad r = 25\sqrt{5}$ m

Radius of curvature at $t = 2$ s is

$$r = \frac{(u\cos\theta)^2}{g}$$

$\Rightarrow \quad r = \dfrac{\left[\left(20\sqrt{2}\right)\left(\dfrac{1}{\sqrt{2}}\right)\right]^2}{10} = \dfrac{(20)^2}{10} = 40$ m

2. A → (r)
B → (s)
C → (p)
D → (q)

Since $v = 2t$

So, distance travelled is

$$s = \int_0^2 v\,dt = \int_0^2 2t\,dt = 2\left(\frac{t^2}{2}\Big|_0^2\right)$$

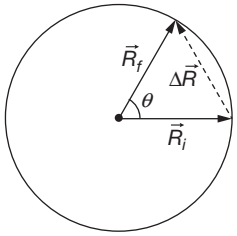
$\Rightarrow \quad s = 4$ m

Hence average speed is

$$v_{av} = \frac{s}{t} = \frac{4}{2} = 2 \text{ ms}^{-1}$$

So, (A) → (r), (C) → (p)
Further since the particle is moving in a circular path, so we have

$$\omega = \frac{v}{r} = 2t \text{ rads}^{-1}$$



$R = 1$ m

Since

$$\theta = \int_0^2 \omega\,dt$$

$\Rightarrow \quad \theta = 2\int_0^2 t\,dt$

$\Rightarrow \quad \theta = 4$ rad

$\Delta\vec{R}$ = Displacement $= \vec{R}_f - \vec{R}_i$

$\Rightarrow \quad |\text{Displacement}| = \sqrt{R_f^2 + R_i^2 + 2R_fR_i\cos(180-\theta)}$

Since $|\vec{R}_f| = |\vec{R}_i| = R$

$\Rightarrow \quad |\text{Displacement}| = \sqrt{R^2 + R^2 - 2R^2\cos\theta}$

$\Rightarrow \quad |\text{Displacement}| = 2R\sin\left(\dfrac{\theta}{2}\right) = 2\sin(2)$

So, average velocity

$$|\vec{v}_{av}| = \frac{|\text{Displacement}|}{\text{Time}} = \frac{2\sin(2)}{2} = \sin(2)$$

Hence (B) → (s) and (D) → (q)

3. A → (r)
B → (s)
C → (p)
D → (q)

Since $\vec{a} = \dfrac{dv}{dt}$

$\Rightarrow \quad \vec{a} = 2\hat{i} + (2t)\hat{j}$

So, at $t = 1$ s, we get

$\vec{v} = 2\hat{i} + \hat{j}$ and $\vec{a} = 2\hat{i} + 2\hat{j}$

$\Rightarrow \quad |\vec{a}| = \sqrt{2^2+2^2} = 2\sqrt{2}$ ms^{-2}

Hence (C) → (p)
Now, since the tangential acceleration is

$$a_T = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$$

$\Rightarrow \quad a_T = \dfrac{\left(2\hat{i}+2\hat{j}\right)\cdot\left(2\hat{i}+\hat{j}\right)}{\sqrt{2^2+1^2}}$

$\Rightarrow \quad a_T = \dfrac{4+2}{\sqrt{5}} = \dfrac{6}{\sqrt{5}}$ ms^{-2}

So, (A) → (r)
Now since we know that

$a^2 = a_C^2 + a_T^2$, where a_C is the radial acceleration

$\Rightarrow \quad a_C = \sqrt{a^2 - a_T^2}$

$\Rightarrow \quad a_C = \sqrt{8 - \dfrac{36}{5}}$

$\Rightarrow \quad a_C = \frac{2}{\sqrt{5}}\ \text{ms}^{-2}$

So, (B) → (s)

Finally, the radius of curvature is given by

$$r = \frac{v^2}{a_C} = \frac{5}{\frac{2}{\sqrt{5}}} = \frac{5\sqrt{5}}{2}\ \text{m}$$

So, (D) → (q)

4. A → (q)
B → (s)
C → (p)
D → (r)

Let us choose the x and y directions along OB and OA respectively. Then

$$u_x = u = 10\sqrt{3}\ \text{ms}^{-1},\ u_y = 0$$

$$a_x = -g\sin(60°) = -5\sqrt{3}\ \text{ms}^{-2}$$

and $a_y = -g\cos(60°) = -5\ \text{ms}^{-2}$

At point Q, x-component of velocity is zero. Hence, substituting in

$$v_x = u_x + a_x t$$

$\Rightarrow \quad 0 = 10\sqrt{3} - 5\sqrt{3}t$

$\Rightarrow \quad t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2s$

So, (A) → (q)

At point Q, $v = v_y = u_y + a_y t$

$\Rightarrow \quad v = 0 - (5)(2) = -10\ \text{ms}^{-1}$

Here, negative sign implies that velocity of particle at Q is along negative y direction. So, $v = 10\ \text{ms}^{-1}$, along $-y$ direction.

Hence (B) → (s)

$$\text{Distance } PO = \left|\begin{matrix}\text{Displacement of partice}\\ \text{along } y\text{-direction}\end{matrix}\right| = |s_y|$$

Since, $s_y = u_y t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}(5)(2)^2 = -10\ \text{m}$

$\Rightarrow \quad PO = 10\ \text{m}$

Since, $h = PO\sin(30°) = (10)\left(\frac{1}{2}\right)$

$\Rightarrow \quad h = 5\ \text{m}$

So, (C) → (p)

$$\text{Distance } OQ = \left|\begin{matrix}\text{Displacement of partice}\\ \text{along } x\text{-direction}\end{matrix}\right| = |s_x|$$

Since, $s_x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow \quad s_x = (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2 = 10\sqrt{3}\ \text{m}$

$\Rightarrow \quad OQ = 10\sqrt{3}\ \text{m}$

Since, $PQ = \sqrt{(PO)^2 + (OQ)^2}$

$\Rightarrow \quad PQ = \sqrt{(10)^2 + (10\sqrt{3})^2} = \sqrt{400}$

$\Rightarrow \quad PQ = 20\ \text{m}$

5. A → (q)
B → (r)
C → (s)
D → (t)
E → (p)

$$t = \frac{x}{\alpha} \qquad \ldots(1)$$

$$y = \alpha\left(\frac{x}{\alpha}\right)\left[1 - \beta\left(\frac{x}{\alpha}\right)\right]$$

$\Rightarrow \quad y = x - \left(\frac{\beta}{\alpha}\right)x^2 \qquad \ldots(2)$

$y = 0$ at $x = R$

$\Rightarrow \quad 0 = R - \frac{\beta}{\alpha}R^2$

$\Rightarrow \quad R\left(1 - \frac{\beta}{\alpha}R\right) = 0$

$\Rightarrow \quad R = \frac{\alpha}{\beta} = 2\left(\frac{\alpha}{2\beta}\right)$

So, (A) → (q)

At maximum height, $\frac{dy}{dx} = 0$

$\Rightarrow \quad 1 - \frac{\beta}{\alpha}(2x) = 0$

$\Rightarrow \quad x = \frac{\alpha}{2\beta}$

$$H_{\max} = \frac{\alpha}{2\beta} - \frac{\beta}{\alpha}\left(\frac{\alpha^2}{4\beta^2}\right)$$

$\Rightarrow \quad H_{\max} = \frac{\alpha}{2\beta} - \frac{\alpha}{4\beta} = \frac{\alpha}{4\beta} = 4\left(\frac{\alpha}{16\beta}\right)$

So, (B) → (r)

$$T = \frac{R}{u_x} \qquad \left\{\text{or from (1), we have } t = \frac{x}{\alpha}\right\}$$

Since $x = \alpha t$, so $v_x = \frac{dx}{dt} = \alpha = \text{constant}$

$\Rightarrow \quad v_x = u_x = \alpha$

$$\Rightarrow \quad T = \frac{\left(\frac{\alpha}{\beta}\right)}{\alpha} = \frac{1}{\beta} = 8\left(\frac{1}{8\beta}\right)$$

So, (C) → (s)

Also, we observe that $y = \alpha t - (\alpha\beta)t^2$

$$\Rightarrow \quad \frac{dy}{dt} = \alpha - 2\alpha\beta t = \alpha(1 - 2\beta t)$$

$\Rightarrow \quad v_y = \alpha(1 - 2\beta t)$

$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j}$

$\Rightarrow \quad \vec{v} = \alpha\hat{i} + \alpha(1 - 2\beta t)\hat{j}$

Now, $a_x = \frac{dv_x}{dt} = 0$ and $a_y = -2(\alpha\beta)\hat{j} = \text{constant}$

$\Rightarrow \quad \vec{a} = a_x\hat{i} + a_y\hat{j} = -(2\alpha\beta)\hat{j}$

$$\Rightarrow \quad |\vec{a}| = 2\alpha\beta = 16\left(\frac{\alpha\beta}{8}\right)$$

So, (D) → (t)

Now at $t = \frac{1}{2\beta}$, we have

$$\vec{v} = \alpha\hat{i} + \alpha\left[1 - (2\beta)\left(\frac{1}{2\beta}\right)\right]\hat{j} = \alpha\hat{i}$$

$\Rightarrow \quad |\vec{v}| = \alpha = 1(\alpha)$

So, (E) → (p)

7. A → (q)
B → (p)
C → (r)
D → (s)

Since range is same for complimentary angles, so

$$R_1 = R_2$$

$$\Rightarrow \quad \frac{R_1}{R_2} = 1$$

$$\frac{H_1}{H_2} = \frac{u^2\sin^2(30°)}{u^2\sin^2(60°)} = \frac{1}{3}$$

$$\frac{T_2}{T_1} = \frac{u\sin(60°)}{u\sin(30°)} = \sqrt{3}$$

$$\frac{T_1H_1R_1}{T_2H_2R_2} = \frac{T_1H_1}{T_2H_2} = \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{3\sqrt{3}}$$

8. A → (q)
B → (r)
C → (p)
D → (s)

$\vec{u} = 20\hat{i} + 20\hat{j}$, $\vec{a} = -10\hat{j}$ and $t = 1$ s

Since, $\Delta\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2 = 20\hat{i} + 15\hat{j}$ …(1)

and $\vec{v} = \vec{u} + \vec{a}t = 20\hat{i} + 10\hat{j}$ …(2)

$$\Rightarrow \quad |\vec{v}_{av}| = \left|\frac{\Delta\vec{r}}{t}\right| = \sqrt{(20)^2 + (15)^2} = 25 \text{ ms}^{-1} \quad \{\text{from (1)}\}$$

$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v} - \vec{u}| = 10 \text{ ms}^{-1}$ {from (2)}

$$\Rightarrow \quad |\vec{v}_{\text{inst}}| = |\vec{v}| = \sqrt{(20)^2 + (10)^2} = 10\sqrt{5} \text{ ms}^{-1}$$

$\Rightarrow \quad \Delta|\vec{v}| = |\vec{v}| - |\vec{u}| = 20\sqrt{2} - 10\sqrt{5} = 6 \text{ ms}^{-1}$

9. A → (r)
B → (r)
C → (p)
D → (q)

Since, $y = x - \frac{x^2}{80}$

Comparing with the standard equation of projectile, i.e.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}, \text{ we get}$$

$$\tan\theta = 1$$

$\Rightarrow \quad \theta = 45°$

and $\frac{1}{80} = \frac{g}{2u^2\cos^2\theta}$

$\Rightarrow \quad u = 20\sqrt{2} \text{ ms}^{-1}$

Since, $\vec{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j} = 20\hat{i} - 20\hat{j}$

When $x = R$, $y = 0$

$$\Rightarrow \quad 0 = R - \frac{R^2}{80}$$

$\Rightarrow \quad R = 80$ m

CHAPTER 5

10. A → (p, r)
B → (q, s)
C → (p, q, r, s)
D → (p, q, r, s)

For a particle to move in circular path, $\vec{a} \perp \vec{v}$
For a particle to undergo projectile motion, angle between $\vec{a}$ and $\vec{v}$ must be acute.

11. A → (q)
B → (r)
C → (s)
D → (p)

(A) Change in momentum is $\Delta p = (mg)t$ in time t.

(B) Angle at highest point is $0°$.

(C) Kinetic energy of body is minimum at highest point.

(D) Horizontal component remains unchanged.

Integer/Numerical Answer Type Questions

1. Since $R = \dfrac{2u^2 \sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$

$$\Rightarrow \quad R = \frac{2(21)^2 \sin(60°-30°)\cos(60°)}{9.8\cos^2(30°)}$$

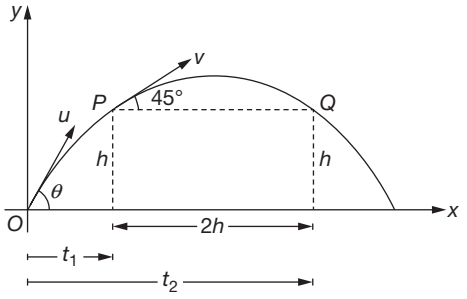
$$\Rightarrow \quad R = \frac{(2)(441)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{(9.8)\left(\frac{3}{4}\right)} = 30 \text{ m}$$

2. At P, at height h,

$$v^2 = u^2 - 2gh$$

$$\Rightarrow \quad v^2 = 4gh - 2gh \qquad \{\because u = 2\sqrt{gh}\}$$

$$\Rightarrow \quad v = \sqrt{2gh} \qquad \ldots(1)$$



Assuming PQ to be horizontal, then Range $= 2h = PQ$

$$\Rightarrow \quad \frac{v^2 \sin(2\theta)}{g} = 2h$$

$$\Rightarrow \quad \frac{2(gh)\sin(2\theta)}{g} = 2h$$

$$\Rightarrow \quad \sin(2\theta) = 1$$

$$\Rightarrow \quad \theta = 45°$$

So, to fly from P to Q, the projectile will take a time t given by

$$t = \frac{2v\sin(45°)}{g} = \left(\frac{2\sqrt{2gh}}{g}\right)\left(\frac{1}{\sqrt{2}}\right) = 2\sqrt{\frac{h}{g}}$$

So, $* = 2$

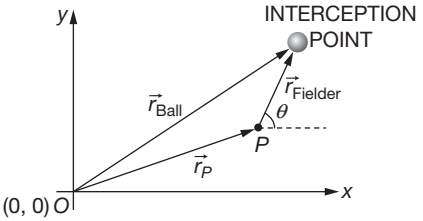
3. From the diagram, we observe that

$$\vec{r}_P + \vec{r}_{\text{Fields}} = \vec{r}_{\text{Ball}} \qquad \ldots(1)$$

where $\vec{r}_P = \left(46\hat{i} + 28\hat{j}\right)$ m $\qquad \ldots(2)$

Let fielder run for the ball making an angle θ with the horizontal.

Then $\vec{r}_{\text{Fielder}} = 5t\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) \qquad \ldots(3)$



where t is the time taken by the fielder to go from point P to the interception point which equals the time taken by the ball to go from the origin to the interception point with a velocity of $\left(7.5\hat{i} + 10\hat{j}\right)$ ms^{-1}. So

$$\vec{r}_{\text{Ball}} = \left(7.5t\hat{i} + 10t\hat{j}\right) \qquad \ldots(4)$$

Substituting (2), (3) and (4) in (1), we get

$$(46 + 5t\cos\theta)\hat{i} + (28 + 5t\sin\theta)\hat{j} = (7.5t)\hat{i} + (10t)\hat{j}$$

$$\Rightarrow \quad 46 + 5t\cos\theta = 7.5t \qquad \ldots(5)$$

$$28 + 5t\sin\theta = 10t \qquad \ldots(6)$$

From (5) and (6), we get

$$\cos\theta = \frac{7.5t - 46}{5t} \text{ and } \sin\theta = \frac{10t - 28}{5t}$$

Since, $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \quad \left(\frac{7.5t - 46}{5t}\right)^2 + \left(\frac{10t - 28}{5t}\right)^2 = 1$$

Solving to get

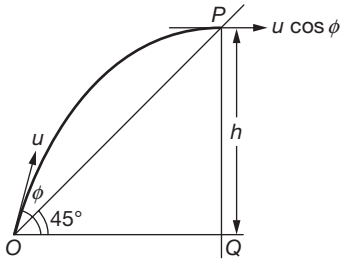
$$t = 4 \text{ s or } t = \frac{116}{21} \text{ s}$$

So, the shortest time for interception is

$$t = 4 \text{ s}$$

4. Let the particle be projected from O with velocity u and strike the plane at a point P horizontally. Then

$PQ = OQ$.



$\Rightarrow$ Maximum height $= \dfrac{\text{Horizontal range}}{2}$

$\Rightarrow \dfrac{u^2 \sin^2\phi}{2g} = \dfrac{u^2 \sin 2\phi}{2g} = \dfrac{u^2 \sin\phi\cos\phi}{g}$

$\Rightarrow \tan\phi = 2$

5. The angular speed is

$$\omega = \frac{v}{r} = \frac{2}{0.25} = 8 \text{ rads}^{-1}$$

6. Since $y = u_y t + \dfrac{1}{2}a_y t^2$, where $u_y = 0$ and $a_y = g$, $y = h$.

So $h = \dfrac{1}{2}gt^2$

$\Rightarrow t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(20)}{10}} = 2$ s

Further since $x = u_x t + \dfrac{1}{2}a_x t^2$ where $u_x = v$ (say), $a_x = 0$, $x = 10$ m, so

$10 = vt = v(2)$

$\Rightarrow v = 5 \text{ ms}^{-1}$

Since $a = \dfrac{v^2}{R}$

$\Rightarrow a = \dfrac{(5)^2}{0.5} = \dfrac{25}{0.5} = 50 \text{ ms}^{-2}$

$\Rightarrow a = 50 \text{ ms}^{-2}$

7. Since the car is travelling with a constant speed, its tangential component of acceleration is zero, i.e., $a_T = 0$. Thus,

$$a = a_N = \frac{v^2}{r}$$

$\Rightarrow 3 = \dfrac{25^2}{r}$

$\Rightarrow r = 208$ m

8. Here, the car's tangential component of acceleration of $a_T = -3 \text{ ms}^{-2}$. Thus,

$a = \sqrt{a_T^2 + a_N^2}$

$\Rightarrow 5 = \sqrt{(-3)^2 + a_N^2}$

$\Rightarrow a_N = 4 \text{ ms}^{-2}$

Since $a_N = \dfrac{v^2}{r}$

$\Rightarrow 4 = \dfrac{20^2}{r}$

$\Rightarrow r = 100$ m

9. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 0.8}{10}} = 0.4$ s

$$u_{MIN} = \frac{(120 - 30 + 10)\times 10^{-3}}{0.4} = 0.25 \text{ ms}^{-1}$$

and $u_{MAX} = \dfrac{(120 + 30 - 10)\times 10^{-3}}{0.4} = 0.35 \text{ ms}^{-1}$

$\Rightarrow u_{MIN} = 25 \text{ cms}^{-1}$ and $u_{MAX} = 35 \text{ cms}^{-1}$

10. Let the ball clear the point C at time t_1. Then

$y = u_y t + \dfrac{1}{2}a_y t^2$

$\Rightarrow (1 + 3.9) = 0 + \dfrac{1}{2}(9.8)t_1^2$ $\{\because u_y = 0\}$

$\Rightarrow 4.9 = \dfrac{1}{2}(9.8)t_1^2$

$\Rightarrow t_1 = 1$ s

Since $BC = u_x t$

$\Rightarrow 6 = u(1)$

$\Rightarrow u = 6 \text{ ms}^{-1}$

Let the ball hit the ground at D in time t.
Then

$6 + x = 6t$ ...(1)

Further $(1 + 3.9 + 14.7) = \dfrac{1}{2}(9.8)t^2$

$\Rightarrow t = 2$ s

$\Rightarrow 6 + x = 12$

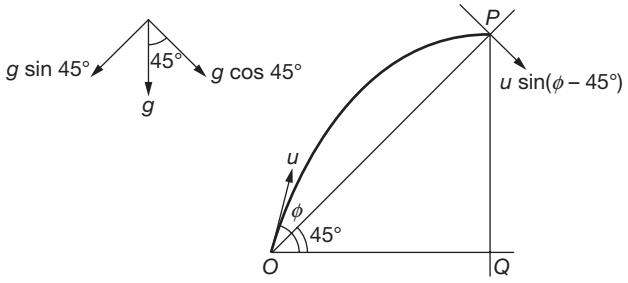
$\Rightarrow x = 6$ m

CHAPTER 5

11. At time $t = T = \dfrac{2u\sin(\phi - 45°)}{g\cos 45°}$

component of velocity along the plane is zero.

$\Rightarrow \quad 0 = u\cos(\phi - 45°) - (g\sin 45°)t$



$\Rightarrow \quad u\cos(\phi - 45°) = (g\sin 45°)\left(\dfrac{2u\sin(\phi - 45°)}{g\cos 45°}\right)$

$\Rightarrow \quad 2\tan(\phi - 45°) = \cot 45° = 1$

$\Rightarrow \quad 2\left(\dfrac{\tan\phi - \tan 45°}{1 + \tan\phi\tan 45°}\right) = 1$

$\Rightarrow \quad \tan\phi = 3$

12. As $y = x\tan\theta - \dfrac{gx^2}{2u^2}(1 + \tan^2\theta)$

for $(a,\ b)$, i.e. when $x = a$ and $y = b$, we have

$$ga^2\tan^2\theta - 2au^2\tan\theta + (ga^2 + 2bu^2) = 0$$

This is a quadratic in $\tan\theta$ and since discriminant of a quadratic must be positive, so we have

$$4a^2u^2 - 4ga^2(ga^2 + 2bu^2) \geq 0$$

Solving, we get

$$u \geq \sqrt{bg + g\sqrt{a^2 + b^2}}$$

On substituting the values, we get

$$u_{\min} = 30 \text{ ms}^{-1}$$

13. $v_x = 8\sqrt{2}$ ms^{-1} is the relative velocity along x-axis

$\Rightarrow \quad x = 22 - (8\sqrt{2})t$

$v_y = 6\sqrt{2}$ ms^{-1} is the relative velocity along y-axis

$\Rightarrow \quad y = 9 - (6\sqrt{2})t$

$\Rightarrow \quad r^2 = x^2 + y^2 \qquad \ldots(1)$

For minimum r, we have r^2 to be minimum, so

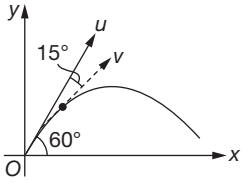
$$\frac{d}{dt}(r^2) = 0$$

$\Rightarrow \quad t = \dfrac{23}{10\sqrt{2}}$ s

Substituting the value of t in equation (1), we get

$$r_{\min} = 6 \text{ m}$$

14. $v = \dfrac{u\cos(60°)}{\cos(45°)}$



$\Rightarrow \quad K_f = K_i\left(\dfrac{\cos^2(60°)}{\cos^2(45°)}\right)$

$\Rightarrow \quad \dfrac{K_i}{K_f} = 2$

15. For Range to be maximum, the projectile must be launched at an angle

$$\alpha = \frac{\pi}{4} + \frac{\beta}{2}$$

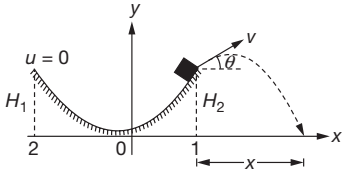
$\Rightarrow \quad \alpha = 45° + 15° = 60°$

16. Since, $H_1 = (2)^2 = 4$ m and

$$H_2 = (1)^2 = 1 \text{ m}$$

By Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in gravitational}\\ \text{potential energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{kinetic energy}\end{pmatrix}$$



$\Rightarrow \quad mg(H_1 - H_2) = \dfrac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{2g(H_1 - H_2)}$

$\Rightarrow \quad v = \sqrt{2 \times g \times (H_2 - H_1)}$

$\Rightarrow \quad v = \sqrt{20 \times 3}$

$\Rightarrow \quad v = \sqrt{60}$

Now $\tan\theta = \left.\frac{dy}{dx}\right|_{x=1} = 2x|_{x=1} = 2$

$\Rightarrow \quad \tan\theta = 2$

Since, $y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$

$\Rightarrow \quad -1 = x(2) - \frac{10x^2}{2v^2}(1+4)$

$\Rightarrow \quad -1 = 2x - \frac{10x^2}{2(60)}(5)$

$\Rightarrow \quad -1 = 2x - \frac{5}{12}x^2$

$\Rightarrow \quad -12 = 24x - 5x^2$

$\Rightarrow \quad 5x^2 - 24x - 12 = 0$

17. Time of flight is

$$T = \frac{2u\sin\theta}{g} = 2\sqrt{2} \text{ s}$$

Range (along north) is

$$R_1 = \frac{u^2\sin 2\theta}{g} = 40 \text{ m}$$

Range (along east) is

$$R_2 = \frac{1}{2}aT^2 = 30 \text{ m}$$

So, range $R = \sqrt{R_1^2 + R_2^2} = \sqrt{30^2 + 40^2} = 50$ m

18. $10 - v\cos(60°) = 0$

$\Rightarrow \quad H = \frac{v^2\sin^2(60°)}{2g} = 15 \text{ m}$

$\Rightarrow \quad H - 10 = 5 \text{ m}$

19. Let angle made by $\vec{V}$ initially and after time t be θ and α respectively.

$$\tan(30°) = \frac{u\sin\theta - g \times 2}{u\cos\theta} \quad \ldots(1)$$

$\Rightarrow \quad \tan(0°) = \frac{u\sin\theta - g \times 3}{u\cos\theta} \quad \ldots(2)$

$\Rightarrow \quad \theta = 60°$

$\Rightarrow \quad \frac{\theta}{10} = 6$

20. Coordinates of the point C are

$$C\left(\frac{10}{3} + x,\ y\right)$$

Since, $\frac{y}{x} = \tan 37°$

$\Rightarrow \quad y = \frac{3}{4}x$

$\Rightarrow \quad u_y t - \frac{1}{2}gt^2 = \frac{3}{4}\left[u_x t - \frac{10}{3}\right]$

$\Rightarrow \quad t = 1.06$

$\Rightarrow \quad x = \frac{3}{4} \times 10 \times 1.06 - \frac{10}{3} = 4.64$

21. Since, $u_{\min} = \sqrt{g\left(h + \sqrt{d^2 + h^2}\right)} = 10\sqrt{3}$ ms^{-1}

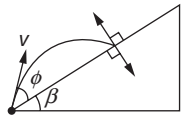
and $\tan\theta = \frac{h + \sqrt{d^2 + h^2}}{d}$

$\Rightarrow \quad \theta = 60°$

$\Rightarrow \quad t = \frac{d}{u\cos\theta} = \frac{10\sqrt{3}}{10\sqrt{3} \times \frac{1}{2}} = 2 \text{ s}$

22. Since, $\tan\phi = \frac{\cot\beta}{2}$

$\Rightarrow \quad t = \frac{2v}{g\sqrt{1 + 3\sin^2\beta}}$



23. $T = \frac{2u\sin\theta}{g} = \frac{2 \times 10 \times \frac{1}{2}}{10} = 1 \text{ s}$

CHAPTER 5

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1. Since $R=\dfrac{2u\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$, where $u=2\ \text{ms}^{-1}$

$\alpha=30+15=45°$

$\beta=30°$

$\Rightarrow\quad R=\dfrac{2(2)^2\sin(15°)\cos(45°)}{g\cos^2(30°)}$

$\Rightarrow\quad R=\dfrac{8\sqrt{2}}{15}\sin(15°)$

$\Rightarrow\quad R=\dfrac{8\sqrt{2}}{15}\sin(45°-30°)$

$\Rightarrow\quad R=\dfrac{8\sqrt{2}}{15}\left(\dfrac{1}{2\sqrt{2}}\right)(\sqrt{3}-1)=\dfrac{4}{15}(\sqrt{3}-1)$

$\Rightarrow\quad R\approx 20\ \text{cm}$

Hence, the correct answer is (B).

2. For same horizontal range, we must have angles as complimentary. So

For $\theta_1=\theta$ and $\theta_2=(90-\theta)$

$$R=\frac{2u^2\sin\theta\cos\theta}{g}$$

$\Rightarrow\quad t_1=\dfrac{2u\sin\theta}{g}$ and $t_2=\dfrac{2u\cos\theta}{g}$

$\Rightarrow\quad t_1t_2=\dfrac{u^2 4\sin\theta\cos\theta}{g^2}$

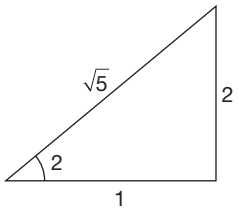
$\Rightarrow\quad t_1t_2=\dfrac{2u^2\sin2\theta}{g^2}=\dfrac{2R}{g}$

Hence, the correct answer is (B).

3. $y=2x-9x^2$

Comparing it with standard equation of trajectory of projectile, i.e.,

$$y=x\tan\theta-\frac{gx^2}{24^2\cos^2\theta}$$



we get, $\tan\theta=2$

and $9=\dfrac{10\times5}{2v_0^2}$

$\Rightarrow\quad v_0=\dfrac{5}{3}\ \text{ms}^{-1}$

Hence, the correct answer is (C).

4. For complementary angles, ranges are equal.

So, for $\theta_1=\theta$ and $\theta_2=(90-\theta)$, we have

$$R=\frac{2u^2\sin\theta\cos\theta}{g}$$

Also, $h_1=\dfrac{u^2\sin^2\theta}{2g}$, $h_2=\dfrac{u^2\cos^2\theta}{2g}$

$\Rightarrow\quad h_1h_2=\left(\dfrac{2u^2\sin\theta\cos\theta}{g}\right)^2\times\left(\dfrac{1}{16}\right)$

$\Rightarrow\quad 16h_1h_2=R^2$

$\Rightarrow\quad R^2=16(h_1h_2)$

Hence, the correct answer is (B).

5. Maximum areas will be covered when range is maximum, so

$$R_{\max}=\frac{u^2}{g}$$

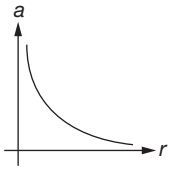
$\Rightarrow\quad \dfrac{A_1}{A_2}=\dfrac{\pi(R_{\max})_1^2}{\pi(R_{\max})_2^2}=\left(\dfrac{u_1^2}{u_2^2}\right)^2=\dfrac{u_1^4}{u_2^4}=\dfrac{1}{16}$

Hence, the correct answer is (D).

6. Speed $v=10\ \text{ms}^{-1}$

Since, centripetal acceleration is given by,

$$a=\frac{v^2}{r}$$



and $|\vec{v}|=\text{constant}$

$\Rightarrow\quad a\propto\dfrac{1}{r}$

$\Rightarrow \quad ar = \text{constant}$

This represents a rectangular hyperbola.

Hence, the correct answer is (C).

7. Since, $\vec{u} = \hat{i} + 2\hat{j}$

Also, $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\Rightarrow \quad u_x = 1$ and $u_y = 2$

Now $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$

$\Rightarrow \quad x = t$

and $y = 2t - \frac{1}{2}\times 10 \times t^2 = 2t - 5t^2$

$\Rightarrow \quad y = 2x - 5x^2$

So, equation of trajectory is $y = 2x - 5x^2$

Hence, the correct answer is (C).

8. Let u be the velocity of projection of the stone.
The maximum height a boy can throw a stone is

$$H_{\max} = \frac{u^2}{2g} = 10 \text{ m} \qquad \ldots(1)$$

The maximum horizontal distance the boy can throw the same stone is

$$R_{\max} = \frac{u^2}{g} = 20 \text{ m} \qquad \{\text{Using (1)}\}$$

Hence, the correct answer is (C).

9. $R_{\max} = \frac{v^2 \sin 90^\circ}{g} = \frac{v^2}{g}$

Area $= \pi\left(R_{\max}\right)^2 = \frac{\pi v^4}{g^2}$

Hence, the correct answer is (B).

10. The position vector of the particle from the origin at any time t is

$$\vec{r} = v_0 t\cos\theta\hat{i} + \left(v_0 t\sin\theta - \frac{1}{2}gt^2\right)\hat{j}$$

Velocity vector, $\vec{v} = \frac{d\vec{r}}{dt}$

$$\Rightarrow \quad \vec{v} = \frac{d}{dt}\left(v_0 t\cos\theta\hat{i} + \left(v_0 t\sin\theta - \frac{1}{2}gt^2\right)\hat{j}\right)$$

$$\Rightarrow \quad \vec{v} = v_0\cos\theta\hat{i} + \left(v_0\sin\theta - gt\right)\hat{j}$$

The angular momentum of the particle about the origin is

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$\Rightarrow \quad \vec{L} = m(\vec{r} \times \vec{v})$$

$$\Rightarrow \quad \vec{L} = m\left[\left(v_0 t\cos\theta\hat{i} + \left(v_0 t\sin\theta - \frac{1}{2}gt^2\right)\hat{j}\right)\times \left(v_0\cos\theta\hat{i} + \left(v_0\sin\theta - gt\right)\hat{j}\right)\right]$$

$$\Rightarrow \quad \vec{L} = m\left[\left(v_0^2 t\cos\theta\sin\theta - v_0 gt^2\cos\theta\right)\hat{k} + \left(v_0^2 t\sin\theta\cos\theta - \frac{1}{2}gt^2 v_0\cos\theta\right)(-\hat{k})\right]$$

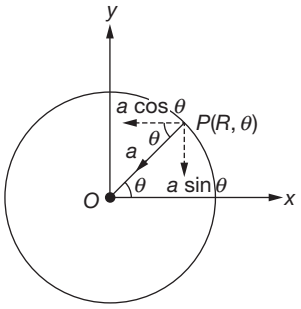
$$\Rightarrow \quad \vec{L} = m\left[v_0^2 t\sin\theta\cos\theta\hat{k} - v_0 gt^2\cos\theta\hat{k} - v_0^2 t\sin\theta\cos\theta\hat{k} + \frac{1}{2}v_0 gt^2\cos\theta\hat{k}\right]$$

$$\Rightarrow \quad \vec{L} = m\left[-\frac{1}{2}v_0 gt^2\cos\theta\hat{k}\right] = -\frac{1}{2}mgv_0 t^2\cos\theta\hat{k}$$

Hence, the correct answer is (D).

11. For a particle in uniform circular motion, acceleration is

$$a = \frac{v^2}{R} \text{ (towards the centre)}$$



From figure, we get

$$\vec{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$$

Hence, the correct answer is (D).

12. $s = t^3 + 3$

$$\Rightarrow \quad v = \frac{ds}{dt} = \frac{d}{dt}\left(t^3 + 3\right) = 3t^2$$

Tangential acceleration, $a_t = \frac{dv}{dt} = \frac{d}{dt}\left(3t^2\right) = 6t$

At $t = 2$ s, we have

$$v = 3(2)^2 = 12 \text{ ms}^{-1}$$

and $a_t = 6(2) = 12 \text{ ms}^{-2}$

Also, centripetal acceleration is given by

$$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ ms}^{-2}$$

Net acceleration,

$$a = \sqrt{(a_c)^2 + (a_t)^2} = \sqrt{(7.2)^2 + (12)^2} = 14 \text{ ms}^{-2}$$

Hence, the correct answer is (A).

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Integer/Numerical Answer Type Questions

1. Average velocity $\langle v \rangle$ is given by

$$\langle v \rangle = \frac{\Sigma R}{\Sigma T}$$

where $\Sigma R = \left(\frac{2u_0^2 \sin\theta\cos\theta}{g}\right)\left(1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + ...\right)$

$$\Sigma R = \left(\frac{2u_0^2 \sin\theta\cos\theta}{g}\right)\left(\frac{1}{1 - \frac{1}{\alpha^2}}\right)$$

and $\Sigma T = \left(\frac{2u_0 \sin\theta}{g}\right)\left(1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + ...\right)$

$$\Rightarrow \quad \Sigma T = \left(\frac{2u_0 \sin\theta}{g}\right)\left(\frac{1}{1 - \frac{1}{\alpha}}\right)$$

Since $\langle v \rangle = 0.8V_1$, where $V_1 = u_0 \cos\theta$

$$\Rightarrow \quad 0.8V_1 = \frac{u_0 \cos\theta\left(\frac{1}{1 - \frac{1}{\alpha^2}}\right)}{\frac{1}{1 - \frac{1}{\alpha}}}$$

$$\Rightarrow \quad \left(\frac{\alpha^2}{\alpha^2 - 1}\right)\left(\frac{\alpha - 1}{\alpha}\right) = 0.8$$

$$\Rightarrow \quad \frac{\alpha}{\alpha + 1} = 0.8$$

$$\Rightarrow \quad 5\alpha = 4\alpha + 4$$

$$\Rightarrow \quad \alpha = 4$$

2. $H = \frac{u^2 \sin^2(45°)}{2g} = 120 \text{ m}$

$$\Rightarrow \quad \frac{u^2}{4g} = 120 \text{ m}$$

If speed is v after the first collision, then speed should remain $\frac{1}{\sqrt{2}}$ times, because kinetic energy has reduced to half.

$$\Rightarrow \quad v = \frac{u}{\sqrt{2}}$$

$$\Rightarrow \quad h_{\max} = \frac{v^2 \sin^2(30°)}{2g}$$

$$\Rightarrow \quad h_{\max} = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30°}{2g}$$

$$\Rightarrow \quad h_{\max} = \left(\frac{u^2/4g}{4}\right) = \frac{120}{4}$$

$$\Rightarrow \quad h_{\max} = 30 \text{ m}$$

3. $T = \frac{2u_y}{g} = \frac{2 \times 10 \sin(60°)}{g} = \sqrt{3} \text{ s}$

Since $R = 1.15$ m and

$$R = u_x T - \frac{1}{2} a_x T^2$$

$$\Rightarrow \quad 1.15 = 10\cos(60°)\sqrt{3} - \frac{1}{2} a\left(\sqrt{3}\right)^2$$

$$\Rightarrow \quad a = 5 \text{ ms}^{-2}$$

CHAPTER 6: NEWTON'S LAWS OF MOTION

Test Your Concepts-I (Based on Impulse Momentum)

1. Since $\Delta\vec{p} = \vec{F}\Delta t$

Now, $\Delta p_y = m(v_y - u_y) = mv[\cos(30°) - \cos(30°)]$

$\Rightarrow \quad \Delta p_y = 0$

Similarly, $\Delta p_x = m(v_x - u_x)$

$\Rightarrow \quad \Delta p_x = mv[-\sin(30°) - \sin(30°)]$

$\Rightarrow \quad \Delta p_x = -2mv\sin(30°) = -30 \text{ kgms}^{-1}$

So, $F_{av} = \dfrac{30}{0.2} = 150 \text{ N}$

2. Velocity of the ball just before hitting the floor is

$v_1 = \sqrt{2gh_1}$, downwards $\quad \{\because v_1^2 - 0^2 = 2gh_1\}$

Velocity of the ball just after impact with the floor is

$v_2 = \sqrt{2gh_2}$, upwards $\quad \{\because 0^2 - v_2^2 = 2(-g)h_2\}$

From Impulse Momentum theorem,

Impulse = Change in Momentum

$\Rightarrow \quad I = m(v_2 + v_1)$

$\Rightarrow \quad I = \dfrac{150}{1000}\left(\sqrt{2(10)(20)} + \sqrt{2(10)(5)}\right)$

$\Rightarrow \quad I = \dfrac{150}{1000}(20 + 10) = 4.5 \text{ kgms}^{-1}$

So, impulse is 4.5 kgms^{-1}, upwards

3. (a) Since impulse is equal to the area under the F-t graph

$I = \int F dt$ = Area under F-t graph

$\Rightarrow \quad I = \dfrac{1}{2}(3.5 - 1)(10^{-3})(16000)$

$\Rightarrow \quad I = 20 \text{ Ns}$

(b) Since we know that $F_{av}\Delta t = \Delta I$

$\Rightarrow \quad F_{av}(2.5 \times 10^{-3}) = 20$

$\Rightarrow \quad F_{av} = 8000 \text{ N} = 8 \text{ kN}$

(c) From the graph we observe that the peak force exerted on the ball is 16000 N = 16 kN.

4. $F = \dfrac{\Delta p_{\text{water}}}{\Delta t} = \dfrac{m(v-u)}{\Delta t}$

$\Rightarrow \quad F = (0.6)(25 - 0) \quad \left\{\because \dfrac{m}{\Delta t} = 0.6\right\}$

$\Rightarrow \quad F = 15 \text{ N}$

Now due to Newton's Third Law, the water exerts a force of equal magnitude back on the hose, hence the gardener must apply a 15 N force in the direction of the velocity of water stream to hold the hose in its position (stationary).

5. When the diver falls freely, then the velocity of the diver just before he hits the surface of water is

$u = \sqrt{2gh} = \sqrt{2(9.8)(10)} = 14 \text{ ms}^{-1}$

Now $|F| = \left|\dfrac{\Delta p}{\Delta t}\right| = \left|\dfrac{m(v-u)}{\Delta t}\right|$

$\Rightarrow \quad |F| = \left|\dfrac{(60)(4 - 14)}{1}\right|$

$\Rightarrow \quad |F| = 600 \text{ N}$

Test Your Concepts-II (Based on Constraints)

1. As already done in an Illustration, we obtained

$a_B + a_C + 2a_A = 0$

Similarly, we can find

$v_B + v_C + 2v_A = 0$

Taking, upward direction as positive we are given:

$v_A = v_B = 1 \text{ ms}^{-1}$

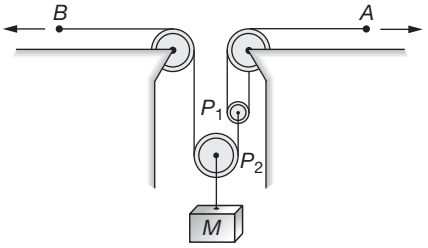
$\Rightarrow \quad v_C = -3 \text{ ms}^{-1}$

i.e., velocity of block C is 3 ms^{-1} (downwards).

2. $v_A = 2 \text{ ms}^{-1}$ {towards right}

$\Rightarrow \quad v_{P_1} = \dfrac{v_A}{2} = 1 \text{ ms}^{-1}$ {upwards}

$v_B = 2 \text{ ms}^{-1}$ {towards left}



Now $2v_{P_2} = v_B + v_{P_1}$

$$\Rightarrow \quad v_{P_2} = \frac{v_B + v_{P_1}}{2} = \frac{2+1}{2} = 1.5 \text{ ms}^{-1}$$

3. Let B and C both move upwards (alongwith their pulleys) with speeds v_B and v_C, then we observe that, A will move downward with speed, $2v_B + 2v_C$. So, with sign we write velocity of A as

$$v_A = 2v_B - 2v_C$$

$$\Rightarrow \quad v_B = -\frac{v_A}{2} - v_C$$

Substituting $v_A = -2 \text{ ms}^{-1}$ and $v_C = 1 \text{ ms}^{-1}$, we get

$$v_B = 0$$

4. $OB = x = \left(L^2 - y^2\right)^{1/2}$

Since, $v_x = \dfrac{dx}{dt} = \dfrac{(-2y)\left(\dfrac{dy}{dt}\right)}{2\left(L^2 - y^2\right)^{1/2}} = -\dfrac{yv_A}{\left(L^2 - y^2\right)^{1/2}}$

$$\Rightarrow \quad a_x = \frac{dv_x}{dt}$$

$$\Rightarrow \quad a_x = -\left[\frac{\sqrt{L^2 - y^2}\left(y\dfrac{dv_A}{dt} + v_A\dfrac{dy}{dt}\right) - \dfrac{yv_A(-2y)\dfrac{dy}{dt}}{2\left(L^2 - y^2\right)^{1/2}}}{\left(L^2 - y^2\right)}\right]$$

Substituting

$$\frac{dv_A}{dt} = 0 \text{ and } \frac{dy}{dt} = v_A \qquad \{\because v_A = \text{constant}\}$$

$$\Rightarrow \quad a_x = -\frac{L^2 v_A^2}{\left(L^2 - y^2\right)^{3/2}}$$

5. $a_A = \dfrac{d^2y}{dt^2} = \dfrac{1}{2} \text{ ms}^{-2}$

From constraint equations, we get $a_B = 8a_A$ (in opposite direction). Hence, if a_A is $\dfrac{1}{2} \text{ ms}^{-2}$ upwards, a_B will be 4 ms^{-2} downwards.

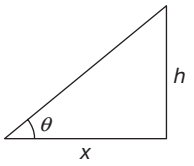
6. Since $h = x\tan\theta$

$$\Rightarrow \quad x = h\cot\theta$$

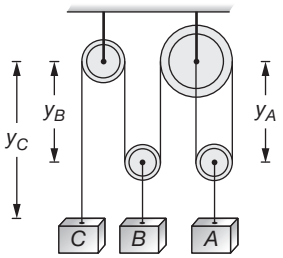
$$\Rightarrow \quad \frac{dx}{dt} = -h\,\text{cosec}^2\theta\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \quad \left|\frac{d\theta}{dt}\right| = \omega = \frac{1}{h\,\text{cosec}^2\theta}\frac{dx}{dt}$$

$$\Rightarrow \quad \omega = \frac{v}{h}\sin^2\theta \qquad \left\{\because v = \frac{dx}{dt}\right\}$$



7. Length of cable $L = 2y_A + 2y_B + y_C + \text{constant}$



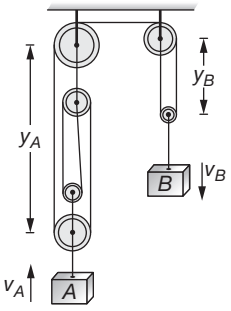
$$0 = 2\dot{y}_A + 2\dot{y}_B + \dot{y}_C$$

$$0 = 2\ddot{y}_A + 2\ddot{y}_B + \ddot{y}_C$$

So, $2a_A + 2a_B + a_C = 0$

2 degrees of freedom

8. Total length of cable to within constants is



$$L = 4y_A + 2y_B$$

$$0 = 4\dot{y}_A + 2\dot{y}_B$$

$$0 = 4\ddot{y}_A + 2\ddot{y}_B$$

Upward acceleration of A is

$$a_A = -\ddot{y}_A = \frac{1}{2}\ddot{y}_B = \frac{1}{2}a_B$$

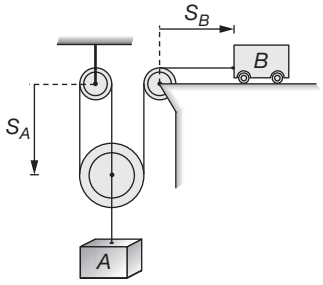
$$v_B = \frac{t^2}{2} + \frac{t^3}{6},\ a_B = \dot{v}_B = t + \frac{t^2}{2}$$

For $t = 2$ s, $a_B = 2 + \frac{4}{2} = 4$ ms^{-2}

So, $a_A = \frac{1}{2}a_B = 2$ ms^{-2}

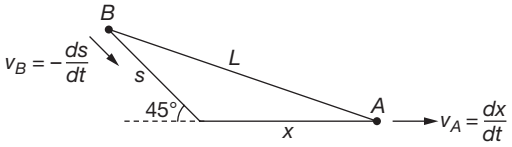
9. Length of cable $L = S_B + 3S_A + \text{constant}$

$$0 = v_B + 3v_A,\ v_A = -\frac{v_B}{3} = -\left(\frac{-1.2}{3}\right) = 0.4 \text{ ms}^{-1} \text{ (down)}$$



10. $L^2 = \left(x + \frac{s}{\sqrt{2}}\right)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = x^2 + \frac{2xs}{\sqrt{2}} + s^2$

$$\Rightarrow \quad 2x\frac{dx}{dt} + \frac{2\frac{dx}{dt}s}{\sqrt{2}} + \frac{2x\frac{ds}{dt}}{\sqrt{2}} + 2s\frac{ds}{dt} = 0$$



$$\Rightarrow \quad xv_A + \frac{s}{\sqrt{2}}v_A - \frac{x}{\sqrt{2}}v_B - sv_B = 0$$

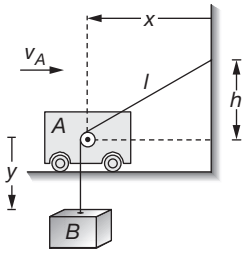
$$\Rightarrow \quad v_B\left(s + \frac{x}{\sqrt{2}}\right) = v_A\left(x + \frac{s}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v_B = \frac{x + \frac{s}{\sqrt{2}}}{s + \frac{x}{\sqrt{2}}}v_A$$

$$\Rightarrow \quad v_B = \frac{s + \sqrt{2}x}{x + \sqrt{2}s}v_A$$

11. $l^2 = x^2 + h^2$

$$\Rightarrow \quad l\frac{dl}{dt} = x\frac{dx}{dt} \qquad \{\because\ h = \text{Constant}\}$$



and $\dot{y} = -\dot{l} = -\frac{x}{l}\dot{x}$

But $v_A = -\frac{dx}{dt}$

So, $(v_B)_y = \frac{dy}{dt} = \frac{x}{l}v_A$

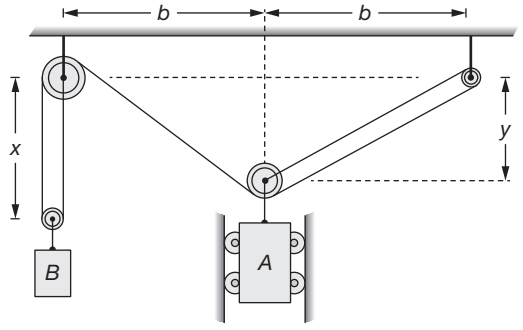
$$(v_B)_x = \frac{dx}{dt} = -v_A$$

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = v_A\sqrt{1 + \frac{x^2}{l^2}}$$

$$\Rightarrow \quad v_B = v_A\sqrt{\frac{2x^2 + h^2}{x^2 + h^2}}$$

12. The total length of the cable is

$$L = 2x + 3\sqrt{y^2 + b^2} + \text{constant}$$



Differentiate to obtain

$$\frac{dL}{dt} = 0 = 2\frac{dx}{dt} + 3\frac{y}{\sqrt{y^2 + b^2}}\left(\frac{dy}{dt}\right)$$

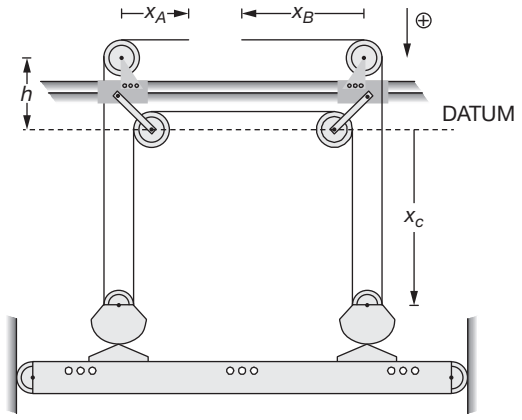
where $\frac{dx}{dt} = v_B$ and $\frac{dy}{dt} = v_A$

So, we get

$$v_B = -\frac{3y}{2\sqrt{y^2 + b^2}}v_A$$

13. Make the constraint equation. 1.8 ms^{-1}, down

14. Datum is established as shown.



The position of point A and B and load C with respect to datum are x_A, x_B and x_C, respectively.

$$4x_C + x_A + x_B + 2h = l \qquad \ldots(1)$$

Since h is a constant, taking the time derivative equation (1), we get

$$4v_C + v_A + v_B = 0 \qquad \ldots(2)$$

Since $v_A = 60\ \text{cms}^{-1}$ and $v_B = 120\ \text{cms}^{-1}$ from equation (2), we get

$$4v_C + 60 + 120 = 0$$

$$v_C = -45\ \text{cms}^{-1} = 45\ \text{cms}^{-1}, \text{ upwards}$$

15. $2s_A + (h - s_C) = l$

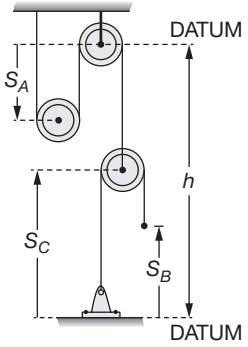
$$2v_A = v_C$$

$$s_C + (s_C - s_B) = l$$

$$2v_C = v_B$$

$$v_B = 4v_A$$

$$a_B = 4a_A$$



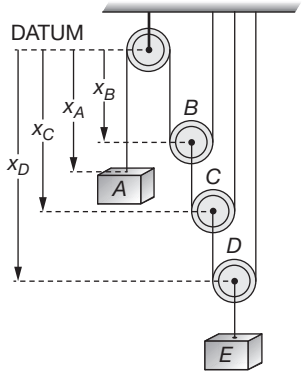
Thus, $-4 = 4v_A$

$$v_A = -1\ \text{ms}^{-1} = 1\ \text{ms}^{-1}, \text{ upwards}$$

$$2 = 4a_A$$

$$a_A = 0.5\ \text{ms}^{-2} = 0.5\ \text{ms}^{-2}, \text{ downwards}$$

16. Datum is located at fixed pulley. The position of point A, pulley B, C and D (or block E) with respect to datum are x_A, x_B, x_C and x_D respectively. Since the system consists of three cords, three position-coordinate equations can be developed.



$$2x_B + x_A = l_1 \qquad \ldots(1)$$

$$x_C + (x_c - x_B) = l_2 \qquad \ldots(2)$$

$$x_D + (x_D - x_C) = l_3 \qquad \ldots(3)$$

Eliminating x_C and x_B from equations (1), (2) and (3), we have

$$x_A + 8x_D = l_1 + 2l_2 + 4l_3$$

Taking the time derivative of the above equation yields

$$v_A + 8v_D = 0 \qquad \ldots(4)$$

Since $v_A = 2\ \text{ms}^{-1}$, from equation (3) and taking downward direction as positive, we get

$$2 + 8v_D = 0$$

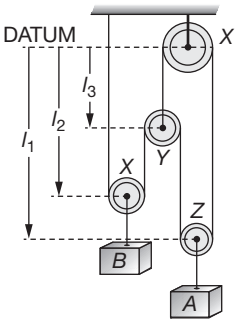
$$\Rightarrow \quad v_D = -0.25\ \text{ms}^{-1} = 0.25\ \text{ms}^{-1}, \text{ upwards}$$

Since the block E is connected directly to the pulley D, so

$$v_E = v_D = 0.25\ \text{ms}^{-1}, \text{ upwards}$$

17. (a) $l_2 + (l_2 - l_3) + (l_1 - l_3) + l_1 + l_3 = \text{constant}$

$$\Rightarrow \quad 2l_2 + 2l_1 - l_3 = \text{constant}$$



$$\Rightarrow \quad 2\left(\frac{dl_2}{dt}\right)+2\left(\frac{dl_1}{dt}\right)-\frac{dl_3}{dt}=0$$

$$\Rightarrow \quad 2v_B+2v_A-v_y=0 \qquad \ldots(1)$$

For $v_B=1\ \text{ms}^{-1}$ and $v_y=2\ \text{ms}^{-1}$, we get

$$v_A=0$$

(b) From (1), we get

$$2a_B+2a_A-a_y=0$$

Taking downward direction as positive, we get

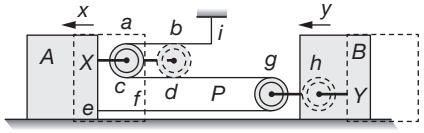
$$a_B=-3\ \text{ms}^{-2} \text{ and } a_y=4\ \text{ms}^{-2}$$

$$\Rightarrow \quad 2(-3)+2a_A-4=0$$

$$\Rightarrow \quad 2a_A=10$$

$$\Rightarrow \quad a_A=5\ \text{ms}^{-2}$$

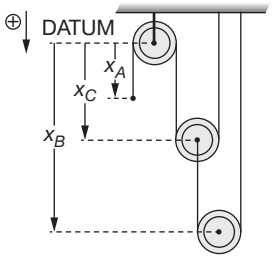
18. Analysis of the problem is shown in figure. If the block A moves towards left by a distance x, the string lengths $ab+cd+ef=3x$ will be pulled towards right and to provide this length block B has to move toward left by a distance y such that the string lengths on the two sides of the pulley Y (twice of ab) i.e., $2y$ will slack to tight the string $3x$. Thus we have $3x=2y$.



As here we are required to find the displacement of the point P, we can start from either end of string e or i. If we start from i, we can see that from i to P there is only pulley X which pulls the thread by a distance $2x$, thus, P will move toward left by $2x$.

Alternatively we can start from e. As we can see that from e to P there is only pulley Y, which slacks the string by $2y$ and point e is pulled toward left by a distance x, thus point P will move toward left by a distance $2y-x$, which is again $2x$. Thus the displacement of point P is twice of x which is 6 m.

19. Let the datum be passing through the fixed pulley D. The position of point A, block B and pulley C with respect to datum are x_A, x_B and x_C, respectively. Since the system consists of two cords, two position coordinate equations can be developed.



$$2x_C+x_A=l_1 \qquad \ldots(1)$$

$$x_B+(x_B-x_C)=l_2 \qquad \ldots(2)$$

Eliminating s_C from equations (1) and (2) yields

$$x_A+4x_B=l_1+2l_2 \qquad \ldots(3)$$

Taking the time derivative of equation (3), we get

$$v_A+4v_B=0 \qquad \ldots(4)$$

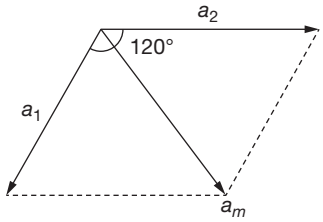
Since $v_A=2\ \text{ms}^{-1}$, from equation (3)

$$\Rightarrow \quad 2+4v_B=0$$

$$v_B=-0.5\ \text{ms}^{-1}=0.5\ \text{ms}^{-1}, \text{ upwards}$$

20. $v_A=2v_0$, downwards

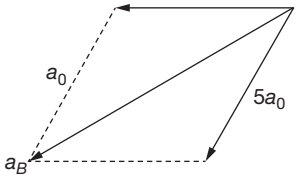
21. As m is also moving down along the incline with M, we can find the net acceleration of m using vector addition of the two acceleration in m, shown in figure.



$$a_m=\sqrt{a_1^2+a_2^2+2a_1a_2\cos(120°)}$$

$$\Rightarrow \quad a_m=\sqrt{25+9-15}=\sqrt{19}=4.36\ \text{ms}^{-2}$$

22. When A moves to the left through x, then a portion $5x$ of the string is loosened which makes the block B to go down by $5x$. So, acceleration of B w.r.t A is $5a_0$. Now, acceleration of B w.r.t. ground is calculated from the diagram



$$\Rightarrow \quad a_B = a_0\sqrt{1+25+10\left(\frac{1}{2}\right)}$$

$$\Rightarrow \quad a_B = \sqrt{31}a_0$$

23. If the distance of mass M from the ceiling is y and the distance of M from each pulley is x and the distance between the two pulleys is l. Then u will be the rate at which x is decreasing. If v is the velocity of M upward, it is the rate at which y is decreasing. Thus we have

$$u = -\frac{dx}{dt}$$

and $v = -\frac{dy}{dt}$

Now we find the relation in x and y as

$$x^2 + \frac{l^2}{4} = y^2$$

On differentiating with respect to t

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$xu = yv$$

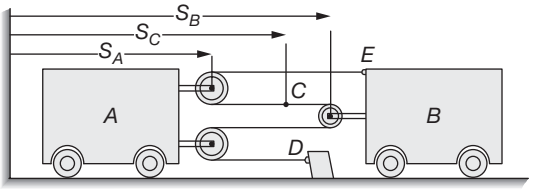
$$v = u\frac{dx}{dt} = u\sec\theta$$

25. Cable length $L = 3(S_B - S_A) + (S_D - S_A)$

$$\Rightarrow \quad 0 = 3v_B - 4v_A \text{ and } 0 = 3a_B - 4a_A$$

$$\Rightarrow \quad v_A = \frac{3}{4}v_B = \frac{3}{4}(2) = 1.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad a_A = \frac{3}{4}a_B = \frac{3}{4}(3) = 2.25 \text{ ms}^{-2}$$



Since $v_{B/A} = v_B - v_A = 2 - (1.5) = 0.5 \text{ ms}^{-1}$

$$a_{B/A} = a_B - a_A = 3 - (2.25) = 0.75 \text{ ms}^{-2}$$

Length of cable between points E and C is

$$L' = (S_B - S_A) + (S_C - S_A) + \text{constants}$$

$$\Rightarrow \quad 0 = v_B - 2v_A + v_C$$

$$\Rightarrow \quad v_C = 2v_A - v_B$$

$$\Rightarrow \quad v_C = 2(1.5) - 2 = 1 \text{ ms}^{-1}$$

(All answers are quantities directed to the right)

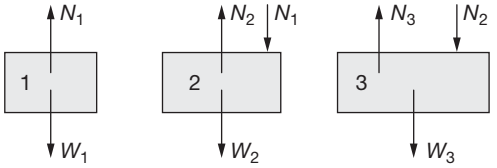
Test Your Concepts-III (Based on F.B.D.)

1. Since, N_1 = normal reaction between 1 and 2

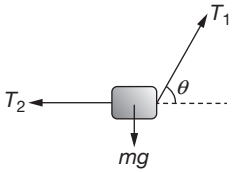
N_2 = normal reaction between 2 and 3

N_3 = normal reaction between 3 and ground

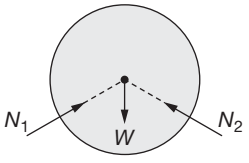
Free body diagrams of 1, 2 and 3 are shown below:



2. The free body diagram of the block is as shown in figure.

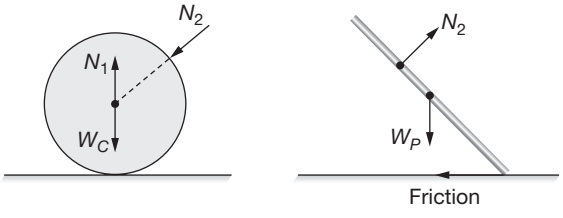


3. The free body diagram of the ball is shown in figure.

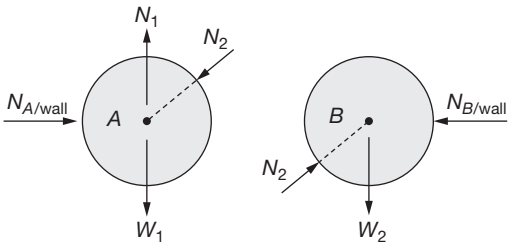


Here, W is weight of ball acting downwards and N_1 and N_2 are the normal reactions between the ball and the two inclined walls.

4.



5.



Test Your Concepts-IV (Based on Newton's Laws of Motion: Accelerated Systems)

1. Since the pull diminishes uniformly at a rate of 1 kgwt per metre for the rope wound up. So, if the length x of the rope is pulled up, then the pull P is given by

$$P = 250\text{ g} - (1)(x)g$$

$\{\because$ pull decreases by 1 kgwt per metre pulled up$\}$

So, if F is the net force required to pull the weight of 200 kg, then

$$F = P - 200\text{ g}$$

$$\Rightarrow \quad F = 250\,g - xg - 200\text{ g}$$

$$\Rightarrow \quad F = (50 - x)g$$

$$\Rightarrow \quad 200a = (50 - x)(10)$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{50 - x}{20}$$

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{5}{2} - \frac{x}{20}$$

$$\Rightarrow \quad vdv = \frac{5}{2}dx - \frac{1}{20}xdx$$

$$\Rightarrow \quad \int_0^v vdv = \frac{5}{2}\int_0^{20} dx - \frac{1}{20}\int_0^{20} x\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \left(\frac{5}{2}x - \frac{x^2}{40}\right)\Bigg|_0^{20}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{5}{2}(20) - \frac{400}{40}$$

$$\Rightarrow \quad \frac{v^2}{2} = 50 - 10$$

$$\Rightarrow \quad v^2 = 80$$

$$\Rightarrow \quad v = 4\sqrt{5}\text{ ms}^{-1}$$

2. Normal reaction between A and B is $N = mg\cos\theta$. Its horizontal component is $N\sin\theta$. Therefore, tension in cord CD is equal to this horizontal component.

Hence, $T = N\sin\theta = (mg\cos\theta)(\sin\theta)$

$$\Rightarrow \quad T = \frac{mg}{2}\sin(2\theta)$$

3. Suppose acceleration of m_1 down the plane is a, then acceleration of m_2, vertically up will be $2a$. Since the pulleys are massless, so, if T is the tension in the string connected to m_2, then the tension in the string connected to mass m_1, is 2T. The equations of motion of the two masses m_1 and m_2 are written for calculating value of a.

For mass m_1,

$$m_1 g\sin\alpha - 2T = m_1 a \qquad \ldots(1)$$

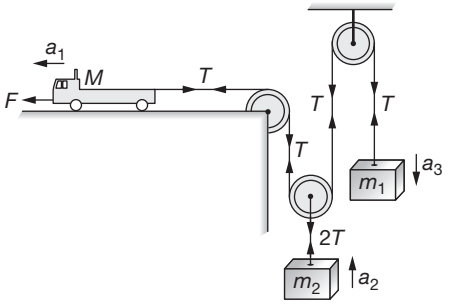
For mass m_2,

$$T - m_2 g = m_2(2a) \qquad \ldots(2)$$

Solving these two equations, we get

$$a = \left(\frac{m_1\sin\alpha - 2m_2}{m_1 + 4m_2}\right)g \text{ and } 2a = 2\left(\frac{m_1\sin\alpha - 2m_2}{m_1 + 4m_2}\right)g$$

4. Tensions in different branches of the string are shown in figure. The direction and magnitude of acceleration, initially assumed by us are also shown.



Let the mass m_2 moves up by a distance x_2, pulley attached to it will also move up by x_2, which will result a slackness of $2x_2$ in the string attached to truck and m_1. If in the same duration, truck moves by x_1 and mass m_1 moves down by x_3, we have $x_1 + x_3 = 2x_2$. Same relation we have among the acceleration of the respective bodies as

$$a_1 + a_3 = 2a_2 \qquad \ldots(1)$$

Writing the equations of the three bodies, we get for truck, we have

$$F - T = Ma_1 \qquad \ldots(2)$$

for mass m_1, we have

$$m_1 g - T = m_1 a_3 \qquad \ldots(3)$$

for mass m_2

$$2T - m_2 g = m_2 a_2 \quad \ldots(4)$$

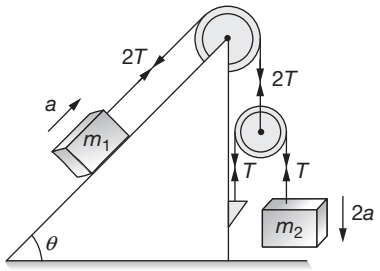
Solving these equations for the given data, we get

$$T = (\text{force acting on truck towards right}) = 4.21 \text{ N}$$

$$a_3 (= \text{acceleration of } m_1) = 5.79 \text{ ms}^{-2}, \text{ downwards}$$

and $a_2 (= \text{acceleration of } m_2) = 6.84 \text{ ms}^{-2}$, upwards

5. As discussed earlier that for a movable pulley fixed at one end, the acceleration of the free end is twice the acceleration of the pulley. So, acceleration of m_2 is two times that of m_1 (attached directly to the pulley through thread). So, we assume if m_1 is moving up the inclined plane with an acceleration a, the acceleration of mass m_2 going down is $2a$. The tensions in different strings are shown in figure.



The dynamic equations can be written as

For mass m_1

$$2T - m_1 g \sin\theta = m_1 a \quad \ldots(1)$$

For mass m_2

$$m_2 g - T = m_2 (2a) \quad \ldots(2)$$

Substituting $m_2 = \eta m_1$ and solving equations (1) and (2), we get acceleration of m_2 as

$$2a = \frac{2g(2\eta - \sin\theta)}{4\eta + 1}$$

6. From our knowledge of constraint relations we get that the acceleration of the rod is double than that of the acceleration of the ball. If ball is going up with an acceleration a, rod will be coming down with the acceleration $2a$, thus, the relative acceleration of the ball with respect to rod is $3a$ in upward direction. If it takes time t second to reach the upper end of the rod, we have

$$2l = \frac{1}{2}(3a)t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2l}{3a}} \quad \ldots(1)$$

Let mass of ball be m and that of rod is M, the dynamic equations of these are

For rod $Mg - T = M(2a)$ …(2)

For ball $2T - mg = ma$ …(3)

Substituting $m = \eta M$ and solving equations (2) and (3), we get

$$a = \left(\frac{2-\eta}{\eta+4}\right) g$$

Substituting the value of a in equation (1), we get

$$t = \sqrt{\frac{2l(\eta+4)}{3g(2-\eta)}}$$

7. (a) In this case net pulling force is

$$F = m_A g \sin(60°) + m_B g \sin(60°) - m_C g \sin(30°)$$

$$\Rightarrow \quad F = (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\left(\frac{\sqrt{3}}{2}\right) - (2)(10)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad F = 24.64 \text{ N}$$

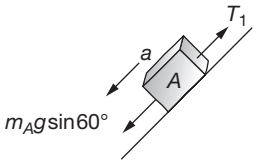
Total mass being pulled $= 1 + 3 + 2 = 6$ kg

So, acceleration of the system is

$$a = \frac{24.64}{6} = 4.1 \text{ ms}^{-2}$$

(b) For the tension in the string between A and B

F.B.D. of A

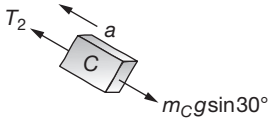


$$m_A g \sin 60° - T_1 = (m_A)(a)$$

$$\Rightarrow \quad T_1 = m_A g \sin(60°) - m_A a = m_A (g \sin(60°) - a)$$

$$\Rightarrow \quad T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56 \text{ N}$$

For the tension in the string between B and C



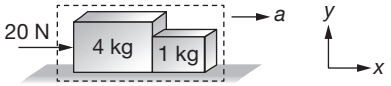
F.B.D. of C

$$T_2 - m_C g \sin 30° = m_C a$$

$$\Rightarrow \quad T_2 = m_C\left(a + g\sin(30°)\right)$$

$$\Rightarrow \quad T_2 = 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2 \text{ N}$$

8. (a) Since, both the blocks will move with same acceleration $(\text{say } a)$ in horizontal direction.



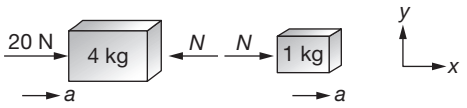
Let us take both the blocks as a system. Net external force on the system is 20 N in horizontal direction.

Using $\Sigma F_x = ma_x$

$$20 = (4+1)a = 5a$$

$$\Rightarrow \quad a = 4 \text{ ms}^{-2}$$

(b) The free body diagram of both the blocks are as shown in figure



Using $\Sigma F_x = ma_x$, we get

For 4 kg block

$$20 - N = 4a = 4 \times 4$$

$$N = 20 - 16 = 4 \text{ N}$$

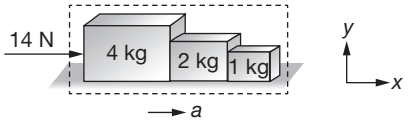
For 1 kg block

$$N = 1a = 1 \times 4 = 4 \text{ N}$$

Here, N is the normal reaction between the two blocks.

Please note that in free body diagram of the blocks we have not shown the forces acting on the blocks in vertical direction, because normal reaction between the blocks and acceleration of the system can be obtained without using $\Sigma F_y = 0$.

9. Since, all the blocks will move with same acceleration $(\text{say } a)$ in horizontal direction. Let us take all the blocks as a system.



Net external force on the system is 12 N in horizontal direction.

Using $\Sigma F_x = ma_x$, we get

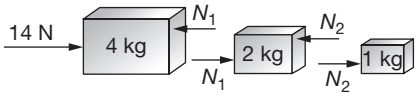
$$14 = (4+2+1)a = 7a$$

$$\Rightarrow \quad a = \frac{14}{7} = 2 \text{ ms}^{-2}$$

Now, let F be the net force on 2 kg block in x-direction, then using $\Sigma F_x = ma_x$ for 2 kg block, we get

$$F = (2)(2) = 4 \text{ N}$$

Also drawing the free body diagram for the arrangement will given same result.



Since, $14 - N_1 = 4(2)$

$$\Rightarrow \quad N_1 = 6 \text{ N}$$

Also, $N_1 - N_2 = 2(2)$

$$\Rightarrow \quad 6 - N_2 = 4$$

$$\Rightarrow \quad N_2 = 2 \text{ N}$$

Please note that here net force F on 2 kg block is the resultant of N_1 and $N_2 (N_1 > N_2)$

where N_1 = normal reaction between 4 kg and 2 kg block.

and N_2 = normal reaction between 2 kg and 1 kg block.

Thus, $F = N_1 - N_2 = 4$ N.

10. Block B will fall vertically downwards and A along the plane.

Writing the equations of motion, we get

for block B, we have

$$m_B g - N = m_B a_B$$

$$\Rightarrow \quad 60 - N = 6a_B \qquad \ldots(1)$$

for block A, we have

$$(N + m_A g)\sin(30°) = m_A a_A$$

$$\Rightarrow \quad (N + 150) = 30a_A \qquad \ldots(2)$$

Further $a_B = a_A \sin(30°)$

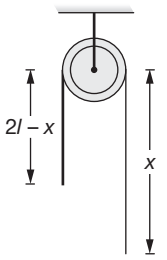
$$\Rightarrow \quad a_A = 2a_B \qquad \ldots(3)$$

Solving equations, (1), (2) and (3), we get

$$a_A = 6.36 \text{ ms}^{-2}$$

$$a_{BA} = a_A \cos(30°) = 5.5 \text{ ms}^{-2}$$

11. Net pulling force is actually due to the difference of the weights of the rope on both sides of the pulley. So,



$$F_{net} = \left[\left(\frac{m}{2l}\right)(x) - \left(\frac{m}{2l}\right)(2l - x)\right]g = \frac{mg}{l}(x - l)$$

$$\Rightarrow \quad ma = \frac{g}{l}(x - l)$$

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{g}{l}(x - l)$$

$$\Rightarrow \quad \int_0^v v\,dv = \frac{g}{l}\int_{(l+c)}^{x}(x - l)dx$$

Since $\int (x-a)dx = \frac{(x-a)^2}{2}$, so

$$\frac{v^2}{2} = \frac{g}{2l}\left[(x-l)^2\right]_{l+c}^{x}$$

$$\Rightarrow \quad v^2 = \frac{g}{l}\left[(x-l)^2 - c^2\right]$$

$$\Rightarrow \quad v = \frac{dx}{dt} = \sqrt{\frac{g}{l}\left[(x-l)^2 - c^2\right]}$$

$$\Rightarrow \quad \int_0^t dt = \int_{(l+c)}^{2l} \frac{dx}{\sqrt{\frac{g}{l}\left[(x-l)^2 - c^2\right]}}$$

$$\Rightarrow \quad t = \sqrt{\frac{l}{g}}\ln\left[\frac{l + \sqrt{l^2 - c^2}}{c}\right]$$

12. Let absolute acceleration of m and M be a_1 and a_2 respectively.

Writing equations of motion

for m, $mg\cos\alpha - N = ma_1$ …(1)

and for M, $N\sin\alpha = Ma_2$ …(2)

Constraint equation can be written as,

$$a_1 = a_2\sin\alpha \qquad \ldots(3)$$

Solving above three equations, we get

Acceleration of rod is

$$a_1 = \frac{mg\cos\alpha\sin\alpha}{\left(m\sin\alpha + \frac{M}{\sin\alpha}\right)}$$

and acceleration of wedge is

$$a_2 = \frac{mg\cos\alpha}{m\sin\alpha + \frac{M}{\sin\alpha}}$$

Please note that here $a_2 \neq a_1\sin\alpha$.

13. $$a = \frac{3T - mg\sin\theta}{m} = \frac{(3)(250) - (100)(10)\sin(30°)}{100}$$

$$\Rightarrow \quad a = \frac{750 - 500}{100}$$

$$\Rightarrow \quad a = \frac{250}{100} = 2.5 \text{ ms}^{-2}$$

14. As done already, from constraint relations we can see that $2a_A = 3a_B$

$$\Rightarrow \quad a_A = \frac{3}{2}a_B$$

So let $a_B = a$ then $a_A = 1.5a$

for block A, we have

$$2T = 70a_A = 105a \qquad \ldots(1)$$

for block B, we have

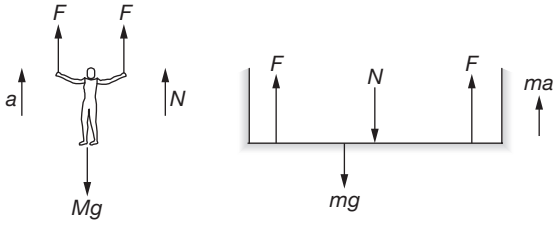
$$300 - 3T = 35a_B = 35a \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$T = 81.8 \text{ N} \text{ and } a = 1.558 \text{ ms}^{-2}$$

$$\Rightarrow \quad T = 81.8 \text{ N},\ a_A = 2.34 \text{ ms}^{-2} \text{ and } a_B = 1.558 \text{ ms}^{-2}$$

15. Let N be the force exerted by the painter on the crate (downwards). Corresponding the same force will be exerted by the crate on the painter (upwards). The free body diagram for both is shown here.



For painter, $2F - Mg + N = Ma$ …(1)

For crate, $2F - mg - N = ma$ …(2)

Adding (1) and (2), we get

$$4F - (M+m)g = (M+m)a$$

$$\Rightarrow \quad a = \frac{4F - (M+m)g}{M+m}$$

16. Time taken by lighter man to reach the pulley is given by

$$h = \frac{1}{2}a_1 t^2 \qquad \ldots(1)$$

For lighter man,

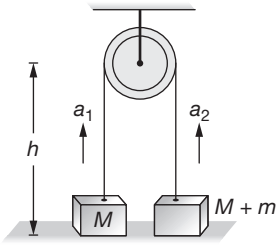
$$T - Mg = Ma_1 \qquad \ldots(2)$$

For heavier man,

$$T - (M+m)g = (M+m)a_2 \qquad \ldots(3)$$

From equations (2) and (3), we get

$$a_2 = \left(\frac{M}{M+m}\right)a_1 - \left(\frac{m}{M+m}\right)g \qquad \ldots(4)$$



In time t, the heavier mass will move a distance, given by

$$s = \frac{1}{2}a_2 t^2$$

$$\Rightarrow \quad s = \frac{1}{2}t^2\left[\left(\frac{M}{M+m}\right)a_1 - \left(\frac{m}{M+m}\right)g\right] \qquad \{\because \text{ of (4)}\}$$

Since, $h = \frac{1}{2}a_1 t^2$ so, we have

$$s = \frac{Mh}{M+m} - \left(\frac{m}{M+m}\right)\frac{gt^2}{2}$$

$$\Rightarrow \quad h - s = \frac{m}{M+m}\left(\frac{gt^2}{2} + h\right)$$

17. Since $a_0\,(=12\ \text{ms}^{-2}) > g\,(=10\ \text{ms}^{-2})$, so the block will leave contact will the floor of the lift and starts falling with an acceleration $a = g = 10\ \text{ms}^{-2}$, so we have

$$s = \frac{1}{2}at^2 = \frac{1}{2}\times 10\times(0.2)^2 = 0.2\ \text{m}$$

$$\Rightarrow \quad s = 20\ \text{cm}$$

18. This motion is just similar to the motion of a projectile moving under the influence of an acceleration $g\sin(45°)$, instead of g.

So, after 2 seconds, we have

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow \quad v = \sqrt{\left(10\sin(45°) - \frac{g}{\sqrt{2}}\times 2\right) + (10\cos 45°)^2}$$

$$\Rightarrow \quad v = 10\ \text{ms}^{-1}$$

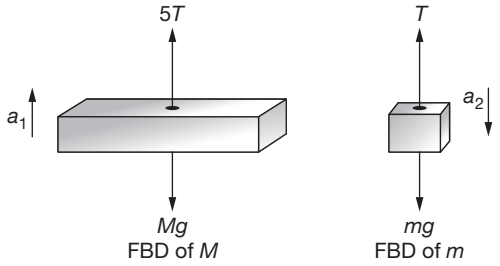
19. Writing equations of motion

for M, $5T - Mg = Ma_1$ $\qquad \ldots(1)$

for m, $mg - T = ma_2$ $\qquad \ldots(2)$

Also, from constraint equation, we get

$$a_2 = 5a_1 \qquad \ldots(3)$$

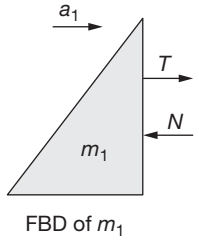


Solving these equations, the acceleration of M and m are given by

$$a_1 = \left(\frac{5m - M}{25m - M}\right)g \text{ and } a_2 = 5\left(\frac{5m - M}{25m - M}\right)g$$

20. FBD of m_1 (showing only the horizontal forces)

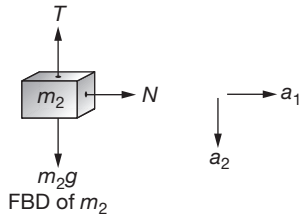
Equation of motion for m_1 is, $T - N = m_1 a_1$ $\qquad \ldots(1)$



Equations of motion for m_2 are given by

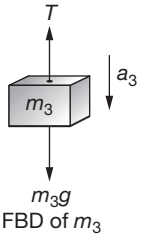
$$N = m_2 a_1 \text{ and} \qquad \ldots(2)$$

$$m_2 g - T = m_2 a_2 \qquad \ldots(3)$$



Equation of motion for m_3 is

$$m_3g - T = m_3a_3 \qquad \ldots(4)$$



Further from constraint equation we get

$$a_1 = a_2 + a_3 \qquad \ldots(5)$$

Solving the five unknowns a_1, a_2, a_3, T and N, we get

$$a_1 = \frac{2m_1m_3g}{(m_2+m_3)(m_1+m_2)+m_2m_3}$$

21. In mode (a), the man applies a force equal to 25 kg wt in upward direction. According to Newton's Third Law of motion, there will be a downward force of reaction on the floor.
So, total action on the floor by the man is

$$N_1 = 50 \text{ kg wt} + 25 \text{ kg wt} = 75 \text{ kg wt}$$

$$N_1 = 75 \times 9.8 \text{ N} = 735 \text{ N}$$

In mode (b), the man applies a downward force equal to 25 kg wt . According to Newton's Third Law, the reaction will be in the upward direction.
So, total action on the floor by the man is

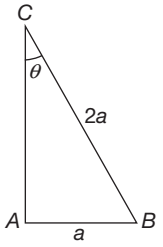
$$N_2 = 50 \text{ kg wt} - 25 \text{ kg wt} = 25 \text{ kg wt}$$

$$N_2 = 25 \times 9.8 \text{ N} = 245 \text{ N}$$

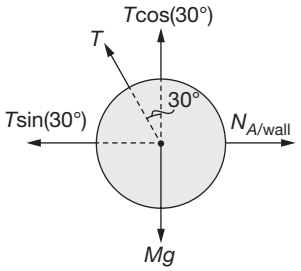
As the floor yields to a downward force of 700 N, so the man should adopt mode (b).

Test Your Concepts-V (Based on Equilibrium)

1. Since $\sin\theta = \frac{a}{2a} = \frac{1}{2}$



$\Rightarrow \quad \theta = 30°$



For equilibrium,

$$T\cos(30°) = Mg$$

$$\Rightarrow \quad T = \frac{2Mg}{\sqrt{3}} \qquad \ldots(1)$$

Since $N_{A/\text{wall}} = T\sin 30° = \frac{T}{2}$

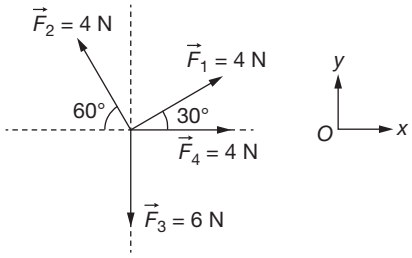
$$\Rightarrow \quad N_{A/\text{wall}} = \frac{Mg}{\sqrt{3}} \qquad \ldots(2)$$

2.

	F_1	F_2	F_3	F_4
x	4 cos (30°)	4 cos (120°)	6 cos (90°)	4 cos (0°)
y	4 sin (30°)	4 sin (120°)	6 sin (90°)	4 sin (0°)

$(F_1)_x = 2\sqrt{3}$ N, $(F_2)_x = -2$ N and

$(F_3)_x = 0$ N, $(F_4)_x = 4$ N



Similarly,

$(F_1)_y = 2$ N, $(F_2)_y = 2\sqrt{3}$ N

$(F_3)_y = 6$ N, $(F_4)_y = 0$ N

So, $(F_{\text{net}})_x = (F_1)_x + (F_2)_x + (F_3)_x + (F_4)_x$

$$\Rightarrow \quad (F_{\text{net}})_x = 2\sqrt{3} - 2 + 0 + 4 = 2(\sqrt{3}+1) \text{ N}$$

Similarly, $(F_{\text{net}})_y = 8 + 2\sqrt{3} \text{ N} = 2(4+\sqrt{3}) \text{ N}$

$$\Rightarrow \quad \vec{F}_{\text{net}} = 2\left[(\sqrt{3}+1)\hat{i} + (4+\sqrt{3})\hat{j}\right] \text{N}$$

3. **METHOD I**

Since, the block is permanently at rest, it is in equilibrium. Net force on it should be zero. In this case only two forces are acting on the block.

(a) Weight $= mg$ (downwards)

(b) Contact force (resultant of normal reaction and friction force) applied by the wedge on the block. For the block to be in equilibrium these two forces should be equal and opposite.
Therefore, force exerted by the wedge on the block is mg (upwards).

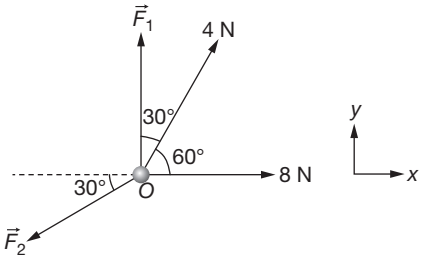
METHOD II

The normal force on the block is $N = mg\cos\theta$ and the friction force on the block is $f = mg\sin\theta = \mu mg\cos\theta$. These two forces are mutually perpendicular. So, net contact force would be

$$\sqrt{N^2 + f^2} = \sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2} = mg$$

4. The object is in equilibrium. Hence,

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$



$$\Rightarrow \quad 8 + 4\cos(60°) - F_2\cos(30°) = 0 \qquad \left\{\because \sum F_x = 0\right\}$$

$$\Rightarrow \quad 8 + 2 - F_2\frac{\sqrt{3}}{2} = 0$$

$$\Rightarrow \quad F_2 = \frac{20}{\sqrt{3}} \text{ N}$$

$$\Rightarrow \quad F_1 + 4\sin(60°) - F_2\sin(30°) = 0 \qquad \left\{\because \sum F_y = 0\right\}$$

$$\Rightarrow \quad F_1 + \frac{4\sqrt{3}}{2} - \frac{F_2}{2} = 0$$

$$\Rightarrow \quad F_1 = \frac{F_2}{2} - 2\sqrt{3} = \frac{10}{\sqrt{3}} - 2\sqrt{3}$$

$$\Rightarrow \quad F_1 = \frac{4}{\sqrt{3}} \text{ N}$$

5. $\vec{F}_{net} = 0$, i.e., $\vec{F}$ and $m\vec{g}$ should be equal and opposite i.e., balance each other, so we have

$$|\vec{F}| = |m\vec{g}|$$

$$\Rightarrow \quad vA\sin\theta = mg$$

$$\Rightarrow \quad v = \frac{mg}{A\sin\theta}$$

v will be MINIMUM, when $\sin\theta$ is maximum. So $\sin\theta =$ MAXIMUM $= 1$

$$\Rightarrow \quad \theta = 90° \text{ and } V_{min} = \frac{mg}{A}$$

6. Various forces acting on the ball are as shown in figure. The three concurrent forces are in equilibrium. Using Lami's theorem

$$\frac{T_1}{\sin(150°)} = \frac{T_2}{\sin(120°)} = \frac{10}{\sin(90°)}$$

$$\Rightarrow \quad \frac{T_1}{\sin(30°)} = \frac{T_2}{\sin(60°)} = \frac{10}{1}$$

So, $T_1 = 10\sin(30°) = 10 \times 0.5 = 5$ N and

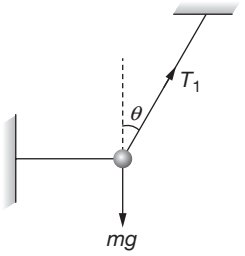
$$T_2 = 10\sin(60°) = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ N}$$

7. In the first case, ball is in equilibrium. Therefore, the net force on ball in any direction should be zero. So,

$$(\Sigma\vec{F}) \text{ in vertical direction} = 0$$

$$\Rightarrow \quad T_1\cos\theta = mg$$

$$\Rightarrow \quad T_1 = \frac{mg}{\cos\theta}$$



Substituting $m_1 = 1$ kg, $g = 10$ ms^{-2} and $\theta = 45°$

$$\Rightarrow \quad T_1 = 10\sqrt{2} \text{ N}$$

In the second case ball is not in equilibrium (temporary rest). After few seconds it will move in a direction perpendicular to OQ. Therefore, net force on the ball at Q is perpendicular to OQ, or net force along $OQ = 0$. So, we get

$$T_2 = mg\cos\theta$$

Substituting the values, we get

$$T_2 = 5\sqrt{2}\ \text{N}$$

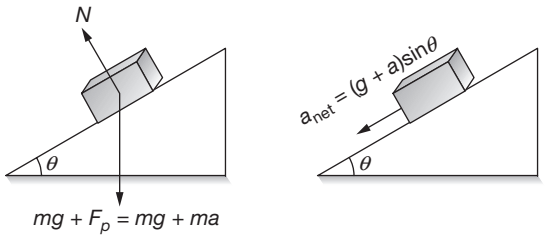
Also, we observe that

$$T_1 \neq T_2$$

Here we deliberately resolved all the forces in vertical direction because component of the tension along the vertical, in the branch RP is zero. Although, since, the ball is in equilibrium, net force on it in any direction is zero. But in a direction other than vertical we will have to consider component of tension in RP also, which will unnecessarily increase the calculation.

Test Your Concepts-VI (Based on Non-inertial Frames: Pseudo Force)

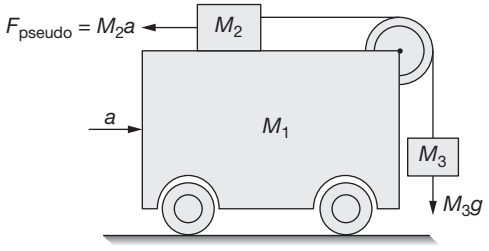
1. Since, the acceleration of block w.r.t. wedge, which happens to be an accelerating or non-inertial frame of reference, is to be find out. So, let us draw the F.B.D. of block w.r.t. wedge is shown in figure.



The acceleration would had been $g\sin\theta$ (down the plane) if the lift were stationary or when only weight (i.e., mg) acts downwards. In this case, however, the downward force is $m(g+a)$.

So, the acceleration of the block w.r.t. wedge will be $(g+a)\sin\theta$, down the plane.

2. The F.B.D. of M_2 and M_3 in accelerated frame of reference is shown in figure



NOTE: Only the necessary forces have been shown.

Mass M_3 will neither rise nor fall if net pulling force is zero.

i.e., $M_2 a = M_3 g$

$$\Rightarrow \quad a = \frac{M_3}{M_2} g$$

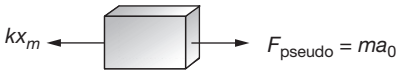
$$\Rightarrow \quad F = (M_1 + M_2 + M_3)a = (M_1 + M_2 + M_3)\frac{M_3}{M_2} g$$

3. In equilibrium, we get

$$kx_m = ma_0$$

$$\Rightarrow \quad x_m = \frac{ma_0}{k}$$

Here, x_m is maximum compression in spring, which is also equal to its amplitudes of oscillation.



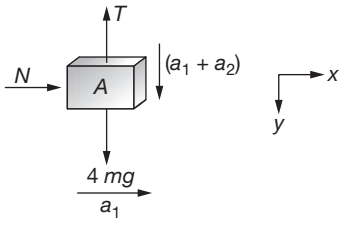
Since $(v_{\text{rel}})_{\max} = (x_m)\omega$

$$\Rightarrow \quad (v_{\text{rel}})_{\max} = \frac{ma_0}{k}\sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad (v_{\text{rel}})_{\max} = a_0\sqrt{\frac{m}{k}}$$

4. Let, acceleration of block C be a_1 (rightwards) and acceleration of block B be a_2 (leftwards)

Then, acceleration of A will be $(a_1 + a_2)$ downwards and a_1 rightwards.

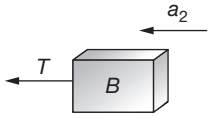


Drawing the free body diagram of A and using $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$, we get

$$N = 4m(a_1) \qquad \ldots(1)$$

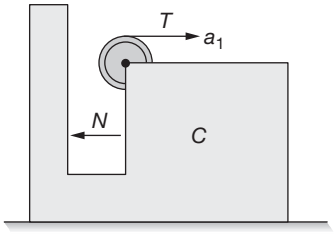
and $4mg - T = 4m(a_1 + a_2)$ …(2)

Drawing the free body diagram of B (showing horizontal forces only) and using $\Sigma F_x = ma_x$, we get



$$T = 3ma_2 \qquad \ldots(3)$$

Again, drawing the free body diagram of C (showing horizontal forces only) and using $\Sigma F_x = ma_x$, we get



$$T - N = 8ma_1 \qquad \ldots(4)$$

Now, we have four unknowns a_1, a_2, T and N. Solving these four equations, we get

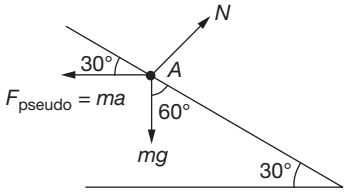
$$a_1 = \frac{g}{8} \text{ and } a_2 = \frac{g}{2}$$

$$\Rightarrow \quad a_1 + a_2 = \frac{5}{8}g$$

So, acceleration of A is $\frac{g}{8}$ in horizontal direction and $\frac{5g}{8}$ in vertical direction.

Acceleration of B is $\frac{g}{2}$ in horizontal direction (leftwards) and acceleration of C is $\frac{g}{8}$ in horizontal direction (rightwards).

5. A is in equilibrium under three concurrent forces shown in figure, so applying Lami's theorem, we get



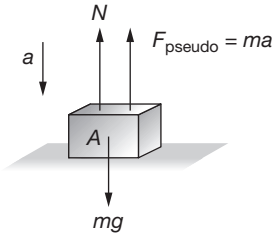
$$\frac{ma}{\sin(90° + 60°)} = \frac{mg}{\sin(90° + 30°)}$$

$$\Rightarrow \quad a = \frac{g\cos(60°)}{\cos(30°)} = 5.66 \text{ ms}^{-2}$$

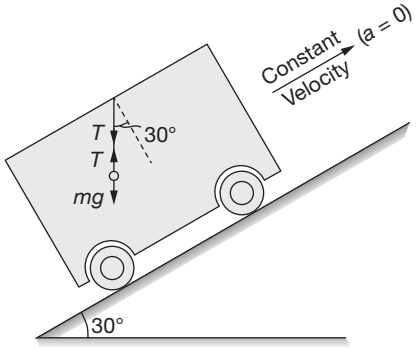
6. Since $N + ma = mg$

$$\frac{mg}{4} + ma = mg$$

$$\Rightarrow \quad a = \frac{3g}{4}$$



7. Since the car is moving up with constant velocity. So it has zero acceleration.



Hence,

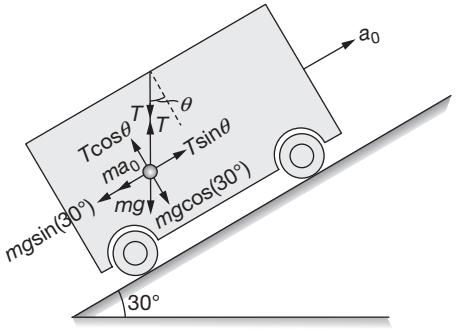
$$\theta = 30°$$

$$T = mg = (2)(10) = 20 \text{ N}$$

8. Let the bob make an angle θ with the normal to the ceiling. Then in the equilibrium state of the bob,

$$T\sin\theta = ma_0 + mg\sin(30°) \text{ and}$$

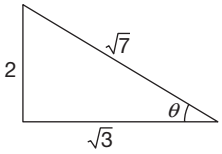
$$T\cos\theta = mg\cos(30°)$$



$$\Rightarrow \quad \tan\theta = \frac{a_0 + g\sin(30°)}{g\cos(30°)} = \frac{\frac{g}{2} + \frac{g}{2}}{\sqrt{3}\frac{g}{2}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Since $T\cos\theta = mg\cos(30°)$



$$\Rightarrow \quad T\cos\theta = mg\left(\frac{\sqrt{3}}{2}\right)$$

Also, $\tan\theta = \frac{2}{\sqrt{3}}$

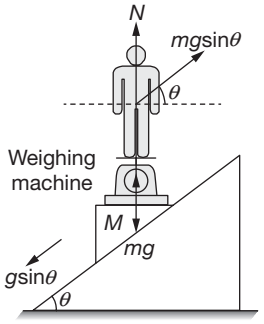
So, we get

$$\cos\theta = \frac{\sqrt{3}}{\sqrt{7}}$$

$$\Rightarrow \quad T\left(\frac{\sqrt{3}}{\sqrt{7}}\right) = (2)(10)\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad T = 10\sqrt{7}\ \text{N}$$

9. If the wedge is in free fall, its acceleration must be $g\sin\theta$. Here we are required to find the weight shown by the weighing machine i.e., the normal force acting between man and the weighing machine. Let us solve the problem in the reference frame of the wedge. The forces acting on man in the reference of wedge block are shown in figure.

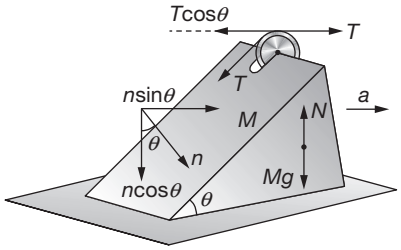


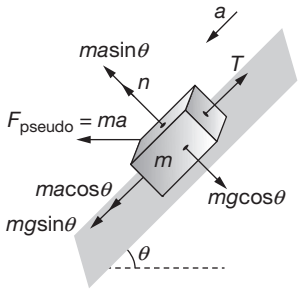
With respect to wedge frame man will experience a pseudo force $mg\sin\theta$ as shown opposite to the acceleration of the wedge. Now for equilibrium of man relative to wedge we have

$$N + mg\sin^2\theta = mg$$

$$\Rightarrow \quad N = mg - mg\sin^2\theta = mg\cos^2\theta$$

10. Using our knowledge of constrained analysis we observe that if the wedge moves toward right by a distance x, then the small bar will travel equal distance x down the inclined plane of wedge. Thus both the bodies will move with same acceleration, wedge towards right on earth and bar downward along incline on wedge. As bar is taken on wedge (a non inertial frame) a pseudo force is applied on bar towards left as shown in its free body diagram. Now we write down the motion equations of m and M according to forces shown in their F.B.D.s in figure.





For *M*

$$T + n\sin\theta - T\cos\theta = Ma \qquad \ldots(1)$$

$$N = Mg + n\cos\theta + T\sin\theta \qquad \ldots(2)$$

For *m*

$$mg\sin\theta + ma\cos\theta - T = ma \qquad \ldots(3)$$

$$ma\sin\theta + n = mg\cos\theta \qquad \ldots(4)$$

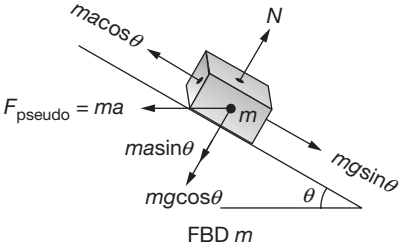
From (3), $T = mg\sin\theta + ma(\cos\theta - 1)$

From (4), $n = mg\cos\theta - ma\sin\theta$

Substitute values of T and n in (1), we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

11. For the smaller block m, not to slip onsurface of m_0, we have



$$mg\sin\theta = ma\cos\theta$$

$$\Rightarrow \quad a = g\tan\theta \qquad \ldots(1)$$

$$N = ma\sin\theta + mg\cos\theta$$

Substituting $a = g\tan\theta$, we get

$$N = mg\sec\theta \qquad \ldots(2)$$

For *M*

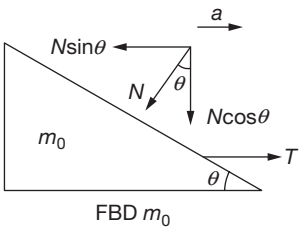
$$Mg - T = ma \qquad \ldots(3)$$

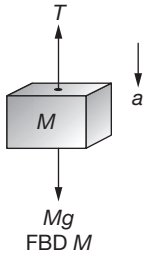
For *m*

$$T - N\sin\theta = m_0 a \qquad \ldots(4)$$

Adding (3) and (4), we get

$$Mg - N\sin\theta = (M + m_0)a$$





$$\Rightarrow \quad M\cancel{g} - m\cancel{g}\frac{\sin\theta}{\cos\theta} = (M + m_0)\cancel{g}\tan\theta$$

$$\Rightarrow \quad M - m\tan\theta = (M + m_0)\tan\theta$$

$$\Rightarrow \quad M - m\tan\theta = M\tan\theta + m_0\tan\theta$$

$$\Rightarrow \quad M(1 - \tan\theta) = (m_0 + m)\tan\theta$$

$$\Rightarrow \quad M = \frac{(m_0 + m)\tan\theta}{1 - \tan\theta}$$

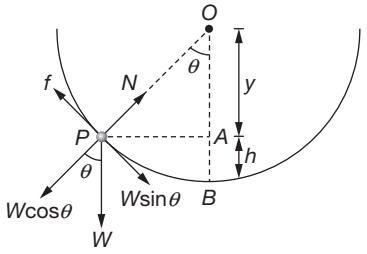
$$\Rightarrow \quad M = \frac{m_0 + m}{\cot\theta - 1}$$

Test Your Concepts-VII (Based on Friction)

1. Let the insect crawl up the bowl upto point P as shown. In doing so, the insect rises through a height $BA = h$, above the bottom B of the bowl of radius r. If W is the weight of insect, then for equilibrium at P,

$$N = W\cos\theta \text{ and}$$

$$f = W\sin\theta$$



From these two equations, we get

$$\tan\theta = \frac{f}{N}$$

In limiting case, we have

$$f = \mu N$$

$$\Rightarrow \quad \frac{f}{N} = \mu$$

Therefore, $\tan\theta = \mu = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad \theta = 30°$$

Since, $y = OP\cos(30°) = \frac{\sqrt{3}r}{2}$

and $h = BA = OB - OA = r - y$

$$\Rightarrow \quad h = r - \frac{\sqrt{3}}{2}r = 0.134r$$

2. The given angle 30° is actually the angle of repose α. So, we get

$$\mu = \tan(30°) = \frac{1}{\sqrt{3}}$$

3. Since, $\mu_1 < \mu_2$, acceleration of 2 kg block down the plane will be more than the acceleration of 4 kg block if allowed to move separately. But as the 2 kg block is behind the 4 kg block both of them will move with same acceleration say a. Considering both the blocks as a system, the force down the plane on the system is

$$F_{\text{down}} = (4 + 2)g\sin(30°)$$

$$\Rightarrow \quad F_{\text{down}} = (6)(10)\left(\frac{1}{2}\right) = 30 \text{ N}$$

Similarly, force up the plane on the system is

$$F_{\text{up}} = \mu_1(2)(g)\cos(30°) + \mu_2(4)(g)\cos(30°)$$

$$\Rightarrow \quad F_{\text{up}} = (2\mu_1 + 4\mu_2)g\cos 30°$$

$\Rightarrow \quad F_{up} = (2\times 0.2 + 4\times 0.3)(10)(0.86)$

$\Rightarrow \quad F_{up} = 13.76 \text{ N}$

So, net force down the plane is

$$(F_{net})_{down} = 30 - 13.76 = 16.24 \text{ N}.$$

Acceleration of both the blocks down the plane will be given by

$$a = \frac{(F_{net})_{down}}{4+2} = \frac{16.24}{6} = 2.7 \text{ ms}^{-2}$$

4. As the box is lying an accelerated frame, so, it experiences a backward force given by

$$F = ma$$

Motion of the box is opposed by the frictional force given by

$$f = \mu mg$$

So, net force on the box in the backward direction is

$$F_{net} = F - f = ma - \mu mg$$

$\Rightarrow \quad F_{net} = 40(2 - (0.1)(10)) = 40 \text{ N}$

Acceleration produced in the box relative to truck in the backward direction is

$$a_r = \frac{F_{net}}{m} = \frac{40}{40} = 1 \text{ ms}^{-2}$$

Since $s = \frac{1}{2}a_r t^2$

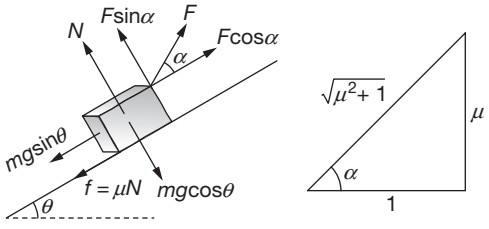
$\Rightarrow \quad t = \sqrt{\frac{2s}{a_r}}$

$\Rightarrow \quad t = \sqrt{\frac{2\times 5}{1}} = 4.34 \text{ s}$

$\Rightarrow \quad t = 3.16 \text{ s}$

5. In the diagram drawn, we have

$$F\cos\alpha = mg\sin\theta + \mu N \qquad \ldots(1)$$



and $N + F\sin\alpha = mg\cos\theta \qquad \ldots(2)$

$\Rightarrow \quad F\cos\alpha = mg\sin\theta + \mu N$

$\Rightarrow \quad F\cos\alpha = mg\sin\theta + \mu(mg\cos\theta - F\sin\alpha)$

$\Rightarrow \quad F = \frac{mg\sin\theta + \mu mg\cos\theta}{\cos\alpha + \mu\sin\alpha}$

For F to be minimum,

$\cos\alpha + \mu\sin\alpha = z$ (say) should be maximum

$\Rightarrow \quad \frac{dz}{d\alpha} = 0$

$\Rightarrow \quad \alpha = \tan^{-1}(\mu)$

and $F_{min} = \frac{mg(\sin\theta + \mu\cos\theta)}{\left(\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu^2}{\sqrt{\mu^2+1}}\right)}$

$\Rightarrow \quad F_{min} = \frac{mg(\sin\theta + \mu\cos\theta)}{\sqrt{\mu^2+1}}$

6. Retardation offered by the belt to the package is given by

$$a = \mu_k g = 0.15\times 9.8 = 1.47 \text{ ms}^{-2}$$

Let s be the distance travelled before the sliding stops then

$$s = \frac{v^2}{2a} = \frac{(5)^2}{2\times 1.47} \approx 8.5 \text{ m}$$

7. Limiting friction between A and wedge

$$f_L = \mu_s m_A g\cos\theta = (0.25)(60)(10)\cos(30°)$$

$\Rightarrow \quad f_L = 130 \text{ N}$

and kinetic friction is

$$f_K = \mu_K m_A g\cos\theta = 104 \text{ N}$$

The system will remain stationary when

$$\frac{W_B}{2} + f_L > W_A\sin 30°$$

$\Rightarrow \quad 100 + 130 > 300$

Since this is not happening, so the system will move. Also, we observe that the acceleration a_A down the plane is two times a_B upwards.

Equations of motion are,

$$W_A\sin(30°) - T - f_K = m_A a_A \qquad \ldots(1)$$

$\Rightarrow \quad 300 - T - 104 = 60a_A$

$$2T - W_B = m_B a_B$$

$\Rightarrow \quad 2T - 200 = 20a_B \qquad \ldots(2)$

and $a_A = 2a_B \qquad \ldots(3)$

Solving these three equations, we get

$T = 107.74$ N

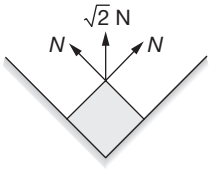
$a_A = 1.489$ ms^{-2}

and $a_B = 0.744$ ms^{-2}

8. Since the block has no motion perpendicular to both the inclines, so we have

$$\sqrt{2}N = mg\cos\theta$$

$$\Rightarrow \quad N = \frac{mg\cos\theta}{\sqrt{2}}$$



Cross sectional view

Down the incline, the acceleration of the block is

$$a = \frac{mg\sin\theta - 2\mu_K N}{m} = g\sin\theta - \sqrt{2}\mu_K g\cos\theta$$

$$\Rightarrow \quad a = g\left(\sin\theta - \sqrt{2}\mu_K\cos\theta\right)$$

9. $$a = v\frac{dv}{dx} = \frac{\text{Net force}}{\text{Mass}} = \frac{F - \mu_K\rho(L-x)g}{\rho L}$$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^L \frac{F - \mu_K\rho(L-x)g}{\rho L}\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{F}{\rho} - \mu_K gL + \frac{\mu_K gL}{2}$$

$$\Rightarrow \quad v = \sqrt{\frac{2F}{\rho} - \mu_K gL}$$

10. (a) Since $u = 0$, so $v = a_1t_1 = 2.6$ ms^{-1} $\quad\{\because v = u + at\}$

$$s_1 = \frac{1}{2}a_1t_1^2 = \frac{1}{2}\times 2\times(1.3)^2 = 1.69 \text{ m} \quad \left\{\because s = ut + \frac{1}{2}at^2\right\}$$

So, $s_2 = (2.2 - 1.69) = 0.51$ m

Now, $s_2 = \dfrac{v^2}{2a_2}$

$$\Rightarrow \quad a_2 = \frac{v^2}{2s_1} = \frac{(2.6)^2}{2\times 0.51} = 6.63 \text{ ms}^{-2}$$

and $t_2 = \dfrac{v}{a_2} = 0.4$ s

(b) Acceleration of package will be 2 ms^{-2} while retardation will be $\mu_K g = 2.5$ ms^{-2}.

For the package, we have, as already calculated,

$$v = a_1t_1 = 2.6 \text{ ms}^{-1}$$

$$s_1 = \frac{1}{2}a_1t_1^2 = 1.69 \text{ m}$$

So, $s_2' = vt_2 - \dfrac{1}{2}a_2^1t_2^2 = 2.6\times 0.4 - \dfrac{1}{2}\times 2.5\times(0.4)^2$

$$s_2' = 0.84 \text{ m}$$

So, total displacement of package $= 2.53$ m

Hence, displacement of package w.r.t. belt is

$$s_r = (2.53 - 2.21) \text{ m} = 0.33 \text{ m}$$

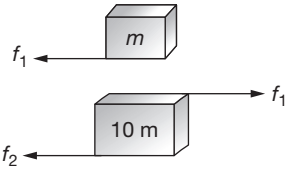
11. (a) Force of friction at different contacts are shown in figure.

Here, $f_1 = \mu_2 mg$ and $f_2 = \mu_1(11mg)$.

Given that $\mu_2 > 11\mu_1$, so

$$f_1 > f_2$$

Retardation of upper block is $a_1 = \dfrac{f_1}{m} = \mu_2 g$



Acceleration of lower block is

$$a_2 = \frac{f_1 - f_2}{m} = \frac{(\mu_2 - 11\mu_1)g}{10}$$

Relative retardation of upper block w.r.t. lower block is

$$a_r = a_1 + a_2$$

$$\Rightarrow \quad a_r = \frac{11}{10}(\mu_2 - \mu_1)g$$

Now, $0 - v_{\min}^2 = 2(-a_r)l \quad \{\because v_r^2 - u_r^2 = 2a_rl_r\}$

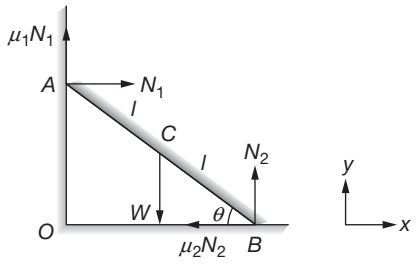
$$\Rightarrow \quad v_{\min} = \sqrt{2a_rl} = \sqrt{\frac{22(\mu_2 - \mu_1)gl}{10}}$$

(b) $0 = v_m - a_rt \quad \{\because v_r = u_r + at\}$

$$\Rightarrow \quad t = \frac{v_m}{a_r} = \sqrt{\frac{20l}{11(\mu_2 - \mu_1)g}}$$

12. For equilibrium of rod, we have

$$\Sigma F_x = 0$$



$$\Rightarrow \quad N_1 = \mu_2 N_2 \qquad \ldots(1)$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad W = N_2 + \mu_1 N_1 \qquad \ldots(2)$$

Also $\Sigma \tau_B = 0$

$$\Rightarrow \quad Wl\cos\theta = N_1(2l\sin\theta) + \mu_1 N_1(2l\cos\theta)$$

$$\Rightarrow \quad \tan\theta = \frac{W - 2\mu_1 N_1}{2N_1} \qquad \ldots(3)$$

From equations (1) and (2), we get

$$N_1 = \frac{\mu_2 W}{1 + \mu_1\mu_2}$$

Substituting in equation (3), we get

$$\tan\theta = \frac{1 + \mu_1\mu_2}{2\mu_2} - \mu_1$$

Substituting the values of $\mu_1 = \frac{1}{2}$ and $\mu_2 = \frac{1}{4}$, we get

$$\theta = \tan^{-1}\left(\frac{7}{4}\right)$$

13. Assuming that mass of truck $\gg$ mass of crate.
Retardation of truck due to friction is

$$a_1 = (0.9)g = 9 \text{ ms}^{-2},$$

Retardation of crate due to friction is

$$a_2 = (0.7)g = 7 \text{ ms}^{-2}$$

So, relative acceleration of crate is $a_r = 2 \text{ ms}^{-2}$.

Time taken by the truck to stop is t_1 (say), so

$$0 = u + (-a_1)t_1$$

$$\Rightarrow \quad t_1 = \frac{u}{a_1} = \frac{15}{9} \text{ s}$$

Truck will stop after time $t_1 = \frac{15}{9} = 1.67$ s and crate will strike the wall at

$$t_2 = \sqrt{\frac{2s}{a_r}} = \sqrt{\frac{2 \times 3.2}{2}} = 1.78 \text{ s}$$

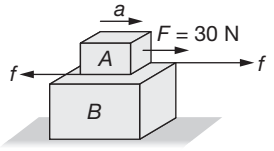
As $t_2 > t_1$, crate will come to rest after travelling a distance

$$s = \frac{1}{2}a_r t_1^2 = \frac{1}{2} \times 2 \times \left(\frac{15}{9}\right)^2 = 2.77 \text{ m}$$

14. Limiting friction between A and B is

$$f_L = \mu N = 0.4 \times 100 = 40 \text{ N}$$

(a) Both the blocks will have a tendency to move together with same acceleration (say a). So, the force diagram is as shown.



Equations of motion are,

$$30 - f = 10 \times a \qquad \ldots(1)$$

$$f = 25 \times a \qquad \ldots(2)$$

Solving these two equations, we get

$$a = 0.857 \text{ ms}^{-2} \text{ and } f = 21.42 \text{ N}$$

Since this force is less than f_L, both the blocks will move together with same acceleration,

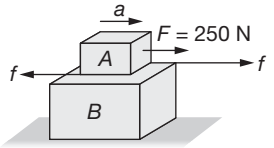
$$a_A = a_B = 0.857 \text{ ms}^{-2}$$

(b) $250 - f = 10a$

$f = 25a$

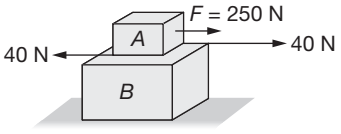
Solving equations (3) and (4), we get

$$f = 178.6 \text{ N}$$



Since, $f > f_L$ so, slipping will take place between the two blocks and hence we have

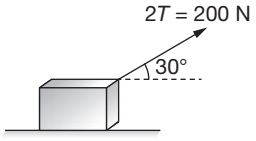
$$f = f_L = 40 \text{ N}$$



$$\Rightarrow \quad a_A = \frac{250-40}{10} = 21 \text{ ms}^{-2}$$

$$\Rightarrow \quad a_B = \frac{40}{25} = 1.6 \text{ ms}^{-2}$$

15. $a = \dfrac{2T\cos(30°) - \mu_K N}{m}$



$$\Rightarrow \quad a = \frac{200\cos(30°) - 0.4(400 - 200\times\sin(30°))}{40}$$

$$\Rightarrow \quad a = 1.33 \text{ ms}^{-2}$$

16. (a) Common acceleration,

$$a = \frac{F\cos\alpha}{m_A + m_B} = \frac{F\cos(30°)}{1+2}$$

$$\Rightarrow \quad a = \frac{F}{2\sqrt{3}}$$

(b) Normal reaction between A and B,

$$N = m_A g + F\sin\alpha = \left(10 + \frac{F}{2}\right)$$

Therefore, maximum friction between A and B is,

$$f_{\max} = \mu N = \frac{1}{2}\left(10 + \frac{F}{2}\right) = \left(5 + \frac{F}{4}\right)$$

The friction helps the block B to move. Hence, maximum acceleration of B can be

$$a_{\max} = \frac{f_{\max}}{m_B} = \frac{5 + \left(\frac{F}{4}\right)}{2} = 2.5 + 0.125F$$

This is also the maximum common acceleration.

Hence, $F\cos(30°) = (m_A + m_B)a_{\max}$

$$\Rightarrow \quad \frac{\sqrt{3}F}{2} = 3(2.5 + 0.125F)$$

$$\Rightarrow \quad F = 15.27N$$

17. $a = \dfrac{m_A g}{m_A + M + m}$

For the equilibrium of B,

$$mg = \mu N = \mu(ma) = \frac{\mu m m_A g}{m_A + M + m}$$

$$\Rightarrow \quad m_A = \frac{(M+m)m}{(\mu - 1)m}$$

$$\Rightarrow \quad m_A = \frac{(M+m)}{\mu - 1}$$

Please note that since $m_A > 0$

So, $\mu > 1$

18. Equations of motion are,

$$T - \mu M_1 g = M_1 a_1 \qquad \ldots(1)$$

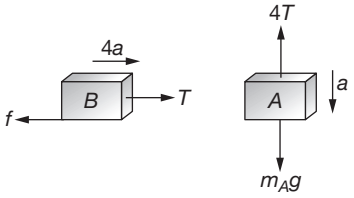
$$T - \mu M_2 g = M_2 a_2 \qquad \ldots(2)$$

$$M_3 g - 2T = M_3\left(\frac{a_1 + a_2}{2}\right) \qquad \ldots(3)$$

We have three unknowns T, a_1 and a_2. Solving these three equations, we get

$$T = \frac{(\mu+1)g}{\frac{2}{M_3} + \frac{1}{2M_1} + \frac{1}{2M_2}}$$

19. Let T be the tension in the string attached to block B. Then tension in the string connected to block A would be $4T$.



Similarly, if a be the acceleration of block A (downwards), then acceleration of block B towards right will be $4a$.

Writing equations of motion

for block A, $m_A g - 4T = m_A a$

$$\Rightarrow \quad 50 - 4T = 5a \qquad \ldots(1)$$

for block B, $T - f = 10(4a)$

$$\Rightarrow \quad T - (0.1)(10)(10) = 40a$$

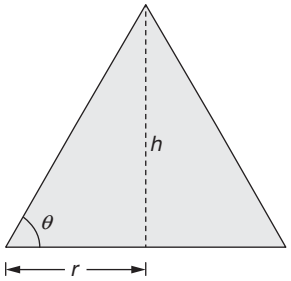
$$\Rightarrow \quad T - 10 = 40a \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$a = 0.06 \text{ ms}^{-2}$$

20. $\tan\theta = \dfrac{h}{r}$ …(1)

Here θ cannot be greater than the angle of repose $\alpha = \tan^{-1}(\mu)$.



Thus, maximum value of θ can be $\tan^{-1}(\mu)$. In critical case, we have

$$\theta = \tan^{-1}(\mu)$$

$$\Rightarrow \quad \tan\theta = \mu \qquad \ldots(2)$$

From equations (1) and (2), we get

$$h = \mu r$$

So, maximum volume is

$$V_{max} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(\mu r)$$

$$\Rightarrow \quad V_{max} = \frac{1}{3}\pi\mu r^3$$

21. Let, f_1 be the limiting friction between A and ground so, $f_1 = \mu(m_1 + m_2)g$

Let f_2 be the limiting friction between A and B so, $f_2 = \mu(m_2)g$

Net pulling force

$$F = m_3 g - f_1 - f_2$$

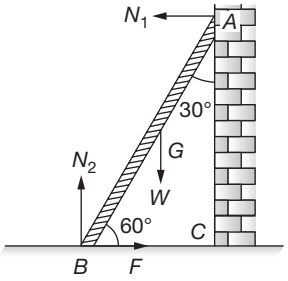
$$\Rightarrow \quad F = m_3 g - \mu(m_1 + 2m_2)g$$

$$\Rightarrow \quad a_A = a_C = \frac{F}{m_1 + m_3} = \frac{g\left(m_3 - \mu(2m_2 + m_1)\right)}{m_1 + m_3}$$

22. In figure AB is a ladder of weight W which acts at its centre of gravity G. Let N_1 be the reaction of the wall and N_2 the reaction of the ground.

$$\angle ABC = 60°$$

$$\Rightarrow \quad \angle BAC = 30°$$



Force of friction f between the ladder and the ground acts along BC.

For horizontal equilibrium $f = N_1$ …(1)

For vertical equilibrium $N_2 = W$ …(2)

Taking moments of the forces about B, we get, for equilibrium,

$$N_1(4\cos 30°) - W(2\cos 60°) = 0 \qquad \ldots(3)$$

where, $W = 250$ N

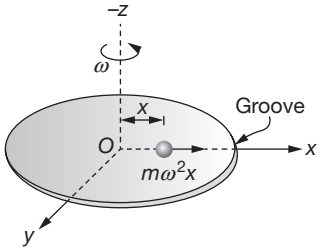
Solving these three equations, we get

$$f = 72.17 \text{ N} \quad \text{and} \quad N_2 = 250 \text{ N}$$

$$\Rightarrow \quad \mu = \frac{f}{N_2} = \frac{72.17}{250} = 0.288$$

Test Your Concepts-VIII (Based on Circular Motion)

1. The situation is shown in figure. Let us work from the frame of reference of the table. Now take the origin at the centre of rotation O and the x-axis along the groove. The y-axis is along the line perpendicular to OX, coplanar with the surface of the table and the z-axis is along the vertical. Let at time t the particle in the groove be at a distance x from the origin and is moving along the x-axis with a speed v. The forces acting on the particle (including the pseudo force) are:



(a) weight mg vertically downward,

(b) normal contact force N_1 vertically upward by the bottom surface of the groove,

(c) normal contact force N_2 parallel to the y-axis by the side walls of the groove,

(d) centrifugal force $m\omega^2 x$ along the x-axis.

As the particle can only move in the groove, its acceleration is along the x-axis. The only force along the x-axis is the centrifugal force $m\omega^2 x$. All the other forces are perpendicular to the x-axis and have no components along the x-axis.
Thus, the acceleration along x-axis is

$$a = \frac{F}{m} = \omega^2 x$$

$$\Rightarrow \quad v\frac{dv}{dx} = \omega^2 x$$

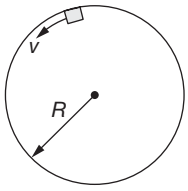
$$\Rightarrow \quad v\,dv = \omega^2 x\,dx$$

$$\Rightarrow \quad \int_0^v v\,dv = \int_a^L \omega^2 x\,dx$$

$$\Rightarrow \quad \left(\frac{1}{2}v^2\right)\Big|_0^v = \left(\frac{1}{2}\omega^2 x^2\right)\Big|_a^L$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{1}{2}\omega^2\left(L^2 - a^2\right)$$

$$\Rightarrow \quad v = \omega\sqrt{L^2 - a^2}$$



2. (a) $N = \dfrac{mv^2}{R}$

(b) $f = \mu N = \dfrac{\mu m v^2}{R}$

(c) $a_t = \dfrac{f}{m} = \dfrac{\mu v^2}{R}$

(d) $v\dfrac{dv}{ds} = -\dfrac{\mu v^2}{R}$

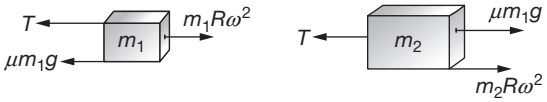
Here negative sign indicates the retarding nature due to friction.

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -\frac{\mu}{R}\int_0^{2\pi R} ds$$

$$\Rightarrow \quad \log_e\left(\frac{v}{v_0}\right) = -2\pi\mu$$

$$\Rightarrow \quad v = v_0 e^{-2\pi\mu}$$

3. Free body diagrams of m_1 and m_2 from rotating frames are as shown for convenience



Please note that only horizontal forces have been shown.
For equilibrium, we have

$$m_1 R\omega^2 = T + \mu m_1 g \qquad \ldots(1)$$

$$T = m_2 R\omega^2 + \mu m_1 g \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\omega = \sqrt{\frac{2m_1\mu g}{(m_1 - m_2)R}}$$

Substituting the values, we get

$$\omega_{\min} = 6.32 \text{ rads}^{-1}$$

Since from (2), we have

$$T = m_2 R\omega^2 + \mu m_1 g$$

$$\Rightarrow \quad T = (1)(0.5)(6.32)^2 + (0.5)(2)(10) \cong 30 \text{ N}$$

4. (a) $mL\omega^2 \le \mu mg$

$$\Rightarrow \quad \omega \le \sqrt{\frac{\mu g}{L}}$$

$$\Rightarrow \quad \omega_{MAX} = \sqrt{\frac{\mu g}{L}}$$

(b) $\mu g = \sqrt{a_T^2 + a_C^2} = \sqrt{(L\alpha)^2 + (L\omega^2)^2}$

$$\Rightarrow \quad L^2\omega^4 = \mu^2 g^2 - L^2\alpha^2$$

$$\Rightarrow \quad \omega = \left(\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right)^{\frac{1}{4}}$$

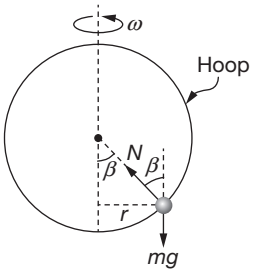
5. (a) $f = \dfrac{mv^2}{R} = \dfrac{85\times(9)^2}{25} = 275.4 \text{ N}$

(b) $N = mg = 833 \text{ N}$

So, net force on bicycle from the road is

$$F_{\text{net}} = \sqrt{N^2 + f^2} = 877.3 \text{ N}$$

6. (a) From the diagram, we get



$$N\sin\beta = mr\omega^2 = m(R\sin\beta)\omega^2 \quad \ldots(1)$$

$$N\cos\beta = mg \quad \ldots(2)$$

From equations (1) and (2), we get

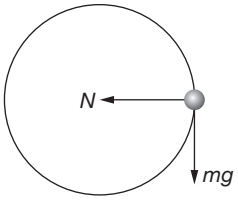
$$\beta = \cos^{-1}\left(\frac{g}{R\omega^2}\right)$$

Since, $\omega = 2\pi f = 6\pi$ rads^{-1}

$$\Rightarrow \quad \beta = \cos^{-1}\left(\frac{10}{0.1\times 36\times \pi^2}\right)$$

$$\beta \cong 74^\circ$$

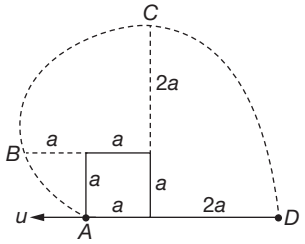
(b) At the same level as the centre of the hoop, component of N in vertical direction is zero, so the bead will fall down due to weight mg and hence it is not possible for the bead to stay at this level.



7. Time $t = \dfrac{AB + BC + CD}{u}$

$$\Rightarrow \quad t = \frac{\left\{\left(\frac{2\pi a}{4}\right) + \frac{2\pi(2a)}{4} + \frac{2\pi(3a)}{4}\right\}}{u}$$

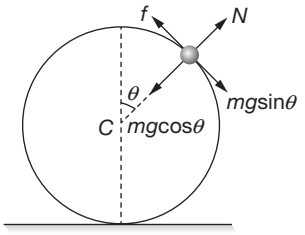
$$\Rightarrow \quad t = \frac{3\pi a}{u}$$



8. (a) $mg = \dfrac{mv_{\min}^2}{R}$

$$\Rightarrow \quad v_{\min} = \sqrt{gR}$$

(b) Velocity at angle θ with respect to point C is $v = R\omega$



Now, $mg\cos\theta - N = \dfrac{mv^2}{R}$

$$\Rightarrow \quad N = mg\cos\theta - \frac{mv^2}{R} \quad \ldots(1)$$

The pebble slides down, when

$$f \le mg\sin\theta$$

$$\Rightarrow \quad \mu N = mg\sin\theta \quad \ldots(2)$$

From equations (1) and (2), we get for $\mu = 1$,

$$\theta = \cos^{-1}\left(\frac{v^2}{\sqrt{2}Rg}\right) - \frac{\pi}{4}$$

9. Since $\tan\theta = \dfrac{v^2}{Rg}$

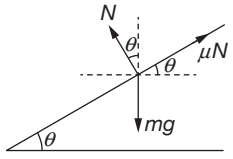
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{v^2}{Rg}\right) = \tan^{-1}\left(\frac{100}{200}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

For $v_{\min}$:

Friction is outwards. Hence the equations of motion are,

$$N\sin\theta - \mu N\cos\theta = \frac{mv^2}{R} \quad \ldots(1)$$

$$N\cos\theta + \mu N\sin\theta = mg \quad \ldots(2)$$



From equations (1) and (2), we get

$$v_{\min} = \sqrt{gR\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$$

Substituting the values, we get

$$v_{\min} = \sqrt{10 \times 20\left(\frac{\frac{1}{2} - 0.4}{1 + 0.4 \times \frac{1}{4}}\right)} = 4.08 \text{ ms}^{-1}$$

For v_{min}:

In this case friction is inwards. Hence the equations of motion are,

$$N\sin\theta + \mu N\cos\theta = \frac{mv_{\max}^2}{R} \quad \ldots(3)$$

$$N\cos\theta - \mu N\sin\theta = mg \quad \ldots(4)$$

From equations (3) and (4), we get

$$v_{\max} = \sqrt{gR\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$$

$$\Rightarrow \quad v_{\max} = \sqrt{10 \times 20\left(\frac{\frac{1}{2} + 0.4}{1 - 0.4 \times \frac{1}{2}}\right)} = 15 \text{ ms}^{-1}$$

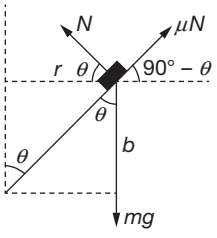
10. $\frac{b}{r} = \cot\theta$

$$\Rightarrow \quad r = b\tan\theta \quad \ldots(1)$$

For $\omega_{\min}$, we have

$$N\cos\theta - \mu N\sin\theta = mr\omega^2 \quad \ldots(2)$$

$$N\sin\theta + \mu N\cos\theta = mg \quad \ldots(3)$$



From the above equations, we get

$$\omega_{\min} = \sqrt{\frac{g(1 - \mu\tan\theta)}{b\tan\theta(\mu + \tan\theta)}}$$

For $\omega_{\max}$, friction μN will act inwards. Equations of motion are,

$$N\cos\theta + \mu N\sin\theta = mr\omega^2 \quad \ldots(4)$$

and $N\sin\theta - \mu N\cos\theta = mg \quad \ldots(5)$

Solving equations (1), (4) and (5), we get

$$\omega_{\max} = \sqrt{\frac{g(1 + \mu\tan\theta)}{b\tan\theta(\tan\theta - \mu)}}$$

11. The maximum speed without slipping is given by

$$v_{MAX} = \sqrt{\mu rg} = \sqrt{(0.9) \times (40) \times (9.8)} = 18.8 \text{ ms}^{-1}$$

The maximum speed without overturning

$$v_1 = \sqrt{\frac{gra}{h}}$$

When the car turns right, then

$$a = \frac{1.15}{2} + 0.075 = 0.65 \text{ m}$$

So, for the first bend

$$v_1 = \sqrt{\frac{(9.8)(40)(0.65)}{0.6}} = 20.6 \text{ ms}^{-1}$$

When the car turns left, then

$$a = \frac{1.15}{2} - 0.075 = 0.5 \text{ m}$$

For second bend the maximum speed without overturning

$$v_2 = \sqrt{\frac{9.8 \times 0.5 \times 40}{0.6}} = 18.1 \text{ ms}^{-1}$$

As for the first bend, $v_{MAX} < v_1$, so the car skids.

For second bend, $v_2 < v_{MAX}$, so the car overturns.

12. $N = \frac{mv^2}{R}$

$$f_{\max} = \mu N = \frac{\mu mv^2}{R}$$

So, retardation $a = \frac{f_{\max}}{m} = \frac{\mu v^2}{R}$

$$\Rightarrow \quad \left(-\frac{dv}{dt}\right) = \frac{\mu v^2}{R}$$

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v^2} = -\frac{\mu}{R}\int_0^t dt$$

$$\Rightarrow \quad v = \frac{v_0}{\left(1 + \frac{\mu v_0 t}{R}\right)}$$

13. Figure shows the funnel with mass m.

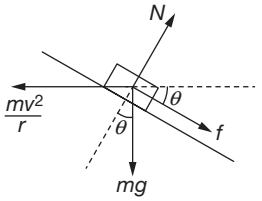
The different forces on mass m (w.r.t. funnel) are:

(i) weight mg acting vertically downwards,

(ii) centrifugal reaction force $\left(\frac{mv^2}{r}\right)$ directed horizontally outwards,

(iii) normal reaction N offered by the wall and

(iv) frictional force f directed along the incline (when the revolution is fast enough, the mass may slide upwards).



Since, the mass m does not move, so we get

$$N - \frac{mv^2}{r}\sin\theta - mg\cos\theta = 0 \qquad \ldots(1)$$

$$f + mg\sin\theta - \frac{mv^2}{r}\cos\theta = 0 \qquad \ldots(2)$$

From equation (1),

$$N = mg\cos\theta + \left(\frac{mv^2}{r}\right)\sin\theta$$

Now, $f = \mu N = \mu\left\{mg\cos\theta + \left(\frac{mv^2}{r}\right)\sin\theta\right\}$ …(3)

Substituting the value of f from equation (3) in equation (2), we get

$$\mu\left[mg\cos\theta + \frac{mv^2}{r}\sin\theta\right] - \frac{mv^2}{r}\cos\theta = -mg\sin\theta$$

$$\Rightarrow \quad \frac{mv^2}{r}(\mu\sin\theta - \cos\theta) = -mg(\sin\theta + \mu\cos\theta)$$

$$\Rightarrow \quad \frac{v^2}{r}(\cos\theta - \mu\sin\theta) = g(\sin\theta + \mu\cos\theta)$$

$$\Rightarrow \quad \frac{v^2}{r} = \frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} \qquad \ldots(4)$$

Since

$$v = r\omega = 2\pi nr \qquad \{\because\ \omega = 2\pi n\}$$

$$\Rightarrow \quad \frac{4\pi^2 n^2 r^2}{r} = \frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

$$\Rightarrow \quad n^2 = \frac{1}{4\pi^2 r}\frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{r(\cos\theta - \mu\sin\theta)}} \qquad \ldots(5)$$

This must be the maximum frequency allowed.

When the revolution is slow enough, the block may slide down. In this case the frictional force f will be directed along the incline upwards.

So, replacing μ by $-\mu$ in equation (5) the minimum frequency allowed will be

$$n = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{r(\cos\theta + \mu\sin\theta)}} \qquad \ldots(6)$$

Hence, $n_{max} = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{r(\cos\theta - \mu\sin\theta)}}$

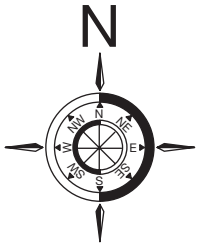
$$n_{min} = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{r(\cos\theta + \mu\sin\theta)}}$$

Single Correct Choice Type Questions

1. $m = 2\ kg$

$\vec{v} = 2\hat{i}$

$\vec{F} = 3\hat{j}$



$$\Rightarrow \quad \vec{a} = \frac{\vec{F}}{m} = \frac{3}{2}\hat{j}$$

$t = 2\ s$

Since $\vec{s} = \vec{v}t + \frac{1}{2}\vec{a}t^2$

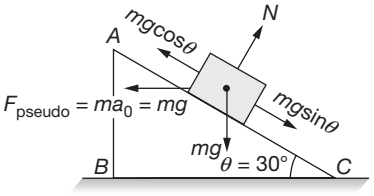
$$\Rightarrow \quad \vec{s} = \left(2\hat{i}\right)2 + \frac{1}{2}\left(\frac{3}{2}\right)(4)\hat{j}$$

$$\Rightarrow \quad \vec{s} = 4\hat{i} + 3\hat{j}$$

$$\Rightarrow \quad |\vec{s}| = \sqrt{4^2 + 3^2} = 5\ \text{m}$$

Hence, the correct answer is (B).

2. Drawing free body diagram of block with respect to plane.



Acceleration of the block up the plane is

$$a = \frac{mg\cos(30°) - mg\sin(30°)}{m}$$

$$\Rightarrow \quad a = \left(\frac{\sqrt{3}-1}{2}\right)g = 3.66 \text{ ms}^{-2}$$

Since, $s = \frac{1}{2}at^2$

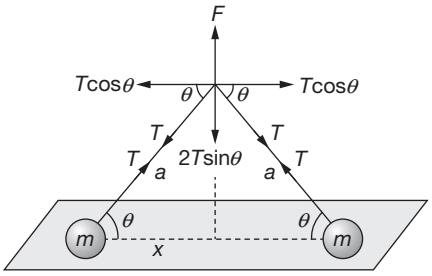
$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 1}{3.66}} = 0.74 \text{ s}$$

Hence, the correct answer is (B).

3. Since the string is light, so at the point of application of force F, and when the threads make an angle θ with the horizontal, we have

$$F - 2T\sin\theta = 0$$

$$\Rightarrow \quad F = 2T\sin\theta$$



For mass m to move, say with acceleration A, we have

$$T\cos\theta = mA$$

$$\Rightarrow \quad \left(\frac{F}{2\sin\theta}\right)\cos\theta = mA$$

$$\Rightarrow \quad A = \left(\frac{F}{2m}\right)\cot\theta$$

where $\tan\theta = \frac{\sqrt{a^2 - x^2}}{x}$

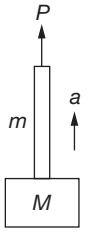
$$\Rightarrow \quad A = \frac{F}{2m}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

Hence, the correct answer is (C).

4. Let f be the force exerted by the rope on the block M. Then

$$f - Mg = Ma \qquad \ldots(1)$$

We must note that once P acts on free end of the rope, then tension due to pull P at the end of rope attached to M is zero. So, M will move up only due to the interaction force f.



Now, for mass m, we have

$$P - f - mg = ma \qquad \ldots(2)$$

Adding (1) and (2), we get

$$P - (M+m)g = (M+m)a$$

$$\Rightarrow \quad P = (M+m)(g+a)$$

$$\Rightarrow \quad a = \frac{P}{M+m} - g$$

$$f = M(g+a)$$

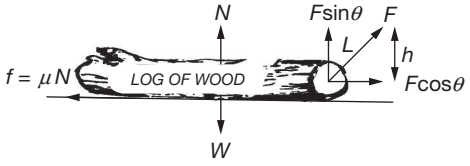
$$\Rightarrow \quad f = \frac{MP}{M+m}$$

Hence, the correct answer is (D).

5. $N + F\sin\theta = W$ and

$F\cos\theta = \mu N$

$$\Rightarrow \quad \mu = \frac{F\cos\theta}{N}$$



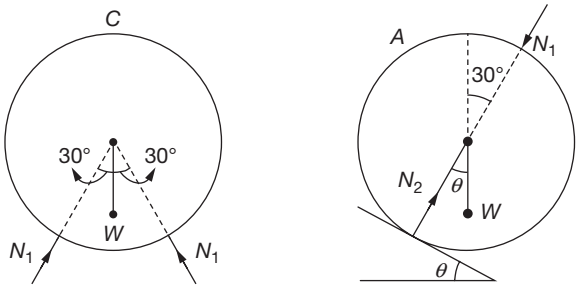
$$\Rightarrow \quad \mu = \frac{F\cos\theta}{W - F\sin\theta}$$

Since $\sin\theta = \frac{h}{L}$ and $\cos\theta = \frac{\sqrt{L^2 - h^2}}{L}$

$$\Rightarrow \quad \mu = \frac{F\sqrt{L^2 - h^2}}{WL - Fh}$$

Hence, the correct answer is (A).

6. Let us draw the free body diagrams of C and A.



W = Weight of each sphere

N_2 = Normal reaction between A and inclined plane

N_1 = Normal reaction between A and C

N_1 = Normal reaction between B and C

Free body diagram of *C*

Resolving vertically $2N_1\cos 30° = W$

$$\Rightarrow \quad N_1 = \frac{W}{\sqrt{3}} \qquad \ldots(1)$$

When the arrangement is on the point of collapsing, the reaction between A and B is zero.

Free body diagram of *A*

Resolving horizontally and vertically

$$N_2\sin\theta = N_1\sin 30°$$

$$\Rightarrow \quad N_2\sin\theta = \frac{W}{2\sqrt{3}} \qquad \ldots(2)$$

$$N_2\cos\theta = W + N_1\cos 30° = \frac{3W}{2} \qquad \ldots(3)$$

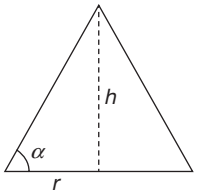
Dividing (2) by (3), we get

$$\tan\theta = \frac{1}{3\sqrt{3}}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)$$

Hence, the correct answer is (C).

7. As soon as the inclination of slanted side reaches the angle of friction, sand begins to slide down.



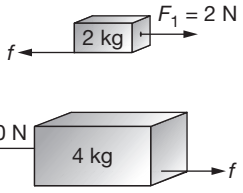
For maximum height, we have

$$\mu = \tan\alpha = \frac{h}{R}$$

$$\Rightarrow \quad h = \mu R$$

Hence, the correct answer is (C).

8. Free body diagram of the two bodies are as follows



Maximum friction between the two blocks is

$$f_{max} = \mu mg \text{ where } m = 2 \text{ kg}$$

$$\Rightarrow \quad f_{max} = (0.5)(2)(10) = 10 \text{ N}$$

Let acceleration of both the blocks be a towards left. Then

$$a = \frac{f-2}{2} = \frac{20-f}{4}$$

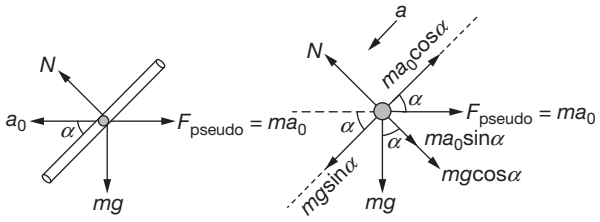
$$\Rightarrow \quad 2f - 4 = 20 - f$$

$$\Rightarrow \quad f = 8 \text{ N}$$

Now since $f < f_{max}$, i.e., static region of 2 kg on 4 kg, hence friction force between the two blocks is 8 N.

Hence, the correct answer is (C).

9.



$$mg\sin\alpha - ma_0\cos\alpha = ma$$

$$\Rightarrow \quad a = g\sin\alpha - a_0\cos\alpha$$

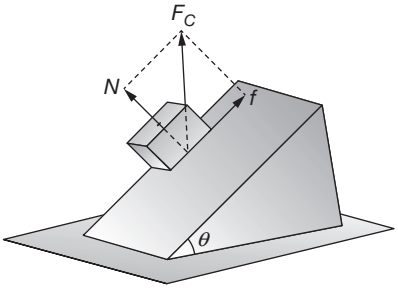
Hence, the correct answer is (D).

10. $\sin\theta = \frac{3}{5}$

$$\Rightarrow \quad \cos\theta = \frac{4}{5}$$

$$mg\sin\theta = (10)(10)\left(\frac{3}{5}\right) = 60 \text{ N}$$

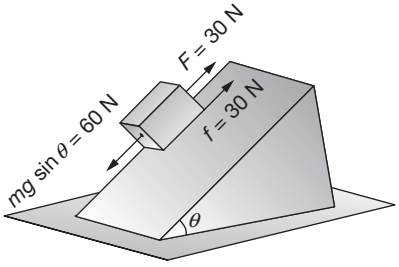
$$\mu mg\cos\theta = \left(\frac{3}{4}\right)(10)(10)\left(\frac{4}{5}\right) = 60 \text{ N}$$



i.e., when a force of 30 N is applied in upward direction friction force of 30 N acts upwards. Normal reaction is perpendicular to plane. Therefore, resultant force will be along OB.

Hence, the correct answer is (B).

11. $mg\sin\theta = 60$ N, $F = 30$ N and $f = 30$ N. So, from the free body diagram, we observe that the net force on the block is zero and hence the acceleration is zero.



Hence, the correct answer is (D).

12. For upward motion

$$-Fh = -\frac{1}{2}mv^2 + mgh$$

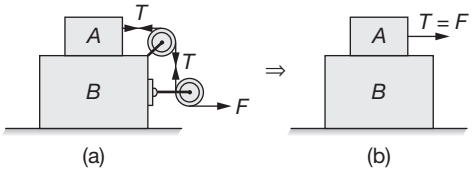
For downward motion

$$-Fh = \frac{1}{2}mv'^2 - mgh$$

$$\Rightarrow \quad v' = v\sqrt{\frac{mg - F}{mg + F}}$$

Hence, the correct answer is (C).

13. Net horizontal force on block B is zero.
Hence, the given figure (a) can be replaced by figure (b)



Maximum value of friction

$$f_{\max} = \mu m_A g = (0.5)(2)(10) = 10 \text{ N}$$

Block B moves due to friction
Therefore, maximum common acceleration of the two blocks can be

$$a_{\max} = \frac{f_{\max}}{m_B} = \frac{10}{2} = 5 \text{ ms}^{-2}$$

$$\Rightarrow \quad F_{\max} = (m_A + m_B)a_{\max}$$

$$\Rightarrow \quad F_{\max} = (2+2)(5) = 20 \text{ N}$$

Hence, the correct answer is (D).

14. $2T\cos\theta = \sqrt{2}\,Mg$ …(1)

$T\sin\theta$ and $T\sin\theta$ cancel out as both are equal in magnitude and are directed opposite to each other. Further,

$$T = Mg \qquad \ldots(2)$$

$$\Rightarrow \quad 2\,Mg\cos\theta = \sqrt{2}\,Mg$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 45°$$

Hence, the correct answer is (C).

15. Mass m_1 moves with constant velocity if tension in the lower string

$$T_1 = m_1 g = (1)(10) = 10 \text{ N} \qquad \ldots(1)$$

So, tension in the upper string is

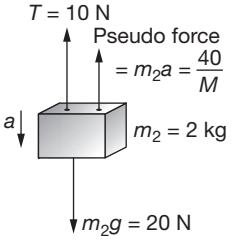
$$T_2 = 2T_1 = 20 \text{ N} \qquad \ldots(2)$$

Acceleration of block M is therefore,

$$a = \frac{T_2}{M} = \frac{20}{M} \qquad \ldots(3)$$

This is also the acceleration of pulley 2

Absolute acceleration of mass m_1 is zero. Thus, acceleration of m_1 relative to pulley 2 is a upwards or acceleration of m_2 with respect to pulley 2 is a downwards. Drawing free body diagram of m_2 with respect to pulley 2.



Equation of motion gives

$$20-\frac{40}{M}-10=2a=\frac{40}{M}$$

Solving this we get $M=8$ kg

Hence, the correct answer is (C).

17. Taking upward direction as positive

Given $a_A=1\ \text{ms}^{-2}$, $a_B=7\ \text{ms}^{-2}$ and $a_C=2\ \text{ms}^{-2}$

Let acceleration of pulley P is a upwards, then acceleration of pulley Q will be a downwards

$\Rightarrow \quad a_P=a$

and $a_Q=-a$

Now $a_{AP}=-a_{BP}$

$\Rightarrow \quad a_A-a_P=a_P-a_B$

$\Rightarrow \quad 1-a=a-7$

$\Rightarrow \quad 2a=8$

$\Rightarrow \quad a=4\ \text{ms}^{-2}$

Further $a_{CQ}=-a_{DQ}$

$\Rightarrow \quad a_C-a_Q=a_Q-a_D$

$\Rightarrow \quad 2-(-a)=-a-a_D$

$\Rightarrow \quad a_D=-2a-2=-10\ \text{ms}^{-2}$

So, acceleration of D is $10\ \text{ms}^{-2}$ downwards.

Hence, the correct answer is (C).

18. $\text{Thrust}=2T=\left(\frac{4mM}{M+m}\right)g$

$\Rightarrow \quad \text{Thrust}=\frac{4mg}{1+\frac{m}{M}}$

Since, $\frac{m}{M}\to 0$

$\Rightarrow \quad \text{Thrust}\approx 4mg$

Hence, the correct answer is (C).

19. From symmetrical behaviour of the arrangement, we observe that tension in both the branches BC and CD to be equal.

$\Rightarrow \quad 2T\cos\beta=mg$

$\Rightarrow \quad T=\frac{mg}{2\cos\beta}=\frac{mg}{2}\sec\beta$

$\Rightarrow \quad T=\frac{mg}{2}\sqrt{1+\tan^2\beta}$

$\Rightarrow \quad T=\frac{mg}{2}\sqrt{1+\left(\frac{12}{5}\right)^2}$

$\Rightarrow \quad T=\frac{13}{10}mg$

Hence, the correct answer is (B).

20. Block starts sliding over the surface when F is the maximum static force of friction

$\Rightarrow \quad Kt=\mu_s mg$

$\Rightarrow \quad t=\frac{\mu_s mg}{K}$

for $t>\frac{\mu_s mg}{K}$, net acceleration of block is

$$a=\frac{F-\mu_k mg}{m}=\frac{Kt}{m}-\mu_k g$$

which is a straight line with positive slope and negative intercept.

So, for $t\le\frac{\mu_s mg}{K}$, acceleration of particle is zero

and for $t>\frac{\mu_s mg}{K}$, acceleration is $a=\frac{Kt}{m}-\mu_k g$.

Hence, the correct answer is (D).

21. $a=\frac{F}{m_1+m_2+m_3}$

For equilibrium $m_1a=m_2g$

$\Rightarrow \quad m_1\left(\frac{F}{m_1+m_2+m_3}\right)=m_2g$

$\Rightarrow \quad F=\frac{m_2g}{m_1}(m_1+m_2+m_3)$

Hence, the correct answer is (C).

22. $kx_1=2g$

$\Rightarrow \quad x_1=\frac{2g}{k}$ …(1)

{k = force constant of spring}

$$\frac{3g - kx_2}{3} = \frac{3g - 2g}{5}$$

$$\Rightarrow \quad x_2 = \frac{12}{5}\frac{g}{k} \qquad \ldots(2)$$

$$\frac{kx_3 - g}{1} = \frac{2g - 1g}{3}$$

$$\Rightarrow \quad x_3 = \frac{4g}{3k} \qquad \ldots(3)$$

From (1), (2) and (3) we see that

$$x_2 > x_1 > x_3$$

Hence, the correct answer is (B).

23. $\text{Impulse} = \begin{pmatrix}\text{Area under force time graph}\\ \text{under the specified interval}\end{pmatrix}$

Hence, the correct answer is (A).

24. From constraint relations we can see that acceleration of block B is

$$a_B = \left(\frac{a_A + a_C}{2}\right) \text{ with proper signs}$$

Hence $a_B = \left(\frac{3 - 12t}{2}\right) = 1.5 - 6t$

$$\Rightarrow \quad \frac{dv_B}{dt} = 1.5 - 6t$$

$$\Rightarrow \quad \int_0^{v_B} dv_B = \int_0^t (1.5 - 6t)\,dt$$

$$\Rightarrow \quad v_B = 1.5t - 3t^2$$

$$\Rightarrow \quad v_B = 0 \text{ at } t = \frac{1}{2} \text{ s}$$

Hence, the correct answer is (D).

25. Let T be the tension in the string then

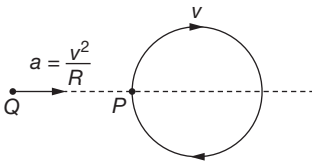
For body $A : F = T + \mu mg$

For body $B : T = \mu mg + \mu(m + M)g$

$$\Rightarrow \quad F = \mu(3m + M)g$$

Hence, the correct answer is (D).

26. At the moment shown in figure net force on P observed by Q is zero.



Hence, the correct answer is (C).

27. $$\frac{g}{2} = \frac{n - (16 - n)\left(\frac{1}{3}\right)}{16} g$$

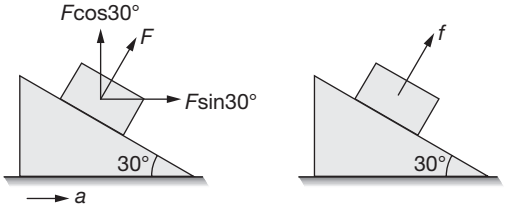
$$\Rightarrow \quad 8 = \frac{3n - (16 - n)}{3}$$

$$\Rightarrow \quad 24 = 3n - 16 + n$$

$$\Rightarrow \quad n = 10$$

Hence, the correct answer is (C).

28. Since the surfaces in contact are smooth, so force of friction between the block and the plane is zero. Hence, contact force is really the normal reaction between the two.



In the first case $F\sin 30° = ma$

and $F\cos 30° = mg \Rightarrow F = \frac{mg}{\cos 30°}$

and in the second case

$$f = mg\cos 30°$$

$$\Rightarrow \quad \frac{F}{f} = \frac{1}{\cos^2 30°} = \frac{4}{3}$$

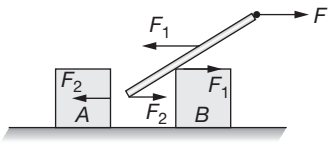
Hence, the correct answer is (B).

29. $$\begin{pmatrix}\text{Scale}\\ \text{reading}\\ \text{at time } t\end{pmatrix} = \begin{pmatrix}\text{Average}\\ \text{force}\\ \text{exerted}\\ \text{by pebbles}\end{pmatrix} + \begin{pmatrix}\text{Weight}\\ \text{of pebbles}\\ \text{collected}\\ \text{in box at time } t\end{pmatrix}$$

$$\begin{pmatrix}\text{Scale}\\ \text{reading}\\ \text{at time } t\end{pmatrix} = F_{av} + (mg)(nt) = mn\left(\sqrt{2gh} + gt\right)$$

Hence, the correct answer is (D).

30. Considering translational equilibrium of rod in horizontal direction.



$$F_1 - F_2 = F \Rightarrow F_1 > F_2$$

Therefore, block B will tend to move first.

Hence, the correct answer is (B).

31. Acceleration of body down the plane is

$$a = (g\sin 30°)(\cos 60°)$$

$$\Rightarrow \quad a = 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2.5 \text{ ms}^{-2}$$

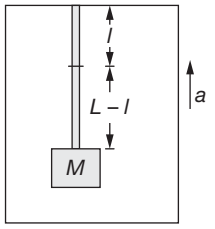
$$\Rightarrow \quad t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2\times 5}{2.5}} = 2 \text{ s} \qquad \left\{\because l = \frac{1}{2}at^2\right\}$$

Hence, the correct answer is (B).

32. Let us consider A and B as a system. Since, there is no vertical force in the upward direction to support their weight, hence the system cannot stay in equilibrium.

Hence, the correct answer is (D).

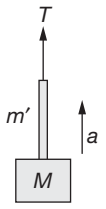
33. Let a be the acceleration of the elevator. Drawing free body diagram of the lower portion.



Let the mass of lower portion of string be $m' = \frac{m}{L}(L-l)$

Equation of motion gives

$$T - Mg - m'g = (M + m')a$$



$$\Rightarrow \quad T - Mg - \frac{mg}{L}(L-l) = \left(M + \frac{m}{L}(L-l)\right)a$$

$$\Rightarrow \quad a = \frac{T}{\left(M + m - \frac{ml}{L}\right)} - g$$

Hence, the correct answer is (C).

34. $T\cos\theta = mg\cos\alpha$ and …(1)

$$T\sin\theta = ma_0 + mg\sin\alpha \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{a_0 + g\sin\alpha}{g\cos\alpha}$$

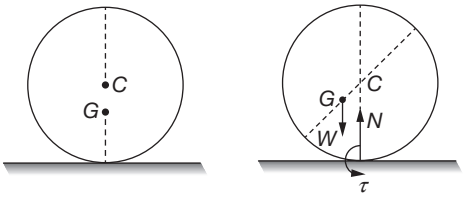
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{\frac{g}{2} + g\sin(30°)}{g\cos(30°)}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Hence, the correct answer is (B).

35. When centre of gravity lies vertically below its centre, the restoring torque while displaced from its equilibrium position, brings it back to its equilibrium position as shown below.



C = centre of sphere

G = centre of gravity

and τ = restoring torque

Hence, the correct answer is (B).

36. $mg = 2T\sin 45$

$$\Rightarrow \quad mg = \sqrt{2}T \qquad \ldots(1)$$

$$T_1\cos\theta = T\cos 45$$

$$\Rightarrow \quad T_1\cos\theta = \frac{T}{\sqrt{2}} = \frac{mg}{2} \qquad \left\{\because T = \frac{mg}{\sqrt{2}}\right\}$$

Further, $Mg + T\cos 45 = T_1\sin\theta$

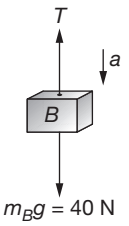
$$\Rightarrow \quad T_1\sin\theta = Mg + \frac{mg}{\sqrt{2}}\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad T_1\sin\theta = Mg + \frac{mg}{2} \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{Mg + \frac{mg}{2}}{\frac{mg}{2}} = 1 + \frac{2M}{m} \qquad \{\text{Divide (2) by (1)}\}$$

Hence, the correct answer is (A).

37.



From constraint relations we can see that

$$a_A = 2a_B$$

so, let $a_B = a$ then $a_A = 2a$

Free body diagram of B gives

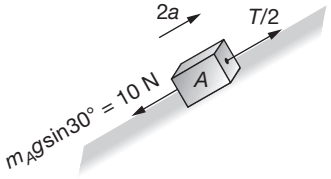
$$40 - T = 4a \qquad \ldots(1)$$

Free body diagram of A is

$$\frac{T}{2} - 10 = 4a$$

$$\Rightarrow \quad T - 20 = 8a \qquad \ldots(2)$$

Solving equations (1) and (2) we get



$$2a = \frac{10}{3}\ \text{ms}^{-2}$$

$$\Rightarrow \quad a_A = \frac{10}{3}\ \text{ms}^{-2}$$

Hence, the correct answer is (A).

38. Let acceleration of A be a_1 downwards and that of B is a_2 upwards and let T be the tension in the string. Then equations of motion of A and B gives

$$m_1g - T = m_1a_1 \qquad \ldots(1)$$

$$2T - m_2g = m_2a_2 \qquad \ldots(2)$$

Equilibrium of pulley C gives

$$2T - T = 0$$

$$\Rightarrow \quad T = 0$$

Substituting in (1) and (2)

$$a_1 = g \ \text{ and } \ a_2 = -g$$

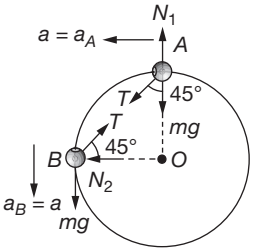
So, both the blocks are falling freely.

Hence, the correct answer is (D).

39. $T\sin 45 = ma$ (For A) $\qquad \ldots(1)$

$$mg - T\sin 45 = ma \ \text{(For } B\text{)} \qquad \ldots(2)$$

$$\Rightarrow \quad T = \frac{mg}{2\sin 45} = \frac{mg}{\sqrt{2}}$$



Hence, the correct answer is (C).

40. For the first 3 second, the acceleration of the combined system is

$$a = \left(\frac{5-m}{5+m}\right)g \qquad \ldots(1)$$

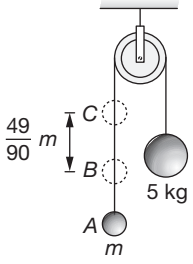
Velocity gained by m (starting from rest) at the end of 3 second is

$$v = 0 + at$$

$$\Rightarrow \quad v = 3\left(\frac{5-m}{5+m}\right)g \qquad \ldots(2)$$

At $t = 3s$, the string is cut (i.e., at position B).

As soon as the string is cut the mass m starts moving under the retardation g and covers a further distance of $\frac{49}{90}$m before its velocity becomes zero (at c) and then it begins to descend.



$$\Rightarrow \quad 0 - v^2 = 2(-g)\frac{49}{90}$$

$$\Rightarrow \quad 9\left(\frac{5-m}{5+m}\right)^2 g^2 = -2g\left(\frac{49}{90}\right)$$

$$\Rightarrow \quad \left(\frac{5-m}{5+m}\right)^2 = \frac{1}{81}$$

$$\Rightarrow \quad \frac{5-m}{5+m} = \pm\frac{1}{9}$$

$$\Rightarrow \quad m = 4 \text{ kg} \text{ or } m = 6.25 \text{ kg}$$

But $m = 6.25$ kg is not acceptable, as the value of lighter mass cannot exceed 5 kg.

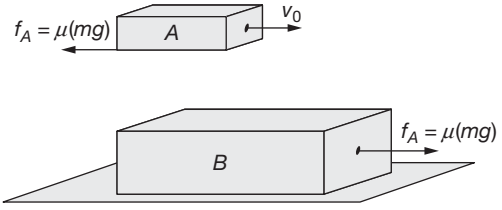
Hence, the correct answer is (A).

41. Force of friction between the two is

$$f_A = \mu mg$$

Retardation of A is $a_A = \frac{\mu mg}{m} = \mu g$

and acceleration of B is $a_B = \frac{\mu mg}{2m} = \frac{\mu g}{2}$



Hence, acceleration of B relative to A is

$$a_{BA} = a_A + a_B = \frac{3\mu g}{2}$$

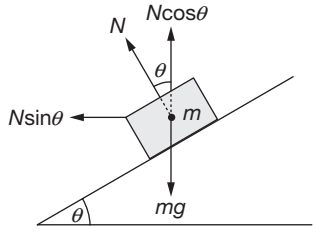
{$\because$ both are in opposite direction}

Since $\mu = \frac{1}{2}$

$$\Rightarrow \quad a_{BA} = \frac{3g}{4}$$

Hence, the correct answer is (C).

42. From the free body diagram, we observe that the component $N\sin\theta$ provides the necessary centripetal force, where as the component $N\cos\theta$ equals mg.



So, $N\cos\theta = mg$ …(1)

$$N\sin\theta = \frac{mv^2}{R} \quad \ldots(2)$$

So, $N = mg\sec\theta$ {$\because$ of (1)}

Hence, the correct answer is (C).

43. Blocks A and C both move due to friction and less friction is available to A as compared to C because normal reaction between A and B is less. Maximum friction between A and B can be

$$f_{\max} = \mu m_A g = \left(\frac{1}{2}\right)mg$$

Maximum acceleration of A can be

$$a_{\max} = \frac{f_{\max}}{m} = \frac{g}{2}$$

Further $a_{\max} = a_D$, because no slipping of A and C has to be on B (which moves forward with an acceleration a_D).

$$\Rightarrow \quad a_{\max} = \frac{m_D g}{3m + m_D}$$

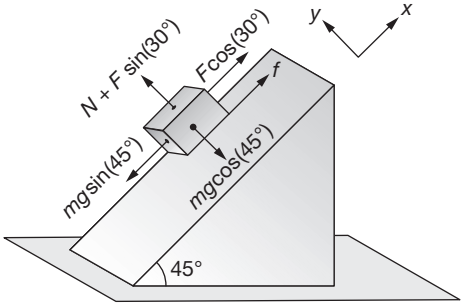
$$\Rightarrow \quad \frac{g}{2} = \frac{m_D g}{3m + m_D}$$

$$\Rightarrow \quad m_D = 3m$$

Hence, the correct answer is (A).

44. Drawing free body diagram of block, we have

$$\Sigma F_y = 0$$



$$\Rightarrow \quad N + F\sin 30° = mg\cos 45°$$

$$\Rightarrow \quad N = mg\cos 45° - F\sin 30°$$

$$\Rightarrow \quad N = (4)(10)\left(\frac{1}{\sqrt{2}}\right) - (10)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad N = 23.28 \text{ N} \quad \ldots(1)$$

$$f_{max} = \mu N = (0.6)(23.28) = 13.97 \text{ N}$$

$$mg\sin(45°) = (4)(10)\left(\frac{1}{\sqrt{2}}\right) = 28.28 \text{ N}$$

and $F\cos(30°) = (10)\left(\frac{\sqrt{3}}{2}\right) = 8.66$ N

Since, $mg\sin 45° > f_{max} + F\cos 30°$, so the block will slide down and friction will be maximum. Therefore, net contact force is

$$F_C = \sqrt{N^2 + (f_{max})^2}$$

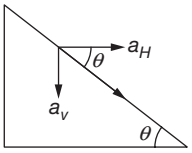
$$\Rightarrow \quad F_C = \sqrt{(23.28)^2 + (13.97)^2}$$

$$\Rightarrow \quad F_C = 27.15 \text{ N}$$

Hence, the correct answer is (C).

45. For no slipping, both A and B move together down the incline with an acceleration $a = g\sin\theta$, down the plane as no slipping exists between them. Horizontal component of this acceleration is $a_H = a\cos\theta$ and vertical component is $a_V = a\sin\theta$

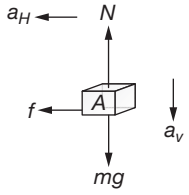
where $a = g\sin\theta$



$$\Rightarrow \quad a_H = a\cos\theta = g\sin\theta\cos\theta \text{ and}$$

$$a_V = a\sin\theta = g\sin^2\theta$$

FBD for A



Let N be the normal reaction between A and B. Equations of motion in horizontal and vertical directions give

$$mg - N = ma_V$$

$$\Rightarrow \quad N = mg - ma_V = mg - mg\sin^2\theta$$

$$\Rightarrow \quad N = mg\cos^2\theta$$

If f be the frictional force between A and B, then

$$ma_H \le \mu N$$

$$\Rightarrow \quad mg\sin\theta\cos\theta \le \mu mg\cos^2\theta$$

$$\Rightarrow \quad \theta \le \tan^{-1}(\mu)$$

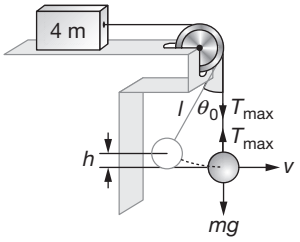
Hence, the correct answer is (D).

46. The tension is maximum in the string when it is in the lowest position. So,

$$h = l(1 - \cos\theta_0)$$

Speed of mass m in its lowest position is

$$v^2 = 2gh = 2gl(1 - \cos\theta_0)$$



Further since, $T_{max} - mg = \frac{mv^2}{l}$

$$\Rightarrow \quad T_{max} = mg + \frac{mv^2}{l}$$

$$\Rightarrow \quad T_{max} = mg + 2mg(1 - \cos\theta_0)$$

$$\Rightarrow \quad T_{max} = mg(3 - 2\cos\theta_0)$$

The block of mass $4m$ will not move, when

$$T_{max} \le \mu(4m)g$$

$$\Rightarrow \quad mg(3 - 2\cos\theta_0) \le 4\ \mu mg$$

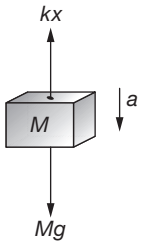
$$\Rightarrow \quad \mu \ge \frac{3 - 2\cos\theta_0}{4}$$

$$\Rightarrow \quad \mu_{MIN} = \frac{3 - 2\cos\theta_0}{4}$$

Hence, the correct answer is (D).

47. Net pulling force on the system is $Mg + mg - mg$ or simply Mg. Total mass being pulled is $M + 2m$. Hence acceleration of system is

$$a = \frac{Mg}{M + 2m}$$



Now since $a < g$, there should be an upward force on M so that its acceleration becomes less than g. Hence for any value of M spring will be elongated.

Hence, the correct answer is (D).

48. Net pulling force $= 2g - 1g = 10$ N

Mass being pulled $= 2 + 1 = 3$ kg

So, acceleration of the system is $a = \frac{10}{3}$ ms^{-2}

Velocity of both the blocks at $t = 1$ s will be

$$v_0 = at = \left(\frac{10}{3}\right)(1) = \frac{10}{3} \text{ ms}^{-1}$$

Now, at this moment velocity of 2 kg block becomes zero, while that of 1 kg block is $\frac{10}{3}$ ms^{-1} upwards. Hence, string becomes tight again when, displacement of 1 kg block = displacement of 2 kg block

$$\Rightarrow \quad v_0 t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \frac{v_0}{g} = \frac{\frac{10}{3}}{10} = \frac{1}{3} \text{ s}$$

Hence, the correct answer is (D).

49. Limiting force of friction between A and B is

$$f_1 = \mu_1 m_A g = (0.3)(30)(10) = 90 \text{ N}$$

Limiting force of friction between B and C is

$$f_2 = \mu_2 (m_A + m_B) g$$

$$f_2 = (0.2)(30 + 10)(10) = 80 \text{ N}$$

and limiting force of friction between C and ground is

$$f_3 = \mu_3 (m_A + m_B + m_C) g$$

$$f_3 = (0.1)(30 + 10 + 20)(10) = 60 \text{ N}$$

As F is gradually increased the force of friction between A and B will increase.

When $F = 60$ N block A will exert a horizontal force of 60 N on B. There will be no relative motion between A and B since this force is less than 90 N. B will also exert a horizontal force of 60 N on C. Hence C will be on the point of motion.

Hence, the least value of F is 60 N. All the three blocks will be on the point of motion as a single body.

Hence, the correct answer is (A).

50. Since $a_{\text{down}} = g(\sin\theta - \mu\cos\theta)$

$$\Rightarrow \quad a_{\text{down}} = a = 5\left(1 - 0.5\sqrt{3}\right) = 0.67 \text{ ms}^{-2}$$

So, minimum speed while reaching the bottom can be

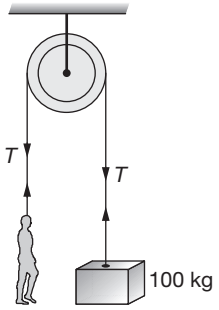
$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad v = (6)^2 + 2(0.67)(15) = 56.1$$

$$\Rightarrow \quad v = 7.49 \text{ ms}^{-1} \cong 7.5 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

51. Let T be the tension in the rope and a the acceleration of rope. The absolute acceleration of man is therefore $\left(\frac{5g}{4} - a\right)$. Equations of motion for mass and man gives



$$T - 100g = 100a \qquad \ldots(1)$$

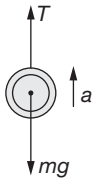
$$T - 60g = 60\left(\frac{5g}{4} - a\right) \qquad \ldots(2)$$

Solving (1) and (2) we get

$$T = 1218 \text{ N}$$

Hence, the correct answer is (C).

52. Let T be the tension in the string. The upward force exerted on the clamp is $T\sin 30° = \frac{T}{2}$.



Given $\frac{T}{2} = 40$ N

$$\Rightarrow \quad T = 80 \text{ N} \qquad \ldots(1)$$

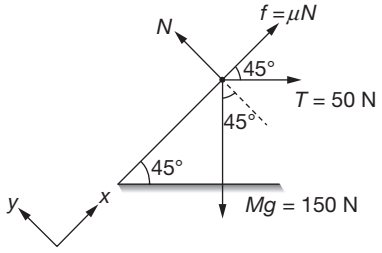
If a is the acceleration of monkey in upward direction. Equation of monkey in upward direction is

$$a = \frac{T - mg}{m}$$

$$a = \frac{80 - (5)(10)}{5} = 6 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

53. The string is under tension, hence there is limiting friction between the block and the plane. Drawing free body diagram of the block.



$$\Sigma F_x = 0$$

$$\Rightarrow \quad \mu N + 50\cos 45° = 150 \sin 45° \qquad \ldots(1)$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad N = 50 \sin 45° + 150 \cos 45° \qquad \ldots(2)$$

Solving (1) and (2) we get

$$\mu = \frac{1}{2}$$

Hence, the correct answer is (A).

54. Acceleration of the system and hence tension (T) on the string attached with the pulley is same in both the cases. Hence

$$F_1 = F_2 = 2T$$

Hence, the correct answer is (C).

55. As long as $mg \sin\theta < \mu mg \cos\theta$, the block will remain stationary.

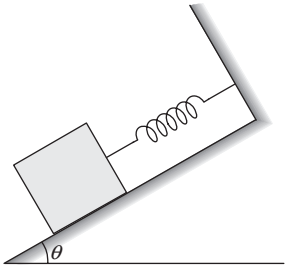
So, $x = 0$, till $\tan\theta < \mu$.

As soon as $\tan\theta > \mu$, then the spring of force constant k gets extended by say x.

So, we have, for equilibrium

$$kx + \mu mg \cos\theta = mg \sin\theta$$

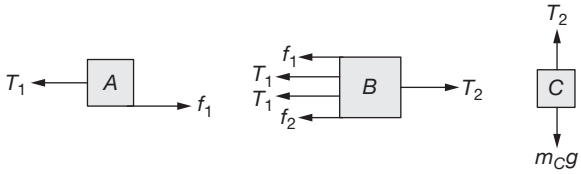
$$\Rightarrow \quad x = \frac{mg \sin\theta - \mu mg \cos\theta}{k}$$



As $\theta \to \frac{\pi}{2}$, $x = \frac{mg}{k}$ $\{\because N = 0\}$

Hence, the correct answer is (A).

56. Maximum friction that can be obtained between A and B is



$$f_1 = \mu m_A g = (0.3)(100)(10) = 300 \text{ N}$$

and maximum friction between B and ground is

$$f_2 = \mu(m_A + m_B)g$$

$$f_2 = (0.3)(100 + 140)(10) = 720 \text{ N}$$

Drawing free body diagrams of A, B and C in limiting case

Equilibrium of A gives

$$T_1 = f_1 = 300 \text{ N} \qquad \ldots(1)$$

Equilibrium of B gives

$$2T_1 + f_1 + f_2 = T_2$$

$$\Rightarrow \quad T_2 = 2(300) + 300 + 720 = 1620 \text{ N} \qquad \ldots(2)$$

and equilibrium of C gives

$$m_C g = T_2$$

$$\Rightarrow \quad 10 m_C = 1620$$

$$\Rightarrow \quad m_C = 162 \text{ kg}$$

Hence, the correct answer is (D).

57. For equilibrium

$$\begin{pmatrix} \text{Weight of} \\ \text{hanging part} \\ \text{of chain} \end{pmatrix} = \begin{pmatrix} \text{Force of friction} \\ \text{on the part resting} \\ \text{on table} \end{pmatrix}$$

Let l be the length of the chain hanging

$$\Rightarrow \quad \left(\frac{M}{L}l\right)g = \mu\left[\frac{M}{L}(L-l)g\right]$$

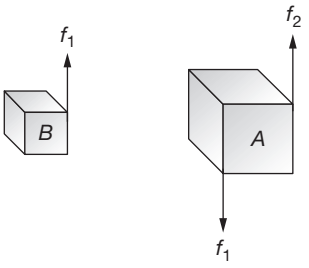
$$\Rightarrow \quad l = \mu(L-l)$$

$$\Rightarrow \quad (\mu+1)l = \mu L$$

$$\Rightarrow \quad l = \frac{\mu L}{\mu+1}$$

Hence, the correct answer is (A).

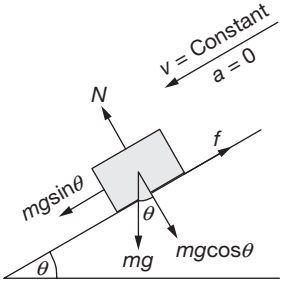
58. The directions of friction forces at different surfaces are shown in figure.



Here, f_1 is the force of friction between A and B and f_2 is the force of friction between A and vertical wall.

Hence, the correct answer is (B).

59. Block slides down with constant velocity. Hence, net force on the block is zero.



So, net contact force F_C on the body is

$$F_C = \sqrt{f^2 + N^2}$$

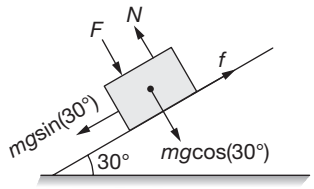
where $f = mg\sin\theta$ $\quad \{\because a = 0 \Rightarrow f = mg\sin\theta\}$

and $N = mg\cos\theta$

$$\Rightarrow \quad F_C = \sqrt{m^2g^2(\cos^2\theta + \sin^2\theta)} = mg$$

Hence, the correct answer is (D).

60. Since, we observe that $mg\sin(30°) > \mu mg\cos(30°)$



So, the block has a tendency to slip downwards. Let F be the minimum force applied on it, such that it does not slip. Then

$$N = F + mg\cos(30°)$$

Also, $mg\sin(30°) = \mu N = \mu\left(F + mg\cos 30°\right)$

$$\Rightarrow \quad F = \frac{mg\sin(30°)}{\mu} - mg\cos(30°)$$

$$\Rightarrow \quad F = \frac{(2)(10)\left(\frac{1}{2}\right)}{0.5} - (2)(10)\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad F = 20 - 17.32$$

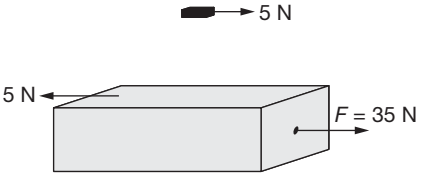
$$\Rightarrow \quad F = 2.68 \text{ N}$$

Hence, the correct answer is (C).

61. Maximum frictional force between the block and the plank is

$$f_{\max} = \mu mg = (0.5)(1)(10) = 5 \text{ N}$$

The free body diagrams of the block and the plank are shown for convenience to solve the problem



Acceleration of block is

$$a_B = \frac{5}{1} = 5 \text{ ms}^{-2}$$

Acceleration of plank is

$$a_P = \frac{35-5}{2} = 15 \text{ ms}^{-2}$$

So, relative acceleration of block with respect to the plank is

$$a_{BP} = a_B - a_P = -10 \text{ ms}^{-2}$$

$$\Rightarrow \quad a_{BP} = 10 \text{ ms}^{-2}, \text{ backwards}$$

Since, $s = \frac{1}{2}at^2$, where $s = 1.25$ m, backwards

$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{(2)(-1.25)}{(-10)}}$$

$$\Rightarrow \quad t = \sqrt{\frac{2.5}{10}} = 0.5 \text{ s}$$

Hence, the correct answer is (B).

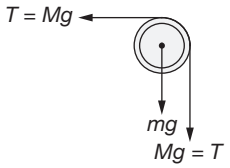
62. $a = g(\sin\theta - \mu\cos\theta)$

$$\Rightarrow \quad a = g(\sin\theta - kx\cos\theta)$$

At $x = 0$, $a = g\sin\theta$. As x increases a decreases and becomes zero at $x_0 = \frac{1}{k}\tan\theta$. For $x > x_0$, the acceleration becomes negative.

Hence, the correct answer is (D).

63.



Three forces are acting on the pulley T (horizontally), T (vertically downward) and the weight of the pulley (also acting vertically downwards) i.e. Force Mg acts horizontally and $(M+m)g$ acts vertically downwards. So, total force acting on the pulley is

$$F_{\text{net}} = \sqrt{M^2g^2 + (M+m)^2 g^2}$$

$$\Rightarrow \quad F_{\text{net}} = \left[\sqrt{M^2 + (M+m)^2}\right]g$$

Hence, the correct answer is (D).

64. $h = R\sin\theta$

Speed of particle at point B is

$$v^2 = 2gh$$

$$\Rightarrow \quad v^2 = 2gR\sin\theta$$

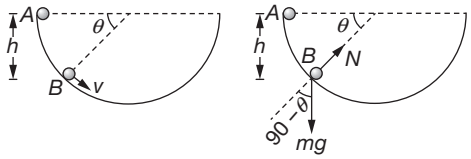
The centripetal force is given by

$$F = \frac{mv^2}{R} = 2mg\sin\theta \qquad \ldots(1)$$

Further since,

$$\Rightarrow \quad N - mg\sin\theta = \frac{mv^2}{R} = 2mg\sin\theta$$

$$\Rightarrow \quad N = 3mg\sin\theta \qquad \ldots(2)$$

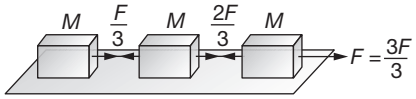


The desired ratio is

$$x = \frac{F}{N} = \frac{2}{3} = \text{constant}$$

Hence, the correct answer is (D).

65.



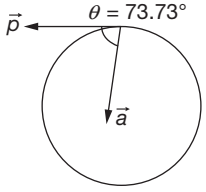
Hence, the correct answer is (A).

66. Angle θ between $\vec{a}$ and $\vec{p}$ (or $\vec{v}$) is given by

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{p}}{|\vec{a}||\vec{p}|}\right)$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{32-18}{\sqrt{(16+9)}\sqrt{64+36}}\right)$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{14}{50}\right)$$



$$\Rightarrow \quad \theta = 73.73°$$

Since $0° < \theta < 90°$, the motion is an accelerated motion.

Hence, the correct answer is (A).

67. $|\vec{F}| = vA\sin\theta$

where θ is angle between $\vec{v}$ and $\vec{A}$.

Particle moves undeflected if $\vec{F}$ is equal and opposite to the weight of the particle. i.e.,

$$|\vec{F}| = mg$$

$$\Rightarrow \quad vA\sin\theta = mg$$

$$\Rightarrow \quad v = \frac{mg}{A\sin\theta} \qquad \ldots(1)$$

At $\theta = 90°$, we have $\sin\theta = 1 = \text{MAXIMUM}$, so

$$v_{\min} = \frac{mg}{A}$$

Now weight is in negative z-direction i.e., $\vec{F}$ should be in positive z-direction. Hence velocity should be in negative y-direction

$$\Rightarrow \quad \vec{v}_{\min} = -\frac{mg}{A}\hat{j}$$

Hence, the correct answer is (D).

68. $$a = \left(\frac{M-m}{M+m}\right)g$$

$$\Rightarrow \quad 1.4 = \frac{1}{2}\left(\frac{M-m}{M+m}\right)g(2)^2$$

$$\Rightarrow \quad \frac{m}{M} = \frac{13}{15}$$

Hence, the correct answer is (A).

69. $$F_{\text{applied}} - f = ma$$

$$\Rightarrow \quad F_{\text{applied}} = \mu mg + ma$$

$$\Rightarrow \quad F_{\text{applied}} = m(\mu g + a) \qquad \ldots(1)$$

Since $v^2 - u^2 = 2as$ $\qquad \ldots(2)$

$$a = \frac{v^2 - u^2}{2s}$$

Put value of a as calculated from (2) in equation (1)
Hence, the correct answer is (C).

70. On displacing m through x towards right, M gets displaced downward by $2x$.
For equilibrium

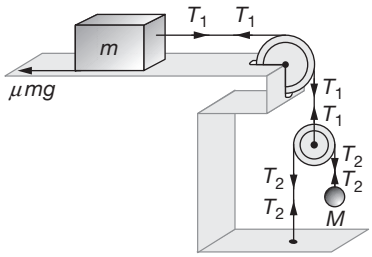
$$T_1 = \mu mg \qquad \ldots(1)$$

$$T_1 = 2T_2 \qquad \ldots(2)$$

$$T_2 = Mg \qquad \ldots(3)$$

$$\Rightarrow \quad 2(Mg) = \mu mg$$

$$\Rightarrow \quad M = \frac{1}{2}\mu m$$



Hence, the correct answer is (A).

71. Acceleration of block till the slipping continues is

$$a = \frac{f_{\max}}{m}$$

$$\Rightarrow \quad a = \frac{\mu mg}{m} = \mu g = (0.5)(10) = 5 \text{ ms}^{-2}$$

Slipping will continue till the velocity of the block also becomes 3 ms^{-1}. So, using $v = u + at$, we get

$$3 = 0 + 5t \qquad \{\because u = 0\}$$

$$\Rightarrow \quad t = 0.6 \text{ s}$$

During this time, the displacement of the block is

$$s_1 = \frac{1}{2}at^2 = \frac{1}{2}(5)(0.6)^2 = 0.9 \text{ m}$$

and the displacement of the belt is

$$s_2 = vt = (3)(0.6) = 1.8 \text{ m}$$

Hence displacement of the block w.r.t., the belt is

$$|s_1 - s_2| = 0.9 \text{ m}$$

Hence, the correct answer is (A).

72. $$mg\sin\alpha - T - f = m\ddot{x}$$

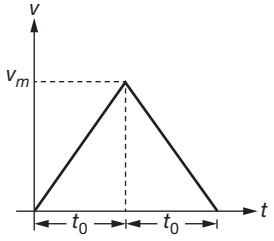
$$\Rightarrow \quad mg\sin\alpha - \frac{\lambda(l - l_0)}{l_0} - k\dot{x} = m\ddot{x}$$

where, k is any known constant.
At Equilibrium Position, $\dot{x} = \ddot{x} = 0$

$$\Rightarrow \quad l = l_0\left(1 + \frac{mg}{\lambda}\sin\alpha\right)$$

Hence, the correct answer is (B).

73. The maximum acceleration and maximum retardation for the car can be $\mu g = \left(\frac{1}{2}\right)(10) = 5 \text{ ms}^{-2}$. The corresponding velocity-time graph is as follows



Let t_0 be the time taken by the car to accelerate. The time taken by it to decelerate is also t_0. So,

$$v_m = (\mu g)t_0 = 5t_0$$

Since, displacement = area under v-t graph. So, we get

$$500 = \frac{1}{2}(2t_0)(5t_0)$$

$\Rightarrow \quad t_0 = 10$ s

Hence total time of journey is $t = 2t_0 = 20$ s.

Hence, the correct answer is (C).

74. Impulse is the area under F-t graph, as well as the change in momentum. So,

$$mu = \text{Area} = \frac{1}{2}F_0T$$

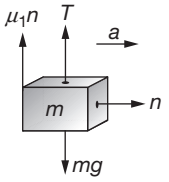
$$\Rightarrow \quad F_0 = \frac{2mu}{T}$$

Hence, the correct answer is (C).

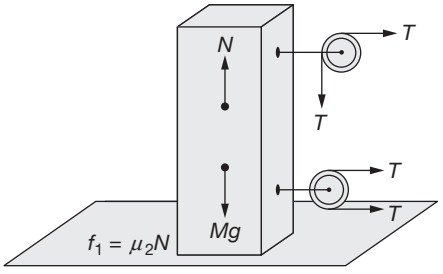
75. For equilibrium of the system i.e., M and m, we have

$$T + \mu_1 n = mg \qquad \ldots(1)$$

$$n = ma \qquad \ldots(2)$$



Since the mass m is at rest, so $n = 0$. This is a justified and an expected answer $(f_2 = 0)$, because when M does not move, then no force is exerted by M on m and hence $n = 0$. Let us now draw free body diagram for M, without taking into account the frictional force on M due to m (as it is zero). So, for M to be at rest we have



$$N = Mg + T \qquad \ldots(3)$$

$$3T = \mu_2 N \qquad \ldots(4)$$

Also, from (1), we get

$$T = mg \qquad \{\because\ n = 0\}$$

From (3), we get

$$N = (M + m)g$$

From (4), we get

$$3mg = \mu_2(M + m)g$$

$$\Rightarrow \quad 3m = \mu_2 M + \mu_2 m$$

$$\Rightarrow \quad m = \frac{\mu_2 M}{3 - \mu_2}$$

$$\Rightarrow \quad m = \frac{\left(\frac{1}{3}\right)(8)}{3 - \left(\frac{1}{3}\right)} = 1 \text{ kg}$$

Hence, the correct answer is (C).

76. For $a \le \mu g$, the blocks will remain stationary with respect to the surface of the plank, so $x = 0$.

For $a \ge \mu g$, both the blocks will move with respect to the surface of the plank with an acceleration $(a - \mu g)$. However at any instant no relative motion between the blocks exists, so still they don't stretch the spring.

Hence, the correct answer is (D).

77. Since the block is about to slide, so we have

$$mg\sin\theta = \mu N$$

$$\Rightarrow \quad \cancel{m}g\sin\theta = \mu(\cancel{m}g\cos\theta)$$

$$\Rightarrow \quad \mu = \tan\theta$$

This condition is independent of the mass, so it will still be on the verge of motion down the incline.

Hence, the correct answer is (D).

78. In CASE-1, sliding starts when

$$mg\sin\theta \ge \mu mg\cos\theta$$

$$\Rightarrow \quad \tan\theta \ge \mu \qquad \ldots(1)$$

While in CASE-2, sliding starts when

$$m(g + a_0)\sin\alpha \ge \mu m(g + a_0)\cos\alpha$$

$$\Rightarrow \quad \tan\alpha \ge \mu \qquad \ldots(2)$$

From (1) and (2), we get

$$\theta = \alpha$$

Hence, the correct answer is (D).

79. $m(2)\omega^2 \leftarrow Q \rightarrow T_1 \qquad T_1 \leftarrow P \rightarrow T_1$ (with $m(1)\omega^2$ at P)

$$T_1 = m(2)\omega^2 \qquad \ldots(1)$$

($\because$ centrifugal force acting on Q is $2m\omega^2$ radially outwards)

$$T_1 + m(1)\omega^2 = T_2 \quad \ldots(2)$$

$$\Rightarrow \quad T_2 = 3m\omega^2 \quad \ldots(3)$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{2}{3}$$

Hence, the correct answer is (C).

80. $T\cos\theta = mg\cos\alpha$ and $\ldots(1)$

$$T\sin\theta = ma_0 + mg\sin\alpha \quad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{a_0 + g\sin\alpha}{g\cos\alpha}$$

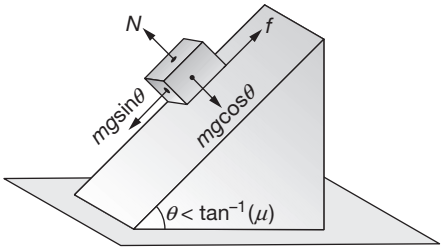
Hence, the correct answer is (D).

81. $\tan\theta = \dfrac{v^2}{rg} = 1$

$$\Rightarrow \quad \theta = 45^\circ$$

Hence, the correct answer is (C).

82. As long as $\tan\theta \le \mu$, the block will not slide down, so we have the contact force $F_C = \sqrt{N^2 + f^2}$



where $N = mg\cos\theta$ and $f = mg\sin\theta$

So, for $\tan\theta \le \mu$, we have

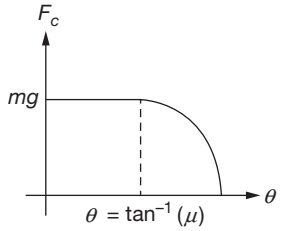
$$F_C = \sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2} = mg$$

However, when $\tan\theta > \mu$, then the block starts sliding down and now $mg\sin\theta > f$. So,

$$F_C = \sqrt{N^2 + f^2} = \sqrt{N^2 + (\mu N)^2}$$

$$\Rightarrow \quad F_C = N\sqrt{1+\mu^2} = mg\cos\theta\sqrt{1+\mu^2}$$

Since, $\cos\theta$ decreases as θ increases hence, the contact force starts decreasing and finally it becomes zero at $\theta = 90^\circ$. Therefore, graph between F_C and θ will be as follows



Hence, the correct answer is (D).

83. Impulse is the area under F-t graph, as well as the change in momentum. So,

$$mu = \pi\left(\frac{F_0}{2}\right)\left(\frac{T}{2}\right)$$

$$\Rightarrow \quad u = \frac{\pi F_0 T}{4m}$$

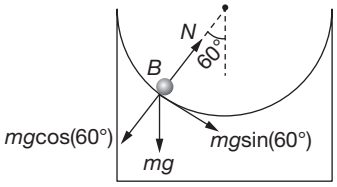
Hence, the correct answer is (C).

84. At position B, speed of ball is $v^2 = 2gh$

where $h = R\cos(60^\circ) = (0.5)\left(\dfrac{1}{2}\right) = 0.25$ m

$$\Rightarrow \quad v^2 = (2)(10)(0.25) = 5\ \text{m}^2\text{s}^{-2} \quad \ldots(1)$$

Let N be the normal reaction between the ball and the wedge in this position. Then



Since, we have

$$N - mg\cos(60^\circ) = \frac{mv^2}{R}$$

$$\Rightarrow \quad N = \frac{mv^2}{R} + mg\cos(60^\circ)$$

Since $v^2 = 5$ $\{\because$ of (1)$\}$

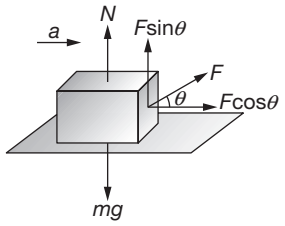
$$\Rightarrow \quad N = \frac{(1)(5)}{(0.5)} + (1)(10)\left(\frac{1}{2}\right) = 15\ \text{N}$$

Now for horizontal equilibrium of wedge
Force exerted by vertical wall on wedge is

$$F = N\sin(60^\circ) = \frac{15\sqrt{3}}{2}\ \text{N}$$

Hence, the correct answer is (C).

85.



$\Rightarrow \quad F\cos\theta = ma$

$\Rightarrow \quad \frac{mg}{3}\cos\theta = m\frac{dv}{dt}$

$\Rightarrow \quad \frac{mg}{3}\cos(ks) = m\frac{dv}{ds}\frac{ds}{dt}$

$\Rightarrow \quad \frac{mg}{3}\cos(ks) = mv\frac{dv}{ds}$

$\Rightarrow \quad vdv = \frac{g}{3}\cos(ks)ds$

$\Rightarrow \quad \frac{v^2}{2} = \frac{g}{3k}\sin(ks)$

$\Rightarrow \quad v = \sqrt{\frac{2g}{3k}\sin\theta}$

Hence, the correct answer is (C).

86. Since, the radii of both the balls is same, so the force of buoyancy is also the same.
When the balls fall with a uniform velocity, then condition of equilibrium is attained, so we get

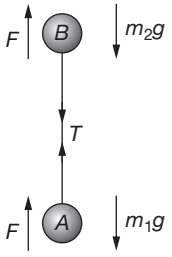
For A, $F + T = m_1 g$ …(1)

For B, $m_2 g + T = F$ …(2)

From (1), equating the value of T, we get

$2F = m_1 g + m_2 g$

$\Rightarrow \quad F = \left(\frac{m_1 + m_2}{2}\right)g$ …(3)



Substitute (3) either in (1) or in (2), we get

$T = m_1 g - \left(\frac{m_1 + m_2}{2}\right)g$

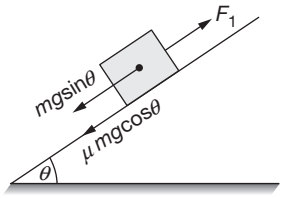
$\Rightarrow \quad T = \left(\frac{m_1 - m_2}{2}\right)g$

Hence, the correct answer is (B).

87. Let the mass of the block be m. Force required to drag it up along the plane is

$F_1 = (mg\sin\theta + \mu mg\cos\theta)$

and force required to lift it is



$F_2 = mg$

Since it is given that $F_1 < F_2$

$\Rightarrow \quad mg\sin\theta + \mu mg\cos\theta < mg$

$\Rightarrow \quad \mu < \frac{1 - \sin\theta}{\cos\theta}$

$\Rightarrow \quad \mu < \frac{1 - \sin 30°}{\cos 30°}$

$\Rightarrow \quad \mu < \frac{1}{\sqrt{3}}$

So, $\mu_{MAX} = \frac{1}{\sqrt{3}}$

Hence, the correct answer is (C).

88. If $m_1 = m_2$, block will slide down when

$\mu g\cos\theta < g\sin\theta$

$\Rightarrow \quad \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) < \frac{1}{2}$ {which is true}

If $m_1 < m_2$, block will slide down when

$\mu(g - a)\cos\theta < (g - a)\sin\theta$

$\Rightarrow \quad \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) < \frac{1}{2}$

Here $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$

If $m_1 > m_2$, block will slide down when

$\mu(g + a)\cos\theta < (g + a)\sin\theta$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) < \frac{1}{2}$$

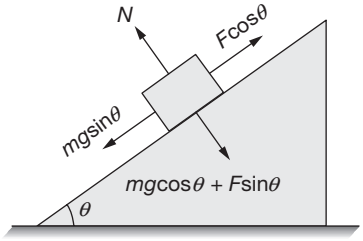
Here $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

So, block can't remain stationary when $m_1 > m_2$.

Therefore, block will slide down under all conditions.

Hence, the correct answer is (D).

89. The free body diagram of the block is shown here.



So, we get

$$N = mg\cos\theta + F\sin\theta$$

and $F_{\text{net}} = F\cos\theta - mg\sin\theta$ {up the plane}

Hence, the correct answer is (D).

90. Let x be the maximum displacement of block downwards. Then from conservation of mechanical energy, we have the decrease in potential energy of 2 kg block must be equal to the increase in elastic potential energy of both the springs. So, we have

$$mgx = \frac{1}{2}(k_1 + k_2)x^2$$

$$\Rightarrow \quad x = \frac{2mg}{k_1 + k_2} = \frac{(2)(4)(10)}{100 + 300} = 0.2 \text{ m}$$

Acceleration of block in this position is

$$a = \frac{(k_1 + k_2)x - mg}{m} \quad \{\text{upwards}\}$$

$$\Rightarrow \quad a = \frac{(400)(0.2) - (4)(10)}{4} = \frac{80 - 40}{4}$$

$$\Rightarrow \quad a = 10 \text{ ms}^{-2} \quad \{\text{upwards}\}$$

Hence, the correct answer is (D).

91. Let T be the tension in the rope, then for the block, we have

$$T = Mg \qquad \ldots(1)$$

If N is the normal reaction between the man and the ground, then for the man, we have

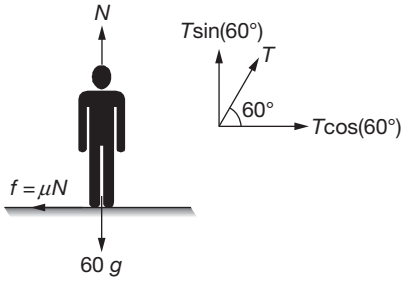
$$N + T\sin(60°) = 60g$$

$$\Rightarrow \quad N = 60g - T\sin(60°) \qquad \ldots(2)$$

Also $T\cos(60°) = \mu N$ $\ldots(3)$

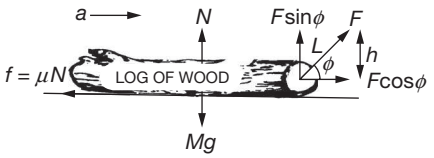
Solving these three equations we get

$$M = 32.15 \text{ kg} \approx 32 \text{ kg}$$



Hence, the correct answer is (D).

92.



$$F\cos\phi - \mu N = Ma \qquad \ldots(1)$$

$$F\sin\phi + N = Mg$$

$$\Rightarrow \quad N = Mg - F\sin\phi \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$F\cos\phi - \mu Mg + \mu F\sin\phi = Ma$$

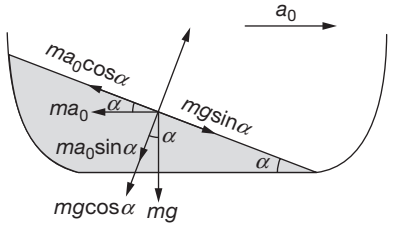
$$\Rightarrow \quad a = \frac{F}{M}(\cos\phi + \mu\sin\phi) - \mu g$$

Hence, the correct answer is (D).

93. According to Newton's Third Law to every action there is equal and opposite reaction and both act on two different bodies. But in this case both are acting on the same body and hence they cancel so that the boat remains stationary.

Hence, the correct answer is (D).

95.



$ma_0 \cos\alpha = mg\sin\alpha$

$\Rightarrow \quad a_0 = g\tan\alpha$

$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{a_0}{g}\right)$

Hence, the correct answer is (A).

96. Thrust on the block $= F = v\frac{dm}{dt}$

$\Rightarrow \quad F = (5)(1) = 5N$

Acceleration of the block $= \frac{F}{M} = \frac{5}{2}\ \text{ms}^{-2}$

Hence, the correct answer is (B).

97. $F = \frac{mv_x^2}{R} = \frac{mv_0^2\cos^2\alpha}{R}$

Hence, the correct answer is (B).

98. $kx = 2mg$

By Law of Conservation of Energy

$Mgx = \frac{1}{2}kx^2$

$\Rightarrow \quad kx = 2Mg$

$\Rightarrow \quad M = m$

Hence, the correct answer is (B).

99. $mg - U = ma_0$ …(1)

and $U - (m - \Delta m)g = (m - \Delta m)a_0$ …(2)

$\Rightarrow \quad \Delta m = \left(\frac{2a_0}{g + a_0}\right)m$ {Adding (1) and (2)}

Hence, the correct answer is (C).

100. Since windows are closed and the balloon is filled with helium which is lighter than air.
The concept of pseudo force cannot be applied here as in this case the density of body is much less than the density of surrounding air. So when car takes a left turn the expected answer is that Helium balloon will be pushed to the right whereas actually it will be pushed to the left.

Hence, the correct answer is (B).

101. The laws of Physics are the same in all inertial frames or the frames moving with constant velocity with respect to an inertial frame.

Hence, the correct answer is (B).

102. $F = \sqrt{T^2 + T^2 - 2T^2\cos 2\theta_0}$

$\Rightarrow \quad F = \sqrt{2}T\sqrt{1 - \cos 2\theta_0}$

$\Rightarrow \quad F = \sqrt{2}T\sqrt{2}\sin\theta_0$

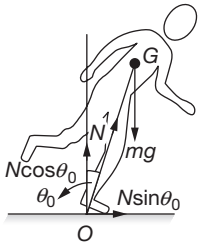
$\Rightarrow \quad F = 2T\sin\theta_0$

Hence, the correct answer is (C).

103. For the man not to slip i.e. for rotational equilibrium to exist, the algebraic sum of the moments of the forces acting on the man must be zero. Let N be the normal reaction exerted by the earth on the man. The following forces are acting on the man.

(a) Weight mg acting vertically downwards from the centre of gravity G of the man.

(b) Normal reaction N exerted by earth on the man along OG for rotational equilibrium to exist.



For the man not to slip.

$N\sin\theta_0 \le f_{\text{limiting}}$ …(1)

But $f_{\text{limiting}} = \mu(N\cos\theta_0)$

$\Rightarrow \quad N\sin\theta_0 \le \mu N\cos\theta_0$

$\tan\theta_0 \le \mu$

Hence, the correct answer is (B).

104. The vertically hanging curtains bend forward thus indicating that the pseudo force acting on the curtain points in the forward direction. For this to happen the plane must be slowing down for landing.

Hence, the correct answer is (C).

105. $F\cos\theta = \mu N$ and …(1)

$N + F\sin\theta = mg$

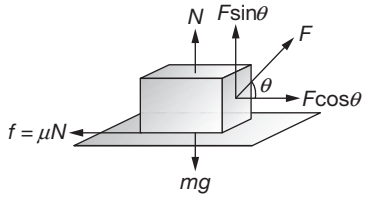
$\Rightarrow \quad N = mg - F\sin\theta$ …(2)

Substituting (2) in (1), we get

$F\cos\theta = \mu mg - \mu F\sin\theta$

$\Rightarrow \quad F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$ …(3)

Let Z = Denominator = $\cos\theta + \mu\sin\theta$



For F to be MINIMUM, Z must be MAXIMUM

$$\Rightarrow \quad \frac{dZ}{d\theta} = 0$$

$$\Rightarrow \quad -\sin\theta + \mu\cos\theta = 0$$

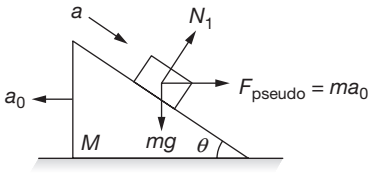
$$\Rightarrow \quad \tan\theta = \mu$$

$$\Rightarrow \quad \theta = \tan^{-1}\mu$$

and $F_{\min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

Hence, the correct answer is (C).

106. Let a be the acceleration of the block with respect to prism. Let the prism move backward with acceleration a_0, then a fictitious (pseudo) force acts on the mass (see figure).



For the mass to move down with acceleration a, we have

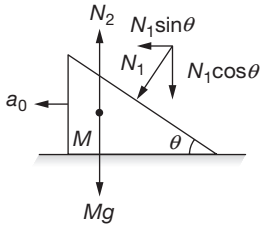
$$ma_0\cos\theta + mg\sin\theta = ma$$

$$\Rightarrow \quad a = a_0\cos\theta + g\sin\theta \qquad \ldots(1)$$

Also the block is in equilibrium perpendicular to the incline i.e.

$$N_1 + ma_0\sin\theta = mg\cos\theta \qquad \ldots(2)$$

For the prism, we have that it is placed on the ground (an inertial frame) with no fictitious force acting on the prism. Following forces are acting on the prism.



(a) Weight of the prism Mg.

(b) Normal reaction N_2 exerted by the surface on the prism.

(c) Normal reaction N_1 exerted by the block placed on the prism.

$N_1\sin\theta$ is the component responsible for horizontal motion of the prism. So,

$$N_1\sin\theta = Ma_0$$

$$\Rightarrow \quad N_1 = \frac{Ma_0}{\sin\theta} \qquad \ldots(3)$$

Substituting (3) in (2), we get

$$\frac{Ma_0}{\sin\theta} + ma_0\sin\theta = mg\cos\theta$$

$$\Rightarrow \quad a_0 = \left(\frac{m\cos\theta}{M + m\sin^2\theta}\right)g\sin\theta \qquad \ldots(4)$$

Substituting a_0 in (1) we get

$$a = \left(\frac{M+m}{M+m\sin^2\theta}\right)g\sin\theta$$

Hence, the correct answer is (C).

OBJECTIVE TRICK

If $\theta \to \dfrac{\pi}{2}$, then m (i.e. the block) will fall freely with acceleration g. This is satisfied by both (A) and (C). To check which one of the two we can further think that if M is very heavy as compared to m, then M must not move and acceleration of m w.r.t. M is $g\sin\theta$. This is satisfied by (C) only. If we can directly think the second way first, then we can even solve the problem without any cumbersome steps.

107. Since $x = kt^2 = 1.732t^2$

$$\Rightarrow \quad x = \sqrt{3}t^2$$

$$\Rightarrow \quad \dot{x} = 2\sqrt{3}t$$

$$\Rightarrow \quad \ddot{x} = a = 2\sqrt{3}\ \text{ms}^{-2}$$

$$\Rightarrow \quad a = g(\sin 45 - \mu\cos 45) = 2\sqrt{3}$$

$$\Rightarrow \quad g\frac{1}{\sqrt{2}}(1-\mu) = 2\sqrt{3}$$

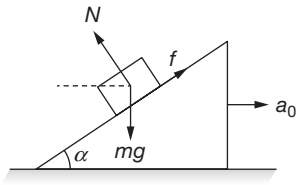
$$\Rightarrow \quad 1 - \mu = \frac{2\sqrt{6}}{g}$$

$$\Rightarrow \quad \mu = 1 - \frac{2\sqrt{6}}{g} = 1 - \frac{2(2.45)}{9.8}$$

$$\Rightarrow \quad \mu = 0.5$$

Hence, the correct answer is (A).

108. Consider the block not to slide, then it must have an acceleration same as that of the plane. Then



$$f\cos\alpha - N\sin\alpha = ma_0 \qquad \ldots(1)$$

and $f\sin\alpha + N\cos\alpha = mg$ …(2)

From (1) and (2), we get

$$f = m(a_0\cos\alpha + g\sin\alpha) \qquad \ldots(3)$$

and $N = m(g\cos\alpha - a_0\sin\alpha)$ …(4)

$$\Rightarrow \quad \frac{f}{N} = \frac{a_0\cos\alpha + g\sin\alpha}{g\cos\alpha - a_0\sin\alpha}$$

$$\Rightarrow \quad \frac{f}{N} = \frac{a_0 + g\tan\alpha}{a_0 - g\tan\alpha} \qquad \ldots(5)$$

Further $\frac{f}{N} = \mu_s = \tan\theta$ in the absence of slipping

$$\Rightarrow \quad \frac{a_0 + g\tan\alpha}{a_0 - g\tan\alpha} \le \tan\theta$$

$$\Rightarrow \quad a_0 \le g\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]$$

$$\Rightarrow \quad a_0 \le g\tan(\theta - \alpha) \qquad \{\text{for no slipping}\}$$

So for the block to slide $a_0 > g\tan(\theta - \alpha)$.

Hence, the correct answer is (C).

109. Let N_1 be normal reaction between the man and the board

Let N_2 be normal reaction between the board and the floor.

For man : $F + N_1 = Mg$ …(1)

For board : $mg + N_1 = N_2$ …(2)

$$F = \mu N_2 \qquad \ldots(3)$$

Hence, the correct answer is (D).

111. $mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$

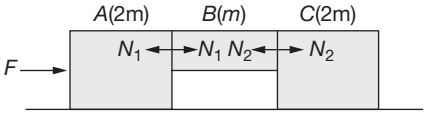
$$\Rightarrow \quad \tan\theta = 3\mu$$

$$\Rightarrow \quad \tan\theta = 3\tan\phi \qquad \{\because \mu = \tan\phi\}$$

Hence, the correct answer is (C).

112. Horizontal acceleration of the system is

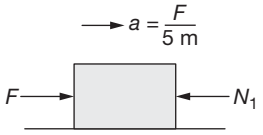
$$a = \frac{F}{2m + m + 2m} = \frac{F}{5m}$$



FBD A

$$F - N_1 = (2m)a$$

$$\Rightarrow \quad N_1 = F - \frac{2F}{5} = \frac{3F}{5}$$

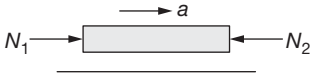


FBD B

$$N_1 - N_2 = ma$$

$$\Rightarrow \quad N_2 = \frac{3F}{5} - \frac{F}{5} = \frac{2F}{5}$$

Now B will not slide downwards if



$$\mu N_2 \ge m_B g$$

$$\Rightarrow \quad (\mu)\left(\frac{2}{5}F\right) \ge mg$$

$$\Rightarrow \quad F \ge \frac{5}{2\mu}mg$$

So, $F_{\min} = \frac{5mg}{2\mu}$

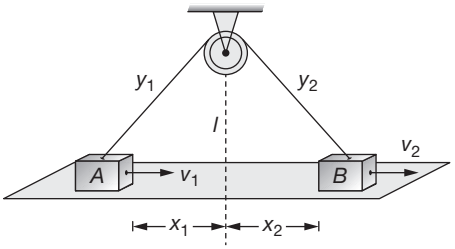
Hence, the correct answer is (B).

113. For A, we have

$$y_1^2 = l^2 + x_1^2$$

$$\Rightarrow \quad 2y_1\frac{dy_1}{dt} = 2x_1\frac{dx_1}{dt}$$

$$\Rightarrow \quad \frac{dy_1}{dt} = \frac{x_1}{y_1}\left(\frac{dx_1}{dt}\right)$$



Since $\frac{x_1}{y_1} = \cos\alpha$

$$\Rightarrow \quad \frac{dy_1}{dt} = v_1\cos\alpha \qquad \ldots(1)$$

Similarly for B, we have

$$y_2^2 = l^2 + x_2^2$$

$$\Rightarrow \quad 2y_2\frac{dy_2}{dt} = 2x_2\frac{dx_2}{dt}$$

$$\Rightarrow \quad \frac{dy_2}{dt} = \frac{x_2}{y_2}\left(\frac{dx_2}{dt}\right)$$

Since $\frac{x_2}{y_2} = \cos\beta$

$$\Rightarrow \quad \frac{dy_2}{dt} = v_2\cos\beta \qquad \ldots(2)$$

Also from our knowledge of constrained motion, we observe that

$$y_1 + y_2 = \text{constant}$$

$$\Rightarrow \quad \frac{dy_1}{dt} + \frac{dy_2}{dt} = 0$$

$$\Rightarrow \quad \left|\frac{dy_1}{dt}\right| = \left|\frac{dy_2}{dt}\right|$$

$$\Rightarrow \quad v_1\cos\alpha = v_2\cos\beta$$

$$\Rightarrow \quad \frac{v_2}{v_1} = \frac{\cos\alpha}{\cos\beta}$$

Hence, the correct answer is (D).

SHORTCUT

We could have directly equated the components of the velocities of blocks A and B along the thread to get $v_1\cos\alpha = v_2\cos\beta$

114. Since one end of pulley P_1 is fixed and the other end moves with an acceleration of $6\ \text{ms}^{-2}$. So, the acceleration of pulley is

$$a_{P_1} = \frac{6+0}{2} = 3\ \text{ms}^{-2}, \text{ towards left}$$

Also, if P_1 moves, say left by x, then B moves up by $\frac{x}{3}$. Hence, acceleration of B must be one third acceleration of P_1. Hence

$$a_B = \frac{(a)_{P_1}}{3} = 1\ \text{ms}^{-2}$$

Hence, the correct answer is (D).

115. $$\frac{1}{K_s} = \frac{1}{K} + \frac{1}{3K} + \frac{1}{9K} + \frac{1}{27K} + \ldots$$

$$\Rightarrow \quad \frac{1}{K_s} = \frac{1}{K}\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\right)$$

$$\Rightarrow \quad \frac{1}{K_s} = \frac{1}{K}\left(\frac{1}{1-\frac{1}{3}}\right)$$

$$\Rightarrow \quad K_s = \frac{2K}{3}$$

Hence, the correct answer is (C).

116. $$a = \left(\frac{10 - 4\sin 30 - (0.2)(4)\cos 30}{10+4}\right)g$$

$$\Rightarrow \quad a = \frac{(10 - 2 - 0.7)}{14}g$$

$$\Rightarrow \quad a = \left(\frac{7.3}{14}\right)9.8$$

$$\Rightarrow \quad a = 5.11\ \text{ms}^{-2}$$

Since $v^2 - u^2 = 2as$

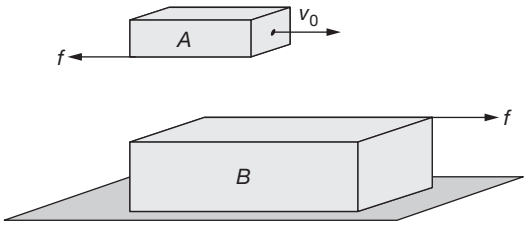
$$\Rightarrow \quad v^2 - 0^2 = 2(5.11)(4)$$

$$\Rightarrow \quad v \simeq 6.5\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

117. Friction force between A and $B(=\mu mg)$ will accelerate B and retard A till slipping stops between A and B. Since mass of both are equal, So we have acceleration of $B =$ retardation of $A = \mu g$.

$$\Rightarrow \quad v_A = v_0 - (\mu g)t \text{ and } v_B = (\mu g)t$$



After slipping ceases, the common velocity of both becomes $\frac{v_0}{2}$.

This can also be obtained from conservation of linear momentum also (to be studied later).

Hence, the correct answer is (B).

119. $\sin\theta = \frac{h}{l}$

$Mg - T = Ma$ and

$\Rightarrow \quad T - mg\sin\theta = ma$

$\Rightarrow \quad a = \left(\frac{M - m\sin\theta}{M+m}\right)g$

Since $v^2 - 0^2 = 2as$, where s is the distance travelled by m before M is detached. After detaching M, m will move the remaining distance of $(l-s)$ up the incline, under a retardation $g\sin\theta$.

$\Rightarrow \quad 0^2 = v^2 - 2(g\sin\theta)(l-s)$

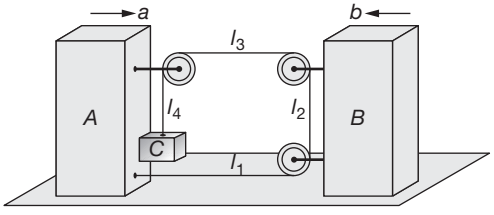
$\Rightarrow \quad 0^2 = 2as - 2g\sin\theta(l-s)$

$\Rightarrow \quad s = \frac{M+m}{M}\left(\frac{hl}{h+l}\right)$

Hence, the correct answer is (D).

120. Let the initial lengths of the respective segments of strings be l_1, l_2, l_3 and l_4. Then

$$\begin{pmatrix}\text{Total length of the}\\ \text{string at } t=0\end{pmatrix} = \begin{pmatrix}\text{Total length of the}\\ \text{string at time } t\end{pmatrix}$$



Let A move forward by x, B move backwards (towards left) by y and correspondingly C moves down, say by z. Then

$$\tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} = \left[\tfrac{1}{1} - (x+y)\right] + \tfrac{1}{2} + \left[\tfrac{1}{3} - (x+y)\right] + \left(\tfrac{1}{4} + z\right)$$

$$z = 2(x+y)$$

$\Rightarrow \quad \ddot{z} = 2(\ddot{x} + \ddot{y})$

$\Rightarrow \quad a_C = 2(a+b)$, downwards

Also, as A moves forward, C also moves forward, hence

$$\vec{a}_C = a\hat{i} - 2(a+b)\hat{j}$$

$\Rightarrow \quad |\vec{a}_C| = \sqrt{a^2 + 4(a+b)^2}$

$\Rightarrow \quad |\vec{a}_C| = \sqrt{5a^2 + 4b^2 + 8ab}$

Hence, the correct answer is (D).

121. $a = \mu g$

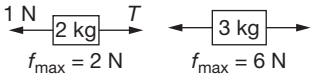
$s = 0 + \frac{1}{2}(\mu g)t_{MIN}^2$

$\Rightarrow \quad t_{MIN} = \sqrt{\frac{2s}{\mu g}}$

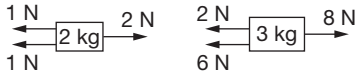
$\Rightarrow \quad t_{MIN} \propto \frac{1}{\sqrt{\mu}}$

Hence, the correct answer is (B).

122. FBD of blocks is shown



Net force without friction on system is 7 N towards right side, so maximum friction will first come on 3 kg block



$\Rightarrow \quad f_1 = 1\text{ N}$, $f_2 = 6\text{ N}$, $T = 2\text{ N}$

Hence, the correct answer is (C).

123. As no force is acting on A,

$$T = 0 \text{ and } a_B = \frac{F}{m_B}$$

Hence, the correct answer is (A).

124. As block moves to the right, N decreases.

Hence, the correct answer is (C).

126. Only magnitude remains constant and direction changes

Hence, the correct answer is (A).

127. $x = 2t$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = 2$$

$$y = 2t^2$$

$$\Rightarrow \quad v_y = \frac{dy}{dx} = 4t$$

$$\Rightarrow \quad \tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t$$

Differentiating with respect to time we get,

$$(\sec^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+\tan^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+4t^2)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{2}{1+4t^2}$$

So, $\frac{d\theta}{dt}$ at $t = 2$ s is

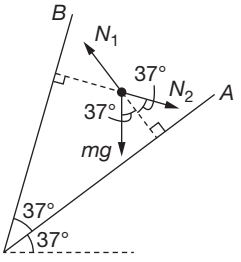
$$\frac{d\theta}{dt} = \frac{2}{1+4(2)^2} = \frac{2}{17} \text{ rads}^{-1}$$

Hence, the correct answer is (A).

128. $2kx\cos(30°) = \left(\frac{4m_1m_2}{m_1+m_2}\right)g$

Hence, the correct answer is (A).

129. Using Lami's Theorem, we get



$$\frac{mg}{\sin(180°-37°)} = \frac{N_2}{\sin(180°-37°)}$$

$$\Rightarrow \quad N_2 = mg$$

Hence, the correct answer is (A).

130. Limiting friction on block B is

$$f_l = (0.5)(2+8)(10) = 50 \text{ N}$$

So, block B will not move.

Hence, the correct answer is (A).

131. Retardation $a = g(\sin\theta + \mu\cos\theta) = 5(\sqrt{3}+\mu)$

Since $v = u - at$

$$\Rightarrow \quad 0 = u - at$$

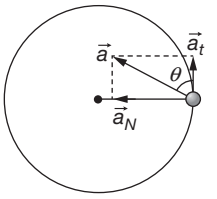
$$\Rightarrow \quad a = \frac{u}{t}$$

$$\Rightarrow \quad 5(\sqrt{3}+\mu) = 10$$

$$\Rightarrow \quad \mu = 0.27$$

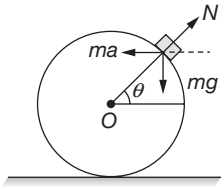
Hence, the correct answer is (C).

132. $\tan\theta = \frac{a_N}{a_t}$



Hence, the correct answer is (C).

134.



Drawing F.B.D of m, we get

$$N\sin\theta = mg \text{ and } N\cos\theta = ma$$

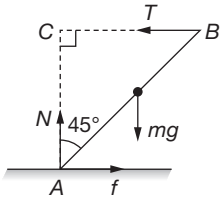
$$\Rightarrow \quad \tan\theta = \frac{g}{a}$$

$$\Rightarrow \quad a = g\cot\theta$$

So, $F = (m+M)g\cot\theta$

Hence, the correct answer is (C).

136.



$$T = f \quad \ldots(1)$$

$$N = mg \quad \ldots(2)$$

Taking torque about A, we get

$$T \times \frac{L}{\sqrt{2}} = mg \times \frac{L/2}{\sqrt{2}}$$

$$\Rightarrow \quad T = \frac{mg}{2} = 50 \text{ N}$$

Hence, the correct answer is (A).

137. $F_{\text{net}} = F - f_k = 2t - \mu mg$

Hence, the correct answer is (C).

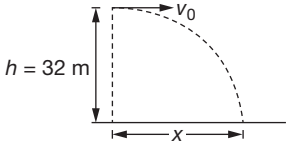
138. $R = u\sqrt{\frac{2h}{g}} = 12\sqrt{\frac{2\times 5}{10}} = 12 \text{ m}$

So, distance from origin is

$$r = \sqrt{5^2 + 12^2} = 13 \text{ m}$$

Hence, the correct answer is (C).

139.



$$x = v_0 t \text{ and } h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 32}{10}} = \frac{8}{\sqrt{10}}$$

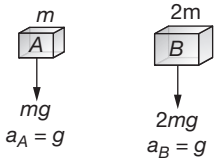
Since, $v_0 = \omega r = \frac{2\pi}{T} \times 12 = \frac{2\pi}{1.2} \times 12 = 20\pi$

$$\Rightarrow \quad x = 20\pi\left(\frac{8}{\sqrt{10}}\right) = \frac{160 \times 3.14}{3.162}$$

$$\Rightarrow \quad x = 160 \text{ m}$$

Hence, the correct answer is (D).

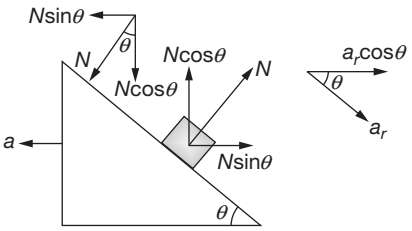
140. In this case spring force is zero initially. F.B.D of A and B are shown below.



Hence, the correct answer is (A).

141. Let acceleration of mass m relative to wedge down the plane is a_r. Its absolute acceleration in horizontal direction is

$$(a_r \cos 60° - a) \qquad \text{(towards right)}$$



Let N be the normal reaction between the mass and the wedge. Then, we have

for the block, $N\sin\theta = m(a_r \cos\theta - a)$ …(1)

for the wedge, $N\sin\theta = Ma$ …(2)

So, from equations (1) and (2), we get

$$N\sin\theta = Ma = m(a_r \cos\theta - a)$$

$$\Rightarrow \quad a_r = \frac{(M+m)a}{m\cos\theta} = \frac{(M+m)a}{m\cos(60°)} = \frac{2(M+m)a}{m}$$

Hence, the correct answer is (C).

142. For both Case (i) and Case (ii), the blocks move together if friction between the blocks is static or limiting. Since we need to calculate the maximum force for which the blocks move together, so friction between the blocks will be limiting in nature.
So, for Case (i), we have

$$F - f_l = ma \quad \ldots(1)$$

$$f_l = Ma \quad \ldots(2)$$

Also, $f_l = \mu mg$ …(3)

From (1), (2) and (3), we get

$$F_{\max} = \mu mg\left(1 + \frac{m}{M}\right)$$

Similarly for Case (ii), we have

$$(F_{\text{pseudo}}) \le f_l$$

$$\Rightarrow \quad ma \le \mu mg$$

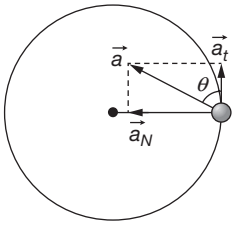
$$\Rightarrow \quad m\frac{F}{M+m} \le \mu mg$$

$$\Rightarrow \quad F \le \mu(M+m)g$$

$$\Rightarrow \quad F_{\max} = \mu(M+m)g$$

Hence, the correct answer is (C).

144.



$$\tan\theta = \frac{a_c}{a_t}$$

$$\Rightarrow \quad \frac{a_c}{a_t} = \tan(30°) = \frac{1}{\sqrt{3}}$$

Hence, the correct answer is (C).

145. Since, $\vec{a} = 6\hat{i} - 8\hat{j}$

$$\Rightarrow \quad a_r = 8 \text{ and } a_t = 6$$

$$\Rightarrow \quad r\omega^2 = 8 \text{ and } r\alpha = 6$$

$$\Rightarrow \quad \omega = 2 \text{ rads}^{-1} \text{ and } \alpha = 3 \text{ rads}^{-2}$$

Since the body is rotating in clockwise sense, so by using Right Hand Thumb Rule, we get

$$\vec{\omega} = -2\hat{k} \text{ rads}^{-1} \text{ and}$$

$$\vec{\alpha} = -3\hat{k} \text{ rads}^{-2}$$

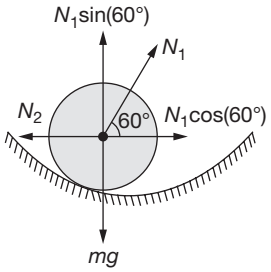
Hence, the correct answer is (A).

146. Since supporting plane is lowered slowly, so

$$N = mg - kx$$

Hence, the correct answer is (C).

147.



$$N_1\cos 60° = N_2$$

$$\Rightarrow \quad \frac{N_1}{N_2} = 2$$

Hence, the correct answer is (B).

149. Force required to stop the body from just sliding down the plane is

$$F_2 = (mg\sin\theta - \mu mg\cos\theta)$$

Force required to just pull the body up the plane is

$$F_1 = (mg\sin\theta + \mu mg\cos\theta)$$

Since $F_1 = 2F_2$

$$\Rightarrow \quad mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow \quad \sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta$$

$$\Rightarrow \quad \sin\theta = 3\mu\cos\theta$$

$$\Rightarrow \quad \tan\theta = 3\mu$$

Hence, the correct answer is (D).

150. Since $r \ll R$, so mass/length can be given by

$$\lambda = \frac{m}{2r} = \left(\frac{\frac{4}{3}\pi r^3\rho}{2r}\right)\left(\frac{2}{3}\pi r^2\rho\right)$$

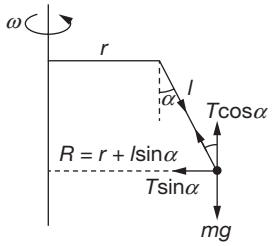
Now, tension in tube is given by

$$T = (\lambda R)(R\omega^2) = \left(\frac{2}{3}\pi r^2\rho\right)(\omega^2)(R^2)$$

$$\Rightarrow \quad T = \frac{2}{3}\pi\rho\omega^2 r^2 R^2$$

Hence, the correct answer is (D).

151.



$$T\cos\alpha = mg$$

$$T\sin\alpha = m(r + l\sin\alpha)\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{g\tan\alpha}{r + l\sin\alpha}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g\tan\alpha}{r + l\sin\alpha}}$$

Hence, the correct answer is (A).

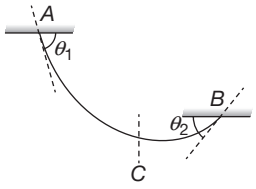
152. Since $T = mg$, so for pulley P, we have

$$T_{\text{net}} = \sqrt{T^2 + T^2 + 2(T)(T)\cos(120°)}$$

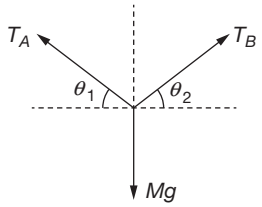
$$\Rightarrow \quad T = mg$$

Hence, the correct answer is (A).

153.



Free body diagram of rope AB is

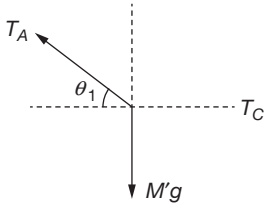


$$T_A\cos\theta_1 = T_B\cos\theta_2$$

$$T_A\sin\theta_1 + T_B\sin\theta_2 = Mg$$

$$\Rightarrow \quad T_A = \frac{Mg\cos\theta_2}{\sin(\theta_1+\theta_2)} \text{ and } T_B = \frac{Mg\cos\theta_1}{\sin(\theta_1+\theta_2)}$$

Free body diagram of portion AC of the rope is



For horizontal equilibrium, we have

$$T_A\cos\theta_1 = T_C$$

$$\Rightarrow \quad T_C = \frac{Mg\cos\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)}$$

So, tension will be maximum at A and minimum at C.

Hence, the correct answer is (C).

154. Net force on m_3 is $F_{\text{net}} = \sqrt{(30)^2+(40)^2} = 50$ N

and limiting friction on m_3 is $f_\ell = \mu m_3 g = 60$ N

Since the applied force is less than the limiting force of friction, so the system remains in equilibrium and friction between m_3 and the surface is equal to the net applied force i.e. 50 N.

Hence, the correct answer is (C).

155. Friction force between A and $B(=\mu mg)$ will accelerate B and retard A till slipping is stopped between the two and since mass of both are equal, so

Acceleration of B is equal to Retardation of A. i.e.

$$|a_B| = |a_A| = \mu g$$

$$\Rightarrow \quad v_1 = v_0 - \mu g t \text{ and } v_2 = \mu g t$$

Hence, the correct graph is (B).

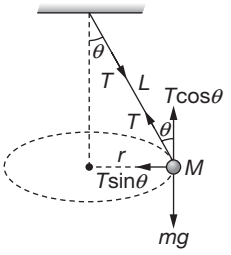
After slipping has ceased, the common velocity of both will become $\frac{v_0}{2}$, which can be also be obtained from Conservation of Linear Momentum also.

Hence, the correct answer is (B).

156. We cannot choose (A) or (B) because the centripetal acceleration vector is not constant it continuously changes in direction. Of the remaining choices, only (C) gives the correct perpendicular relationship between a_c and v.

Hence, the correct answer is (C).

157.



$$T\cos\theta = Mg \qquad \ldots(1)$$

$$T\sin\theta = Mr\omega^2 = M(L\sin\theta)\omega^2$$

$$\Rightarrow \quad T = ML\omega^2 \qquad \ldots(2)$$

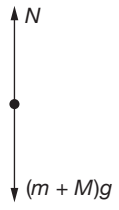
Since, $\omega = \frac{2}{\pi}$ rev per sec

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1}$$

$$\Rightarrow \quad T = M\omega^2 L = M(4)^2 L = 16ML$$

Hence, the correct answer is (D).

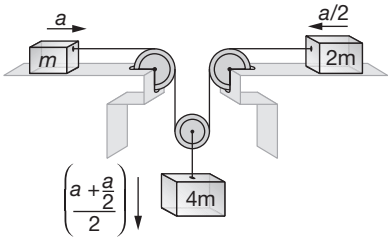
158. Free body diagram of "wedge + mass" system is



$$\Rightarrow \quad N = (m+M)g$$

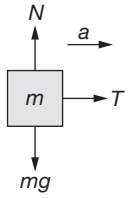
Hence, the correct answer is (B).

159.



Let a be the acceleration of block of mass m, then acceleration of $2m$ will be $\frac{a}{2}$ and that of $4m$ will be $\left(\frac{a+\frac{a}{2}}{2}\right)=\frac{3a}{4}$.

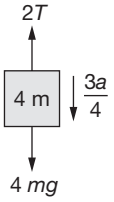
For m, we have



$N = mg$ and

$T = ma$ …(1)

For $4m$, we have



$4mg - 2T = 4m\left(\frac{3a}{4}\right)$ …(2)

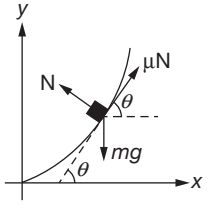
$\Rightarrow \quad a = \frac{4g}{5}$

Hence, the correct answer is (B).

160. Block A moves in horizontal direction only due to friction.

Hence, the correct answer is (B).

161. Since $\frac{dy}{dx}=\frac{x}{10}$



$\Rightarrow \quad \tan\theta = \frac{x}{10}$ …(1)

Equilibrium of mass in horizontal direction gives the equation

$\mu N\cos\theta = N\sin\theta$

$\Rightarrow \quad \tan\theta = \mu = \frac{1}{2}$ …(2)

From (1) and (2), we get

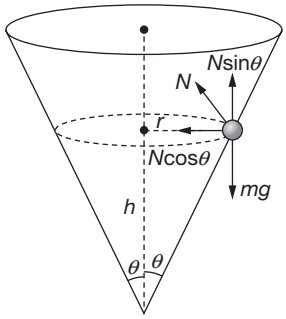
$\frac{x}{10}=\frac{1}{2}$

$\Rightarrow \quad x = 5$ m

$\Rightarrow \quad y = \frac{x^2}{20}=\frac{25}{20}=1.25$ m

Hence, the correct answer is (B).

162.



$N\sin\theta = mg$

$N\cos\theta = \frac{mv^2}{r}$

$\Rightarrow \quad \tan\theta = \frac{gr}{v^2}$

Since $\tan\theta = \frac{r}{h}$

$\Rightarrow \quad \frac{r}{h}=\frac{rg}{v^2}$

$\Rightarrow \quad h = \frac{v^2}{g} = 2.5$ cm

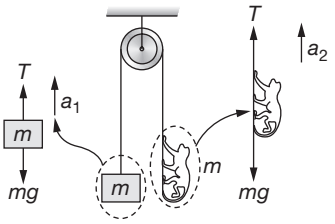
Hence, the correct answer is (D).

163. $\frac{mv^2}{r} = \mu mg$

$\Rightarrow \quad v = \sqrt{\mu rg}$

Hence, the correct answer is (D).

164.



For Bananas, we have

$$T - mg = ma_1$$

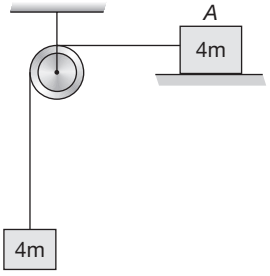
For Monkey, we have

$$T - mg = ma_2$$

Since $a_1 = a_2$ so both bananas and monkey have same acceleration and hence they move up at speed v.

Hence, the correct answer is (D).

165. The system can be redrawn as

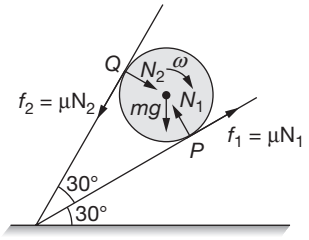


$$\Rightarrow \quad a = \frac{4mg}{8m} = \frac{g}{2} = 5 \text{ ms}^{-2}$$

$$\Rightarrow \quad v = u + at = 2 \times 5 = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

166. Let μ be the friction coefficient between sphere and each wall. Free body diagram of sphere is



Net force on the sphere in horizontal direction is zero. So,

$$N_1 \cos 60° + \mu N_2 \cos 60° = N_2 \cos 30° + \mu N_1 \cos 30°$$

$$\Rightarrow \quad N_1 + \mu N_2 = \sqrt{3}\left(N_2 + \mu N_1\right)$$

$$\Rightarrow \quad N_1\left(1 - \sqrt{3}\mu\right) = N_2\left(\sqrt{3} - \mu\right)$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{\sqrt{3} - \mu}{1 - \sqrt{3}\mu}$$

Substituting $\mu = \frac{1}{3}$ we get,

$$\frac{N_1}{N_2} = \frac{\sqrt{3} - \frac{1}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3\sqrt{3} - 1}{3 - \sqrt{3}} = 1 + \frac{4}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{\mu N_1}{\mu N_2} = 1 + \frac{4}{\sqrt{3}}$$

Hence, the correct answer is (A).

167. Blocks A and C both move due to friction. But less friction is available to A as compared to C because normal reaction between A and B is less. Maximum friction between A and B can be

$$f_{\max} = \mu m_A g = \left(\frac{1}{2}\right) mg$$

So, maximum acceleration of A can be

$$a_{\max} = \frac{f_{\max}}{m} = \frac{g}{2}$$

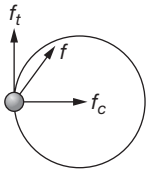
Further $a_{\max} = \dfrac{m_D g}{3m + m_D}$

$$\Rightarrow \quad \frac{g}{2} = \frac{m_D g}{3m + m_D}$$

$$\Rightarrow \quad m_D = 3m$$

Hence, the correct answer is (C).

168.



Let f be the friction force, then

$$f = \sqrt{F_C^2 + F_T^2}$$

After 2 seconds, $\omega = \alpha t = \dfrac{2}{3}$ rads^{-1}

$$\Rightarrow \quad F_T = ma_T = m(\alpha R) = \left(36 \times 10^{-5}\right)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) \text{N}$$

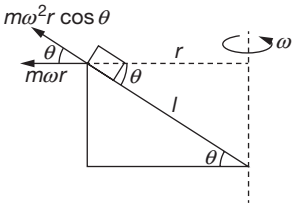
$\Rightarrow \quad F_T = 3\times10^{-5}$ N

$F_C = m\omega^2 R = (36\times10^{-5})\left(\frac{4}{9}\right)\left(\frac{1}{4}\right) = 4\times10^{-5}$ N

$\Rightarrow \quad f = \sqrt{F_C^2 + F_T^2} = 5\times10^{-5}$ N $= 50\ \mu$N

Hence, the correct answer is (C).

169.



$m\omega^2 r\cos\theta = mg\sin\theta$

$\Rightarrow \quad m\omega^2 l\cos^2\theta = mg\sin\theta \qquad \{\because\ r = \ell\cos\theta\}$

$\Rightarrow \quad \omega = \sqrt{\frac{g\sin\theta}{l\cos^2\theta}} = \sqrt{\frac{g\sin\theta}{l}}\sec\theta$

Hence, the correct answer is (A).

170. $a = \frac{a_A + a_B}{2} = \frac{3+4}{2} = \frac{7}{2} = 3.5\ \text{ms}^{-2}$

Hence, the correct answer is (C).

171. Reading of spring balance is

$\text{Reading} = \frac{\text{Thrust}}{g} = \frac{4m_1m_2}{m_1+m_2} = \frac{10}{3}$ kg

$\Rightarrow \quad \text{Reading} = 3.33\ \text{kg}\ \ (< 6\ \text{kg})$

Hence, the correct answer is (B).

172. Net horizontal force on wedge

$F_H = mg\cos\theta\sin\theta$

Net normal reaction from the ground is

$N = 2mg + mg\cos^2\theta$

Since, $\mu N = F_H$

$\Rightarrow \quad \mu = 0.20$

Hence, the correct answer is (B).

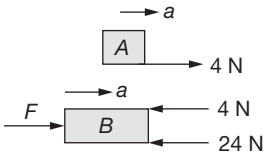
173. Maximum frictional force between A and B is

$f_1 = \mu_1 m_A g = (0.2)(2)(10)$ N

$\Rightarrow \quad f_1 = 4$ N

Limiting friction between B and ground is

$f_2 = (0.4)(6)(10) = 24$ N



For A, $4 = 2a_A = 2a$

$\Rightarrow \quad a = 2\ \text{ms}^{-2}$

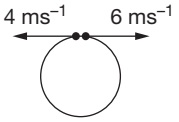
For B, $F - 28 = 4a$

$\Rightarrow \quad F - 28 = 8$

$\Rightarrow \quad F = 36$ N

Hence, the correct answer is (B).

174.



Since $S_1 + S_2 = 2\pi r$

$\Rightarrow \quad 4t + 6t = 2\pi r$

$\Rightarrow \quad t = \frac{2\pi r}{10} = \frac{2\times3.14\times4}{10} = 2.5$ s

Hence, the correct answer is (B).

175. Since $a = g\tan\theta$

$\Rightarrow \quad Mg - T = Ma \qquad \ldots(1)$

$T = (m + M')a \qquad \ldots(2)$

Put (2) in (1), we get

$Mg - (m+M')a = Ma$

$\Rightarrow \quad Mg = (m + M + M')g\tan\theta$

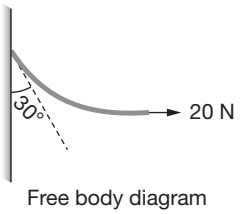
$\Rightarrow \quad Mg(1 - \tan\theta) = (m+M')(g\tan\theta)$

$\Rightarrow \quad M = \frac{(m+M')\tan\theta}{1-\tan\theta}$

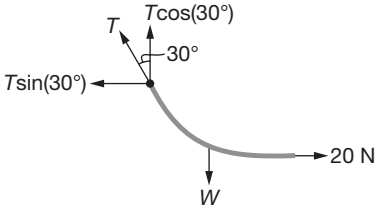
$\Rightarrow \quad M = \frac{m+M'}{\cot\theta - 1}$

Hence, the correct answer is (D).

177.



Free body diagram



For equilibrium, we have

$$T\sin(30°) = 20$$

$$T\cos(30°) = W$$

$$\Rightarrow \quad \tan(30°) = \frac{20}{W}$$

$$\Rightarrow \quad W = 20\cot(30°)$$

$$\Rightarrow \quad W = 20\sqrt{3} = 20 \times 1.732$$

So, weight, $W = 34.64$ N

$$\Rightarrow \quad \text{Mass, } m = \frac{W}{g} = 3.5 \text{ kg}$$

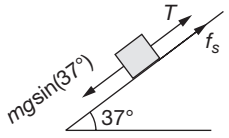
Hence, the correct answer is (C).

179. Since $\mu = 0.8 = \frac{4}{5}$

$$\Rightarrow \quad \text{Angle of repose } \alpha = 39°$$

Since angle which incline makes with the horizontal is $37°(< 39°)$, so the block will not slide down the incline and friction between the block and the incline will be static in nature. So,

$$f_s = F_{\text{applied}} = mg\sin(37°)$$



Now, if T be the tension in the string, then

$$T + f_s = mg\sin(37°)$$

$$\Rightarrow \quad T = 0 \qquad \{\because\ f_s = mg\sin(37°)\}$$

Hence, the correct answer is (D).

180. Mass of hanging part of chain is

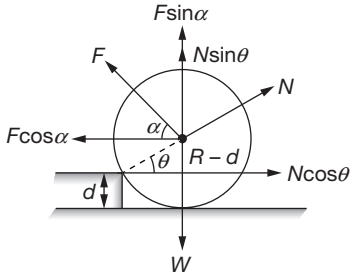
$$m' = \frac{m}{\left(\frac{3\pi R}{2}\right)}\left(\frac{3\pi R}{2} - \pi R\right) = \frac{m}{3}$$

So, acceleration of chain is given by

$$a = \frac{2mg - \frac{mg}{3}}{m} = \frac{5g}{3}$$

Hence, the correct answer is (D).

182.



Since, $F\cos\alpha - N\cos\theta = 0$

$$\Rightarrow \quad F\cos\alpha = N\cos\theta \qquad \ldots(1)$$

Also, $N\sin\theta + F\sin\alpha - W = 0$

$$\Rightarrow \quad N\sin\theta = W - F\sin\alpha \qquad \ldots(2)$$

From (1), $N = \frac{F\cos\alpha}{\cos\theta}$, put in (2), we get

$$F\cos\alpha\tan\theta = W - F\sin\alpha$$

$$\Rightarrow \quad F\left(\cos\alpha\frac{\sin\theta}{\cos\theta} + \sin\alpha\right) = W$$

$$\Rightarrow \quad F\sin(\alpha+\theta) = W\cos\theta$$

$$\Rightarrow \quad F = \frac{W\cos\theta}{\sin(\theta+\alpha)}$$

Now $F = F_{\min}$ when $\theta + \alpha = 90°$

$$\Rightarrow \quad F_{\min} = W\cos\theta$$

Hence, the correct answer is (A).

183. **METHOD I**

For A, $f = ma$

$$\Rightarrow \quad \mu mg = ma$$

$$\Rightarrow \quad a = \mu g \qquad \ldots(1)$$

For B, $T - \mu mg = 2ma$ $\ldots(2)$

For C, $mg - T = ma$ $\ldots(3)$

From (1), (2) and (3), we get

$$mg - \mu mg = 3m(\mu mg)$$

$$\Rightarrow \quad 4\mu mg = mg$$

$$\Rightarrow \quad \mu = \frac{1}{4}$$

METHOD II

Acceleration of the system is

$$a_{\text{system}} = \frac{mg}{4m} = \frac{g}{4}$$

For Block A not to slip on B, we have

$$F_{\text{pseudo}} = f$$

$$\Rightarrow \quad m\left(\frac{g}{4}\right) = \mu mg$$

$$\Rightarrow \quad \mu = \frac{1}{4}$$

Hence, the correct answer is (C).

184. If common maximum acceleration is 'a' then,

$$(m_A + m_B)a = f$$

$$\Rightarrow \quad (2+4)a = 2t$$

$$\Rightarrow \quad 3a = t \qquad \ldots(1)$$

For block B, we have

$$f_L = m_B \times a$$

$$\Rightarrow \quad 0.5 \times 2 \times 10 = 4a$$

$$\Rightarrow \quad a = 2.5 \text{ ms}^{-2}$$

From equation (1), we get

$$t = 7.5 \text{ sec}$$

Hence, the correct answer is (B).

185. Since M is at rest, so the tension in the string is

$$T = \frac{Mg}{2}$$

Let acceleration of m and m' be a, so one will move downward and other will move upward.
For m, we have

$$mg - \frac{Mg}{2} = ma \qquad \ldots(1)$$

For m', we have

$$\frac{Mg}{2} - m'g = m'a \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\frac{4}{M} = \frac{1}{m} + \frac{1}{m'}$$

Hence, the correct answer is (A).

186. $T = 4g + 5(10+2)$

$$\Rightarrow \quad T = 40 + 60 = 100 \text{ N}$$

Hence, the correct answer is (B).

188. For the sliding not to occur, we have $\tan\theta \le \mu$.

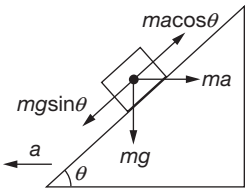
Since $\tan\theta = \frac{dy}{dx} = \frac{2x}{a} = \frac{2\sqrt{ya}}{a} = 2\sqrt{\frac{y}{a}}$

$$\Rightarrow \quad 2\sqrt{\frac{y}{a}} \le \mu$$

$$\Rightarrow \quad y \le \frac{a\mu^2}{4}$$

Hence, the correct answer is (C).

189.



Acceleration of block relative to wedge, is

$$a' = a\cos\theta - g\sin\theta \text{ (up the plane)}$$

$$\Rightarrow \quad a' = 10(\sqrt{3})\left(\frac{\sqrt{3}}{2}\right) - 10\left(\frac{1}{2}\right) = 10 \text{ ms}^{-2}$$

Since $l = \frac{1}{2}a't^2$

$$\Rightarrow \quad t = \sqrt{\frac{2l}{a'}} = \sqrt{\frac{2.5}{10}} = \frac{1}{2} \text{ sec}$$

Hence, the correct answer is (D).

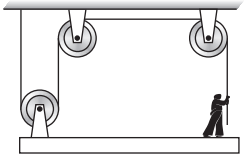
190. Since, acceleration of body w.r.t. lift is $a_{\text{net}} = g + a$.

So, time of flight of body, $t = \frac{2u}{g+a}$

$$\Rightarrow \quad a = \frac{2u - gt}{t} = \frac{(2\times5) - (10\times0.8)}{0.8} = 2.5 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

191. For the system to move at constant velocity, force should be zero. So, we have

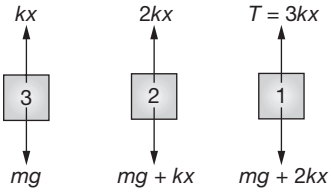


$3T = 750$

$\Rightarrow \quad T = 250$ N

Hence, the correct answer is (B).

192. In equilibrium,



When thread is cut, $T = 0$ so, a_1 is maximum but $a_2 = a_3 = 0$.

Hence, the correct answer is (D).

193. $N = mg\sin\theta + \dfrac{mv^2}{r}$ and

$v^2 = 2gr\sin\theta$

Hence, the correct answer is (A).

194. $N = m\ell\omega^2$

Since $f = \mu N$

$\Rightarrow \quad \mu ml\omega^2 = mg$

$\Rightarrow \quad \omega = \sqrt{\dfrac{g}{\mu l}}$

Hence, the correct answer is (A).

195. Let the thickness of each plank be x.
Then, we have

$0 = u^2 - 2a(2x)$

$\Rightarrow \quad u^2 = 4ax \qquad \ldots(1)$

When velocity is doubled, let n planks be required to stop the bullet, then

$0 = (2u)^2 - 2a(nx)$

$\Rightarrow \quad n = \dfrac{4u^2}{2ax} = \dfrac{4(4ax)}{2ax} = 8$

Hence, the correct answer is (C).

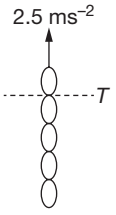
196. Acceleration of system $a = \dfrac{Mg}{M + nm}$

Tension in the last string is

$T = ma = \dfrac{mMg}{M + nm}$

Hence, the correct answer is (A).

197.



$T = mg + ma$

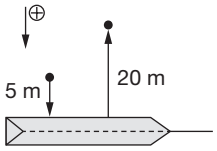
$\Rightarrow \quad T = 0.4g + 0.4a$

$\Rightarrow \quad T = 0.4(g + a)$

$\Rightarrow \quad T = 0.4(9.8 + 2.5) = 4.92$ N

Hence, the correct answer is (B).

198.



Velocity of ball just before collision is

$v_1 = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 5} = 10$ ms^{-1}

Velocity of ball just after collision is

$v_2 = \sqrt{2gh_2} = \sqrt{2 \times 10 \times 20} = 20$ ms^{-1}

$\Rightarrow \quad \Delta p = p_2 - p_1 = -m(v_1 + v_2)$

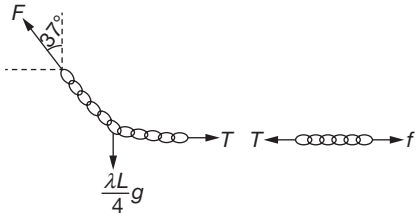
$\Rightarrow \quad F_{av} = \dfrac{\Delta p}{\Delta t} = \dfrac{m(v_1 + v_2)}{\Delta t}$

$\Rightarrow \quad 100 = \dfrac{(0.4)(10 + 20)}{\Delta t}$

$\Rightarrow \quad \Delta t = 0.12$ sec

Hence, the correct answer is (A).

199. Let λ be the mass per unit length of the chain, then



$F\cos(37°) = \dfrac{\lambda L}{4}g$

Also, $F\sin(37°) = T = f$

$$\Rightarrow \quad f = \frac{3\lambda Lg}{16} \le \mu N$$

$$\Rightarrow \quad \mu \ge \frac{1}{4}$$

Hence, the correct answer is (B).

200. Acceleration $a = \frac{dv}{dt}$

$$\Rightarrow \quad a = 2bt$$

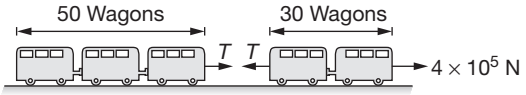
For block to just slide on the plate, we have

$$\mu mg = m(2bt)$$

$$\Rightarrow \quad t = \frac{\mu g}{2b}$$

Hence, the correct answer is (D).

201.



Acceleration of train is

$$a = \frac{4\times10^5}{80\times5\times10^3} = 1 \text{ ms}^{-2}$$

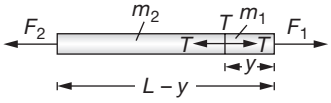
Thus, tension between 30th and 31st wagon is

$$T = ma = 50\times5\times10^3\times1$$

$$\Rightarrow \quad T = 25\times10^4 \text{ N}$$

Hence, the correct answer is (A).

202.



$$m_1 = \frac{M}{L}y \text{ and } m_2 = \frac{M}{L}(L-y)$$

Since, $a = \frac{F_1 - F_2}{M}$

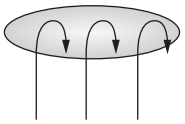
$$F_1 - T = m_1 a$$

$$\Rightarrow \quad T = F_1 - m_1 a = F_1 - \left(\frac{M}{L}y\right)\left(\frac{F_1 - F_2}{M}\right)$$

$$\Rightarrow \quad T = F_1\left(1 - \frac{y}{L}\right) + F_2\left(\frac{y}{L}\right)$$

Hence, the correct answer is (A).

203. $F_{av} = \frac{\Delta p}{\Delta t} = 2mnv$



$$\Rightarrow \quad 2mnv = Mg$$

$$\Rightarrow \quad v = \frac{Mg}{2mn} = \frac{(10\times10^{-3})\times9.8}{2(5\times10^{-3})\times10}$$

$$\Rightarrow \quad v = 0.98 \text{ ms}^{-1} = 98 \text{ cms}^{-1}$$

Hence, the correct answer is (B).

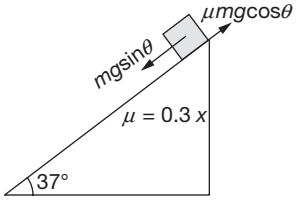
204. Since area under the curve $= \Delta p$

$$\Rightarrow \quad \frac{1}{2}(2)(10) + (2)(10) + \frac{1}{2}(30)(2) + \frac{1}{2}(4)(20) = 2(v_2 - 0)$$

$$\Rightarrow \quad v_2 = 50 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

205. After some time, friction becomes more than $mg\sin\theta$, then body will retard. Thus speed is maximum when, total force or acceleration is zero.



$$\Rightarrow \quad mg\sin\theta - \mu mg\cos\theta = 0$$

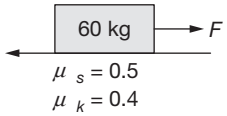
$$\Rightarrow \quad \mu = \tan\theta$$

$$\Rightarrow \quad 0.3x = \frac{3}{4}$$

$$\Rightarrow \quad x = 2.5 \text{ m}$$

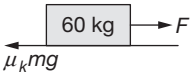
Hence, the correct answer is (D).

206. Force required to start the motion is



$$F = \mu_s mg$$

Once the body starts sliding, friction becomes kinetic, so



$$a = \frac{F - \mu_k mg}{m} = \frac{\mu_s mg - \mu_k mg}{m}$$

$$\Rightarrow \quad a = (\mu_s - \mu_k)g = 1 \text{ ms}^{-2}$$

Hence, the correct answer is (D).

207. Since external force on each block $= mg\sin\theta$ and acceleration of each block $= g\sin\theta$, so force on each block due to another block is zero.

Hence, the correct answer is (B).

208. Thrust force on the rocket,

$$F = v\frac{dm}{dt} = 600 \times 1 = 600 \text{ N}$$

So, acceleration of rocket

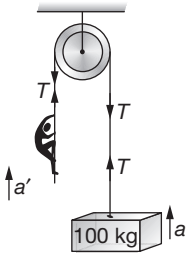
$$a = \frac{F}{M}$$

$$\Rightarrow \quad a = \frac{600}{120} = 5 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

209. If a be the acceleration of body w.r.t. ground, then acceleration of man w.r.t. ground is

$$a' = \frac{5}{4}g - a$$



For body,

$$T - 100g = 100a \qquad \ldots(1)$$

For man,

$$T - 60g = 60\left(\frac{5}{4}g - a\right)$$

$$\Rightarrow \quad T = 135g - 60a \qquad \ldots(2)$$

By solving equations (1) and (2), we get

$$T = 1218 \text{ N}$$

Hence, the correct answer is (C).

210. Change in momentum of bead is

$$\Delta p = 2mv$$

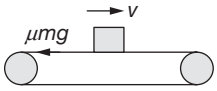
and time to cover the half cycle is

$$\Delta t = \frac{\text{Distance}}{\text{Speed}} = \frac{\pi r}{v} = \frac{\pi d}{2v}$$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{2mv}{\pi d/2v} = \frac{4mv^2}{\pi d}$$

Hence, the correct answer is (B).

211. Relative to belt, $a = \dfrac{\mu mg}{m} = \mu g = 5 \text{ ms}^{-2}$

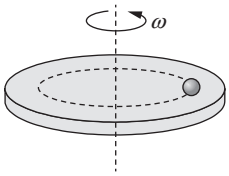


$$\Rightarrow \quad 0 = u^2 - 2as$$

$$\Rightarrow \quad s = \frac{u^2}{2a} = \frac{(3)^2}{2 \times 5} = 0.9 \text{ m}$$

Hence, the correct answer is (C).

212.

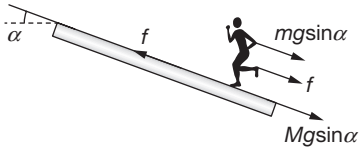


$$mR\omega^2 \le \mu mg$$

$$\Rightarrow \quad R \le \frac{\mu g}{\omega^2}$$

Hence, the correct answer is (D).

213. F.B.D. of man and plank are



For plank to be at rest, applying Newton's Second Law on the plank along the incline, we get

$$Mg\sin\alpha = f \qquad \ldots(1)$$

and applying Newton's Second Law on man along the incline, we get

$$Mg\sin\alpha + f = ma \qquad \ldots(2)$$

$$\Rightarrow \quad a = g\sin\alpha\left(1 + \frac{M}{m}\right), \text{ down the incline}$$

Hence, the correct answer is (B).

214. Limiting friction between A and $B = 60$ N

Limiting friction between B and $C = 90$ N

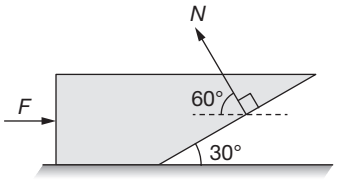
Limiting friction between C and ground $= 50$ N

Since limiting friction is least between C and ground, slipping will occur at first between C and ground. This will occur when $F = 50$ N.

Hence, the correct answer is (D).

215. Acceleration of two mass system is $a = \dfrac{F}{2m}$ leftwards

FBD of block A is drawn, so



$$N\cos(60°) - F = ma = \frac{mF}{2m}$$

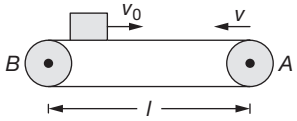
$$\Rightarrow \quad N = 3F$$

Hence, the correct answer is (D).

216. $f = ma_2$

Hence, the correct answer is (B).

217.



Maximum amount of heat is liberated when the friction between the block and belt is kinetic, i.e., when the block moves with a velocity relative to the belt.

Since, $a = \mu g$

Also, $v^2 = u^2 + 2as$

$$\Rightarrow \quad 0 = v_0^2 + 2(-\mu g)\ell$$

$$\Rightarrow \quad v_0 = \sqrt{2\mu g\ell}$$

Hence, the correct answer is (B).

218. Since, $a_A = 6a_B$

$$\Rightarrow \quad a_B = \frac{a_A}{6} = \frac{12}{6} = 2 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

219. Let acceleration of block A and B be a_1 and a_2 respectively and a_p be acceleration of pulley P and let tension in string be T. Then, according to principle of virtual work, we have

$$(3T)a_p = Ta_1 + (8T)a_2$$

$$\Rightarrow \quad a_p = \frac{9g}{3} = 3g \text{ (upwards)}$$

Hence, the correct answer is (D).

220. For first $\dfrac{H}{3}$, we have

$$a_1 = \frac{2mg}{3m} = \frac{2g}{3}$$

$$\Rightarrow \quad v_1^2 = 2\left(\frac{2g}{3}\right)\left(\frac{H}{3}\right)$$

$$\Rightarrow \quad v_1^2 = \frac{4gH}{9}$$

For next H, we have

$$a_2 = \frac{mg}{2m} = \frac{g}{2}$$

$$\Rightarrow \quad v^2 = v_1^2 + 2a_2H$$

$$\Rightarrow \quad v^2 = \frac{4gH}{9} + 2\left(\frac{g}{2}\right)H = \frac{13}{9}gH$$

$$\Rightarrow \quad v = \frac{\sqrt{13gH}}{3}$$

Hence, the correct answer is (C).

221. Acceleration of A w.r.t. ground (G) is

$$\vec{a}_{AG} = a_0\cos\theta\hat{i} + a_0\sin\theta\hat{j}$$

Acceleration of B w.r.t. ground (G) is

$$\vec{a}_{BG} = a_0\sin\theta\hat{j}$$

So, acceleration of A w.r.t. B is

$$\vec{a}_{AB} = \vec{a}_{AG} - \vec{a}_{BG} = a_0\cos\theta\hat{i}$$

Hence, the correct answer is (C).

222. $2V_A\cos\theta = V_0$

Differentiating, we get

$$2a_A\cos\theta - 2V_A\sin\theta\frac{d\theta}{dt} = 0 \qquad \ldots(1)$$

Since $Y_A = b\cot\theta$

$$\Rightarrow \quad V_A = \frac{dY_A}{dt} = (b\,\text{cosec}^2\,\theta)\frac{d\theta}{dt}$$

Put V_A in (1), we get

$$a_A = \frac{V_0^2}{4b}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. (A) is correct when the bicycle is being pedalled because during pedalling, the force is acting in the backward direction on the rear wheel and thus frictional force acts in the forward direction.

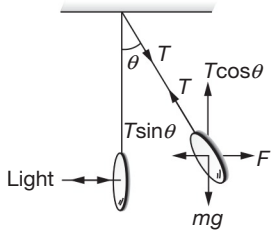
(D) is correct when bicycle is not pedalled.

Hence, (A) and (D) are correct.

2. All accelerated frames are Non-inertial frames. Since earth rotates about its own axis and revolves around the sun, so it is a Non-Inertial frame (STRICTLY SPEAKING), whereas for a good number of cases we assume the earth to be an Inertial frame of reference as the value of acceleration is extremely small.
Hence, (B) and (D) are correct.

3. Let F be the force exerted on the mirror by the photon. Since a photon will be reflected from the mirror with the same value of momentum so, change in momentum $\Delta p = p - (-p) = 2p = \frac{2E}{c}$.

(Because for a photon $E = pc$)



$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t}$$

$$\Rightarrow \quad F = \frac{2E}{c\Delta T} \qquad \ldots(1)$$

Also

$$T\cos\theta = mg \text{ and} \qquad \ldots(2)$$

$$T\sin\theta = F = \frac{2E}{c\Delta T} \qquad \ldots(3)$$

$$\Rightarrow \quad \tan\theta = \frac{2E}{mgc\Delta T} \qquad \ldots(4)$$

Further by definition intensity of a photon beam is defined as the energy incident per unit area of a surface per unit time.

$$\Rightarrow \quad I = \frac{E}{A\Delta t}$$

$$\Rightarrow \quad \frac{E}{\Delta t} = IA \qquad \ldots(5)$$

Substituting (5) in (4), we get

$$\tan\theta = \frac{2IA}{mgc}$$

Also from (1), we get

$$F = \frac{2}{c}(IA)$$

$$\Rightarrow \text{ Radiation Pressure } = \frac{F}{A} = \frac{2I}{c}$$

Hence, (A) and (C) are correct.

4. When P is gradually increased, so till the moment P becomes equal to f_0, mass A will not move and hence the string will not develop any tension due to pull P. So, $T = 0$ for $P < f_0$. Further for A and B to move collectively with acceleration a (say), we must have

$$a = \frac{P - (f_1)_{\lim} - (f_2)_{\lim}}{2m}$$

$$\Rightarrow \quad a = \frac{P - 2f_0}{2m} \qquad \ldots(1)$$

So, the system will not move till $P > 2f_0$, but T will be developed as soon as P becomes greater than f_0. Hence

$$P = T + f_0$$

$$\Rightarrow \quad T = P - f_0 \text{ for } f_0 < P < 2f_0$$

So, **(B)** is also correct.

Once $P > 2f_0$, then for

BLOCK *A*

$$P - T - f_0 = ma \qquad \ldots(2)$$

$$\Rightarrow \quad P - T - f_0 = m\left(\frac{P - 2f_0}{2m}\right) \qquad \{\because \text{of}(1)\}$$

$$\Rightarrow \quad P - T - f_0 = \frac{P}{2} - f_0$$

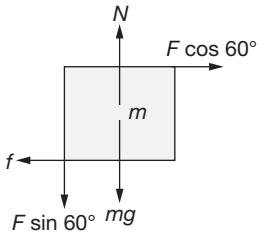
$$\Rightarrow \quad P - T = \frac{P}{2}$$

$$\Rightarrow \quad T = \frac{P}{2}$$

Hence, (A), (B) and (C) are correct.

CHAPTER 6

5.



For no motion $F_{\text{applied}} \le f\,(=\mu N)$

$\Rightarrow \quad F\cos 60° \le \mu\left(\text{mg} + F\sin 60°\right)$

$\Rightarrow \quad \dfrac{F}{2} \le \dfrac{1}{2\sqrt{3}}\left(\sqrt{3}g + \dfrac{F\sqrt{3}}{2}\right)$

$\Rightarrow \quad \dfrac{F}{2} \le g$

$\Rightarrow \quad F_{\max} = 20\ \text{N}$

Hence, (A) is correct.

6. If T is the tension in the string and N is the normal reaction exerted by B on A, then from FBD of A, we get

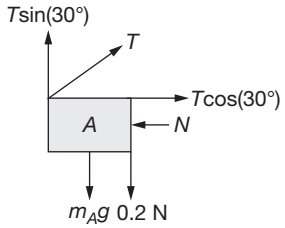
$$T\sin(30°) = m_A g + 0.2\ \text{N} \qquad \ldots(1)$$

$$T\cos(30°) = N \qquad \ldots(2)$$

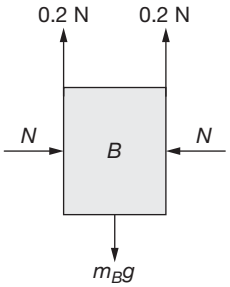
$\Rightarrow \quad T\sin(30°) = m_A g + 0.2T\cos(30°)$

$\Rightarrow \quad T\left[\sin(30°) - 0.2\cos(30°)\right] = 100$

$\Rightarrow \quad T \cong 306\ N$ and $N \cong 265\ N$



Free body diagram for B is shown here. If a be the acceleration of B, then



$$m_B g - 2(0.2\ \text{N}) = m_B a$$

$\Rightarrow \quad a = \dfrac{200 - (0.4)(265)}{20}$

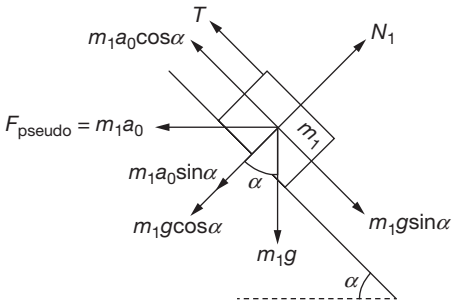
$\Rightarrow \quad a = 4.7\ \text{ms}^{-2}$

Hence, (A), (C) and (D) are correct.

7. **For m_1**

$$T + m_1 a_0 \cos\alpha = m_1 g \sin\alpha$$

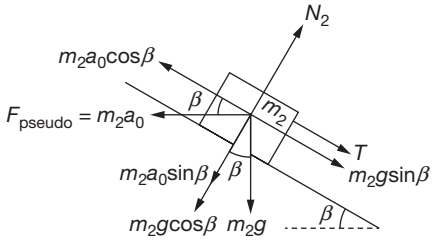
$\Rightarrow \quad T = m_1 g \sin\alpha - m_1 a_0 \cos\alpha \qquad \ldots(1)$



For m_2

$$T + m_2 g \sin\beta = m_2 a_0 \cos\beta$$

$\Rightarrow \quad T = m_2 a_0 \cos\beta - m_2 g \sin\beta \qquad \ldots(2)$



Equating (1) and (2), we get

$$m_1 g\sin\alpha - m_1 a_0\cos\alpha = m_2 a_0\cos\beta - m_2 g\sin\beta$$

$\Rightarrow \quad \left(m_1\sin\alpha + m_2\sin\beta\right)g = \left(m_1\cos\alpha + m_2\cos\beta\right)a_0$

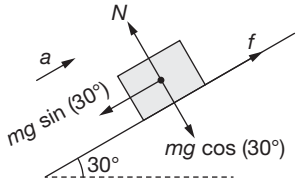
$\Rightarrow \quad a_0 = \left(\dfrac{m_1\sin\alpha + m_2\sin\beta}{m_1\cos\alpha + m_2\cos\beta}\right)g \qquad \ldots(3)$

Substituting (3) either in (2) or in (1), we get

$$T = \left[\frac{m_1 m_2}{m_1\cos\alpha + m_2\cos\beta}\right] g\sin(\alpha - \beta) \qquad \ldots(4)$$

Hence, (B) and (D) are correct.

8. Free body diagram for the block is shown for convenience



According to Newton's Second Law, we have

$$f - mg\sin(30°) = ma$$

$$\Rightarrow \quad f = (1)(10)\left(\frac{1}{2}\right) + (1)(1) = 6 \text{ N}$$

Also, we have $N = mg\cos(30°) = 5\sqrt{3}$ N.

So, net contact force between the block and the belt is

$$F_c = \sqrt{N^2 + f^2} = \sqrt{75 + 36} = \sqrt{111} \text{ N}$$

$$\Rightarrow \quad F_c \cong 10.5 \text{ N}$$

Hence, (A), (B) and (C) are correct.

9. $a = g\tan\alpha$ …(1)

Since, $\sin\alpha = \frac{1}{x}$

$$\Rightarrow \quad \tan\alpha = \frac{1}{\sqrt{x^2 - 1}}$$

$$\Rightarrow \quad a = \frac{g}{\sqrt{x^2 - 1}} \qquad \{\text{Using (1)}\}$$

and $N = \frac{mg}{\cos\alpha} = \frac{mgx}{\sqrt{x^2 - 1}}$

Hence, (B), (C) and (D) are correct.

10. From our knowledge of constraint equations, we observe that $a_A = a_B = a$ (say). Also, since the string is light and inextensible, so tension throughout the string is also the same, say T.

For A,

$$T + mg\sin(30°) = ma$$

$$\Rightarrow \quad T + \frac{mg}{2} = ma \qquad \ldots(1)$$

For B,

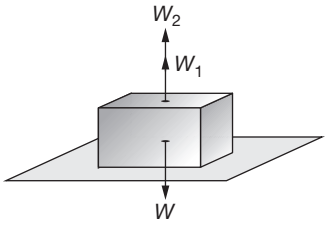
$$mg - T = ma \qquad \ldots(2)$$

Solving (1) and (2), we get

$$a = \frac{3g}{4} \text{ and } T = \frac{mg}{4}$$

Hence, (B) and (D) are correct.

11. The readings of the spring balance (under various situations discussed) and the reading of the weighing machine are equal to the forces exerted by them on the block.



Hence, (A), (B) and (D) are correct.

12. Let m be the mass per unit length of the chain. Then

$$(mx)g - T = (mx)a \qquad \ldots(1)$$

$$T = m(L - x)a \qquad \ldots(2)$$

$$\Rightarrow \quad (mx)g - m(L - x)a = (mx)a$$

$$\Rightarrow \quad xg = La$$

$$\Rightarrow \quad a = g\frac{x}{L}$$

$$\Rightarrow \quad v\frac{dv}{dx} = g\frac{x}{L}$$

$$\Rightarrow \quad vdv = \frac{g}{L}xdx$$

$$\Rightarrow \quad \int_0^v vdv = \frac{g}{L}\int_0^x xdx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{g}{L}\frac{x^2}{2}$$

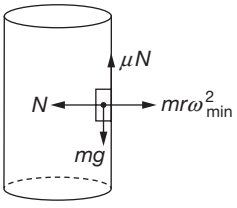
$$\Rightarrow \quad v = x\sqrt{\frac{g}{L}}$$

Hence, (A) and (C) are correct.

13. $mg = \mu N$ and

$$N = mr\omega_{\min}^2$$

$$\Rightarrow \quad mg = \mu mr\omega_{\min}^2$$



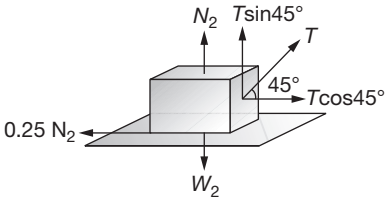
$$\Rightarrow \quad \omega_{\min} = \sqrt{\frac{g}{\mu R}} \text{ and } v_{\min} = R\omega_{\min} = \sqrt{\frac{gR}{\mu}}$$

Hence, (A) and (D) are correct.

14. For W_2:

$$N_2 + T\sin 45 = W_2 = 100 \qquad \ldots(1)$$

$$T\cos 45 = 0.25N_2 \qquad \ldots(2)$$

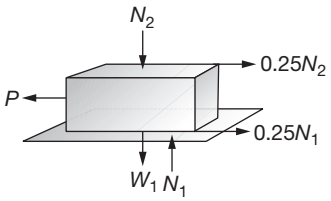


FBD for W_2

For W_1:

$$P = 0.25(N_1 + N_2) \qquad \ldots(3)$$

$$N_2 + W_1 = N_1 \qquad \ldots(4)$$

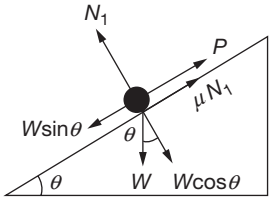


FBD for W_1

Solving, we get $P = 90$ N, $T = 20\sqrt{2}$ N

Hence, (B) and (D) are correct.

15.



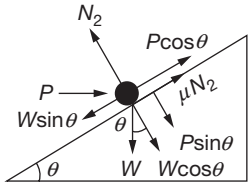
$$W\sin\theta = P + \mu N_1 \text{ and } N_1 = W\cos\theta$$

$$\Rightarrow \quad W\sin\theta = P + \mu W\cos\theta$$

$$\Rightarrow \quad P = W(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow \quad P = W(\sin\theta - \tan\phi\cos\theta) \qquad \{\because \mu = \tan\phi\}$$

$$\Rightarrow \quad \frac{P}{W} = \frac{\sin(\theta-\phi)}{\cos\phi} \qquad \ldots(1)$$



$$P\cos\theta + \mu N_2 = W\sin\theta$$

$$W\cos\theta + P\sin\theta = N_2$$

$$\Rightarrow \quad P\cos\theta + \mu(W\cos\theta + P\sin\theta) = W\sin\theta$$

$$\Rightarrow \quad P(\cos\theta + \mu\sin\theta) = W(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow \quad P(\cos\theta\cos\phi + \sin\theta\sin\phi) = W(\sin\theta\cos\phi - \cos\theta\sin\phi)$$

$$\Rightarrow \quad P\cos(\theta-\phi) = W\sin(\theta-\phi)$$

$$\Rightarrow \quad \frac{P}{W} = \tan(\theta-\phi) \qquad \ldots(2)$$

Equating (1) and (2), we get

$$\cos\phi = \cos(\theta-\phi)$$

$$\Rightarrow \quad \theta - \phi = \phi$$

$$\Rightarrow \quad \theta = 2\phi$$

$$\Rightarrow \quad \frac{P}{W} = \tan(2\phi - \phi)$$

$$\Rightarrow \quad \frac{P}{W} = \tan\phi$$

Hence, (A) and (D) are correct.

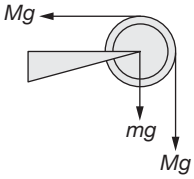
16. $mg\cos\alpha = N \qquad \ldots(1)$

$$mg\sin\alpha = \mu N \qquad \ldots(2)$$

$$\Rightarrow \quad \cot\alpha = \frac{1}{\mu} = 3$$

Hence, (A) is correct.

17. $T = Mg$



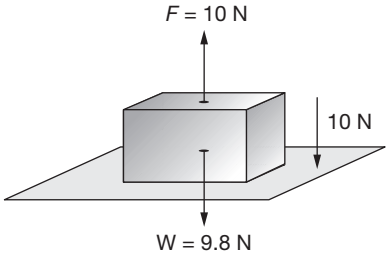
Also the weight of pulley mg is acting vertically downward. So total downward force is $(m+M)g$ and horizontal force is $T = Mg$. The resultant of these two is

$$\sqrt{M^2 + (m+M)^2}\,g$$

Hence, (D) is correct.

18. Reading of the spring balance is equal to the tension in the rope which equals the force of friction between the rope and the boy as well as the force exerted by the rope and the boy on each other.
Hence, (A), (C) and (D) are correct.

19. Due to Newton's Third Law, the block will exert an equal and opposite force on the table. Since $F > W$, so the block will have an upward acceleration.



Hence, (B) and (C) are correct.

20. Net torque must be zero, so

$$\vec{\tau}_N + \vec{\tau}_{fr} = \vec{0}$$

$$\Rightarrow \quad \vec{\tau}_N = -\vec{\tau}_{fr}$$

Hence, the correct answer is (D).

21. When the entire incline is smooth, then both the blocks move down with an acceleration $g\sin(30°) = 5\ \text{ms}^{-2}$. However, if A is smooth and B is rough then initial acceleration of A will be $g\sin\theta$, whereas that of B is less than $g\sin\theta$ and hence there will be elongation in the spring.
Hence, (A) and (D) are correct.

22. Newton's Laws are valid in all inertial frames i.e., frames either at rest or frames moving with constant velocity. Further a frame moving with constant velocity with respect to an inertial frame is also Inertial. Earth is not an inertial frame due to rotation about its axis and revolution round the sun.
Hence, (B) and (C) are correct.

23. $F - fy - Mg = Ma$ {at a distance y}

$$\Rightarrow \quad a = \frac{F - fy - Mg}{M}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{F - fy - Mg}{M}$$

$$\Rightarrow \quad v\frac{dv}{dy} = \frac{F - fy - Mg}{M}$$

$$\Rightarrow \quad \int_0^v v\,dv = \frac{1}{M}\int_0^s (F - fy - Mg)\,dy$$

$$\Rightarrow \quad v = \sqrt{\frac{2s}{M}\left(F - Mg - \frac{fs}{2}\right)}$$

Hence, (A) and (C) are correct.

24. When lift moves with uniform velocity no pseudo force exists and hence $f = \mu mg\,(A \,\&\, B)$. When lift moves down with acceleration of $4.8\ \text{ms}^{-2}$ then $N = m(9.8 - 4.8) = 5$ m.

So, frictional force $= \mu N = 5\ \mu$m

Hence, (A), (B), (C) and (D) are correct.

25. The block with lower value of μ will tend to have greater acceleration down the slope.
Hence, (A) and (B) are correct.

26. $2T\cos\theta = \sqrt{2}\ Mg$ …(1)

$$\Rightarrow \quad 2Mg\cos\theta = \sqrt{2}\ \text{Mg}$$

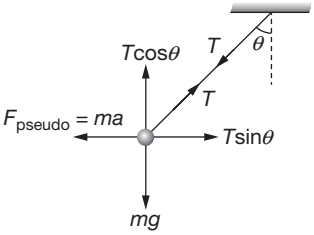
$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 45°$$

Hence, (C) is correct.

27. For equilibrium, we get

$$T\cos\theta = mg \text{ and } T\sin\theta = ma$$



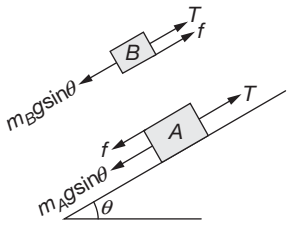
$$\Rightarrow \quad \tan\theta = \frac{a}{g}$$

Also, $T = \dfrac{mg}{\cos\theta} = mg\sec\theta = mg\sqrt{1 + \tan^2\theta}$

$$\Rightarrow \quad T = m\sqrt{a^2 + g^2}$$

Hence, (B) and (D) are correct.

28. Since friction always has a tendency to stop the relative motion between the blocks, so when $m_B > m_A$, then for a frictional force f between A and B, we have



$$m_Bg\sin\theta = T + f \quad \ldots(1)$$

$$m_Ag\sin\theta = T - f \quad \ldots(2)$$

Subtracting (2) from (1), we get

$$(m_B - m_A)g\sin\theta = 2f$$

$$\Rightarrow \quad f = \frac{1}{2}(m_B - m_A)g\sin\theta$$

Now, $f = 0$, when $m_A = m_B$

So, (B) is correct.

Also, for $f \le f_{\max}$, the blocks will remain stationary.

$$\Rightarrow \quad \frac{1}{2}(m_B - m_A)g\sin\theta \le \mu(m_Bg\cos\theta)$$

$$\Rightarrow \quad \mu \ge \left(\frac{m_B - m_A}{2m_B}\right)\tan\theta$$

Hence the acceleration of the system is zero if $\mu \ge \left(\frac{m_B - m_A}{2m_B}\right)\tan\theta$ and $m_B > m_A$.

Also, from (1) and (2), we get

$$T = \left(\frac{m_A + m_B}{2}\right)g\sin\theta$$

$$\Rightarrow \quad T = m_Ag\sin\theta = mg\sin\theta$$

Hence, (A), (B) and (D) are correct.

29. $a = \dfrac{m_Bg}{m_A + m_B} = \dfrac{2}{7}(9.8)\ \text{ms}^{-2}$

$$\Rightarrow \quad a = 2.8\ \text{ms}^{-2} = 280\ \text{cms}^{-2}$$

Since the initial velocity of A is towards left whereas acceleration is towards the right. So this acceleration acts as retardation for motion of A towards left. (Take left to right as positive).

$$v|_{t=1s} = -50 + (280)(1)$$

$$\Rightarrow \quad v|_{t=1s} = +230\ \text{cms}^{-1} \text{ (towards right) and}$$

$$s|_{t=1s} = -(50)(1) + \frac{1}{2}(280)(1)$$

$$\Rightarrow \quad s|_{t=1s} = +90\ \text{cm} \qquad \{\text{displacement towards right}\}$$

So distance from pulley = 200 – 90 = 110 cm

Hence, (B) and (D) are correct.

30. $a = \left(\dfrac{M - m}{M + m}\right)g$,

$$T = \left(\frac{2Mm}{M + m}\right)g$$

Thrust $= 2T = \left(\dfrac{4Mm}{M + m}\right)g$ and

$$(M + m)g - 2T = (M + m)a_{cm}$$

$$\Rightarrow \quad a_{cm} = \left(\frac{M - m}{M + m}\right)^2 g$$

or $\quad a_{cm} = \dfrac{Ma + m(-a)}{M + m} \qquad \left\{\because a_{cm} = \dfrac{m_1a_1 + m_2a_2}{m_1 + m_2}\right\}$

$$\Rightarrow \quad a_{cm} = \left(\frac{M - m}{M + m}\right)a = \left(\frac{M - m}{M + m}\right)^2 g$$

Hence, (A), (B), (C) and (D) are correct.

31. Since the surfaces are smooth, so for any value of θ both A and B will have the same acceleration downwards, as a result of which neither of them exerts a force on each other.

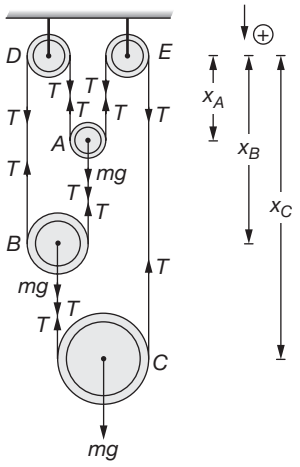
Hence, (B) and (D) are correct.

32. According to Impulse-Momentum Theorem
Impulse = Change in momentum

$$\Rightarrow \quad F(\Delta t) = m[v - (-u)]$$

Hence, (A) and (B) are correct.

33.
- Take downward as positive direction.
- The total length of the thread is a constant.
- Starting from centre of A and going to centre of B we have



$$(x_B - x_A) + x_B + 2x_A + x_C + (x_C - x_B) = \text{constant}$$

$\Rightarrow \quad x_B + x_A + 2x_C = \text{constant}$

$\Rightarrow \quad \ddot{x}_B + \ddot{x}_A + 2\ddot{x}_C = 0$

$\Rightarrow \quad a_B + a_A + 2a_C = 0 \qquad \ldots(1)$

Applying cause-effect equations we get

$mg - 2T = ma_C$ (For C) $\qquad \ldots(2)$

$mg + T - 2T = ma_B$ (For B)

$\Rightarrow \quad mg - T = ma_B \qquad \ldots(3)$

$mg + T - 2T = ma_A$ (For A)

$\Rightarrow \quad mg - T = ma_A \qquad \ldots(4)$

From (3) and (4), $a_A = a_B$ so (1) gives

$\Rightarrow \quad a_A = a_B = -a_C \qquad \ldots(5)$

Adding (3) and (4), we get

$2mg - 2T = 2ma_A \qquad \ldots(6)$

Subtract (2) from (6), we get

$mg = 2ma_A - ma_C$

$\Rightarrow \quad mg = 2ma_A + ma_A \qquad \{\because a_A = -a_C\}$

$\Rightarrow \quad mg = 3ma_A$

$\Rightarrow \quad a_A = \frac{g}{3}$

So, $a_A = a_B = \frac{g}{3}$; Downwards

$a_C = -\frac{g}{3}$ Upwards

Also we get T by putting value of a_A in (4) i.e. $T = \frac{2}{3}$mg.

Hence, (A) and (B) are correct.

34. $200 - kx = 20a_1$

and $kx = 10(12) = 120$

$\Rightarrow \quad a_1 = 4\ ms^{-2}$

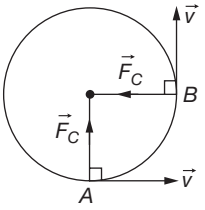
$$a = \frac{F}{m_1} + \frac{F}{m_2} = \frac{F}{\left(\frac{m_1 m_2}{m_1 + m_2}\right)} = \frac{F}{\mu}$$

$\Rightarrow \quad a = \frac{200}{(20/3)}$

$\Rightarrow \quad a = 30\ \text{ms}^{-2}$

Hence, (B), (C) and (D) are correct.

35. Centripetal force is a force which is directed radially inwards, but this does not imply that is a force constant in magnitude and direction.



So $\quad |\vec{F}_C| = \frac{mv^2}{r}$

Also in uniform circular motion magnitude of velocity is constant but velocity is not constant as it changes its direction continuously. Further, $\vec{F} \perp \vec{v}$ implies work done is zero, so according to **Work Energy Theorem**

Work done = Change in K.E.

$\Rightarrow \quad 0 = \Delta(K.E.)$

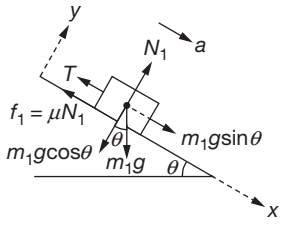
$\Rightarrow \quad$ K.E. = constant

Hence, (C) and (D) are correct.

36. Concept of Pseudo-Force.

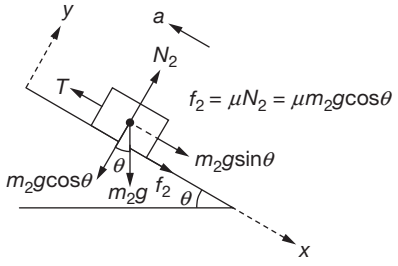
Hence, (A), (C) and (D) are correct.

37.



$m_1 g\sin\theta - T - \mu N_1 = m_1 a$

$\Rightarrow \quad m_1 g\sin\theta - T - \mu m_1 g\cos\theta = m_1 a \qquad \ldots(1)$



$T - m_2 g\sin\theta - \mu N_2 = m_2 a$

$\Rightarrow \quad T - m_2 g\sin\theta - \mu m_2 g\cos\theta = m_2 a \qquad \ldots(2)$

Adding (1) and (2), we get

$$a=\left[\frac{(m_1-m_2)\sin\theta-\mu(m_1+m_2)\cos\theta}{m_1+m_2}\right]g$$

for m_1 downwards along the incline.

So for m_2 acceleration is directed up the incline i.e.

$$a_2=-a$$

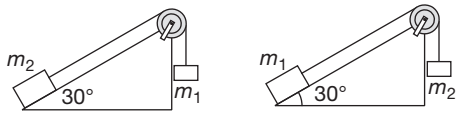
Hence, (A) and (B) are correct.

OBJECTIVE TRICK

If $\theta=\frac{\pi}{2}$ and no friction exists i.e. $\mu=0$, then 'a' must be that for an ideal Atwood's machine i.e. $\left(\frac{m_1-m_2}{m_1+m_2}\right)g$ for m_1 downwards.

This condition is met by option **(A)** for m_1 and option **(B)** for m_2.

38.



$$\begin{pmatrix}\text{Time taken by}\\ m_2 \text{ to reach}\\ \text{maximum height}\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\text{Time taken by}\\ m_1 \text{ to reach}\\ \text{maximum height}\end{pmatrix}$$

$$a_1=\left(\frac{m_1-m_2\sin 30}{m_1+m_2}\right)g \text{ and}$$

$$a_2=\left(\frac{m_2-m_1\sin 30}{m_1+m_2}\right)g$$

Since $l=\frac{1}{2}a_1t_1^2=\frac{1}{2}a_2t_2^2$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{2m_1-m_2}{m_1+m_2}\right)\frac{t_2^2}{4}=\frac{1}{2}\left(\frac{2m_2-m_1}{m_1+m_2}\right)t_2^2$$

$$\Rightarrow \quad \left(\frac{2m_1-m_2}{m_1+m_2}\right)\frac{1}{4}=\left(\frac{2m_2-m_1}{m_1+m_2}\right)$$

$$\Rightarrow \quad 8m_2-4m_1=2m_1-m_2$$

$$\Rightarrow \quad 9m_2=6m_1$$

$$\Rightarrow \quad \frac{m_1}{m_2}=\frac{3}{2}$$

Hence, (A) and (C) are correct.

40. $T_{\text{upper}}=(2.9+0.2+1.9)(9.8+0.2)$

$$\Rightarrow \quad T_{\text{upper}}=50 \text{ N and}$$

$$T_{\text{lower}}=(1.9+0.1)(9.8+0.2)$$

$$\Rightarrow \quad T_{\text{lower}}=20 \text{ N}$$

Hence, (A) and (D) are correct.

41. $4500-500-T=5\times10^4a$ …(1)

$T-400=4\times10^4a$ …(2)

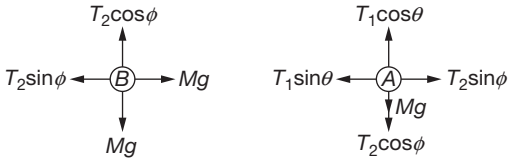
Adding (1) and (2), we get

$$a=\frac{3600}{9\times10^4}$$

$$\Rightarrow \quad a=0.04 \text{ ms}^{-2} \text{ and } T=2000 \text{ N}$$

Hence, (A) and (C) are correct.

42.



$T_2\cos\phi=Mg$ …(1)

$T_2\sin\phi=Mg$ …(2)

$T_1\sin\theta=T_2\sin\phi$ …(3)

$T_1\cos\theta=T_2\cos\phi+Mg$ …(4)

From (1) and (2), we get

$\tan\phi=1$ {OPTION **(A)**}

$T_2=\sqrt{2}\,Mg$ {OPTION **(C)**}

From (3), we get

$$T_1\sin\theta=\sqrt{2}\,Mg\frac{1}{\sqrt{2}}$$

$T_1\sin\theta=Mg$ …(5)

From (4), we get

$$T_1\cos\theta=\sqrt{2}\,Mg\left(\frac{1}{\sqrt{2}}\right)+Mg$$

$$\Rightarrow \quad T_1\cos\theta=2Mg \qquad \ldots(6)$$

From (5) and (6), we get

$\tan\theta=\frac{1}{2}=0.5$ {OPTION **(B)**}

and $T_1=\sqrt{5}\,Mg$ {OPTION **(D)**}

Hence, (A), (B), (C) and (D) are correct.

43. Concept of Pseudo Force.

Hence, (A) and (C) are correct.

44. $a_{BA} = 3a_0$ {using constraint relations}

$a_{BG} = \sqrt{a_0^2 + 9a_0^2 + 6a_0^2\cos\theta}$

$a_0\sqrt{10 + 6\cos\theta}$

Hence, (A) and (D) are correct.

45. With respect to wedge, the acceleration of block is same as that of wedge with respect to ground.

Hence, (A) and (B) are correct.

46. $F = kt$

$\Rightarrow \quad m\dfrac{dv}{dt} = kt$

$\Rightarrow \quad mv = \dfrac{kt^2}{2}$ …(1)

Since, $a = \dfrac{F}{m}$

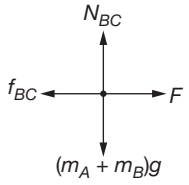
$\Rightarrow \quad a = \dfrac{k}{m}t$ …(2)

From (1) and (2), we get

$$mv = \frac{k}{2}\times\left(\frac{ma}{k}\right)^2$$

Hence, (A), (B) and (C) are correct.

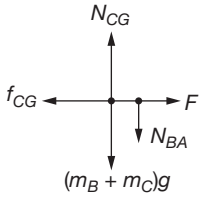
48. Free body diagram of $(A+B)$ system



$f_{BC} = F \neq 0$

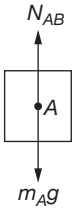
$\Rightarrow \quad f_{CB} \neq 0$

Free body diagram of $(B+C)$ system



$f_{CG} = F = f_{BC}$

Free body diagram of A is



No force acts on A parallel to the surface in contact, so

$f_{AB} = 0$

Hence, (B) and (C) are correct.

49. (A) Magnitude of velocity is changing hence acceleration is present

(C) Velocity is changing hence it can happen by change in direction also as in a uniform circular motion. Hence acceleration is present.

Hence, (A) and (C) are correct.

50. Since, $v = 2t$ and if the radius of circular path is r, then we have

$$a_T = \frac{dv}{dt} = 2$$

$$a_r = \frac{v^2}{r} = \frac{4t^2}{r}$$

$\Rightarrow \quad a = \sqrt{a_T^2 + a_r^2}$

$\Rightarrow \quad a = \sqrt{4 + \dfrac{16t^4}{r^2}}$

Hence, (B) and (D) are correct.

51. $N_A = mg\cos\theta$ and $N_B = mg\cos(53°)$

Since, $\dfrac{N_A}{N_B} = \dfrac{4}{3}$

$\Rightarrow \quad \dfrac{\cos\theta}{\cos 53°} = \dfrac{4}{3}$

$\Rightarrow \quad \cos\theta = \dfrac{4}{3}\times\dfrac{3}{5}$

$\Rightarrow \quad \cos\theta = \dfrac{4}{5}$

$\Rightarrow \quad a = \dfrac{mg[\sin(53°) - \sin\theta]}{2m} = \dfrac{g}{10}$

Hence, (A) and (C) are correct.

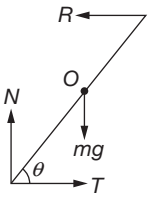
52. $R = T$ …(1)

$N = mg$ …(2)

For equilibrium, we have

$$\Sigma\vec{\tau} = \vec{0}$$

So, lets calculate torque about centre of mass of ladder i.e., O.



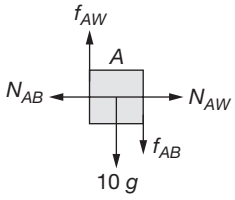
$$N \times \frac{L}{2}\cos\theta = R \times \frac{L}{2}\sin\theta + T \times \frac{L}{2}\sin\theta$$

$$\Rightarrow \quad \frac{mg}{2}\cos\theta = T\sin\theta$$

$$\Rightarrow \quad T = \frac{mg}{2}\cot\theta$$

Hence, (A) and (B) are correct.

53. **Free body diagram of block *A*:**

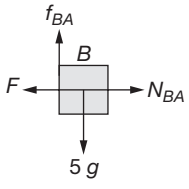


Friction on A due to B, f_{AB} acts downwards and friction on A due to Wall (W), f_{AW} acts upwards. So, we have

$f_{AW} = f_{AB} + 10$ g …(1)

$N_{AW} = N_{AB}$ …(2)

Free body diagram of block *B*:



$f_{BA} = 5$ g …(3)

$N_{BA} = F$ …(4)

From (1) and (3), we get

$f_{AW} = 150$ N

From (2) and (4), we get

$N_{AW} = F$

$\Rightarrow \quad \mu N_{AW} \geq 150$ N

$\Rightarrow \quad 0.4 \times F \geq 150$ N

$\Rightarrow \quad F \geq 375$ N

Hence, (A), (B) and (C) are correct.

54. Acceleration of block after application of force

$$a = \frac{2t}{m} - \mu g$$

Hence, (B), (C) and (D) are correct.

55. Given that, $a_T = a_N$

$$\Rightarrow \quad a_T = \frac{v^2}{R}$$

$$\Rightarrow \quad -\frac{dv}{dt} = \frac{v^2}{R}$$

$$\Rightarrow \quad dv = -\frac{v^2}{R}dt$$

$$\Rightarrow \quad \int_{v_0}^{v}\frac{dv}{v^2} = -\frac{1}{R}\int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{v}\Big|_{v_0}^{v} = -\frac{1}{R}t$$

$$\Rightarrow \quad -\frac{1}{v}\Big|_{v_0}^{v} = -\frac{1}{R}t$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{v_0} + \frac{t}{R}$$

$$\Rightarrow \quad v = \frac{v_0}{1 + \frac{v_0 t}{R}}$$

Since, $a_T = a_N = \frac{v^2}{R}$ and $a_T = v\frac{dv}{dS}$

$$\Rightarrow \quad v\frac{dv}{dS} = -\frac{v^2}{R}$$

$$\Rightarrow \quad \int_{v_0}^{v}\frac{dv}{v} = -\frac{1}{R}\int_0^S dS$$

$$\Rightarrow \quad \log_e v\Big|_{v_0}^{v} = -\frac{S}{R}$$

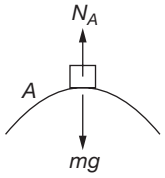
$$\Rightarrow \quad \log_e\left(\frac{v}{v_0}\right) = -\frac{S}{R}$$

$$\Rightarrow \quad \frac{v}{v_0} = e^{-\frac{S}{R}}$$

$$\Rightarrow \quad v = v_0 e^{-\frac{S}{R}}$$

Hence, (A) and (B) are correct.

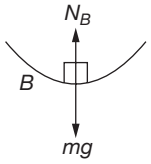
56. **At *A*:**



$$mg - N_A = \frac{mv^2}{r_A}$$

$$N_A = mg - \frac{mv^2}{r_A}$$

At *B*:



$$N_B - mg = \frac{mv^2}{r_B}$$

$$N_B = mg + \frac{mv^2}{r_B}$$

At *C*:

$$N_C = mg - \frac{mv^2}{r_C}$$

At *D*:

$$N_D = mg + \frac{mv^2}{r_D}$$

From figure $r_B < r_D$, so $N_B > N_D$

Hence N_B is greatest

Since, $r_C < r_A$, so $N_C < N_A$

Hence N_C is least

At *A* and *C*; $N_A < mg$

$N_C < mg$

At *B* and *D*; $N_B > mg$

$N_D > mg$

Hence, (A), (B) and (C) are correct.

57. For 20 kg block, we have

$$20g\sin(37°) = \mu\left[20g\cos(37°)\right] + T$$

$$\Rightarrow \quad 20\times 10\times\frac{3}{5} = 0.5\times 20\times 10\times\frac{4}{5} + T$$

$$\Rightarrow \quad 120 = 80 + T$$

$$\Rightarrow \quad T = 40 \text{ N}$$

Also, $2mg\cos(37°) = T$

$$\Rightarrow \quad 2m\times 10\times\frac{4}{5} = 40$$

$$\Rightarrow \quad m = 2.5 \text{ kg}$$

Also, force applied by 20 kg block on inclined is

$$R = \sqrt{N^2 + f^2} = 20g\cos(37°)\sqrt{1+\mu^2} = 178.8 \text{ N}$$

Hence, (A) and (C) are correct.

58. Free body diagram of blocks are drawn.

For m_1:

$$a_1 = \frac{m_1 g - T}{m_1}$$

$$\Rightarrow \quad a_1 = g - \frac{T}{m_1}$$

For m_2:

$$a_2 = \frac{m_2 g - T}{m_2}$$

$$\Rightarrow \quad a_2 = g - \frac{T}{m_2}$$

For $t = 0, T = 0$ $\{\because x = 0\}$

$$\Rightarrow \quad a_1 = a_2$$

For $t > 0, T > 0$

$$\Rightarrow \quad a_1 < a_2$$

Hence, (A) and (D) are correct.

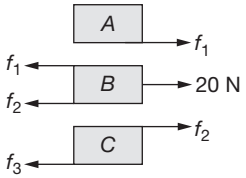
59. $\mu m_3 g = \sqrt{(m_1 g)^2 + (m_2 g)^2}$

Hence, (A), (C) and (D) are correct.

60. (f_1) maximum $= 0.2 \times 10 = 2$ N

(f_2) maximum $= 0.8 \times 20 = 16$ N

(f_3) maximum $= 0.3 \times 30 = 9$ N



So, acceleration of the lowest block is

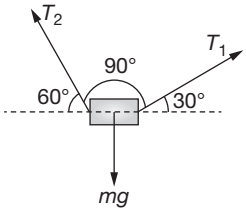
$$a_3 = \frac{f_2 - f_3}{1} = 4.5 \text{ ms}^{-2}$$

Acceleration of upper most block is

$$a_1 = \frac{f_1}{1} = 2 \text{ ms}^{-2}$$

Hence, (A), (B) and (D) are correct.

63.



Applying Lami's Theorem, we get

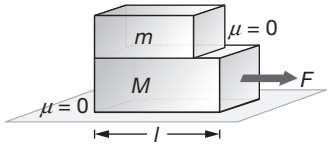
$$\frac{mg}{\sin 90°} = \frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°}$$

$$\Rightarrow \quad T_1 = mg \sin(150°) = 50 \times \frac{1}{2} = 25 \text{ N}$$

$$\Rightarrow \quad T_2 = mg \sin 120° = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3} \text{ N}$$

Hence, (A) and (D) are correct.

64.



Since friction is absent everywhere, so

$$a = \frac{F}{M}$$

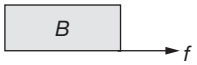
Since $s = \frac{1}{2}at^2$

$$\Rightarrow \quad l = \frac{1}{2}\left(\frac{F}{M}\right)t^2$$

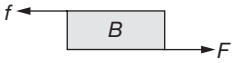
$$\Rightarrow \quad t = \sqrt{\frac{2lM}{F}}$$

Hence, (B) and (D) are correct.

65.



$$f = m_A a = 40 \text{ N}$$

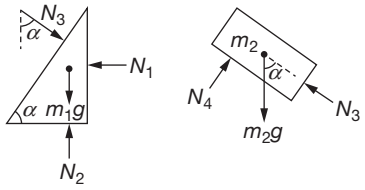


$$F - f = m_B a = 50 \times 3 \text{ N} = 150 \text{ N}$$

$$\Rightarrow \quad F = 150 \text{ N} + 40 \text{ N} = 190 \text{ N}$$

Hence, (A), (B) and (C) are correct.

66. The normal reaction on the wedge can be due to the wall, ground and the block



From free body diagrams of m_1 and m_2, we get

$$N_3 = m_2 g \cos\alpha$$

$$N_3 \sin\alpha = N_1$$

$$N_2 = m_1 g + N_3 \cos\alpha$$

$$\Rightarrow \quad N_1 = m_2 g \sin\alpha \cos\alpha$$

$\Rightarrow \quad N_2 = m_1 g + m_2 g \cos^2 \alpha$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. (i) $F = \dfrac{dp}{dt}$ (Newton's Second Law)

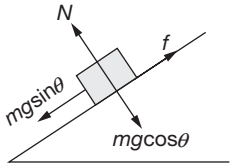
(ii) Newton's Third Law

Hence, the correct answer is (B).

2. $f = mg \sin\theta$ …(1)

$M = mg \cos\theta$

$R = \sqrt{N^2 + f^2} = mg$ …(2)



Hence, the correct answer is (A).

3. Conceptual

Hence, the correct answer is (D).

4. Law of inertia.

Hence, the correct answer is (A).

5. Force needed when breaks are applied

$$f_1 = ma = \frac{mv^2}{d}$$

$\{v$: initial speed, d : distance from wall$\}$

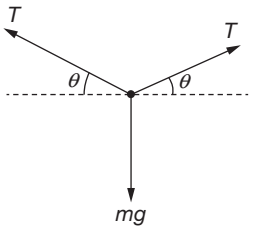
when turn is taken

$$f_2 = ma = \frac{mv^2}{d}$$

Hence brakes must be applied.

Hence, the correct answer is (B).

7. $2T \sin\theta = mg$



$T = \dfrac{mg}{2\sin\theta} > mg$ for $\sin\theta < \dfrac{1}{2}$ i.e., $\theta < 30°$

Hence, the correct answer is (A).

8. Here the Statement-1 is based on the idea that if you substitute $\vec{F} = \vec{0}$ in $\vec{F} = m\vec{a}$ (because there happens to be a particle on which net force $\vec{F} = \vec{0}$)

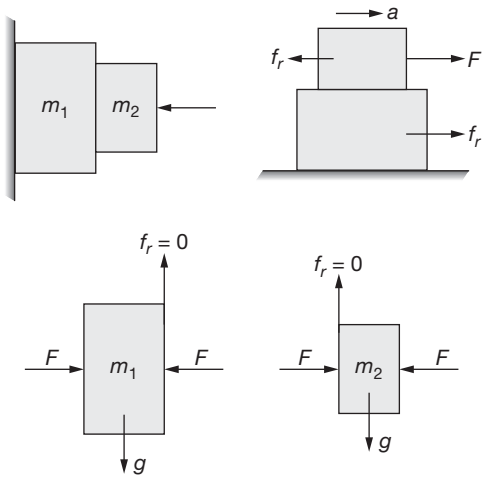
You get $\vec{a} = \vec{0}$

$\Rightarrow \quad \vec{v} =$ constant

$\Rightarrow$ such a particle will move with constant speed along a fixed direction which is Newton's First Law. But the point is, you cannot employ $\vec{F} = m\vec{a}$, without first ascertaining that it is valid in this form (i.e., without pseudo forces). And that can be done only by applying Newton's First Law and checking the behaviour of your frame against the description laid down in the First Law.

Hence, the correct answer is (D).

10.



Hence, the correct answer is (B).

11. According to Newton's Third Law of motion action and reactions are equal and opposite.

Hence, the correct answer is (D).

12. Horizontal range also depends upon the velocity of projection.

Hence, the correct answer is (D).

13. In the direction of normal reaction net acceleration is zero. Hence forces in this direction will be balanced. Hence $N = mg \cos\theta$.

Hence, the correct answer is (A).

14. By the definition of inertial and non-inertial frame.

Hence, the correct answer is (B).

15. Coefficient of friction $\mu = \tan(\theta)$. The value of $\tan\theta$ way exceed unity.

Hence, the correct answer is (C).

16. $F = \dfrac{\Delta P}{\Delta t}$

If Δt is more, then F will be less.

Hence, the correct answer is (A).

17. $a_1 = g - \dfrac{F}{M}$

$a_2 = g - \dfrac{F}{m}$

$\Rightarrow \quad a_1 > a_2$

Hence, the correct answer is (A).

18. While running boy pushes the ground in backward direction and available friction pushed him in forward direction.

Hence, the correct answer is (D).

19. In equilibrium, net force on the body = 0, therefore, it acceleration $a = 0$. If the body is at rest it will remain at rest. If the body is moving with constant speed along a straight line, it will continue to do so.

Hence, the correct answer is (D).

20. Inertia is the property by virtue of which the body is unable to change by itself not only the state of rest but also the state of motion.

Hence, the correct answer is (A).

21. Due to change in normal reaction pulling is easier.

Hence, the correct answer is (D).

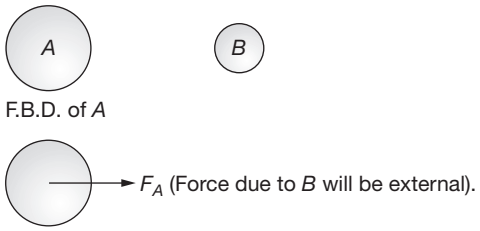
22. Contact force is sum of friction and normal reaction.

Hence, the correct answer is (D).

23. Static frictional force is self adjusting.

Hence, the correct answer is (A).

24.

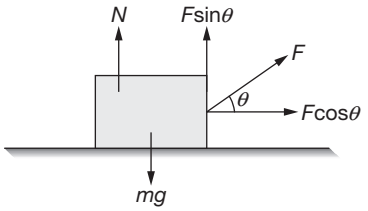


Hence, the correct answer is (B).

25. For pulling condition:

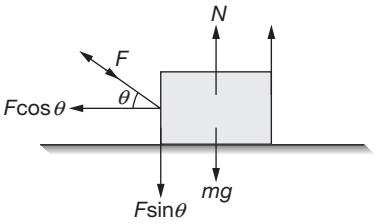
$N + F\sin\theta = mg$

$N = Mg - F\sin\theta \qquad \ldots(1)$



For pushing condition:

$N = F\sin\theta + Mg \qquad \ldots(2)$



Hence, the correct answer is (D).

Linked Comprehension Type Questions

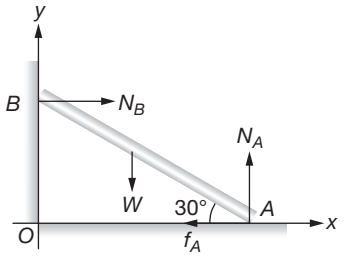
1. **The correct answer is (B).**

2. **The correct answer is (A).**

3. **The correct answer is (B).**

Combined solution to 1, 2, 3

Let length of the rod be $2l$. Using the three conditions of equilibrium. Anticlockwise moment is taken as positive.



For translational equilibrium, we have

	$\Sigma F_x = 0$		$\Sigma F_y = 0$
$\Rightarrow$	$N_B - f_A = 0$	$\Rightarrow$	$N_A - W = 0$
$\Rightarrow$	$N_B = f_A \qquad \ldots(1)$	$\Rightarrow$	$N_A = W \qquad \ldots(2)$

For rotational equilibrium, we have

$\Sigma\tau_0 = 0$

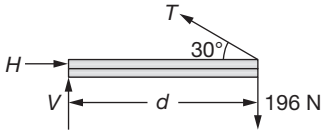
$\Rightarrow \quad N_A(2l\cos 30°) - N_B(2l\sin 30°) - W(l\cos 30°) = 0$

$$\Rightarrow \quad \sqrt{3}N_A - N_B - \frac{\sqrt{3}}{2}W = 0 \qquad \ldots(3)$$

Solving these three equations, we get

$$f_A = \frac{\sqrt{3}}{2}W\,,\; N_A = W\,,\; N_B = \frac{\sqrt{3}}{2}W$$

4. Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.



$$\sum \tau = +(T\sin(30°))d - (196\text{ N})d = 0$$

$$\Rightarrow \quad T = 392\text{ N}$$

Hence, the correct answer is (D).

5. Since $\sum F_x = 0$

$$H - T\cos(30°) = 0$$

$$\Rightarrow \quad H = (392)\cos(30°) = 339\text{ N} \text{ to the right}$$

Hence, the correct answer is (C).

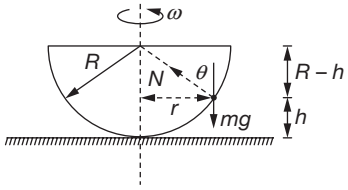
6. Since $\sum F_y = 0$

$$V + T\sin(30°) - 200 = 0$$

$$\Rightarrow \quad V = 196 - (392)\sin(30°) = 0$$

Hence, the correct answer is (A).

7. FBD of particle in ground frame of reference is shown in figure. We observe that



$$\tan\theta = \frac{r}{R-h}$$

Also, $N\cos\theta = mg$ and $\qquad \ldots(1)$

$$N\sin\theta = mr\omega^2 \qquad \ldots(2)$$

Dividing equation (2) by (1), we get

$$\tan\theta = \frac{r\omega^2}{g}$$

$$\Rightarrow \quad \frac{r}{R-h} = \frac{r\omega^2}{g}$$

$$\Rightarrow \quad \omega^2 = \frac{g}{R-h} \qquad \ldots(3)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R-h}}$$

Hence, the correct answer is (D).

8. From equation (3),

$$h = R - \frac{g}{\omega^2}$$

For non-zero value of h, we must have

$$R > \frac{g}{\omega^2}$$

$$\Rightarrow \quad \omega > \sqrt{g/R}$$

So, minimum value of ω should be

$$\omega_{\min} = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{0.1}}\text{ rads}^{-1}$$

$$\Rightarrow \quad \omega_{\min} = 9.9\text{ rads}^{-1}$$

Hence, the correct answer is (C).

9. Since, $h = R - \frac{g}{\omega^2}$

If R and ω are known precisely, then

$$\Delta h = -\frac{\Delta g}{\omega^2}$$

$$\Rightarrow \quad \Delta g = \omega^2 \Delta h \qquad \text{(neglecting the negative sign)}$$

$$\Rightarrow \quad (\Delta g)_{\min} = (\omega_{\min})^2\,\Delta h = (9.89)^2 \times 10^{-4}\,\text{ms}^{-2}$$

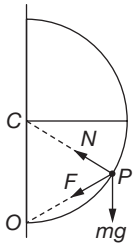
$$\Rightarrow \quad (\Delta g)_{\min} = 9.78 \times 10^{-3}\text{ ms}^{-2}$$

Hence, the correct answer is (B).

10. $CP = CO =$ Radius of circle (R)

$\therefore \quad \angle CPO = \angle POC = 60°$

$\therefore \quad \angle OCP$ is also $60°$.



Therefore, ΔOCP is an equilateral triangle.

Hence, $OP = R$

Natural length of spring is $3R/4$.

So, extension in the spring is

$$x = R - \frac{3R}{4} = \frac{R}{4}$$

$$\Rightarrow \text{ Spring force, } F = kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$$

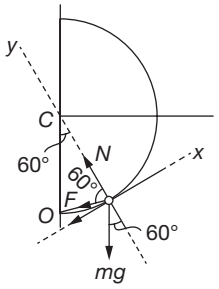
The free body diagram of the ring will be as shown.

Here, $F = k\Delta x = \dfrac{mg}{4}$ $\qquad \left\{\because \Delta x = \dfrac{R}{4}\right\}$

And $N =$ normal reaction.

Hence, the correct answer is (C).

11. Tangential acceleration a_T :



The ring will move towards the x-axis just after the release. So, net force along x-axis i.e. the tangential force is

$$F_T = F_x = F\sin 60° + mg\sin 60°$$

$$\Rightarrow \quad F_T = \left(\frac{mg}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad F_T = \frac{5\sqrt{3}}{8}mg$$

$$\Rightarrow \quad a_T = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8}g$$

Hence, the correct answer is (D).

12. Since, $N + F\cos 60° = mg\cos 60°$

$$\Rightarrow \quad N = mg\cos 60° - F\cos 60°$$

$$\Rightarrow \quad N = \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right) = \frac{mg}{2} - \frac{mg}{8}$$

$$\Rightarrow \quad N = \frac{3}{8}\,mg$$

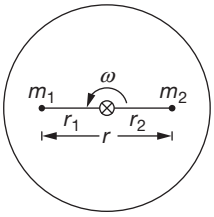
Hence, the correct answer is (C).

13. Given $m_1 = 10$ kg, $m_2 = 5$ kg, $\omega = 10$ rads^{-1}

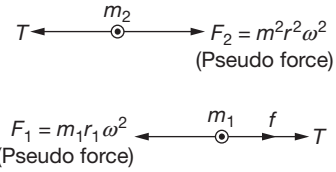
$r = 0.3$ m, $r_1 = 0.124$ m

$$\Rightarrow \quad r_2 = r - r_1 = 0.176 \text{ m}$$

Masses m_1 and m_2 are at rest with respect to rotating table. Let f be the friction between mass m_1 and table.



Free body diagram of m_1 and m_2 with respect to table (non-inertial frame of reference are shown in figures).



Equilibrium of m_2 gives

$$T = m_2 r_2 \omega^2 \qquad \ldots(1)$$

Since, $m_2 r_2 \omega^2 < m_1 r_1 \omega^2$

Therefore, $m_1 r_1 \omega^2 > T$

and friction of M_1 will be inward (toward centre)

Equilibrium of m_1 gives

$$f + T = m_1 r_1 \omega^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$f = m_1 r_1 \omega^2 - m_2 r_2 \omega^2 \qquad \ldots(3)$$

$$f = (m_1 r_1 - m_2 r_2)\omega^2$$

$$\Rightarrow \quad f = (10 \times 0.124 - 5 \times 0.176)(10)^2 \text{ N} = 36 \text{ N}$$

Therefore, frictional force on m_1 is 36 N (inwards)

Hence, the correct answer is (D).

14. From equation (3),

$$f = (m_1 r_1 - m_2 r_2)\omega^2$$

Masses will start slipping when this force is greater than f_{max} or $(m_1 r_1 - m_2 r_2)\omega^2 > f_{max} > \mu m_1 g$.

Minimum values of ω is

$$\omega_{min} = \sqrt{\frac{\mu m_1 g}{m_1 r_1 - m_2 r_2}} = \sqrt{\frac{0.5 \times 10 \times 9.8}{10 \times 0.124 - 5 \times 0.176}}$$

$$\Rightarrow \quad \omega_{min} = 11.67 \text{ rads}^{-1}$$

Hence, the correct answer is (B).

15. From equation (3), frictional force $f = 0$ where $m_1 r_1 = m_2 r_2$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{5}{10} = \frac{1}{2}$$

and $r = r_1 + r_2 = 0.3$ m

$$\Rightarrow \quad r_1 = 0.1 \text{ m and } r_2 = 0.2 \text{ m}$$

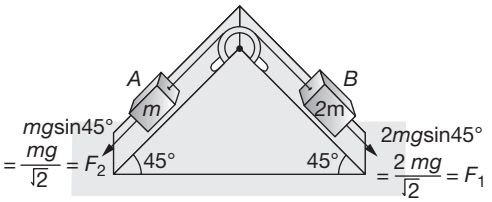
i.e. mass m_2 should placed at 0.2 m and m_1 at 0.1 m from the centre O.

Hence, the correct answer is (C).

16. Acceleration of block A

Maximum friction force that can be obtained at A is

$$(f_{max})_A = \mu_A (mg\cos 45°) = \frac{2}{3}\left(\frac{mg}{\sqrt{2}}\right) = \frac{\sqrt{2}\,mg}{3}$$



Similarly,

$$(f_{max})_B = \mu_B (2mg\cos 45°) = \frac{1}{3}\left(\frac{2mg}{\sqrt{2}}\right)$$

$$\Rightarrow \quad (f_{max})_B = \frac{\sqrt{2}\,mg}{3}$$

Therefore, maximum value of friction that can be obtained on the system is

$$(f_{max}) = (f_{max})_A + (f_{max})_B = \frac{2\sqrt{2}\,mg}{3} \qquad \ldots(1)$$

Net pulling force on the system is

$$F = F_1 - F_2 = \frac{2mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = \frac{mg}{\sqrt{2}} \qquad \ldots(2)$$

From (1) and (2), we can see that

Net pulling force $< f_{max}$

Therefore, the system will not move or the acceleration of block A will be zero.

Hence, the correct answer is (D).

17. The correct answer is (B).

18. The correct answer is (C).

Combined solution to 17, 18

Tension in the string and friction at A

Net external force on the system (block A and B)

$$F = F_1 - F_2 = \frac{mg}{\sqrt{2}}$$

Therefore, total friction force on the blocks should also be equal to $\frac{mg}{\sqrt{2}}$

$$\Rightarrow \quad f_A + f_B = F = \frac{mg}{\sqrt{2}}$$

Now since the blocks will start moving from block B first (if they move), therefore, f_B will reach its limiting value first and if still some force is needed, it will be provided by f_A.

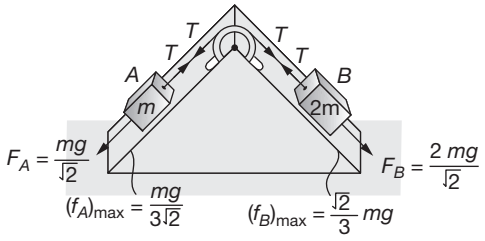
Here $(f_{max})_B < F$.

Therefore, f_B will be in its limiting value and rest will be provided by f_A.

Hence $f_B = (f_{max})_B = \frac{\sqrt{2}\,mg}{3}$

and $f_A = F - f_B = \frac{mg}{\sqrt{2}} - \frac{\sqrt{2}\,mg}{3} = \frac{mg}{3\sqrt{2}}$

The FBD of the whole system will be as shown in the figure.



Therefore, friction on A is

$$f_A = \frac{mg}{3\sqrt{2}} \qquad \text{(down the plane)}$$

Now for tension T in the string, we may consider either equilibrium of A or B.
Equilibrium of A gives

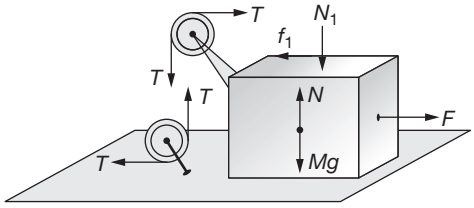
$$T = F_2 + f_A = \frac{mg}{\sqrt{2}} + \frac{mg}{3\sqrt{2}} = \frac{4mg}{3\sqrt{2}} \text{ or } \frac{2\sqrt{2}mg}{3}$$

Similarly, equilibrium of B gives $T + f_B = F_1$.

$$T = F_1 - f_B = \frac{2\,mg}{\sqrt{2}} - \frac{\sqrt{2}\,mg}{3} = \frac{4\,mg}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}\ \text{mg}$$

Therefore, tension in the string is $\frac{2\sqrt{2}}{3}$ mg.

19. Free body diagram of mass M is



Hence, the correct answer is (B).

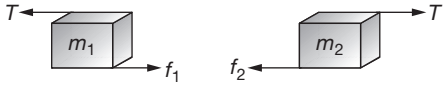
20. The maximum value of f_1 is

$$(f_1)_{max} = (0.3)(20)(10) = 60 \text{ N}$$

The maximum value of f_2 is

$$(f_2)_{max} = (0.3)(5)(10) = 15 \text{ N}$$

Forces on m_1 and m_2 in horizontal direction are as follows



Now there are only two possibilities

(1) Either both m_1 and m_2 will remain stationary (with respect to ground) or

(2) Both m_1 and m_2 will move (with respect to ground)

First case is possible when

$$T \le (f_1)_{max} \text{ i.e. } \quad T \le 60 \text{ N}$$

and $T \le (f_2)_{max}$ i.e. $\quad T \le 15$ N

These conditions will be satisfied when $T \le 15$ N

say $T = 14$ N then $f_1 = f_2 = 14$ N

Therefore, the condition $f_1 = 2f_2$ will not be satisfied.
So, m_1 and m_2 both can't remain stationary.
In the second case, when m_1 and m_2 both move

$$f_2 = (f_2)_{max} = 15 \text{ N}$$

$$\Rightarrow \quad f_1 = 2f_2 = 30 \text{ N}$$

Now since $f_1 < (f_2)_{max}$, there is no relative motion between m_1 and M. i.e., all the masses move with same acceleration, say a.

$$\Rightarrow \quad f_2 = 15 \text{ N}$$

and $f_1 = 30$ N

Hence, the correct answer is (D).

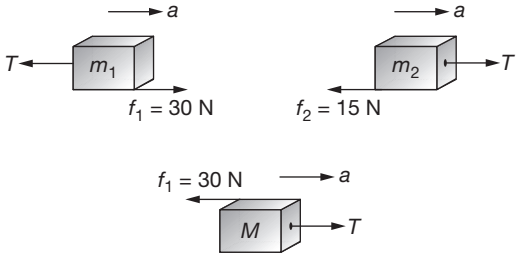
21. The correct answer is (A).

22. The correct answer is (D).

23. The correct answer is (C).

Combined solution to 21, 22, 23

Free body diagrams and equations of motion are as shown



For m_1, $30 - T = 20a$ …(1)

For m_2, $T - 15 = 5a$ …(2)

For M, $F - 30 = 50a$ …(3)

Solving these three equations, we get

$$F = 60 \text{ N}$$

$$T = 18 \text{ N and } a = \frac{3}{5} \text{ ms}^{-2}$$

24. $F_x = \frac{d}{dt}(p_x) = -2\sin t$

$$F_y = \frac{d}{dt}(p_y) = 2\cos t$$

$$\Rightarrow \quad F = \sqrt{F_x^2 + F_y^2}$$

$\Rightarrow \quad F = \sqrt{4\sin^2 t + 4\cos^2 t}$

$\Rightarrow \quad F = \sqrt{4\left(\sin^2 t + \cos^2 t\right)} = 2$ unit

Hence, the correct answer is (B).

25. $\vec{F} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$

$\vec{p} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$\vec{F}\cdot\vec{p} = \left(-2\sin t\,\hat{i} + 2\cos t\,\hat{j}\right)\cdot\left(2\cos t\hat{i} + 2\sin t\,\hat{j}\right)$

$\Rightarrow \quad \vec{F}\cdot\vec{p} = -4\sin t\cos t + 4\sin t\cos t = 0$

So, the angle between $\vec{F}$ and $\vec{p}$ is $90°$.

Hence, the correct answer is (D).

26. $p = \sqrt{p_x^2 + p_y^2}$

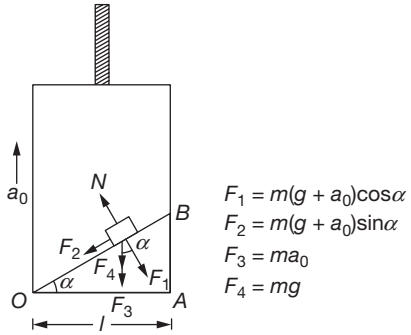
$p = \sqrt{4\cos^2 t + 4\sin^2 t} = 2$ units

Hence, the correct answer is (B).

27. $N = m\left(g + a_0\right)\cos\alpha$ and

$ma = m\left(g + a_0\right)\sin\alpha$

$\Rightarrow \quad a = \left(g + a_0\right)\sin\alpha$



$F_1 = m(g + a_0)\cos\alpha$
$F_2 = m(g + a_0)\sin\alpha$
$F_3 = ma_0$
$F_4 = mg$

Hence, the correct answer is (B).

29. Since $OB = \dfrac{l}{\cos\alpha} = \dfrac{1}{2}at^2$

$\Rightarrow \quad \dfrac{l}{\cos\alpha} = \dfrac{1}{2}\left(g + a_0\right)\sin\alpha t^2$

$\Rightarrow \quad T = \sqrt{\dfrac{2l}{\left(g + a_0\right)\sin\alpha\cos\alpha}}$

$\Rightarrow \quad T = \sqrt{\dfrac{4l}{\left(g + a_0\right)\sin 2\alpha}}$

Hence, the correct answer is (B).

31. The correct answer is (A).

32. The correct answer is (B).

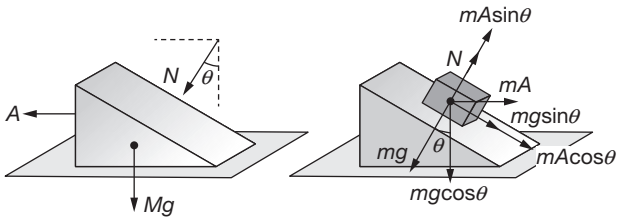
33. The correct answer is (B).

Combined solution to 31, 32, 33

$N\sin\theta = MA$ …(1)

$N + mA\sin\theta = mg\cos\theta$ …(2)

$mg\sin\theta + mA\cos\theta = ma$ …(3)



From (1) and (2)

$\dfrac{MA}{\sin\theta} + mA\sin\theta = mg\cos\theta$

$\Rightarrow \quad A\left(\dfrac{M}{\sin\theta} + m\sin\theta\right) = mg\cos\theta$

$\Rightarrow \quad A = \dfrac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}$

From (3)

$a = g\sin\theta + A\cos\theta$

$\Rightarrow \quad a = g\sin\theta + \dfrac{mg\sin\theta\cos^2\theta}{M + m\sin^2\theta}$

$\Rightarrow \quad a = \dfrac{g\sin\theta\left(M + m\sin^2\theta + m\cos^2\theta\right)}{M + m\sin^2\theta}$

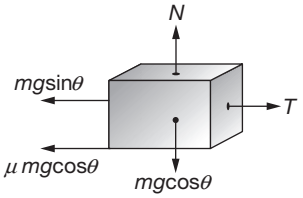
$\Rightarrow \quad a = \dfrac{(M + m)g\sin\theta}{M + m\sin^2\theta}$

$\Rightarrow \quad N = \dfrac{MA}{\sin\theta} = \dfrac{Mmg\cos\theta}{M + m\sin^2\theta}$

34. The correct answer is (B).

35. The correct answer is (D).

Combined solution to 34, 35



The block is moving with constant velocity, so

$T = mg\left(\sin\theta + \mu\cos\theta\right)$

Since, $P = Tv$

$\Rightarrow \quad 5\times10^3 = 250\times10\left(\frac{1}{2}+\mu\frac{\sqrt{3}}{2}\right)\times2$

$\Rightarrow \quad 5\times10^3 = 1250\left(1+\sqrt{3}\mu\right)\times2$

$\Rightarrow \quad 1+\sqrt{3}\,\mu = \frac{5000}{1250} = 4$

$\Rightarrow \quad \mu = \frac{1}{\sqrt{3}}$

$F_C = \sqrt{N^2+(\mu N)^2}$

$\Rightarrow \quad F_C = N\sqrt{1+\mu^2} = mg\cos\theta\sqrt{1+\mu^2}$

$\Rightarrow \quad F_C = 250\times10\times\frac{\sqrt{3}}{2}\sqrt{1+\frac{1}{3}}$

$\Rightarrow \quad F_C = 2500\text{ N}$

36. $T = mg(\sin\theta+\mu\cos\theta)$

$\Rightarrow \quad T = 2500\left(\frac{1}{2}+\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \quad T = 2500\text{ N}$

Hence, the correct answer is (C).

37. $f_k = \mu mg\cos\theta$

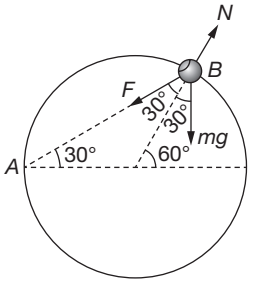
$\Rightarrow \quad f_k = \frac{1}{\sqrt{3}}\times2500\frac{\sqrt{3}}{2}$

$\Rightarrow \quad f_k = 1250\text{ N}$

Hence, the correct answer is (C).

38. Extension in the spring is

$x = AB - R = 2R\cos(30°) - R = \left(\sqrt{3}-1\right)R$



The spring force is given by

$F = kx = \frac{\left(\sqrt{3}+1\right)\text{ mg}}{R}\left(\sqrt{3}-1\right)R = 2\text{mg}$

Free body diagram of bead is shown here.

$N = (F+mg)\cos(30°)$

$\Rightarrow \quad N = (2mg+mg)\frac{\sqrt{3}}{2}$

$\Rightarrow \quad N = \frac{3\sqrt{3}mg}{2}$

Hence, the correct answer is (D).

39. Tangential force

$F_t = F\sin(30°) - mg\sin(30°)$

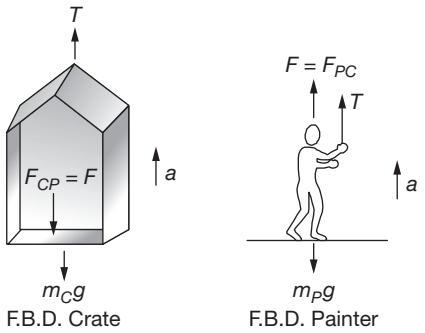
$\Rightarrow \quad F_t = (2mg-mg)\sin(30°) = \frac{mg}{2}$

So, tangential acceleration is given by

$a_t = \frac{F_t}{m} = \frac{g}{2}$

Hence, the correct answer is (A).

40.



$T + F - m_Pg = m_Pa$ (For Painter) …(1)

$T - F - m_Cg = m_Ca$ (For crate) …(2)

Subtract (2) from (1), we get

$2F + (m_C - m_P)g = (m_P - m_C)a$

$\Rightarrow \quad 900 + (25-100)(10) = 75a$

$\Rightarrow \quad 900 - 750 = 75a$

$\Rightarrow \quad a = 2\text{ ms}^{-2}$

Hence, the correct answer is (C).

41. Substituting in (2), we get

$T = 750\text{ N}$

Hence, the correct answer is (D).

42. $a = 0$

$\Rightarrow \quad T + F - Mg = 0$ {from (1)}

and $T - F - mg = 0$ [from (2)]

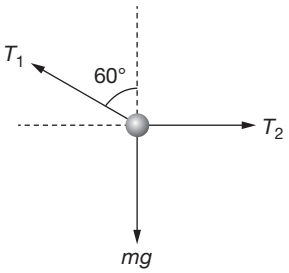
Subtracting, we get

$$2F = (M - m)g$$

$$\Rightarrow \quad F = \frac{1}{2}(M - m)g$$

Hence, the correct answer is (D).

43.



$$T_1 \cos(60°) = mg$$

$$\Rightarrow \quad T_1 = 2\text{mg}$$

Hence, the correct answer is (A).

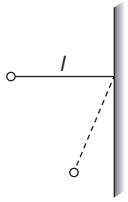
44. $T_1 \sin(60°) = T_2$

$$\Rightarrow \quad 2mg\frac{\sqrt{3}}{2} = T_2$$

$$\Rightarrow \quad T_2 = \sqrt{3}\,mg$$

Hence, the correct answer is (B).

45. Loss in PE = Gain in KE



$$\Rightarrow \quad mg\ell = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{2g\ell}$$

Hence, the correct answer is (B).

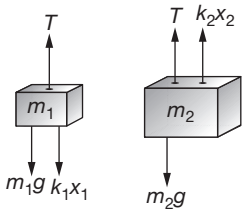
46. For equilibrium of m_1 and m_2, we have

$$T = m_1 g + k_1 x_1 \qquad \ldots(1)$$

$$T + k_2 x_2 = m_2 g \qquad \ldots(2)$$

From (1) and (2), we get

$$m_1 g + k_1 x_1 = m_2 g - k_2 x_2$$



$$\Rightarrow \quad k_1 x_1 = (m_2 - m_1)g - k_2 x_2$$

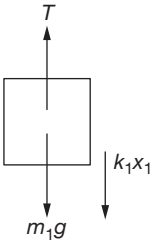
$$\Rightarrow \quad k_1 x_1 = (8 - 2)(10) - (70)(0.5)$$

$$\Rightarrow \quad k_1 x_1 = 25$$

$$\Rightarrow \quad x_1 = \frac{25}{50} = 0.5 \text{ m}$$

Hence, the correct answer is (C).

47. Since, $T = m_1 g + k_1 x_1$ $\{\because$ of (1)$\}$



$$\Rightarrow \quad T = 50 \times 0.5 + 20$$

$$\Rightarrow \quad T = 45 \text{ N}$$

Hence, the correct answer is (A).

49. No friction between M and m_0, so acceleration of m_0 is zero

Hence, the correct answer is (B).

50. $\mu(M + m_0)g = mg$

$$\Rightarrow \quad \mu = \frac{m}{M + m_0}$$

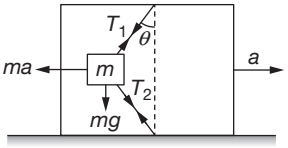
Hence, the correct answer is (B).

51. $$a = \frac{mg - \mu(M + m_0)g}{M + m}$$

$$a = \left[\frac{m - \mu(M + m_0)}{M + m}\right]g$$

Hence, the correct answer is (C).

52. For minimum force F, the lower string just remains tight i.e. $T_2 \to 0$.



$\Rightarrow \quad T_1\cos\theta = mg$ and $T_1\sin\theta = ma$

$\Rightarrow \quad a = g\tan\theta$

$\Rightarrow \quad F = (m+M)g\tan\theta$

Hence, the correct answer is (C).

53. When the lower string is just tight, then

$T_2 \to 0$

$\Rightarrow \quad T_1\cos\theta = mg$

$\Rightarrow \quad T_1 = \dfrac{mg}{\cos\theta}$

Hence, the correct answer is (B).

54. $T_1\cos\theta = mg + T_2\cos\theta$

$\Rightarrow \quad (T_1 - T_2)\cos\theta = mg$ …(1)

Also $(T_1 + T_2)\sin\theta = ma$ …(2)

From (1) and (2), we get

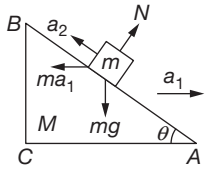
$$\left(\frac{T_1+T_2}{T_1-T_2}\right)\tan\theta = \frac{a}{g}$$

Since $T_1 = 2T_2$

$\Rightarrow \quad a = 3g\tan\theta$

Hence, the correct answer is (A).

55.



Let a_2 be acceleration of m w.r.t. M

Then, $ma_1\cos\theta - mg\sin\theta = ma_2$

$$\frac{a_1}{\sqrt{2}} - \frac{g}{\sqrt{2}} = a_2$$

$\Rightarrow \quad a_1 - g = a_2\sqrt{2}$ …(1)

Also, $N = ma_1\sin\theta + mg\cos\theta$ …(2)

and $F - N\sin\theta = Ma_1$ …(3)

Put (2) in (3), we get

$$F - ma_1\sin^2\theta - mg\sin\theta\cos\theta = Ma_1$$

$\Rightarrow \quad F - mg\sin\theta\cos\theta = (M + m\sin^2\theta)a_1$

$\Rightarrow \quad 210 - 20\left(\dfrac{1}{2}\right) = \left[9 + 2\left(\dfrac{1}{2}\right)\right]a_1$

$\Rightarrow \quad a_1 = 20 \text{ ms}^{-2}$

Hence, the correct answer is (C).

56. Since, $AB = l = \dfrac{1}{2}a_2t^2$, where

$$a_2 = \frac{a_1 - g}{\sqrt{2}} = \frac{20-10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ ms}^{-2}$$

$\Rightarrow \quad \dfrac{10}{\cos(45°)} = \dfrac{1}{2}\left(\dfrac{10}{\sqrt{2}}\right)t^2$

$\Rightarrow \quad 10\sqrt{2} = \dfrac{10}{2\sqrt{2}}t^2$

$\Rightarrow \quad t = 2 \text{ s}$

Since $v = u + at$

$\Rightarrow \quad v = a_2t = \left(\dfrac{10}{\sqrt{2}}\right)^2 = \dfrac{20}{\sqrt{2}} = 10\sqrt{2} \text{ ms}^{-1}$

Hence, the correct answer is (C).

57. $l' = \dfrac{1}{2}a_1t^2$

$\Rightarrow \quad l' = \dfrac{1}{2}(20)(4) = 40 \text{ m}$

Hence, the correct answer is (D).

58. $F\sin\theta + f_2 = mg$

$\Rightarrow \quad F\sin\theta + \mu F\cos\theta = mg$

$\Rightarrow \quad F = \dfrac{mg}{\sin\theta + \mu\cos\theta}$

Hence, the correct answer is (B).

59. $F\sin\theta = mg + f_2$

$\Rightarrow \quad F\sin\theta - \mu F\cos\theta = mg$

$\Rightarrow \quad F = \dfrac{mg}{\sin\theta - \mu\sin\theta}$

Hence, the correct answer is (C).

60. $F\sin\theta = mg$

Hence, the correct answer is (B).

61. Since there is no force on upper block of mass 10 kg and no tendency of slipping over 20 kg block therefore friction on it will be zero.

Hence, the correct answer is (A).

62. Maximum acceleration or retardation upto which both blocks will move together is

$$a_{max} = \mu_1 g = 5 \text{ ms}^{-2}$$

$$\Rightarrow \quad \mu_2 (m_1 + m_2) g = (m_1 + m_2) a$$

$$\Rightarrow \quad \mu_2 = 0.5$$

Also kinetic friction on 20 kg is

$$f_k = \mu_2 (m_1 + m_2) g = 150 \text{ N}$$

Hence, the correct answer is (A).

63. Since, retardation is 5 ms^{-2}, so applying

$v^2 = u^2 - 2as$, we get

$$0^2 = 400 - 2 \times 5 \times s$$

$$\Rightarrow \quad s = 100 \text{ m}$$

Hence, the correct answer is (B).

64. $\omega = at^2$

Linear speed $v = \omega r = 1 \times 2 \times t^2 = (2 \text{ ms}^{-3})(t^2)$

Since, $a_T = \dfrac{dv}{dt} = \dfrac{d(2t^2)}{dt} = 4t$ and

$$a_C = \frac{v^2}{r} = \omega^2 r$$

Net acceleration is

$$a = a_{net} = \sqrt{a_T^2 + a_C^2}$$

Force $= ma = m\sqrt{a_T^2 + a_C^2}$

Hence, the correct answer is (B).

66. Since rate of change of speed is tangential acceleration, so we have

$$a_T = \frac{a}{\sqrt{2}}$$

$$\Rightarrow \quad a_T^2 + a_C^2 = a^2$$

$$\Rightarrow \quad \frac{a^2}{2} + a_C^2 = a^2$$

$$\Rightarrow \quad a_C = \frac{a}{\sqrt{2}}$$

$$\Rightarrow \quad a_C = a_T = \frac{a}{\sqrt{2}}$$

$$\Rightarrow \quad 4t = 4t^3$$

$$\Rightarrow \quad t = 1 \text{ s}$$

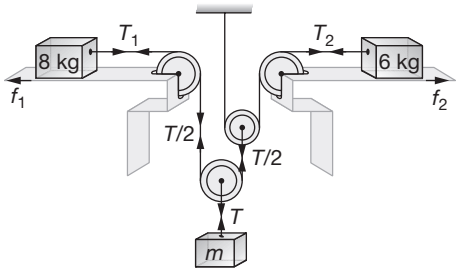
Hence, the correct answer is (B).

67. **The correct answer is (B).**

68. **The correct answer is (B).**

69. **The correct answer is (C).**

Combined solution to 67, 68, 69



For equilibrium of

8 kg block, $T_1 \le 40$ N

6 kg block, $T_2 \le 24$ N

Also, $T_1 = \dfrac{T}{2}$ and $T_2 = \dfrac{T}{4}$

So, for equilibrium of

8 kg block, we have $T \le 80$ N and

6 kg block, we have $T \le 96$ N

However, system will be equilibrium if all the three blocks are in equilibrium. Hence

$$T \le 80 \text{ N}$$

$$\Rightarrow \quad m \le 8 \text{ kg}$$

The 6 kg block will start moving if

$$T_2 > 96 \text{ N}$$

$$\Rightarrow \quad m > 9.6 \text{ kg}$$

When 9 kg block is hanged, 6 kg block remains at rest, but 8 kg block moves. Let acceleration of 9 kg block be a, then acceleration of 8 kg block will be $2a$. For 9 kg block, we have

$$90 - T = 9a \qquad \ldots(1)$$

For 8 kg block, we have

$$\frac{T}{2} - f = 8(2a)$$

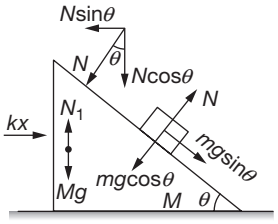
$$\Rightarrow \quad \frac{T}{2} - 40 = 8(2a) \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{90 - 9a}{2} = 16a + 40$$

$$\Rightarrow \quad a = \frac{10}{41} \text{ ms}^{-2}$$

70.



$N\sin\theta = kx$

$N = mg\cos\theta$

$\Rightarrow \quad x = \dfrac{N\sin\theta}{k} = \dfrac{mg\sin\theta\cos\theta}{k} = \dfrac{mg\sin(2\theta)}{2k}$

Hence, the correct answer is (C).

71. $N\cos\theta + Mg = N_1$

$\Rightarrow \quad N_1 = (mg\cos\theta)\cos\theta + Mg$

$\Rightarrow \quad N_1 = mg\cos^2\theta + Mg$

$\Rightarrow \quad N_1 = (m\cos^2\theta + M)g$

Hence, the correct answer is (D).

72. Net unbalanced force on the block is $mg\sin\theta$.

Hence, the correct answer is (A).

73. Friction force will be static in nature. So

$f = 30$ N

Hence, the correct answer is (C).

74. Slipping will start if acceleration a of system is

$a > \dfrac{0.3\times 25\times 10}{5}$

$\Rightarrow \quad \left(\dfrac{M}{M+30}\right)g > \dfrac{0.3\times 25\times 10}{5}$

Hence, the correct answer is (D).

75. $a = \left(\dfrac{M}{M+30}\right)g = 6\ \text{ms}^{-2}$

Hence, the correct answer is (A).

76. $v = kt_1$

Tangential acceleration is

$a_T = \dfrac{dv}{dt} = k$

Centripetal acceleration is

$a_C = \dfrac{v^2}{r}$

$\Rightarrow \quad \dfrac{v^2}{r} = k$

$\Rightarrow \quad \dfrac{k^2t_1^2}{r} = k$

$\Rightarrow \quad t_1 = \sqrt{\dfrac{r}{k}}$

Hence, the correct answer is (C).

77. Net force $F = \sqrt{mk^2 + mk^2} = \sqrt{2}mk$

Hence, the correct answer is (B).

78. $\tan\theta = \dfrac{a_T}{a_C} = \dfrac{K}{\left(\dfrac{v^2}{r}\right)} = \dfrac{K}{\left(\dfrac{a_T^2t^2}{r}\right)}$

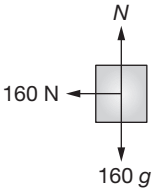
$\Rightarrow \quad \tan\theta = \dfrac{K}{\left(\dfrac{4}{3}\right)\dfrac{K^2t_1^2}{r}} = \dfrac{3}{4}$

$\Rightarrow \quad \tan\theta = \dfrac{3}{4}$

$\Rightarrow \quad \theta = 37°$

Hence, the correct answer is (B).

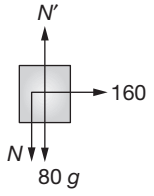
79. **Free body diagram of block**



$\Rightarrow \quad a_{\text{block}} = \dfrac{160}{160} = 1\ \text{ms}^{-2}$ {leftwards}

Free body diagram (platform + mass) system

Assuming no friction between platform and surface, we get acceleration of pulley to be equal to acceleration of platform. So



$a_{\text{pulley}} = \dfrac{160}{80} = 2\ \text{ms}^{-2}$ {rightwards}

Relative acceleration of pulley and block is

$(2+1) = 3\ \text{ms}^{-2}$

So, time taken by block to reach pulley is

$$l = \frac{1}{2} a_{\text{rel}} t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2 \times 6}{3}} = 2 \text{ s}$$

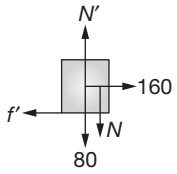
Hence, the correct answer is (B).

80. Length of string pulled out in 1 s is four times the distance moved by block relative to pulley. So

$$l = 4 \times 1.5 = 6 \text{ m}$$

Hence, the correct answer is (B).

81. Assuming friction between platform and surface,



$$f' = 160$$

$$\Rightarrow \quad \mu N' \geq 160$$

$$\Rightarrow \quad \mu \geq \frac{160}{2400} = \frac{2}{30} = 0.066$$

Hence, the correct answer is (D).

83. $mg = 5v$

$$\Rightarrow \quad 50 = 5v$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

84. At any, instant, $a = g - kv$, where k is constant

$$\Rightarrow \quad \frac{dv}{dt} = g - kv$$

$$\Rightarrow \quad \int_0^v \frac{dv}{g - kv} = \int_0^v dt$$

$$\Rightarrow \quad \frac{\left[ln(g - kv) \right]_0^v}{-k} = t$$

$$\Rightarrow \quad \ln(g - kv) - \ln(g) = -kt$$

$$\Rightarrow \quad \ln\left[1 - \frac{kv}{g} \right] = -kt$$

$$\Rightarrow \quad 1 - \frac{kv}{g} = e^{-kt}$$

$$\Rightarrow \quad v = \frac{g}{k}(1 - e^{-kt})$$

Hence, the correct answer is (B).

85. $$a = \frac{m_3 g + (m_1 + m_2) g \sin\theta - F}{m_1 + m_2 + m_3}$$

Hence, the correct answer is (A).

86. Elongation in string is maximum when relative velocity of 1 w.r.t. 2 is zero.

Hence, the correct answer is (B).

87. $$a_3 = \frac{m_3 g + m_2 g \sin\theta - k x_{\max}}{m_2 + m_3}$$

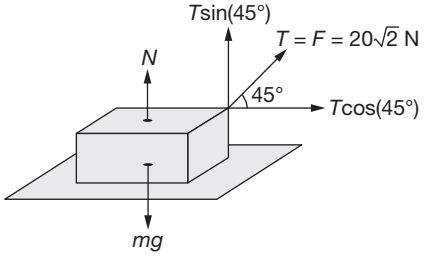
Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (q)
B → (r)
C → (s)
D → (p)

$N + T\sin(45°) = mg$, where $T = 20\sqrt{2}$ N

$$\Rightarrow \quad N = 40 - 20 = 20 \text{ N}$$



So, $f_s = \mu_s N = 16$ N and $f_k = \mu_k N = 12$ N

Since $T\cos(45°) = 20$ N which happens to be greater than f_s. So, the block will move and hence we have

$$T\cos(45°) - f_k = ma$$

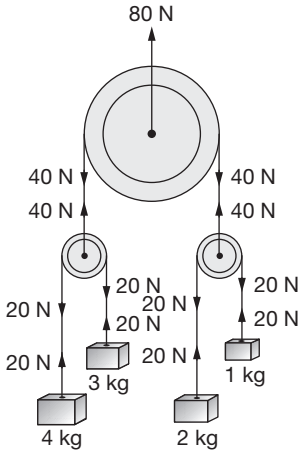
$$\Rightarrow \quad 20 - 12 = 4a$$

$$\Rightarrow \quad a = 2 \text{ ms}^{-2} = 200 \text{ cms}^{-2}$$

2. A → (p, u)
B → (t)
C → (r, s)
D → (q, r)

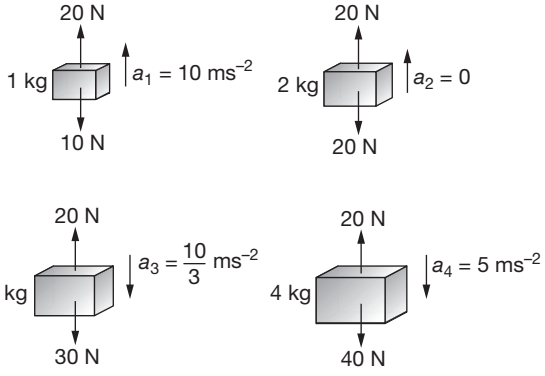
Since the pulleys are smooth and massless, so net force on each pulley (even when accelerating) is zero (because mass of each pulley tends to be zero).

Let acceleration of 1 kg, 2 kg, 3 kg and 4 kg blocks be a_1, a_2, a_3 and a_4 respectively. Then drawing the FBD's of all we get the values of a_1, a_2, a_3 and a_4.



So, $a_1 = 10\ \text{ms}^{-2}$ (up), $a_2 = 0$,

$a_3 = \frac{10}{3}\ \text{ms}^{-2}$ (down), $a_4 = 5\ \text{ms}^{-2}$ (down)



3. A → (r)
B → (s)
C → (t)
D → (q)

If a be the acceleration of the system, then

$$a = \frac{60 - 18 - (1+2+3)g\sin(30°)}{1+2+3}$$

$\Rightarrow \quad a = 2\ \text{ms}^{-2}$

Net force on the 3 kg block is $F = m_3 a = 6$ N

So, (A) → (r) and (B) → (s)

Let N_{21} be the force on 2 due to 1 and N_{32} be the force on 3 due to 2.

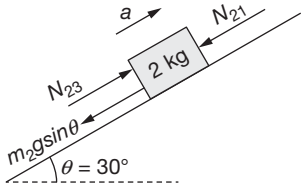
$$N_{23} - N_{21} - m_2 g\sin\theta = m_2 a \qquad \ldots(1)$$

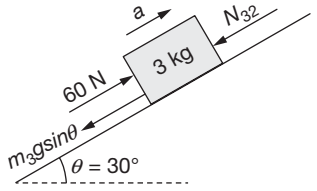
$$60 - N_{32} - m_3 g\sin\theta = m_3 a \qquad \ldots(2)$$

Since $a = 2\ \text{ms}^{-2}$, so from (2), we get

$$60 - N_{32} - 30\left(\frac{1}{2}\right) = 3(2)$$

$\Rightarrow \quad N_{32} = 60 - 15 - 6 = 39\ \text{N} = N_{23}$





Substituting the value of N_{32} in equation (1), we get

$$39 - N_{21} - (2)(10)\left(\frac{1}{2}\right) = (2)(2)$$

$\Rightarrow \quad N_{21} = 25$ N

4. A → (t)
B → (r)
C → (p)
D → (s)
E → (q)

Pseudo force comes into play, when the frame of reference (from where the phenomenon is being observed) is accelerating. This pseudo force is then directed opposite to the acceleration of the observation frame. So for the observation frames A, B, C, D and E, respective values of accelerations are

$\vec{a}_A = 8\hat{i}$, $\vec{a}_B = \vec{0}$

$\vec{a}_C = -4\hat{j}$, $\vec{a}_D = -3\hat{i}$ and

$\vec{a}_E = 5\hat{j}$

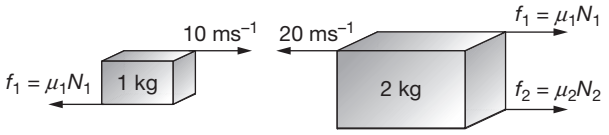
Hence, the pseudo force on A as observed by B is zero. The pseudo force on B as observed by C is along $+y$ direction.

The pseudo force on C as observed by D is along $+x$ direction.

The pseudo force on D as observed by E is along $-y$ direction.

The pseudo force on E as observed by A is along $-x$ direction.

5. A → (q)
B → (s)
C → (p, t)
D → (r)



$f_1 = \mu_1 N_1 = (0.4)(1)(10) = 4$ N

$f_2 = \mu_2 N_2 = (0.6)(1+2)(10) = 18$ N

So, net force acting on 1 is 4 N, hence acceleration of 1 is $a_1 = \frac{f_1}{1} = 4$ ms^{-2}, towards left.

Similarly, net force acting on 2 is $f_1 + f_2 = 22$ N.

So, acceleration of 2 is

$$a_2 = \frac{f_1 + f_2}{2} = 11 \text{ ms}^{-2}, \text{ towards right}$$

Finally, relative acceleration of one w.r.t. other is 15 ms^{-2}.

6. A → (r, s)
B → (p, s)
C → (p, s)
D → (p, r, s)

For an object moving along $+x$ axis with constant velocity, there may be no net force acting on the object or there may not be any force acting along $+x$ axis. So, (A) → (r, s).

For both the accelerated motions expressed in (B) and (C), we have either the net force on the object to act along $+x$ axis or also no net force along $+x$ axis such that components of force along $+x$ axis happen to be zero. Hence (B) → (p, s) and (C) → (p, s).

Finally, for the motion of object along $+x$ axis (accelerated or non-uniform) all (p, r, s) satisfy the condition. So (D) → (p, r, s).

7. A → (q, r)
B → (p, s)
C → (p, s)
D → (q, r)

As per the question, we are given that

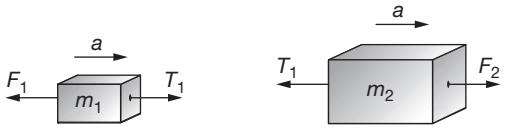
$$a_1 = \frac{F_1}{m_1} \text{ and } a_2 = \frac{F_2}{m_2}$$

For CASE-1 and 4:

If a be the combined acceleration of the system, then

$T_1 - F_1 = m_1 a$ and $F_2 - T_1 = m_2 a$

$$\Rightarrow \quad a = \frac{F_2 - F_1}{m_1 + m_2}$$



Substituting in either of the equations, we get

$$T_1 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_1 + a_2)$$

So, (A) → (q, r)

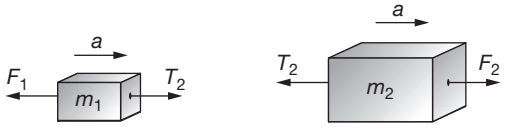
Also,

$$N_2 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_1 + a_2)$$

So, (D) → (q, r)

For CASE-2 and 3:

If a be the combined acceleration of the system, then



$F_1 + T_2 = m_1 a$ and $F_2 - T_2 = m_2 a$

$$\Rightarrow \quad a = \frac{F_1 + F_2}{m_1 + m_2}$$

Substituting in either of the equations, we get

$$T_2 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_2 - a_1)$$

So, (C) → (p, s)

Also,

$$N_1 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_2 - a_1)$$

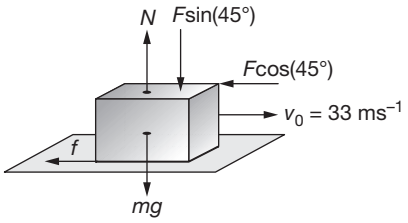
So, (B) → (p, s)

8. A → (r)
B → (t)
C → (q)
D → (p)

$$N = mg + F\sin(45°)$$

$$\Rightarrow \quad N = 50 + (20\sqrt{2})\frac{1}{\sqrt{2}} = 70 \text{ N}$$

$$\Rightarrow \quad f_{\max} = \mu N = (0.5)(70) = 35 \text{ N}$$



So, retardation offered to the block is given by

$$a = \frac{F\cos(45°) + f}{m} = \frac{35 + 20}{5} = 11 \text{ ms}^{-2}$$

Since $v = u + at$, so the block attains zero velocity, say at time t_0, we have

$$0 = 33 - 11t_0$$

$$\Rightarrow \quad t_0 = 3 \text{ s}$$

Till $t < t_0 (= 3 \text{ s})$, we have the following two facts

(a) the force of friction acting on the block is $f_{\max} = \mu N_1 = 35$ N

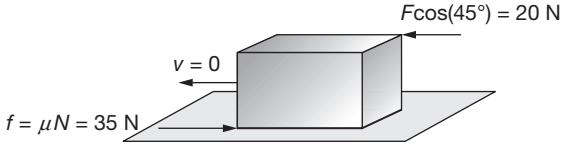
(b) and the net retarding force acting on the block is

$$(F_{\text{net}})_{\text{retarding}} = F\cos(45°) + f_{\max} = 55 \text{ N}$$

So, we have (A) → (r) and (C) → (q)

Now, for $t = 4.5$ s, i.e., $t > t_0$, since the block has zero velocity at $t_0 = 3$ s. At this very instant the friction will be opposite to the applied force and happens to be more than the applied force, so the block will remain at rest after $t_0 = 3$ s and in the **Static Region**, we have the force of static friction to self adjust to a value equal to the applied force, so for $t > t_0$, i.e., $t = 4.5$ s, we have

$$f_{\text{static}} = F_{\text{applied}} = F\cos(45°) = 20 \text{ N}$$



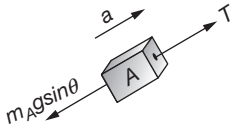
Hence (B) → (t)

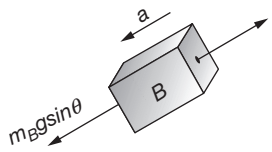
Also, at $t = 4.5$ s, the net force on the block is zero as it happens to be in static mode. Hence (D) → (p)

9. A → (r)
B → (t)
C → (p)
D → (q)

For A, $T - m_A g\sin\theta = m_A a$ …(1)

For B, $m_B g\sin\theta - T = m_B a$ …(2)





Add (1) and (2), we get

$$(m_B - m_A)g\sin\theta = (m_A + m_B)a$$

$$\Rightarrow \quad a = \left(\frac{m_B - m_A}{m_A + m_B}\right)g\sin\theta$$

$$\Rightarrow \quad a = \left(\frac{4-1}{4+1}\right)(10)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad a = 3 \text{ ms}^{-2}$$

So, (A) → (r)

Substitute the value of a in (1), we get

$$T - (10)\left(\frac{1}{2}\right) = (1)(3)$$

$$\Rightarrow \quad T = 8 \text{ N}$$

So, Thrust $= 2T = 16$ N, i.e., (B) → (t)

Force exerted by A on B is $N_A = m_A g\cos\theta = 5\sqrt{3}$ N and force exerted by B on incline is $N_B = (m_A + m_B)g\cos\theta = 25\sqrt{3}$ N

Hence (C) → (p) and (D) → (q)

10. A → (s)
B → (p)
C → (t)
D → (q)

For 10 kg block, $10g - T = 10a$ …(1)

For A, $f = \mu N$ and $N = 4a$ …(2)

$$\Rightarrow \quad f = (0.5)(4)a$$

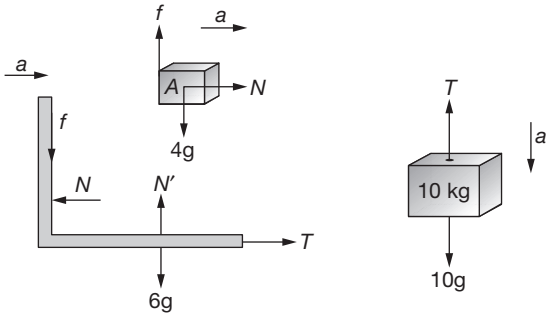
$$\Rightarrow \quad f = 2a \quad \text{…(3)}$$

For L shaped block B,

$$T - N = 6a \qquad \ldots(4)$$

From (2) and (4), we get

$$T = 10a \qquad \ldots(5)$$



Substituting in (1), we get

$$100 - 10a = 10a$$

$$\Rightarrow \quad a = 5 \text{ ms}^{-2}$$

$$\Rightarrow \quad f = 10 \text{ N}$$

$\Rightarrow \quad N = 20$ N, i.e., normal reaction between A and B. Also, from diagram, we see the normal reaction on the block B by the table is N' given by

$$N' = 6g + f$$

$$\Rightarrow \quad N' = 60 + 10 = 70 \text{ N}$$

and finally from (5), we get

$$T = 50 \text{ N}$$

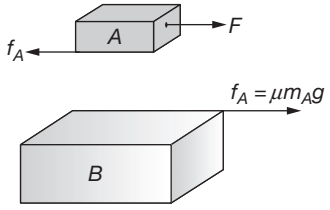
Please note that, the block A will have both the horizontal and vertical accelerations however the asked questions have nothing to do with the vertical acceleration of A, which happens to be

$$a_y = \frac{m_A g - f}{m_A} = \frac{40 - 10}{4} = 7.5 \text{ ms}^{-2}$$

11. A → (t, u)
B → (p, u)
C → (q, u)
D → (s, u)

Since $f_A = \mu m_A g = (0.5)(10)(10) = 50$ N

So, friction on B due to A is 50 N, towards right
Hence (A) → (t, u)



Now, when $F = 35 \text{ N} < f_A$, so no slipping of A on B and since B lies on a frictionless surface, hence both A and B move together with a common acceleration a (say). Then

$$a = \frac{F}{m_A + m_B} = \frac{35}{35} = 1 \text{ ms}^{-2}, \text{ towards right}$$

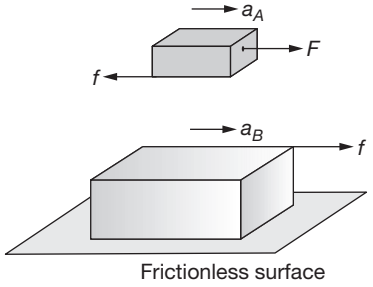
So, we get (B) → (p, u)
Now, when $F = 250 \text{ N} > 50 \text{ N}$, then for block A

$$F - \mu m_A g = m_A a_A$$

$$\Rightarrow \quad 250 - 50 = (10)a_A$$

$$\Rightarrow \quad a_A = 20 \text{ ms}^{-2}, \text{ towards right}$$

So, (C) → (q, u)



Also, for B, we get

$$f = m_B a_B$$

$$\Rightarrow \quad 50 = 25a_B$$

$$\Rightarrow \quad a_B = 2 \text{ ms}^{-2}, \text{ towards right}$$

Hence, finally (D) → (s, u)

12. A → (p, q, r)
B → (p, q, r)
C → (p, q)
D → (s)
The directions of $\vec{F}$ and $\vec{p}$ may be different, so $\vec{p}$ may change its direction.
If direction of $\vec{F}$ and $\vec{p}$ are the same, then direction of $\vec{p}$ would not change.

If $\vec{F}$ is non-zero in magnitude then magnitude of $\vec{p}$ will change.

13. A → (s)
B → (r)
C → (p)
D → (q)

Since $N = m(g + a_y) = 20(10 + 10) = 400$ N

and $F_{\text{pseudo}} = m\sqrt{(3t)^2 + (4t)^2} = 100t$

$\Rightarrow \quad f_\ell = \mu N = 0.5 \times 400 = 200$ N

Slipping starts when, $100t = 200$

$\Rightarrow \quad t = 2$ s

So, $\vec{a}$ at $t = 2$ s is $\vec{a} = \left(6\hat{i} + 10\hat{j} + 8\hat{k}\right)$ ms^{-2}

$\Rightarrow \quad |\vec{a}| = 10\sqrt{2}$ ms^{-2}

Total force F applied by the surface on the block at $t = 1$ s is $\sqrt{N^2 + f_l^2}$ (also called as Contact Force (C.F.)

$\Rightarrow$ Contact Force $= \sqrt{(400)^2 + (100)^2}$

$\Rightarrow$ C.F. $= 100\sqrt{17}$ N

15. A → (r)
B → (p)
C → (q)
D → (s)

$a_1 \cos(37°) = a_2$

$\Rightarrow \quad 4a_1 = 5a_2 \qquad \ldots(1)$

Since, $100 - T = 10a_2 \qquad \ldots(2)$

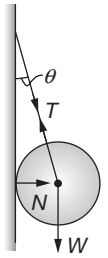
and $T\cos(37°) = 5a_1$

$\Rightarrow \quad T = \dfrac{25}{4}a_1 \qquad \ldots(3)$

Force on pulley by the string is

$$F = \sqrt{T^2 + T^2 + 2T^2\cos(53°)}$$

16. A → (p)
B → (r)
C → (q)
D → (s)



Since the system is in equilibrium, so

$$\vec{T} + \vec{W} + \vec{N} = 0$$

$\Rightarrow \quad \vec{T} = -\vec{W} - \vec{N}$

Also, $T\cos\theta = W$ and $T\sin\theta = N$

$\Rightarrow \quad T^2 = N^2 + W^2$ and $N = W\tan\theta$

17. A → (q, s)
B → (p, s)
C → (r)
D → (r)

$(f_1)_{\max} = (f_1)_{\lim} = (0.8)(50) = 40$ N

$(f_2)_{\max} = (f_2)_{\lim} = (0.2)(150) = 30$ N

$F_1 = 35$ N, $F_2 = 0$: Block A will not slip over block B, because $F_{\text{app}} < (f_1)_{\lim}$

$F_1 = 50$ N, $F_2 = 0$: Block B will slip over horizontal surface, because $F_{\text{app}} > (f_1)_{\lim}$

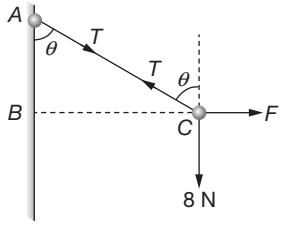
$F_1 = 0$, $F_2 = 30$ N : Block B will be at rest, because $F_{\text{app}} = (f_2)_{\lim}$

$F_1 = 10$ N, $F_2 = 50$ N : Block A have no tendency of slipping over block B.

Integer/Numerical Answer Type Questions

1. $AC = 0.5$ m, $BC = 0.3$ m

$\Rightarrow \quad AB = 0.4$ m



Let $\angle BAC = \theta$. Then

$$\cos\theta = \frac{AB}{AC} = \frac{0.4}{0.5} = \frac{4}{5} \text{ and } \sin\theta = \frac{BC}{AC} = \frac{0.3}{0.5} = \frac{3}{5}$$

We observe that the object is in equilibrium under three concurrent forces. So, applying Lami's theorem, we get

$$\frac{F}{\sin(180^\circ - \theta)} = \frac{8}{\sin(90^\circ + \theta)} = \frac{T}{\sin 90^\circ}$$

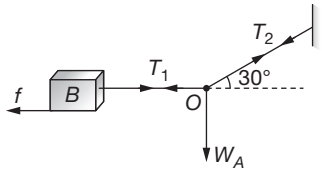
$$\Rightarrow \quad \frac{F}{\sin\theta} = \frac{8}{\cos\theta} = T$$

$$\Rightarrow \quad T = \frac{8}{\cos\theta} = \frac{8}{4/5} = 10 \text{ N} \text{ and}$$

$$F = \frac{8\sin\theta}{\cos\theta} = \frac{(8)\left(\frac{3}{5}\right)}{\left(\frac{4}{5}\right)} = 6 \text{ N}$$

2. Equilibrium of B gives,

$$T_1 = f_1 = \mu W_B = 0.25 \times 692 = 173 \text{ N} \qquad \ldots(1)$$



Equation of knot (Point O) gives,

$$T_1\cos(30^\circ) = T_1 \text{ and} \qquad \ldots(2)$$

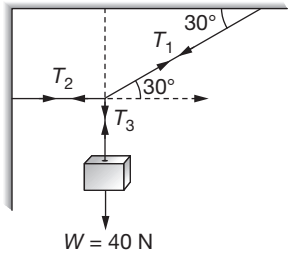
$$T_2\sin(30^\circ) = W_A \qquad \ldots(3)$$

From equations (2) and (3), we get

$$W_A = T_1\tan(30^\circ) = \frac{173}{\sqrt{3}} = \frac{173}{1.73} = 100 \text{ N}$$

$$\Rightarrow \quad W_A = 100 \text{ N} = 10 \text{ kgwt}$$

3. Resolving the tension T_1 along horizontal and vertical directions. As the body is in equilibrium,



$$T_1\sin(30^\circ) = T_3 = W = 40 \text{ N} \qquad \ldots(1)$$

$$\Rightarrow \quad T_3 = 40 \text{ N} \text{ and } T_1 = 80 \text{ N}$$

Also, $T_1\cos(30^\circ) = T_2 \qquad \ldots(2)$

$$\Rightarrow \quad T_2 = 80\cos(30^\circ)$$

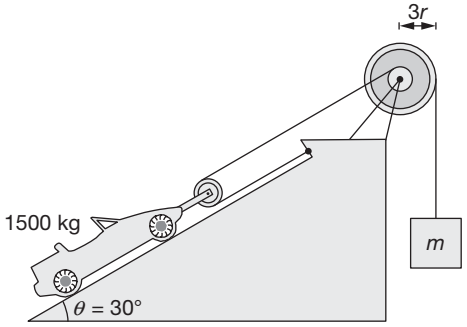
$$\Rightarrow \quad T_2 = 80\frac{\sqrt{3}}{2} = 40\sqrt{3} \text{ N}$$

$$\Rightarrow \quad T_2 = 40(1.7) = 68 \text{ N}$$

So, $T_1 = 80$ N , $T_2 = 68$ N and $T_3 = 40$ N

4. $\sum \tau = 0 = mg(3r) - Tr$

$$2T - Mg\sin(30^\circ) = 0$$



$$\Rightarrow \quad T = \frac{Mg\sin(30^\circ)}{2}$$

$$\Rightarrow \quad T = \frac{(1500)(10)}{2}\left(\frac{1}{2}\right) = 3750 \text{ N}$$

$$m = \frac{T}{3g} = \frac{3750}{30} = 125 \text{ kg}$$

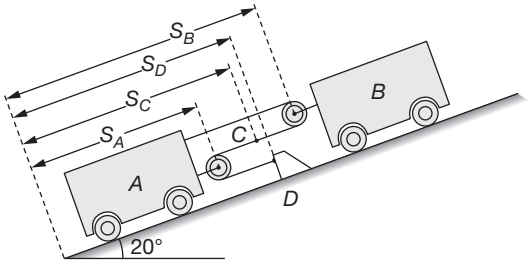
5. The cable length is

$$L = 2(S_B - S_A) + S_D - S_A + \text{constants}$$

Differentiating,

$$0 = 2v_B - 3v_A$$

$$0 = 2a_B - 3a_A$$



So, $v_A = \frac{2}{3}v_B = \frac{2}{3}(3) = 2$ ftsec^{-1}

$a_A = \frac{2}{3}a_B = \frac{2}{3}(6) = 4$ ftsec^{-2}

$v_{B/A} = v_B - v_A = 3 - 2 = 1$ ftsec^{-1}

$a_{B/A} = a_B - a_A = 6 - 4 = 2$ ftsec^{-2}

The length of cable between A and C is

$L' = (S_B - S_A) + (S_B - S_C) = 2S_B - S_A - S_C + \text{constants}$

$\Rightarrow \quad 0 = 2v_B - v_A - v_C$

$v_C = 2v_B - v_A = 2(3) - 2 = 4$ ftsec^{-1}

(All answers are quantities directed up incline).

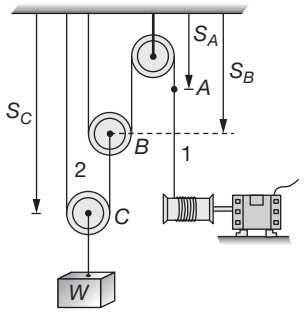
6. Let A be a point on cable 1

$L_1 = S_A + 2S_B$

$0 = v_A + 2v_B$ …(1)

$L_2 = S_C + (S_C - S_B)$

$0 = 2v_C - v_B$ …(2)

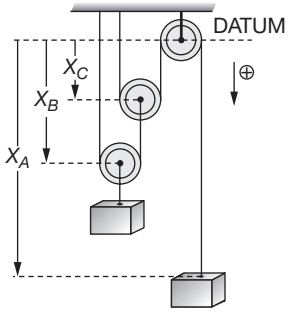


Combine (1) and (2) to obtain

$v_C = -\frac{1}{4}v_A = -\frac{1}{4}(320) = -80$ mms^{-1}

So, W rises by $h = 80(5) = 400$ mm

7. Let the datum be passing through the fixed pulley as shown



The length of the two ropes in terms of the position coordinates x_A, x_B and x_C are

$x_A + 2x_C = l_1$ …(1)

and $x_B + (x_B - x_C) = l_2$ …(2)

Eliminating s_C from equations (1) and (2),

$4x_B + x_A = 2l_2 + l_1$ …(3)

Taking the time derivative of equation (3), we get

$4v_B + v_A = 0$

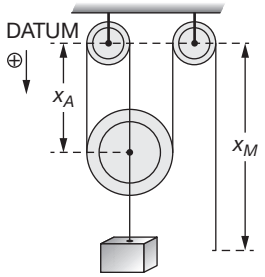
Since $v_A = 4$ ms^{-1}, so we get

$4v_B + 4 = 0$

$\Rightarrow \quad v_B = -1$ ms^{-1} $= 1$ ms^{-1}, upwards

$\Rightarrow \quad v_B = 100$ cms^{-1}, upwards

8. Let the datum be passing through both the fixed pulleys as shown.



The length of the rope in terms of the position coordinates x_A and x_M is

$3x_A + x_M = l$ …(1)

Taking the time derivative of equation (1), we get

$3v_A + v_M = 0$

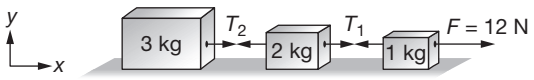
Taking downward direction as positive, so we have

$v_M = 10$ ms^{-1}

$\Rightarrow \quad 3v_A + 9 = 0$

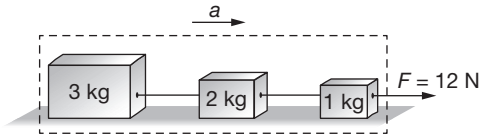
$\Rightarrow \quad v_A = -3$ ms^{-1} $= 3$ ms^{-1}, upwards.

9. (a) Let a be the acceleration of each block and T_1 and T_2 be the tensions, in the two strings as shown in figure.



Taking the three blocks and the two strings as the system.

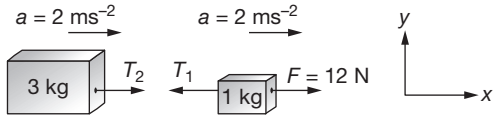
Using $\Sigma F_x = ma_x$



$\Rightarrow \quad 12 = (3+2+1)a$

$\Rightarrow \quad a = \dfrac{12}{6} = 2 \text{ ms}^{-2}$

(b) Free body diagram (showing the forces in x-direction only) of 3 kg block and 1 kg block are shown in figure.



Using $\Sigma F_x = ma_x$

For 1 kg block, $F - T_1 = (1)(a)$

$\Rightarrow \quad 12 - T_1 = (1)(2) = 2$

$\Rightarrow \quad T_1 = 12 - 2 = 10 \text{ N}$

For 3 kg block,

$T_2 = (3)(a)$

$\Rightarrow \quad T_2 = (3)(2) = 6 \text{ N}$

10. Let a be the acceleration with which the masses move and T_1 and T_2 be the tensions in left and right strings. Friction on mass A is $\mu mg = 8$ N. Then equations of motion of masses A, B and C are

For mass *A*

$T_1 - 8 = 4a \qquad \ldots(1)$

For mass *B*

$T_2 = 8a \qquad \ldots(2)$

For mass *C*

$200 - T_1 - T_2 = 20a \qquad \ldots(3)$

Adding the above three equations, we get

$32a = 192$

$\Rightarrow \quad a = 6 \text{ ms}^{-2}$

From equations (1) and (2), we have

$T_2 = 48 \text{ N}$ and $T_1 = 32 \text{ N}$

11. When force F is applied at the end of the string, the tension in the lower part of the string is also F as shown. If T is the tension in string connecting the pulley and the block, then,

$T = 2F$

Also, $T = ma = (200)(1) = 200 \text{ N}$

$\Rightarrow \quad 2F = 200 \text{ N}$

$\Rightarrow \quad F = 100 \text{ N}$

12. Free body diagram of crate A w.r.t. ground is shown in figure.

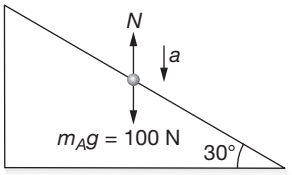
Equation of motion is given by

$100 - N = 10a_A \qquad \ldots(1)$

Further, we have $a_A \operatorname{cosec} 30° =$ acceleration of crate

$\Rightarrow \quad 2a_A = 2$

$\Rightarrow \quad a_A = 1 \text{ ms}^{-2}$



Substituting in equation (1) we get

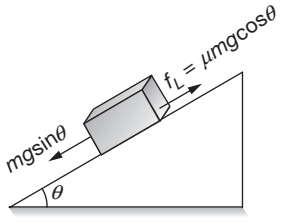
$N = 90 \text{ N}$

13. Let θ be the required angle, then maximum retardation is

$a = \dfrac{f_L - mg\sin\theta}{m}$

$\Rightarrow \quad a = \dfrac{\mu mg\cos\theta - mg\sin\theta}{m}$

$\Rightarrow \quad a = \mu g\cos\theta - g\sin\theta = g(\mu\cos\theta - \sin\theta)$



Now, using $v^2 = u^2 - 2as$, we get

$0 = (10)^2 - 2g(\mu\cos\theta - \sin\theta)(5)$

$\Rightarrow \quad \mu g\cos\theta - g\sin\theta = 10$

$$\Rightarrow \left(\frac{4}{3}\right)(10)\cos\theta-(10)\sin\theta=10$$

$$\Rightarrow 4\cos\theta-3\sin\theta=3$$

$$\Rightarrow 4\cos\theta=3(1+\sin\theta)$$

$$\Rightarrow 16\cos^2\theta=9(1+\sin^2\theta+2\sin\theta)$$

$$\Rightarrow 16(1-\sin^2\theta)=9(1+\sin^2\theta+2\sin\theta)$$

$$\Rightarrow 25\sin^2\theta+18\sin\theta-7=0$$

$$\Rightarrow \sin\theta=\frac{-18\pm\sqrt{324+700}}{50}$$

$$\Rightarrow \sin\theta=\frac{\sqrt{1024}-18}{50}=\frac{32-18}{50}=\frac{14}{50}$$

$\{\because$ Rejecting negative sign$\}$

$$\Rightarrow \theta\approx 16^\circ$$

14. Since the man is standing stationary w.r.t. the belt, so acceleration of the man equals the acceleration of the belt i.e. $a=1\ \text{ms}^{-2}$

Net force on the man is $F_{\text{net}}=Ma=65\times1=65\ \text{N}$

Limiting friction is

$$f_L=\mu mg$$

Let the man remains stationary with respect to the belt for maximum acceleration a_0, then

$$ma_0=f_L=\mu mg$$

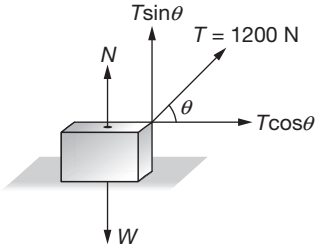
$$\Rightarrow a_0=\mu g=(0.2)(10)=2\ \text{ms}^{-2}$$

$$\Rightarrow a_0=200\ \text{cms}^{-2}$$

15. Equations of motion in this case are,

$$T\cos\theta=\mu N \qquad \ldots(1)$$

and $N=W-T\sin\theta \qquad \ldots(2)$



(a) From equations (1) and (2), we get

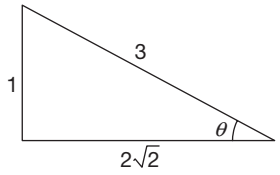
$$W=\frac{T(\cos\theta+\mu\sin\theta)}{\mu} \qquad \ldots(3)$$

(b) For W to be maximum

$$\frac{d}{d\theta}(\cos\theta+\mu\sin\theta)=0$$

$$\Rightarrow \theta=\tan^{-1}(\mu)=\tan^{-1}\left(\frac{1}{2\sqrt{2}}\right)$$

Since $\sin(10^\circ)\cos(10^\circ)=\frac{1}{6}$



$$\Rightarrow 2\sin(10^\circ)\cos(10^\circ)=\frac{1}{3}$$

$$\Rightarrow \sin(20^\circ)=\frac{1}{3}$$

$$\Rightarrow \tan(20^\circ)=\frac{1}{2\sqrt{2}}$$

So, $\theta=20^\circ$

From equation (3), we get

$$W=\frac{1200\left(\cos(20^\circ)+\frac{1}{2\sqrt{2}}\sin(20^\circ)\right)}{\frac{1}{2\sqrt{2}}}$$

$$\Rightarrow W=\frac{1200\left(\frac{2\sqrt{2}}{3}+\frac{1}{2\sqrt{2}}\frac{1}{3}\right)}{\frac{1}{2\sqrt{2}}}$$

$$\Rightarrow W=400(2\sqrt{2})\left(2\sqrt{2}+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow W=400(8+1)$$

$$\Rightarrow W=3600\ \text{N}$$

16. (a) When the truck accelerates eastwards, force of friction on mass is eastwards, because the block has a tendency to move westwards w.r.t. the truck. So,

$$f_{\text{required}}=\text{mass}\times\text{acceleration}=30\times1.8=54\ \text{N}$$

Since it is less than $\mu_s mg$, hence

$$f=54\ \text{N}\ \text{(eastwards)}$$

(b) When the truck accelerates westwards then similar to the argument given earlier, force of friction is westwards. So,

$f_{\text{required}} = \text{Mass} \times \text{Acceleration} = 30 \times 3.8 = 114 \text{ N}$

Since it is greater than $\mu_s mg$, hence

$f = f_k = \mu_k mg \approx 59 \text{ N}$ (westwards)

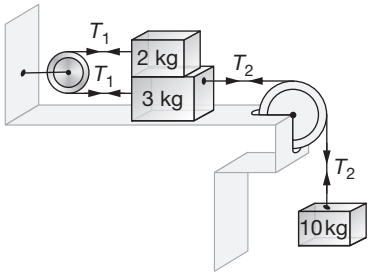
17. $100 = \dfrac{mv^2}{r} = \dfrac{(4)(v^2)}{(1)}$

$\Rightarrow \quad v = 5 \text{ ms}^{-1}$

Now, $5 = (\mu_k g)t$ $\quad \{\because v = at\}$

$\Rightarrow \quad t = \dfrac{5}{(0.1)(10)} = 5 \text{ s}$

18.



For 10 kg , $10g - T_2 = 10a$ …(1)

For 3 kg , $T_2 - (T_1 + 0.6g) = 3a$ …(2)

For 2 kg , $T_1 - 0.6g = 2a$ …(3)

Solving these equations, we get

$a = 6 \text{ ms}^{-2}$

19. Since, $v_C^2 = v_A^2 + 2a_T s$

Tangential acceleration in interval AC is

$a_T = \dfrac{v_C^2 - v_A^2}{2s} = 1.447 \text{ ms}^{-2}$

Centripetal i.e. Normal acceleration a_N at C is

$a_N = \dfrac{v^2}{r_C} = 2.41 \text{ ms}^{-2}$

Tangential force F_T at C is

$F_T = ma_T = 1500 \times 1.447 = 2170 \text{ N}$

Normal force F_N at C is

$F_N = ma_N = 1500 \times 2.41 = 3620 \text{ N}$

Total force at C is

$F = \sqrt{F_N^2 + F_T^2}$

$\Rightarrow \quad F = \sqrt{(2170)^2 + (3620)^2}$

$\Rightarrow \quad F = 4220 \text{ N}$

20. $R = \dfrac{v^2}{\mu g} = \dfrac{5 \times 5}{0.5 \times 10} = 5 \text{ m}$

21. $m_A = m_B = m_C = m$

Since, $F = (m_A + m_B + m_C)g \sin\theta$ and $N = m_C g \sin\theta$

$\Rightarrow \quad N = \dfrac{m_C F}{m_A + m_B + m_C} = \left(\dfrac{m}{3m}\right)15 = 5 \text{ N}$

22. There is no tendency of relative slipping between blocks.

23. For equilibrium,

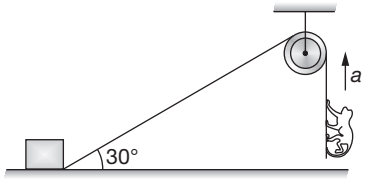
$10 = 8 + T$ …(1)

$T + f_2 = 20$ …(2)

$\Rightarrow \quad f_2 = 18 \text{ N}$

24. Let T be the tension in the string. The upward force exerted on the clamp is

$T \sin(30°) = \dfrac{T}{2}$



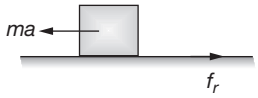
Now $\dfrac{T}{2} = 40 \text{ N}$

$\Rightarrow \quad T = 80 \text{ N}$

$\Rightarrow \quad a = \dfrac{T - mg}{m} = \dfrac{80 - 50}{5} = 6 \text{ ms}^{-2}$

25. $v = 2t^2$

$\Rightarrow \quad a = 4t$



At $t = 1$ s , slipping occurs, so we have

$\Rightarrow \quad ma = \mu_s mg$

$\Rightarrow \quad 4t = \mu_s g$

$\Rightarrow \quad \mu_s = 0.4$

So in the interval from $t = 1$ to $t = 3$ s, we observe that the slipping occurs between $t = 1$ and $t = 2$ s.

$$\Rightarrow \quad 4t - \mu_k g = \frac{dv}{dt}$$

Integrating, we get

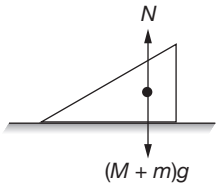
$$v = \left(2t^2 - \mu_k g t\right)\Big|_1^2$$

$$\Rightarrow \quad \mu_k = 0.3$$

$$\Rightarrow \quad \frac{3\mu_s}{\mu_k} = 4$$

26. $a = \dfrac{160-120}{40} = 1 \text{ ms}^{-2}$

27. Since wedge is stationary and block is slipping down with a constant velocity, so normal force on the wedge due to the table is



$N = (m + M)g = 8$ N

28. Let tension the thread connecting m_2 to the pulley P_1 be T and acceleration of m_2 be a. Then tension in the thread connecting lowest pulley P_3 to m_1 is $4T$ and acceleration of m_1 is $\frac{a}{4}$.

For m_1: $40 - 4T = 4\left(\dfrac{a}{4}\right)$

For m_2: $T = 1a$

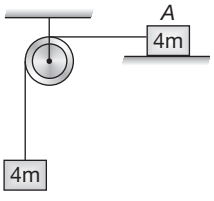
$$\Rightarrow \quad 40 - 4a = a$$

$$\Rightarrow \quad 5a = 40$$

$$\Rightarrow \quad a = 8 \text{ ms}^{-2}$$

29. Pseudo force is zero w.r.t. an observer in an Inertial or ground frame.

30. The system can be redrawn as



$$\Rightarrow \quad a = \frac{4mg}{8m} = \frac{g}{2} = 5 \text{ ms}^{-2}$$

Since, $v = u + at$

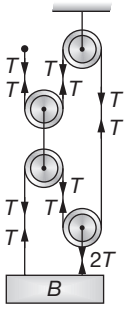
$$\Rightarrow \quad v = 0 + (2)(5) = 10 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 1 \text{ decametre per second}$$

31. According to Principle of Virtual Work, we have

$$Tx = 3Tx_B$$

$$\Rightarrow \quad x = 3x_B$$



$$\Rightarrow \quad \frac{dx}{dt} = 3\frac{dx_B}{dt}$$

$$\Rightarrow \quad v = 3v_B$$

$$\Rightarrow \quad v_B = \frac{v}{3}$$

32. By constraint relation knowledge, we have

$$V_A \sin(60°) = V_{P_2}\left[1 + \cos(60°)\right] + V_B$$

$$\Rightarrow \quad V_{P_2} = 4 \text{ ms}^{-1}$$

33. $a = \dfrac{2 \times 10}{2 + \frac{1}{3} + 1} = 6 \text{ ms}^{-2}$

Also, $2 \times g - T = 2 \times 6$

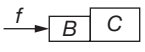
$$\Rightarrow \quad T = 8 \text{ N}$$

34. Since the block is not moving, so frictional force acting on the block is opposite to $\vec{F}_{\text{pseudo}} = -(ma)\hat{i}$

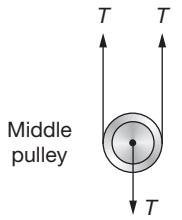
$$\Rightarrow \quad \vec{f} = -\vec{F}_{\text{pseudo}} = (2\hat{i}) \text{ newton}$$

35. $a = \dfrac{10}{5} = 2 \text{ ms}^{-2}$

$f = (M_B + M_C) \times a = 8$ N



36. Free body diagram for the middle pulley is shown.



So, we observe that

$2T - T = 0 \quad \{\because$ pulley is light, so $m_{pulley} \cong 0\}$

$\Rightarrow \quad T = 0$

37. **For B,** $mg - T = ma$ …(1)

For A, $T + mg\sin\theta = ma$ …(2)

Adding, (1) and (2), we get

$$a = \frac{g}{2}(1 + \sin\theta)$$

$$\Rightarrow \quad a = \frac{3g}{4}$$

Put $a = \frac{3g}{4}$ in (1), we get

$$mg - T = \frac{3mg}{4}$$

$$\Rightarrow \quad T = \frac{mg}{4} = 5 \text{ N}$$

38. $a_C = \frac{a_A + a_B}{2}$

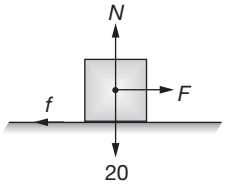
$$\Rightarrow \quad a_C = \frac{7-3}{2} = 2 \text{ ms}^{-2}$$

39. $f_l = (0.2)(20) = 4$ N

So, till $2t = 4 \Rightarrow t = 2$ s, block will be at rest.

For $2 \le t \le 4$ s, the block will move with an acceleration

$$a = \frac{F - f_l}{m}$$



$$\Rightarrow \quad \frac{dv}{dt} = (2t - 4)$$

$$\Rightarrow \quad \int_0^v dv = 2\int_2^4 t\,dt - 4\int_2^4 dt$$

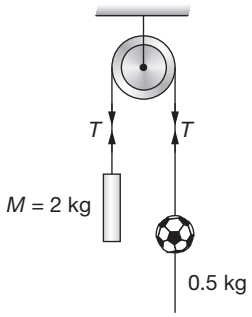
$$\Rightarrow \quad v = 2\left(\frac{t^2}{2}\Big|_2^4\right) - 4\left(t\Big|_2^4\right)$$

$$\Rightarrow \quad v = (16 - 4) - 4(4 - 2)$$

$$\Rightarrow \quad v = 12 - 8$$

$$\Rightarrow \quad v = 4 \text{ ms}^{-1}$$

40.



For rod of mass 2 kg

$20 - T = 2a_1$ …(1)

For ball

$5 - f = 5a_2$ …(2)

Since thread is mass less

$T = f$ …(3)

$$l = 0.3 = \frac{1}{2}a_{rel}t^2$$

41. If C doesn't move, then

$a_A = 4a_B$ …(1)

For *A*

$P - T - \mu mg = m(4a_B)$ …(2)

For *B*

$4T - mg = ma_B$ …(3)

$$\Rightarrow \quad \frac{P}{mg} = 5$$

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1. Since $P = \frac{F}{A}$

$$\Rightarrow \quad P = \frac{N \times (2mv)}{A\Delta t}$$

$$\Rightarrow \quad P = \frac{10^{22} \times 2 \times 10^{-26} \times 10^{4}}{1 \times 1} = 2 \text{ Nm}^{-2}$$

Hence, the correct answer is (B).

2. $p = mV = m\sqrt{V_0^2 - 2gh}$

Direction of momentum changes at top most point

Hence, the correct answer is (D).

3. Retardation of the particle

$$a = -\left(g + \gamma v^2\right)$$

For $H_{\max}$, $v = 0$

$$\Rightarrow \quad \int_{V_0}^{0} \frac{-dv}{g + \gamma v^2} = \int_{0}^{t} dt$$

$$\Rightarrow \quad \frac{1}{\sqrt{\gamma g}} \tan^{-1}\left(\frac{\sqrt{\gamma} V_0}{\sqrt{g}}\right) = t$$

Hence, the correct answer is (C).

4. $a = \left(\frac{F - f}{M + m}\right)$

$$\Rightarrow \quad a = \frac{F - (0.2)4 \times 10}{4} = \left(\frac{F - 8}{4}\right)$$

Since $a \le \mu g$

$$\Rightarrow \quad \frac{F - 8}{4} \le (0.2)10$$

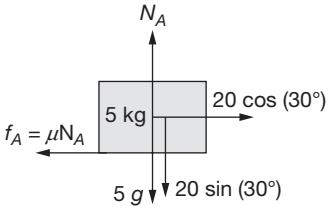
$$\Rightarrow \quad F - 8 \le 8$$

$$\Rightarrow \quad F \le 16$$

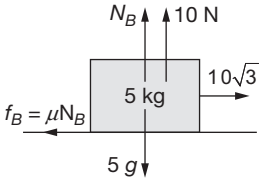
Hence, the correct answer is (C).

5. $N_A = 5g + 20\sin(30°) = 60$ N

Since, $f_A = 0.2 \times 60 = 12$ N



$$\Rightarrow \quad a_A = \frac{\left(\frac{20\sqrt{3}}{2} - 12\right)}{5} = \frac{5.3}{5} = 1.06 \text{ ms}^{-2}$$



For B,

$$N_B + 10 = 50$$

$$\Rightarrow \quad N_B = 40 \text{ N}$$

Since $f_B = \mu N_B$

$$\Rightarrow \quad f_B = 8 \text{ N}$$

$$\Rightarrow \quad \frac{20\sqrt{3}}{2} - 8 = 5a_B$$

$$\Rightarrow \quad a_B = \frac{17.3 - 8}{5} = \frac{9.3}{5} = 1.86 \text{ ms}^{-2}$$

$$\Rightarrow \quad a_B - a_A = 0.8 \text{ ms}^{-2}$$

Hence, the correct answer is (D).

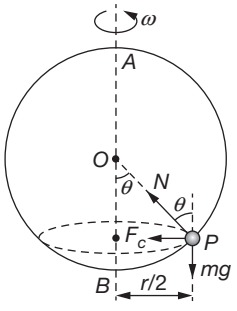
6. $l_1 = nl_2$

Since $k \propto \frac{1}{l}$

$$\Rightarrow \quad \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{1}{n}$$

Hence, the correct answer is (D).

7.



Let θ be the angle with the vertical, then

$$\sin\theta = \frac{r/2}{r} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

$$N\cos\theta = mg \qquad \ldots(1)$$

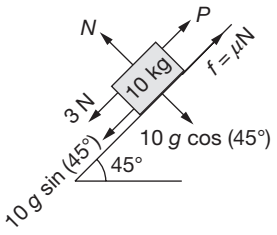
$$N\sin\theta = F_c = \frac{m\omega^2 r}{2} \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{\omega^2 r}{2g}$$

$$\Rightarrow \quad \omega^2 = \frac{2g\tan\theta}{r} = \frac{2g}{r\sqrt{3}}$$

Hence, the correct answer is (C).

8.



Friction force should be acting upward along the plane. For the state of impending motion, we have

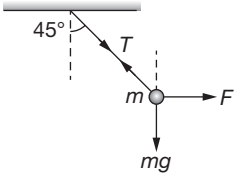
$$3 + (10)(10)\frac{1}{\sqrt{2}} = P + \left(\frac{6}{10}\right)(10)(10)\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad P = 73.71 - 42.42$$

$$\Rightarrow \quad P = 31.28 \approx 32 \text{ N}$$

Hence, the correct answer is (B).

9.



$$T\cos(45°) = mg$$

$$T\sin(45°) = F$$

$$\Rightarrow \quad F = mg$$

$$\Rightarrow \quad F = 10 \times 10 = 100 \text{ N}$$

Hence, the correct answer is (A).

10. $F = kt$

Since $F = \dfrac{dp}{dt}$

$$\Rightarrow \quad \frac{dp}{dt} = kt$$

$$\Rightarrow \quad \int_p^{3p} dp = k\int_0^T t\,dt$$

$$\Rightarrow \quad 2p = \frac{kT^2}{2}$$

$$\Rightarrow \quad T = 2\sqrt{\frac{p}{k}}$$

Hence, the correct answer is (B).

11. Since, $\vec{r}_{\text{final}} = \vec{r}_{\text{initial}} + \vec{u}t + \dfrac{1}{2}\vec{a}t^2$

$$\Rightarrow \quad \vec{r}_{\text{final}} = \left(2\hat{i} + 4\hat{j}\right) + \left(5\hat{i} + 4\hat{j}\right) \times 2 + \frac{1}{2}\left[4\hat{i} + 4\hat{j}\right] \times 2^2$$

$$\Rightarrow \quad \vec{r}_{\text{final}} = \left(20\hat{i} + 20\hat{j}\right) \text{ m}$$

$$\Rightarrow \quad |\vec{r}_{\text{final}}| = 20\sqrt{2} \text{ m}$$

Hence, the correct answer is (A).

12. Since the block remains at rest on the rough incline upto a maximum force of 2 N down the incline, so we get

$$2 + mg\sin(30°) = \mu N$$

$$\Rightarrow \quad 2 + \frac{mg}{2} = \mu mg\frac{\sqrt{3}}{2} \qquad \ldots(1)$$

When only, 10 N is applied up the incline, then we have

$$\frac{mg}{2} + \mu mg\frac{\sqrt{3}}{2} = 10 \qquad \ldots(2)$$

From (1) and (2), we get

$$2 + mg = 10$$

$$\Rightarrow \quad mg = 8$$

From (1), we get

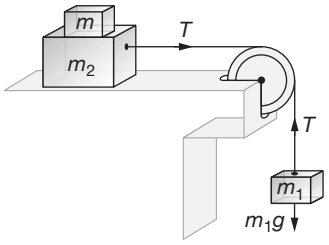
$$6 = \mu \times 8 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \mu = \frac{2 \times 6}{8\sqrt{3}}$$

$$\Rightarrow \quad \mu = \frac{\sqrt{3}}{2}$$

Hence, the correct answer is (B).

13. To stop the moving blocks, the frictional force between m_2 and surface is increased by placing some extra mass m on top of mass m_2.



Condition for stopping moving blocks is $f \geq T$

$\Rightarrow \quad \mu N \geq T$

$\Rightarrow \quad \mu(m+m_2)g \geq m_1 g$

For minimum value of m, we have

$\mu(m+m_2)g = m_1 g$

$\Rightarrow \quad m = \dfrac{m_1}{\mu} - m_2 = \dfrac{5}{0.15} - 10$

$\Rightarrow \quad m = 33.33 - 10 = 23.33 \text{ kg}$

From given options the best suitable answer will be 27.3 kg.

Hence, the correct answer is (B).

14. Time taken to slide along smooth surface is given by

$$l = \frac{1}{2}\left[g\sin(45°)\right]t_1^2$$

$\Rightarrow \quad t_1 = \sqrt{\dfrac{2\sqrt{2}l}{g}}$

Time taken to slide along rough surface is given by

$$l = \frac{1}{2}g\left[\sin(45°) - \mu\cos(45°)\right]t_2^2$$

$\Rightarrow \quad t_2 = \sqrt{\dfrac{2\sqrt{2}l}{g(1-\mu)}}$

As per question, $t_2 = nt_1$

$\Rightarrow \quad t_2^2 = n^2 t_1^2$

$\Rightarrow \quad \dfrac{2\sqrt{2}l}{g(1-\mu)} = n^2 \times \dfrac{2\sqrt{2}l}{g}$

$\Rightarrow \quad 1-\mu = \dfrac{1}{n^2}$

$\Rightarrow \quad \mu = 1 - \dfrac{1}{n^2}$

Hence, the correct answer is (B).

15. Downward acceleration on the incline is

$$a = g(\sin 30° - \mu\cos 30°)$$

Since body moves up with an acceleration a upwards due to external force F, so we have

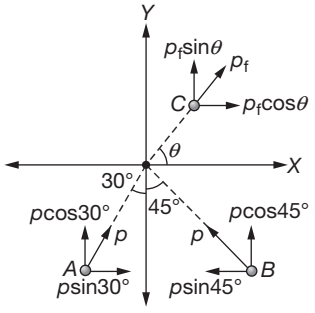
$$F - mg(\sin 30° + \mu\cos 30°) = ma$$

$\Rightarrow \quad F = mg(\sin 30° + \mu\cos 30°) + mg(\sin 30° - \mu\cos 30°)$

$\Rightarrow \quad F = 2mg\sin 30° = 2 \times 2 \times 10 \times \dfrac{1}{2} = 20 \text{ N}$

Hence, the correct answer is (D).

16. During completely inelastic collision both particles A and B stick together.



Here, p_i = initial momentum of each particle

p_f = final momentum of the system

Using conservation of linear momentum,

Along X-axis, $p_f\cos\theta = p\sin 30° - p\sin 45°$ …(1)

Along Y-axis, $p_f\sin\theta = p\cos 30° + p\cos 45°$ …(2)

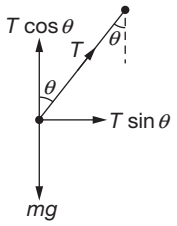
Dividing equation (2) by equation (1), we get

$$\frac{\sin\theta}{\cos\theta} = \frac{\cos 30° + \cos 45°}{\sin 30° - \sin 45°} = \frac{\dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}}}{\dfrac{1}{2} - \dfrac{1}{\sqrt{2}}}$$

$\Rightarrow \quad \tan\theta = \dfrac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$

Hence, the correct answer is (A).

17. FBD of the pendulum is shown in the figure. So, we have



$$T\sin\theta = \frac{mv^2}{r} \text{ and}$$

$$T\cos\theta = mg$$

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg}$$

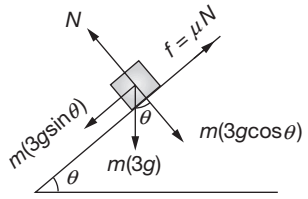
Since, $\theta = 45°$, $r = 0.4$ m

$$\Rightarrow \quad v^2 = rg$$

$$\Rightarrow \quad v = \sqrt{rg} = \sqrt{0.4\times 10} = 2 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

18. Since the rocket is moving vertically upwards with acceleration $2g$, therefore the apparent acceleration experienced by the point object is $g + 2g = 3g$ vertically downwards.



From figure, we have $N = 3mg\cos\theta$

Point object does not move on inclined surface, so

$$\mu N = 3mg\sin\theta$$

$$\Rightarrow \quad \mu(3mg\cos\theta) = 3mg\sin\theta$$

$$\Rightarrow \quad \mu = \tan\theta$$

Hence, the correct answer is (B).

19. Here, $F = \frac{R}{t^2}v(t)$

$$\Rightarrow \quad m\frac{dv}{dt} = \frac{R}{t^2}v(t)$$

$$\Rightarrow \quad \frac{dv}{v(t)} = \frac{R}{m}\frac{dt}{t^2}$$

Integrating both sides,

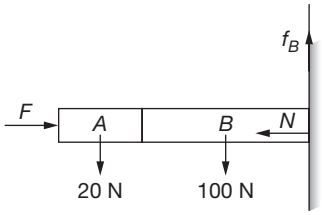
$$\int\frac{dv}{v(t)} = \frac{R}{m}\int\frac{dt}{t^2}$$

$$\Rightarrow \quad \ln v = -\left(\frac{R}{m}\right)\left(\frac{1}{t}\right) + C$$

Graph between $\ln v$ and $\left(\frac{1}{t}\right)$ is a straight line.

Hence, the correct answer is (A).

20. Various forces acting on the system are shown in the figure.



For vertical equilibrium of the system,

$$f_B = 100 \text{ N} + 20 \text{ N} = 120 \text{ N}$$

Hence, the correct answer is (A).

21. System (block + bullet) comes to rest after moving

$$2 \text{ m},\ s = 2 \text{ m},\ v_2 = 0,\ v_1 = ?$$

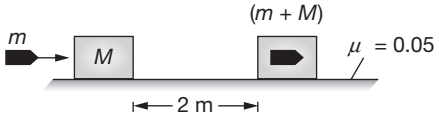
$$a = -\mu g = -0.05\times 10 = -0.5 \text{ ms}^{-2}$$

Using $v^2 = u^2 + 2as$, we get

$$0^2 = v_1^2 - 2(0.5)\times 2$$

$$\Rightarrow \quad v_1 = \sqrt{2} \text{ ms}^{-1}$$

By Conservation of Linear Momentum, we have



$$\begin{pmatrix}\text{Momentum of}\\ \text{the system}\\ \text{after collision}\end{pmatrix} = \begin{pmatrix}\text{Momentum of}\\ \text{the system}\\ \text{before collision}\end{pmatrix}$$

$$mv + 0 = (M + m)V$$

$$\Rightarrow \quad \sqrt{2}(10 + 50\times 10^{-3}) = (50\times 10^{-3})\times v + 0$$

$$\Rightarrow \quad v = \frac{\sqrt{2}\times 10}{50\times 10^{-3}} = 200\sqrt{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad \frac{v}{10} = 20\sqrt{2} \text{ ms}^{-1}$$

For a freely falling body, to acquire

$$v' = \frac{v}{10} = 20\sqrt{2} \text{ ms}^{-1}, \text{ we have}$$

$$v'^2 = 2gH$$

$$\Rightarrow \quad H = \frac{v'^2}{2g}$$

$$\Rightarrow \quad H = \frac{800}{20} = 40 \text{ m} = 0.04 \text{ km}$$

Hence, the correct answer is (C).

22. Since collisions are elastic and masses are equal, velocities of colliding particles get exchanged.
Change in momentum Δp in each collision with the support is $\Delta p = 2mv$.
Time interval between consecutive collisions with one support

$$\Delta t = \frac{(L - 2nr) \times 2}{v}$$

So, average force experienced by each support is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{(L-2nr)2}{v}} = \frac{mv^2}{L - 2nr}$$

Hence, the correct answer is (B).

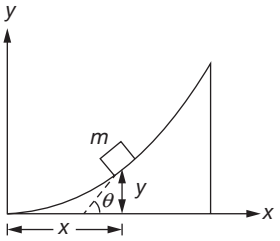
23. Block is under limiting friction, so

$$\mu = \tan\theta \qquad \ldots(1)$$

Equation of the surface is

$$y = \frac{x^3}{6}$$

So, slope is $\frac{dy}{dx} = \frac{x^2}{2}$. $\qquad \ldots(2)$



From equations (1) and (2), we get

$$\mu = \frac{x^2}{2}$$

$$\Rightarrow \quad 0.5 = \frac{x^2}{2}$$

$$\Rightarrow \quad x^2 = 1$$

$$\Rightarrow \quad x = 1$$

$$\Rightarrow \quad y = \frac{1}{6} \text{ m}$$

Hence, the correct answer is (B).

24. Centripetal acceleration, $a_c = \omega^2 r$, where $\omega = \frac{2\pi}{T}$
Since, $T_1 = T_2$

$$\Rightarrow \quad \omega_1 = \omega_2$$

$$\Rightarrow \quad \frac{a_{c_1}}{a_{c_2}} = \frac{r_1}{r_2}$$

Hence, the correct answer is (B).

25. $F(t) = F_0 e^{-bt}$ {Given}

$$\Rightarrow \quad ma = F_0 e^{-bt}$$

$$\Rightarrow \quad a = \frac{F_0}{m} e^{-bt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{F_0}{m} e^{-bt}$$

$$\Rightarrow \quad dv = \frac{F_0}{m} e^{-bt} dt$$

Integrating both sides, we get

$$\int_0^v dv = \int_0^t \frac{F_0}{m} e^{-bt} dt$$

$$\Rightarrow \quad v = \frac{F_0}{m}\left(\frac{e^{-bt}}{-b}\right)\Big|_0^t = \frac{F_0}{mb}(1 - e^{-bt})$$

Hence, the correct answer is (B).

26. The acceleration of the body down the smooth inclined plane is $a = g\sin\theta$ along the inclined plane, where θ is the angle of inclination.
The vertical component of acceleration a is

$$a_{(\text{along vertical})} = (g \sin\theta)\sin\theta = g\sin^2\theta$$

For block A

$$a_{A(\text{along vertical})} = g\sin^2(60°)$$

For block B

$$a_{B(\text{along vertical})} = g\sin^2(30°)$$

The relative vertical acceleration of A with respect to B is

$$a_{AB(\text{along vertical})} = a_{A(\text{along vertical})} - a_{B(\text{along vertical})}$$

$$\Rightarrow \quad a_{AB(\text{along vertical})} = g\sin^2(60°) - g\sin^2(30°)$$

$$\Rightarrow \quad a_{AB(\text{along vertical})} = g\left(\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)$$

$$\Rightarrow \quad a_{AB(\text{along vertical})} = \frac{g}{2} = 4.9 \text{ ms}^{-2}$$

Hence, the correct answer is (A).

27. Since position-time $(x\text{-}t)$ graph is a straight line, so motion is uniform. Because of impulse, direction of velocity changes as can be seen from slopes of the graph.
From graph,

Initial velocity, $u = \frac{(2-0)}{(2-0)} = 1 \text{ ms}^{-1}$

Final velocity, $v = \frac{(0-2)}{(4-2)} = -1 \text{ ms}^{-1}$

So, initial momentum, $p_i = mu = 0.4 \times 1 = 0.4$ Ns and

Final momentum, $p_f = mv = 0.4 \times (-1) = -0.4$ Ns

$\Rightarrow$ Impulse = Change in momentum $= p_f - p_i$

$\Rightarrow$ Impulse $= -0.4 - (0.4)$ Ns $= -0.8$ Ns

$\Rightarrow$ $|\text{Impulse}| = 0.8$ Ns

Hence, the correct answer is (C).

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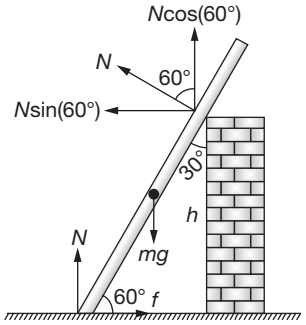
Single Correct Choice Type Problems

1.



For Translational Equilibrium of the stick, we have

$$N + N\cos(60°) = mg \qquad \ldots(1)$$

$$f = N\sin(60°) \qquad \ldots(2)$$

$$\Rightarrow \quad N = \frac{2mg}{3}$$

For Rotational Equilibrium of stick about the lowest point, we have

$$mg\left[\frac{l}{2}\cos(60°)\right] = N\left(\frac{h}{\sin 60°}\right)$$

$$\Rightarrow \quad \frac{mgl}{4} = \frac{2mg}{3}\left(\frac{2h}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \frac{h}{l} = \frac{3\sqrt{3}}{16}$$

Also, $f = N\sin(60°)$

$$\Rightarrow \quad f = \frac{2mg}{3}\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad f = \frac{mg}{\sqrt{3}} = \frac{(1.6)(10)}{\sqrt{3}} = \frac{16}{\sqrt{3}} \text{ N} = \frac{16\sqrt{3}}{3} \text{ N}$$

Hence, the correct answer is (D).

2. Block will not slip if

$$(m_1 + m_2)g\sin\theta \le \mu m_2 g\cos\theta$$

$$\Rightarrow \quad 3\sin\theta \le \left(\frac{3}{10}\right)(2)\cos\theta$$

$$\tan\theta \le \frac{1}{5}$$

$$\Rightarrow \quad \theta \le 11.5°$$

For (P), $\theta = 5°$ friction is static

$$f = (m_1 + m_2)g\sin\theta$$

For (Q), $\theta = 10°$ friction is static

$$f = (m_1 + m_2)g\sin\theta$$

For (R), $\theta = 15°$ friction is kinetic

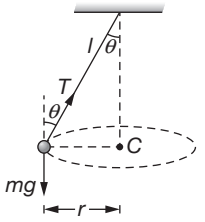
$$f = \mu m_2 g\cos\theta$$

For (S), $\theta = 20°$ friction is kinetic

$\Rightarrow$ $f = \mu m_2 g\cos\theta$

Hence, the correct answer is (D).

3. $T\cos\theta$ component will cancel mg.



$T\sin\theta$ component will provide necessary centripetal force to the ball towards centre C.

$\Rightarrow \quad T\sin\theta = mr\omega^2 = m(l\sin\theta)\omega^2$

$\Rightarrow \quad T = ml\omega^2$

$\Rightarrow \quad \omega = \sqrt{\dfrac{T}{ml}}$

$\Rightarrow \quad \omega_{max} = \sqrt{\dfrac{T_{max}}{ml}}$

$\Rightarrow \quad \omega_{max} = \sqrt{\dfrac{324}{0.5\times 0.5}}$

$\Rightarrow \quad \omega_{max} = 36 \text{ rads}^{-1}$

Hence, the correct answer is (D).

4. When $P = mg(\sin\theta - \mu\cos\theta)$

$f = \mu mg\cos\theta$ {upwards}

when $P = mg\sin\theta$; $f = 0$

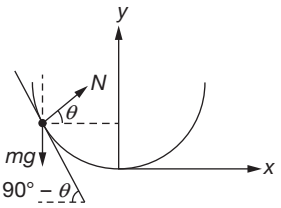
and when $P = mg(\sin\theta + \mu\cos\theta)$

$f = \mu mg\cos\theta$ {downwards}

Hence, friction is first positive, then zero and then negative.

Hence, the correct answer is (A).

5.



$N\sin\theta = mg$

$N\cos\theta = ma$

$\Rightarrow \quad \tan\theta = \dfrac{g}{a}$

$\Rightarrow \quad \cot\theta = \dfrac{a}{g} = \tan(90° - \theta) = \dfrac{dy}{dx}$

$\Rightarrow \quad \cot\theta = \dfrac{dy}{dx}$ …(1)

Since $y = kx^2$

$\Rightarrow \quad \dfrac{dy}{dx} = 2kx$

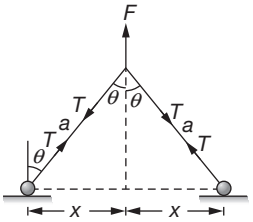
$\Rightarrow \quad \dfrac{a}{g} = 2kx$

$\Rightarrow \quad x = \dfrac{a}{2 \text{ kg}}$

Hence, the correct answer is (B).

6. $2T\cos\theta = F$

$\Rightarrow \quad T = \dfrac{F}{2}\sec\theta$



Acceleration of particle a_0 is given by

$a_0 = \dfrac{T\sin\theta}{m}$

$\Rightarrow \quad a_0 = \dfrac{F\tan\theta}{2m}$

$\Rightarrow \quad a_0 = \dfrac{F}{2m}\dfrac{x}{\sqrt{a^2 - x^2}}$

Hence, the correct answer is (B).

7. Initially under equilibrium of mass m

$T = mg$

Now, the string is cut. Therefore, $T = mg$ force is decreased on mass m upwards and downwards on mass $2m$.

$\Rightarrow \quad a_m = \dfrac{mg}{m} = g$ (downwards) and

$a_{2m} = \dfrac{mg}{2m} = \dfrac{g}{2}$ (upwards)

Hence, the correct answer is (A).

8. This is the equilibrium of coplanar forces. Hence,

$\Sigma F_x = 0$

$\Rightarrow \quad F = N$

$\Rightarrow \quad \Sigma F_y = 0$,

$f = mg$

$\Sigma\tau_c = 0$

$\Rightarrow \quad \vec{\tau}_N + \vec{\tau}_f = 0$

Since, $\vec{\tau}_f \neq 0$

$\Rightarrow \quad \vec{\tau}_N \neq 0$

Hence, the correct answer is (D).

9. Angular frequency of the system,

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

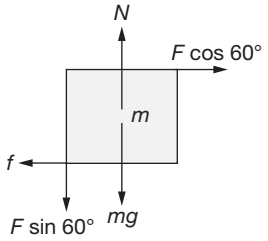
Maximum acceleration of the system will be, $\omega^2 A$ or $\frac{kA}{2m}$.

This acceleration to the lower block is provided by friction.

Hence, $f_{max} = ma_{max} = m\omega^2 A = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$

Hence, the correct answer is (A).

10.



For no motion $F_{applied} \le f\,(= \mu N)$

$\Rightarrow \quad F\cos 60° \le \mu(\text{mg} + F\sin 60°)$

$\Rightarrow \quad \frac{F}{2} \le \frac{1}{2\sqrt{3}}\left(\sqrt{3}g + \frac{F\sqrt{3}}{2}\right)$

$\Rightarrow \quad \frac{F}{2} \le g$

$\Rightarrow \quad F_{max} = 20\ \text{N}$

Hence, the correct answer is (A).

11. $2T\cos\theta = \sqrt{2}\ Mg$ …(1)

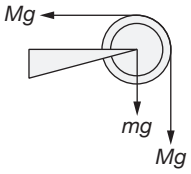
$\Rightarrow \quad 2Mg\cos\theta = \sqrt{2}\ \text{Mg}$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = 45°$

Hence, the correct answer is (C).

12. $T = Mg$



Also the weight of pulley mg is acting vertically downward. So total downward force is $(m+M)g$ and horizontal force is $T = Mg$. The resultant of these two is

$$\sqrt{M^2 + (m+M)^2}\,g$$

Hence, the correct answer is (D).

13. $mg\cos\alpha = N$ …(1)

$mg\sin\alpha = \mu N$ …(2)

$\Rightarrow \quad \cot\alpha = \frac{1}{\mu} = 3$

Hence, the correct answer is (A).

14. At the highest point all blocks will possess equal speeds (as we assume friction to be absent) so, at the highest point

$$mg + N = \frac{mv'^2}{R}$$

$\Rightarrow \quad N = \frac{mv'^2}{R} - mg$

R is MINIMUM in the first case, so

N is MAXIMUM.

Hence, the correct answer is (A).

15. Tangential Force = $F_T = ma = m(\alpha L) = N$

Limiting value of friction $(f_s)_{max} = \mu N = \mu F_T$

$\Rightarrow \quad (f_s)_{max} = \mu N = \mu m\alpha L$ …(1)

Further if ω is the angular velocity at time t, then

$\omega = \alpha t$ …(2)

Also, the centripetal force is

$$F_C = mL\omega^2 = mL(\alpha^2 t^2)$$ …(3)

For bead to slide $F_C > (f_s)_{max}$

$\Rightarrow \quad mL(\alpha^2 t^2) > \mu m\alpha L$

$\Rightarrow \quad t > \sqrt{\frac{\mu}{\alpha}}$

So, the minimum time after which the bead begins to start is $\sqrt{\frac{\mu}{\alpha}}$.

Hence, the correct answer is (A).

16. Limiting value of static friction is given by,

$$f_s = \mu_s N = (0.5)(5) = 2.5 \text{ N}$$

Weight of the block = 0.98 N

Since the block remains stationary, we have

Force of friction = Weight of block

$\Rightarrow \quad f = 0.98$ N

Hence, the correct answer is (B).

17. $\tan\theta = \dfrac{v^2}{rg} = 1$

$\Rightarrow \quad \theta = 45°$

Hence, the correct answer is (C).

19. For equilibrium $f = mg\sin\theta = (2)(9.8)\sin 30$

$\Rightarrow \quad f = 9.8$ N

Here, we have not used the formula $f = \mu mg\cos\theta$ as we are provided with coefficient of static friction which may or may not be the limiting value of static friction.

Hence, the correct answer is (A).

20. $a = \dfrac{5\times10^4}{3\times10^7}$ ms^{-2}

$\Rightarrow \quad a = \dfrac{5}{3}\times10^{-3}$ ms^{-2}

Since $v^2 - u^2 = 2as$

$$v^2 - 0^2 = 2\left(\frac{5}{3}\times10^{-3}\right)3$$

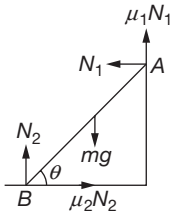
$\Rightarrow \quad v^2 = 10^{-2}$

$\Rightarrow \quad v = 0.1$ m

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. μ_2 can never be zero for equilibrium.



When $\mu_1 = 0$, we have

$$N_1 = \mu_2 N_2 \qquad \ldots(1)$$

$$N_2 = mg \qquad \ldots(2)$$

$\tau_B = 0$

$\Rightarrow \quad mg\dfrac{L}{2}\cos\theta = N_1 L\sin\theta$

$\Rightarrow \quad N_1 = \dfrac{mg\cot\theta}{2}$

$\Rightarrow \quad N_1\tan\theta = \dfrac{mg}{2}$

When $\mu_1 \neq 0$ we have

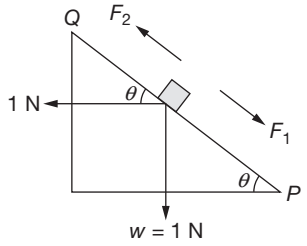
$$\mu_1 N_1 + N_2 = mg \qquad \ldots(3)$$

$$\mu_2 N_2 = N_1 \qquad \ldots(4)$$

$\Rightarrow \quad N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

Hence, (C) and (D) are correct.

2. $w = mg = 0.1 \times 10 = 1$ N



$F_1 =$ component of weight $= 1\sin\theta = \sin\theta$

$F_2 =$ component of applied force $= 1\cos\theta = \cos\theta$

Now, at $\theta = 45°$: $F_1 = F_2$ and block remains stationary without the help of friction.

For $\theta > 45°$, $F_1 > F_2$, so friction will act towards Q.

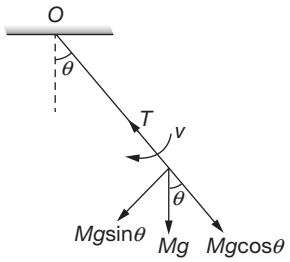
For $\theta < 45°$, $F_2 > F_1$ and friction will act towards P.

Hence, (A) and (C) are correct.

3. Motion of pendulum is the part of a circular motion. In circular motion it is better to resolve the forces in two perpendicular directions. First along radius (towards centre) and second along tangential. Along radius, net force should be equal to mv^2/R and along tangent it should be equal to ma_T, where a_T is the tangential acceleration in the figure.

$$T - Mg\cos\theta = \frac{Mv^2}{L} \text{ and } Mg\sin\theta = Ma_T$$

$\Rightarrow \quad a_T = g\sin\theta$



Hence, (B) and (C) are correct.

4. All accelerated frames are Non-inertial frames. Since earth rotates about its own axis and revolves around the sun, so it is a Non-Inertial frame (STRICTLY SPEAKING), whereas for a good number of cases we assume the earth to be an Inertial frame of reference as the value of acceleration is extremely small.
Hence, (B) and (D) are correct.

Integer/Numerical Answer Type Problems

1. Linear impulse, $J = mv_0$

$\Rightarrow \quad v_0 = \dfrac{J}{m} = 2.5 \text{ ms}^{-1}$

$\Rightarrow \quad v = v_0 e^{-t/\tau}$

$\Rightarrow \quad \dfrac{dx}{dt} = v_0 e^{-t/\tau}$

$\Rightarrow \quad \displaystyle\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$

$\Rightarrow \quad x = v_0 \left[\dfrac{e^{-t/\tau}}{-1/\tau} \right]_0^\tau$

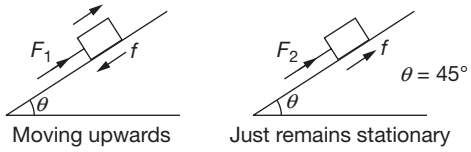
$\Rightarrow \quad x = 2.5(-4)\left(e^{-1} - e^{0}\right)$

$\Rightarrow \quad x = 2.5(-4)(0.37 - 1)$

$\Rightarrow \quad x = 6.30 \text{ m}$

$\Rightarrow \quad x = 6 \text{ m}$

2.



$F_1 = mg\sin\theta + \mu mg\cos\theta$

$F_2 = mg\sin\theta - \mu mg\cos\theta$

Given that $F_1 = 3F_2$

$\Rightarrow \quad (\sin 45° + \mu\cos 45°)$

$\Rightarrow \quad F_1 = 3(\sin 45° - \mu\cos 45°)$

On solving, we get

$\mu = 0.5$

$\Rightarrow \quad N = 10\mu = 5$

Assertion and Reasoning Type Problems

1. Both Statements are correct. But Statement-II, does not explain correctly, Statement-I.
Correct explanation: There is an increase in normal reaction when the object is pushed and there is a decrease in normal reaction when the object is pulled (but strictly not horizontally).

2. The cloth can be pulled out without dislodging the dishes from the table due to law of inertia, which is Newton's First Law. While, the Statement-II is true, but it is Newton's Third Law.

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